Abstract

Title: A study of local symmetric properties and geometric constants in Banach spaces

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This dissertation explores the symmetricity of elements in Banach spaces and in the space of all bounded linear operators with respect to various notions of orthogonality. We analyze left and right symmetric elements in Banach spaces with respect to T-orthogonality, and extend our investigation to operator spaces through numerical radius orthogonality (nr-orthogonality). It is shown that the only nr-left symmetric bounded operator on a Hilbert space is the zero operator, while no nonzero compact normal operator on an infinite-dimensional Hilbert space can be nr-right symmetric. Some necessary and sufficient conditions are established separately for nr-left and nr-right symmetric operators defined on Banach spaces. We further examine symmetricity with respect to norm derivative orthogonality, i.e., ρ-orthogonality. In this context, we prove that a two-dimensional Banach space is a strictly convex Radon plane if every element is ρ-symmetric. Additionally, we characterize Hilbert spaces via ρ-symmetricity for the spaces with dimensions greater than or equal to three. A complete classification of ρ-symmetric elements in ℓ_1^n and ℓ_∞^n is also provided. Apart from symmetricity we also study several important geometric constants in Banach spaces. We show that elements attaining the James constant must be isosceles orthogonal to each other. We explore the relationship between the modulus of convexity and approximate isosceles orthogonality. To measure the quantitative difference between Birkhoff-James orthogonality and ρ-orthogonality, we introduce a new constant revealing that a Banach space is uniformly non-square if this constant is less than 0.5. We compute the exact value of this constant for spaces whose unit ball is a regular $2n$-gon. In the end, we explore another local geometric constant both in general Banach spaces and in the space of all bounded linear operators. By investigating the properties of this local constant, we obtain sufficient conditions under which the collection of smooth points forms an open set in a Banach space and characterize approximate smoothness in the space of all bounded linear operators.

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