Analysis of Cosmological models through Dynamical system

Abstract

In this thesis, we have considered a flat homogeneous and isotropic FLRW model of universe. We have investigated here the qualitative behavior of cosmological models by deploying dynamical system tools. To study the cosmological models, one needs to solve the field equations pertaining to the respective cosmological models which are not always possible analytically as the field equations form a complex system of non linear differential equations. Therefore, by deploying dynamical system method, we have converted the field equations obtained from respective cosmological models into a system of autonomous differential equations. Critical points from an autonomous system of differential equations have been computed in order to study the qualitative nature of the system around those points. Stability of the hyperbolic critical points are thoroughly analyzed with special attention to stable critical points as the nature of the cosmological models around those points depicts our universe as global attractor.

In chapter-1, we have discussed key cosmological components as well as various theoretical models which have been developed earlier to explain the observational evidences.

In chapter-2, we have reviewed the components of dynamical system method briefly along with various techniques, used to analyze the stability of the critical points. We have showed how autonomous system of differential equations can be framed from the field equations pertaining to a cosmological model.

In chapter-3, we have considered a conformally coupled massless scalar field in semiclassical gravity where matter is represented by a quantum field in curved spacetime, while gravity is described by the classical spacetime metric, governed by Einstein's field equations. Here, matter content in the gravitational field equations is expressed as the expected value of the energy-momentum tensor operator in a given quantum state, thereby incorporating quantum effects of matter on the classical geometry. The dark energy component is modeled as a massless, conformally coupled scalar field. By employing this setup, evolution equations have been derived and reformulated into an autonomous system through the introduction of appropriate dimensionless variables. Subsequently, by performing a dynamical systems analysis, stability properties of the universe near the critical points have been analyzed. The cosmological implications

around these critical points have been explored.
In chapter-4, we have considered non-minimally coupled $f(Q)$ gravity model. Some challenges have been observed in the framework of General Relativity to address the late time acceleration of the universe. To address this issue, geometric components of general gravity have been modified. One such approach is to change the geometric components of general gravity where gravitational interaction is denoted by Q, Q being the non metricity. Here, we have considered the linear combination of two functions of Q, namely $f_1(Q) = \alpha Q^n$, $n \neq 1$ and $f_2(Q) = Q$ where α is a constant. Forming the autonomous system, we have analyzed stable state of universe using dynamical system tools. These techniques help us to study the behavior of the universe under several circumstances. We have studied the stability around critical points and considering the recent observational data available for some cosmological parameters, feasible solutions are noted.
Lastly, we have studied Rényi holographic dark energy model where a new idea of dark energy has been studied depending on the holographic principle of quantum gravity, called as the holographic dark energy(HDE). Later on modifying Bekestein-Hawking entropy, different generalized entropies have been proposed, one of them being Rényi entropy which leads to Rényi holographic dark energy model (RHDE). We have considered RHDE model with Hubble horizon as the IR cut off and have studied the cosmological behaviour under non interacting, linear and non-linear interacting scenarios with the help of dynamical systems analysis. We have also investigated the stability of the system around hyperbolic critical points along with the type of fluid description, evolution of equation of state parameter as well as matter and energy density parameters.

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