ABSTRACT

Title of the thesis: "Mathematical Study on Biodiesel Production Considering Ultrasound Effect and Special Emphasis on Ecological Issues"

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Biodiesel has gained global attention as a sustainable alternative to fossil fuels due to its renewable origin, environmental benefits, and potential to reduce greenhouse gas emissions. Biodiesel mainly produced from various vegetable oil like *Jatropha curcas* oil, Rapeseed oil, Palm oil, Soybean oil; animal fat, and waste cooking oil collected from various households and restaurants and recently from laboratory generated microalgae. Biodiesel is produced using the transesterification reaction of alcohol and vegetable oil. Despite the availability of feedstocks, large-scale biodiesel production is hindered by two major challenges: enhancing the efficiency of the transesterification process and ensuring a reliable supply of feedstock. This thesis addresses both issues through mathematical modeling, with the first part focusing on process optimization in biodiesel production and the second part on ecological sustainability of *Jatropha curcas* plantations.

In the first part of the thesis, ultrasound-assisted biodiesel production is investigated using different feedstocks including *Jatropha curcas* oil, rapeseed oil, and waste cooking oil. A series of mathematical models are developed to capture the role of ultrasound frequency in overcoming mass transfer resistance, a major limitation in conventional transesterification. The models incorporate process-specific correlations, such as the modified Dittus–Boelter relation, and employ optimal control theory to determine ultrasound frequency profiles that maximize biodiesel yield while minimizing processing time and cost. Comparative analyses with mechanical stirring reveal that ultrasound substantially enhances reaction rates, with numerical simulations showing yields exceeding 97% within industrially feasible timeframes. Furthermore, fractional-order models using Caputo–Fabrizio and Atangana–Baleanu operators are formulated to account for memory effects in biodiesel reactions, providing deeper insights into the nonlocal dynamics of the process. Rigorous analyses of the fractional models are carried out in terms of existence, uniqueness, sensitivity, and optimal control, revealing that fractional dynamics capture the system's behaviour more realistically than their integer-order counterparts. Additionally, a mathematical framework is developed for enzymatic biodiesel production with ultrasound application, aiming to overcome the limitations of conventional stirring method by reducing reaction time and enhancing cost-effectiveness through optimal control.

The second part of the thesis focuses on addressing ecological constraints in biodiesel feedstock production, particularly pest-induced yield losses in *Jatropha curcas* plantation. The pest *Pempelia morosalis*, known for causing up to 40% biomass reduction, is modeled under different management scenarios using the Sterile Insect Technique (SIT). A system of nonlinear differential equations is constructed to capture interactions between wild and sterile pest populations, with analytical and numerical analysis of both continuous and impulsive sterile male release strategies. Results indicate that SIT can effectively suppress pest populations while maintaining ecological sustainability, ensuring stable biomass growth for biodiesel production. Cost-effective protocols are identified, highlighting the potential of periodic release strategies to optimize resource use.

Together, the mathematical and ecological models developed in this thesis form an integrated framework that advances both technological and environmental aspects of biodiesel production. By combining ultrasound-assisted process optimization with sustainable pest management, this work contributes to enhancing biodiesel yield, reducing production costs, and securing long-term feedstock availability. The findings underscore the value of interdisciplinary mathematical modeling in addressing the dual challenges of clean energy generation and ecological resilience.

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