ANALYSIS OF TURBULENT CHARACTERISTICS IN FLOW PAST BLUFF BODIES

Synopsis submitted by

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SYNOPSIS

The present work attempts to address the problems mentioned chapter-wise in the following. Apart from the introduction and literature review chapter, there are five chapters in the dissertation devoted to the following topics-

Chapter 3: Description of the experimental set up and data collection procedure together with uncertainty estimations of data have been discussed.

Chapter 4: Basic turbulent features, length scales and kinetic energy budget analysis for turbulent flow past two horizontal cylinders one above the other.

Chapter 5: Stress and dissipation rate turbulent anisotropy analysis for flow past two horizontal cylinders one above the other.

Chapter 6: Turbulent bursting events and its correlation with higher order moments has been investigated for flow past two horizontal cylinders.

Chapter 7: Turbulent mixed convection in a square cavity with two cylindrical blockages has been investigated by finite element method and heat transfer rate has been predicted via several machine learning algorithms.

Chapter 8: Summary and conclusion from the works done in the previous chapters.

In **chapter 3**, the details of the experimental setup, the experimental scheme, the procedure followed, and the methods used for measurements are discussed. All the experiments were carried out at the Fluvial Mechanics Laboratory of the Indian Statistical Institute, Kolkata, India. At different streamwise distances, the instantaneous streamwise (u), spanwise (v), and vertical (w) velocity components along the flume centerline were measured using a Nortek Vectrino Plus, a 5-cm down-looking four-beam acoustic Doppler velocimeter. To get a time-invariant time-average velocity profile, the data have been collected at each point for a duration of 240s. The data collection is being executed in the rectangular section of the flume; hence, our analysis does not include curvature effects.

In **chapter 4**, turbulence in wall-wake flow past two horizontal cylinder is studied. In order to validate the collected data, the velocity power spectral density function $F_{ii}(f)$ are plotted with respect to frequency, which shows a reasonable agreement with the '−5/3 scaling-law' within the inertial subrange. Moreoever, to verify the fully developed flow over the gravel bed, the equation of time-averaged streamwise velocities is used, known as logarithmic law (Dey 2014).

Time averaged velocity, shear stress intensity plots are depicted with respect to vertical distances for cylinders of three diiferent diameters. Turbulent kinetic energy disssipation rate has been evaluated with different methodologies, from using velocity power spectra to incorporating structure functions of second order (Penna et al. 2020, Padhi et al. 2019):

$$\langle \varepsilon \rangle = (1/r)\,(\langle \Delta u \rangle^2 / C_2)^{3/2}$$

To measure the dimension of eddies in different flow zones, several length scales have been depicted namely Taylor length scale and Kolmogorov length scale. In order to incorporate the effects of higher order moments and its correlation with bursting effects, turbulent kinetic energy fluxes have been illustrated being defined as (Dey et al. 2018):

$$f_{ku} = 0.5(\overline{u'^3} + \overline{u'}\overline{v'^2} + \overline{u'}\overline{w'^2})$$

Finally, the TKE budget have been shown which quantifies the impacts of several turbulent processes, including viscous diffusion rate (v_D), pressure energy diffusion rate (p_D), TKE production rate (t_P), TKE dissipation rate (ε), and TKE diffusion rate (t_D), in order to preserve dynamic stability in turbulent flow (Vreman et al. 2018):

$$t_p = \varepsilon + t_{\mathrm{D}} + p_{\mathrm{D}} - v_D$$

We end the discussion by leaving the following conclusion: wall-wake flow is being identified by highly negative pressure energy diffusion rate together with a sufficient TKE production rate, stabilized by the amplified TKE diffusion and dissipation rates.

In **chapter 5**, the anisotropic behaviour has been illustrated in wall-wake flow past two horizontal cylinders. It has been observed from velocity power spectra plot that the spectral density functions of u, v and w are satisfying the following relation: $F_{uu} > F_{vv} > F_{ww}$, at immediate downstream of the cylinders indicating high anisotropy. Gradually, at $z/D = 2.25$ and above, spectral density values tend to merge i.e., $F_{uu} \approx F_{vv} \approx F_{ww}$, which suggests a tendency of converging towards three-dimensional isotropy above the horizontal cylinders (Sarkar et al. 2021).

The turbulence correlation coefficient (ρ_{uw}) is described as the ratio of Reynolds stress (with respect to ρ) to the product of vertical and streamwise turbulence intensities (Hinze 1987; Schlichting 1979). Mathematically, it is expressed as ρ_{uw} [$= -\overline{u'w'}/(\overline{u'u'} \times \overline{w'w'})^{0.5}$]. The non-uniformity of correlation coefficient values in near bed zone is suggesting the violation of isotropic turbulence (Sarkar et al. 2016) i.e., existence of high anisotropy immediate downstream of the cylinders.

Isotropic turbulence is an idealized turbulent state in which the turbulent fluctuations are considered to be statistically equivalent in all directions; mathematically, $\overline{u'u'} = \overline{v'v'} = \overline{w'w'}$; where u', v' and w' represent its usual meaning (Dey & Nath 2010). But in experimental point of view, every turbulence is anisotropic in nature. To visualize the anisotropic nature, stress and dissipation tensor have been evaluated and Anisotropic Invariant Map (AIM) has been plotted, which is bounded inside a triangle, known as Lumley triangle (Smalley et al. 2002):

$$b_{ik} = \frac{a_{ik}}{2q} = \overline{u_i' u_k'}/(2q) - (\delta_{ik}/3) \qquad\qquad d_{ik} = \varepsilon_{ik}/(2\varepsilon) - (\delta_{ik}/3)$$

There is one unique characteristic being observed in the AIM plots, especially in near-wake zone that due to the presence of two bluff-bodies, huge anisotropy is being produced which results in occupying most spaces in the triangle. Another simple and effective way to identify whether turbulence is two-dimensional or three-dimensional is by using the invariant function **F,** because of its simplified representation. **F** = 0 indicates two-dimensional turbulence, whereas **F** = 1 denotes an isotropic three-dimensional condition. It is graphically more intriguing to observe that in near wake flow, data sets have values close to x-axis in mid flow which indicates existence of strong turbulence anisotropy.

The Lumley triangle has the disadvantage that its nonlinear behaviours distort these data randomly over the map and make analysis less comprehensible. By using a convex combination of the three limiting phases (1C, 2C, and 3C) represented by barycentric coordinates (Banerjee et al. 2007; Longo et al. 2017), this problem was resolved. For plotting a point, the convex combination of the limiting states used is described as below:

$$x_{new} = C_{1c}\, x_{1c} + C_{2c} x_{2c} + C_{3c} x_{3c}$$

$$y_{new} = C_{1c} y_{1c} + C_{2c} y_{2c} + C_{3c} y_{3c}$$

These C_{1c}, C_{2c}, C_{3c} are the coefficients of the basis tensors required to represent anisotropy tensor which satisfies the following: $C_{1c} + C_{2c} + C_{3c} = 1$. In near-wake region, the data set trend is observed to be from left to right towards the vertex 1c; meaning the anisotropy is being shifted from two-component to one-component which indicates strong generation of anisotropy behaviour in near-wall zone. In Lumley triangle the bottom cusp being the indication of 3D isotropy, is similar to the concept of three-component turbulence 3c present in barycentric triangle. Moreover, we have shown in our work that there is a one-one correspondence between AIM and barycentric coordinates.

In **chapter 6**, to extend our previous works, analysis of turbulent bursting and its correlation with higher order bursting is presented with the same experimental environment. Studying the conditional statistics of the velocity variations (u' and w') on an $u'w'$ -plane is essential to comprehend the features of the bursting events, which are being characterized by four quadrants, namely outward interactions Q_1 $(u', w' > 0)$, ejections Q_2 $(u' < 0, w' > 0)$, inward interactions Q_3 $(u', w' < 0)$, and sweeps Q_4 $(u' > 0, w' < 0)$. A hole-size threshold H is used to determine which quadrant's stronger contributions to $-\overline{u'w'}$ are (Dey et al. 2011). The following equation can be used to calculate the time-averaged contribution to the overall Reynolds shear stress for a particular quadrant *i* beyond the hole size *H*:

$$\overline{u'w'}^{i,H} = \lim_{T\to\infty} \frac{1}{T} \sum_0^T u'w'\lambda_{i,H} dt$$

The proportionate fractional contributions to the formation of RSS are provided by (Sarkar 2016; Demare, et al. 1999):

$$S_{i,H} = \frac{\overline{u'w'}^{i,H}}{\overline{u'w'}}$$

One intriguing observation is that in the near-wake zone, sweep events contribute to the formation of RSS in the order of 600% ($S_{4,0} \approx 6$) at $x/D = 1$ near the wall, resulting in a surge of accelerating fluid streaks. When the distributions of Q_2 and Q_4 events intersect at $z/D \approx 2$, Q_2 events take precedence over Q_4 events with the increment in z/D. Consequently, because of the surge of fast-moving fluid stripes, sweep events serve as the controlling mechanism in wall-wake flows; while ejection prevails over sweep far away from the bed.

For a particular *H*, the numbers of Q_2 or Q_4 event occurrences in a recorded sample are gathered in order to analyze the durations and frequencies of these events. This allowed for the estimation of the mean time-intervals between ejections (i_E) and sweeps (i_S) as well as the mean duration of ejections (t_E) or sweeps (t_S). In the immediate downstream positions, two sudden peaks are visible in each of the time profiles just behind both the cylinders, indicating prolonged bursting events to be taken place. Another interesting observation is that the highest peak is visible in between two cylinders ($z/D \approx 1.5$), which means bursting cycle is longest at the recirculation zone.

The third-order moments of velocity fluctuations are crucial for understanding turbulence due to their ability to represent the schematic coherent structures within the flow. That being said, according to their signs, they directly correlate with the temporal bursting events; being defined as (Raupach 1981):

$$M_{ij} = \overline{\hat{u}^i \hat{w}^j}$$

Where $\hat{u} = u'/\sqrt{\overline{u'u'}}$ and $\hat{w} = w'/\sqrt{\overline{w'w'}}$, and i, j being non-negative integers with $i + j = 3$. The thorough analysis of the third-order moments M_{ij} provides insights about the prevalence of bursting events within distinct flow zones. The sweep events are caused by the preponderance of the combinations of (positive M_{30} and negative M_{03}) and (negative M_{21} and positive M_{12}) in near wall zone; whereas, the ejection events are detected by the prominence of the opposite sign combinations of third-order moments in far-wall zone.

The primary contributions to the overall RSS are sweeps and ejections, therefore it's intriguing to see how important each is in the flow from the residual conditional shear stress: $\Delta S_{4-2} = S_{4,0} - S_{2,0}$. This ΔS_{4-2} exhibits a correlation with 3rd order moment M_{30}. More specifically, there is a simplified linear relationship (Raupach 1981; Mignot et al. 2009):

$$\Delta S_{4-2} = 0.37 M_{30}$$

In our present setup with such complex flow patterns being observed due to the presence of two bluff bodies, it is expected to obtain some under or overestimation to a certain extent in near-wake zone.

In **chapter 7**, turbulent mixed convective flow in a square cavity with two cylindrical blockages has been studied. The flow under consideration is assumed to be two-dimensional, turbulent, steady, and incompressible. All cavity walls are subject to an isothermal cold wall temperature of T_c (300 K), while the cylindrical surfaces are subject to an isothermal hot wall temperature of T_h (301 K). The following equations indicate normalised Reynolds Averaged Navier Stokes equations with heat transfer in y-momentum (Rodi 1997):

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial}{\partial x}(\overline{u'u'}) - \frac{\partial}{\partial y}(\overline{u'v'})$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial}{\partial x}(\overline{u'v'}) - \frac{\partial}{\partial y}(\overline{v'v'}) + \frac{Gr}{Re^2}\theta$$

Normalised Reynolds averaged heat transfer equation:

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{RePr}(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2})-c_p\frac{\partial}{\partial x}(\overline{\theta' u'})-c_p\frac{\partial}{\partial y}(\overline{\theta' v'})$$

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

In case of isotropic turbulence, streamwise and nomal velocity fluctuations vary in the same order in both directions (Kafoussias et al. 1999). Therefore,

$$\epsilon_h\approx\frac{-\overline{u'v'}}{\frac{\partial v}{\partial x}}\approx\frac{-\overline{u'u'}}{\frac{\partial u}{\partial x}} \qquad \epsilon_v\approx\frac{-\overline{u'v'}}{\frac{\partial u}{\partial y}}\approx\frac{-\overline{v'v'}}{\frac{\partial v}{\partial y}}$$

After incorporation of turbulent prandtl number along with these assumptions, the final governing equations takes the following form:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+(\frac{1}{Re}+\epsilon_h)\frac{\partial^2 u}{\partial x^2}+(\frac{1}{Re}+\epsilon_v)\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+(\frac{1}{Re}+\epsilon_h)\frac{\partial^2 v}{\partial x^2}+(\frac{1}{Re}+\epsilon_v)\frac{\partial^2 v}{\partial y^2}+\frac{Gr}{Re^2}\theta$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=(\frac{1}{RePr}+\frac{c_p\epsilon_h}{Pr_t})\frac{\partial^2\theta}{\partial x^2}+(\frac{1}{RePr}+\frac{c_p\epsilon_v}{Pr_t})\frac{\partial^2\theta}{\partial y^2}$$

with boundary conditions:

$$\text{at}, y=0: u=0, v=0, \theta=0 \qquad \text{at}, y=1: u=1, v=0, \theta=0$$

$$\text{at}, x=0: u=1, v=0, \theta=0 \qquad \text{at}, x=1: u=0, v=0, \theta=0$$

$$\text{at both cylinder walls}: u=0, v=0, \theta=1$$

A Galerkin finite element method has been incoporated to solve the aforementioned equations. Both experimental and numerical validation have been performed with the lab facilities of our institute and previous literatures as well, showing good agreement with acceptable error rates with our findings.

The impact of having two circular cylinders in the cavity is discussed by isotherms and streamlines for a range of Reynolds and Richardson values. Due to the presence of two cylinders and displacement of left wall with constant velocity, the vortex region is being formed downstream of two cylindrical blockages, also known as recirculation region in the literature. The streamwise u and vertical v velocity profiles in the horizontal and vertical specified planes, respectively, provide a summary of the hydrological characteristics of the mixed convective flow. The u-velocities are all zero and negative in the vicinity of the cylindrical blockages, indicating some kind of channel flow feature. Moreover, to elucidate the heat transfer rate along the walls of cavity and blockages, local and average Nusselt number have been evaluated and depicted. The aim of our current research is to develop a predictive model that uses the Nusselt number to estimate the thermal distribution of a cavity flow with cylindrical blockages. To do this, two widely used machine learning algorithms Artificial Neural Network (ANN) and Support Vector Regression (SVR) have been implemented. The following parameter values, when used throughout model training, validation, and testing, will ensure the accuracy of the $\overline{Nu}$ prediction:

$$e_i = Y_{ANN/GPR}^i - Y_{FEM}^i$$

$$MSE = \frac{1}{n}\sum \left(Y_{ANN/GPR}^i - Y_{FEM}^i\right)^2$$

Due to a slightly higher testing accuracy, ANN is more preferred than GPR. Moreover, the model performs more robustly in the case of ANN, as evidenced by the smaller difference between training and testing accuracies $(0.94 - 0.89 < 0.97 - 0.87)$, which shows that the model generalizes to new data better.

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