Ref. No. *Ex/PG/EE/T/127E/2019*

Master of Electrical Engg. & Control System Engg. 2nd Semester Examination, 2019

Optimal and Robust Control

Time: Three Hours; Full Marks: 100

Answer any ***four*** questions
All questions carry equal marks

1. a) Explain the terms 'supremum' and 'infimum' with the help of appropriate examples. 5+8+12

   b) Find the 2-norm and ∞-norm of the following signal:

$$u(t) = \begin{cases} 0, & if \quad t \le 0 \\ 4t^3, & if \quad 0 < t \le 1 \\ 0, & if \quad t > 1 \end{cases}$$

   c) For the system with transfer function $G(s) = \dfrac{as+1}{bs+1}$, find $\|G\|_\infty$ when (i) $a > b$, (ii) $a = b$, (iii) $a < b$.

2. a) Distinguish between structured and unstructured uncertainty. With the help of diagrams, explain the terms additive and multiplicative unstructured uncertainty. 8+2+15

   b) State one limitation of Kharitonov's theorem.

   c) For a system whose characteristic polynomial is given by
   $s^3 + (2a-1)s^2 + a^2 s + a = 0$,
   where $a = 3$, check for the robust stability of the system for $\pm 10\%$ variation in each of the coefficients.

3. For a unity feedback control system, the forward path transfer function is given by $P(s) = \dfrac{5000}{s(1+50s)}$. 10+5+10

   Find

   (i) the largest value of the complementary sensitivity M_t and the corresponding frequency ω_{mt}.

   (ii) the allowable size of the process uncertainty ΔP

   (iii) permissible values of gain and phase variations when a controller is designed to provide the largest value of the complementary sensitivity M_t.

4. The regulator shown in Fig. F-4 contains a plant described by 10+8+7

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad ; \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

and has the performance index

$$J = \int_0^{\infty} \left\{ x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + u^2 \right\} dt .$$

Determine

(a) the Riccati matrix P
(b) the state feedback matrix K
(c) the closed loop eigenvalues.

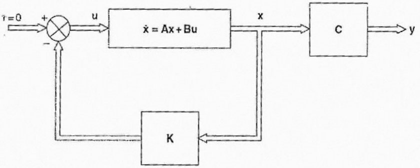

Fig. F-4

5 a) State the Brachistochrone problem and derive an expression for the time of descent of a particle in a Brachistochrone problem. 12+13

b) Find the curve which is the solution to the above Brachistochrone problem, given initial position is A (x_1, y_1) and final position is B (x_2, y_2).

6 a) Given a functional $J = \int_a^b F\left(x; y_1, y_2, \ldots, y_n; y_1', y_2', \ldots, y_n'\right) dx$. 6+4+15

Derive the condition(s) necessary to achieve an extremum of the functional.

b) State Legendre condition for functionals dependent on several unknown functions.

c) (i) Obtain the functional used to express the length of the curve $y = y(x);\ \ z = z(x)$ in three-dimensional space.

(ii) Find the extremum of the above functional.

(iii) Check whether the above extremum is a maximum or a minimum.

7 a) Given a functional $J = \int_a^b F(x; y; y')\,dx$. For the fixed end-point problem, prove that the necessary condition to achieve an extremum of the functional is $F_y - \frac{d}{dx} F_{y'} = 0$. 5+5+15

b) Determine the shortest curve y connecting two points (0, 0) and (1, 1) from the calculus of variations.

c) A plane curve, $y = y(x)$ passes through the points $(0, b)$ and (a, c), $(b \neq c)$ in the $x - y$ plane (a, b, c are positive constants) and the curve lies above the x-axis as shown in Fig. F-7.
Determine the curve $y(x)$ which, when rotated about the x-axis, yields a surface of revolution with minimum surface area.

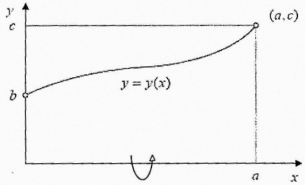

Fig. F-7