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Application of Multi-Criteria Decision Making in Real Life Optimization Problems in Fuzzy Environment

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DECLERATION

This is to certify that the thesis entitled "***Application of Multi-Criteria Decision Making in Real Life Optimization Problems in Fuzzy Environment***", submitted by me to the *Jadavpur University, Kolkata- 700032, West Bengal*, for the award for the degree of Doctor of Philosophy (Science) is an authentic work carried out by me under the supervision of **Prof. Bibhas Chandra Giri** and co-supervision of **Dr. Arijit Ghosh**. The content of this thesis, in full or in parts, has not been submitted to any other University or Institute for the award of any degree or diploma.

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Date: 01.07.2025

Dedicated to my guiding star: My father, a celestial light watching over me from above. My Mother, Husband and Son whose strength and love inspire my every step.

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CERTIFICATE FROM THE SUPERVISORS

This is to certify that thesis entitled "*Application of Multi-Criteria Decision Making in Real Life Optimization Problems in Fuzzy Environment*", submitted by Neha Ghorui Nee Mundhra who got her name registered on 7th February, 2022 (INDEX NO: 22/22/Maths./27) For the award of Ph.D (Science) degree of Jadavpur University, is absolutely based upon her own work under the supervision of **Prof. Bibhas Chandra Giri** and co-supervision of **Dr. Arijit Ghosh**. The thesis has fulfilled all requirements as per the regulations of the institute and in our opinion neither this thesis nor any part of it has been submitted for either any degree/ diploma or any other academic award anywhere before.

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Neha Ghorui nee Mundhra

Contents

Acronyms

DM	Decision Making
MCDM	Multi Criteria Decision Making
MCDA	Multi Criteria Decision Analysis
AHP	Analytic Hierarchy Process
ANP	Analytic Network Process
ANN	Artificial Neural Network
WASPAS	Aggregated Sum Product Assessment
ELECTRE	Election et Choix Traduisant La Realite
PROMETHEE	Preference Ranking Organization Method for Enrichment Evaluations
VIKOR	VIekriterijumskoKOmpromisnoRangiranje
TOPSIS	Techniques for Order Preference by Simmilarity to Identical solution
DEMATEL	Decision-Making Trial and Evaluation Laboratory Method
COPRAS	Complex Proportional Assessment
CRITIC	Criteria Importance Through Inter-criteria Correlation
MABAC	Multi-Attributive Border Approximation area Comparison
PIS	Positive Ideal Solution
NIS	Negative Ideal Solution
GIS	Geographic Information System
TFN	Triangular Fuzzy Number
PFN	Pentagonal Fuzzy Number
HFN	Hexagonal Fuzzy Number
HFS	Hesitant Fuzzy Sets
GFN	Generalized Fuzzy Number
GTHFN	Generalized Triangular Hesitant Fuzzy Number
GTrHFN	Generalized Trapezoidal Hesitant Fuzzy Number
GHPFN	Generalized Hesitant Pentagonal Fuzzy Number
IFS	Intuitionistic Fuzzy Set
TIFN	Triangular Intuitionistic Fuzzy Number
TrIFN	Trapezoidal Intuitionistic Fuzzy Number
PIFN	Pentagonal Intuitionistic Fuzzy Number
NFS	Neutrosophic Fuzzy Set
TrNS	Trapezoidal Neutrosophic Set
TrNN	Trapezoidal Neutrosophic Number
SVNS	Single Valued Neutrosophic Set
PyFN	Pythagorean Fuzzy Number
HPyFN	Hexagonal Pythagorean Fuzzy Number
HPyFS	Hexagonal Pythagorean Fuzzy Sets

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1 Introduction

'"Nothing is more difficult, and therefore more precious, than to be able to decide." **- Napoleon Bonaparte**

Decision making is one of the fundamental cognitive processes of human practices by which the most preferred option is selected from a set of alternatives based on certain factor. In day to day life, we have to make decisions. The art of making right decisions in life is quite significant and to emphasize this, here are a few insightful quotes:

"The art of decision making includes the art of questioning." – Pearl Zhu
"Every decision you make reflects your evaluation of who you are." – Marianne Williamson

According to Harris (1998), "Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case, we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on."
Several steps constitute the effective decision making model. According to Baker et al. (2001) and Fulop (2005), decision making model initiates with the identification of the problem followed by certain generalized steps which helps to achieve an optimal alternative from a given set of options. The steps involved to get an efficient decision making are as follows:

- **Identification of the problem:** Identifying the problem, defining its root causes, and limiting the assumption as per the requirement are the primary concerns of decision making. Decision makers should have expertise of the problem.

- **Establishment of the goal:** Goals are comprehensive statements which should always state as what should be done and what it shouldn't. Goals are developed prior to identification of the alternatives. Goals at times might be conflicting in nature but this should not be a reason of concern. Identification of alternatives requires significant understanding of the goals and requirements.

- **Selection of criteria:** Identifying criteria and alternatives with reference to a particular problem should be intended to meet the goal. Criterion or criteria selected for a specific decision making problem should be able to distinguish the performance of different alternatives. Criteria can be further classified into sub-criteria as per the need of the problem. Decision criteria should be able to discriminate among alternatives in a substantial way. It should cover all the goals.

- **Identification of alternatives:** The identification and assessment of alternatives are significant. Alternatives can be assessed with qualitative methods, quantitative methods or combination of both the methods. The alternatives selected must be able to meet the requirement and goals. If any alternative does not meet the requirement, then it can be discarded or changed.

- **Selection of appropriate decision making tool:** Decision-making tools are widely used to calculate the weight of the criteria, which is needed in the next step for evaluation of alternatives. To justify, some decision-making tools such as AHP and DEMATEL can be applied. These tools works on the principle of construction of pairwise comparison matrix, based on their relative importance. These techniques make decision and calculation simple because of the delight analogy.

- **Evaluation of alternatives with respect to the criteria:** Correct decision making requires evaluation of alternatives with respect to the criteria. Assessment of the alternatives with respect to criterion can be based on objective (fixed data, quantitative), subjective (quantitative), certain scale of measurement (depending on the hesitancy and indeterminacy of the experts). The established MCDM tools such as TOPSIS, COPRAS, VIKOR, PROMETHEE can be applied for optimal ranking of the alternatives.

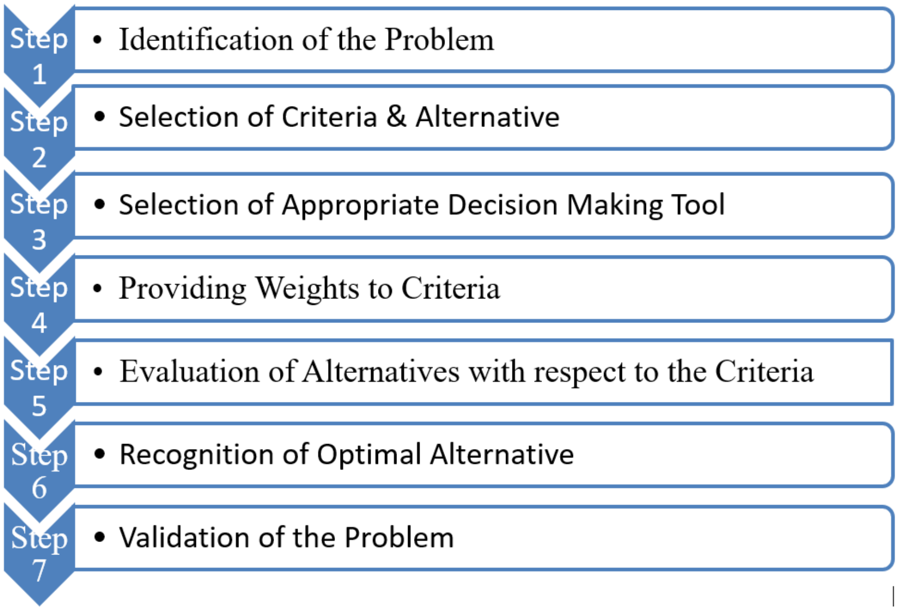

Figure 1.1: Flow chart depicting steps of decision making process

- **Validation of alternatives:** Ranking obtained after the application of decision tools should always be validated. Comparative and sensitivity analysis can be conducted to check the steadiness of the process used. It may happen that the MCDM tools are not applied according to the requirement of the problem or may give inappropriate ranking. In complex problems containing multiple conflicting criteria or sub-criteria, decision makers can feel the need to add or eliminate alternatives in order to attain the desired goal.

1.1 Multi-Criteria Decision Making (MCDM)

Decision making implies selection of the most preferred alternative from a set of alternatives. Additionally, it can be interpreted as clustering of alternatives into different preference sets.

MCDM delivers strong decision making when conflicting or several criteria are involved. In real life situations, MCDM is more often used to provide solution for planning problems in which multiple criteria are involved. According to Brugha (2004), MCDM is the process of molding the information which can lead to better decision making. The knowledge in this regard should be conceptual, accessible, differentiable, quantifiable, comprehensible, sustainable, improvable and applicable. MCDM is a useful tool in domains such as site selection, medical diagnosis problem, cloud computing services, material selection, military etc. The process of MCDM is based on the following steps:

Step 1: *Identification of the problem*: Identification of relevant criteria pertaining to the problem.

Step 2: *Construction of preferences by the decision makers*: Collection of appropriate information for the relevant criteria so that the decision experts can give preferences and execute strong decision analysis.

Step 3: *Selection of a set of feasible alternatives*: Alternatives corresponding to the criteria are selected which help us in understanding the motivation of the work. This further ensures that the desired goal will be achieved.

Step 4: *Determination of the most preferred alternative*: The execution of the first three steps leads to develop an appropriate MCDM method which will help in optimal ranking of the alternatives.

There are different MCDM methods and each method is unique in its own way. Every individual MCDM method has its purpose, uniqueness, advantages and disadvantages (Aruldoss et al., 2013). In the following sub-section, we discuss some of the MCDM techniques which have been applied in various chapters of the thesis.

1.1.1 *Brief overview of several MCDM techniques*

- **Analytic Hierarchy Process (AHP)**
  The AHP method developed by Satty (1980), is a powerful tool in handling quantitative and qualitative data. It is a logically structured analytical framework used in MCDM problems, aiding complex decision-making through heuristic methods. Evaluation of criteria weights is significant in decision making problem. AHP constructs the decision matrix in the form of pairwise comparison matrices, which delivers more precise judgment about the attributes relevant for ranking of the alternatives. The steps for AHP are given below.

Step 1: Formation of a comparison matrix in linguistic terms by a decision expert or a group of experts:

$$(S_{kl}^{\alpha}) = \begin{bmatrix} 1 & (S_{12}^{\alpha}) & \dots & (S_{1n}^{\alpha}) \\ (S_{21}^{\alpha}) & 1 & \dots & (S_{2n}^{\alpha}) \\ \vdots & \vdots & \ddots & \vdots \\ (S_{n1}^{\alpha}) & (S_{n2}^{\alpha}) & \dots & 1 \end{bmatrix} \tag{1.1}$$

Step 2: Normalization of each element of the matrix

$$N_f = \frac{p_f}{\sum_{f=1}^{m} p_f} \tag{1.2}$$

Step 3: Estimation of criteria Priority Weight (PW)

$$E = \frac{s^{th}\ root\ value}{\Sigma s^{th}\ root} \tag{1.3}$$

Step 4: Determination of the Consistence Index (CI) of the matrix

$$CI = \frac{\gamma_{max} - n}{n - 1} \tag{1.4}$$

where n is the size of the matrix.

Step 5: Determination of Consistence Ratio (CR)

$$CR = \frac{CI}{RI} \tag{1.5}$$

where, Random Index (RI) varies with the size of the matrix "n". The value of $CR \leq 0.1$ means a consistent matrix. Table 1.1 represents RI values for different sizes of the matrix considered for the problem.

Table 1.1: Random Index (RI) values for different sizes of matrix

Matrix size (p)	1	2	3	4	5	6	7	8	9	10
Random Index (RI)	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

- **Decision-Making Trial and Evaluation Laboratory (DEMATEL) Method**

  DEMATEL method developed by Gabus and Fontela (1972) is based on the concept of graph theory. The investigation and solution of a problem in this methodology is executed with the help of visualization technique. It is a comprehensive technique which analyses the direct and indirect relationship between factors and help in differentiating between system factors and stressing core driving factors (Du et al., 2021). Generalizing, this segmentation of factors enables the researcher to identify the components as causal group and effect group. DEMATEL method has been applied to many optimization problems by researchers because of its ability to solve complex problem and come up with the best possible solution. The steps of DEMATEL method comprises of the following steps:

  Step 1: Defining the dominant feature in the research methodology, the linguistic measurement scale is set for pairwise comparison among all characteristics. The initial direct relation matrix $E = [\alpha_{ij}]_{n\times n}$ is obtained by pairwise comparison between criteria, in which α_{ij} denotes the degree to which the criteria i affects the criteria j.

  Step 2: Normalization of direct relation matrix: On the basis of direct relation matrix α, the normalized direct relation matrix can be obtained as

$$U = k \times E, \tag{1.6}$$

$$\text{where, } k = \frac{1}{\max\limits_{1\le i\le n} \sum_{j=1}^{n} \alpha_{ij}}.$$

  Step 3: The total relation matrix is evaluated using the formula given below:

$$V = U(I-U)^{-1}, \text{where } I \text{ is the } n \times n \text{ identity matrix.} \tag{1.7}$$

  Step 4: Construct the DEMATEL map with respect to the total relation matrix $TR = [\tilde{t}_{ij}]_{n\times n}$. The row sum and the column sum are represented by the vectors β_j $(j = 1, 2..., n)$ and α_i $(i = 1, 2, ..., n)$, respectively and are given by

$$\alpha_i = \left[\sum_{i=1}^{n} \tilde{t}_{ij}\right]_{n\times 1} \tag{1.8}$$

$$\beta_j = \Big[\sum_{j=1}^{n} \tilde{t}_{ij} \Big]_{1\times n} \tag{1.9}$$

where $\alpha_i + \beta_j$ symbolises horizontal axis vector or 'prominence' which indicates the relative importance of the criterion, and the vertical axis $\alpha_i - \beta_j$ implies 'relation'.

- value of $(\alpha_i - \beta_j) > 0$ implies that the criteria falls into causal group
- value of $(\alpha_i - \beta_j) < 0$ implies that the criteria falls into effect group

Step 5: The sum of each column of the total relation matrix is 1 by normalized method, which gives the inner dependency of the matrix.

- **Technique of Order Preference by a Similarity to Ideal Solution (TOPSIS) Method**

The TOPSIS method is a widely used MCDM tool because of its unique characteristics. It was introduced by Hwang et al. (1981). The main concept of TOPSIS methodology is to rank the corresponding alternatives selected for the problem. The methodology is classified as a distance measure method in which the optimal alternative obtained is farthest from the NIS and closest to the PIS. The approach is useful in handling the complexity of the situation involving several factors and their sub-factors. Finally, the alternative with higher relative closeness is considered as the best one. The steps of TOPSIS methodology includes the following:

Step 1: Construction of the decision matrix: TOPSIS method begins with the construction of decision matrix. In this step, decision experts assign preferential rating of alternatives with respect to the criteria in matrix notation.

Step 2: Evaluation of the normalized decision matrix:

The common method used to obtain normalized value is calculated using vector normalization technique, as shown below:

$$N_{ij}^{*} = \frac{n_{ij}}{\sum_{i=1}^{m} {n_{ij}}^2}, \;\; j = 1, 2, \ldots n \tag{1.10}$$

Step 3: Determination of the weighted normalized matrix:

Product of criteria weight and the normalized value results in formulation of weighted normalized value

$$\tilde{V} = (\tilde{v}_{ij})_{n\times m} \tag{1.11}$$

where $\tilde{v}_{ij} = w_j \times N^*_{ij}$

Step 4: Calculation of PIS (IS^+) and NIS (IS^-):

PIS and NIS for each alternative are computed as given below:

$$IS^+ = s_1^+, s_2^+, ..., s_m^+ = \left(\max(s_{ab}) | b \in F_b, \min(s_{ab}) | b \in F_{nb} \right) \tag{1.12}$$

$$IS^- = s_1^-, s_2^-, ..., s_m^- = \left(\min(s_{ab}) | b \in F_a, \max(s_{ab}) | b \in F_{nb} \right) \tag{1.13}$$

where (s_b^+) denotes the maximum value of (s_{ab}), and (s_b^-) denotes the minimum value of (s_{ab}). F_b indicates benefit factor and F_{nb} indicates the non-benefit factor or the cost factor.

Step 5: Calculation of the distance measure of alternatives from PIS and NIS:

The classical Euclidean distance for individual alternatives from PIS and NIS is calculated as follows:

$$I_c^+ = \sqrt{\sum_{b=1}^{m} (s_{ab} - s_b^+)^2}, \quad a = 1, 2, \ldots l \tag{1.14}$$

$$I_c^- = \sqrt{\sum_{b=1}^{m} (s_{ab} - s_b^-)^2}, \quad a = 1, 2, \ldots l \tag{1.15}$$

Step 6: Determination of the relative closeness:

$$R_c = \frac{I_c^-}{I_c^- + I_c^+}, a = 1, 2, \ldots l \tag{1.16}$$

Step 7: Ranking of the alternatives:

The alternatives are scored on the basis of R_c. A higher value of R_c indicates the desired alternative.

- **COmplex PRoportional ASsessment (COPRAS) Method**

  COPRAS is an extensive MCDM tool used for optimal ranking of the alternatives. It is a simple and easy technique which is used by researchers for application in different application fields. This method was developed by Zavadskas et al. (1994). It is based on evaluating alternatives in terms of significant and utility degree. It works on both qualitative and quantitative data. COPRAS tool compromises of six steps which are given below:

  Step 1: Construction of the decision matrix:

  The first step of COPRAS method begins with the construction of decision matrix which is obtained by the information collected from the decision makers.
  Step 2: Normalization of the decision matrix:

  In this step, the decision matrix is normalized using the formula given below:

$$N_{ij}^{*} = \frac{n_{ij}}{\sum_{i=1}^{n} n_{ij}}, \quad j = 1, 2, \ldots n \tag{1.17}$$

  Here, N_{ij}^{*} depicts the normalized value of the decision matrix of ith alternative with respect to jth criterion.

  Step 3: Determination of weighted normalized matrix:

  The weighted normalized matrix is calculated using the formula:

$$w_{ij}^{n} = N_{ij}^{*} * w_j, \quad i = 1, 2, \ldots m; j = 1, 2, \ldots n \tag{1.18}$$

  Step 4: Determination of maximizing and minimizing indices

  The maximizing and minimizing indices for each criterion are computed based on the categorization of criteria whether they are negative (non-benefit criteria) or positive (benefit criteria).

$$S_i^{+} = \sum_{j=1}^{k} w_{ij}^{n}, \quad i = 1, 2, \ldots m \tag{1.19}$$

$$S_i^{-} = \sum_{j=k+1}^{n} w_{ij}^{n}, \quad i = 1, 2, \ldots m \tag{1.20}$$

where, k denotes the number of benefit criteria and $(n-k)$ depicts the number of non-benefit criteria. S_i^+ and S_i^- represent the maximizing and minimizing indices of the ith criteria based on it's characteristic (positive or negative).

Step 5: Evaluation of relative significance degree

The significance of each alternative is determined as follows:

$$Q_i = S_i^+ + \frac{S_{min}^- \times \sum_{i=1}^{m} S_i^-}{S_i^- \times \sum_{i=1}^{m} \left(\frac{S_{min}^-}{S_i^-} \right)} \tag{1.21}$$

where $S_{min}^- = min\{S_i^- : i = 1, 2, 3, \ldots, m\}$.

Step 6: Ranking of alternatives

The ranking of alternatives is calculated as

$$R_i = \frac{Q_i}{Q_{max}} \times 100\% \tag{1.22}$$

where $Q_{max} = \{Q_i : i = 1, 2, 3, \ldots, m\}$.

Finally, ranking of the alternative is executed in descending order of the R_i score. The highest value of R_i means rank of order 1.

1.2 Fuzzy Logic in MCDM techniques

The classical MCDM techniques use crisp or fixed number for the evaluation of weight and ranking of the alternatives. Crisp values don't consider the impreciseness, hesitancy and uncertainty of the situation or problem. Moreover, the data collected need not always have the fixed value. For example, if literacy rate or number of graduates in a particular area may not have every year's data, then previous year's data can act as a forecasting for current and future years. In this regard, fuzzy logic can be applied to capture the impreciseness of the decision expert. The fuzzy system software has also been developed as the most effective and practical application of the theory and models with fuzzy sets (Chang and Zadeh, 1972). These systems can deal with the representation of knowledge and reasoning subject to vagueness and uncertainties for the solution of various kinds of real-life problems.

Fuzzy sets and systems are most effective in modeling complex systems and can integrate the human expert system and knowledge with uncertain information. A fuzzy set consists of the collection of elements with calculating unlikelihood in the set and has various degrees of membership values which vary from 0 to 1. The linguistic variables are the representation values in words rather than numbers (e.g., low, moderate, high). The rule base system contains rules which explain the operation of the controller.

1.3 Fuzzy Logic in Uncertain environment

Definition 1. *A fuzzy set T is defined as a set of ordered pairs:*

$$T = \{< (\tau, \mu_T(\tau)) >, \ \tau \in T, \mu_T(\tau) \in (0,1)\} \tag{1.23}$$

where $\mu_T(\tau)$ represents the membership function of T, which takes value from 0 to 1.

1.3.1 *Triangular fuzzy number (TFN)*

A TFN, denoted by $S = (p, r, t)$, consists of the following membership function:

$$\mu_{\tilde{S}}(x)) = \begin{cases} \frac{x-p}{r-p} & \text{for } p \leq x \leq r \\ \frac{t-x}{t-r} & \text{for } r \leq x \leq t \\ 0 & otherwise \end{cases} \tag{1.24}$$

Figure 1.2 represents the pictorial representation of a triangular fuzzy number. The variable r represents the maximum membership value of 1 and is considered to be the most promising value. The variables p and t denote the smallest viable value and the largest viable value, respectively. The triplet (p, r, t) which describes the fuzziness of a particular event enables the field of practicable evaluation.

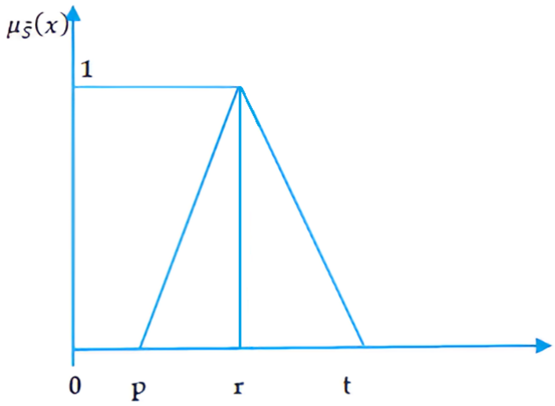

Figure 1.2: Diagram representing a Triangular Fuzzy Number (TFN)

Note 1. *When $p = r = t$, the TFN becomes a crisp value.*

Arithmetic operations on TFN

Assume two TFN's $S_1 = (p_1, r_1, t_1)$ and $S_2 = (p_2, r_2, t_2)$. The arithmetic operations (Wang et al., 2018) on two TFNs, S_1 and S_2, are presented below:

$$(S_1 + S_2) = (p_1 + p_2, r_1 + r_2, t_1 + t_2) \tag{1.25}$$

$$(S_1 - S_2) = (p_1 - p_2, r_1 - r_2, t_1 - t_2) \tag{1.26}$$

$$(S_1 \times S_2) = (p_1 p_2, r_1 r_2, t_1 t_2) \tag{1.27}$$

$$(S_1 / S_2) = \left(\frac{p_1}{t_2}, \frac{r_1}{r_2}, \frac{t_1}{p_2}\right) \tag{1.28}$$

$$S^{-1} = \left(\frac{1}{t_1}, \frac{1}{r_1}, \frac{1}{p_1}\right) \tag{1.29}$$

Definition 2. *Let $S_1 = (p_1, r_1, t_1)$ and $S_2 = (p_2, r_2, t_2)$ be two TFN's. Then the distance between these two triangular fuzzy numbers is determined by the vertex method as*

$$d = \sqrt{\frac{1}{3}\{(p_1 - p_2)^2 + (r_1 - r_2)^2 + (t_1 - t_2)^2\}} \tag{1.30}$$

TFN is widely used for its efficiency to deal with applications where vagueness and uncertainty surround the decision making. The hesitancy or fuzziness of the DM can be expressed in TFN while constructing a comparison matrix and decision matrix to rank the best alternative.

1.3.2 *Pentagonal fuzzy number* (PFN)

A PFN is represented by $I =< [l_1, m_1, n_1, o_1, p_1]; w >$, where $l_1, m_1, n_1, o_1, p_1 \in \mathbb{R}$, the set of real numbers such that $l_1 \leq m_1 \leq n_1 \leq o_1 \leq p_1$. The membership function of PFN is denoted by

$$\mu = f(x) = \begin{cases} 0 & \text{for } x \leq l_1 \\ \frac{W(x-l_1)}{(m_1-l_1)} & \text{for } l_1 \leq x \leq m_1 \\ W - \frac{(1-W)(x-m_1)}{(n_1-m_1)} & \text{for } m_1 \leq x \leq n_1 \\ 1 & \text{for } x = n_1 \\ 1 - \frac{(1-W)(o_1-x)}{(o_1-n_1)} & \text{for } n_1 \leq x \leq o_1 \\ \frac{W(p_1-x)}{(p_1-o_1)} & \text{for } o_1 \leq x \leq p_1 \\ 0 & \text{for } x \geq p_1 \end{cases} \tag{1.31}$$

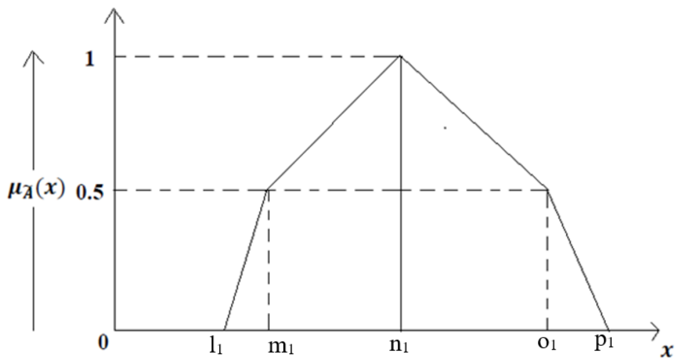

Figure 1.3: Graphical representation of a PFN

Note 2. *If $m_1 = n_1 = o_1$, then the PFN is reduced to a TFN.*

Note 3. *Figure 1.3 represents a PFN, where $l_1 \leq m_1 \leq n_1 \leq o_1 \leq p_1$. The variable n_1 possess the maximum degree of membership i.e. $\mu_{\tilde{\mathfrak{U}}}(x) = 1$. The variables m_1 and o_1 attain equal membership $\mu_{\tilde{\mathfrak{U}}}(x) = 0.5$, whereas the variables l_1 and p_1 have 0 membership value. In other words, membership increases from l_1, reaches the maximum value at n_1 and then start diminishing till p_1 .*

Arithmetic operations on PFN

Definition 3. *Let two PFNs be* $I = [l_1, m_1, n_1, o_1, p_1]$ *and* $J =< [l_2, m_2, n_2, o_2, p_2]$. *Then the following arithmetic operations can be defined:*

$$(I + J) = (l_1 + l_2, m_1 + m_2, n_1 + n_2, o_1 + o_2, p_1 + p_2) \quad (1.32)$$

$$(I - J) = (l_1 - p_2, m_1 - o_2, n_1 - n_2, o_1 - m_2, p_1 l_2) \quad (1.33)$$

$$(I \times J) = (l_1 l_2, m_1 m_2, n_1 n_2, o_1 o_2, p_1 p_2) \quad (1.34)$$

$$aI = (al_1, am_1, an_1, ao_1, ap_1) \quad (1.35)$$

$$(I/J) = \left(\frac{l_1}{p_2}, \frac{m_1}{o_2}, \frac{n_1}{n_2}, \frac{o_1}{m_2}, \frac{p_1}{l_2}\right) \quad (1.36)$$

$$I^{-1} = \left(\frac{1}{p_1}, \frac{1}{o_1}, \frac{1}{n_1}, \frac{1}{m_1}, \frac{1}{l_1}\right) \quad (1.37)$$

1.3.3 *Hexagonal fuzzy number (HFN)*

A number $\alpha_{HFN} = (h_1, h_2, h_3, h_4, h_5, h_6; \mu(\alpha))$ is defined as HFN if it satisfies the following properties:

(i) $\mu(\alpha)$ is a continuous function in $[0, 1]$.

(ii) $\mu(\alpha)$ is a strictly increasing continuous function in $[h_1, h_2]$ and $[h_2, h_3]$.

(iii) $\mu(\alpha)$ attains the maximum value 1 in $[h_3, h_4]$.

(iv) $\mu(\alpha)$ is a strictly decreasing continuous function in $[h_4, h_5]$ and $[h_5, h_6]$.

Figure 1.4 represents the membership function of a symmetric HFN.

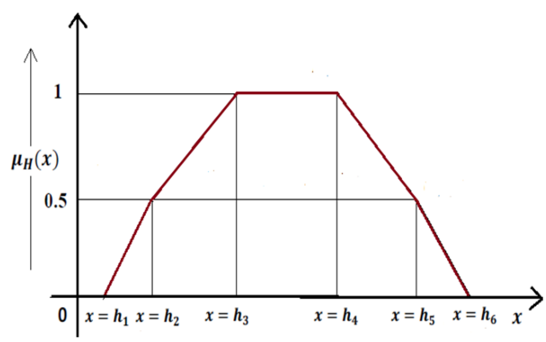

Figure 1.4: Representation of the membership function of a HFN

$$\mu(\alpha) = \begin{cases} 0 & \text{for } x \leq h_1 \\ 0.5\frac{x-h_1}{h_2-h_1} & \text{for } h_1 \leq x \leq h_2 \\ 0.5 + 0.5\frac{x-h_2}{h_3-h_2} & \text{for } h_2 \leq x \leq h_3 \\ 1 & \text{for } h_3 \leq x \leq h_4 \\ 1 - 0.5\frac{x-h_4}{h_5-h_4} & \text{for } h_4 \leq x \leq h_5 \\ 0.5 - 0.5\frac{x-h_5}{h_6-h_5} & \text{for } h_5 \leq x \leq h_6 \\ 0 & \text{for } x \geq h_6 \end{cases} \quad (1.38)$$

A HFN captures the hesitancy and uncertainty in broader aspect compared to TFN, PFN as the later undertakes three, and five numbers, respectively to represent the impreciseness and ambiguity of the decision maker (DM). If we consider the linguistic term corresponding to TFN then it becomes [Low, Medium, High]; if PFN then [Very low, Low, Medium, High, Very high]; but in HFN then it is [Very very low, Very low, Low, High, Very high, Very very high] i.e. maximum possible spread can be accommodated in HFN.

- **Arithmetic operations of linear symmetric HFN** (Çetinkaya et al. 2016)

Let $U = (u_1, u_2, u_3, u_4, u_5, u_6)$ and $V = (v_1, v_2, v_3, v_4, v_5, v_6)$ be two $HFNs$. Then their general arithmetic operations can be defined as follows:

$$(U + V) = (u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4, u_5 + v_5, u_6 + v_6) \quad (1.39)$$

$$(U - V) = (u_1 - v_6, u_2 - v_5, u_3 - v_4, u_4 - v_3, u_5 - v_2, u_6 - v_1) \quad (1.40)$$

$$(U \times V) = (u_1v_1, u_2v_2, u_3v_3, u_4v_4, u_5v_5, u_6v_6) \quad (1.41)$$

$$\alpha U = (\alpha u_1, \alpha u_2, \alpha u_3, \alpha u_4, \alpha u_5, \alpha u_6) \quad (1.42)$$

$$(U/V) = \left(\frac{u_1}{v_6}, \frac{u_2}{v_5}, \frac{u_3}{v_4}, \frac{u_4}{v_3}, \frac{u_5}{v_2}, \frac{u_6}{v_1}\right) \quad (1.43)$$

$$U^{-1} = \left(\frac{1}{u_6}, \frac{1}{u_5}, \frac{1}{u_4}, \frac{1}{u_3}, \frac{1}{u_2}, \frac{1}{u_1}\right) \quad (1.44)$$

- **Distance measure between two HFNs**

Let $A_d = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $B_d = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two HFNs. Then the distance between the two HFNs can be determined as

$$d(A_d, B_d) = \frac{1}{6}\sqrt{\begin{matrix}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + \\ (a_4 - b_4)^2 + (a_5 - b_5)^2 + (a_6 - b_6)^2]\end{matrix}} \tag{1.45}$$

- **Centroid based method for defuzzification of HFN:**

A HFN can be considered as union of two triangles and two trapeziums. In Figure 1.5, $\triangle ADH$, $\triangle BCG$, $ABCD$, and $CDEF$ together form a HFN.

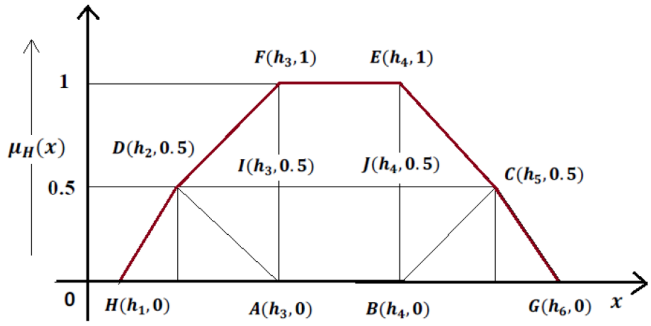

Figure 1.5: Hexagonal fuzzy number as union of different regions

Further, the trapezium is a union of two triangles and one rectangle. Applying centroid based method on triangles and rectangles, and finally summing them we obtain the centroid of HFN. Since the defuzzified value should remain within the range of a HFN, the given formulae below provides the required defuzzified value. Derivation of HFN is executed in the following way:

$$\text{Centroid of } \triangle \text{ BCG} = \frac{h_4 + h_5 + h_6}{3}$$

$$\text{Centroid of } \triangle \text{ ADH} = \frac{h_1 + h_2 + h_3}{3}$$

Centroid of trapezium ABCD:

$$\text{Centroid of } \triangle \text{ ADI} = \frac{h_2 + 2h_3}{3}$$

$$\text{Centroid of } \triangle \text{ BCD} = \frac{2h_4 + h_5}{3}$$

$$\text{Centroid of rectangle ABIJ} = \frac{h_3 + h_4}{2}$$

$$C_{ABCD} = \frac{2h_2 + 7h_3 + 7h_4 + 2h_5}{18}$$
$$\text{Centroid of trapezium } CDEF = \frac{2h_2 + 7h_3 + 7h_4 + 2h_5}{18}$$

The defuzzified value is given by

$$C_{HGCEFD} = \frac{3h_1 + 3h_2 + 10h_3 + 10h_4 + 5h_5 + 3h_6}{34}$$

1.3.4 *Hesitant fuzzy set (HFS)*

Hesitant Fuzzy Set (HFS), introduced by Aliahmadipour et al. (2016), is applicable when uncertainties and hesitancy exists in assigning the degree of belongingness of the elements. The idea of decision making process involves the hesitancy or uncertainty of the decision makers in terms of preferences. Thus HFS can be flexibly used to represent the preferences (Xu & Zhang, 2013).

Definition 4. *Let X be fixed set. A HFS on X is defined in terms of a function, which when applied to X returns a subset of* $[0, 1]$*. To make it simpler, Xia & Xu (2011), represent the HFS in mathematical symbol as follows:*

$$F = (<x|hF(x)>|x \in X) \tag{1.46}$$

where $h_F(x)$ *belongs to a set of some values in* $[0, 1]$*, which denotes the degree of belongingness of the element* $x \in X$ *to the set* F*. For convenience, Xia and Xu (2011), called* $h = h_F(x)$*, a HFE and H the set of all HFEs. Let three HFEs be represented by* h, h_1 *and* h_2*. Then the following operational rules are defined by Saeed et al. (2021):*

$$h' = \bigcup_{\beta \in h} \{1 - \beta\} \tag{1.47}$$

$$h_1 \bigcup h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} max\{\beta_1, \beta_2\} \tag{1.48}$$

$$h_1 \bigcap h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} min\{\beta_1, \beta_2\} \tag{1.49}$$

Assuming three HFE's h, h_1 and h_2, and $\lambda > 0$, Xia and Xu (2011) introduced some operations as follows:

$$h^{\lambda} = \bigcup_{\beta \in h} \{\beta^{\lambda}\} \tag{1.50}$$

$$\lambda \times h = \bigcup_{\beta \in h} \{1 - (1 - \beta)^{\lambda}\} \tag{1.51}$$

$$h_1 \oplus h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} \{\beta_1 + \beta_2 - \beta_1 \beta_2\} \tag{1.52}$$

$$h_1 \otimes h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} \{\beta_1 \beta_2\} \tag{1.53}$$

Definition 5. *Xia and Xu (2011) defined ranking method for two HFEs. For a particular HFE 'h', the score function is defined as*

$$s_f(h) = \frac{1}{n(h)} \sum_{\beta \in h} \beta \tag{1.54}$$

where $n(h)$ represents the number of elements in h. Let two HFEs be 'h_1' and 'h_2'. If

i.)

$$s_f(h_1) > s_f(h_2), \text{then } h_1 > h_2 \tag{1.55}$$

ii.)

$$s_f(h_1) = s_f(h_2), \text{then } h_1 = h_2 \tag{1.56}$$

The definition 5 has some drawbacks, which is illustrated in the example below.

Example 1. *Let $h_1 = \{0.2, 0.4\}, h_2 = \{0.3\}, h_3 = \{0.1, 0.2, 0.4, 0.5\}$ be three HFEs. According to definition 5, $s_F(h_1) = s_F(h_2) = s_F(h_3) = 0.3$, which implies $h_1 = h_2 = h_3$. But it is not true.*

Farhadinia (2013), proposed a new definition for score function as follows:

Definition 6. *Let $h = \bigcup_{\beta \in h}\{\beta\} = \{\beta_s | s = 1, 2, \cdots, n(h)\}$ be a HFE, where $'n(h)'$ is the number of elements in $'h'$. Then the new score function $s_F(h)$ is defined as*

$$s_F(h) = \frac{\sum_{s=1}^{n(h)} \rho(s)(\beta_s)}{\sum_{s=1}^{n(h)} \rho(s)} \tag{1.57}$$

where $\{\rho(s)|s = 1, 2, \cdots, n(h)\}$ is a positive valued monotonic increasing sequence of index $'s'$.

1.3.4.1 *Generalized hesitant fuzzy numbers (GHFN)*

It is a set of some different generalized fuzzy numbers in the set of real numbers R, which represents the possible membership functions of the element $x \in X$. Different example of this form are $GTHFN$, $GTrHFN$, $GPHFN$ etc.

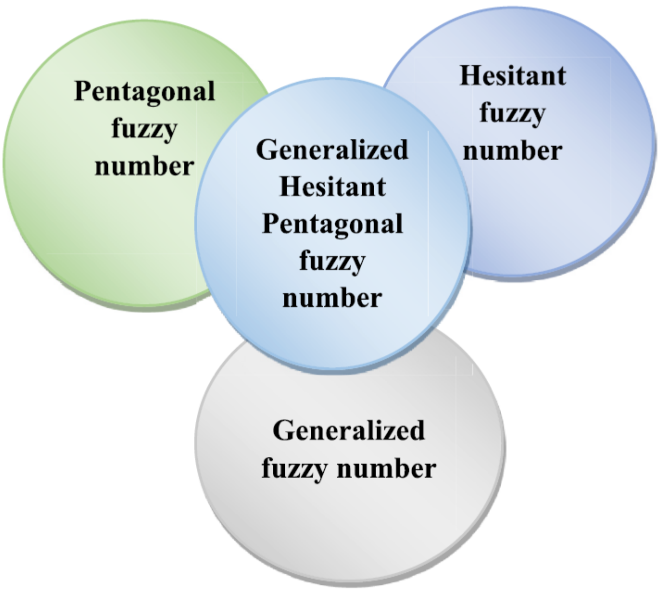

Figure 1.6: Representation of different types of fuzzy number as venn diagram.

Definition 7. *Generalized hesitant pentagonal fuzzy numbers (GHPFN). Let $'X'$ be a fixed set and $p_i, q_i, r_i, s_i, t_i \in R$ such that $p_i \leq q_i \leq r_i \leq s_i \leq t_i (i \in I_m = 1, 2, \ldots, m)$. Then a pentagonal hesitant fuzzy set on $'X'$ is defined as: $\psi = < x, (p_i, q_i, r_i, s_i, t_i) : i \in I_m x \in X >$ where $(p_i, q_i, r_i, s_i, t_i) : i \in I_m x$ is a set of different pentagonal fuzzy numbers denoting the set of real numbers, that represents the membership value of the element $x \in X$.*

Definition 8. *Assume 'X' to be a fixed set, $\alpha_i \in [0,1]$ $(i \in I = \{1,2,\ldots,m\}$ or $\{1,2,\ldots,n\})$ and $p_i, q_i, r_i, s_i, t_i \in \mathbb{R}$ such that the relation among them are $p_i \leq q_i \leq r_i \leq s_i \leq t_i (i \in I)$. A GHPFN is then defined as: $\psi = <x, (p_i, q_i, r_i, s_i, t_i); \delta_i : i \in I> : x \in X$ where $(p_i, q_i, r_i, s_i, t_i); \delta_i : i \in I$ represents a set of different GPFN in a set of real numbers, denoting the possible membership functions of the element $x \in X$.*

From the above definitions, we construct two particular forms of generalized hesitant pentagonal fuzzy numbers. In the first type of *GHPFN*, the membership function is fixed but the *PFN* can vary. In the second type, the *PFN* are considered to be fixed but the membership for a specific *PFN* can vary depending on the preference of DMs.

Definition 9. *Let $\psi = <x, (p_i, q_i, r_i, s_i, t_i); \delta_i : i \in I> : x \in X$ represents a set of different GHPFN. If $\delta_i = \delta \forall i \in I$, then GHPFN is reduced to a single membership function for different GHPFN. This form of GHPFN is said to be first type. Symbolically it can be expressed as $\beta^a GHPFN = <x, (p_i, q_i, r_i, s_i, t_i) : i \in I; \beta>$*

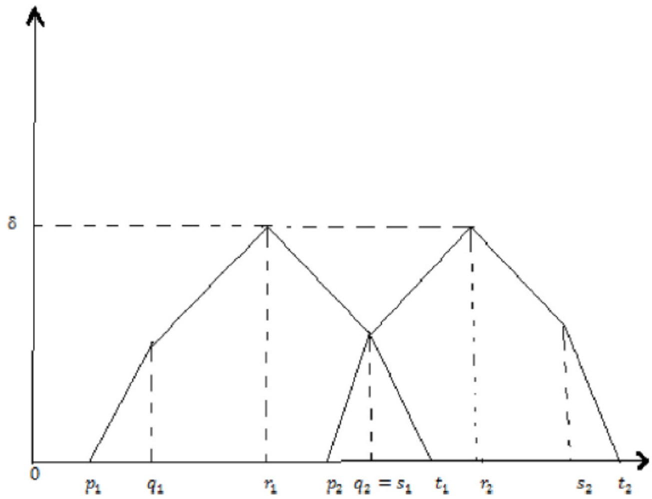

Figure 1.7: Representation of GHPFN 1st type.

Remarks 1. *GHPFN can be used to describe the hesitancy or vagueness of a DM or group of DMs in different possible ways. Unlike the GHTFN or GHTrFN which represents the impreciseness in three numbers or four numbers respectively, GHPFN considers five numbers and thus captures the uncertainty in a better way. Fig. 1.7 represents the GHPFN 1st type.*

Note 4. *The GHPFN 1st type considers the same degree of hesitancy for different sets of PFN. In this case, the DM's preference for rating can be a different PFN but the degree of membership for a particular event is the same. Figure 1.7 captures the concept clearly. The diagram signifies that the PFN can be different by the DMs but their hesitancy/ confidence 'δ' of assigning that particular PFN for the linguistic term remains the same.*

Arithmetic operations and score function on *GHPFN* **(1st Type).**
Let $\beta^1_{GHPFN} = <(p_1, q_1, r_1, s_1, t_1), (p_2, q_2, r_2, s_2, t_2); \beta>$,
$\beta^2_{GHPFN} = <(p_3, q_3, r_3, s_3, t_3), (p_4, q_4, r_4, s_4, t_4); \beta>$ *and* $\epsilon \geq 0$. Then

$$\beta^1_{GHPFN} + \beta^2_{GHPFN} = \begin{matrix} <(p_1 + p_3, q_1 + q_3, r_1 + r_3, s_1 + s_3, t_1 + t_3), \\ (p_2 + p_4, q_2 + q_4, r_2 + r_4, s_2 + s_4, t_2 + t_4); \beta> \end{matrix} \tag{1.58}$$

$$\beta^1_{GHPFN} - \beta^2_{GHPFN} = \begin{matrix} <(p_1 - t_3, q_1 - s_3, r_1 - r_3, s_1 - q_3, t_1 - p_3), \\ (p_2 - t_4, q_2 - s_4, r_2 - r_4, s_2 - q_4, t_2 - p_4); \{\beta\}> \end{matrix} \tag{1.59}$$

$$\beta^1_{GHPFN} \times \beta^2_{GHPFN} = \begin{cases} (p_1p_3, q_1q_3, r_1r_3, s_1s_3, t_1t_3), (p_2p_4, q_2q_4, r_2r_4, s_2s_4, t_2t_4); \\ \{\beta\}(t_1 > 0, t_2 > 0, t_3 > 0, t_4 > 0) \\ (p_1t_3, q_1s_3, r_1r_3, s_1q_3, t_1p_3), (p_2t_4, q_2s_4, r_2r_4, s_2q_4, t_2p_4); \\ \{\beta\}(t_1 < 0, t_2 < 0, t_3 > 0, t_4 > 0) \\ (t_1t_3, s_1s_3, r_1r_3, q_1q_3, p_1p_3), (t_2t_4, s_2s_4, r_2r_4, q_2q_4, p_2p_4); \\ \{\beta\}(t_1 < 0, t_2 < 0, t_3 < 0, t_4 < 0) \end{cases} \tag{1.60}$$

$$\beta^1_{GHPFN} \div \beta^2_{GHPFN} = \begin{cases} (p_1/t_3, q_1/s_3, r_1/r_3, s_1/q_3, t_1/p_3), \\ (p_2/t_4, q_2/s_4, r_2/r_4, s_2/q_4, t_2/p_4); \\ \{\beta\}(t_1 > 0, t_2 > 0, t_3 > 0, t_4 > 0) \\ (t_1/t_3, s_1/s_3, r_1/r_3, q_1/q_3, p_1/p_3), \\ (t_2/t_4, s_2/s_4, r_2/r_4, q_2/q_4, p_2/p_4); \\ \{\beta\}(t_1 < 0, t_2 < 0, t_3 < 0, t_4 < 0) \\ (t_1/p_3, s_1/q_3, r_1/r_3, q_1/s_3, p_1/t_3), \\ (t_2/p_4, s_2/q_4, r_2/r_4, q_2/s_4, p_2/t_4); \\ \{\beta\}(t_1 < 0, t_2 < 0, t_3 < 0, t_4 < 0) \end{cases} \quad (1.61)$$

$$e\beta^1_{GHPFN} = < (ep_1, eq_1, er_1, es_1, et_1), (ep_2, eq_2, er_2, es_2, et_2); \{1 - (1-\beta)^e\} > \quad (1.62)$$

$$(\beta^1_{GHPFN})^e = < (p_1^e, q_1^e, r_1^e, s_1^e, t_1^e), (p_2^e, q_2^e, r_2^e, s_2^e, t_2^e); \{\beta)^e\} > \quad (1.63)$$

Score function on GHPFN (1st type):

Definition 10. *Let* $\beta^1_{GHPFN} = <(p_1, q_1, r_1, s_1, t_1), (p_2, q_2, r_2, s_2, t_2); \beta^1>$ *be a GHPFN (1st type). Then the score function of* β^1_{GHPFN} *is defined as*

$$S(\beta^1_{GHPFN}) = \frac{(r_1+r_2)^2 + (s_1+s_2)^2 + (t_1+t_2)^2 - (p_1+p_2)^2 - (q_1+q_2)^2}{3(\#h)} \times \beta^1 \quad (1.64)$$

where $\#h$ denotes the number of elements in β^1_{GHPFN}.

Definition 11. *Let* $\psi = <x, (p_i, q_i, r_i, s_i, t_i); \delta_i : i \in I> : x \in X$ *represents set of different GHPFN. If* $p = p_i, q = q_i, r = r_i, s = s_i, t = t_i \forall i \in I$*, then GPFN is reduced to single valued generalized hesitant pentagonal fuzzy number with different membership value. This form of GHPFN is said to be 2nd type. Symbolically it can be expressed as* $\beta_{1GHPFN} = <(p, q, r, s, t); \beta_i : \beta_i \in \beta(x)>, \beta(x)$ *is a set of some values in* $[0, 1]$*. This is a special form of hesitant fuzzy set on the set of real numbers R. The membership functions are defined as follows:*

$$f^i(x) = \begin{cases} (x-p)\beta_i/(q-p), & p \leq x < q \\ (x-q)\beta_i/(r-q), & q \leq x < r \\ \beta_i, & x = r \\ \beta_i \frac{s-x}{s-r} & \text{for } r \leq x < s \\ \beta_i \frac{t-x}{t-s} & \text{for } s \leq x < t \\ 0 & \text{for Otherwise} \end{cases} \tag{1.65}$$

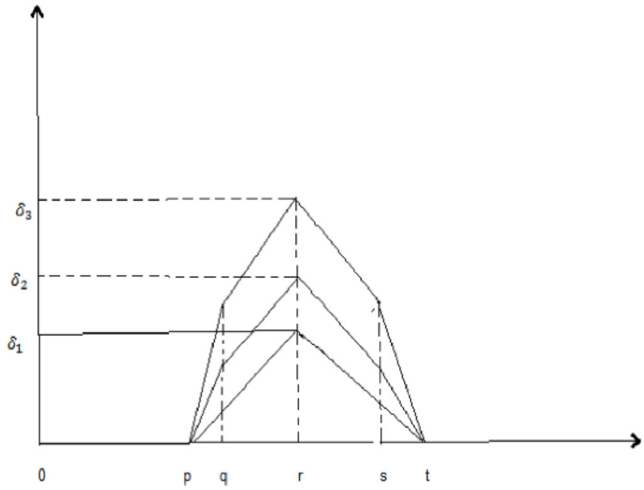

Figure 1.8: Representation of GHPFN 2nd type.

Arithmetic operations and score function on $GHPFN$ **(2nd Type)**

Let $\beta^1_{GHPFN} = <(p_1, q_1, r_1, s_1, t_1); \beta^1>(p_2, q_2, r_2, s_2, t_2)$,
$\beta^2_{GHPFN} = <(p_2, q_2, r_2, s_2, t_2); \beta^1>$ and $\epsilon \geq 0$. Then

$$\beta^1_{GHPFN} + \beta^2_{GHPFN} =<(p_1 + p_2, q_1 + q_2, r_1 + r_2, s_1 + s_2, t_1 + t_2); \bigcup_{\beta^1_1 \in \beta_1, \beta^2_1 \in \beta^2} \{\beta^1_1 + \beta^2_1 - \beta^1_1\beta^2_1\}>; \tag{1.66}$$

$$\beta^1_{GHPFN} - \beta^2_{GHPFN} =<(p_1 - t_2, q_1 - s_2, r_1 - r_2, s_1 - q_2, t_1 - p_2); \bigcup_{\beta^1_1 \in \beta_1, \beta^2_1 \in \beta^2} \{\beta\}>; \tag{1.67}$$

where, $\beta = \begin{cases} \frac{\beta_1^1-\beta_1^2}{1-(\beta_1)^2} \;\; if \;\; \beta_1^1 \geq \beta_1^2 \text{ and } \beta_1^2 \neq 1 \\ 0, \text{ otherwise} \end{cases}$.

$$\beta^1_{GHPFN} \times \beta^2_{GHPFN} = \begin{cases} (p_1p_2, q_1q_2, r_1r_2, s_1s_2, t_1t_2); \\ <\bigcup_{\beta_1^1\in\beta_1^1,\beta_1^2\in\beta^2}\{\beta_1^1\beta_1^2\}>(t_1>0,t_2>0) \\ (p_1t_2, q_1s_2, r_1r_2, s_1q_2, t_1p_2); \\ <\bigcup_{\beta_1^1\in\beta_1^1,\beta_1^2\in\beta^2}\{\beta_1^1\beta_1^2\}>(t_1<0,t_2>0) \\ (t_1t_2, s_1s_2, r_1r_2, q_1q_2, p_1p_2); \\ <\bigcup_{\beta_1^1\in\beta_1^1,\beta_1^2\in\beta^2}\{\beta_1^1\beta_1^2\}>(t_1<0,t_2<0) \end{cases} \tag{1.68}$$

$$\beta^1_{GHPFN} \div \beta^2_{GHPFN} = \begin{cases} (p1/t2, q1/s2, r1/r2, s1/q2, t1/p2); \\ <\bigcup_{\beta_1^1\in\beta_1^1,\beta_1^2\in\beta^2}\{\beta^*\}>(t_1>0,t_2>0) \\ (t1/t2, s1/s2, r1/r2, q1/q2, p1/p2); \\ <\bigcup_{\beta_1^1\in\beta_1^1,\beta_1^2\in\beta^2}\{\beta^*\}>(t_1<0,t_2>0) \\ (t1/p2, s1/q2, r1/r2, q1/s2, p1/t2); \\ <\bigcup_{\beta_1^1\in\beta_1^1,\beta_1^2\in\beta^2}\{\beta^*\}>(t_1<0,t_2<0) \end{cases} \tag{1.69}$$

where,

$$\beta^* = \begin{cases} \beta_1^1/\beta_1^2 \;\; if \;\; \beta_1^1 \leq \beta_1^2 \;\; and \;\; \beta_1^2 \neq 0 \\ 1, \;\; otherwise \end{cases} \tag{1.70}$$

$$e\beta_{GHPFN} =< (ep, eq, er, es, et); \bigcup_{\beta\in\beta(x)} \{1-(1-\beta)^e\} >, \epsilon \geq 0 \tag{1.71}$$

$$(\beta_{GHPFN})^e =< (p^e, q^e, r^e, s^e, t^e); \bigcup_{\beta\in\beta(x)} \{\beta)^e\} >, \epsilon \geq 0 \tag{1.72}$$

Score function on GHPFN (2nd type)

Definition 12. *Let $\beta_{2GHPFN} = <(p,q,r,s,t);\beta(x)>$ be a GHPFN (2nd type). Then the score function of β_{2GHPFN} is defined as*

$$S(\beta_{2GHPFN}) = \frac{(2r)^2 + (s)^2 + (t)^2 - (p)^2 - (q)^2}{3(\#h)} \times \sum \beta \tag{1.73}$$

where $\#h$ denotes the number of elements in β_{2GHPFN}.

1.3.5 *Intuitionistic fuzzy set (IFS)*

A major expansion of the fuzzy theory was carried out by the preface of the conception of intuitionistic fuzzy set (IFS) by Atanassov (1994), which integrated the design to count the "degree of belongingness" and "degree of non-belongingness" as well. In IFS, as an alternative to one membership grade, there is also a non-membership grade found with each element. Further, there is a limit that the sum of these two grades is less or equal to unity. In IFS, the degree of non-belongingness is not independent but it is dependent on the degree of belongingness also.

1.3.5.1 *Pentagonal intuitionistic fuzzy number ($PIFN$)*

Figure 1.9 shows a pictorial representation of PIFN.

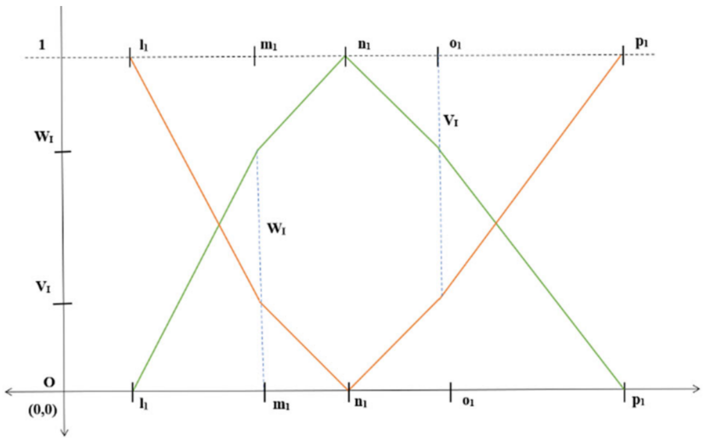

Figure 1.9: Graphical representation of $PIFN$

Definition 13. *A PIFN $< (l_1, m_1, n_1, 0_1, p_1); w_I, v_I >$ is a distinct fuzzy number on the real set $\mathbb{R}$. The membership and non-membership functions of $PIFN$ are defined as follows:*

$$\mu_I(x) = \begin{cases} 0 & for\ \ x \leq l_1 \\ \frac{W(x-l_1)}{(m_1-l_1)} & for\ \ l_1 \leq x \leq m_1 \\ W + \frac{(W_I-0.5)(x-m_1)}{(n_1-m_1)} & for\ \ m_1 \leq x \leq n_1 \\ W_I & for\ \ x = n_1 \\ W + \frac{(W_I-W)(o_1)-x)}{(o_1-n_1)} & for\ \ n_1 \leq x \leq o_1 \\ \frac{W(p_1-x)}{(p_1-o_1)} & for\ \ o_1 \leq x \leq p_1 \\ 0 & for\ \ x \geq p_1 \end{cases} \tag{1.74}$$

$$\nu_I(x) = \begin{cases} 0 & for\ \ x \leq l_1 \\ 1 - \frac{W(x-l_1)}{(m_1-l_1)} & for\ \ l_1 \leq x \leq m_1 \\ \frac{(W-(W-v_I)(x-m_1)}{(n_1-m_1)} & for\ \ m_1 \leq x \leq n_1 \\ v_I & for\ \ x = n_1 \\ \frac{W-(W-v_I)(o_1-x)}{(o_1-n_1)} & for\ \ n_1 \leq x \leq o_1 \\ 1 - \frac{W(p_1-x)}{(p_1-o_1)} & for\ \ o_1 \leq x \leq p_1 \\ 0 & for\ \ x \geq p_1 \end{cases} \tag{1.75}$$

Arithmetic operations on PIFN

Definition 14. *(Nasir and Beenu, 2021) Let us assume two PIFNs* $A =< (l_1, m_1, n_1, o_1, p_1); w_a, v_{a'} >$ *and* $B =< (l_2, m_2, n_2, o_2, p_2); w_b, v_{b'} >$*. Then the arithmetic properties between them are defined as follows:*

$$\begin{aligned}(A + B) =& [l_1 + l_2, m_1 + m_2, n_1 + n_2, o_1 + o_2, p_1 + p_2], \\ & \min(w_a, w_b), \max(v_{a'}, v_{b'});\end{aligned} \tag{1.76}$$

$$\begin{aligned}(A - B) =& [l_1 - p_2, m_1 - 0_2, n_1 - n_2, o_1 - m_2, p_1 - l_2], \\ & \min(w_a, w_b), \max(v_{a'}, v_{b'})\end{aligned} \tag{1.77}$$

$$(A \times B) = [l_1 l_2, m_1 m_2, n_1 n_2, o_1 o_2, p_1 p_2] \tag{1.78}$$

$$\lambda A = [\lambda l_1, \lambda m_1, \lambda n_1, \lambda o_1, \lambda p_1] \tag{1.79}$$

$$(A/B) = \left[\frac{l_1}{p_2}, \frac{m_1}{o_2}, \frac{n_1}{n_2}, \frac{o_1}{m_2}, \frac{p_1}{l_2}\right] \tag{1.80}$$

$$I^{-1} = \left[\frac{1}{p_1}, \frac{1}{o_1}, \frac{1}{n_1}, \frac{1}{m_1}, \frac{1}{l_1}\right] \tag{1.81}$$

Distance measure between two PIFNs

Definition 15. *Let* $A = < (\tilde{l}_1, \tilde{m}_1, \tilde{n}_1, \tilde{o}_1, \tilde{p}_1); w_a, v_{a'} >$ *and* $B =< (\tilde{l}_2, \tilde{m}_2, \tilde{n}_2, \tilde{o}_2, \tilde{p}_2); w_b, v_{b'}$ *be two PIFNs. Then the distance between two PIFNs can be determined by using Hamming distance as*

$$d(A,B) = \left\{ \begin{array}{c} |(1+w_a - v_{a'})\tilde{l}_1 - (1+w_b - v_{b'})\tilde{l}_2| \\ +|(1+w_a - v_{a'})\tilde{m}_1 - (1+w_b - v_{b'})\tilde{m}_2| \\ +|(1+w_a - v_{a'})\tilde{n}_1 - (1+w_b - v_{b'})\tilde{n}_2| \\ +|(1+w_a - v_{a'})\tilde{p}_1 - (1+w_b - v_{b'})\tilde{p}_2| \end{array} \right\} \tag{1.82}$$

1.3.6 *Neutrosophic fuzzy set*

In neutrosophic logic, each proposition is represented by the percentage of occurrence for the truth factor, indeterminacy factor, and falsity factor altogether (Smarandache, 2005; Smarandache and Vladareanu, 2011).

The range of true membership function/T value, indeterminacy membership function/I value, and falsity membership function/F value are a subset of $]^-0, 1^+[$ and the supremum of neutrosophic logic and infimum of neutrosophic logic are $n_{sup} = \sup T + \sup I + \sup F \leq 3$ and $n_{inf} = \inf T + \inf I + \inf F \geq 0$ respectively.

Let X be the universe of discourse with arbitrary subset $A \subset X$ and an arbitrary element $y \in X$ belongs to the set A. Then a neutrosophic set (Smarandache and Vladareanu, 2011; Smarandache 1999; Li et al., 2010) is denoted as $y(t,i,f)$ and described as follows: y is t% true, i% is indeterminate (undetermined, it may or may not be true) and f% false on the basis of the set A subset of X, where the variables $t \in T, i \in I \,\&\, f \in F$.

Here, the subsets of $]^{-}0, 1^{+}[$ are T, I & F, which are not only fixed sets, but also operators/functions calculated on the basis of y and various known/unknown variables. The concept of Neutrosophic sets was given by Smarandache and Vlădăreanu (2005).

Definition 16. *(Smarandache and Vlădăreanu, 1998) Assume X to be a universal set of discourse. Let neutrosophic set N^{+} is presented in the form: $N^{+}(y) = \{<y; T(y), I(y), F(y) : y \in X>\}$, where $T(y)$ is truth, $I(y)$ is indeterminacy and $F(y)$ is falsity functional component of an arbitrary element $y \in X$ with the mapping: $T, I, F : y \rightarrow]0^{-}, 1^{+}[$ and satisfy the following conditions: $0 \leq T(y) \leq 1$, $0 \leq I(y) \leq 1$, $0 \leq F(y) \leq 1$ so that $0 \leq T(y) + I(y) + F(y) \leq 3$.*

Example 2. *If $x(0.7, 0.5, 0.3)$ represents a neutrosophic number with $x \in X$, then the element x is 70% true, 50% indeterminate, and 30% false.*

Consider three real-valued standard category subsets or non-standard category subsets T, I & I of $]^{-}0, 1^{+}[$ with supremum and infimum defined as true $\sup T = t\_\sup$, $\inf T = t\_\inf$, for indeterminate $\sup I = i\_\sup$, $\inf I = i\_\inf$ and for false $\sup F = f\_\sup$, $\inf F = f\_\inf$. Neutrosophic supremum and neutrosophic infimum are $n_{sup} = \sup T + \sup I + \sup F$ and $n_{inf} = \inf T + \inf I + \inf F$, respectively.
Let us consider the universe of discourse denoted by X with arbitrary subset $A \subset X$. An arbitrary element $y \in X$ belongs to the set A. On the basis of the neutrosophic set, denoted as $y(t, i, f)$ and described as follows: y is t% true, i% is indeterminate (undetermined, it may or may not be true) and f% false on the basis of the set A subset of X, where the variables $t \in T, i \in I$ and $f \in F$.
Here, the subsets of $]^{-}0, 1^{+}[$ are T, I, and F, are not only fixed set, but are operators/functions calculated on the basis of y and various known/unknown variables. The concept of Neutrosophic sets was given by Smarandache (1999).

- Single Valued Neutrosophic Set (SVNS) (Wang et al., 2010; Peng et al., 2015)

  Single Valued Neutrosophic Set (SVNS) is a set that consists of element(s) only one element in each membership value. Let $\tilde{\Gamma}(\zeta) = \{\zeta; T_{\Gamma}(\zeta), I_{\Gamma}(\zeta), F_{\Gamma}(\zeta) | \zeta \in \tilde{\Gamma}\}$ be a Neutrosophic Set (Jin et al., 2021) with each element $\zeta \in \tilde{\Gamma}$ and the true membership function $T_{\Gamma}(\zeta)$, indeterminacy membership function $I_{\Gamma}(\zeta)$ and false membership function $F_{\Gamma}(\zeta)$ are unique value for $\zeta \in \tilde{\Gamma}$. The range of membership functions lie between $[0,1]$, i.e., $(0 \leq T_\Gamma(\zeta), I_\Gamma(\zeta), F_\Gamma(\zeta) \leq 1)$.

Example 3. *Let $\tilde{A}_1 = \{x; 1, 0.5, 0.6\}$, $\tilde{A}_2 = \{x; 0.8, 0.3, 0.5\}$ and $\tilde{A}_3 = \{x; 0.7, 0.2, 0\}$ be three SVNSs. Neutrosophic set $\tilde{A}_1$ contains one element x with true membership value 1, indeterminacy membership value 0.5, and false membership value 0.6. Similar rule exists for the set $\tilde{A}_2$ and $\tilde{A}_3$. All these three membership values of the neutrosophic set lie between $[0,1]$.*

Example 4. *Let $\tilde{B}_1 = \{\{y; 0.9, 0.35, 0.42\}, \{z; 0.7, 0.25, 0.15\}\}$ and $\tilde{B}_2 = \{x; 1, 0.1, 0.2\}$ be two SVNSs. Here Neutrosophic set $\tilde{B}_1$ consists of two elements, their three membership values, i.e., true, indeterminacy & falsity membership values of the element $y, z \in \tilde{B}_1$. The true membership value is 0.9 and indeterminacy and falsity membership values are 0.35 and 0.42, respectively for the element $y \in \tilde{B}_1$, and true membership value is 0.7 and indeterminacy and falsity membership values are 0.25 and 0.15, respectively for the element $z \in \tilde{B}_1$. Similarly, for the neutrosophic set $\tilde{B}_2$, the true membership value is 1, and indeterminate and false membership values are 0.1 and 0.2, respectively.*

❖ Trapezoidal Neutrosophic Number (*TrNN*)

Definition 17. *Trapezoidal Neutrosophic Set (TrNS)*
Let $\tilde{\Gamma}(\zeta) = \{(\rho_1, \rho_2, \rho_3, \rho_4; \sigma_1, \sigma_2, \sigma_3, \sigma_4; \phi_1, \phi_2, \phi_3, \phi_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ be TrNS with ζ be an element on it. Then it's true membership function $T_\Gamma(\zeta)$, indeterminacy membership function $I_\Gamma(\zeta)$ and false membership function $F_\Gamma(\zeta)$ are represented respectively as

$$T_\Gamma(\zeta) = \begin{cases} 0 & \text{if } \zeta \leq \rho_1 \\ t_\Gamma \frac{(\zeta - \rho_1)}{\rho_2 - \rho_1} & \text{if } \rho_1 \leq \zeta \leq \rho_2 \\ t_\Gamma & \text{if } \rho_2 \leq \zeta \leq \rho_3 \\ t_\Gamma \frac{(\rho_4 - \zeta)}{\rho_4 - \rho_3} & \text{if } \rho_3 \leq \zeta \leq \rho_4 \\ 0 & \text{if } \rho_4 \leq \zeta \end{cases} \tag{1.83}$$

$$I_\Gamma(\zeta) = \begin{cases} 1 & \text{if } \zeta \leq \sigma_1 \\ \frac{(\sigma_2-\zeta)+i_\Gamma(\zeta-\sigma_1)}{\sigma_2-\sigma_1} & \text{if } \sigma_1 \leq \zeta \leq \sigma_2 \\ i_\Gamma & \text{if } \sigma_2 \leq \zeta \leq \sigma_3 \\ \frac{(\zeta-\sigma_3)+i_\Gamma(\sigma_4-\zeta)}{\sigma_4-\sigma_3} & \text{if } \sigma_3 \leq \zeta \leq \sigma_4 \\ 1 & \text{if } \sigma_4 \leq \zeta \end{cases} \tag{1.84}$$

$$F_\Gamma(\zeta) = \begin{cases} 1 & \text{if } \zeta \leq \phi_1 \\ \frac{(\phi_2-\zeta)+f_\Gamma(\zeta-\phi_1)}{\phi_2-\phi_1} & \text{if } \phi_1 \leq \zeta \leq \phi_2 \\ f_\Gamma & \text{if } \phi_2 \leq \zeta \leq \phi_3 \\ \frac{(\zeta-\phi_3)+f_\Gamma(\phi_4-\zeta)}{\phi_4-\phi_3} & \text{if } \phi_3 \leq \zeta \leq \phi_4 \\ 1 & \text{if } \phi_4 \leq \zeta \end{cases} \tag{1.85}$$

where $0 \leq T_\Gamma(\zeta) \leq 1$, $0 \leq I_\Gamma(\zeta) \leq 1$, $0 \leq F_\Gamma(\zeta) \leq 1$ *and* $0 \leq T_\Gamma(\zeta) + I_\Gamma(\zeta) + F_\Gamma(\zeta) \leq 3$. *Then* $\tilde{\Gamma}(\zeta) = \{(\rho_1,\rho_2,\rho_3,\rho_4;\sigma_1,\sigma_2,\sigma_3,\sigma_4;\phi_1,\phi_2,\phi_3,\phi_4);\ t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ *is TrNS when* $\rho_1,\rho_2,\rho_3,\rho_4 \in \mathbb{R}$; $\sigma_1,\sigma_2,\sigma_3,\sigma_4 \in \mathbb{R}$; $\phi_1,\phi_2,\phi_3,\phi_4 \in \mathbb{R}$ *and* $\rho_1 \leq \rho_2 \leq \rho_3 \leq \rho_4$; $\sigma_1 \leq \sigma_2 \leq \sigma_3 \leq \sigma_4$; $\phi_1 \leq \phi_2 \leq \phi_3 \leq \phi_4$.

Definition 18. *Trapezoidal Neutrosophic Set (TrNS)*
Let $\tilde{\Gamma}(\zeta) = \{(\mu_1,\mu_2,\mu_3,\mu_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ *be a TrNS with* ζ *be an element on it. Then it's true membership function* $T_\Gamma(\zeta)$*, indeterminacy membership function* $I_\Gamma(\zeta)$ *and false membership function* $F_\Gamma(\zeta)$ *are represented respectively as*

$$T_\Gamma(\zeta) = \begin{cases} 0 & \text{if } \zeta \leq \mu_1 \\ t_\Gamma\frac{(\zeta-\mu_1)}{\mu_2-\mu_1} & \text{if } \mu_1 \leq \zeta \leq \mu_2 \\ t_\Gamma & \text{if } \mu_2 \leq \zeta \leq \mu_3 \\ t_\Gamma\frac{(\mu_4-\zeta)}{\mu_4-\mu_3} & \text{if } \mu_3 \leq \zeta \leq \mu_4 \\ 0 & \text{if } \mu_4 \leq \zeta \end{cases} \tag{1.86}$$

$$I_\Gamma(\zeta) = \begin{cases} 1 & if\ \zeta \leq \mu_1 \\ \frac{(\mu_2-\zeta)+i_\Gamma(\zeta-\mu_1)}{\mu_2-\mu_1} & if\ \mu_1 \leq \zeta \leq \mu_2 \\ i_\Gamma & if\ \mu_2 \leq \zeta \leq \mu_3 \\ \frac{(\zeta-\mu_3)+i_\Gamma(\mu_4-\zeta)}{\mu_4-\mu_3} & if\ \mu_3 \leq \zeta \leq \mu_4 \\ 1 & if\ \mu_4 \leq \zeta \end{cases} \tag{1.87}$$

$$F_\Gamma(\zeta) = \begin{cases} 1 & if\ \zeta \leq \mu_1 \\ \frac{(\mu_2-\zeta)+f_\Gamma(\zeta-\mu_1)}{\mu_2-\mu_1} & if\ \mu_1 \leq \zeta \leq \mu_2 \\ f_\Gamma & if\ \mu_2 \leq \zeta \leq \mu_3 \\ \frac{(\zeta-\mu_3)+f_\Gamma(\mu_4-\zeta)}{\mu_4-\mu_3} & if\ \mu_3 \leq \zeta \leq \mu_4 \\ 1 & if\ \mu_4 \leq \zeta \end{cases} \tag{1.88}$$

where $0 \leq T_\Gamma(\zeta) \leq 1$, $0 \leq I_\Gamma(\zeta) \leq 1$, $0 \leq F_\Gamma(\zeta) \leq 1$ *and* $0 \leq T_\Gamma(\zeta) + I_\Gamma(\zeta) + F_\Gamma(\zeta) \leq 3$. *Then* $\tilde{\Gamma}(\zeta) = \{(\mu_1, \mu_2, \mu_3, \mu_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ *is TrNS when* $\mu_1, \mu_2, \mu_3, \mu_4 \in \mathbb{R}$ *and* $\mu_1 \leq \mu_2 \leq \mu_3 \leq \mu_4$.

Definition 17 is generalized definition of *TrNS*.

Note 5. *Geometric presentation of TrNN,* $\tilde{\Gamma}(\zeta) = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ *is shown in Figure 1.10 where* $\beta_1, \beta_2, \beta_3$ *and* β_4 *are first, second, third and fourth entries of the TrNN, respectively and* $t = t_\Gamma(\zeta)$ *for maximum true membership value,* $i = i_\Gamma(\zeta)$ *for maximum indeterminacy membership value and* $f = f_\Gamma(\zeta)$ *for maximum false membership value of TrNN* $\tilde{\Gamma}(\zeta)$.

Example 5. *Let* $\tilde{\Gamma}(\zeta) = \{(2, 3, 4, 5); 0.7, 0.3, 0.5\}$ *be a TrNS with* ζ *be an element on it. Then it's true membership function* $T_\Gamma(\zeta)$*, indeterminacy membership function* $I_\Gamma(\zeta)$ *and false membership function* $F_\Gamma(\zeta)$ *are represented respectively as*

$$T_{\Gamma}(\zeta) = \begin{cases} 0 & \textit{if } \zeta \leq 2 \\ 0.7 \times \frac{(\zeta-2)}{3-2} = 0.7 \times (\zeta - 2) & \textit{if } 2 \leq \zeta \leq 3 \\ 0.7 & \textit{if } 3 \leq \zeta \leq 4 \\ 0.7 \times \frac{(5-\zeta)}{5-4} = 0.7 \times (5 - \zeta) & \textit{if } 4 \leq \zeta \leq 5 \\ 0 & \textit{if } 5 \leq \zeta \end{cases} \tag{1.89}$$

$$I_{\Gamma}(\zeta) = \begin{cases} 1 & \textit{if } \zeta \leq 2 \\ \frac{(3-\zeta)+0.3\times(\zeta-2)}{3-2} = (3 - \zeta) + 0.3 \times (\zeta - 2) & \textit{if } 2 \leq \zeta \leq 3 \\ 0.3 & \textit{if } 3 \leq \zeta \leq 4 \\ \frac{(\zeta-4)+0.3\times(5-\zeta)}{5-4} = (\zeta - 4) + 0.3 \times (5 - \zeta) & \textit{if } 4 \leq \zeta \leq 5 \\ 1 & \textit{if } 5 \leq \zeta \end{cases} \tag{1.90}$$

$$F_{\Gamma}(\zeta) = \begin{cases} 1 & \textit{if } \zeta \leq 2 \\ \frac{(3-\zeta)+0.5\times(\zeta-2)}{3-2} = (3 - \zeta) + 0.5 \times (\zeta - 2) & \textit{if } 2 \leq \zeta \leq 3 \\ 0.5 & \textit{if } 3 \leq \zeta \leq 4 \\ \frac{(\zeta-4)+0.5\times(5-\zeta)}{5-4} = (\zeta - 4) + 0.5 \times (5 - \zeta) & \textit{if } 4 \leq \zeta \leq 5 \\ 1 & \textit{if } 5 \leq \zeta \end{cases} \tag{1.91}$$

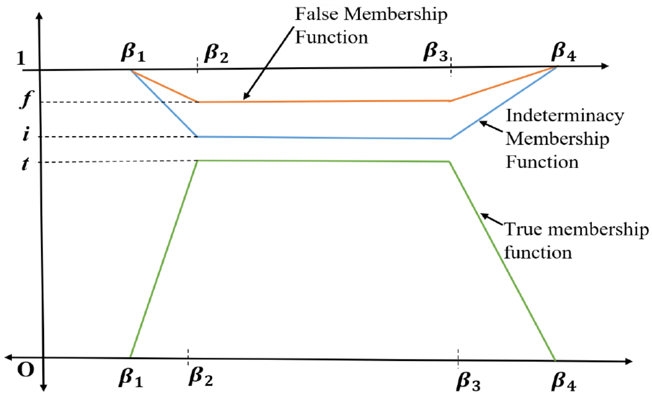

Figure 1.10: Geometric representation of a *TrNN*.

Arithmetic operations on *TrNN*

Let $\tilde{\Gamma} = \{(\sigma_1, \sigma_2, \sigma_3, \sigma_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ and $\tilde{\Lambda} = \{(\tau_1, \tau_2, \tau_3, \tau_4); t_\Lambda, i_\Lambda, f_\Lambda\}$ be two single-valued *TrNNs* where t_ξ denotes true membership function, i_ξ denotes indeterminacy membership function and f_ξ denotes false membership function where $\xi \in \{\Gamma, \Lambda\}$. Then arithmetic operations on *TrNN* are defined as

$$\tilde{\Sigma} = \tilde{\Gamma} \oplus \tilde{\Lambda} = \left\{ \begin{array}{l} (\sigma_1 + \tau_1, \sigma_2 + \tau_2, \sigma_3 + \tau_3, \sigma_4 + \tau_4); \\ t_\Gamma + t_\Lambda - t_\Gamma t_\Lambda,\ i_\Gamma i_\Lambda,\ f_\Gamma f_\Lambda \end{array} \right\}$$

$$\tilde{N} = -\tilde{\Gamma} = \{(-\sigma_4, -\sigma_3, -\sigma_2, -\sigma_1); 1 - t_\Gamma, 1 - i_\Gamma, 1 - f_\Gamma\} \tag{1.92}$$

$$\tilde{\Omega} = \tilde{\Gamma} \ominus \tilde{\Lambda} = \left\{ \begin{array}{l} (\sigma_1 - \tau_1, \sigma_2 - \tau_2, \sigma_3 - \tau_3, \sigma_4 - \tau_4); \\ 1 - t_\Lambda(1 - t_\Gamma), i_\Gamma(1 - i_\Lambda), f_\Gamma(1 - f_\Lambda) \end{array} \right\} \tag{1.93}$$

$$\tilde{M} = k \times \tilde{\Gamma} = \left\{ (k\sigma_1, k\sigma_2, k\sigma_3, k\sigma_4); 1 - (1 - t_\Gamma)^k,\ i_\Gamma^k,\ f_\Gamma^k \right\} \tag{1.94}$$

$$\tilde{\Pi} = \tilde{\Gamma} \otimes \tilde{\Lambda} = \left\{ \begin{array}{l} (\sigma_1\tau_1, \sigma_2\tau_2, \sigma_3\tau_3, \sigma_4\tau_4); \\ t_\Gamma t_\Lambda,\ i_\Gamma + i_\Lambda - i_\Gamma i_\Lambda,\ f_\Gamma + f_\Lambda - f_\Gamma f_\Lambda \end{array} \right\}$$

$$\begin{aligned} \tilde{I} &= \tilde{\Gamma}^{-1} = \frac{1}{\{(\sigma_1, \sigma_2, \sigma_3, \sigma_4); t_\Gamma, i_\Gamma, f_\Gamma\}} \\ &= \left\{ \left(\frac{1}{\sigma_4}, \frac{1}{\sigma_3}, \frac{1}{\sigma_2}, \frac{1}{\sigma_1} \right); t_\Gamma, i_\Gamma, f_\Gamma \right\} \end{aligned} \tag{1.95}$$

Distance between Two *TrNNs*

Distance measuring between two neutrosophic numbers plays a significant role in MCDM techniques. It gives an idea of ranking of the alternatives.

Definition 19. *(Hamming distance)*

Let $\tilde{\Gamma} = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ *and* $\tilde{\Lambda} = \{(\tau_1, \tau_2, \tau_3, \tau_4); t_\Lambda, i_\Lambda, f_\Lambda\}$ *be two TrNNs. Then the Hamming distance between* $\tilde{\Gamma}$ *and* $\tilde{\Lambda}$ *is denoted by* $d(\tilde{\Gamma}, \tilde{\Lambda})$ *and defined by*

$$d(\tilde{\Gamma}, \tilde{\Lambda}) = \left\{ \begin{array}{c} |\beta_1(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_1(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_2(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_2(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_3(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_3(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_4(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_4(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \end{array} \right\} \tag{1.96}$$

Definition 20. *(Normalized Hamming distance)*

Let $\tilde{\Gamma} = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ *and* $\tilde{\Lambda} = \{(\tau_1, \tau_2, \tau_3, \tau_4); t_\Lambda, i_\Lambda, f_\Lambda\}$ *be two TrNNs. Then the normalized Hamming distance between* $\tilde{\Gamma}$ *and* $\tilde{\Lambda}$ *is denoted by* $d_N(\tilde{\Gamma}, \tilde{\Lambda})$ *and defined by*

$$d_N(\tilde{\Gamma}, \tilde{\Lambda}) = \frac{1}{12} \times \left\{ \begin{array}{c} |\beta_1(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_1(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_2(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_2(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_3(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_3(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_4(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_4(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \end{array} \right\} \tag{1.97}$$

De-neutrosophication of neutrosophic number

Let $\tilde{\Gamma} = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ be a *TrNN* with $0 \leq \beta_1 \leq \beta_2 \leq \beta_3 \leq \beta_4 \leq 1$ and $0 \leq t_\Gamma, i_\Gamma, f_\Gamma \leq 1$. Then de-neutrosophication of neutrosophic number $\tilde{\Gamma}$ is denoted by by $\mathcal{N}(\tilde{\Gamma})$ and defined by

$$\mathcal{N}(\tilde{\Gamma}) = \frac{1}{10}(\beta_1 + \beta_2 + \beta_3 + \beta_4) \times (2 + t_\Gamma - i_\Gamma - f_\Gamma) \tag{1.98}$$

Note: We have constructed the formula in Equation (1.98) in a new way. Basically, the method stands for the transformation of a *TrNN* to a crisp number. In a similar way, we may construct de-neutrosophication of other numbers, such as triangular neutrosophic numbers, pentagonal neutrosophic numbers, and hexagonal neutrosophic numbers.

1.3.7 Pythagorean fuzzy number ($PyFN$)

Definition 21. *(Yager, 2013) Let X be a universe of discourse. Then a Pythagorean fuzzy set defined on X is of the form*

$$P = \{< \theta, \mu_p(\theta), \nu_p(\theta) > | \theta \in X\}$$

where $\mu_p : X \to [0,1]$ *and* $\nu_p : X \to [0,1]$ *are membership and non-membership functions, respectively and satisfy the following relation:*

$$0 \leq (\mu_p(\theta))^2 + (\nu_p(\theta))^2 \leq 1 \quad \forall\ \theta \in X$$

Then the degree of indeterminacy membership $\pi_p(\theta)$ *is defined as*

$$\pi_p(\theta) = \sqrt{1 - (\mu_p(\theta))^2 - (\nu_p(\theta))^2} \tag{1.99}$$

Zhang and Xu (2014) considered $\beta =< \mu_p, \nu_p >$ as a Pythagorean fuzzy number(PFN) where $\mu_p \in [0,1]$ and $\nu_p \in [0,1]$ are membership and non-membership values, respectively and $\pi_p = \sqrt{1 - \mu_p^2 - \nu_p^2}$ and $0 \leq \mu_p^2 + \nu_p^2 \leq 1$.

Some basic operations on $PyFN$

Definition 22. *(Zhang and Xu, 2014) Let* $\alpha_1 =< \mu_{p_1}, \nu_{p_1} >$ *and* $\alpha_2 =< \mu_{p_2}, \nu_{p_2} >$ *be two PyFNs. Then the ordering between these two PyFNs is described as* $\alpha_1 \geq \alpha_2 \Leftrightarrow \mu_{p_1} \geq \mu_{p_2}$ *and* $\nu_{p_1} \leq \nu_{p_2}$

Let $\beta_1 =< \mu_{a_1}, \nu_{a_1} >$ and $\beta_2 =< \mu_{a_2}, \nu_{a_2} >$ be two PFNs. Zhang and Xu (2014) defined the following operations:

$$\beta_1 \cup \beta_2 = < \max\{\mu_{a_1}, \mu_{a_2}\}, \min\{\nu_{a_1}, \nu_{a_2}\} > \quad (1.100)$$

$$\beta_1 \cap \beta_2 = < \min\{\mu_{a_1}, \mu_{a_2}\}, \max\{\nu_{a_1}, \nu_{a_2}\} > \quad (1.101)$$

$$\beta_1^c = < \nu_{a_1}, \mu_{a_1} > \quad (1.102)$$

$$\beta_1 \oplus \beta_2 = < \sqrt{\mu_{a_1}^2 + \mu_{a_2}^2 - \mu_{a_1}^2 \mu_{a_2}^2}, \nu_{a_1}\nu_{a_2} > \quad (1.103)$$

$$\beta_1 \otimes \beta_2 = < \mu_{a_1}\mu_{a_2}, \sqrt{\nu_{a_1}^2 + \nu_{a_2}^2 - \nu_{a_1}^2 \nu_{a_2}^2} > \quad (1.104)$$

$$\delta\beta_1 = < \sqrt{1-(1-\mu_{a_1}^2)^\delta}, (\nu_{a_1})^\delta >, \delta > 0 \quad (1.105)$$

$$\beta_1^\delta = < (\mu_{a_1})^\delta, \sqrt{1-(1-\nu_{a_1}^2)^\delta} >, \delta > 0 \quad (1.106)$$

Definition 23. *(Zhang and Xu, 2014) Let $\alpha_1 =< \mu_{p_1}, \nu_{p_1} >$ and $\alpha_2 =< \mu_{p_2}, \nu_{p_2} >$ be two PFNs. Then the ordering between these two PFNs is described as follows:*

$$\alpha_1 \geq \alpha_2 \Leftrightarrow \mu_{p_1} \geq \mu_{p_2} \text{ and } \nu_{p_1} \leq \nu_{p_2}$$

Definition 24. *(Zhang and Xu, 2014) If $\alpha =< \mu, \nu >$ be a PFN then the score function of $\alpha =< \mu, \nu >$ is denoted by $s(\alpha)$ and is defined by*

$$s(\alpha) = \mu^2 - \nu^2 \quad (1.107)$$

The following propositions hold:

- $s(\alpha) \in [-1, 1]$
- *For two PFNs $\alpha_1 =< \mu_{p_1}, \nu_{p_1} >$ and $\alpha_2 =< \mu_{p_2}, \nu_{p_2} >$, if $s(\alpha_1) > s(\alpha_2)$ then $\alpha_1 > \alpha_2$.*

Definition 25. *(Zhang and Xu, 2014) Let $\alpha_1 =< \mu_{p_1}, \nu_{p_1} >$ and $\alpha_2 =< \mu_{p_2}, \nu_{p_2} >$ be two PFNs and $s(\alpha_1)$ and $s(\alpha_2)$ be the score values of α_1 and α_2, respectively. Then the following relations hold for the two PFNs:*

- $s(\alpha_1) < s(\alpha_2) \Rightarrow \alpha_1 \prec \alpha_2$

- $s(\alpha_1) > s(\alpha_2) \Rightarrow \alpha_1 \succ \alpha_2$
- $s(\alpha_1) = s(\alpha_2) \Rightarrow \alpha_1 \sim \alpha_2$, *where* $\alpha_1 \sim \alpha_2$ *means that* α_1 *and* α_2 *are not comparable.*

The membership value of IFN satisfies $0 \leq \mu_I + \nu_I \leq 1$, whereas the membership value of PFN satisfies $0 \leq \mu_P^2 + \nu_P^2 \leq 1$. Yager (2013), showed that the space of intuitionistic fuzzy membership grade is a subspace of the space of Pythagorean membership grade, Therefore, every IFN is PFN but the converse is not true. With this advantage, the decision maker can express preference values of alternatives in a more flexible way with PFN than IFN.

1.3.7.1 *Hexagonal pythagorean fuzzy set* ($HPyFS$)

Definition 26. *(Xian et al., 2018) Let* $\tilde{S} = [s_1, s_2, s_3, s_4; \mu_i, \nu_i]$ *where* $s_1, s_2, s_3, s_4 \in X, s_1 \leq s_2 \leq s_3 \leq s_4$, μ_i *and* ν_i *denote the maximum degree of membership and non-membership, respectively satisfying* $0 \leq \mu_i \leq 1$, $0 \leq \nu_i \leq 1$ *and* $0 \leq \mu_P^2 + \nu_P^2 \leq 1$. *Then this set is called Trapezoidal Pythagorean fuzzy set (TPyFS).*

We extend this definition of $TPFS$ to $HPyFS$.

Definition 27. *Let* $\tilde{A} = [a_1, a_2, a_3, a_4, a_5, a_6; \mu_1, \nu_1]$ *where* $a_1, a_2, a_3, a_4, a_5, a_6 \in X$, *such that* $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6$. *The membership and non-membership values satisfy* $0 \leq \mu_P^2 + \nu_P^2 \leq 1$. *This set is known as hexagonal Pythagorean fuzzy set.*

Arithmetic operations on $HPyFN$

Let $\tilde{A} = [a_1, a_2, a_3, a_4, a_5, a_6; \mu_1, \nu_1]$ and $\tilde{B} = [b_1, b_2, b_3, b_4, b_5, b_6; \mu_2, \nu_2]$ be two $HPyFNs$, and $\delta(\geq 0)$ be a scalar. Then

$$\tilde{A} + \tilde{B} = \begin{bmatrix} a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6; \\ \sqrt{\mu_1^2 + \mu_2^2 - \mu_1\mu_2}, \nu_1\nu_2 \end{bmatrix} \tag{1.108}$$

$$\tilde{A} \times \tilde{B} = \begin{bmatrix} a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6; \\ \mu_1\mu_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1\nu_2} \end{bmatrix} \tag{1.109}$$

$$a\tilde{A} = [aa_1, aa_2, aa_3, aa_4, aa_5, aa_6; \sqrt{1 - (1 - \mu_1^2)^a}, \nu_1^a] \tag{1.110}$$

$$\tilde{A}^a = [a_1^a, a_2^a, a_3^a, a_4^a, a_5^a, a_6^a; \mu_1^a, \sqrt{1 - (1 - \nu_1^2)^a}] \tag{1.111}$$

Using the above set of operations, one can easily check and verify that the followings laws hold.

- $\tilde{A} \oplus \tilde{B} = \tilde{B} \oplus \tilde{A}$
- $\tilde{A} \otimes \tilde{B} = \tilde{B} \otimes \tilde{A}$
- $\beta(\tilde{A} \oplus \tilde{B}) = \beta\tilde{A} \oplus \beta\tilde{B},\ \beta > 0$
- $\beta_1\tilde{A} \oplus \beta_2\tilde{A} = (\beta_1 \oplus \beta_2)\tilde{A},\ \beta_1, \beta_2 > 0$
- $(\tilde{A} \otimes \tilde{B})^{\beta} = \tilde{A}^{\beta} \otimes \tilde{B}^{\beta},\ \beta > 0$
- $\tilde{A}_1^{\beta_1} \otimes \tilde{A}_1^{\beta_2} = \tilde{A}_1^{(\beta_1+\beta_2)},\ \beta_1, \beta_2 > 0$

1.4 Scope of the Study

The aim of this doctoral study is to focus on real life multi-criteria decision making problems in fuzzy environment. The research deals with finding the best possible alternatives or ranking of the alternatives when set of multiple conflicting criteria or sub-criteria are involved. To effectively manage the inherent uncertainty and hesitation in decision-making, diverse forms of fuzzy number have been employed, each adapted to the environmental context relevant to the specific problem. Consideration of criteria, sub-criteria and alternatives has been done in such a way that it brings idea or knowledge to the readers. This thesis deals with MCDM problems where impreciseness of data is prevailing and there is uncertainty and hesitancy in the mind of the decision maker for making choices. The methodology evolved through the research work is capable of addressing such imprecise, uncertain and hesitant environment. In the domains of medical science and cloud computing, data analytics and multi-criteria decision-making (MCDM) play a pivotal role. This research investigates these areas and proposes problem-specific solutions tailored to their unique challenges. Among the critical factors in project planning, site selection stands out as particularly important. Accordingly, this study includes three dedicated chapters that address site selection cases across the sectors of commerce, energy infrastructure, and education.

1.5 Framework of the Thesis

The thesis is organized into ten chapters, each summarized below to provide an overview of the overall structure and content.

Chapter 1: ***Introduction***

Being an introductory chapter, it presents a concise overview of decision-making and multi-criteria decision-making (MCDM). It also summarizes various MCDM tools commonly employed by researchers to address real-world problems. The concept of fuzzy set theory is introduced, along with a discussion on the significance of integrating fuzzy logic with MCDM techniques, as relevant to the objectives of this thesis.

Chapter 2: ***Literature review***

This chapter presents a concise review of the literature relevant to the study.

Chapter 3: ***AHP-TOPSIS inspired shopping mall site selection problem with fuzzy data***

Site selection is an important issue as the optimal selection involves several complex factors and sub-factors for a successful investment venture. The MCDM techniques AHP and TOPSIS, integrated with fuzzy logic, have been employed to address the problem of shopping mall site selection. AHP is used to determine the crisp weights of the evaluation factors, while the imprecise linguistic terms provided by decision-makers are translated into fuzzy values. To better capture decision-makers' hesitancy, Triangular Fuzzy Numbers (TFNs) are utilized. The fuzzy weights of sub-factors, derived using Fuzzy AHP (FAHP), are then incorporated into Fuzzy TOPSIS (FTOPSIS) to rank the alternatives. This study considers seven main factors and seventeen sub-factors. Data were collected from various locations with the assistance of municipal authorities and architects.

Chapter 4: ***Identifying dominant risk factors involved in spread of COVID-19 using hesitant fuzzy MCDM methodology***

This chapter explores the risk factors associated with the COVID-19 pandemic. The objective of this study is to evaluate and rank the key factors contributing to the spread of the virus using MCDM techniques. FAHP is employed to determine the relative weights of the identified risk factors, followed by the application of Hesitant

Fuzzy Sets (HFS) integrated with TOPSIS to prioritize them. Additionally, a sensitivity analysis is conducted to assess the robustness and reliability of the applied methodology.

Chapter 5: ***Application of hexagonal fuzzy MCDM methodology for site selection of electric vehicle charging station***
This chapter introduces a centroid-based method for the defuzzification and distance measurement between two Hesitant Fuzzy Numbers (HFNs). Geographic Information Systems (GIS) are integrated with MCDM techniques to facilitate the site selection of electric vehicle charging stations. A practical example is considered to demonstrate the applicability and effectiveness of the proposed model.

Chapter 6: ***Selection of cloud service providers using MCDM methodology under intuitionistic fuzzy uncertainty***
Cloud computing has experienced remarkable growth over the past decade. With a wide range of Cloud Service Providers (CSPs) now available, selecting the most suitable option has become challenging. This chapter highlights on choosing the optimal CSP based on multiple criteria and specific organizational or individual requirements. The study employs Pentagonal Intuitionistic Fuzzy Numbers (PIFNs) in conjunction with the MCDM techniques AHP and TOPSIS. Initially, the weights of the evaluation criteria are determined using AHP under the PIFN environment. These weights are then used in the Fuzzy TOPSIS (FTOPSIS) method to derive the final ranking of CSPs. Moreover, sensitivity and comparative analyses are conducted to evaluate the stability and reliability of the resulting rankings.

Chapter 7: ***Optimal site selection for women university using neutrosophic MCDM technique***
The selection of a site for establishing a new university involves evaluating multiple criteria. Certain factors that may be less critical for a co-educational university become significantly more relevant when selecting a site for a women's university. This chapter presents a decision-making model specifically designed for identifying optimal location for a women's university, incorporating various uncertainties inherent in the site selection process. Ten important criteria are chosen for the selection of the site. To capture the uncertainty of the problem, $TrNNs$ are used along with the MCDM tool AHP for obtaining criteria weights. Finally, the TOPSIS and

COPRAS are applied for ranking of the alternatives. Comparative and sensitivity analysis are conducted to check the robustness and consistency of the techniques used.

Chapter 8: ***Evaluation of the treatment options for COVID-19 patients using generalized hesitant fuzzy MCDM technique***
This chapter focuses on the evaluation and ranking of treatment options for COVID-19 using a MCDM approach. To support this process, a Generalized Hexagonal Pythagorean Fuzzy Number (GHPFN) framework is developed, and its properties are thoroughly demonstrated. Two types of GHPFNs are considered in the study. The second type of GHPFN is integrated with the TOPSIS method to rank the available COVID-19 treatment options effectively.

Chapter 9: ***Identification of dominant factor of cardiac arrest by DEMATEL and AHP methods under Pythagorean fuzzy environment***
Cardiac arrest has emerged as a significant health concern, particularly among the younger population, and is associated with a high mortality rate. This chapter focuses on identifying and evaluating the key factors contributing to out-of-hospital cardiac arrest. Determining the weights of these factors is essential for public awareness and informing preventive strategies. This chapter employs MCDM techniques, specifically AHP and DEMATEL, to assess the importance of each factor and explore their interrelationships. To effectively address the inherent uncertainty and hesitancy in expert evaluations, Pythagorean Fuzzy Sets (PFS) are integrated into the analysis. Furthermore, Hexagonal Pythagorean Fuzzy Numbers (HPFNs), an extension of PFS, are utilized due to their enhanced ability to manage complex real-world decision-making scenarios.

Chapter 10: ***Conclusion***
This chapter provides a summary of the key findings and contributions of the thesis. It highlights the methodologies employed, the outcomes derived from the application of MCDM techniques in fuzzy environment, and the practical implications of the research across various real-world scenarios. It also outlines several scopes for future research.

2 Literature Review

This chapter presents a concise literature review of various fuzzy numbers and fuzzy MCDM techniques, highlighting their application across different domains such as site selection, COVID-19 treatment option evaluation, and cloud service provider selection, as explored in the subsequent chapters of this thesis.

2.1 Fuzzy numbers

The concept of fuzzy set was first introduced by Zadeh (1996) and through this pioneering work a new area of uncertainty modeling with fuzzy theory has opened up. From Fuzzy set, the idea of fuzzy number (Dubois 1980; Dubois and Prade, 1993) evolved which has become a tool to represent uncertain parameter where the data is not precisely known due to measurement difficulty, system behavior or lack of reliable data. There exists literature where researchers used different types of fuzzy number in their theoretical framework: triangular fuzzy number (Buckley, 1988; Ghorui et al., 2020), trapezoidal fuzzy number (chen and Chen, 2007; Abbasbandy & Hajjari, 2009), Pentagonal fuzzy number (Mondal and Mandal, 2017; Chakraborty et al., 2019), hexagonal fuzzy number (Chakraborty et al., 2021; Parveen and Kamble, 2020), heptagonal fuzzy number (Mhaske & Bondar, 2020; Maity et al., 2020), type 2 fuzzy number (Mitchell, 2006; Mazandarani and Najariyan, 2014), etc.

- *Generalized Fuzzy Number (GFN)*

  In fuzzy sets,the maximum value for degree of belongingness is 1. But this maximum degree of membership may vary from one decision maker to another if more than one decision maker is involved. In that context, the concept of generalized fuzzy set concept comes. Suppose that DM1 opined that in a pond there is about 100 big fish but he is not fully confident (he give his confident level as 0.9 in $[0,1]$ scale), in same question, DM2 replies that there is about 100 big fish but he is not fully confident (he give his confident level as 0.8 in $[0,1]$ scale). So for DM1, recommended fuzzy number is like $100; \omega \in [0,0.9]$, and for DM2, it is $100; \omega \in [0,0.8]$. The generalized fuzzy number was first defined by Chen (1999). It has many application fields such as management science (Kaufmann and Gupta, 1988), ranking (Vincent et al., 2013; Molinari, 2016; Rouhparvar and Panahi, 2015), fault diagnosis (Zuo et al., 2013), risk analysis (Chen and Chen, 2003, 2007; Chen et al., 2012), transportation problems (Ebrahimnejad, 2014), decision making (Vincent et al., 2013), and pattern recognition (Yong et al., 2004) etc. For better understanding, the readers are referred to the articles by (Amirfakhrian and Yeganehmanesh, 2018; Vincent et al., 2013)

❖ *Hesitant Fuzzy Number* (HFN)

Torra and Narukawa (2013) first proposed the idea of the Hesitant Fuzzy Set (HFS), where the membership degree value of an element for given fuzzy set, having different values. The author also gave brief idea how hesitant fuzzy set is connected with the other extended fuzzy sets concept like intuitionistic fuzzy set (Atanassov 2016; Mondal et al., 2019), type-2 fuzzy set (Dubois, 1980; Zadeh, 1975) and fuzzy multi-set (Yager, 2015). In decision making theory, Hesitant fuzzy number can be a good choice to deal with real-life problems. Several MCDM methods are tagged with hesitant fuzzy uncertainty such as HF-AHP (Kahraman et al., 2016; Öztaysi et al., 2015), HF-TOPSIS (Xu and Zhang, 2013; Li, 2014), HF-MOORA (Shouzen et al., 2013; Li 2014), HF-COPRAS (Zheng et al., 2018; Peng et al., 2017), HF-VIKOR (Liao and Xu, 2013; Ren et al., 2017) etc.

❖ **Intuitionistic Fuzzy Number (**IFN**)**

Unlike a classic fuzzy number where only the membership degree ($\mu(x)$) is defined, intuitionistic fuzzy numbers allow explicit representation of non-membership and hesitation, making them more flexible in modeling uncertainty. There exist different types of intuitionistic fuzzy numbers such as triangular intuitionistic fuzzy number (Mondal and Roy, 2014), generalized trapezoidal intuitionistic fuzzy number (Mondal and Roy, 2015), cloud-type intuitionistic dense fuzzy number (Maity et al., 2020), Interval-valued intuitionistic fuzzy number (Mondal 2018), Pentagonal intuitionistic fuzzy number (Mondal et al., 2018a), non-linear intuitionistic fuzzy number (Mondal et al., 2019) etc. Xu et al. (2018) applied Delphi method with intuitionistic fuzzy numbers for evaluating the comprehensive product quality for customer satisfaction. Wang et al. (2021) proposed interval 2- tuple linguistic intuitionistic fuzzy numbers and regret theory for scheme selection of design for disassembly (DFD) based on sustainability. Maiti et al. (2020) used a triangular intuitionistic fuzzy number and proposed defuzzification technique. Giri et al. (2021) considered non-linear intuitionistic fuzzy numbers and developed a possibilistic mean by the possibility measure. The implementation of the proposed arithmetic operations is explained by taking a case study of the inventory model. Zhang (2018) used interval-valued intuitionistic fuzzy numbers and introduced schweizer-sklari-norm and schweizer-sklari-conorm. Alshammari et al. (2020) applied the concept of residual series algorithm to depict the approximate result to non-linear fuzzy Duffling oscillator. Sahu et al. (2020) used hesitant fuzzy sets with MCDM tool AHP and TOPSIS for evaluating and estimating the durability of web application.

2.2 Fuzzy MCDM approaches

MCDM makes the decision on the basis of multiple criteria. The maker of the decision is supposed to choose between the non-quantifiable multiple criteria and quantifiable multiple criteria. The preference of the decision-makers (DMs) are of the foremost importance since the options are very close and their personal preferences plays an important role in finding out the optimal solution. In order to tackle the uncertainties of the data, fuzzy methods can be integrated with the MCDM methods. Cheng et al. (2002) solved a landfill selection problem using a fuzzy MCDM approach. Önüt and Soner (2008) applied the fuzzy TOPSIS approach for transshipment site selection in Istanbul,Turkey. Aktas and Kabak (2016) used MCDM on hesitant fuzzy linguistic terms for site selection of wind turbines. Senvar et al. (2016) applied an integrated approach of HFSs to TOPSIS to select the best hospital site in Istanbul. Wang et al. (2018) used hybrid Fuzzy AHP (FAHP) to calculate the weights of varying locations in Vietnam and then TOPSIS was applied for ranking of the alternatives. Table 2.1 presents various types of site selection problems addressed using different methodologies. Table 2.2 outlines some theoretical developments related to MCDM tools in fuzzy environments that have emerged in the last few years.

Table 2.1: Different types of site selection problems with solution methodologies.

Author(s)	Types of Site	MCDM Method
Onut et al. (2010)	Shopping center site	Fuzzy AHP and TOPSIS
Zhou et al. (2012)	Biofuel refinery location	Fuzzy TOPSIS
Kharat et al. (2016)	Landfill site	Fuzzy AHP and fuzzy TOPSIS
Wang et al. (2018)	Renewable energy plants location	Fuzzy AHP and TOPSIS
Wang et al. (2018)	Solar power plant location	DEA, fuzzy AHP and TOPSIS
Karasan et al. (2020)	Electric vehicles charging stations	AHP, DEMATEL and TOPSIS
Rezaeisabzevar et al. (2020)	Landfill site	AHP, TODIM and ANP
Boyacı and Şişman (2022)	Pandemic hospital location	Fuzzy AHP and TOPSIS
Mozaffari et al. (2023)	Landfill site	GIS, fuzzy AHP and SAW
Zewdie and Yeshanew (2023)	Waste disposal site	GIS and fuzzy AHP
Çolak (2024)	Wind power plant site	fuzzy AHP and DEMATEL
Yaman (2024)	Onshore wind farm site	GIS and Fuzzy AHP
Topaloğlu (2024)	Facility location	Fuzzy AHP and SSM

Table 2.2: Different fuzzy MCDM techniques and applications

Author(s)	MCDM Method	Application Area
Zain (2018)	Fuzzy TOPSIS	Evaluation of the quality of online information on breast cancer
Chattopadhyay and Bose (2018)	Fuzzy TOPSIS	Supplier selection
Oo and Hnin (2019)	Fuzzy AHP, TOPSIS	Destination selection
Singh et al. (2020)	Fuzzy TOPSIS	Raw material selection in pulp and paper making industry
Lata et al. (2021)	Fuzzy TOPSIS	Selection of machine tool
Basaran and Homsi (2022)	Fuzzy TOPSIS	Mobile mathematics learning application selection
Alhakami (2023)	Fuzzy TOPSIS	Risk Evaluation in Energy Management and Control Systems

The COVID-19 infection has caused severe respiratory illness syndrome which led to the admission of critical patients largely to the ICU and high mortality among people with comorbidity has been observed. Anticipation and early detection of pandemic trend enable administration to formulate strategies and managerial decision making. Evaluation of different treatment options depends on several criteria, hence MCDM based optimization technique is an useful tool in ranking the treatment options. Yang et al. (2020) used MCDM technique along with algorithm in spherical normal fuzzy environment (SpNoF) for the selection of face mask to protect COVID-19 disease. Mardani et al. (2020) used SWARA and WASPAS with Fuzzy HFS for evaluation and ranking the complexity challenges of Digital Technology (DT) intervention to control the COVID-19 pandemic. Özkan et al. (2021) considered fuzzy AHP along with multi-objective optimization method by ratio analysis for intensive care unit admission of patients in the era of pandemic.

A neutrosophic set is an advanced mathematical concept introduced by Florentin Smarandache as a generalization of fuzzy sets and intuitionistic fuzzy sets. It is designed to better model uncertainty, imprecision, vagueness, and incomplete or inconsistent information. Neutrosophic Sets are valuable in MCDM because they provide a more comprehensive framework to handle these complexities. A few real-life problems are treated with MCDM methodology combined with neutrosophic set. Table 2.3 provides a brief review based on the theoretical developments and applications of neutrosophic MCDM technique in the last five years.

Cloud computing technology (CCT) is a popular subject matter among researchers and industries. CCT has become a possible choice for businesses and personal services to replace the on-premise IT infrastructure. This technology makes the information technology (IT) industries in a remarkable infrastructure and application-oriented services on an online subscription basis ideology. It has changed the understanding of how to acquire computing resources with much adaptability, accessibility, and less organizational effort (Zadeh, 2020). Many research works apply the fuzzy logic methodology with MCDM in cloud computing problem to address different types of research challenges and issues with proper solution. Table 2.4 reviews various works conducted on cloud computing services using fuzzy MCDM methodology.

Table 2.3: Neutrosophic MCDM techniques along with application area

Author(s)	Neutrosophic Set/Number	MCDM Method	Application Area
Ren (2018)	SVNS	Prioritized Weighted Geometric (SVNPWG) operator based MCDM	Selection of an investment company
Garg and Nancy (2018)	SVNN	Prioritized MuirheGarg and Nancy (2018)ad Mean based MCDM	Finding an appropriate IT software company
Nabeeh et al. (2019)	Triangular neutrosophic numbers	AHP	Selection of IoT based enterprises
Wang et al. (2019)	Interval neutrosophic sets	Improved cosine similarity measure based MCDM method	Supplier selection
Zeng et al. (2020)	SVNS	Correlation based TOPSIS	Finding an appropriate IT software company
Hezam et al. (2021)	Generalized triangular neutrosophic number	Neutrosophic AHP-TOPSIS	Prioritized peoples group selection for vaccine
Jafar et al. (2021)	Neutrosophic hypersoft sets	Similarity measures based MCDM	Renewable energy source selection
Rani et al. (2021)	SVNS	SWARA & CoCoSo	Renewal energy source selection
Abdullah et al. (2021)	SVNS	DEMATEL	Identification of influential criteria in sub-contractors selection
Elhosiny et al. (2021)	SVNS	TOPSIS, PROMETHEE	Selection of wind energy power plant location
Deveci et al. (2021)	Type 2 Neutrosophic sets	MABAC	Site selection of offshore wind farm location
Duong and Thao (2021)	Entropy based neutrosophic numbers	TOPSIS	Market segment selection and evaluation
Ye et al. (2022)	Neutrosophic enthalpy set	Aggregation operator and score function based MCDM method using algebraic, Einstein t-norms and t-conorms	Car selection
El et al. (2023)	Neutrosophic Environment	AHP	Health assessment in COVID-19 pandemic
Ali (2023)	Triangular Neutrosophic number	TOPSIS	Renewable energy alternatives selection
Alshehri (2023)	Type-2 neutrosophic numbers	AHP and MABAC	Security assessment in IoT framework
samasti et al. (2024)	Interval value neutrosophic numbers	EDAS	Medical waste disposable site
Chakraborty and Saha (2024)	SVTNn	MOORA	Forklift unit selection
Biswas et al. (2024)	Neutrosophic numbers	CRITIC and COPRAS	Canteen location selection

Table 2.4: Fuzzy MCDM technique in cloud computing

Author(s)	**Criteria/sub-criteria**	**Uncertain environment**	**MCDM Method**
Le et al. (2014)	9 criteria	*TFN*	Fuzzy ANP
Wibowo et al. (2016)	5 criteria	IFN	Fuzzy TOPSIS
Boutkhoum et al. (2016)	3 criteria, 10 sub-criteria	TFN	Fuzzy AHP
Lee and Seo (2016)	4 criteria	*TFN*	Fuzzy Delphi, Fuzzy AHP
Subramanian and Savarimuthu, (2016)	6 criteria, 13 sub-criteria	*TFN*	Fuzzy ANP, Fuzzy TOPSIS, Fuzzy ELECTRE
Kumar et al. (2017)	10 criteria	*TFN*	Fuzzy TOPSIS
Tanoumand et al. (2017)	6 criteria	*TFN*	Fuzzy AHP
Büyüközkan et al. (2018)	6 criteria, 3-6 sub-criteria	Interval-valued IFN	IVIF AHP, IVIF COPRAS, IVIF MULTIMOORA, IVIF VIKOR
Alam et al. (2018)	9 criteria, 30 sub-criteria	*TFN*	Fuzzy AHP, WASPAS
Jatoth et al. (2019)	7 criteria	Grey Theory	AHP, Grey TOPSIS
Monika and Sangwan (2022)	6 criteria	Spherical fuzzy set (SFS)	AHP, TOPSIS
Faiz and Daniel (2024)	5 criteria	Trapezoidal membership function (TMF)	AHP, ANP

3 AHP-TOPSIS Inspired Shopping Mall Site Selection Problem with Fuzzy Data

3.1 Introduction

Shopping malls can be considered as one of the most important growth points for business strategies. There exist multiple factors which are responsible for a decision-maker (DM) to select the optimal site for shopping mall construction. Detailed study has been conducted to identify all the factors and sub-factors related to site selection. The weights are assigned to each of the factors and sub-factors with the help of an expert decision-maker (DM). The needs for shopping malls are increasing throughout the country. As several factors influence the selection of the best site, it can be considered as an application of MCDM. MCDM is considered as the most significant branch of Operation Research, as it incorporates complex decisions of people's lives. There exists multiple MCDM models. Researchers use MCDM techniques depending on the problem of the decision making. The MCDM model Analytic Hierarchy Process (AHP) introduced by Saaty (1980) is one of the powerful techniques to obtain the factors' and sub-factors' weights. The model is widely used in numerous fields of engineering, economics, and operations management. The factors' and sub-factors' importance is calculated by the pair-wise comparison matrix. The pure AHP model lacks the ability to capture uncertainty, so several researchers have integrated Fuzzy with AHP to capture the impreciseness in decision making. In this

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chapter, we use fuzzy AHP (FAHP) to determine the factors' and sub-factors' fuzzy weights.

TOPSIS method developed by Hwang and Yoon (1981) is a logistic approach to select the best alternatives in real life problems, when several conflicting qualitative and quantitative criteria exist for the evaluation. The idea of this technique is that the best alternative is closest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). Decision making problems with uncertainty nowadays play an important role (Garg 2016; Sarkar 2012; Sarkar et al., 2011; Selvachandran et al., 2018; Kumar and Garg 2018; Basset et al., 2018). The FTOPSIS, an extension of classical TOPSIS to fuzzy domain, was introduced by Sodhi and T V (2012). They used fuzzy numbers instead of crisp values.

The fuzzy domain is a suitable approach in handling real life problems. This study compares and evaluates the proposed sites for the shopping mall using the fuzzy MCDM model. During the course of research, the opinion was taken from the people so as to bring the real customer needs and wants into the picture. Moreover, the study also explains the direct and indirect relation between the factors and sub-factors. Attention has been given on the minute specifications of the sub-factors and applies Triangular Fuzzy Numbers (TFNs) to give proper weightage to factors and sub-factors using Fuzzy AHP. Finally, the best site is assessed using the Fuzzy TOPSIS approach. The use of fuzzy set theory with the MCDM technique enables the decision-making problem to deal with vagueness and uncertainty.

The aims of the study are as follows:

- To study the important factors and sub-factors in a detailed way. These sub-factors capture minute detail considered by developers for new shopping mall site selection.
- Questionnaires will be prepared, and interviews will be conducted with municipal authorities and architects to gain a clearer understanding of uncertain factors such as land cost, population density, and population growth rate.
- TFN FAHP and TFN FTOPSIS will be employed to get the proper weightage and optimal site.

- The linguistic terms provided by the decision experts will be converted into Triangular Fuzzy Numbers (TFNs) using an effective and efficient method.
- A sensitivity analysis will be performed, along with a comparative analysis of the proposed problem, to demonstrate how the rankings vary with changes or the removal of specific factors and sub-factors.

The remainder of the chapter is constructed in the following way: Section 3.2 reflects the design and methodology of the proposed research. The FAHP and FTOPSIS methodologies are also covered in this section. Section 3.4 discusses the application, data source, and numerical problem of the study. A brief discussion of the sub-factors is executed and the final ranking is evaluated. Section 3.5 discusses the sensitivity analysis and numerical simulation. Finally, the conclusion is presented in Section 3.6. The flow of the proposed study is depicted in Figure 3.1.

3.2 Fuzzy analytic hierarchy process

AHP was first developed by Saaty (1980), and is a widely used scientific method in MCDM. AHP helps the decision-makers to solve the complex decisions with heuristic methods. Evaluation of factors' and sub-factors' weights are important for ranking the optimal site selection. AHP creates a framework of the problem hierarchy with the construction of comparison matrices to give subjective judgments about the factor's which are considered highly responsible in ranking the best.

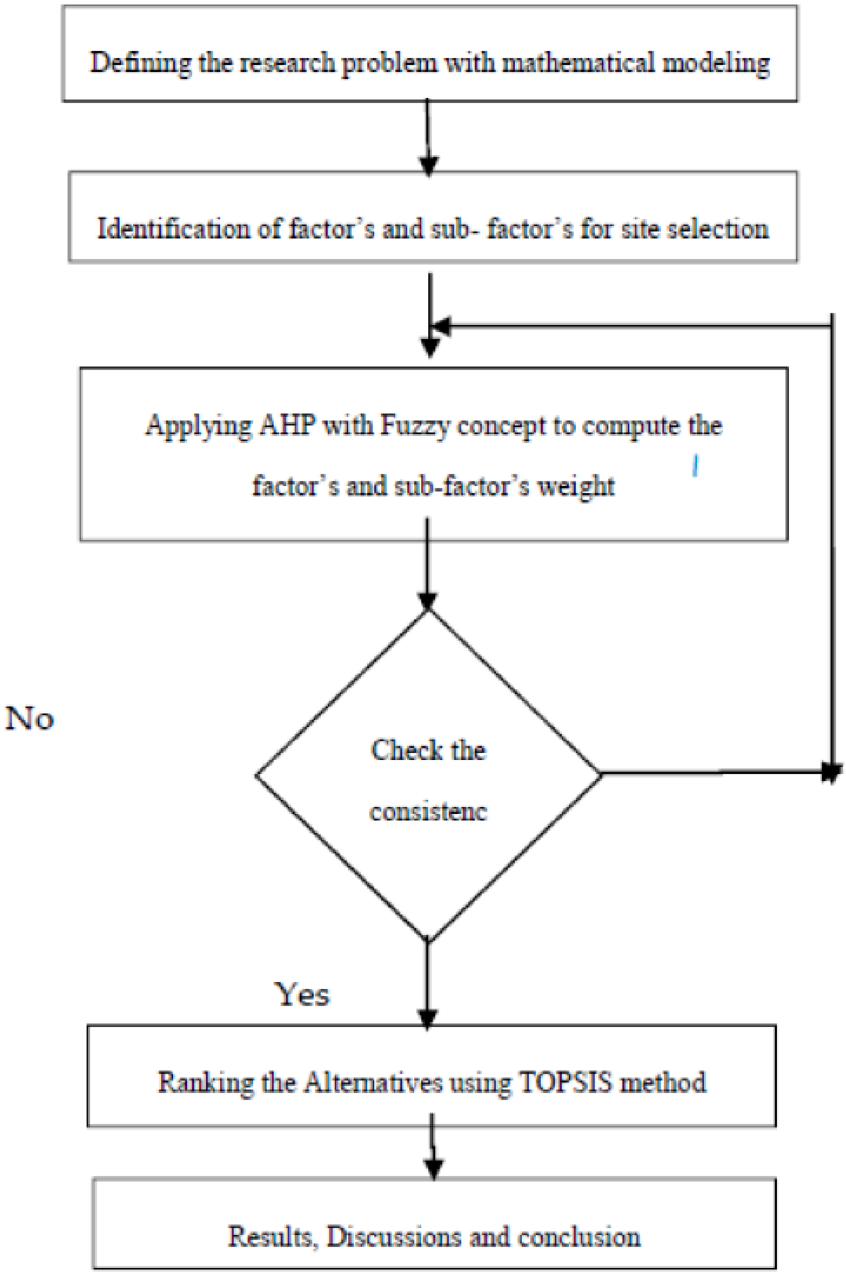

Figure 3.1: Flow chart for the proposed study

In this chapter, FAHP is used rather than AHP as the fuzzy environment takes into consideration the uncertainty and impreciseness of the decision experts. The fuzzy AHP methodology is based on the pairwise comparison approach, applied to understand the relative importance of factors and sub-factors. The AHP coupled with fuzzy logic enables the decision-makers a suitable approach in obtaining more realistic results in decision-making problems. The steps of $FAHP$ are described below.

Step 1: Construction of a comparison matrix in terms of TFN by a decision expert or a group of decision experts. Assume a group of m decision-makers involved in the pairwise comparison of factors and sub-factors. Thus, m set of matrices are obtained, $S_m = \{s_{klm}\}$; where $s_{klm} = (p_{klm}, r_{klm}, t_{klm})$ expresses the relative preference of k factor to l factor as decided by the m decision experts.

Step 2: Defuzzification of TFN. A TFN $s_{klm} = (p_{klm}, r_{klm}, t_{klm})$ can be defuzzified to a crisp value using the method proposed by Chang et al. (2009): $(S^{\alpha}_{kl})^{\beta} = [\beta.(p_{kl})^{\alpha} + (1-\beta).(t_{kl})^{\alpha}], 0 \leq \beta \leq 1, 0 \leq \alpha \leq 1$, where α signifies the preference display of the evaluator and β signifies the risk factor of the uncertain conditions. The method explicitly express fuzzy logic owing to the uncertainty of the decision makers. The uncertainty is maximum when $\alpha = 0$ and stability increases in decision making with increasing α. Moreover, β can be defined as the risk factor of the decision maker. $\beta = 1$ implies highly pessimistic whereas $\beta = 0$ implies highly optimistic. $p^{\alpha}_{kl} = (r_{kl} - p_{kl}) \times \alpha + p_{kl}$ denotes the lower bound of α-cut for s_{kl}, and $t^{\alpha}_{kl} = t_{kl} - \alpha(t_{kl} - r_{kl})$ denotes the upper bound of α-cut for s_{kl}

Step 3: Construction of a comparison matrix in terms of crisp values. Generalized representation of defuzzified comparison matrix:

$$((S^{\alpha}_{kl})^{\beta}) = \begin{bmatrix} 1 & (S^{\alpha}_{12})^{\beta}) & \dots & (S^{\alpha}_{1n})^{\beta}) \\ (S^{\alpha}_{21})^{\beta}) & 1 & \dots & (S^{\alpha}_{2n})^{\beta}) \\ \vdots & \vdots & \ddots & \vdots \\ (S^{\alpha}_{n1})^{\beta}) & (S^{\alpha}_{n2})^{\beta}) & \dots & 1 \end{bmatrix} \tag{3.1}$$

The subsequent steps of the AHP method are calculated using equations (1.2), (1.3), (1.4), and (1.5).

3.3 Fuzzy TOPSIS under TFN

In this research, it is assumed that the selection of the optimal site for constructing a shopping mall depends on multiple, often conflicting, factors and sub-factors. The MCDM method fuzzy TOPSIS (FTOPSIS) introduced by Sodhi and TV (2012) is one of the most suitable and reliable methods for the problem. The linguistic human decisions can be reflected better with FTOPSIS. The fuzzy logic extends our goal to obtain more sensitive results in this regard. The steps of FTOPSIS are described below.

Step 1: Construct the decision matrix.

Step 2: Evaluate the normalized fuzzy decision matrix.

Step 3: Obtain the weighted fuzzy normalized matrix by multiplying the sub-factors' fuzzy weights with the normalized fuzzy value:

$$\begin{aligned} \tilde{N} &= [n_{cd}]_{ij}, c = 1,2,....i; d = 1,2...j; \tilde{n}_{cd} = \left(\frac{p_{cd}}{t_d^*}, \frac{r_{cd}}{t_d^*}, \frac{t_{cd}}{t_d^*}\right), d \in B, \\ & t_d^* = max t_{cd} \end{aligned} \tag{3.2}$$

Step 4: Calculate the Fuzzy Positive Ideal Solution (FPIS) (T^+) and Fuzzy Negative Ideal Solution (FNIS) (T^-), where (h_c^+) denotes the maximum value of (h_{cd}) and (h_c^-) denotes the minimum value of (h_{cd}):

$$T^+ = h_1^+, h_2^+, ..., h_j^+ = \Big(\max(h_{cd})|d \in K_a, \min(h_{cd})|d \in K_b\Big) \tag{3.3}$$

$$T^- = h_1^-, h_2^-, ..., h_j^- = \Big(\min(h_{cd})|d \in K_a, \max(h_{cd})|d \in K_b\Big) \tag{3.4}$$

where K_a relates to the benefit factors and K_b relates to the non-benefit factors.

Step 5: Determine the distance of the alternatives from the PIS and NIS. The two Euclidean distances for individual alternatives can be calculated as follows:

$$U_c^+ = \left(\sum_{d=1}^{j} d(h_{cd}, h_c^+)\right), c = 1,2,\dots i \tag{3.5}$$

$$U_c^- = \left(\sum_{d=1}^{j} d(h_{cd}, h_c^-\right), c = 1,2,\dots i \tag{3.6}$$

where $d(.,.)$ denotes the distance between two fuzzy numbers.

Step 6: Determine the relative closeness to the ideal alternatives:

$$R_c = \frac{U_c^-}{U_c^- + U_c^+}, c = 1,2,\dots i \tag{3.7}$$

Step 7: Rank the alternatives:

The alternatives are ranked based on the score obtained by R_c, and the larger value of R_c signifies the better alternative.

Now, we fix the linguistic terms and the corresponding TFN in Table 3.1 as follows:

Table 3.1: Linguistic term and the corresponding scale in TFN

Scale Definition	**TFN Scale**	**TFN Reciprocal Scale**
Just Equal	(1,1,1)	(1,1,1)
Equally Important (EI)	(1/2,1,3/2)	(2/3,1,2)
Fairly Important (FI)	(3/2,2,5/2)	(2/5,1/2,2/3)
Strongly Important (SI)	(5/2,3,7/2)	(2/7,1/3,2/5)
Very Strongly Important (VSI)	(7/2,4,9/2)	(2/9,1/4,2/7)
Absolutely Important (AI)	(5,5,5)	(1/5,1/5,1/5)

3.4 Empirical study of shopping mall site selection problem

Enhanced modern lifestyle has changed the shopping style of people from small independent retail shops to shopping malls. The key importance lies in the fact that shopping malls provide many variations which the population demands. The customer finds comfort while shopping in malls, as under one roof they get exclusive clothing brands, department stores, food courts or restaurants, game section for children, multiplex, family salon and spa, etc. People get attracted towards shopping malls as it benefits the customer with window shopping too. Numerous conflicting factors and sub-factors impact the evaluation of the best site.

The main aim of the proposed research is to find the best location for the construction of a shopping mall from a given set of alternatives. The emerging choice of people over shopping malls builds great interest in investors to invest in shopping mall businesses. Choosing the best site from a set of different locations is tough, as individual locations have the corresponding robustness and flaws. As maximum return is the main aim of an investor, the selection of the best site requires scientific as well as mathematical modeling in an uncertain and imprecise environment.

Accessibility is one of the major factors of site selection. One of the factors of selecting a shopping mall site is based on how visible it is from the highway and how much road connectivity it has. In case there is already huge traffic on the site, it will become diffcult to construct anything new since traffic congestion is not liked by shoppers. In case the shopping mall site is faraway, then in order to increase footfall, a shuttle service, promotions, and reward programs will go a long way. A destination shopping mall is also a good concept since the land price can be optimized. To make it a success, the characteristics of the future customers such as age, income, and brand preferences need to be studied in detail since only then can the shopping mall be made to cater to the retail and leisure offerings alike. Configuration and size are also important factors for site selection since proper use of the catchment area will lead to maximization of resources. Mixed use developments are one of the most liked structures by the developers since the various segments complement each other and a synergy is achieved which helps in appropriate circulation of all asset classes. In the long run, the scope for further expansion must also be kept in mind. The shopping mall should have such an infrastructure which enables maximum utilization of space. Current and future competition, performance of the retail spaces, and the consumption patterns are some of the other factors which influence the site selection.

Building on the previously discussed factors and sub-factors, a more detailed and comprehensive discussion of these elements is presented below.

- **Population density**
  The population of a particular place is an important factor (Cheng et al., 2005; Erdin & Akbas, 2019; Kazemi & Amiri, 2017), when considering the place as a building site for a shopping mall. The primary reason is that the greater the possibility of consumption in an area, the greater the predicted sales from the shopping mall, which in turn will increase the growth of the area and eventually lead to an increase in the happiness index. In case the area consists of people from the young and middle generations, then there are greater chances

of good population growth, which in turn will lead to higher numbers of customers for the mall, thereby leading to a boom in the business.

- **Transportation** (Cheng et al., 2005; Kazemi & Amiri, 2017)
  Proximity to Metro —Having access to the metro will lead to an increase in the footfall in the mall. The reason is that greater connectivity of the mall acts as a catalyst for people who have the money and are looking for places to purchase from.
  Proximity to Railway — Railways are still one of the most widely used modes of transportation in India. If the shopping mall is situated near a Railway station, it will help people who have to do emergency buying or last-minute shopping.
  Proximity to Highway —Highways are visualized as roads with long roads and trees on both sides. The opening up of a shopping mall on the highway will attract customers who go out for short journeys and wish to shop and relax themselves.

- **Regional Growth**
  Administrative offices — Administrative offices are placed where people are usually stuck at a 9–5 job. They do not wish to go to different standalone stores at the end of the day in order to purchase their necessities and prefer a shopping mall which has stores of different kinds under its roof.
  Business Hub — Shopping malls which are large in size and which have too many kinds of stores in it make the mall a business hub. For example, if the mall has a Cafe Coffee Day store, then it attracts people who wish to do a business meet or recruitment while sipping coffee.

- **Cost** (Cheng et al., 2005; Erdin & Akbas, 2019; Kazemi & Amiri, 2017)
  Land—Cost of the land needs to be such that the revenue earned from the mall can cover it many times over in a few years after its inception.
  Construction —The construction cost of the land should also be considered as a fixed cost since it is not every day that the mall will be constructed. Renovations, expansion, and extra floors might be added, but it will not be the same

as a new construction, it will be considered as add-on's.

- **Attractive design**
  External design—The external design of the mall should be such that it is attractive, pleasing, and eye-catching at the first glance. The first thing the prospective customer looks at is the external design, which lays down the fact of whether it is a premium mall or a common one.
  Internal design—The internal design should be spacious as well as well-optimized. It means that in case a store needs a larger area and has the scope for an increase in sales, it should be provided with it if space is available, while simultaneously being smart enough to close down the stores which are performing poorly.

- **Provision for nearby Parking** —Proper parking space availability serves as a major determinant for many buyers. In cases where adequate parking space is unavailable, it causes the customer to go and probably never come back.

- **Environmental health** (Cheng et al., 2005; Erdin & Akbas, 2019; Kazemi & Amiri, 2017)
  Noise Pollution—If the shopping mall area has a continuous noise polusion, it forms a bad impression on the prospective customers and they might not come for a revisit, which will lead to a loss of customers and revenue.
  Air Pollution— If the shopping mall is built in such a place which has air pollution, it will lead to difficulty in breathing for the customers and employees and will not be a hit for sure.

Table 3.2 represents the factors and sub-factors of the associated problem.

Table 3.2: Representation of the factors and sub-factors

Factors	**Sub-Factors**
Population ($\tilde{f}^1$)	Current Population ($\tilde{f}^{11}$), Socioe conomic state ($\tilde{f}^{12}$), Population growth rate ($\tilde{f}^{13}$)
Transportation ($\tilde{f}^2$)	Accessibility ($\tilde{f}^{21}$), Distance to transportation vehicles ($\tilde{f}^{22}$)
Regional Growth ($\tilde{f}^3$)	Administrative offices ($\tilde{f}^{31}$), Business hub($\tilde{f}^{32}$), School and college ($\tilde{f}^{33}$)
Cost($\tilde{f}^4$)	Land ($\tilde{f}^{41}$) Construction ($\tilde{f}^{42}$)
Building structure (Design)($\tilde{f}^5$)	External design(Architecture) ($\tilde{f}^{51}$) Internal design(Infrastructure)($\tilde{f}^{52}$) Parking ($\tilde{f}^{53}$)
Provision of nearby Parking ($\tilde{f}^6$)	Capacity of the parking ($\tilde{f}^{61}$)space Parking rate per hour ($\tilde{f}^{62}$)
Environmental health ($\tilde{f}^7$)	Noise Pollution ($\tilde{f}^{71}$) Air Pollution ($\tilde{f}^{72}$)

In the present study, different locations in and around Kolkata (Calcutta) are taken as alternatives.

i) Howrah (A_1)

ii) Chinsurah (A_2)

iii) Uttarpara (A_3)

iv) Dunlop (A_4)

v) Ballygunge (A_5)

vi) Behala (A_6)

vii) New Town (A_7)

viii) Uttarpara(A_8)

ix) Dunlop (A_9)

x) Ballygunge (A_{10})

3.4.1 *Data source for the study*

The data was collected from the various municipal authorities in the related locations of West Bengal. The municipal representatives were interviewed about the questions related to the factors and sub-factors of the shopping mall site selection problem. They provided information regarding population density and several important factors relevant for this research. The registrar of land, revenue, and expert architects were asked about the prevailing land price as it plays a vital role in site selection.

3.4.2 *Numerical study*

Step 1: We now construct Table 3.3 based on the comparison matrix with respect to factors given in Table 3.1.

Table 3.3: Comparison matrix with respect to factors

Factors	$\tilde{f}_1$	$\tilde{f}_2$	$\tilde{f}_3$	$\tilde{f}_4$	$\tilde{f}_5$	$\tilde{f}_6$	$\tilde{f}_7$
$(\tilde{f}_1)$	1	$(\frac{2}{5},\frac{1}{2}\frac{2}{3})$	$(\frac{2}{7},\frac{1}{3},\frac{2}{5})$	$(\frac{7}{2},4,\frac{9}{2})$	$(\frac{3}{2},2,\frac{5}{2})$	$(\frac{2}{3},1,2)$	$(\frac{1}{2},1,\frac{3}{2})$
$(\tilde{f}_2)$	$(\frac{3}{2},2,\frac{5}{2})$	1	$(\frac{1}{2},1,\frac{3}{2})$	$(\frac{2}{5},\frac{1}{2}\frac{2}{3})$	$(\frac{3}{2},2,\frac{5}{2})$	$(\frac{3}{2},2,\frac{5}{2})$	$(\frac{2}{5},\frac{1}{2}\frac{2}{3})$
$(\tilde{f}_3)$	$(\frac{5}{2},3,\frac{7}{2})$	$(\frac{2}{3},1,2)$	1	$(5,5,5)$	$(\frac{5}{2},3,\frac{7}{2})$	$(\frac{3}{2},2,\frac{5}{2})$	$(\frac{2}{3},1,2)$
$(\tilde{f}_4)$	$(\frac{2}{9},\frac{1}{4},\frac{2}{7})$	$(\frac{3}{2},2,\frac{5}{2})$	$(\frac{1}{5},\frac{1}{5},\frac{1}{5})$	1	$(\frac{3}{2},2,\frac{5}{2})$	$(\frac{1}{2},1,\frac{3}{2})$	$(\frac{3}{2},2,\frac{5}{2})$
$(\tilde{f}_5)$	$(\frac{2}{5},\frac{1}{2},\frac{2}{3})$	$(\frac{2}{7},\frac{1}{3},\frac{2}{5})$	$(\frac{2}{9},\frac{1}{4},\frac{2}{7})$	$(\frac{2}{5},\frac{1}{2}\frac{2}{3})$	1	$(\frac{2}{9},\frac{1}{4},\frac{2}{7})$	$(\frac{1}{2},1,\frac{3}{2})$
$(\tilde{f}_6)$	$(\frac{1}{2},1,\frac{3}{2})$	$(\frac{2}{5},\frac{1}{2},\frac{2}{3})$	$(\frac{2}{5},\frac{1}{2},\frac{2}{3})$	$(\frac{2}{3},1,2)$	$(\frac{7}{2},4,\frac{9}{2})$	1	$(\frac{3}{2},2,\frac{5}{2})$
$(\tilde{f}_7)$	$(\frac{2}{3},1,2)$	$(\frac{3}{2},2,\frac{5}{2})$	$(\frac{1}{2},1,\frac{3}{2})$	$(\frac{2}{5},\frac{1}{2},\frac{2}{3})$	$(\frac{2}{3},1,2)$	$(\frac{2}{5},\frac{1}{2},\frac{2}{3})$	1

Step 2: Defuzzification of TFN

Here, $\alpha = 0.5$ signifies the preference display of the evaluator and $\beta = 0.5$ signifies the risk factor of the uncertain conditions. The defuzzification step is performed as follows:

$p_{31}^{\alpha} = (3 - 2.5) \times 0.5 + 2.5t_{31}^{\alpha} = 3.5 - (3.5 - 3) \times 0.5 \times (s_{31}^{0.5})^{0.5} = 3$

Applying the above-mentioned process, the calculation is conducted for all other elements and the defuzzified matrix is represented in Table 3.4.

Table 3.4: Defuzzified comparison matrix.

Factors	$\tilde{f}_1$	$\tilde{f}_2$	$\tilde{f}_3$	$\tilde{f}_4$	$\tilde{f}_5$	$\tilde{f}_6$	$\tilde{f}_7$
$\tilde{f}_1$	1	0.54	0.34	4	2	1.17	1
$\tilde{f}_2$	2	1	1	0.52	2	2	0.52
$\tilde{f}_3$	3	1.17	1	5	3	2	1.17
$\tilde{f}_4$	0.25	2	0.2	1	2	1	2
$\tilde{f}_5$	0.52	0.34	0.25	0.52	1	0.25	1
$\tilde{f}_6$	1	0.52	0.52	1.17	4	1	2
$\tilde{f}_7$	1.17	2	1	0.52	1.17	0.52	1

The matrix is normalized using Eq. (1.2) and then the priority weights for the factors are calculated using Eq. (1.3). Finally, the Consistency Ratio (CR) values are obtained to determine the consistency of the matrix. Table 3.5 depicts weights of the factors.

Table 3.5: The weights of the factors

Factors	$\tilde{f}_1$	$\tilde{f}_2$	$\tilde{f}_3$	$\tilde{f}_4$	$\tilde{f}_5$	$\tilde{f}_6$	$\tilde{f}_7$
Priority Weight	0.141	0.15	0.26	0.115	0.063	0.15	0.125

The factor "Regional Growth" scores the maximum weight of 0.26, whereas the factor "Provisions of nearby parking" scores the lowest weight. In the similar order, the sub-factors' matrices are obtained and their respective fuzzy weights are calculated in the exact process as above. After obtaining the sub-factor fuzzy weights, global weight is computed by the product of individual sub-factor weight with the respective factor fuzzy weight. The global fuzzy weights obtained are ultimately required for ranking the best alternative using the FTOPSIS method. Table 3.6 represents global fuzzy weights.

Table 3.6: Representation of global fuzzy weights

Factors' fuzzy weight	Sub-factors' fuzzy Weight	Global fuzzy weight
W_1= (0.08, 0.14, 0.24)	W_{11}= (0.1, 0.12, 0.16)	W_{11}= (0.01, 0.02, 0.04)
	W_{12}= (0.28, 0.36, 0.47)	W_{12}= (0.02, 0.05, 0.11)
	W_{13}= (0.39, 0.52, 0.67)	W_{13}= (0.03, 0.07, 0.16)
W_2= (0.086, 0.15, 0.24)	W_{21}= (0.31, 0.5, 0.92)	W_{21}= (0.03, 0.07, 0.22)
	W_{22}= (0.27, 0.5, 0.80)	W_{22}=(0.02, 0.07, 0.19)
W_3= (0.16, 0.26, 0.44)	W_{31}= (0.12, 0.15, 0.20)	W_{31}= (0.02, 0.04, 0.09)
	W_{32}= (0.29, 0.38, 0.5)	W_{32}= (0.05, 0.10, 0.22)
	W_{33}= (0.34, 0.47, 0.64)	W_{33}= (0.05, 0.12, 0.28)
W_4= (0.07, 0.12, 0.18)	W_{41}= (0.18, 0.2, 0.23)	W_{41}= (0.01, 0.024, 0.04)
	W_{42}= (0.70, 0.8, 0.91)	W_{42}= (0.049, 0.096, 0.164)
W_5= (0.04, 0.06, 0.10)	W_{51}= (0.1, 0.12, 0.15)	W_{51}= (0.004, 0.007, 0.015)
	W_{52}= (0.30, 0.39, 0.51)	W_{52}= (0.012, 0.02, 0.051)
	W_{53}= (0.35, 0.49, 0.66)	W_{53}= (0.014, 0.029, 0.066)
W_6= (0.08, 0.15, 0.26)	W_{61}= (0.18, 0.2, 0.22)	W_{61}= (0.014, 0.03, 0.057)
	W_{62}= (0.72, 0.8, 0.9)	W_{62}= (0.058, 0.12, 0.234)
W_7= (0.07, 0.12, 0.23)	W_{71}= (0.31, 0.5, 0.80)	W_{71}= (0.022, 0.06, 0.212)
	W_{72}= (0.27, 0.5, 0.80)	W_{72}= (0.019, 0.06, 0.184)

The global fuzzy sub-factors obtained will be used in ranking of the alternatives using TOPSIS. Here, the factor ($\tilde{f}^4$) "Cost", ($\tilde{f}^7$) "Environmental health" and the sub-factor ($\tilde{f}^{62}$) "Parking rate per hour" are non-beneficial attributes, whereas the other factors and sub-factors are beneficial attributes. The steps involved in the numerical FTOPSIS method are outlined below.

Step 1: The preference of the alternatives with respect to the sub-factors is expressed in linguistic terms, as shown in Table 3.7.

Table 3.7: Linguistic preference in terms of TFN.

Linguistic Variable	**TFN Scale**
Excellent (E)	(7,9,9)
Good (G)	(5,7,9)
Fair (F)	(3,5,7)
Poor (P)	(1,3,5)
Very Poor (VP)	(1,1,3)

The table constructed using linguistic preference in terms of TFN of Table 3.7 is represented in Table 3.8.

Table 3.8: Defuzzified comparison matrix.

Alternatives	$\tilde{f}_{11}$	$\tilde{f}_{12}$	$\tilde{f}_{13}$	$\tilde{f}_{21}$	$\tilde{f}_{22}$	$\tilde{f}_{31}$	$\tilde{f}_{32}$	$\tilde{f}_{33}$	$\tilde{f}_{41}$	$\tilde{f}_{42}$	$\tilde{f}_{51}$	$\tilde{f}_{52}$	$\tilde{f}_{53}$	$\tilde{f}_{61}$	$\tilde{f}_{62}$	$\tilde{f}_{71}$	$\tilde{f}_{72}$
A_1	E	F	E	E	E	E	G	G	G	G	F	G	P	VP	F	E	E
A_2	F	F	F	F	G	E	F	F	F	G	G	F	P	VP	P	P	VP
A_3	G	E	F	F	G	P	VP	G	P	F	G	F	F	P	F	P	F
A_4	E	G	G	E	G	P	F	G	G	F	G	F	G	G	P	E	E
A_5	E	E	F	E	E	F	F	G	E	E	E	E	G	E	E	G	G
A_6	E	G	E	G	G	F	F	G	G	G	G	G	F	P	G	F	G
A_7	P	G	F	F	G	P	F	P	F	F	E	E	E	G	F	VP	VP

Step 2: Normalizing of the matrix using Eq. (1.2).

Step 3: Computation of weighted normalized matrix using Eq. (3.2).

The weighted normalized matrix of all the sub-factors is divided into three tables (Tables 3.9, 3.10 and 3.11).

Table 3.9: Weighted normalized matrix for factors 1 and 2.

	$\tilde{f}_{11}$	$\tilde{f}_{12}$	$\tilde{f}_{13}$	$\tilde{f}_{21}$	$\tilde{f}_{22}$
A_1	.007, 0.02, 0.04	.006,0.03, 0.08	0.02, 0.07, 0.2	0.02, 0.07, 0.2	0.02, 0.07, 0.19
A_2	0.003, .01, .03	0.006,0.03,0.08	0.009,0.04,0.1	0.009,0.04,0.2	0.01,0.05,0.19
A_3	.005,0.015,0.04	0.015,0.05,0.1	0.009,0.04,0.12	0.009,0.04,0.17	0.01,0.05,0.19
A_4	0.007,0.02,0.04	0.01,0.04,0.11	0.016,0.05,0.12	0.023,0.07,0.22	0.01,0.05,0.19
A_5	0.007,0.02,0.04	0.015,0.05,0.11	0.009,0.04,0.12	0.023,0.07,0.22	0.015,0.07,0.19
A_6	0.007,0.02,0.04	0.01,0.04,0.11	0.023,0.07,0.16	0.016,0.05,0.22	0.01,0.05,0.19
A_7	0.001,0.006,0.022	0.01,0.04,0.11	0.009,0.04,0.12	0.009,0.04,0.17	0.01,0.05,0.19

Table 3.10: Weighted normalized matrix for factors 3, 4 and 5.

	$\tilde{f}_{31}$	$\tilde{f}_{32}$	$\tilde{f}_{33}$	$\tilde{f}_{41}$	$\tilde{f}_{42}$	$\tilde{f}_{43}$
A_1	.01,.04,.09	.03,.07,0.2	0.03,.09,0.3	.001,.003,.01	.02, 0.04, 0.1	.001,.004,0.1
A_2	.01,.04,.09	0.02,.05,0.2	.02,.06,0.2	.001,.005,.01	.02,.04,.10	.002,.005,.015
A_3	.002,.013,.05	.005,.01,.07	.03,.09,.28	.002,.008,.01	.02,.06,.164	.002,.005,.015
A_4	.002,.013,.05	.02,.05,0.2	.03,0.1,0.3	.001,.003,.01	.02,.06,0.2	.002,.005,0.01
A_5	.006,.02,.07	.02,0.05,0.2	.03,.09,.28	0.001,.001,.003	.02,.02,.02	.003,.007,.015
A_6	.01,.02,.07	.02,.05,0.2	.03,0.1,0.3	.001,.003,.01	.02,.04,.10	.002,.005,.015
A_7	.002,.01,.05	.02,.05,.2	.005,.04,.15	.001,.005,.01	.02,.06,.16	.001,.004, 0.12

Table 3.11: Weighted normalized matrix for factors 5,6 and 7.

	$\tilde{f}_{52}$	$\tilde{f}_{53}$	$\tilde{f}_{61}$	$\tilde{f}_{62}$	$\tilde{f}_{71}$	$\tilde{f}_{72}$
A_1	.01,.01,.05	.001,.01,.04	.001,.003,.02	.008,.024,.08	.002,0.01,.03	.002,.007,.02
A_2	.004,.01,.04	.001,.01,.04	0.001,.003,.02	0.01,.04,.23	.004,.02,.21	0.01,.1,0.2
A_3	0.004,.01,.04	0.005,.02,.05	0.001,.01,.03	.008,.024,0.1	0.004,.02,.21	0.003,.01,.01
A_4	0.004,.011,.04	0.01,.02,.07	.01,.02,.06	.01,.04,.234	.002,.01,.03	.002,0.01,.02
A_5	.01,.02,.051	0.01,.02,.07	0.01,.03,.06	0.01, .01,.03	0.002,.01,.04	0.002,.01,.04
A_6	0.007,.01,.051	0.005,.02,.05	0.001,.01,.03	.01,.02,.05	.003,.012,.07	0.002,.01,.04
A_7	.01,.02,.051	.01,.03,.07	.01,.03,.06	.01,.024,.08	.007,.06,.21	0.01,.01,0.2

Step 4: Determination of the Fuzzy Positive Ideal Solution (FPIS) ($T+$) and Fuzzy Negative Ideal Solution (FNIS) ($T-$), where (h_c^+) denotes the maximum value of (h_{cd}) hcd and (h_c^-) denotes the minimum value of (h_{cd}). The Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) are divided into three tables (Tables 3.12, 3.13, 3.14).

Table 3.12: First part of FPIS and FNIS.

T_+	.007,.02,.04	.015,.05,.11	.023,.07,.16	.023,.07,.22	.015,.07,.19	.01,.04,.09
T_-	0.001, 0.01,.02	0.01,.03,.08	0.01, 0.04,.12	0.01,.04,.17	0.01,.05,.19	.002,.013,.05

Table 3.13: Second part of FPIS and FNIS.

T_+	.03,.07,.22	.03,.09,.28	.001,.001,.003	.02,.02,.02	0.003,.007,.12	0.009,.02,.051
T_-	.005,0.01,.07	.005,.04,.154	.002, 0.01, 0.01	0.02,0.1,.164	0.001,.004,.015	0.004,.01,.04

Table 3.14: Third part of FPIS and FNIS.

T_+	0.01,.029,.07	0.01,.03,.06	0.01,0.01, 0.03	0.002,.007,.03	0.002,.01,.02
T_-	0.001,.01,.04	0.001,.003,.02	0.01, 0.04,.234	0.01,.06,.21	0.006,.06,.2

Table 3.15 presents the distance measures from the FPIS and FNIS, the relative closeness coefficients, and the final rankings.

Table 3.15: Calculation of relative closeness coefficient.

Alternatives	U_c^+	U_c^-	$R_C = \frac{U_c^-}{U_c^- + U_c^+}$	**Rank**
A_1	0.157	0.65	0.805	2
A_2	0.66	0.168	0.203	7
A_3	0.547	0.38	0.410	5
A_4	0.39	0.425	0.521	4
A_5	0.153	0.675	0.815	1
A_6	0.275	0.553	0.668	3
A_7	0.610	0.251	0.291	6

The Relative Closeness (R_c) depicts the ranking. From Figure 5, we can see that the larger value of R_c indicates the most preferred alternatives. In this study, the alternative $\tilde{A}_5$ scores the maximum R_c value of 0.815, whereas, the alternative $\tilde{A}_2$ scores the lowest value of 0.203. Thus, according to our study, Ballygunge (A_5), located on the heart of the city of Kolkata, is the best site for shopping mall construction as it has the maximum relative closeness coefficient, followed by Howrah (A_1), Behala (A_6), Dunlop (A_4), Uttarpara (A_3), New Town (A_7), and Chinsurah (A_2). This ranking is obtained on the basis of surveys conducted and data analysis performed. A sensitivity analysis is conducted to assess the impact of modified weights for different factors and sub-factors.

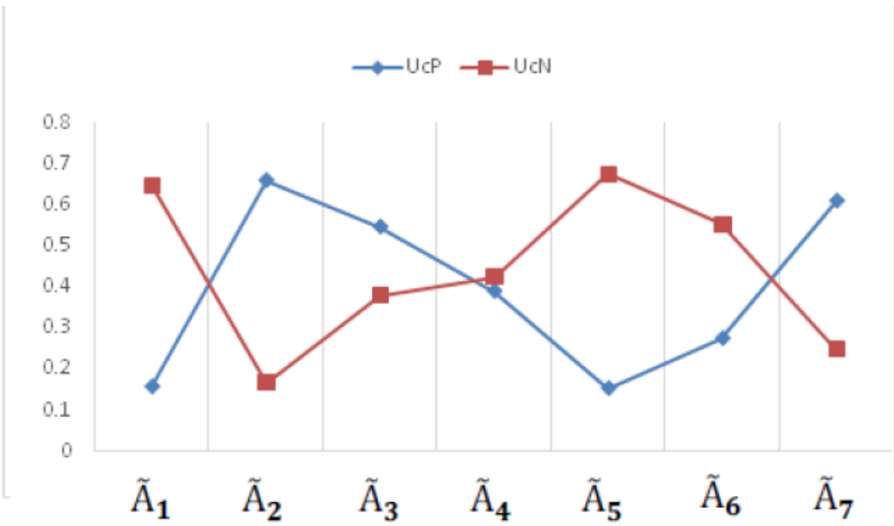

Figure 3.2: Geometric distances from FPIS and FNIS using Table 3.15

3.5 Sensitivity analysis

The results of the final ranking are summarized in Table 3.16. The best shopping mall site selection is ($\tilde{A}_5$). To analyze the impact, due to change in sub-factors' weights or elimination of certain sub-factors, a sensitivity analysis is conducted. In the analysis, the following three cases are considered:

Case 1: The fuzzy weights of the sub-factors "Current population density ($\tilde{f}^{11}$)" and "Administrative offices ($\tilde{f}^{31}$)" are interchanged.

Case 2: The sub-factors "Capacity of the parking space ($\tilde{f}^{61}$)" and "Parking rate per hour ($\tilde{f}^{62}$)" are eliminated.

Case 3: The sub-factors "Noise Pollution ($\tilde{f}^{71}$)" and "Air Pollution ($\tilde{f}^{72}$)" are eliminated.

The different rankings obtained are depicted in Table 3.16 and Figure 3.3. The sensitivity analysis reveals that varying the weights of sub-factors results in changes to the ranking. This clearly highlights the sensitivity of the optimal ranking to market dynamics. Shifts in environmental conditions and the socio-economic landscape of the proposed area under consideration also contribute to fluctuations in the ranking.

Table 3.16: Rankings obtained in the three cases

Alternatives	Case 1	Case 2	Case 3
$\tilde{A}_1$	2	2	1
$\tilde{A}_2$	7	7	6
$\tilde{A}_3$	5	6	5
$\tilde{A}_4$	4	4	4
$\tilde{A}_5$	1	1	2
$\tilde{A}_6$	3	3	3
$\tilde{A}_7$	6	5	7

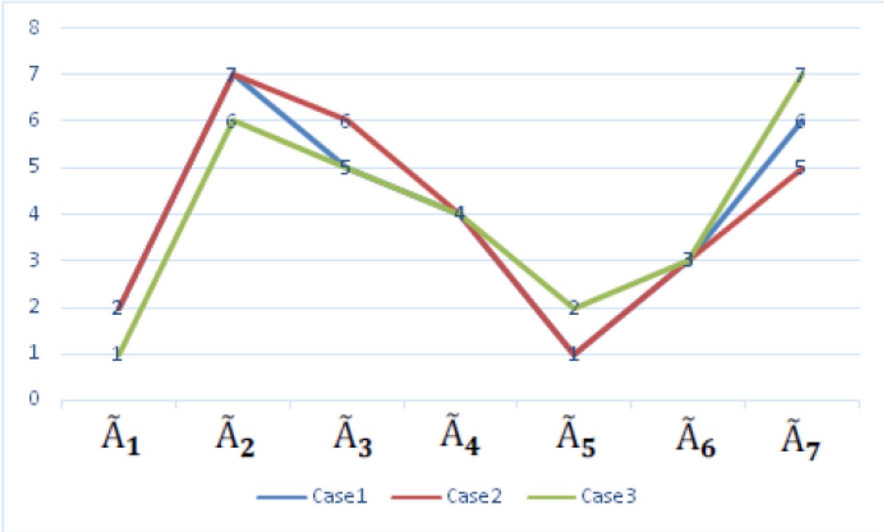

Figure 3.3: Ranking of alternate sites in different cases.

3.6 Conclusions

Shopping mall site selection is a critical process, as it significantly impacts both environmental and socio-economic factors. Identifying the optimal location is inherently complex, requiring decision-makers to have a comprehensive understanding of numerous factors and sub-factors—some of which are quantitative, while others are imprecise and qualitative. The methodology used in this study places a greater emphasis on sub-factors, recognizing their vital role in the decision-making process. A detailed study has been carried out in respect of the attributes. A highly knowledgeable, skilled, experienced expert's opinion has been taken into consideration regarding factors and sub-factors for the preferential ranking of the alternatives. The MCDM techniques FAHP and FTOPSIS have been used to select the best location. Seven locations around the city of Kolkata (Calcutta), West Bengal have been considered in the study, where conflicting preferences regarding criteria and sub-criteria exist. These MCDM techniques can be considered as a practical approach to rank the best site, when multiple complex imprecise constraints exist. Sensitivity analysis gives a clear idea regarding influence of market dynamics in the ranking process. The result of FTOPSIS and sensitivity analysis obtained show that the 'Ballygunge' region of Kolkata is the superior alternative for shopping mall site selection. The possible explanation for the best ranking is that 'Ballygunge' is one of the most developed regions in Kolkata and excels in a majority of the factors and sub-factors considered in this study. Thus, the rank of 'Ballygunge' region scores a spot of '1' with our ranking method and '1' or '2' under sensitivity analysis. This research can be extended as a practical approach for ranking the alternatives considering various conflicting factors and sub-factors for real-life problems like super-specialty hospital site selection, sports academy site selection, etc.

4

Identification of Dominant Risk Factor Involved in Spread of COVID-19 using Hesitant Fuzzy MCDM Methodology

4.1 Introduction

COVID-19 was first reported in Wuhan, China, in December 2019 [1]. Shortly thereafter, the number of infections began to rise rapidly, leading the World Health Organization (WHO) to officially declare COVID-19 a global pandemic. As of August 7, 2020, approximately 18,902,735 COVID-19 cases had been reported across 216 countries, with 709,511 confirmed deaths[2]. WHO had been in a responsible mode to bring awareness and updated the countries to take urgent necessary action since the origin of the pandemic. Several preventive measures had been taken by the people and government of different countries. Given the lack of immunity to the disease, the WHO issued urgent warnings about its severity and called on individuals to act responsibly, emphasizing the importance of empathy by acknowledging the struggles of those around them who were suffering and fighting for their lives.

In this study, with input from medical professionals, literature reviews, and media surveys, a mathematical model has been applied to identify the most dominant

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[1]https://www.who.int/emergencies/diseases/novel-coronavirus-2019.

[2]https://www.who.int/emergencies/diseases/novel-coronavirus-2019.

risk factor contributing to the spread of the disease. To achieve this, the Hesitant Fuzzy Set approach combined with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has been used for analyzing and ranking the key risk factors.

4.1.1 *Objectives of the present research*

The present study focuses on:

a. Identifying the most imperative risk factors for the spread of COVID-19. The factors included in this study are from doctor's opinion, literature review and media survey.

b. Applying Hesitant Fuzzy sets (HFS) in combination with the TOPSIS approach to rank various risk factors. HFS provides greater flexibility by allowing multiple membership degrees for an alternative with respect to a given criterion. To the best of our knowledge, no prior research has employed an integrated methodology combining FAHP and HFS-TOPSIS for identifying the most significant risk factors associated with COVID-19.

The remaining of the chapter is organized as follows: Section 4.1.2 outlines the scientific contributions of the study. Section 4.2 details the methodological approach, including the use of HFS-AHP and HFS-TOPSIS. Section 4.3 presents the selection criteria and the risk factors considered. Section 4.4 provides the empirical analysis and the ranking of the identified risk factors. Section 4.5 focuses on the sensitivity analysis. The results and discussion are presented in Section 4.6, and Section 4.7 concludes the chapter.

4.1.2 *Contributions of the research*

This study has identified and ranked the key risk factors contributing to the spread of COVID-19. The findings aim to assist the government in developing targeted administrative strategies to mitigate these high-priority risks. Based on the results, the government should prioritize the implementation of policies that minimize prolonged physical contact by strictly enforcing social distancing measures. Adequate transportation should be arranged for frontline COVID-19 workers, including those in essential services such as electricity, banking, postal work, and sanitation, ensuring all conveyance methods comply with social distancing guidelines. Additionally, enhanced safety protocols must be implemented in hospitals, diagnostic centers, and clinics to curb transmission. The safe and proper disposal of contaminated materials, such as PPE kits, along with thorough sanitization of high-risk areas, must be rigorously maintained.

4.2 Methodology

The study focuses on finding the most significant risk factor for the spread of the pandemic coronavirus disease using MCDM technique. It uses FAHP to obtain weights of the criteria. The Hesitant Fuzzy Set theory coupled with TOPSIS tool is applied to finding the most significant risk factor for the spread of the COVID-19 disease. Fig. 4.1 represents the design of the proposed model.

4.2.1 *Fuzzy analytic hierarchy process*

A logic-based analytical framework is employed to address this problem, proving particularly useful for complex decision-making involving heuristic methods. The evaluation of criteria weights plays a critical role in accurately ranking the risk factors. Traditionally, the Analytic Hierarchy Process (AHP) is used to structure the problem into a hierarchy by constructing pairwise comparison matrices based on subjective judgments of relevant attributes. However, in this study, the Fuzzy Analytic Hierarchy Process (FAHP) is utilized instead of the conventional AHP to account for uncertainties inherent in the decision-making process. The steps of FAHP are given below:

Step 1: Prepare a comparison matrix associated with TFN by a decision expert or a group of experts using Table 4.1.

Let a cluster of 'T' decision-makers concerned in the pairwise comparison of criteria weights. Thus, 't' set of matrices are obtained, $p_t = p_{flt}$ where $p_{flt} = (q_{flt}, r_{flt}, s_{flt})$ represent the relative preference of 'f' factor to 'l' factor as decided by the 't' experts.

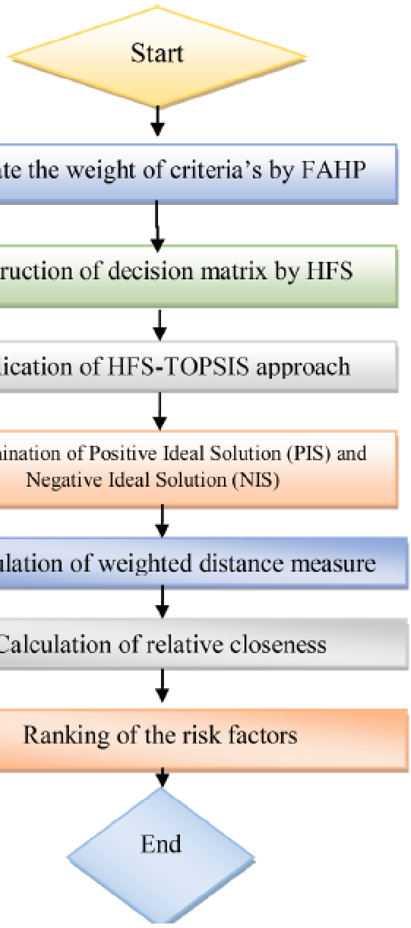

Figure 4.1: Flow chart for the proposed study.

$$\begin{cases} q_{fl} = \min_{t=1,2,\ldots,T} q_{flt} \\ r_{fl} = \sqrt[T]{\prod_{t=1}^{T} r_{flt}} \\ s_{fl} = \max_{t=1,2,\ldots,T} (s_{flt}) \end{cases} \tag{4.1}$$

Table 4.1: Linguistic terms in TFN for the assessment of criteria weights.

Linguistic Term	**TFNs**
Highly Important	(4.5,5,5.5)
Important	(3.5,4,4.5)
Fairly Important	(2.5,3,3.5)
Less Important	(1.5,2,2.5)
Very Less Important	(0.5,1,1.5)

Step 2: Defuzzification of TFN: A TFN $p_{flt} = (q_{flt}, r_{flt}, s_{flt})$ can be defuzzified according to the methodology proposed by Chang et al. (2009)

$$(p_{fl}^{\gamma})^{\delta} = [\delta.(q_{fl})^{\gamma} + (1-\delta).(s_{fl})^{\gamma}], 0 \leq \delta \leq 1, 0 \leq \gamma \leq 1 \tag{4.2}$$

where γ denotes the preference value of the evaluator and δ is defined as the risk aspect existing in uncertainty. $\delta = 1$ implies extremely pessimistic whereas $\delta = 0$ implies highly optimistic. The stability increases in decision making with the increase in γ and diminishes when $\gamma = 0$. The expression $(q_{fl}^{\gamma}) = [(r_{fl} - q_{fl}) \times \gamma + (q_{fl})]$ means the lower bound of γ- cut for p_{fl} and $(s_{fl}^{\gamma}) = s_{fl} - \gamma \times [(s_{fl} - r_{fl})$ means the upper bound of γ- cut for p_{fl}.

Step 3: Construct a comparison matrix in terms of crisp values.
The remaining steps of the AHP method are derived using formulas (1.2), (1.3), (1.4), and (1.5).

4.2.2 *HFS-TOPSIS method*

The TOPSIS method, introduced by Hwang and Yoon (1981), is one of the most widely used multi-criteria decision-making (MCDM) techniques. In this study, the optimal strategy is identified using the Hesitant Fuzzy Set-based TOPSIS (HFS-TOPSIS) approach, incorporating the previously calculated weights. Based on earlier research, the procedural steps for implementing HFS-TOPSIS are outlined as follows:

Step 1: Determine the PIS and NIS:

$$A_p^+ = h_1^+, h_2^+, \cdots, h_n^+; where \tag{4.3}$$

$h_c^+ = \bigcup_{b=1}^{m} h_{bc} = \bigcup_{\beta_{1c} \in h_{1c}, \cdots, \beta_{mc} \in h_{mc}} \max\{\beta_{1c}, \cdots, \beta_{mc}\}, c = 1, 2, \cdots, n.$

$$A_n^- = h_1^-, h_2^-, \cdots, h_n^-; where \tag{4.4}$$

$h_c^- = \bigcap_{b=1}^{m} h_{bc} = \bigcap_{\beta_{1c} \in h_{1c}, \cdots, \beta_{mc} \in h_{mc}} \max\{\beta_{1c}, \cdots, \beta_{mc}\}, c = 1, 2, \cdots, n.$

Step 2: Measure the distances from PIS and NIS for the alternatives. The present study uses weighted hesitant normalized Hamming distance introduced by Zhang and Xu (2012). The distance of an alternative from PIS can be calculated as follows:

$$D_b^+ = \sum_{c=1}^{n} W_c ||h_{bc} - h_c^+|| \tag{4.5}$$

where w_c denotes the weight of the c^{th} criterion, which is calculated by FAHP. Similarly, the distance of an alternative from NIS can be calculated as follows:

$$D_b^- = \sum_{c=1}^{n} W_c ||h_{bc} - h_c^-|| \tag{4.6}$$

The literature offers various methods for calculating the distance between two Hesitant Fuzzy Elements (HFEs). Xu and Xia (2012) proposed the use of Euclidean distance for this purpose, defined as follows:

$$d(h_1, h_2) = \sqrt{\frac{1}{n(h)} \sum_{c=1}^{n(h)} |h_{1_{\sigma(c)}} - h_{2_{\sigma(c)}}|^2} \tag{4.7}$$

Zhang and Wei (2013) defined Hamming distance between two different HFEs as follows:

$$d(h_1, h_2) = \frac{1}{n(h)} \sum_{c=1}^{n(h)} |h_{1_{\sigma(c)}} - h_{2_{\sigma(c)}}| \tag{4.8}$$

Here h_1 and h_2 are HFEs, and $n(h)$ denotes the number of elements present in a HFE, also known as length of HFE. Generally, the lengths of HFEs are diverse and the values in it frequently out of order. To calculate the distance between two Hesitant Fuzzy Elements (HFEs), the following procedure should be followed:

Assemble the elements in increasing order or declining order.

When the lengths of two HFEs are dissimilar, the shorter length HFE is supposed to be extended by adding the minimal value or maximum value, which depends on the preference of resolution makers. The optimistic DM prefers addition of maximum value while the pessimistic DM prefers minimum value, according to Xu and Zhang (2013).

Step 3: The relative closeness to the ideal solution is determined with the following equation:

$$RC_b = \frac{D_b^-}{D_b^- + D_b^+} \tag{4.9}$$

Step 4: Ranking of the alternatives is done on the values of RC_b. The greater value signifies the optimal alternative.

Note 6. *HFS-TOPSIS is particularly effective in addressing the inherent vagueness and uncertainty of subjective evaluations. Hesitant Fuzzy Set (HFS) theory enables the representation of the membership degree of an element to a given set through multiple possible numerical values, reflecting hesitation or ambiguity in judgment.*

Table 4.2: Relative closeness of the alternatives.

Risk Factors	D_b^+	D_b^-	$RC_b = \frac{D_b^-}{D_b^- + D_b^+}$
$\tilde{\theta}_1$	0.122	0.5225	0.811
$\tilde{\theta}_2$	0.0715	0.704	0.91
$\tilde{\theta}_3$	0.482	0.357	0.42
$\tilde{\theta}_4$	0.556	0.1725	0.24
$\tilde{\theta}_5$	0.3255	0.3735	0.53
$\tilde{\theta}_6$	0.1375	0.615	0.82

4.3 Selection of criteria and risk factors

The objective of this study is to rank the key risk factors contributing to the spread of the coronavirus. The selection of criteria was guided by expert input, primarily based on medical professionals' recommendations. The literature review served as a key source in identifying relevant risk factors. Additionally, media surveys played a significant role in the selection of these factors. In this study, the most significant risk factors contributing to the spread of COVID-19 have been identified based on three key criteria: doctors' opinions (ψ_1), literature review (ψ_2), and media reports (ψ_3), as referenced by Majumder et al. (2020). The selected risk factors are presented in Table 4.3. The hierarchical structure illustrated in Fig. 4.2 represents the three criteria used to determine the six chosen risk factors.

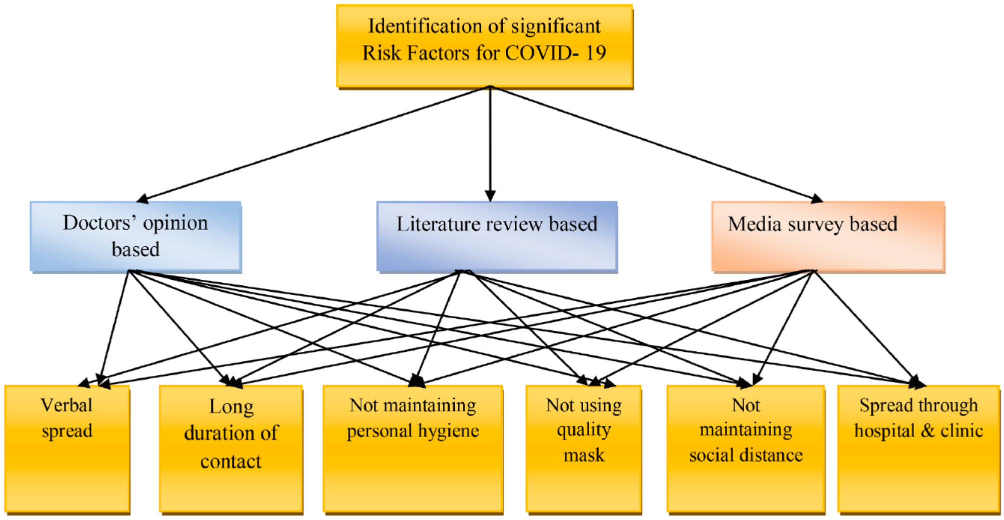

Figure 4.2: Hierarchical structure of the study.

Table 4.3: Representation of different risk factors

Risk factors associated with the disease	**Brief description**
Verbal spread (Θ^1)	Verbal spread is due to the cough, sneezing and viral load of an infected person. (Cliver 2009)
Long duration of contact with the infected person (Θ^2)	Time factor significantly depends when a normal person is with an infected person.
Not maintaining personal hygiene (Θ^3)	Frequently usage of sanitizer, washing hands, changing clothes if a person comes from outside is highly recommended.
Not using quality mask(Θ^4)	N-95 and surgical mask are recommended by doctors, as these masks lowers down the risk, whereas it is seen that the normal mask doesn't prevent the disease and probability is higher if not using quality masks.
Not maintaining distance in public transport or public places (Θ^5)	Maintaining social distance is very important in order to prevent the pandemic.
Infection spread through hospitals and clinic (Θ^6)	A person has higher probability of getting infected, if he/she visits hospitals and clinic.

4.4 Empirical study

Data sources for our study are the opinion of doctors, literature review (Cliver, 2009; Majumder et al., 2020) and media survey. The DMs comparative preferences for the criteria in terms of TFN are represented in Tables 4.4 and 4.5 using Table 4.1. The entire process of FAHP used in the study is described below.

Step 1: Construction of comparison matrix

Table 4.4: Comparison matrix by DM 1

Comparison Matrix (DM1)	(ψ_1)	(ψ_2)	(ψ_3)
(ψ_1)	1	(3.5,4,4.5)	(4.5,5,5.5)
(ψ_2)	(0.22,0.25,0.286)	1	(3.5,4,4.5)
(ψ_3)	(0.18,0.2,0.22)	(0.22,0.25,0.286)	1

Table 4.5: Comparison matrix by DM 2

Comparison Matrix (DM2)	(ψ_1)	(ψ_2)	(ψ_3)
(ψ_1)	1	(0.22,0.25,0.286)	(4.5,5,5.5)
(ψ_2)	(3.5,4,4.5)	1	(4.5,5,5.5)
(ψ_3)	(0.18,0.2,0.22)	(0.18,0.2,0.22)	1

Step 2: Aggregation of TFN using Eq. (4.1).
Step 3: Defuzzification of TFN using Eq. (4.2).
Step 4: Normalization of the matrix using Eq. (1.2).
Step 5: Determination of criteria weights using Eq. (1.3).

Table 4.6: Representation of criteria weights.

Criteria	(ψ_1)	(ψ_2)	(ψ_3)
Weight	0.47	0.45	0.08

Table 4.6 depicts the criteria weights obtained by FAHP. Table 4.7 represents the hesitant fuzzy decision matrix used in TOPSIS approach for expressing opinion about the risk factors on the basis of criteria used in the study.

Table 4.7: Hesitant decision matrix.

Risk Factors	(ψ_1)	(ψ_2)	(ψ_3)
Verbal spread	(0.5,0.7,0.9)	(0.6,0.7)	(0.8)
Long duration of contact	(0.9)	(0.6,0.7,0.8,0.9)	(0.7,0.8)
Not maintaining personal hygiene	(0.4,0.5,0.6)	(0.1,0.3,0.5)	(0.5)
Not using quality mask	(0.2,0.3,0.4,0.5)	(0.3)	(0.4,0.5)
Not maintaining distance in public transport or public places	(0.6,0.7)	(0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)
Infection spread through hospitals and clinic	(0.8,0.9)	(0.6,0.7,0.8)	(0.4,0.5,0.6)

Step 1: Identification of PIS and NIS using Eqs. (4.3) and (4.4):

$A_p^+ = \{0.9, 0.9, 0.8\}$

$A_p^- = \{0.2, 0.1, 0.3\}$

Step 2: Separation measures d_b^+ and d_b^- are calculated for each alternative using Eqs. (4.5) and (4.6):

$d_1^+ = 0.47 \times ||\{0.5, 0.7, 0.9\} - \{0.9\}|| + 0.45 \times ||\{0.6, 0.7\} - \{0.9\}|| + 0.08 \times ||\{0.8\} - \{0.8\}||$

The Hamming distance between two HFE is calculated using Eq. (4.8):

$||\{0.5, 0.7, 0.9\} - \{0.9\}|| = \{\frac{1}{3}[|0.5 - 0.9| + |0.7 - 0.9| + |0.9 - 0.9|]\} = 0.2$

Table 4.8 represents the separation measures and subsequent relative closeness calculation.

Table 4.8: Relative closeness of the alternatives.

Risk Factors	(D_b^+)	(D_b^-)	$RC_b = \frac{D_b^-}{D_b^- + D_b^+}$	**Rank**
θ_1	0.122	0.522	0.811	3
θ_2	0.0715	0.704	0.91	1
θ_3	0.482	0.357	0.42	5
θ_4	0.556	0.1725	0.24	6
θ_5	0.3255	0.3735	0.53	4
θ_6	0.1375	0.615	0.82	2

From the analysis, it is seen that the risk factor 'long duration of contact (θ_2)' is the most significant risk factor. Arranging the ranks in descending order, we see $\theta_2 > \theta_6 > \theta_1 > \theta_5 > \theta_3 > \theta_4$. This shows 'long duration of contact with the infected person' is the most significant risk factor, followed by 'spread through hospitals and clinic', 'verbal spread', 'not maintaining distance in public transport or public places', 'not maintaining personal hygiene' and 'not using quality mask'.

4.5 Sensitivity analysis

To assess the sensitivity of the obtained ranking, several alternative ranking techniques are employed. Specifically, we apply AHP-TOPSIS using crisp values, the Parametric Form of Interval Numbers (PIVN) AHP-TOPSIS, and the Triangular Fuzzy Number (TFN) FAHP-FTOPSIS methods.

a.) **Crisp AHP-TOPSIS**- In this MCDM technique, the linguistic terms are considered as crisp numbers. The linguistic terms used for the comparison matrix are crisp values. The linguistic terms for preferential rating of the alternatives are also crisp values. Karmaker et al. (2018) used AHP-TOPSIS to check the preference order of the teachers in an educational institute.

b.) **PIVN AHP-TOPSIS**- In this method, parametric form of interval numbers is considered. Pal and Mahapatra (2017) represented the interval numbers in various functional forms and explained the concept with arithmetic operations.

c.) **FAHP-FTOPSIS**- In FAHP, fuzzy numbers are used for the construction of comparison matrix and the weights obtained are fuzzy weights. The FTOPSIS is an extension of classical TOPSIS approach as it solves the decision-making problem with fuzzy uncertainty. Sun (2010) used FAHP-FTOPSIS for performance evaluation of industrial practitioner in terms of TFN.

Table 4.9: Rankings obtained under sensitivity analysis

Risk Factors	FAHP HFS-TOPSIS ($M1$)	Crisp AHP-TOPSIS ($M2$)	PIVN AHP-TOPSIS ($M3$)	FAHP-FTOPSIS ($M4$)
$\tilde{\theta}_1(R1)$	3	4	5	3
$\tilde{\theta}_2(R2)$	1	5	3	2
$\tilde{\theta}_3(R3)$	5	6	6	6
$\tilde{\theta}_4(R4)$	6	3	4	5
$\tilde{\theta}_5(R5)$	4	2	2	1
$\tilde{\theta}_6(R6)$	2	1	1	4

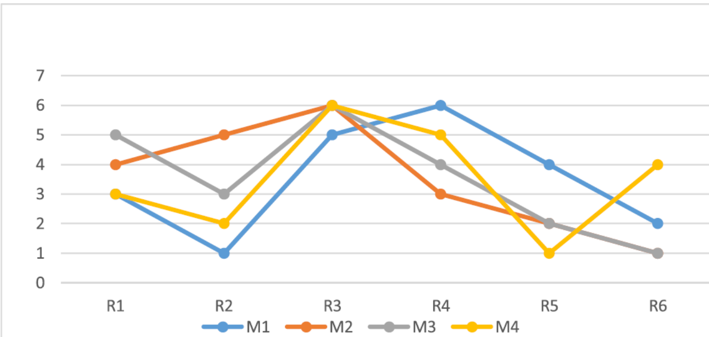

Figure 4.3: Line Chart depicting sensitivity of the ranking based on different methods.

4.6 Results discussion

Based on the results obtained from FAHP, HFS-TOPSIS, and sensitivity analysis, it is observed that both FAHP and HFS-TOPSIS consistently ranked "long duration of contact with an infected person" as the most significant risk factor. This is followed by “spread through hospitals and clinics,” “verbal transmission,” “not maintaining social distance in public places or public transport,” “lack of personal hygiene,” and “not using a quality mask.” In contrast, the crisp AHP-TOPSIS and PIVN AHP-TOPSIS methods identified “spread through hospitals and clinics” as the most critical factor. Meanwhile, the FAHP-FTOPSIS method ranked “not maintaining social distance in public places and transport” as the top risk factor.

4.7 Conclusions

In this study, the most significant risk factors contributing to the spread of COVID-19 have been analyzed using the FAHP and HFS-TOPSIS methods. The selection of risk factors has been guided by three key criteria: doctors’ opinions, literature reviews, and media surveys. The weights of these criteria have been determined through FAHP, while HFS-TOPSIS has been employed to identify and rank the most critical risk factors. One of the key advantages of using HFS-TOPSIS is its ability to provide a comprehensive analysis, effectively capturing the uncertainty and hesitation in expert evaluations.

Future research could expand upon this work by incorporating additional risk factors. Moreover, future studies could focus on scientifically determining optimal locations for quarantine centers, planning for isolation facilities, designing safe homes, developing standards for protective masks, and constructing models for epidemic control and hospital bed augmentation. The development of mathematical models that account for varying levels of intervention during lockdowns would also aid in formulating optimal lockdown strategies.

5

Application of Hexagonal Fuzzy MCDM Methodology for Site Selection of Electric Vehicle Charging Station

5.1 Introduction

Transportation is a significant contributor to urban air pollution and reduction of urban city emissions is the need of the hour. High levels of pollution are degrading the environment and have made the concept of sustainable development a fairy-tale phenomenon (Bilgen, 2014; Zhao et al., 2014). Sustainable consumption needs to be adopted by conducting timely environmental and sustainability assessments to prevent any large-scale ecological disaster from happening. This can be done by utilizing cleaner production and technical processes and by employing sustainable products and services. This is where electrical vehicles come into the picture with sustainability, environment protection and pocket-friendly being a few of its rewards.

Electric vehicles use a minimum of one electric motor or traction motor for propulsion. They may be self-contained with a generator or battery for converting the fuel into electricity or they may be powered via a collector scheme by using electricity from off-vehicle sources (Hernández et al., 2018). The problem of energy crisis in the world can be tackled in the future using this option. The ever-rising gas prices force people to look for alternative modes of energy supply or to even switch over

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to walking or availing public transportation. The other reason why electrical vehicles are preferred over fossil-fuel ones is due to their eco-friendly nature and the cheaper cost of driving them (Schoettle and Sivak, 2016; Singh et al., 2013). Filling up an electric car costs fewer pennies than filling up a full tank. Internal combustion engines emit more CO_2 emissions than electric vehicles (Graham et al., 2012; Mak et al., 2013; Hernández et al., 2017). Moreover, to promote the ratio of charging location, we need to focus on two facets: the power of the government and the role of the market. The government influences taxation policies and subsidy policies directly on the stations, thus affecting the income of the charging station. On the other hand, from the market viewpoint, the key is the demand and supply relationship i.e. the consumers who demand the electric vehicles and the supply of electric vehicle charging and the electric vehicle charging stations (Fang et al. 2020). Electric vehicles can usually be charged in three ways i.e. using inductive charging, conductive charging, and battery replacement method (Hernández et al., 2017). Inductive method of charging works through electromagnetic transmission keeping no contact between the charging station and the electric vehicle. Conductive method has a battery connected by a cable which is directly plugged into an electricity provider whereas the battery replacement method replaces the discharged batteries with new batteries in a charging station by keeping in mind the internal connections and the dimensions of the batteries resulting in it being the least used method. The charging station operators mostly use conductive charging since it is more efficient and cheaper (Dericioglu et al., 2018).

5.1.1 *Objectives of the study*

The present research has the following objectives:

- Identification of the most preferred site for the construction of E-vehicle charging station.
- Application of Hexagonal Fuzzy Numbers (HFN) in AHP-TOPSIS and AHP-COPRAS to obtain ranking of the selected sites.

5.1.2 *Novelty of the study*

Several researchers have explored fuzzy numbers with MCDM techniques AHP, TOPSIS, COPRAS. Hardly any research has been done using hexagonal fuzzy numbers under MCDM methodology. In this research, HFN defuzzification formulae has been developed and utilized. Distance measure between two HFN is also defined. Formulae has been derived to calculate hexagonal fuzzy weight of factors and sub-factors. Technique has been developed to incorporate more than one decision maker's opinion into a single comprehensive value in terms of HFN. Two different ranking methods Fuzzy AHP-TOPSIS and Fuzzy AHP-COPRAS have been used in this research. GIS software has been used for distance measurement and graphical presentation of the selected sites.

5.1.3 *Structure of the chapter*

The remainder of this chapter is organized as follows: Section 5.2 introduces the MCDM techniques—AHP, Fuzzy TOPSIS, and Fuzzy COPRAS. Section 5.3 presents the numerical application along with a detailed description of the selected factors and sub-factors. Section 5.4 provides a comparison of results and sensitivity analysis. The Section 5.5 outlines the future scope of research and concludes the chapter. The structured framework of the study, illustrated in Figure 5.1, outlines the sequential steps followed in this analysis. After the initial selection of factors and sub-factors, their weights are determined using the FAHP method. Subsequently, the fuzzy TOPSIS and fuzzy COPRAS techniques are applied to identify preferred locations, followed by comparative and sensitivity analyses.

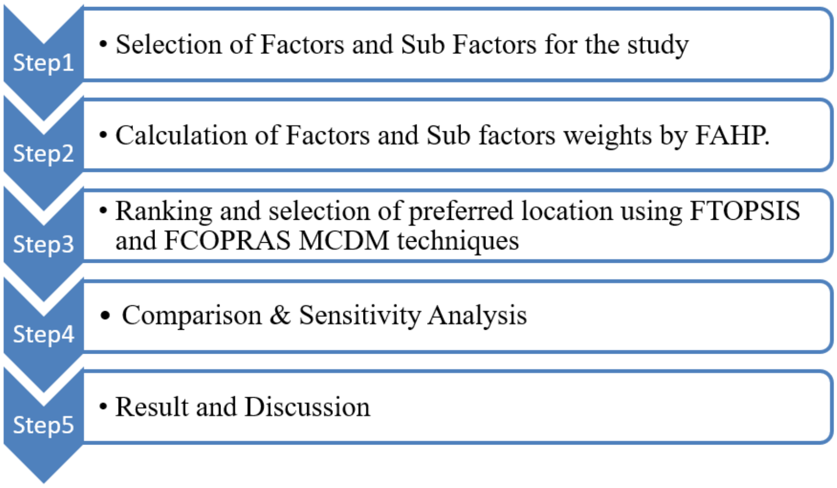

Figure 5.1: Structural framework of the study

5.2 MCDM techniques used in this study

(i) **Fuzzy Analytic Hierarchy Process (FAHP)**

This method is widely used for evaluating the weights of factors and sub-factors. The process of comparing these elements and assigning preferences in linguistic terms can often be a challenging and hesitant task for decision-makers (DMs). To address this uncertainty, the integration of Hesitant Fuzzy Numbers (HFNs) with the AHP methodology effectively captures the inherent vagueness of the decision-making problem. Accurate determination of the weights of factors and sub-factors is essential for ranking electric vehicle charging stations. AHP structures the problem into a hierarchy, where pairwise comparison matrices are developed to reflect subjective judgments about criteria and sub-criteria. By incorporating fuzzy logic, the FAHP approach enables decision-makers to derive more reliable and robust evaluations. The steps involved in the FAHP method are outlined below.

- Construction of a comparison matrix in terms of HFN by group of decision experts. Let there be a group of H decision-makers assigned to perform the comparisons of factors and sub-factors. Let DMs express their preference in the pairwise comparison of factors and sub-factors. Thus 'h' set of matrices are obtained $T^h = \{t^h_{cd}\}$ where $t^h_{cd} = (m^h_{cd}, n^h_{cd}, o^h_{cd}, p^h_{cd}, q^h_{cd})$

denotes the HFN of c factor to d factor as expressed by the 'h' DM.

$$m_{cd} = \min_{h=1,2,\ldots,H} m^h_{cd} \tag{5.1}$$

$$n_{cd} = \min_{h=1,2,\ldots,H} n^h_{cd} \tag{5.2}$$

$$o_{cd} = \frac{1}{H}\sum_{h=1}^{H} o^h_{cd} \tag{5.3}$$

$$p_{cd} = \frac{1}{H}\sum_{h=1}^{H} p^h_{cd} \tag{5.4}$$

$$q_{cd} = \max_{h=1,2,\ldots,H} q^h_{cd} \tag{5.5}$$

$$r_{cd} = \max_{h=1,2,\ldots,H} r^h_{cd} \tag{5.6}$$

Determination of hexagonal fuzzy weights of factors and sub-factors
We extend the methodology developed by Buckley (1985), for TFN in the context of determining hexagonal fuzzy weight.

- The geometric mean value of HFN is obtained as

$$k_c = \left(\prod_{d=1}^{j} y_{cd}\right)^{\frac{1}{c}} \quad \text{where } c = 1, 2, \ldots, i \tag{5.7}$$

- Compute the sum of each k_c value.
- Calculate the inverse of each k_c and arrange it in increasing order.
- Find the Hexagonal fuzzy weights of factors and sub-factors using the following formula:

$$w_c = k_c \times \left(\sum_{i=1}^{i} k_i\right)^{-1} \tag{5.8}$$

- Compute global hexagonal fuzzy weights of sub-factors by the product of factor weight with the respective sub-factor fuzzy weight.

- Defuzzification of HFN: A HFN can be defuzzified by using the centroid based method. Using eq. (1.46), HFN is transformed to a crisp value. After defuzzification of HFN, the other steps of AHP can be computed using Eqs. (1.2), (1.3), (1.4) and (1.5).

(ii) **FTOPSIS**

The linguistic human decisions can be reflected suitably with Fuzzy TOPSIS (FTOPSIS). The approach is useful in handling the complexity of the situation involving several factors and their sub-factors. The steps of FTOPSIS are described below.

- Construct the decision matrix by the help of decision experts in terms of linguistic terms. The linguistic terms are then converted to HFN.
- Evaluate the normalized HFN fuzzy decision matrix:

$$ND = [n_{gh}]_{s\times t}, \quad g = 1,2,\ldots,s; h = 1,2,\ldots,t \tag{5.9}$$

$$n_{gh} = \left(\frac{a_{1gh}}{a_{6gh}}, \frac{a_{2gh}}{a_{6gh}}, \frac{a_{3gh}}{a_{6gh}}, \frac{a_{4gh}}{a_{6gh}}, \frac{a_{5gh}}{a_{6gh}}, \frac{a_{6gh}}{a_{6gh}}\right), d \in BA \tag{5.10}$$

$$a_6^* = \max(a_{gh}) \tag{5.11}$$

$$n_{gh} = \left(\frac{a_h^*}{a_{1gh}}, \frac{a_h^*}{a_{2gh}}, \frac{a_h^*}{a_{3gh}}, \frac{a_h^*}{a_{4gh}}, \frac{a_h^*}{a_{5gh}}, \frac{a_h^*}{a_{6gh}}\right), d \in NBA \tag{5.12}$$

$$a_h^* = \min(a_{1gh}) \tag{5.13}$$

  where BA and NBA signifies the benefit attributes and non-benefit attributes, respectively.

- Evaluate the weighted fuzzy normalized matrix by multiplying the sub-factors' fuzzy weights with the normalized fuzzy value:

$$WN = [P_{gh}]_{s\times t} \quad g = 1,2,\ldots,s; h = 1,2,\ldots,t \tag{5.14}$$

  where

$$P_{gh} = N_{gh} \times W_h \quad g = 1,2,\ldots,s; h = 1,2,\ldots,t \tag{5.15}$$

- Calculate the Fuzzy Positive Ideal Solution (FPIS) (PIS+) and Fuzzy Negative Ideal Solution (FNIS) (NIS-) where h_g^+ denotes the maximum value of h_g, and h_g^- denotes the minimum value of h_g:

$$\begin{aligned} PIS^+ &= \{a_1^+, a_2^+, \ldots, a_t^+\} \\ &= \{\max(a_{gh} \mid h \in M_B), \min(a_{gh} \mid h \in M_{NB})\} \qquad (5.16) \\ NIS^- &= \{a_1^-, a_2^-, \ldots, a_t^-\} \\ &= \{\min(a_{gh} \mid h \in M_B), \max(a_{gh} \mid h \in M_{NB})\} \end{aligned} \qquad (5.17)$$

where MB denotes the benefit attribute and MNB denotes the non-benefit attribute.

- Calculate the distance measure of all alternatives from the PIS and NIS. The two Euclidean distances for individual alternatives can be calculated as follows:

$$L_g^+ = \sum_{h=1}^{t} d(P_{gh}, h_g^+), g = 1, 2, \ldots, s \qquad (5.18)$$

$$L_g^- = \sum_{h=1}^{t} d(P_{gh}, h_g^-), g = 1, 2, \ldots, s \qquad (5.19)$$

where $d(\cdot)$ denotes the Euclidean distance between two fuzzy numbers.

- Determine the relative closeness to the ideal alternatives:

$$R_g = \frac{L_g^-}{L_g^- + L_g^+}, \quad g = 1, 2, \ldots, s \qquad (5.20)$$

- Rank the alternatives: The alternatives are ranked based on the scores obtained by R_g. The larger value of R_g signifies the better alternative.

(iii) **Fuzzy COPRAS**

The Complex Proportional Assessment (COPRAS) method was first introduced by Zavadskas et al. (1994). Fuzzy COPRAS is an extended approach of COPRAS technique widely used for decision making problems. It uses step-wise ranking and evaluation procedure for the alternatives with reference to significance and utility degree. An extension of COPRAS method is Fuzzy COPRAS which is frequently used in decision making problems. Ghose et al. (2019), used a hybrid fuzzy COPRAS method for selecting the optimal material to be used for a solar car. The reason for using fuzzy-based MCDM technique was that it helps decision makers to get over the problems of ambiguous data. The steps of COPRAS method are illustrated below:

- Construct the decision matrix in terms of HFN, the alternatives are given linguistic terms by the decision experts with respect to the criteria.
- Normalize the decision matrix using Eq. (1) in the similar way we constructed for TOPSIS normalized matrix.
- Construct weighted normalized matrix by multiplying the criteria weights with fuzzy normalized matrix using Eq. (5.15).
- Aggregate the beneficial (B_g^+) and non-beneficial (NB_g^-) indices for each alternative evaluated.

$$\begin{aligned} B_g^+ &= \sum_{h=1}^{m} NW_{a1} + \sum_{h=1}^{m} NW_{a2} + \sum_{h=1}^{m} NW_{a3} + \sum_{h=1}^{m} NW_{a4} \\ &+ \sum_{h=1}^{m} NW_{a5} + \sum_{h=1}^{m} NW_{a6} \end{aligned} \tag{5.21}$$

$$\begin{aligned} NB_g^- &= \sum_{h=m+1}^{t} NW_{a1} + \sum_{h=m+1}^{t} NW_{a2} + \sum_{h=m+1}^{t} NW_{a3} + \sum_{h=m+1}^{t} NW_{a4} \\ &+ \sum_{h=m+1}^{t} NW_{a5} + \sum_{h=m+1}^{t} NW_{a6} \end{aligned} \tag{5.22}$$

  where each $h = 1, 2, \ldots, m$ represents the benefit attribute of the alternatives and each $h = m+1, m+2, \ldots, t$ represents the non-benefit attribute of the alternative.

- Finally, defuzzify the aggregated beneficial and non-beneficial indices using Eq. (9) and determine R_g^+ and R_g^-.
- Calculate R_g using the following formulae:

$$R_g = R_g^+ + \left(\min_{g=1}^{l} R_g^-\right) / R_g^- \left(\sum_{g=1}^{l} R_g^-\right) / \left(\min_{g=1}^{l} R_g^-\right) \tag{5.23}$$

- Rank the alternatives using the formulae:

$$R = \left(R_g / R_{\max}\right) \times 100\% \tag{5.24}$$

where R_g represents the g-th defuzzified value and $R_{\max}$ represents the maximum defuzzified value of individual alternative.

5.3 Numerical example

The factors and sub-factors considered in this study are explained as follows:

Economic factors (C_1)

Assessment of the economic factors can reveal the feasibility of the present study. The factors considered are:

- **Land cost (C_{11})** - Since the purpose is to build a charging station we can minimize the land cost by utilizing an already existing utility station. If the money is saved on the cost of procuring the land then it can be utilized for setting up the station.

- **Operating and management cost (OMC) (C_{12})** - The charging station should be so executed such that the operations can be systematically planned which will reduce the operating and management costs. Since electric vehicles will reduce the air pollution hence initial operating costs are understandable since the long run implications outweighs the costs.

- **Consumption level (C_{13})** - A charging station can be built in a high consumption area since the throng of people will have more ways of traveling and procuring their wants.

- **Construction cost (C_{14})** - Construction cost varies with the location. If the location is well connected by various transportation facilities then the cost of transferring the construction materials will decrease which will decrease the construction cost and the overall profitability of the charging station will increase initially.

- **Public facilities (C_{15})** - In case a charging station is built near a location having a large density of public facilities it will act as a boon, since money will frequently change hands and thereby develop the area.

Environmental factors (C_2)

It refers to those factors which will influence the immediate surroundings of the charging station. A clean and green environment helps in resonating the theme of the electric vehicles and thereby makes a charging station built in such a location a success.

- **Generation of noise and air pollution (GNAP) (C_{21})** - Electric vehicles contribute to reducing both air and noise pollution, as their battery-powered operation produces no emissions and runs quietly on the road.
- **Petrol stations (C_{22})** - Building a charging station near a petrol station will cause an increase in the number of vehicles and people visiting the area and thereby turn out to be more profitable since the cost will be less.
- **Transportation stations (C_{23})** - The greater the frequency of the transportation vehicles, the more will be its impact on the environment. Electric vehicles help in the easy transportation of people without harming the environment.

Traffic factors (C_3)

It refers to those set of factors which are only noticeable when there is a huge population in the area. A charging station which takes into consideration the traffic factors is one which will be able to have huge implications on the state of travel in the area.

- **Number of roads (C_{31})** - If the charging station is strategically built near the intersection of the heavy traffic roads then it will help the drivers more at ease while driving since they will have a backup nearby.
- **Road potency (C_{32})** - More the number of vehicles in the region greater will be the footfall which will eventually increase the success rate of the charging station.
- **Parking areas (C_{33})** - If the charging station is built near the parking area then the vehicle owners can directly charge and park it.

Societal factors (C_4)

The societal factors point out the major problems which may be looked at to improve the quality of the society at large.

- **Adverse impact of noise and electromagnetic field (AI) (C_{41})** - An electric vehicle charging station has a constant aura of noise and electromagnetic field surrounding it during the construction phase which might cause certain category of people to develop problems. If proper measures can be taken in the initial stage then this impact maybe minimized since public health is of utmost importance.

- **Population density (C_{42})** - The need of transportation in high density population area is usually very high and an electric vehicle charging station constructed in such an area may just be what the people need.

Table 5.1: Factors and sub-factors considered in the present study

Factors	**Sub-factors**
Economic factors (C_1)	Land Cost (C_{11}), Operating and Management Cost (OMC) (C_{12}), Consumption Level (C_{13}), Construction Cost (C_{14}), Public facilities (C_{15})
Environmental factors (C_2)	GNAP (C_{21}), Petrol stations (C_{22}), Transportation Station (C_{23})
Traffic factors (C_3)	Number of Roads (C_{31}), Road Potency (C_{32}), Parking Areas (C_{33})
Societal factors (C_4)	Adverse Impact of noise and electromagnetic field (AI) (C_{41}), Population Density (C_{42})

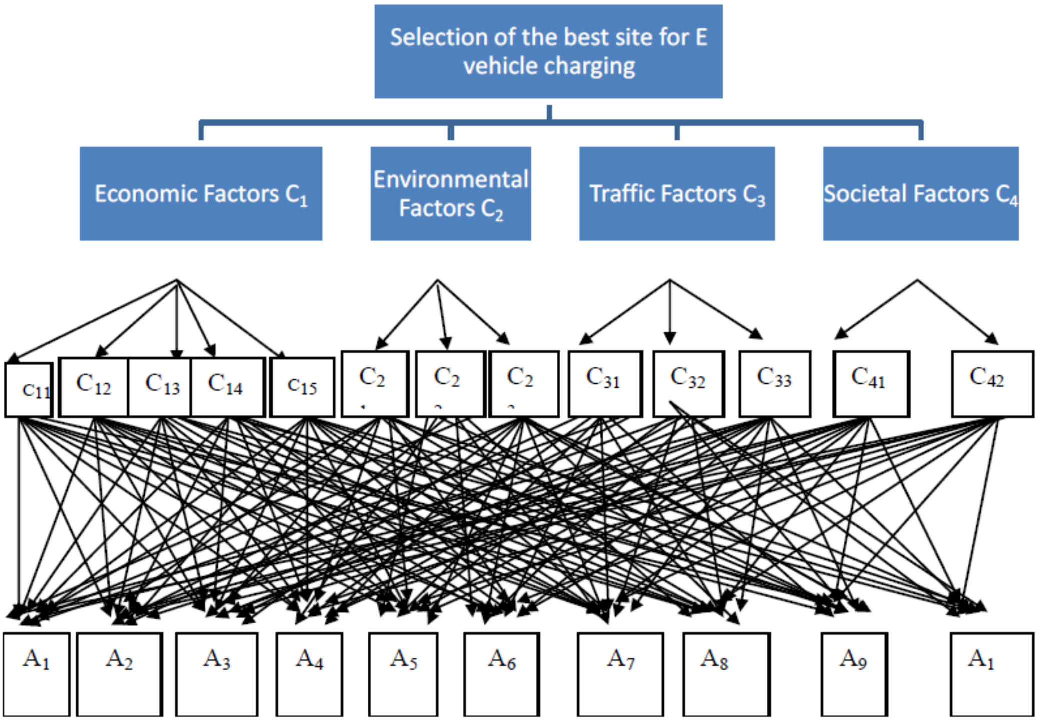

Figure 5.2: Hierarchical structure of the problem

Table 5.2: Alternatives and their correspondence nearby location, latitude, and longitude

Alternative	Nearby location	Latitude and longitude
Dasnagar	HP petrol pump (Debi Service station)	22.599152, 88.307854
Santragachi	HPCL petrol pump	22.586515, 88.276026
Belgachia	Petrol pump	22.603168, 88.323001
Howrah Maidan	Near Kabra stores	22.581972, 88.332230
Liluah	Sur petrol pump	22.625105, 88.350044
Kadamtala	HP petrol pump	22.587778, 88.320151
Shibpur	Chowrabasti Shibpur	22.562826, 88.326159
Salkia	Malipanchghara	22.600887, 88.349325
Bakultala	Botanical Garden west end	22.564016, 88.288795
Belur	SSBPCL petrol pump	22.639311, 88.350857

The rural, urban and total population for the census years tarting from 2001 and 2011 has been represented in a graph (Fig 5.3). In this figure, the projected rural, urban and total population also have been forecasted for the year 2021, 2031, 2041 and 2051 in a chronological way. The graph clearly indicates a consistent increase in population from 4.23 million in 2001 to 6.60 in 2051 (projected) with a sharp increase of urban population and the urban population growth is very higher than the rural,

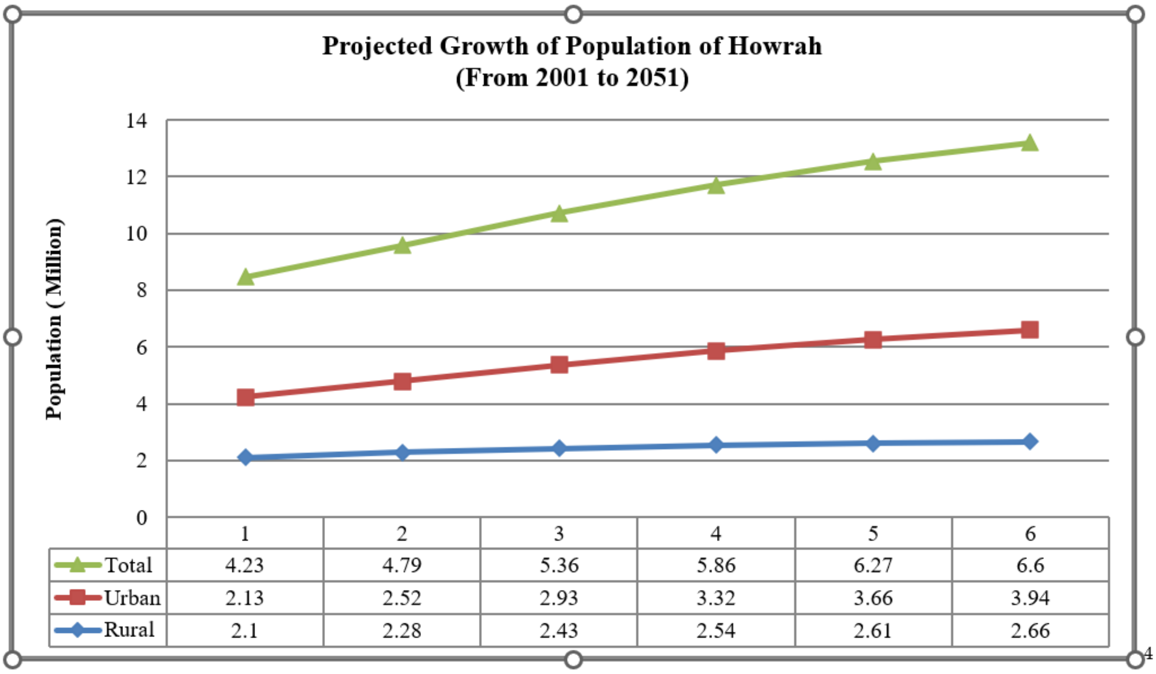

Figure 5.3: Projected growth of population of rural and urban area of Howrah district

which indicates higher increasing infrastructural demand. The projected population made us think about the future transport services in the area.

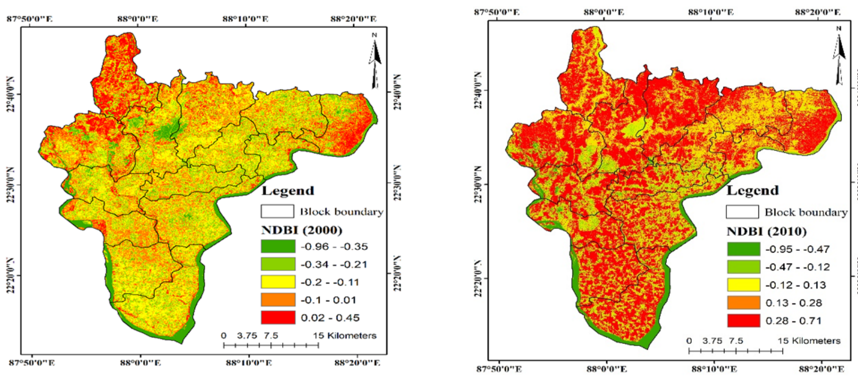

Figure 5.4: Normalized difference built-up index mapping for level of urbanization during 2000 to 2010

The present study considers the spatio-temporal characteristics of urban growth and its inference in the transport of Howrah. The built-up land (Fig 5.4) has been generated using the NDBI(Normalized Difference Built-Up Index) with the following equation: $NDBI = \frac{MIR-NIR}{MIR+NIR}$ (Zha et al., 2003). Here, NIR is a near-infrared band such as ETM + and TM and LISSIII is a band no.4, MIR is a middle infrared band such as ETM+ and TM and LISSIII is a band no.5. The index is based on the unique spectral response of built-up lands that have a higher reflectance in the MIR wavelength range as compared to that in the NIR wavelength range. Thereafter, the (NDBI) mapping has been prepared to understand the level of urbanization from 2000 to 2010 in the study area. It helps to correlate the changes in land use patterns and its consequences to the water storage of the study area. The NDBI values range from -1 to $+1$. Very low values of the NDBI (0.1 and below) correspond to non-urban features, while higher value indicate covering of areas of impervious surfaces such as asphalt and concrete. To understand the levels of urbanization the NDBI values have categorized into five zone, ranges from -0.96 to 0.45 in 2000 and from –0.95 to 0.71 in 2010.

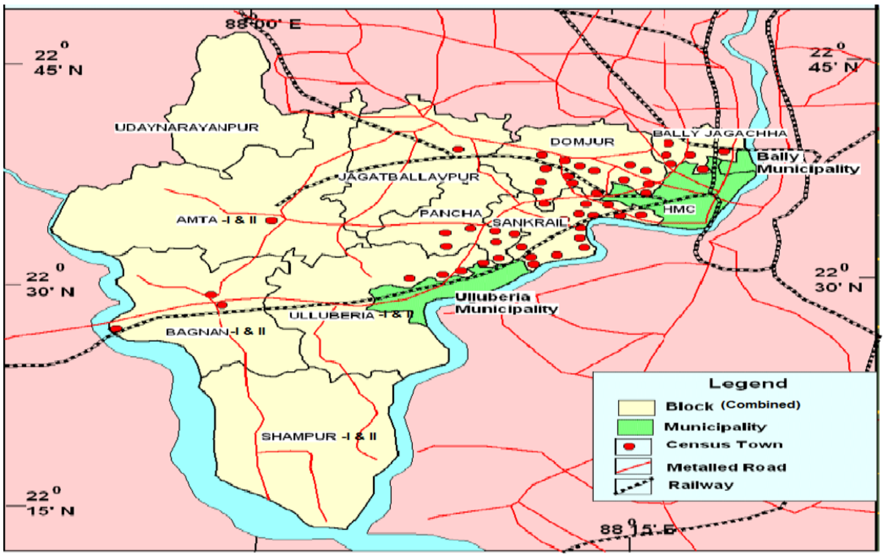

Figure 5.5: Transport network in and around Howrah along with the census town

The transport network of the Howrah district is mapped (Fig 5.6) and the closest census towns (Fig 5.5) to the HMC were also plotted to understand the importance of daily communication with the Howrah station or surrounding areas. A large number of daily commute are coming to the area, mostly for their economic and education purpose. It indicates the concentration of traffic in the area, in turn increase the public transport connectivity as well as requirement for improving the local transport system.

Figure 5.6: Transport Network of Howrah district

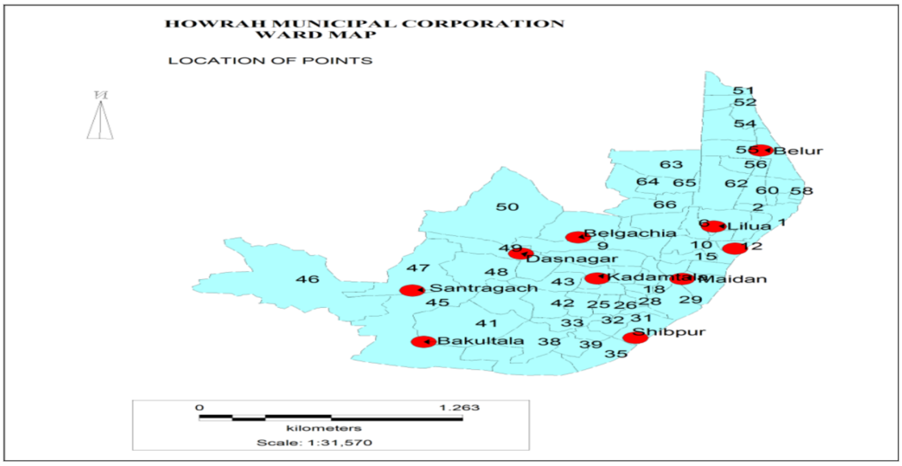

Figure 5.7: Location of Points in Howrah Municipal Corporation (66 wards)

The ten selected points/ locations (Fig 5.7) are mapped in the HMC to understand the spatial coverage and important transport nodes in the area of the current study. The population distribution of each ward is also mapped (Fig. 5.8) to observe the population pressure which is also able to justify the present selection of the ten locations for the study. Most of the selected locations are densely populated area, where the public and local transport services becoming very essential. Furthermore, the Howrah Maidan (S_4) and Salkia (S_8) are highly densely populated and thus very important in terms of transport services, whereas, Liluah (S_5), Belur (S_{10}) and Shibpur (S_7) belong to highly densely populated and Kadamtala (S_6), Bakultala (S_9), Belgachia (S_3) belong to moderately populated and Dasnagar (S_1) and Santragachi

(S_2) belong to comparatively less populated among the ten points. But all these points are almost equally important and essential in terms of either population or transport or both.

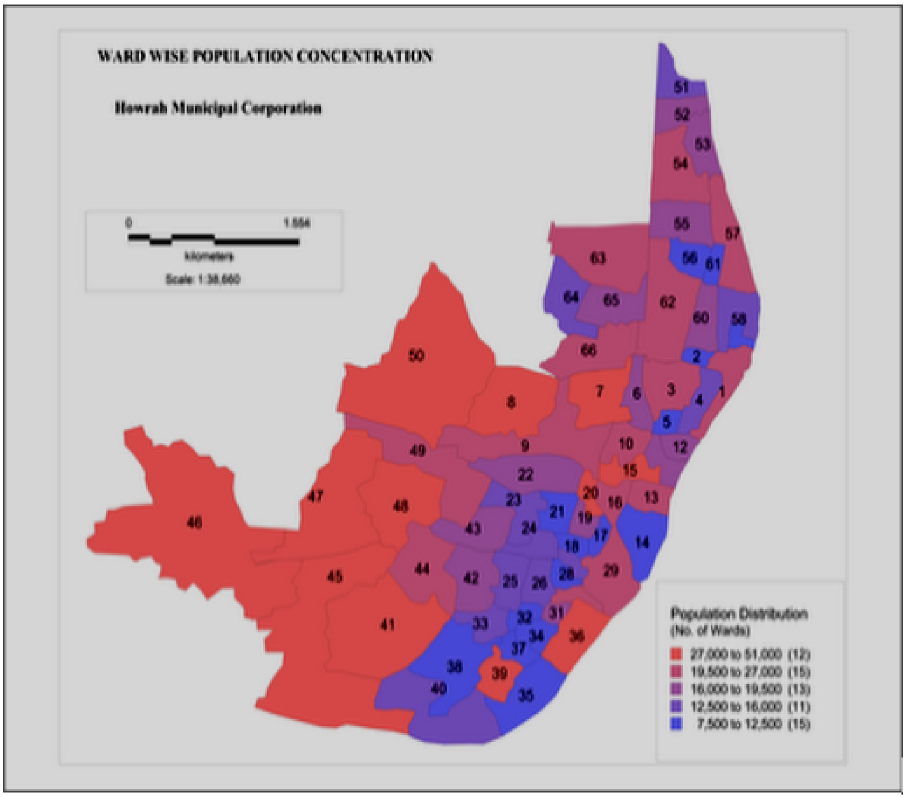

Figure 5.8: Ward wise population distribution of Howrah Municipal Corporation (66 wards)

Linguistic variables in HFN required for the comparison of factors and sub-factors are shown in Table 5.3.

Table 5.3: Linguistic term in HFN 1-9 scale

Linguistic Terms	**1-9 Scale**	**Hexagonal fuzzy number (HFN)**
Equally Important (EI)	1	1
Weakly Important (WI)	2	(1.1, 1.2, 1.3, 1.4, 1.5, 1.6)
Moderately Important (MI)	3	(1.8, 2.2, 2.5, 2.7, 3)
Strongly Important (SI)	5	(2.9, 3.2, 3.3, 3.5, 3.9)
Very Strongly Important (VSI)	7	(3.6, 4.1, 4.4, 4.5, 4.8)
Absolutely Important (AI)	9	(4.6, 4.8, 5, 5.2, 5.4, 5.7)
Absolutely Unimportant (AUI)	1/9	(0.17, 0.18, 0.19, 0.2, 0.21, 0.22)
Very Strongly Unimportant	1/7	(0.21, 0.22, 0.23, 0.24, 0.25, 0.26)
Strongly Unimportant (SUI)	1/5	(0.25, 0.27, 0.28, 0.3, 0.32, 0.33)
Moderately Unimportant (MUI)	1/3	(0.33, 0.36, 0.37, 0.38, 0.39, 0.42)
Weakly Unimportant (WUI)	1/2	(0.43, 0.45, 0.48, 0.5, 0.53, 0.54)

Table 5.4: A comparison among factors

Factors	Economic (C_1)		Environmental (C_2)		Traffic (C_3)		Societal (C_4)	
Decision maker	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
C_1	EI	EI	AUI	VSUI	SUI	VSUI	SUI	AUI
C_2	AI	VSI	EI	EI	SI	VSI	EI	MUI
C_3	SI	VSI	SUI	VSUI	EI	EI	SUI	VSUI
C_4	SI	AI	EI	MI	SI	VSI	EI	EI

Table 5.5: Preference of factors in defuzzified form

Factors	Economic	Environmental	Traffic	Societal
Economic	1	0.22	0.27	0.22
Environmental	4.7	1	3.8	0.74
Traffic	3.8	0.27	1	0.27
Societal	3.8	1.74	3.8	1

A comparison among factors in linguistic variables given by two DMs is presented in Table 5.3. Table 5.5 represents preference of factors in defuzzified form using Eq. (5.8). Normalized matrix is obtained using Eq. (1.2), and priority weights of factors are calculated using Eq. (1.3). Societal factors obtain the maximum weight of 0.430, followed by environmental factor 0.37, followed by traffic factor 0.139 and economic factor 0.065. The weights of the factors obtained are shown in Table 5.6. Using Eq. (1.4), CI is calculated as 0.08. As $n = 4$, the RI value is 0.09. Thus using Eq. (1.5), we find CR = $\frac{0.08}{0.9} = 0.09 < 0.1$. Hence the matrix is consistent. In the similar way, the comparison analysis of sub-factors can be calculated. The fuzzy weights of factors, sub-factors and global fuzzy weight are presented in Table 5.7.

Table 5.6: Priority weights of factors

Factors	C_1	C_2	C_3	C_4	Sum	E/Sum
C_1	0.065	0.081	0.038	0.095	0.278	4.277
C_2	0.305	0.366	0.529	0.318	1.518	4.15
C_3	0.247	0.099	0.139	0.116	0.601	4.313
C_4	0.247	0.637	0.529	0.430	1.843	4.29

Table 5.7: Hexagonal fuzzy weights of factors, sub- factors and global weight

Factors' fuzzy weight	Sub-factors' fuzzy weight	Global weight
C_1= (.04,.05,.07, .07,.10,.11)	C_{11}=(0.06,0.07,0.1,0.11,0.18,0.20)	C_{11}= (.003 ,.003,.007,.008,.018,.021)
	C_{12}= (.09,.11,.16,.17,.29,.32)	C_{12}= (.004,.005,.011,.012,.028,.035)
	C_{13}= (.13,.15,.24 ,.25,.41,.46)	C_{13}= (.006 ,.007,.016 ,.018,.040,.050)
	C_{14}= (.03,.04,.06,.06,.09,.10)	C_{14}= (.001,.002,.004,.004,.009,.011)
	C_{15}= (.23,.27,.41,.43,.68,.76)	C_{15}= (.01,.012,.03,.030,.07,0.08)
C_2= (.21,.23,.35,0.4,0.5,0.6)	C_{21}= (.08, .09, .11, .125,.16, .18)	C_{21}= (.02,.02, .04, .05,.08, .10)
	C_{22}= (.17,.19,.24,.3,.34,0.4)	C_{22}=(.04,.04,.08,.10,.19,.23)
	C_{23}= (.41,.46,.59,.65,.83,.93)	C_{23}= (.08,.11,.21,.24,.45,.54)
C_3=(0.09,0.10,0.14,0.15,0.20,0.22)	C_{31}= (.07,0.08,0.1.0.11,0.14,0.16)	C_{31}= (.01,.01,.01,0.02,.03,.035)
	C_{32}= (.49,.55,.66,.71,.9,.96)	C_{32}=(.045,.054,.09,.104,.18,.21)
	C_{33}= (.142,.16,.197,.214,.27,.31)	C_{33}= (.013,.015,0.03,.03,.06,.07)
C_4= (.26,.27,.42,0.45,0.69,0.76)	C_{41}= (.3,.3,.49,.50,.82,.87)	C_{41}= (.08,.082,.202,.23,.57,.66)
	C_{42}= (0.3,0.3,0.43,0.43,0.61,0.63)	C_{42}=(.08,.09,.18,.19,.42,.48)

Table 5.8: Comparison analysis in linguistic variables for preference of alternatives with respect to sub- factors

Locations	Sub-factors												
	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{31}	C_{32}	C_{33}	C_{41}	C_{42}
Dasnagar (S_1)	M	L	L	M	M	VL	9	9	3	H	M	VL	L
Santragachi (S_2)	H	L	L	M	L	VL	9	7	1	VH	H	VL	L
Belgachia (S_3)	M	M	VL	M	VL	H	9	3	2	H	L	H	M
Howrah Maidan (S_4)	VH	VH	VH	VH	VH	VH	7	7	4	VH	L	VH	VH
Liluah (S_5)	H	H	M	VH	VH	H	9	5	1	M	L	H	H
Kadamtala (S_6)	H	H	H	H	H	H	9	1	2	M	L	VH	M
Shibpur (S_7)	H	H	VH	H	H	M	9	1	1	M	L	M	H
Salkia (S_8)	H	VH	VH	VH	H	VH	5	1	2	M	L	VH	VH
Bakultala (S_9)	M	L	H	H	L	L	9	1	1	L	L	L	M
Belur (S_{1_0})	M	M	M	M	L	H	9	3	1	M	L	H	H

Note 7. *: (C_{22}), (C_{23}) and (C_{31}) are represented with crisp numbers.*

5.3.1 Ranking of alternatives using fuzzy AHP-TOPSIS method

Following the steps outlined in section 5.2, distance measure, relative closeness and ranking of sites are computed as depicted in Table 5.9.

Table 5.9: Distance measure, relative closeness and ranking of sites

Alternatives	(L_G^+)	(L_G^-)	$R_G = \frac{L_G^-}{L_G^+ + L_G^-}$	**Ranking**
Dasnagar(S_1)	0.441	0.264	0.375	8
Santragachi (S_2)	0.468	0.238	0.337	9
Belgachia (S_3)	0.271	0.435	0.616	4
Howrah Maidan (S_4)	0.058	0.647	0.918	1
Liluah (S_5)	0.184	0.522	0.739	2
Kadamtala (S_6)	0.302	0.404	0.572	6
Shibpur (S_7)	0.285	0.422	0.596	5
Salkia (S_8)	0.259	0.446	0.633	3
Bakultala (S_9)	0.379	0.328	0.464	4
Belur (S_{10})	0.239	0.468	0.662	3

5.3.2 Ranking of alternatives using fuzzy AHP-COPRAS method

Following the steps outlined in section 5.2, the values of R_g^+, R_g^-, R_g, R and ranking are computed as depicted in Table 5.10.

Table 5.10: Values of R_g^+, R_g^-, R_g, R and ranking

Alternatives	R_g^+	R_g^-	R_g	R	**Ranking**
Dasnagar(S_1)	0.68	0.19	0.691	77.98	7
Santragachi (S_2)	0.66	0.19	0.703	79.35	6
Belgachia (S_3)	0.59	0.08	0.707	79.82	5
Howrah Maidan (S_4)	0.75	0.06	0.886	100	1
Liluah (S_5)	0.65	0.07	0.778	87.82	2
Kadamtala (S_6)	0.55	0.07	0.680	76.81	9
Shibpur (S_7)	0.58	0.08	0.685	77.33	8
Salkia (S_8)	0.58	0.06	0.716	80.82	4
Bakultala (S_9)	0.52	0.10	0.614	69.34	10
Belur (S_{10})	0.62	0.08	0.732	82.66	3

5.4 Comparison and sensitivity analysis

Two different MCDM techniques fuzzy AHP-TOPSIS and fuzzy AHP-COPRAS are employed for the selection of optimal site for electric vehicle charging station in and around the city of Howrah, West- Bengal, India. Figure 5.9 shows the comparative ranking obtained under the two methodologies.

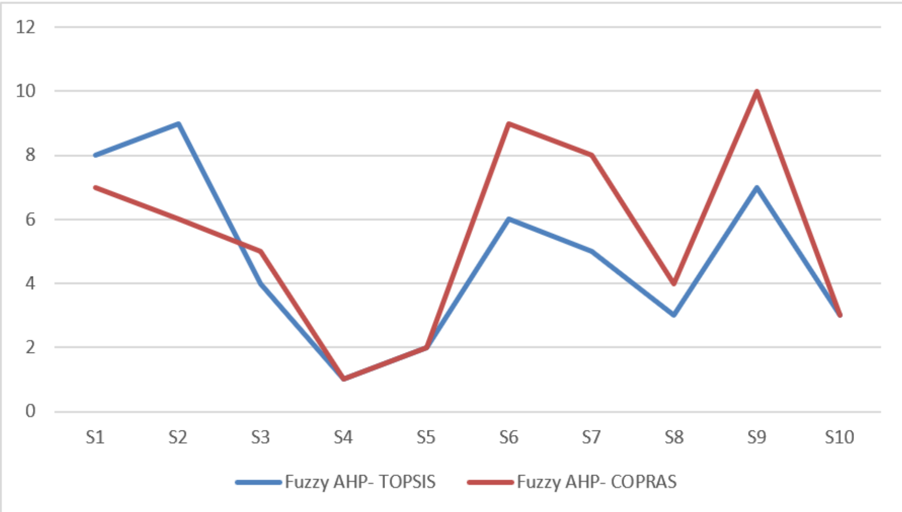

Figure 5.9: Representation of the ranking obtained under the two MCDM techniques.

Sensitivity analysis is conducted to see the ranking obtained under different changing conditions. Figures 5.10 and 5.11 represent the clustered column chart to compare the ranking while interchanging the sub-factors' weights. Two different cases are considered. In the first case, weights of sub-factors parking facilities (C_{15}) and population density (C_{42}) are interchanged. In the second case, weights of land cost (C_{11}) and generation of noise and air pollution (C_{21}) are interchanged. For both the cases, two different methodologies - fuzzy AHP-TOPSIS and fuzzy AHP-COPRAS are used.

Figure 5.11 in which ranking obtained by fuzzy AHP-TOPSIS under sensitivity analysis shows that the alternatives (S_4), (S_5), (S_6), (S_{10}), (S_3) and (S_2) are consistent with first, second, third, fourth, fifth and sixth position, respectively, under the considered two cases, whereas Figure 5.11 in which ranking yields by fuzzy AHP-COPRAS shows that the sites (S_4), (S_5), (S_2), (S_{10}), (S_3) and (S_9) score the rank of first, second, fifth, seventh, ninth and tenth position, respectively. The other sites' variation of rank is noticed under the sensitivity analysis and depicted in the mentioned

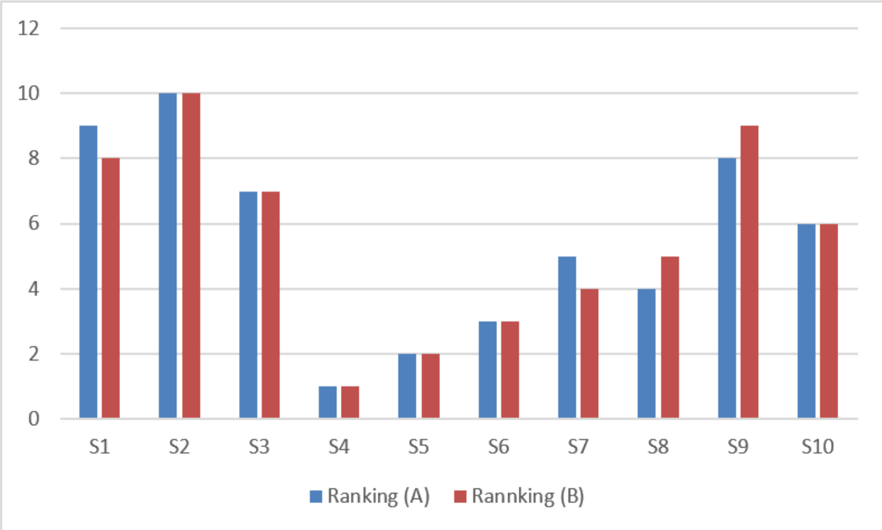

Figure 5.10: Sensitivity analysis in which ranking obtained under Fuzzy AHP-TOPSIS

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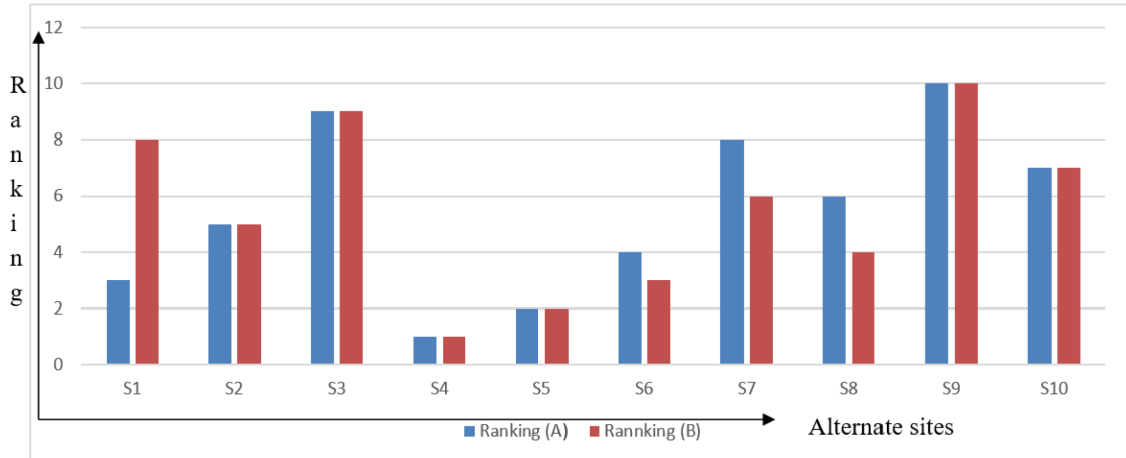

Figure 5.11: Sensitivity analysis in which ranking obtained under fuzzy AHP-COPRAS.

The ranking obtained under the two MCDM techniques yield the site 'Howrah Maidan' (S_4) as the best alternative for E- Vehicle site selection followed by 'Liluah' (S_5) and 'Belur'(S_{1_0}). The FAHP-TOPSIS ranked the alternative 'Belur' (S_{1_0}) and 'Salkia'(S_8) equally at the third position. Ranking obtained for all the sites are presented in Tables 5.9 and 5.10. In sensitivity analysis, where the sub-factors' weights are interchanged as discussed in section 5.4, it is seen that the site 'Howrah Maidan' (S_4) consistently remains in the first position. The rankings obtained under sensitivity analysis by the two methods are depicted in Figures 5.9 and 5.10.

5.5 Conclusions

Electric vehicles (E-vehicles) offer both direct and indirect benefits, including ease of commutation, pollution-free transportation, and employment generation. In developing countries, where pollution levels are high and road infrastructure is limited, E-vehicles have the potential to be transformative. Based on our study of ten locations across the city, 'Howrah Maidan' emerged as the top-ranked site due to its proximity to India's largest railway station, Howrah, along with several other strategic advantages. It was followed by' Liluah' and 'Belur', securing the second and third positions, respectively. These rankings remained consistent regardless of the two MCDM methodologies employed in the study.

This chapter utilizes GIS along with advanced MCDM techniques such as FAHP, FTOPSIS, and FCOPRAS to determine the optimal locations for E-vehicle charging stations. The HFN (Hesitant Fuzzy Numbers) method has been applied to assign preferential ratings to factors, sub-factors, and alternatives. The rankings derived from these MCDM tools are both logical and scientifically sound. HFN has been used as it captures the hesitancy and vagueness in an efficient way. To practice the qualitative criteria evaluation for imprecise information, FAHP, FTOPSIS and FCOPRAS are used. Comparative analysis which uses FTOPSIS and FCOPRAS in our example has depicted consistent results. The reliability, robustness and efficiency of this methodology have also been tested through sensitivity analysis. Though HFN captures wider range of linguistic terms but usage of HFN makes computation a bit longer. The current research offers valuable insights that can serve as a reference for future related studies and problem-solving efforts.

6 Selection of Cloud Service Providers using MCDM Methodology under Intuitionistic Fuzzy Uncertainty

6.1 Introduction

We are living in the age of the digital era, where a massive amount of data is available online and offline. The requirement to store, analyze and access the necessary data securely and safely arises every day. We all need a tool that can help any individual or organization to store data online. This modern age requirement had thus necessitated the birth of "Cloud computing," which is basically storing the data on the web cloud, i.e., on shared data servers, and provides services like faster access to data, lower operating costs of managing and storing of data, security, and protection of data, etc.

The main objective of this study is the solution to the cloud computing service provider selection problem by proposing a decision-making framework under uncertainty that assesses alternatives in the view of different decision criteria.

6.1.1 *Concept of cloud computing*

Different types of companies nowadays give attention to their core functions by parting Cloud Service Providers (CSPs) to switch their computing belongings. CSPs

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are vendors who rent to their clients, diverse types of services that are enthusiastically provisioned based on customer's orders in a pay-as-you-go basis strategy. In terms of applicability and adaptation cloud computing services are structured as follows:

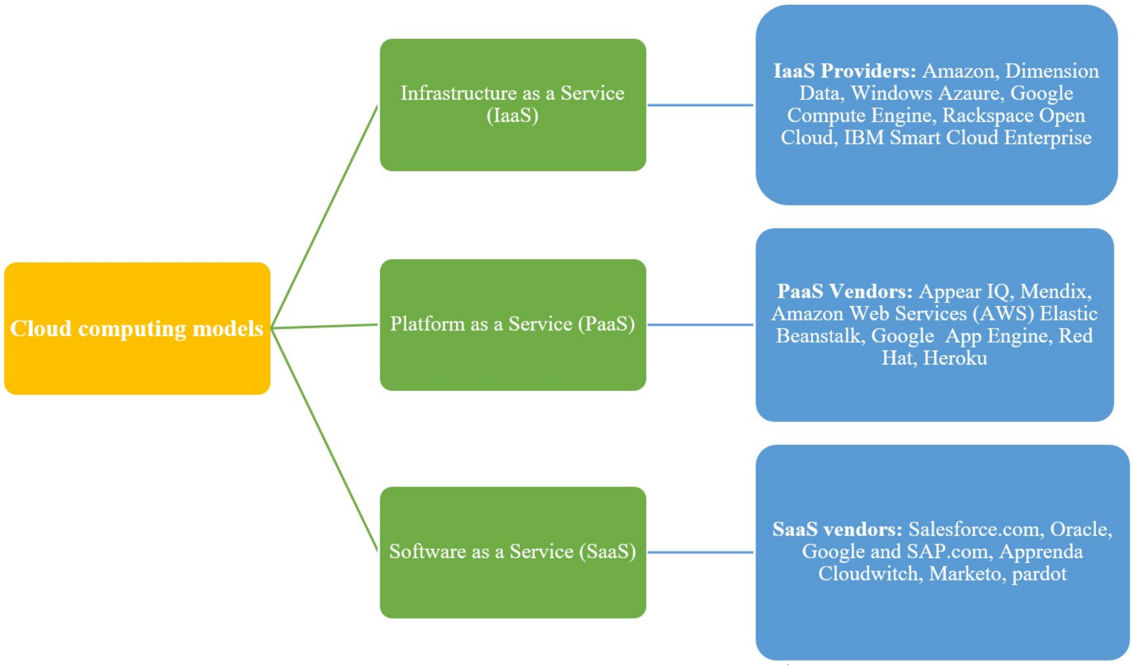

Figure 6.1: Model representing the types of Cloud computing services

- Infrastructures as a Service (IaaS): IaaS refers to services of cloud computing that offer storage, networking resources, servers, and virtual machines on-demand to the customers. These services are provided over the internet on a pay-per-use basis. Using IaaS, helps the business organization to reduce their expenditure on maintenance of onsite data centres and hardware costs and enables them to increase or decrease resources as per their organizational requirement. IaaS solutions aid the organization in gaining real-time insights into business and bypass the cost and complexity of managing huge data servers. It is a very useful tool for organizations where the demand for storage is not predictable, and it can increase or decrease the storage or backup and recovery as per the need.

- Platform as a Service (PaaS): PaaS refers to using a platform or environment for software development through a web browser. It includes software development tools, database management systems, business intelligence services and web development applications. PaaS allows to build, deploy, test, maintain and update web applications. So, the consumer has to manage only the apps and their services and the cloud service provider takes care of the rest of the things. Developers use PaaS to create applications or cloud-based customized apps which can be created using built-in components of the software.

- Software as a Service(SaaS): SaaS refers to connecting consumers with cloud-based apps using the internet. It offers facilities such as emails, help desk applications, Microsoft office suite, Customer Relationship Management applications, online calendars etc. SaaS is software that is deployed on the server of the Cloud service provider. It enables the consumer to take the app on rent and use it as per his requirement. All the data collected through the app is stored in the data centre of the cloud service provider. In turn, the cloud service provider manages the data and also takes care of its security. Some common form of SaaS which is used by individuals is outlook, yahoo mail, Gmail, Hotmail, etc. This service is provided to individuals free of cost by creating an account with the provider. All the emails that the individual sends or receives are stored in the server of the cloud service provider, which can be accessed anytime, anywhere via the internet.

6.1.2 *Objectives of this research*

The motivation behind this study is to select a good cloud service provider that has a top performance matrix. There are many competitors in the market of Cloud Service Providers, and each provider comes up with a variety of services and features, which makes them somewhat better than their competitors. Selecting the best provider among so many available providers is an enormous task for any business house. Evaluating the various available options and analyzing their services is the objective of this study. This chapter aims at the following:

- Selecting different alternatives for Cloud Service Providers and comparing them based on multiple criteria or services provided by them.
- Selecting different criteria and creating a comparison matrix using PIFN to analyze its result.

- Applying AHP method to support decision-making based on multiple criteria (MCDM technique).
- To compare and analyze the best alternative through the application of numerical methods.

6.1.3 *Novelties of the study*

The novelties of the study are as follows:

i.) Distance measure between two PIFN has been defined and used for the numerical illustration in this research.

ii.) Formulae have been developed to calculate the pentagonal intuitionistic fuzzy weight of criteria.

iii.) Technique has been developed to aggregate the decision-maker's opinions into a single comprehensive value in terms of PIFN.

iv.) Two different ranking methods, Fuzzy AHP- TOPSIS and Fuzzy AHP- COPRAS have been used in this research. Fuzzy AHP- COPRAS has been used for the comparative study; also, a comparative analysis has been conducted with two different fuzzy numbers to understand the difference of PIFN used in this study.

v.) Comparative and sensitivity analysis have been conducted to check the robustness and steadiness of the techniques used.

The remainder of the chapter is organized as follows: Section 6.2 briefly describes the concept of MCDM techniques - fuzzy AHP, and Fuzzy TOPSIS. Section 6.3 provides a detailed description of the criteria, alternatives, and the numerical application used in the study. Sensitivity analysis and comparative analysis are presented in Sections 6.4 and 6.5, respectively. The results and their interpretation are discussed in Section 6.6, while Section 6.7 concludes the chapter and outlines the scope for future research.

6.1.4 *Rationale for using $PIFN$ over other fuzzy numbers*

Ranking of CSPs under the influence of various conflicting criteria includes hesitancy, uncertainty, and vagueness of the problem. PFNs capture the ambiguity and uncertainty more compared to triangular fuzzy numbers (TFNs) and trapezoidal fuzzy numbers (TRFNs). The integration of PFNs with intuitionistic fuzzy numbers, resulting in PIFNs, enhances the expression of confidence levels. This allows experts or decision-makers to convey not only their degree of assurance but also their degree of non-assurance. In real-life scenarios, allowing experts or a group of experts to express both their confidence and hesitation reflects rational judgment, ultimately contributing to more effective decision-making. Therefore, in this chapter, PIFNs have been integrated with the MCDM methods—AHP, TOPSIS, and COPRAS—to determine the weights of the criteria and ranking of the CSPs.

6.2 Fuzzy MCDM techniques

6.2.1 PIFN- AHP method

In this chapter, FAHP is employed instead of traditional AHP, as the fuzzy approach accounts for the ambiguity and imprecision inherent in expert judgments. The steps involved in the FAHP process are outlined below.

Step A: Construction of a comparison matrix in terms of PIFN by a decision expert or a group of decision experts using Figure 6.1. The linguistic ratings assigned by DMs are transformed into PIFN. The aggregation of the opinions is done by using Eq. (6.3).

Step B: Defuzzification of PIFN or score value of PIFN can be calculated using the following formula:

$$S_p = (l + m + 3n + o + p)/6(1 + w_a - v_{a'}) \tag{6.1}$$

The remaining steps of the AHP methodology are determined using Eqs. (1.2), (1.3), (1.4) and (1.5).

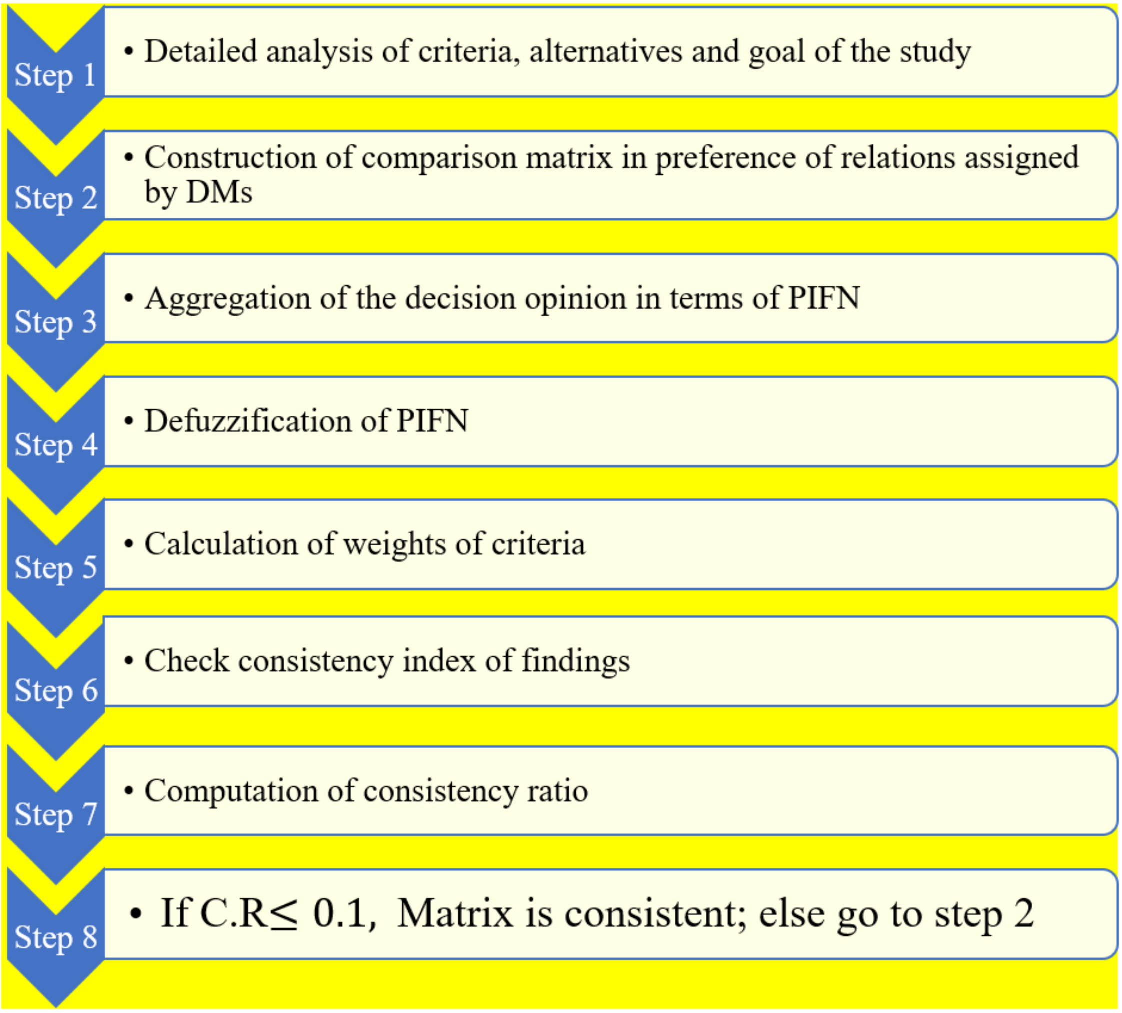

Figure 6.2: Schematic structure of AHP for strategy

6.2.2 *Derivation of Pentagonal intuitionistic fuzzy Weights of criteria*

Step 1: The geometric mean value of PIFN is as follows:

$$M_g = \left\langle \left(\prod_{h=1}^{n} y_{gh} \right); \min_{h=1,2,\ldots n} w_{gh}, \max_{h=1,2,\ldots n} v_{gh} \right\rangle \tag{6.2}$$

Step 2: Find the summation of each M_g using Eq.(1.76).

Step 3: Calculate the inverse of each M_g using Eq. (1.81), and then arrange the results in ascending order.

6.2.3 PIFN-TOPSIS approach

The TOPSIS MCDM technique is one of the extensively used methodologies, introduced by Hwang and Yoon (1981), to rank the alternatives. The linguistic terms assigned by the DMs can be captured better with fuzzy TOPSIS ($FTOPSIS$). The fuzzy approach is useful in dealing with the hesitancy and uncertainty of the DMs and the conflicting criteria. The PIFN is incorporated with the classical TOPSIS to obtain FTOPSIS formulae.

The PIFN-TOPSIS methodology is described in the following steps:

Step 1: Determination of alternatives and their preferential linguistic ratings in terms of PIFN. Let us assume a set of m $CSPs$ $(p_1, p_2, \dots, p_m)$ and n criteria $(f_1, f_2, \dots, f_n)$. Let $(D_1, D_2, \dots, D_k)$ be the number of decision-makers (DMs). The DMs assign $PIFNs$ as their decisions for the alternatives depend on different criteria.

Step 2: Aggregation of decisions of DMs assigned in $PIFN$ using the formulae:

$$\begin{cases} \tilde{l}_{gh} = \min_{k=1,2,\dots,K} \tilde{l}_{hik} \\ \tilde{m}_{gh} = \min_{k=1,2,\dots,K} \tilde{m}_{hik} \\ \tilde{n}_{gh} = \min_{k=1,2,\dots,K} \tilde{n}_{hik} \\ \tilde{o}_{gh} = \min_{k=1,2,\dots,K} \tilde{o}_{hik} \\ \tilde{p}_{gh} = \min_{k=1,2,\dots,K} \tilde{p}_{hik} \\ \tilde{w}_{gh} = \min_{k=1,2,\dots,K} \tilde{w}_{hik} \\ \tilde{v}_{gh} = \min_{k=1,2,\dots,K} \tilde{v}_{hik} \end{cases} \tag{6.3}$$

Step 3: Normalization of the PIFNs, using the formulae:

$\tilde{N} = [n_{gh}]_{mn}, g = 1, 2, ...m; h = 1, 2, ...n$

$$N_{gh}^{B} = \left\langle \left(\frac{\tilde{l}_{gh}}{\tilde{p}^*}, \frac{\tilde{m}_{gh}}{\tilde{p}^*}, \frac{\tilde{n}_{gh}}{\tilde{p}^*}, \frac{\tilde{o}_{gh}}{\tilde{p}^*}, \frac{\tilde{p}_{gh}}{\tilde{p}^*} \right); w_{hi}, v_{hi} \right\rangle \\ d \in B.C \quad \tilde{p}^* = \max(p_{gh}) \tag{6.4}$$

$$N_{gh}^{NB} = \left\langle \left(\frac{\tilde{l}^*}{\tilde{p}_{gh}}, \frac{\tilde{l}^*}{\tilde{o}_{gh}}, \frac{\tilde{l}^*}{\tilde{n}_{gh}}, \frac{\tilde{l}^*}{\tilde{m}_{gh}}, \frac{\tilde{l}^*}{\tilde{l}_{gh}} \right); w_{hi}, v_{hi} \right\rangle \\ d \in N.B.C \quad \tilde{l}_h^* = \min(\tilde{l}_{gh}) \tag{6.5}$$

where BC and NBC signify the Benefit Criteria and Non-Benefit Criteria, respectively.

Step 4: Determine the weighted fuzzy normalized matrix by multiplying the fuzzy weights of the criteria with their corresponding normalized fuzzy values. For the product of two PIFNs, Eq. (1.78) can be used.

$W = [WN_{gh}]_{mn} \quad g = 1, 2 \ldots m; h = 1, 2 \ldots n$, where

$$FW_{gh} = \tilde{N_{gh}} \times \tilde{w}_h \quad g = 1, 2 \ldots m; h = 1, 2 \ldots n \tag{6.6}$$

Step 5: Computation of the Fuzzy Positive Ideal Solution $(FPIS)$ (P^+) and Fuzzy Negative Ideal Solution $(FNIS)$ (N^-), where t_{g^+} denotes the maximum value of t_{gh} and t_{g^-} denotes the minimum value of t_{gh}:

$$\begin{aligned} P^+ &= \Big\langle (a_1^+, \min_g W_{g1}, \max_g v_{g1}), (a_2^+, \min_g w_{g2}, \max_g v_{g2}), \ldots, \\ &\quad (a_n^+, \min_g W_{mn}, \max_g v_{mn}) \Big\rangle \\ &= \Big(\max_{t_g h} |h \in M_{BC}, \min_{t_g h} |h \in M_{NBC} \Big) \end{aligned} \tag{6.7}$$

$$\begin{aligned} N^- &= \Big\langle (a_1^-, \min_g W_{g1}, \max_g v_{g1}), (a_2^-, \min_g W_{g2}, \max_g v_{g2}), \ldots, \\ &\quad (a_n^-, \min_g W_{mn}, \max_g v_{mn}) \Big\rangle \\ &= \Big(\min_{t_g h} |h \in M_{BC}, \max_{t_g h} |h \in M_{NBC} \Big) \end{aligned} \tag{6.8}$$

where M_B denotes the benefit criteria, and M_{NB} denotes the non-benefit criteria.

Step 6: Determination of the distance measure of all alternatives from the PIS and NIS. The two Hamming distances for an individual alternative can be computed as follows:

$$\tilde{M}_g^+ = \sum_{h=1}^{n} d(t_{gh}, t_g^+), g = 1, 2, ..., m \tag{6.9}$$

$$\tilde{M}_g^- = \sum_{h=1}^{n} d(t_{gh}, t_g^-), g = 1, 2, ..., m \tag{6.10}$$

where (.,.) denotes the Hamming distance between two fuzzy numbers. Using Eq. (1.81), the distance between two PIFNs can be computed.

Step 7: Determination of the relative closeness to the ideal alternatives:

$$RC_g = \frac{\tilde{M}_g^-}{\tilde{M}_g^- + \tilde{M}_g^+}, g = 1, 2, \ldots m \tag{6.11}$$

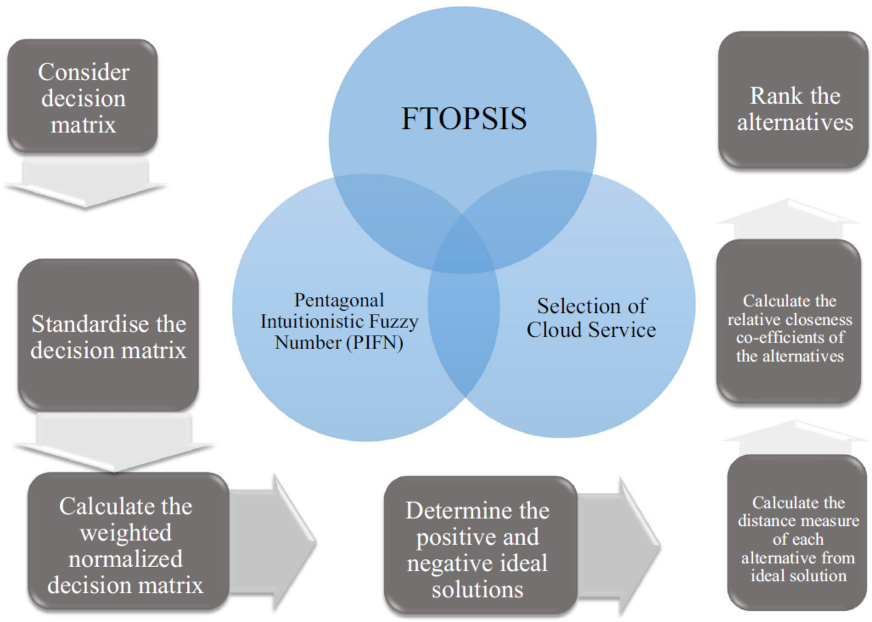

Figure 6.3: A schematic structure of FTOPSIS strategy

6.3 Numerical example

6.3.1 *Cloud service providers (CSPs)*

Cloud Space is a networked storage, or disk space, available over a specific network - the Internet. Cloud Service Providers are organizations who rent out the cloud space to other organizations or individuals and provide them with cloud-based platforms where data could be stored or accessed. The requirement of CSP arises when a business unit wants to reduce the cost of hardware, servers and storage space and transfer all the data online so that it can be accessed from anywhere and anytime. Cloud Service Technologies are required to ascertain the best suitable services for the consumers. It empowers individuals or organizations to develop computing solutions quickly at reasonable cost. Among the pool of CSPs available which provides services globally, we select 6 top alternatives for Cloud Service Providers and compare them on various parameters.

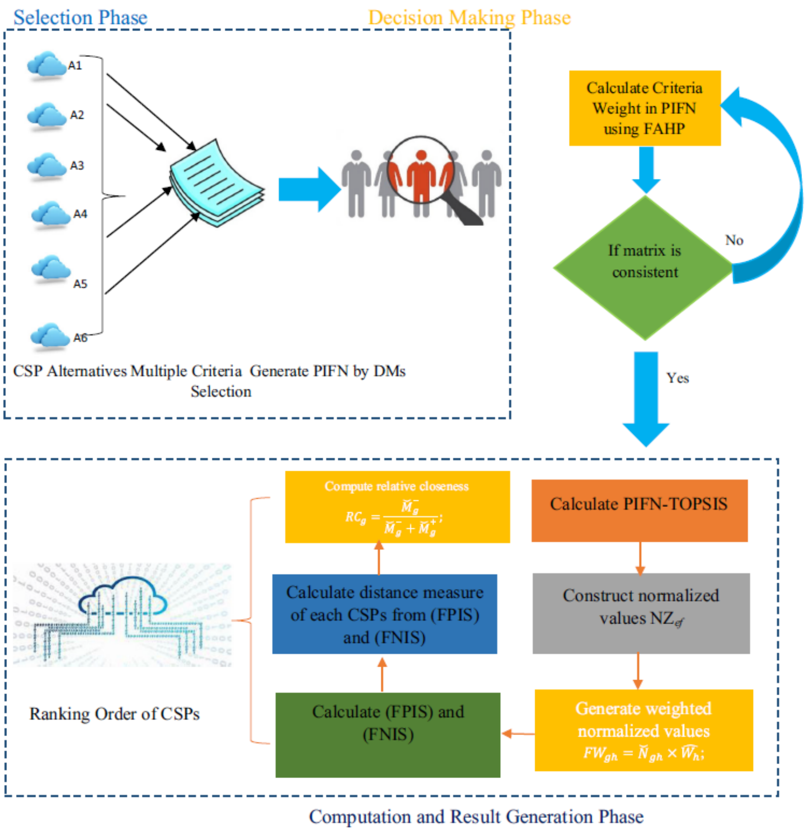

Figure 6.4: Flowchart depicting the methodology used in the study

6.3.2 *Cloud service provider selection criteria*

Cloud Service Provider selection criteria are taken into consideration after a detailed evaluation by a group of decision expert. All the criteria have been taken keeping in mind the general requirement of cloud users by doing a survey with few decision-makers and industry colleagues. We also use secondary data in order to validate our findings through literature studies and Internet research. In the following, we briefly explain the criteria taken in this study.

i.) Cloud Security/Privacy- Cloud security consists of policies, procedures and technologies that work to protect the data and customers privacy. Data privacy and data security are considered to be the top priority for any organization.

ii.) Pricing - Cost of cloud services and discounts, if any, are based on business models and their framework. One needs to look into the pricing structure and its various components charged by CSP.

iii.) Downtime - The unavailability of services being provided by the CSP may increase the costing of client. It may affect their normal working routines and they might incur heavy losses due to it. It also impacts the reputation of the provider as well as the customer.

iv.) Support services are important in deployment of CSP in any organization. One has to check that whether CSP's cloud architecture and support services offered are suitable to the organization's workload and roadmap.

v.) Portability- There may be chances that the data of customer might be lost due to moving the data from one platform to another. To avoid this, CSP should provide an environment or an API (Application Program Interface) which helps to retrieve the data from the old platform/source to new platform (Sahu et al., 2020).

vi.) Scalability- It refers to ability to increase workload with existing hardware resources. This is important to evaluate in order to determine whether a system can handle large number of application requests (Büyüközkan et al., 2018; Wibowo et al., 2016).

vii.) Disaster recovery - Any organization going for CSP implementation needs to understand about the disaster recovery and data recovery options and CSPs ability to support data preservations.

viii.) Deployment and upgrades - For deployment, there are mainly three basic variants - Public Cloud , Private Cloud and Hybrid Cloud . Also, it's important to check whether the CSPs are providing regular updates and whether those updates are included in the service or charged separately.

ix.) Service Level Agreements ($SLAs$)- It describes all the negotiations and terms and conditions of the contract (Jatoth et al., 2019). $SLAs$ could be very complex as there are no standards available in the cloud industry that how to construct SLAs and define them.

Note 8. *In the criteria given above, Pricing and Downtime are non-benefit criteria while Security/ Privacy, Support Services, Portability, Scalability, Upgrades, Disaster recovery, and SLA are benefit criteria.*

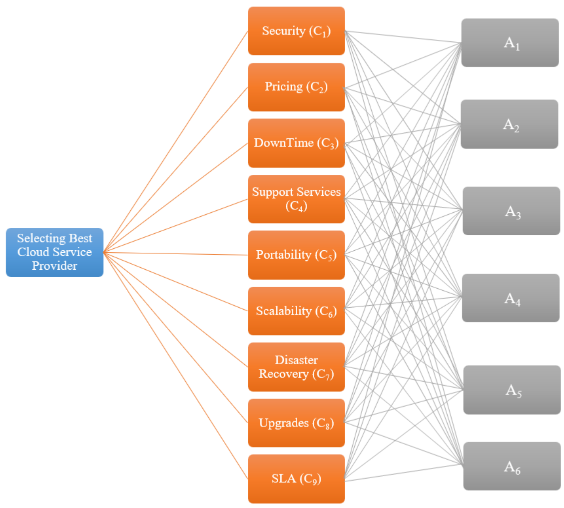

Figure 6.5: Hierarchical framework representing the criteria and alternatives taken in this research

Table 6.1 illustrates the PIFN linguistic terms taken in this study for the calculation of the criteria's weights. Since the membership degree and non-membership value may vary for each DM, the generalized value $(w_a, v_{a'})$ is represented in Table 6.1.

Table 6.1: Linguistic variables' rating in terms of $PIFN$

Linguistic Terms	$1-9$ **Scale**	**Pentagonal Intuitionistic fuzzy number (**$PIFNs$**)**
Equally Important (EI)	1	$<(1,1,1,1,1);w_a,v_{a'}>$
Moderately Important (MI)	3	$<(2,4,5,6,6.5)w_a,v_{a'}>$
Strongly Important(SI)	5	$<(3,5,6,7,8);w_a,v_{a'}>$
Very Strongly Important (VSI)	7	$<(6,7,7,8,9);w_a,v_{a'}>$
Absolutely Important (AI)	9	$<(7,8,9,9,10);w_a,v_{a'}>$
Moderately not Important (MNI)	$\frac{1}{3}$	$<(\frac{1}{6.5}\frac{1}{6},\frac{1}{5},\frac{1}{4},\frac{1}{2});w_a,v_{a'}>$
Strongly not Important (SNI)	$\frac{1}{5}$	$<(\frac{1}{8},\frac{1}{7},\frac{1}{6},\frac{1}{5},\frac{1}{3});w_a,v_{a'}>$
Very Strongly not Important ($VSNI$)	$\frac{1}{7}$	$<(\frac{1}{9},\frac{1}{8},\frac{1}{7},\frac{1}{7},\frac{1}{6});w_a,v_{a'}>$

Note 9. *The transpose of the matrix is represented in Table 6.2.*

Table 6.2: Criteria to criteria comparison were conducted by three DMs.

Criteria	Decision Makers	Securit (c^1)	Pricing (c^2)	Downti (c^3)	Suppor services (c^4)	Portabi (c^5)	Scalabili (c^6)	Disaste recovery (c^7)	Upgrac (c^8)	SLA (c^9)
	DM1	EI	1/AI	EI	1/AI	EI	1/AI	EI	1/AI	1/AI
Security (c^1)	DM2	EI	1/AI	1/AI	1/AI	1/AI	1/AI	EI	1/AI	1/AI
	DM3	EI	1/AI	1/AI	1/AI	1/AI	EI	EI	EI	1/AI
	DM1	AI	EI	EI	VSI	VSI	1/SI	AI	VSI	EI
Pricing (c^2)	DM2	AI	EI	VSI	VSI	SI	1/SI	AI	VSI	VSI
	DM3	AI	EI	VSI	SI	VSI	VSI	AI	AI	VSI
	DM1	AI	EI	EI	EI	EI	1/VSI	AI	EI	EI
Downtime (c^3)	DM2	AI	1/VSI	EI	1/VSI	VSI	1/VSI	EI	EI	EI
	DM3	AI	1/VSI	EI	SI	EI	VSI	AI	AI	EI
	DM1	AI	EI	EI	EI	EI	1/VSI	AI	EI	EI
Support services (c^4)	DM2	AI	1/VSI	VSI	EI	EI	EI	AI	VSI	EI
	DM3	AI	1/SI	EI	EI	EI	VSI	AI	AI	EI
	DM1	AI	1/VSI	EI	EI	EI	1/VSI	EI	EI	1/VSI
Portability (c^5)	DM2	AI	1/SI	EI	EI	EI	1/SI	AI	VSI	1/VSI
	DM3	AI	1/VSI	EI	EI	EI	AI	AI	AI	EI
	DM1	AI	SI	VSI	VSI	VSI	EI	AI	VSI	VSI
Scalability (c^6)	DM2	AI	SI	VSI	EI	SI	EI	AI	VSI	SI
	DM3	AI	1/VSI	1/VSI	1/VSI	1/AI	EI	EI	EI	VSI
	DM1	EI	1/AI	1/AI	1/AI	EI	I/AI	EI	1/AI	1/AI
Disaster Recovery (c^7)	DM2	EI	1/AI	EI	1/AI	1/AI	1/AI	EI	1/AI	1/AI
	DM3	EI	1/AI	1/AI	1/AI	1/AI	EI	EI	EI	1/AI
	DM1	AI	1/VSI	VSI	EI	EI	1/VSI	AI	EI	EI
Upgrades (c^8)	DM2	AI	1/VSI	EI	1/VSI	1/VSI	1/VSI	AI	EI	EI
	DM3	EI	1/AI	1/AI	1/AI	1/AI	EI	EI	EI	1/AI
	DM1	AI	EI	EI	EI	VSI	1/VSI	AI	EI	EI
SLA (c^9)	DM2	AI	1/VSI	EI	EI	VSI	1/SI	AI	EI	EI
	DM3	AI	1/VSI	EI	EI	EI	1/VSI	AI	AI	EI

The linguistic data collected by the DMs in Table 5 are transformed to PIFN. The individual PIFN assigned by the DMS is integrated into a single PIFN by using Eq. (6.3).

Table 6.3: Linguistic variables rating in terms of $PIFN$

Criteria	PIFN weights of Criteria
Security (c^1)	$<(0.182, 0.217, 0.376, 0.574, 0.677); 0.5, 0.4>$
Pricing (c^2)	$<(0.005, 0.006, 0.017, 0.046, 0.059), 0.5, 0.4>$
Down time (c^3)	$<(0.009, 0.010, 0.042, 0.165, 0.189); 0.5, 0.4>$
Support services (c^4)	$<(0.013, 0.016, 0.044, 0.129, 0.151); 0.5, 0.4>$
Portability (c^5)	$<(0.016, 0.019, 0.063, 0.208, 0.242); 0.5, 0.4>$
Scalability (c^6)	$<(0.006, 0.007, 0.030, 0.176, 0.214); 0.5, 0.4>$
Disaster recovery (c^7)	$<(0.077, 0.086, 0.261, 0.560, 0.662); 0.5, 0.4>$
Upgrades (c^8)	$<(0.029, 0.033, 0.114, 0.444, 0.518); 0.5, 0.4>$
SLA (c^9)	$<(0.016, 0.020, 0.047, 0.104, 0.120); 0.5, 0.4>$

Table 6.4: Criteria weights were obtained by using $FAHP$

Criteria	Weight
Security (c^1)	0.294
Pricing (c^2)	0.013
Down time (c^3)	0.055
Support services (c^4)	0.049
Portability (c^5)	0.073
Scalability (c^6)	0.053
Disaster recovery (c^7)	0.278
Upgrades (c^8)	0.141
SLA (c^9)	0.043

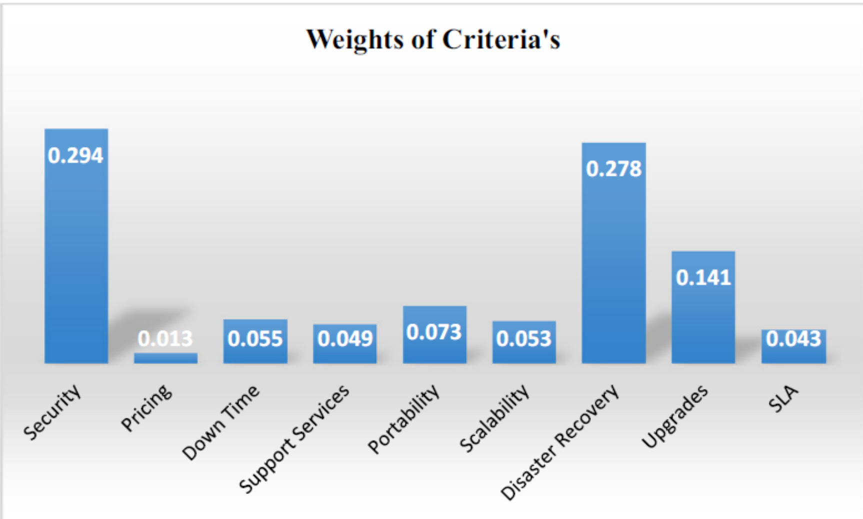

Figure 6.6: Clustered column chart depicting the weights of criteria obtained using FAHP

Table 6.3 depicts the final PIFN weights which will be used further for ranking the alternatives using the FTOPSIS approach. Table 6.4 represents the weights of the criteria calculated by using the MCDM tool FAHP. For the calculation of criteria's crisp weights, firstly, the PIFN is defuzzified using Eq. (6.1), and then, the AHP tool is applied to get the priority weights. Table 6.4 and Fig. 6.6 represent the crispified weights of criteria calculated by using FAHP.

Table 6.5: Alternatives linguistic ratings assigned by DMs w.r.t. criteria

Alternative	(A_1)		(A_2)		(A_3)		(A_4)		(A_5)		(A_6)	
Decision maker	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Security	EG	EG	EG	EG	EG	EG	EG	EG	VG	VG	VG	VG
Pricing	L	H	H	H	H	H	H	H	L	H	L	L
Downtime	VL	VL	L	L	L	L	H	L	L	L	H	H
Support services	EG	EG	EG	EG	EG	EG	VG	VG	VG	VG	EG	VG
Portability	EG	EG	VG	VG	EG	EG	EG	EG	VG	VG	VG	VG
Scalability	EG	EG	G	G	VG	VG	EG	VG	VG	VG	VG	VG
Disaster recovery	EG	EG	EG	EG	VG	VG	G	VG	G	VG	VG	G
Upgrades	EG	EG	EG	EG	EG	EG	VG	VG	VG	VG	G	G
SLA	EG	EG	EG	EG	EG	EG	EG	EG	VG	VG	G	G

Step 1: Formation of integrated PIFN using equation 6.3.

Step 2: Calculation of normalized matrix using equation 6.4 and 6.5 .

Step 3: Computation of weighted normalized matrix using equation 6.5 .

Step 4: PIS and NIS are calculated using Eqs. 6.7 and 6.8.

Step 5: Distance measures of each CSP's are calculated from the PIS and NIS using Eqs. 6.9 and 6.10.

Step 6: Lastly, the relative closeness is determined for each alternative using Eq. 6.11. The highest value of RC represents the optimal CSP.

Note 10. *The weighted normalized matrix is constructed using the product of PIFN weights of criteria obtained and the normalized matrix.*

Table 6.6: Representation of distance measure from PIS and NIS, relative closeness and ranks of the alternatives

Alternatives	D+	D-	Relative Closeness (RC)	Rank
A_1	0.059	0.293	0.833	1
A_2	0.063	0.288	0.820	2
A_3	0.080	0.272	0.773	3
A_4	0.123	0.228	0.649	4
A_5	0.232	0.119	0.339	5
A_6	0.269	0.083	0.235	6

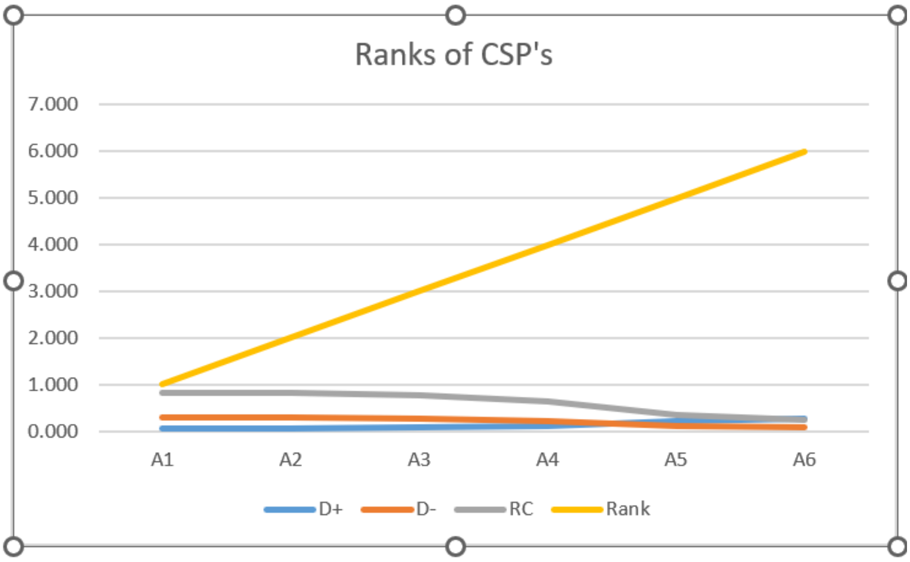

Figure 6.7: Line chart representing the distance measure, relative closeness and rankings of the CSPs

6.4 Sensitivity analysis

Sensitivity analysis is carried out by interchanging the PIFN weights of the most sensitive criterion. Three different cases are undertaken and the rankings obtained under these cases are represented in Table 6.7. Graphically, these rankings are illustrated with the help of a line chart in Figure 6.8.

Case 1: Interchange of PIFN weights of Support services and pricing – On interchanging the given values by decision-makers for support services and pricing we found that the ranking of Alternative '1' has changed while the rest others are not showing any change. This shows that pricing is one of the factors which is responsible to change the selection of cloud service providers A_1 and A_2.

Case 2: Interchange of PIFN weights of upgrades and Downtime – This case shows a drastic change in the ranking of all the alternatives except A_2. Minimal downtime and regular updates should be the main criteria of cloud service provider but according to the decision-makers, these two criteria has different values if we consider different CSP alternatives. As, downtime is considered a vital factor for selecting CSP, interchanging its value with other criteria shows a substantial change.

Case 3: Interchange of PIFN weights of portability and scalability – As both the criteria portability and scalability are at par, so the changes in the ranking of alternatives are very negligible. The ranking obtained under the first two cases depict that the alternative A_1 ranking order is changed from the first position to 2nd and 4th places in the two cases respectively. This change in ranking under the change of two sensitive criteria represents the importance of these criteria. Considering the case 3, it is easy to understand from Table 11 and Figure 10 that the same ranking is obtained as the methodlogy used in this chapter. The interchange of weight of criteria's portability and scalability shows no alteration in ranking of the alternatives. This indicates the equivalency of these two criteria.

Remarks 2. *The three cases are considered as per the opinion of the decision experts. Sensitive criteria are taken into account and accordingly, their weights are interchanged to obtain the ranking. This mechanism is utilized to analyse the change in the pattern of ranking and to get a clear and deeper concept of the the most important criteria.*

Table 6.7: Rankings obtained under sensitivity analysis

Alternatives	Case 1	Case 2	Case 3
A_1	2	4	1
A_2	1	1	2
A_3	3	2	3
A_4	4	3	4
A_5	5	6	5
A_6	6	5	6

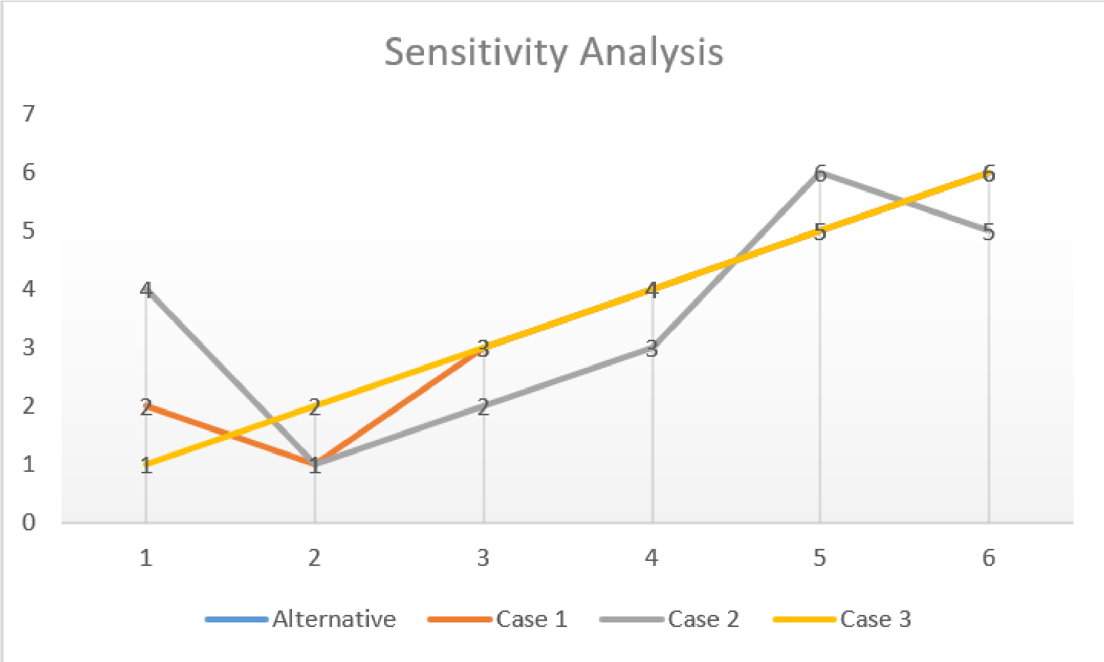

Figure 6.8: Line chart illustration of rankings obtained under sensitivity analysis.

6.5 Comparative analysis

The proposed comparative study is divided into two segments. In the first segment, our methodology i.e., $FTOPSIS$ ranking, is compared with the $FCOPRAS$ MCDM ranking tool. In the second segment, the fuzzy number i.e., $PIFN$ is compared with the trapezoidal intuitionistic fuzzy number ($TRIFN$), triangular intuitionistic fuzzy number ($TIFN$). The ranking obtained under these two different concepts is discussed in the following subsections.

6.5.1 *Comparative study with $FCOPRAS$ technique*

The Complex Proportional Assessment ($COPRAS$) methodology developed by Zavadskas et al. (1994) is a comprehensive method of $COPRAS$ technique, widely applicable to decision-making problems. It is a step-wise ranking and evaluation technique for the alternatives with reference to significance and utility degree. Table 6.7 and Figure 6.9 demonstrate the ranking obtained using the existing MCDM tool $FTOPSIS$ with $FCOPRAS$.

Table 6.8: Ranking of the alternatives using FCOPRAS and FTOPSIS

Alternatives	Rank (FCOPRAS)	Rank (FTOPSIS)
A_1	1	1
A_2	2	2
A_3	3	3
A_4	4	4
A_5	5	5
A_6	6	6

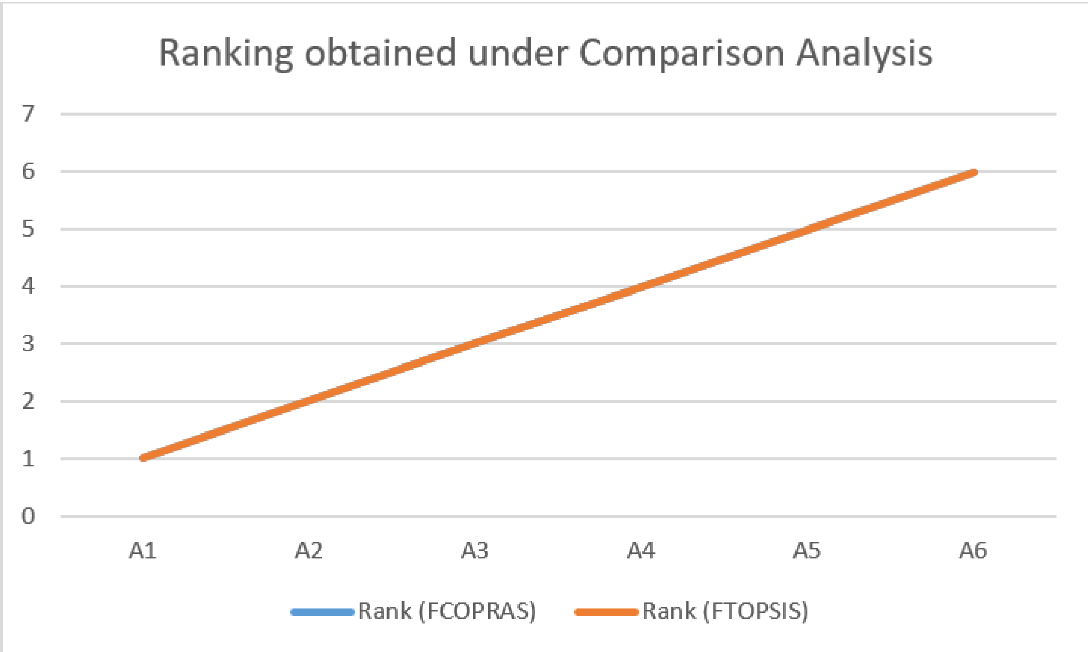

Figure 6.9: Line chart representation of rankings obtained under comparative analysis

6.5.2 *Comparative study with different fuzzy numbers*

This section depicts Table 6.7 and Figures (6.10 and 6.11), which are obtained under the ranking of CSPs under $TrIFN$ (Parvathi and Malathi, 2012; Rezvani 2013) and $TIFN$ (Wang et al., 2013; Li, 2010) and the ranking is compared with $PIFN$ which is used in this research. Vashishtha and Susan (2022), developed multi lexicons adaptive neuro-fuzzy- inference system (MultiLexANIFS), which integrates inputs from different lexicons to carry out the sentiment analysis of social media posts. The existing method i.e., by use of $PIFN$, the alternative A_1 ranks the highest, followed by $A_2 > A_3 > A_4 > A_5 > A_6$. Under TrIFN it is observed that the ranks of alternatives A_1 and A_2 remain the consistent position of 1st and 2nd followed by $A_3 > A_4 > A_6 > A_5$. In $TIFN$, the ranks of alternatives are interchanged, A_2 being the first followed by $A_1 > A_3 > A_4 > A_6 > A_5$.

Table 6.9: Ranking of alternatives using different types of intuitionistic fuzzy numbers

Alternatives	PIFN	TrIFN	TIFN
A_1	1	1	2
A_2	2	2	1
A_3	3	3	3
A_4	4	4	4
A_5	5	6	6
A_6	6	5	5

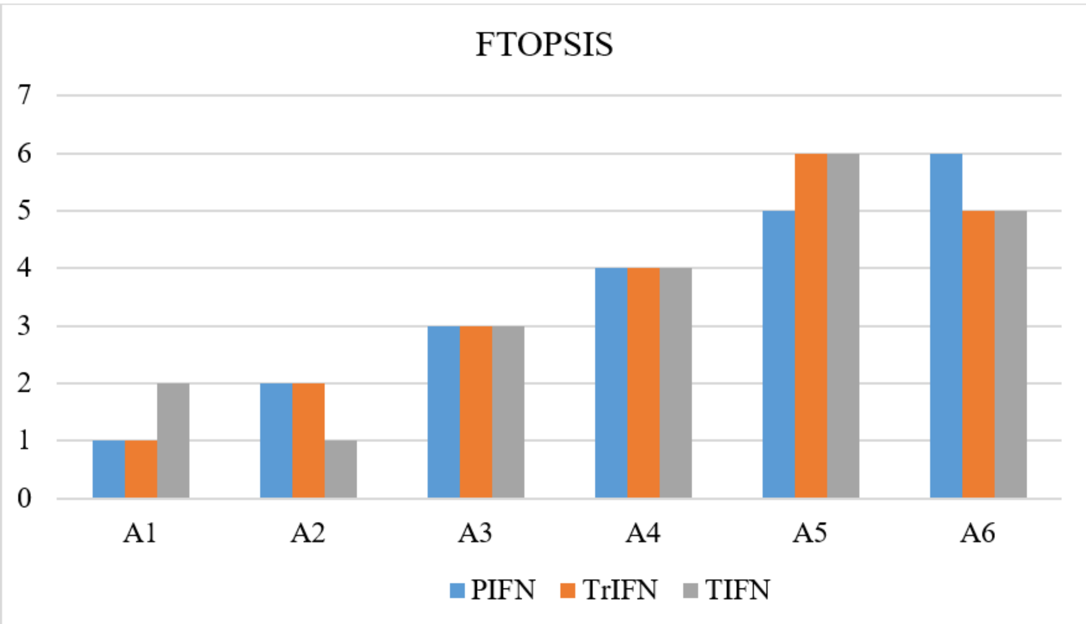

Figure 6.10: Bar graph representation of rankings obtained under different types of intuitionistic fuzzy numbers.

Remarks 3. *The ranking obtained under comparative analysis in this section depicts the same ranking when FTOPSIS is compared with FCOPRAS. This indicates the equivalency of the two methods. In the second segment, where PIFN is compared with other types of intuitionistic fuzzy numbers reveals that the ranking obtained under TrFN holds the same position for alternatives* A_1*,* A_2*,* A_3 *and* A_4 *but the ranks are interchanged for alternatives*

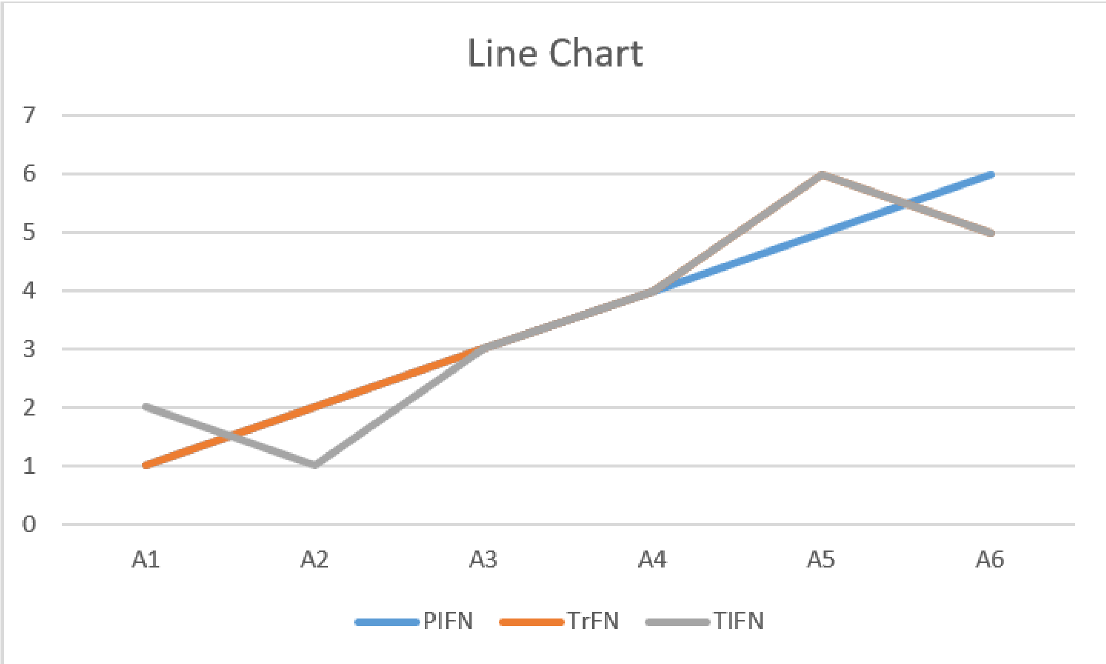

Figure 6.11: Line chart representation of rankings obtained under different types of intuitionistic fuzzy numbers.

A_5 and A_6. In comparison of PIFN with TFN, it is seen that the alternative A_2 scored the first rank followed by A_1. The ranking of other alternatives obtained under TFN holds consistent rank with TrFN. The ranking obtained under PIFN with TOPSIS can be said as the best technique because PIFN incorporates the degree of hesitancy and vagueness of the DMs in an optimal way compared to other types of Intuitionistic fuzzy numbers.

6.6 Main findings

This chapter reveals the ranking of Cloud Service Provider alternatives using fuzzy AHP and fuzzy TOPSIS methods. The ranking technique $FTOPSIS$ is compared with $FCOPRAS$. The comparative study is presented in section 6.5. The comparative study consists of two sub-sections. The first sub-section 6.5.1 provides a comparison of $FTOPSIS$ with
$FCOPRAS$. The ranking obtained under these two MCDM shows that the alternatives A_1 and A_2 obtain the ranks 1 and 2, respectively. Thus the $FTOPSIS$ technique for ranking used in this study and the $FCOPRAS$ for comparison yields consistent

rankings of the $CSPs$. In the second sub-section 6.5.2, a comparison analysis is conducted with different types of intuitionistic fuzzy numbers. viz. $TrIFN$ and $TIFN$. Using $TOPSIS$, the final ranking has been done, which shows that alternatives 1 and 2 are the best Cloud Service Providers taking into account all the factors responsible for selecting the best alternative. The sensitivity analysis shows that, in cases 1 and 3 where the fuzzy weights of most sensitive criteria are interchanged, then also the first and second alternatives obtain consistent ranks of 1 and 2, which implies that they are the top giants of the market due to the fact that it provides excellent services along with customer satisfaction and any business house can consider it reliable to store and access their important data.

6.7 Conclusions

This chapter primarily focuses on the use of intuitionistic fuzzy numbers with MCDM tools fuzzy AHP and fuzzy TOPSIS for obtaining the weights of criteria and ranking of CSPs, respectively. Formulae that are developed using $PIFN$ determine the weights of criteria and also aggregate the decision maker's estimation into a single comprehensive value. Further, fuzzy AHP-TOPSIS and fuzzy AHP-COPRAS have been used to determine the final ranking of the alternatives. The opinion of different decision-makers have been taken to determine the best alternative. Although the views of decision-makers are highly influenced by their working environment and the size and severity of business, the attempt to determine the alternative ranking is done. Comparative and sensitivity analysis have been conducted to check the robustness and steadiness of the techniques used.

7 Optimal Site Selection for Women University using Neutrosophic MCDM Technique

7.1 Introduction

The site selection problem is quite significant in today's world. Location selection for setting up an industry, real estate, hospitality management, or other cases require proper data, decision experts, future perspective, establishment cost, etc. Moreover, there exist several criteria which make the Decision Maker (DM) select the optimal alternative. Ranking the sites requires a mathematical understanding of the problem. In this context, multi-criteria decision-making (MCDM) can play an important role. Optimal selection or ranking of various sites is complicated as it depends on multiple conflicting criteria. Obtaining weights of criteria is a major part of the DMs. The first step in decision-making is to integrate the opinion of decision experts in linguistic rating. The linguistic rating may not always be transformed to a fixed scale, as the decision experts may consider uncertainty, hesitancy, and vagueness. In this context, researchers utilize MCDM techniques to solve site selection problem in an uncertain, hesitant environment.

This chapter is based on the work published in *Buildings*, 2023, 13(1), 152. https://doi.org/10.3390/buildings13010152

7.1.1 *Objectives of the study*

- MCDM tools in the neutrosophic environment have been applied in different areas, but the literature survey reflects that less work has been done on site selection problem. So, this study focuses on this problem in neutrosophic environment.
- To develop a new de-neutrosophication technique for trapezoidal neutrosophic number and use it in the proposed model.
- To find the TrNNs, weights are to be constructed in a new way and applied in the proposed model.
- To propose a model for Women University site selection by considering important criteria, and conduct ranking of the sites using an uncertain MCDM method.
- Questionnaires are to be made regarding different locations and their attributes. Different experts' opinion about the criteria will be taken into consideration as the input.
- To obtain the crisp weight of the criteria, AHP technique will be used and to check consistency, two MCDM tools, viz. neutrosophic TOPSIS and neutrosophic COPRAS will be applied for ranking of the sites.
- To measure the change in ranking and check the robustness and steadiness of the proposed method, sensitivity analysis will be carried out.

7.1.2 *Structure of the chapter*

The mathematical formulation of MCDM techniques viz. NAHP, NTOPSIS and NCOPRAS are described in Section 7.2. The model setup and criteria for the application problem are presented in Sections 7.3 and 7.4, respectively. Numerical results are shown in Section 7.5. Sensitivity analysis is performed in Section 7.6. Finally, the conclusion and future research scopes are discussed in Section 7.7.

7.2 MCDM methods in neutrosophic environment

The MCDM methods AHP, TOPSIS and COPRAS in the neutrosophic environment are introduced in this section. The NAHP is used to check the consistency of the decision matrix. The trapezoidal neutrosophic number is computed for criteria weight, and the ranking of different alternatives is done by NTOPSIS and NCOPRAS methods. Graphical representation of complete ranking flowchart is shown in Figure 7.1

(i) **Neutrosophic AHP (NAHP)**

The steps of NAHP method (Canco et al., 2021; Taherdoost, 2017; Kumar and Kumar, 2019) are as follows:

I. Recognition of the criteria and their sub-criteria.

II. On the basis of opinions of DMs, construction of a pairwise comparison matrix with the trapezoidal neutrosophic numbers (TrNNs). Let N number of DMs give their decisions. Individual DMs reveal their own views in terms of the pairwise comparison matrix of criteria. Let t be the number of criteria. Then the comparison matrix is a $t \times t$ square matrix. Now a set of N matrices are obtained: $D_c = \{d_{ijc}\}$ where $c = 1, 2, \ldots, N$ & $i, j = 1, 2, \ldots, t$ and $d_{ijc} = \{(\alpha_{ijc}, \beta_{ijc}, \gamma_{ijc}, \delta_{ijc}); t_{ijc}, i_{ijc}, f_{ijc}\}$ indicates TrNN of i criteria to j criteria as communicated by the DM 'c'.

$$\begin{cases} \alpha_{ij} = \min_{c=1,2,\ldots,N} \alpha_{ijc} \\ \beta_{ij} = \sqrt[N]{\prod_{c=1}^{N} \beta_{ijc}} \\ \gamma_{ij} = \sqrt[N]{\prod_{c=1}^{N} \gamma_{ijc}} \\ \delta_{ij} = \max_{c=1,2,\ldots,N} \delta_{ijc} \\ t_{ij} = \min_{c=1,2,\ldots,N} t_{ijc} \\ i_{ij} = \max_{c=1,2,\ldots,N} i_{ijc} \\ f_{ij} = \max_{c=1,2,\ldots,N} f_{ijc} \end{cases} \tag{7.1}$$

III. De-neutrosophication of TrNN:

De-neutrosophication of the TrNN is done by using Eq. (1.98) of the matrix

$$A_{t\times t} = [a_{ij}]_{t\times t} \tag{7.2}$$

where $i, j = 1, 2, \ldots, t$.

Further steps of AHP are computed using Eqs. (1.2), (1.3), (1.4) and (1.5).

Determination of Trapezoidal Neutrosophic Number (TrNN) Weights of Criteria

(a) Construction of pairwise comparison matrix in terms of TrNNs given by the DMs.

(b) Aggregation of the opinions of 'N' DMs using the operator followed by the Eq. (7.1).

(c) The geometric mean of the TrNNs in the comparison matrix is calculated as follows:

$$\{(\alpha_j, \beta_j, \gamma_j, \delta_j), t_j, i_j, f_j\} = \left\{ \begin{array}{c} \left((\prod_{i=1}^{t} \alpha_{ij})^{\frac{1}{t}}, (\prod_{i=1}^{t} \beta_{ij})^{\frac{1}{t}}, (\prod_{i=1}^{t} \gamma_{ij})^{\frac{1}{t}}, (\prod_{i=1}^{t} \delta_{ij})^{\frac{1}{t}}\right), \\ \min_{i=1}^{t} t_{ij}, \max_{i=1}^{t} i_{ij}, \max_{i=1}^{t} f_{ij} \end{array} \right\} \tag{7.3}$$

(d) Addition of trapezoidal numbers and optimization of membership numbers are done as follows:

$$\{(\alpha^+, \beta^+, \gamma^+, \delta^+), t^+, i^+, f^+\} = \left\{ \begin{array}{c} \left(\sum_{j=1}^{t} \alpha_j, \sum_{j=1}^{t} \beta_j, \sum_{j=1}^{t} \gamma_j, \sum_{j=1}^{t} \delta_j\right), \\ \min_{j=1}^{t} t_j, \max_{j=1}^{t} i_j, \max_{j=1}^{t} f_j \end{array} \right\} \tag{7.4}$$

(e) Calculation of the inverse of TrNN is done as follows:

$$\{(\alpha^-, \beta^-, \gamma^-, \delta^-), t^-, i^-, f^-\} = \left\{ \left(\frac{1}{\delta_j}, \frac{1}{\gamma_j}, \frac{1}{\beta_j}, \frac{1}{\alpha_j}\right), t_j, i_j, f_j \right\} \tag{7.5}$$

(f) The TrNN weights of the criteria are calculated as follows:

$$\{(\alpha_j^w, \beta_j^w, \gamma_j^w, \delta_j^w), t_j^w, i_j^w, f_j^w\}$$
$$= \{(\alpha_j\alpha^-, \beta_j\beta^-, \gamma_j\gamma^-, \delta_j\delta^-), min\{t_j, t^-\}, max\{i_j, i^-\}, max\{f_j, f^-\}\} \tag{7.6}$$

Finally, the TrNN weights of the criteria are obtained from Eq. (7.6).

(ii) **Neutrosophic TOPSIS (NTOPSIS)**

In this technique, the decision matrix is created in linguistic terms which are assigned by decision experts. These linguistic ratings are then transformed to TrNNs (Biswas et al., 2018, 2018a). The TrNNs are standardized, and then the PIS and NIS are computed for each alternative separately. The procedure of NTOPSIS method is as follows:

Step 1: Decision matrices are constructed on the basis of DMs' linguistic ratings. The linguistic assignments are then transformed into TrNNs.

Step 2: Aggregation of the opinions of 'N' DMs using the operator followed by Eq. (7.1).

Step 3: Standardization of TrNNs, using the formula:

$$\tilde{S} = [S_{ij}]_{k\times l} \tag{7.7}$$

where $i = 1, 2, 3, \ldots, k$; $j = 1, 2, 3, \ldots, l$;
$S_{ij}^B = \left\langle \left(\frac{\alpha_{ij}}{\delta_j^+}, \frac{\beta_{ij}}{\delta_j^+}, \frac{\gamma_{ij}}{\delta_j^+}, \frac{\delta_{ij}}{\delta_j^+} \right), t_{ij}, i_{ij}, f_{ij} \right\rangle$; $\delta_j^+ = \max \delta_{ij}$ and $i \in$ Beneficent Criteria (BC)
and $S_{ij}^{N.B} = \left\langle \left(\frac{\alpha_j^-}{\delta_{ij}}, \frac{\alpha_j^-}{\gamma_{ij}}, \frac{\alpha_j^-}{\beta_{ij}}, \frac{\alpha_j^-}{\alpha_{ij}} \right), t_{ij}, i_{ij}, f_{ij} \right\rangle$; $\alpha_j^- = \min \alpha_{ij}$ and $i \in$ Non-beneficent Criteria (NBC).

Step 4: Weighted standardized matrix is determined by the product of criteria's TrNN weight (wc) and standardized TrNN value (S_{ij}).

$$\tilde{WS} = [wcS_{ij}]_{k\times l};\ i = 1, 2, 3, \ldots, k \text{ and } j = 1, 2, 3, \ldots, l \tag{7.8}$$

where
$TrNNW_{ij} = S_{ij} \times wc_j; i = 1, 2, 3, \ldots, k$ and $j = 1, 2, 3, \ldots, l$.

Trapezoidal neutrosophic number (TrNN) weights of criteria are calculated by the Eq. (7.6).

Step 5: Determination of TrNNs positive ideal solution (TP^+) and TrNNs negative ideal solution TN^-. Here p_i^+ signifies the maximum value of p_{ij} and p_i^- denotes the minimum value of p_{ij}.

$(TP^+) =$
$\langle (r_1^+, \max_i t_{ij}, \min_i i_{ji}, \min_i f_{ij}), (r_2^+, \max_i t_{ij}, \min_i i_{ji}, \min_i f_{ij}), \ldots,$
$(r_l^+, \max_i t_{ij}, \min_i i_{ji}, \min_i f_{ij}) \rangle$

$$= \{(\max p_{ij} | j \in B.C), (\min p_{ij} | j \in N.B.C)\} \tag{7.9}$$

$(TN^-) =$
$\langle (r_1^-, \min_i t_{ij}, \max_i i_{ji}, \max_i f_{ij}), (r_2^-, \min_i t_{ij}, \max_i i_{ji}, \max_i f_{ij}), \ldots,$
$(r_l^-, \min_i t_{ij}, \max_i i_{ji}, \max_i f_{ij}) \rangle$

$$= \{(\min p_{ij} | j \in B.C), (\max p_{ij} | j \in N.B.C)\} \tag{7.10}$$

where

$$\begin{cases} r_j^+ = \{(r_j^{+1}, r_j^{+2}, r_j^{+3}, r_j^{+4})\} = \{\max_i(r_{ij}^1), \max_i(r_{ij}^2), \max_i(r_{ij}^3), \max_i(r_{ij}^4)\} \\ r_j^- = \{(r_j^{-1}, r_j^{-2}, r_j^{-3}, r_j^{-4})\} = \{\min_i(r_{ij}^1), \min_i(r_{ij}^2), \min_i(r_{ij}^3), \min_i(r_{ij}^4)\} \end{cases} \tag{7.11}$$

Step 6: Relative distance is calculated for each alternative in term of TrNNs i.e., from positive ideal solution (TP^+) and negative ideal solution (TN^-), respectively.

$$\begin{cases} DP_j^+ = \sum_{j=1}^{l} d(p_{ij}, p_i^+), i = 1, 2, \ldots, k \\ DP_j^- = \sum_{j=1}^{l} d(p_{ij}, p_i^-), i = 1, 2, \ldots, k \end{cases} \tag{7.12}$$

where DP_j^+ and DP_j^- denote the Hamming distance. The distance measure used here is given in Eq. (1.96).

Step 7: Finally, the relative closeness of the alternatives is calculated as

$$R_j = \frac{DP_j^-}{DP_j^+ + DP_j^-} \tag{7.13}$$

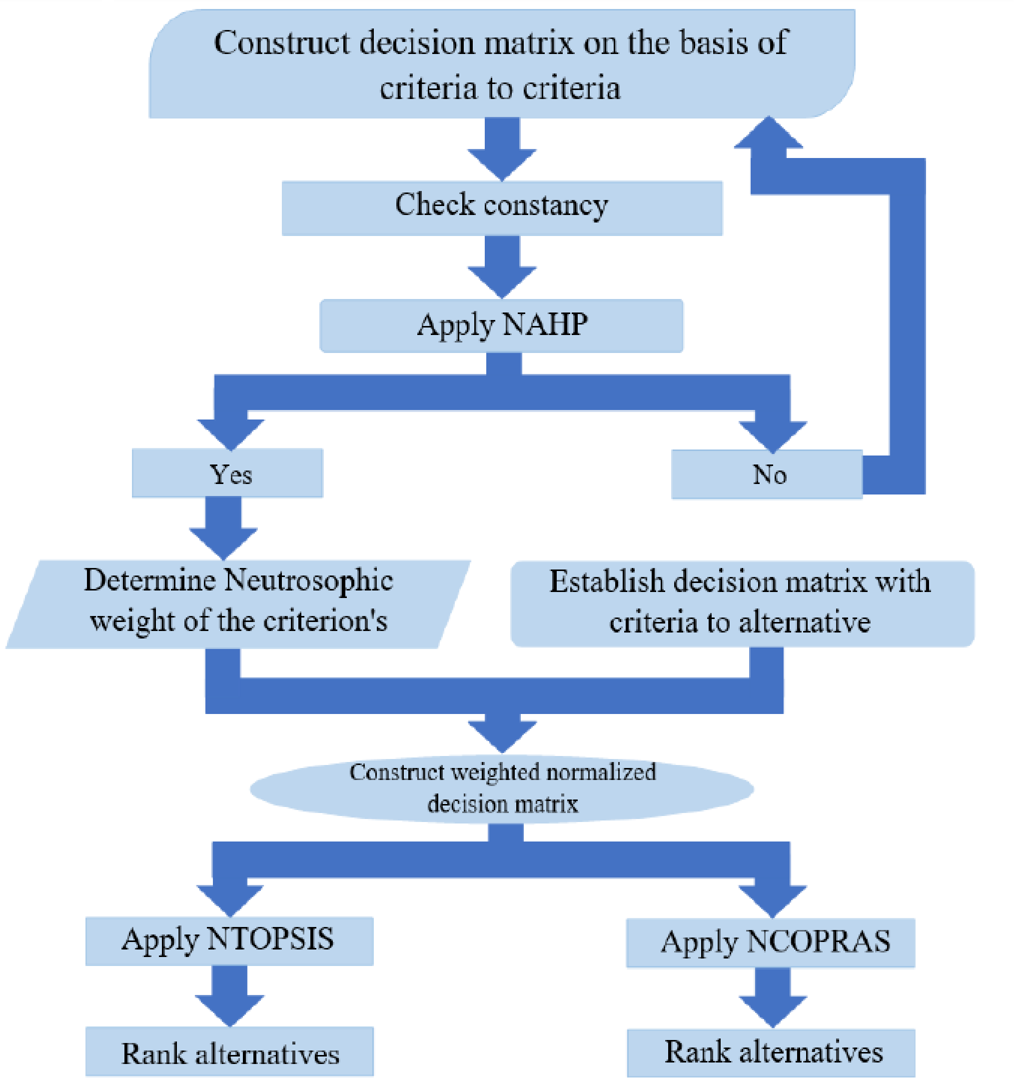

Figure 7.1: Diagrammatic structure of finding the best alternative.

Remarks 4. *Here, the criteria 'investment costs* $(\tilde{m}_2)$*' is only non-beneficiary criterion (NBC), and all other criteria are beneficiary criteria (BC). Beneficiary criteria are those criteria that are beneficial for selectors, and non beneficiary criteria are the ones whose declination is beneficial for selectors.*

(iii) **Neutrosophic COPRAS (NCOPRAS)**

The steps of NCOPRAS are as follows:

- Determination of TrNN comparison matrix by expert decision makers (DMs). The DMs allocate in linguistic terms depending on criteria.
- Integration of the decision matrix using the operator followed by the Eq. (7.1).
- Standardization of decision matrix is done in the same way as computed in the TOPSIS method using Eq. (7.7).
- Construction of weighted standardization decision matrix is done by taking product of the TrNN criteria weight and standardization decision matrix.
- Calculation of BC and NBC) denoted by BC^+ and NBC^- respectively as follows:

$$BC^+ = \left\{ \sum_{j \in B.C} \alpha_{ij}, \sum_{j \in B.C} \beta_{ij}, \sum_{j \in B.C} \gamma_{ij}, \sum_{j \in B.C} \delta_{ij}, \min_{j \in B.C} t_{ij}, \max_{j \in B.C} i_{ij}, \max_{j \in B.C} f_{ij} \right\} \tag{7.14}$$

$$NBC^- = \left\{ \sum_{j \in N.B.C} \alpha_{ij}, \sum_{j \in N.B.C} \beta_{ij}, \sum_{j \in N.B.C} \gamma_{ij}, \sum_{j \in N.B.C} \delta_{ij}, \max_{j \in N.B.C} t_{ij}, \min_{j \in N.B.C} i_{ij}, \min_{j \in N.B.C} f_{ij} \right\} \tag{7.15}$$

where $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij}, t_{ij}, i_{ij}$ & f_{ij} $(i = 1, 2, 3, \ldots, k$ and $j = 1, 2, 3, \ldots, l)$ come from the weighted standardization matrix in Eq. (7.6).

- De-neutrosophication of the TrNN using Eq. (1.98). De-deutrosophication of the BC is denoted by S_i^+, and de-neutrosophication of the NBC is denoted by S_i^-.
- Calculate

$$Q_i = S_i^+ + \frac{S_{min}^- \times \sum_{i=1}^{k} S_i^-}{S_i^- \times \sum_{i=1}^{k} \left(\frac{S_{min}^-}{S_i^-} \right)} \tag{7.16}$$

where $S_{min}^- = min\{S_i^- : i = 1, 2, 3, \ldots, k\}$ and i indicates alternatives.

– Calculation of the value of R_i

$$R_i = \frac{Q_i}{Q_{max}} \times 100\% \tag{7.17}$$

where $Q_{max} = \{Q_i : i = 1, 2, 3, \ldots, k\}$.
Finally, ranking of alternatives is done based R_i scores.

7.2.1 *Pseudo code of the model*

Suppose that the model is constructed taking k number of criteria and l number of alternatives with N DMs. The input are given by DMs in the form of linguistic terms. The linguistic terms are transformed to TrNN to get the output, i.e., ranking of alternatives by neutrosophic MCDM method. The comparison matrix is of order $k \times k$, and the decision matrix is of order $l \times k$.

INPUT: Comparison matrix & Decision matrix

OUTPUT: Ranking of alternatives
COMPUTE: Consistency ratio, weights of the criteria in TrNN
INITIALIZE: TrNN
OPERATION: NAHP, weights in TrNN, NTOPSIS & NCOPRAS

```
FOR NAHP
    MERGE N DMs' inputs of comparison matrix
    IF comparison matrix is inconsistent (CR ≥ 0.1)
        THEN reconstruct the comparison matrix
    ELSE comparison matrix is consistent (CR < 0.1)
END FOR

COMPUTE TrNN weights of the criteria
    CONSTRUCT comparison matrix
    THEN compute the weighted normalized comparison matrix
FIND the weights of the criteria in TrNN

THEN consider the N decision matrix given by DMs
    MERGE N DMs' inputs of decision matrix
COMPUTE the weighted normalised decision matrix
```

BEGIN NTOPSIS
    COMPUTE ranking of alternatives using weighted normalized decision matrix
END NTOPSIS

BEGIN NCOPRAS
    COMPUTE ranking of alternatives using weighted normalized decision matrix
END NCOPRAS

7.3 Empirical study

In under privileged area, people's perception towards the co-educational universities and women universities are different. Poor families with their conservative mentality possess inhibitions while sending their girl child to co-educational universities as they have concerns about safety, security and family honour. Hence many attributes which are not so important for co-educational universities are more pertinent for women university. The problem of selecting a women university site in the state of eastern India, namely West Bengal, is chosen for the current study. From the literature (Farid and Riaz 2022; Wu et al., 2023), it is found that there are only two women universities currently in this state. In our study, the identification and evaluation of criteria are carried out based on expert opinions. Subsequently, district headquarters that meet the established criteria are selected as potential sites. Two key criteria—safety and sex ratio—are carefully chosen with specific consideration for establishing a women's university. The criteria weights are determined using the Fuzzy Analytic Hierarchy Process (FAHP), and the selected sites were then ranked using Multi-Criteria Decision-Making (MCDM) methods, namely TOPSIS and COPRAS.

Table 7.1: Used location (alternative) details.

Location	District	Latitude & Longitude	Location details
Kolkata (L_{A1})	Kolkata	22.5726°N, 88.3639°E	Capital city of West Bengal.
Howrah (L_{A2})	Howrah	22.5958°N, 88.2636°E	Capital city of Howrah district.
Berhampore (L_{A3})	Murshidabad	24.0983°N, 88.2684°E	Capital city of Murshidabad district.
Siliguri (L_{A4})	Darjeeling	26.7271°N, 88.3953°E	This city is in Darjeeling district on base of the Himalaya mountains and the side of Mahananda river. It is second largest city of West Bengal.
Midnapore (L_{A5})	Paschim Medinipur	22.4257°N, 87.3199°E	This city is located along the Kangsabati River and serves as the capital of the Midnapore district.
Durgapur (L_{A6})	Paschim Bardhaman	23.5204°N, 87.3119°E	Capital city of Burdwan district.

Presently in West Bengal, India, there is two women's university. The first one is "Diamond Harbour Women's University" in Diamond Harbour, South 24 Parganas district, and the other one is "Kanyashree University" in Krishnanagar, Nadia district. According to the 2011 census, [1] population of West Bengal is 91,276,115, where the female population is 44,467,088, and the male population is 46,809,027. The literacy rate for females is 70.54% and for males 81.69% and overall 76.36%. Female literacy is 10.93% more than the 2001 census. Six locations are considered as alternatives for choosing the best site for women university. Table 7.1 gives their details and Figure 7.2 shows their positions in West Bengal, India's map.

[1] https://www.census2011.co.in/census/state/west+bengal.html

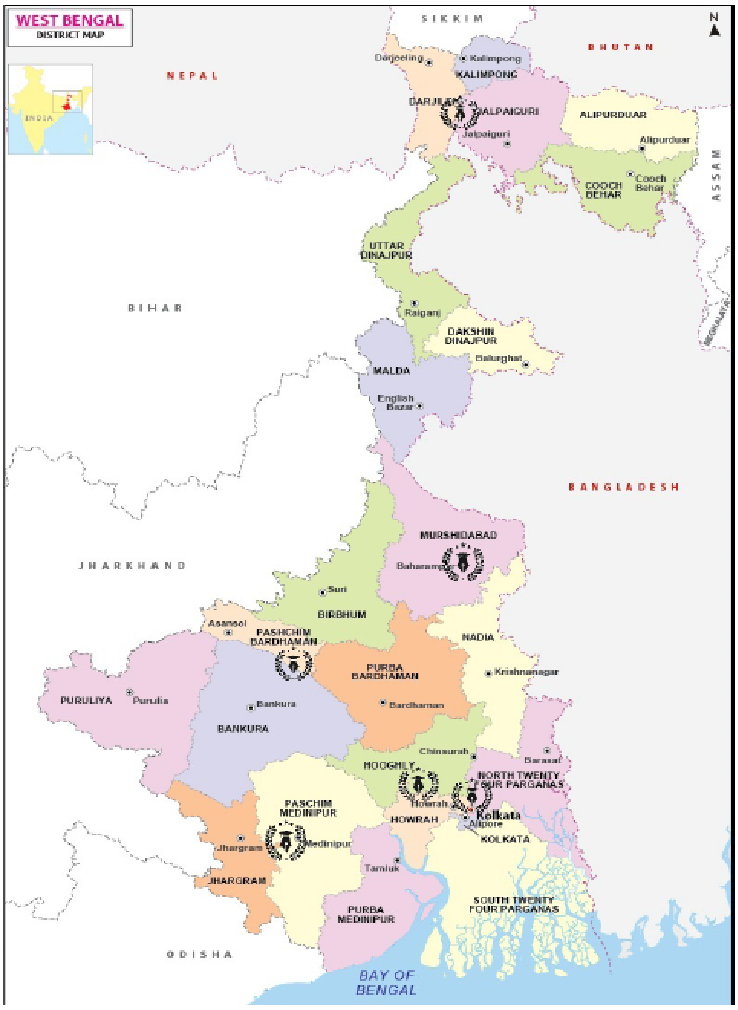

Figure 7.2: Locations for Women's University in West Bengal, India

7.4 Criteria of Women University site selection

After consulting a few experts, we finalize the following ten criteria. The hierarchical structure of the Women University site selection problem is shown in Figure 7.3.

- Population Density ($\tilde{m}_1$) (fixed data): The large population density of a specific zone implies a greater probability of students enrolling for higher studies. The probability becomes obvious, if the literacy rate of the related area is higher.

- Investment Costs ($\tilde{m}_2$): Investment cost implies the aggregate cost connected with the project of selecting a site for University construction. This cost includes the price of the land, construction costs, operating costs, and management costs.

- Rate of Literacy ($\tilde{m}_3$) (fixed data):

  Educational opportunities depend on Literacy. Literacy is at the heart of basic education for all and essential for eradicating poverty, reducing child mortality, curbing population growth, achieving gender equality, and ensuring sustainable development. The rate of literacy is an important attribute when one thinks about the construction of higher educational institutions.

- Number of Graduates ($\tilde{m}_4$:) This criterion implies the number of persons who have completed their graduation degree and are interested in higher studies. If the number of graduates in a particular location is more, then the possibility of university construction in that specific site is high.

- Yearly Income per Person ($\tilde{m}_5$): Income and education have a strong relationship. Higher education leads to multiple professions with good income and vice versa. The person with higher education has low average unemployment than those with less or no education. Thus, data on yearly income per person in the area strongly signifies a greater number of students who can pursue higher studies. The flowchart of the selection process is also shown in Figure 7.1

- Accessibility ($\tilde{m}_6$): Accessible facilities include trains, buses, and premises accommodation/ hostels for professors, students, and non-teaching staff are quintessential for university site selection.

- Public Services ($\tilde{m}_7$): Fire safety, such as fire alarms and fire devices, is important to avoid fire dangers in educational institutions.

- Safety ($\tilde{m}_8$): Safety for the students and Professors is equally important. Places where social hazards in the neighborhood, such as high incidence of crime and drug or alcohol abuse, are not to be considered safe for University construction.

- Proximity of Educational Institutes ($\tilde{m}_9$): Sites that are selected close to libraries, educational institutions/research institutes are preferred more than those sites which don't have these facilities nearby.

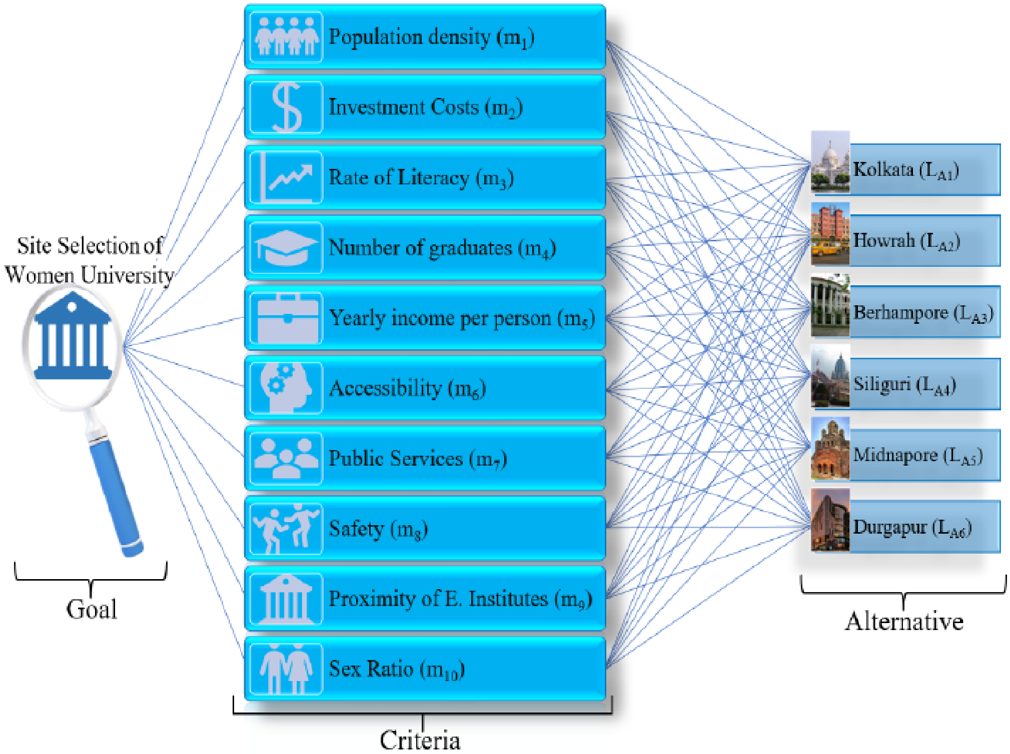

Figure 7.3: The hierarchical structure of women's university site selection problem.

- Sex Ratio ($\tilde{m_{10}}$) (fixed data): The sex ratio or gender ratio is the ratio of females to males in a population. Since this research is about women's university site selection, this factor is quite significant for finding out the optimal site.

7.5 Numerical illustration

7.5.1 *Data collection*

Data is obtained from a government portal. Past records are analyzed to develop a scale, based on which appropriate linguistic terms are assigned. In criteria-to-criteria comparison, linguistic rating to TrNN transformation by DMs are shown in Table 7.1. We also give Table 7.6 for related criteria with their units and data collection procedure. In Table 7.2, the linguistic rating for criteria to alternative with TrNN is given. The fixed data scaling and linguistic conversion is performed in our own way (see Tables 7.5 and 7.6). Without the fixed data cases, all the linguistic ratings for criteria to criteria and criteria to an alternative are shown in Tables 7.2 and 7.4, respectively.

Table 7.2: Linguistic terms and their corresponding TrNN.

Linguistic Terms	Trapezoidal Neutrosophic Numbers (TrNN)
Equally Important (EI)	$\{(0.4, 0.5, 0.6, 0.7); 0.85, 0.20, 0.15\}$
Moderately Important (MI)	$\{(0.5, 0.6, 0.7, 0.8); 0.85, 0.15, 0.10\}$
Strongly Important (SI)	$\{(0.6, 0.7, 0.8, 0.9); 0.90, 0.15, 0.10\}$
Very Strongly Important (VSI)	$\{(0.7, 0.8, 0.9, 0.95); 0.90, 0.10, 0.05\}$
Absolutely Important (AI)	$\{(0.8, 0.9, 0.95, 1.0); 0.95, 0.10, 0.00\}$
Moderately Not Important (MNI)	$\{(0.3, 0.4, 0.5, 0.6); 0.80, 0.20, 0.15\}$
Strongly Not Important (SNI)	$\{(0.2, 0.3, 0.4, 0.5); 0.80, 0.25, 0.15\}$
Very Strongly Not Important (VSNI)	$\{(0.1, 0.2, 0.3, 0.4); 0.80, 0.25, 0.20\}$
Absolutely Not Important (ANI)	$\{(0.0, 0.1, 0.2, 0.3); 0.75, 0.30, 0.20\}$

Table 7.3: Related criteria with their units and data sources.

Criteria	Scales (Units)	Source of Data
Population density ($\tilde{m}_1$)	This is calculated by average number of population per square kilometer.	This is fixed data collected from Census 2011 [2]
Investment Costs ($\tilde{m}_2$)	In Rupees.	After considering the investment costs different location.
Rate of Literacy ($\tilde{m}_3$)	Divide the number of literates of a given age range by the corresponding age group population and then multiply the result by 100.	This is fixed data collected from Census 2011 [3].
Number of graduates ($\tilde{m}_4$)	Number of people.	Based on literacy rate.
Yearly income per person ($\tilde{m}_5$)	In Rupees.	After considering the average income of the district where the sites are located.
Accessibility ($\tilde{m}_6$)	Linguistics term (good, bad etc.).	After seeing the transport and related system of the sites.
Public Services ($\tilde{m}_7$)	Linguistics term.	After seeing whether there is public service active or not nearby the sites.
Safety ($\tilde{m}_8$)	Linguistics term	After analysing the crime agents women and crime rate nearby the sites.
Proximity of E. Institutes ($\tilde{m}_9$)	Linguistics term	After seeing nearby educational institutes exist or not.
Sex Ratio ($\tilde{m}_{10}$)	Number of women per 1000 men.	This is fixed data collected from Census 2011 [4].

Table 7.4: Linguistic terms and their corresponding TrNN for rating alternatives.

Linguistic Terms	Trapezoidal Neutrosophic Numbers (TrNN)
Low priority (LP)	$\{(0.0, 0.1, 0.2, 0.3); 0.75, 0.20, 0.20\}$
Below priority (BP)	$\{(0.2, 0.3, 0.4, 0.5); 0.80, 0.20, 0.15\}$
Medium priority (MP)	$\{(0.4, 0.5, 0.6, 0.7); 0.85, 0.15, 0.15\}$
Very priority (VP)	$\{(0.6, 0.7, 0.8, 0.9); 0.90, 0.10, 0.15\}$
Extremely priority (EP)	$\{(0.7, 0.8, 0.9, 1.0); 0.95, 0.15, 0.00\}$

Step 1: Using data given in Table 7.2, the consistency of the decision matrix is examined. The decision matrix is found to be consistent (< 0.1). The criteria weights in crisp value are described in Table 7.8.

Step 2: Using the decision-makers' data presented in Table 7.3, and converting the associated linguistic terms into TrNN values as per Table 7.2, we apply the neutrosophic weight calculation formulas described in Section 7.2. The neutrosophic weights of all criteria are shown in Table 7.10 where β_1, β_2, β_3, and β_4 are the first, second, third, and forth entries of the trapezoidal neutrosophic numbers, respectively, and t is used for true, i for indeterminacy and f for false memberships.

Remarks 5. *The linguistic rating by TrNN is given in Table 7.2. This numerical rating is done in a scientific manner.*

Table 7.5: Fixed data for population density $(\tilde{m_1})$, rate of literacy $(\tilde{m_3})$ & sex ratio $(\tilde{m_{10}})$.

Alternative	Population Density $(\tilde{m_1})$	Rate of Literacy $(\tilde{m_3})$	Sex Ratio $(\tilde{m_{10}})$
Kolkata (L_{A1})	24306	86.31%	908
Howrah (L_{A2})	3306	83.31%	939
Berhampore (L_{A3})	1334	66.59%	958
Siliguri (L_{A4})	586	79.56%	970
Midnapore (L_{A5})	631	78.00%	966
Durgapur (L_{A6})	1099	76.21%	945

Table 7.6: Conversion of fixed data into corresponding linguistic terms

Linguistic Terms	**Population Density ($\tilde{m}_1$)**	**Rate of Literacy ($\tilde{m}_3$)**	**Sex Ratio ($\tilde{m}_{10}$)**
Low priority (LP)	$\tilde{m}_1 < 500$	$\tilde{m}_3 < 65$	$\tilde{m}_{10} < 910$
Below priority (BP)	$500 \leq \tilde{m}_1 < 1000$	$65 \leq \tilde{m}_3 < 70$	$910 \leq \tilde{m}_{10} < 935$
Medium priority (MP)	$1000 \leq \tilde{m}_1 < 3000$	$70 \leq \tilde{m}_3 < 75$	$935 \leq \tilde{m}_{10} < 960$
Very priority (VP)	$3000 \leq \tilde{m}_1 < 15000$	$75 \leq \tilde{m}_3 < 80$	$960 \leq \tilde{m}_{10} < 985$
Extremely priority (EP)	$15000 \leq \tilde{m}_1$	$80 \leq \tilde{m}_3$	$985 \leq \tilde{m}_{10}$

Table 7.7: Linguistic comparison matrix of criteria from the three decision-makers.

	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 1	Population density ($\tilde{m}_1$)	EI	VSNI	VSNI	SNI	EI	SNI	ANI	ANI	MNI	AI
	Investment Costs ($\tilde{m}_2$)	VSI	EI	VSNI	SNI	SI	VSNI	SNI	ANI	EI	ANI
	Rate of Literacy ($\tilde{m}_3$)	VSI	VSI	EI	EI	AI	EI	MI	EI	SI	SI
	Number of graduates ($\tilde{m}_4$)	SI	SI	EI	EI	SI	EI	EI	SNI	EI	SI
	Yearly income per person ($\tilde{m}_5$)	EI	SNI	ANI	SNI	EI	ANI	VSNI	ANI	SI	ANI
	Accessibility ($\tilde{m}_6$)	SI	VSI	EI	EI	AI	EI	SI	EI	SI	ANI
	Public Services ($\tilde{m}_7$)	AI	SI	MNI	EI	VSI	SNI	EI	SNI	EI	ANI
	Safety ($\tilde{m}_8$)	AI	AI	EI	SI	AI	EI	SI	EI	AI	AI
	Proximity of E. Institutes ($\tilde{m}_9$)	MI	EI	SNI	EI	SNI	SNI	EI	ANI	EI	MNI
	Sex Ratio ($\tilde{m}_{10}$)	ANI	AI	SNI	SNI	AI	AI	AI	ANI	MI	EI
	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 2	Population density ($\tilde{m}_1$)	EI	SNI	ANI	VSNI	MNI	SNI	SNI	ANI	MNI	AI
	Investment Costs ($\tilde{m}_2$)	SI	EI	ANI	SNI	VSI	SNI	SNI	VSNI	MNI	VSNI
	Rate of Literacy ($\tilde{m}_3$)	AI	AI	EI	MI	VSI	EI	SI	EI	EI	VSI
	Number of graduates ($\tilde{m}_4$)	VSI	SI	MNI	EI	AI	MI	SI	ANI	MNI	SI
	Yearly income per person ($\tilde{m}_5$)	MI	VSNI	VSNI	ANI	EI	ANI	VSNI	ANI	MI	VSNI
	Accessibility ($\tilde{m}_6$)	SI	SI	EI	MNI	AI	EI	EI	EI	SI	ANI
	Public Services ($\tilde{m}_7$)	SI	SI	SNI	SNI	VSI	EI	EI	SNI	EI	VSNI
	Safety ($\tilde{m}_8$)	AI	VSI	EI	AI	AI	EI	SI	EI	VSI	VSI
	Proximity of E. Institutes ($\tilde{m}_9$)	MI	MI	EI	MI	MNI	SNI	EI	VSNI	EI	EI
	Sex Ratio ($\tilde{m}_{10}$)	ANI	VSI	VSNI	SNI	VSI	AI	VSI	VSNI	EI	EI

Remarks 6. *All data about population density ($\tilde{m}_1$) & sex ratio ($\tilde{m}_{10}$) are collected from*

Census 2011 [5] *and rate of literacy* $(\tilde{m}_3)$ *data from Wikipedia* [6] *shown in Table 7.5. Those data are taken from authorised sources in the year 2011. Transformation of fixed data to linguistic term is shown in Table 7.6.*

Table 7.7: *Cont.*

	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 3	Population density $(\tilde{m}_1)$	EI	MNI	MNI	SNI	MI	ANI	ANI	VSNI	EI	VSI
	Investment Costs $(\tilde{m}_2)$	MI	EI	VSNI	ANI	SNI	MI	VSNI	ANI	SI	ANI
	Rate of Literacy $(\tilde{m}_3)$	MI	VSI	EI	MI	AI	SI	EI	EI	MI	MI
	Number of graduates $(\tilde{m}_4)$	SI	AI	MNI	EI	AI	SI	EI	EI	SI	VSI
	Yearly income per person $(\tilde{m}_5)$	MNI	SI	ANI	ANI	EI	VSNI	SNI	SNI	SNI	VSNI
	Accessibility $(\tilde{m}_6)$	AI	MNI	SNI	SNI	VSI	EI	MI	SNI	MI	VSNI
	Public Services $(\tilde{m}_7)$	AI	VSI	EI	EI	SI	MNI	EI	EI	SNI	ANI
	Safety $(\tilde{m}_8)$	VSI	AI	EI	EI	SI	SI	EI	EI	MNI	AI
	Proximity of E. Institutes $(\tilde{m}_9)$	EI	SNI	MNI	SNI	SI	MNI	SI	MNI	EI	MNI
	Sex Ratio $(\tilde{m}_{10})$	VSNI	AI	MNI	VSNI	VSI	VSI	AI	ANI	MI	EI

Table 7.8: Normalized criterion's weight using neutrosophic AHP.

Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Criteria weight	0.0696	0.0677	0.1400	0.1197	0.0578	0.1074	0.0989	0.1478	0.0932	0.0979

Remarks 7. *The crisp weights of all criteria are shown in Table 7.8 using NAHP. The values reflect that the criteria 'safety'* $(\tilde{m}_8)$ *is the most important followed by 'rate of literacy'* $(\tilde{m}_3)$*, 'number of graduates'* $(\tilde{m}_4)$*, 'accessibility'* $(\tilde{m}_6)$*, 'public services'* $(\tilde{m}_7)$*, 'sex ratio'* $(\tilde{m}_{10})$*, 'proximity of E. institutes'* $(\tilde{m}_9)$*, 'population density'* $(\tilde{m}_1)$*, 'investment costs'* $(\tilde{m}_2)$ *and 'yearly income per person'* $(\tilde{m}_5)$ *is the least significant criteria amongst the site selection criteria.*

[5] https://www.census2011.co.in/census/state/districtlist/west+bengal.html
[6] https://en.wikipedia.org/wiki/List_of_West_Bengal_districts_ranked_by_literacy_rate

Table 7.9: Linguistic comparison matrix between criteria and alternatives as provided by the three decision-makers

	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 1	Kolkata (L_{A1})	EP	EP	EP	MP	MP	EP	EP	MP	EP	LP
	Howrah (L_{A2})	VP	MP	EP	LP	BP	VP	EP	LP	BP	MP
	Berhampore (L_{A3})	MP	BP	BP	LP	BP	BP	LP	LP	LP	MP
	Siliguri (L_{A4})	BP	MP	VP	LP	BP	MP	BP	BP	LP	VP
	Midnapore (L_{A5})	BP	BP	VP	MP	BP	MP	MP	MP	MP	VP
	Durgapur (L_{A6})	MP	MP	VP	MP	MP	BP	BP	MP	BP	MP
	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 2	Kolkata (L_{A1})	EP	EP	EP	MP	VP	EP	EP	VP	VP	LP
	Howrah (L_{A2})	VP	VP	EP	BP	MP	EP	VP	BP	MP	MP
	Berhampore (L_{A3})	MP	BP	BP	LP	LP	MP	LP	MP	LP	MP
	Siliguri (L_{A4})	BP	BP	VP	LP	MP	VP	MP	BP	LP	VP
	Midnapore (L_{A5})	BP	MP	VP	EP	MP	VP	BP	VP	MP	VP
	Durgapur (L_{A6})	MP	BP	VP	BP	BP	MP	BP	BP	BP	MP
	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 3	Kolkata (L_{A1})	EP	VP	EP	VP	VP	VP	EP	VP	EP	LP
	Howrah (L_{A2})	VP	VP	EP	MP	MP	VP	EP	BP	BP	MP
	Berhampore (L_{A3})	MP	BP	BP	BP	LP	BP	LP	BP	BP	MP
	Siliguri (L_{A4})	BP	MP	VP	LP	BP	MP	BP	MP	BP	VP
	Midnapore (L_{A5})	BP	MP	VP	VP	MP	BP	MP	VP	MP	VP
	Durgapur (L_{A6})	MP	BP	VP	BP	BP	BP	LP	MP	LP	MP

Step 3: Application of NTOPSIS ranking model which is described in Section 7.2. The positive ideal solution (TP_j^+), negative ideal solution (TN_j^-), and relative closeness R_j with ranking on the basis of R_j values are represented in Table 7.11.

Step 4: We perform an analysis using by NCOPRAS method for ranking, which is discussed in Section 7.2. The de-neutrosophication sum of beneficiary criteria (BC) and sum of non-beneficiary criteria (NBC) is denoted by S_i^+ and S_i^-, respectively. The factor Q_i is mentioned in Equation (7.16), and its percentage is denoted by R_i. All the computation values and ranking of the alternatives on ascending order of R_i values are described in Table 7.12.

Table 7.10: Depiction of neutrosophic criteria weights.

Criteria	β_1	β_2	β_3	β_4	t	i	f
Population density ($\tilde{m}_1$)	0.0000	0.0536	0.0929	0.6438	0.7500	0.3000	0.2000
Investment costs ($\tilde{m}_2$)	0.0000	0.0528	0.0925	0.6458	0.7500	0.3000	0.2000
Rate of literacy ($\tilde{m}_3$)	0.0634	0.1146	0.1647	0.9274	0.8500	0.2000	0.1500
Number of graduates ($\tilde{m}_4$)	0.0000	0.0972	0.1454	0.9104	0.7500	0.3000	0.2000
Yearly income per person ($\tilde{m}_5$)	0.0000	0.0425	0.0791	0.6024	0.7500	0.3000	0.2000
Accessibility ($\tilde{m}_6$)	0.0000	0.0873	0.1340	0.8142	0.7500	0.3000	0.2000
Public services ($\tilde{m}_7$)	0.0000	0.0798	0.1247	0.7703	0.7500	0.3000	0.2000
Safety ($\tilde{m}_8$)	0.0649	0.1228	0.1734	0.9633	0.8000	0.2000	0.1500
Proximity of E. institutes ($\tilde{m}_9$)	0.0000	0.0751	0.1178	0.7849	0.7500	0.3000	0.2000
Sex ratio ($\tilde{m}_{10}$)	0.0000	0.0764	0.1222	0.7359	0.7500	0.3000	0.2000

Table 7.11: Ranking of alternatives using **NTOPSIS** method along with the corresponding data

Alternatives	TP_j^+	TN_j^-	$R_j = \frac{TN_j^-}{TP_j^+ + TN_j^-}$	Ranking
Kolkata (L_{A1})	0.1382	0.8824	0.8646	**1**
Howrah (L_{A2})	0.3527	0.6680	0.6545	**2**
Berhampore (L_{A3})	0.8321	0.1886	0.1847	**6**
Siliguri (L_{A4})	0.6078	0.4160	0.4063	**5**
Midnapore (L_{A5})	0.3555	0.6656	0.6518	**3**
Durgapur (L_{A6})	0.6058	0.4178	0.4082	**4**

Table 7.12: Ranking of alternatives using **NCOPRAS** method along with the corresponding data

Alternatives	S_i^+	S_i^-	Q_i	$R_i(\%)$	Ranking
Kolkata (L_{A1})	1.807561	0.056614	2.062153	100.0	**1**
Howrah (L_{A2})	1.58208	0.082565	1.756652	85.18	**2**
Berhampore (L_{A3})	1.099746	0.165103	1.187046	57.56	**6**
Siliguri (L_{A4})	1.356608	0.159694	1.446866	70.16	**5**
Midnapore (L_{A5})	1.653031	0.159694	1.743288	84.53	**3**
Durgapur (L_{A6})	1.361117	0.162181	1.44999	70.31	**4**

Remarks 8. *From Tables 7.8 and 7.10, and Figure 7.4, we see that the ranking for alternatives gives the same result for two methods NTOPSIS and NCOPRAS, respectively. So the decision maker can easily take the preferable sites for the mentioned alternatives. The site 'Kolkata' comes to the first position, Howrah becomes the second position, Midnapore*

becomes the third position, Durgapur becomes the fourth position, Siliguri becomes the fifth position, and Berhampore becomes the sixth position.

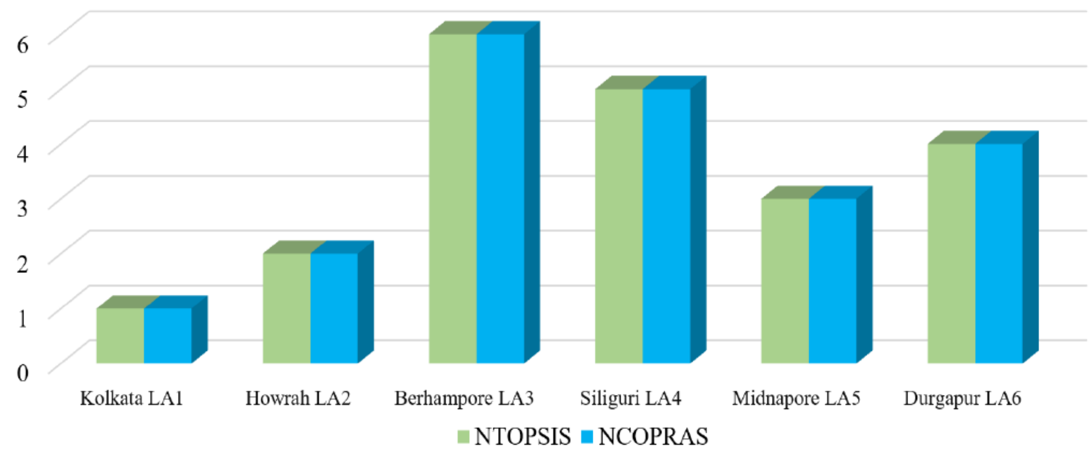

Figure 7.4: Comparative ranking diagram between NTOPSIS and NCOPRAS methods.

7.5.2 Computational complexity

The computational complexity of the proposed fuzzy MCDM methodology is described in this section. The concept of computational complexity is described in Chakraborty and Saha (2022, 2022a); Ghaleb et al. (2020) . The number of mathematical operations performed to get the result is determined by time complexity which is denoted by TC in this study. We also assume l as the number of factors, k as the number of alternatives with N as the number of decision-makers. Thereafter, the following steps are taken to compute the computational complexity.

- For NAHP, the comparison matrix is of l^2 entries; therefore, N DMs give $N \times l^2$ entries. For finding a comparison matrix, Nl^2 operations are needed. Then for de-enutrosophic process, l^2 number of operations performed and to get normalized de-neutrosophic comparison matrix, l^2 operations conducted. Thereafter, to find the nth root and factor weight, $2l + 1$ operations are needed. Also, factor weight needs l^2 operations. Then factor sum and sum/weight are calculated by $2l$ operations. Finally, the consistency ratio is calculated by 3 more operations. Therefore, the total operations conducted for NAHP is $N \times l^2 + l^2 + l^2 + 2l + 1 + l^2 + 2l + 3 = (N + 3)l^2 + 4(l + 1)$.

- For Trapezoidal Neutrosophic Number (TrNN), factor weights are calculated on comparison matrix with given N DMs' total $N \times l^2$ entries. Calculation of geometric mean is done by $7l$ operations. The sum and inverse operations are performed by 2 operations. Finally, factor weight calculation requires $7l$ operations. Therefore, the total operations performed is $7l + 2 + 7l = 14l + 2$.

- In the NTOPSIS method, the decision matrix contains $k \times l$ entries. With N DMs the total number of entries becomes Nkl, requiring Nkl operations to construct the aggregated decision matrix. Next, the normalized and weighted normalized decision matrices are computed, involving $2kl + l$ operations. Determining the positive and negative ideal solutions requires an additional $2l$ operations. To evaluate the relative closeness to the ideal solutions, $2kl$ operations are needed. The total sum for each alternative is then calculated using $2k$ operations. Finally, the comparison ratio and ranking of the alternatives require another $2k$ operations. Summing all these steps, the total number of operations performed is: $Nkl + 2kl + l + 2l + 2kl + 2k + 2k = (N + 4)kl + 4k + 3l$.

- In the NCOPRAS technique, the operations up to the construction of the weighted normalized decision matrix require $Nkl + 2kl + l$ operations, which are already completed during the NTOPSIS process. Next, computing the sums of beneficial and non-beneficial attributes requires $2k$ operations. For the de-neutrosophic transformation, an additional $2k$ operations are performed. Calculating Q_i values requires k operations, followed by another k operations for ranking the alternatives. Therefore, the total number of operations is: $Nkl + kl + l + 2k + 2k + k + k = (N + 1)kl + 6k + l$.

Time complexity (TC) of this study is calculated with number of factors $l = 10$, number of alternatives $k = 6$ and number of decision makers $N = 3$ as follows:

- For NAHP, number of calculations is $(3 + 3) \times 10^2 + 4 \times 10 + 4 = 644$.

- For weight, number of operations is $14 \times 10 + 2 = 142$.

- For FTOPSIS, number of operations is $(3 + 4) \times 6 \times 10 + 4 \times 6 + 3 \times 10 = 474$.

- For FCOPRAS, number of calculations is $(3 + 1) \times 6 \times 10 + 6 \times 6 + 10 = 286$.

Therefore, the time complexity $TC = 644 + 142 + 474 + 286 = 1546$.

7.6 Sensitivity analysis

Sensitivity analysis generally expresses the different ranking of the alternatives in a different environment. As it is known that decision-making depends on various conflicting criteria, in sensitivity analysis, removal of criteria or interchange of criterion's weight with respect to some conditions can be executed. Thus, in this study, three different cases have been considered. Different rankings obtained under these cases using two MCDM tools NTOPSIS and NCOPRAS are represented graphically.

- Removing Investment Cost $(\tilde{m}_2)$ - Several times, it is seen that during the construction of a University, a government or charitable trust offers funds for the construction. Thus, in this scenario, the investor need not necessarily think about the investment cost.

  Remarks 9. *Ranking obtained under removal of investment cost shows Table 7.13 and Figure 7.5 that the alternatives 'Kolkata', 'Berhampore' and 'Siliguri' remained consistent with positions first, sixth and fifth, respectively.*

Table 7.13: Ranking alternatives by removing Investment Cost $(\tilde{m}_2)$.

Alternatives	NTOPSIS	NCOPRAS
Kolkata (L_{A1})	1	1
Howrah (L_{A2})	4	4
Berhampore (L_{A3})	6	6
Siliguri (L_{A4})	5	5
Midnapore (L_{A5})	2	2
Durgapur (L_{A6})	3	3

- Removing Accessibility $(\tilde{m}_6)$ - The criteria 'accessibility' can removed under the following considerations:

  1. Government has proposed public accessibility.
  2. The investors might set up its own accessibility.
  3. Fully residential university need not require accessibility.

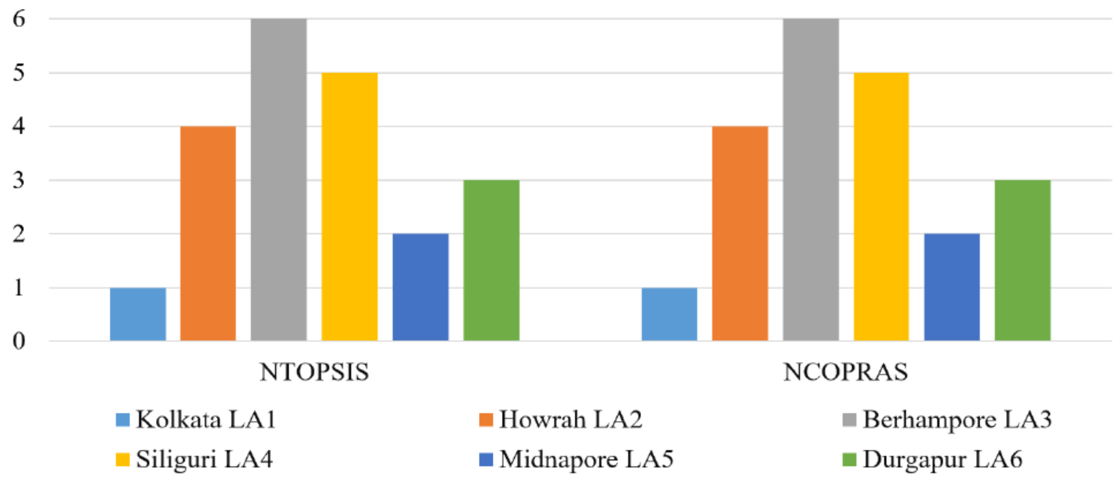

Figure 7.5: Depiction of ranking alternatives by removing Investment Cost $(\tilde{m}_2)$.

Remarks 10. *Ranking obtained under this environment reveals in Table 7.14 and Figure 7.6 which show that the locations 'Kolkata', 'Durgapur', 'Siliguri' and 'Berhampore' are at the same position whereas the alternatives 'Howrah', and 'Midnapore' rankings have been interchanged.*

Table 7.14: Ranking of alternatives by removing criteria Accessibility $(\tilde{m}_6)$.

Alternatives	**Ranking Using NTOPSIS**	**Ranking Using NCOPRAS**
Kolkata (L_{A1})	1	1
Howrah (L_{A2})	3	3
Berhampore (L_{A3})	6	6
Siliguri (L_{A4})	5	5
Midnapore (L_{A5})	2	2
Durgapur (L_{A6})	4	4

- Removing Proximity of Educational Institute $(\tilde{m}_9)$ - Individual University has it's own infrastructure, such as libraries, research cell, academic development cell, etc. So the proximity of educational institutes to the University may not be considered necessary.

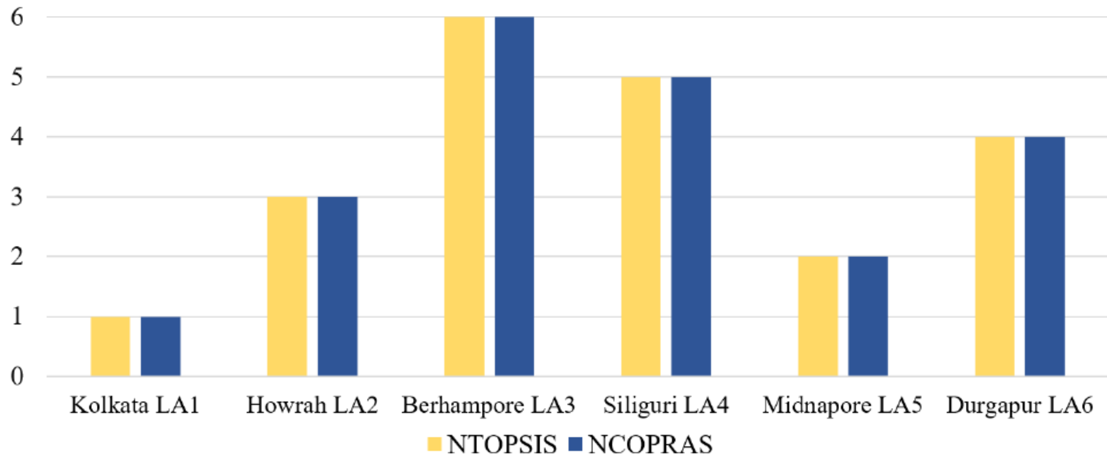

Figure 7.6: Ranking of alternatives by removing Accessibility $(\tilde{m}_6)$.

Remarks 11. *Ranking obtained under this condition represented in Table 7.15 and Figure 7.7 depicts that the same ranking is obtained for the locations 'Kolkata', 'Durgapur', 'Siliguri' and 'Berhampore'.*

Table 7.15: Ranking of alternatives by removing Proximity of Educational institute $(\tilde{m}_9)$.

Alternatives	Ranking Using NTOPSIS	Ranking Using NCOPRAS
Kolkata (L_{A1})	1	1
Howrah (L_{A2})	2	2
Berhampore (L_{A3})	6	6
Siliguri (L_{A4})	5	5
Midnapore (L_{A5})	3	3
Durgapur (L_{A6})	4	4

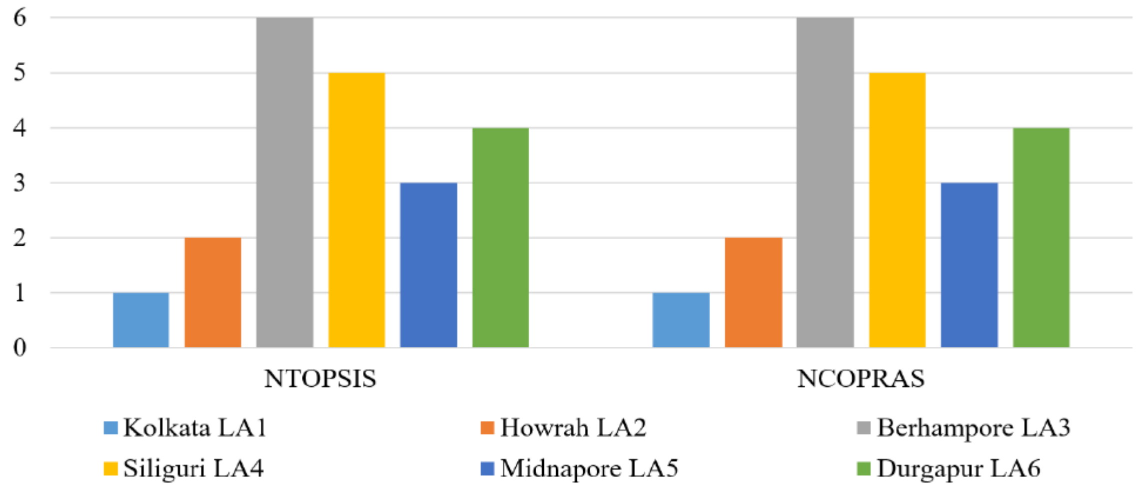

Figure 7.7: Ranking of alternatives by removing Proximity of Educational institute $(\tilde{m}_9)$.

7.7 Conclusions

The neutrosophic MCDM method uses a screening methodology to find the solution for different complex problems with uncertain data. It is capable of finding valuable information for the decision-makers by comparing a host of different parameters before making the final conclusion. Finding the best location for setting up a women's university is a major social and economic concern requiring the trade-off and weighting of various factors. The core aim of this study is the selection of the best location for women's universities by considering social and national needs.

In this work, two different selection methods have been used for choosing an optimum site. The most commonly used methods that have been elaborated upon include AHP, TOPSIS, COPRAS, and neutrosophic set theory. A new de-neutrosophication technique has been introduced and applied in this present work. Comparisons between the two methods NTOPSIS and NCOPRAS have been carried out. This comparison showed that the proposed methodology is reliable. Sensitivity analysis has been carried out by incorporating different possible weights and using a combination of influencing factors to accommodate different organizational needs.

The following steps are taken for doing the numerical study:

- Check the consistency of the decision matrix using neutrosophic AHP.

- Obtain the neutrosophic weight of the criteria to evaluate the weighted decision matrix.
- Calculate the nearest distance from the positive ideal solution, and the farthest distance from the negative ideal solution using neutrosophic TOPSIS.
- Determine the maximizing and minimizing index values. The attributes of maximizing and minimizing indexes are the assessment of the results examined individually using the neutrosophic COPRAS method.

The findings of the work presented here may be helpful for a decision-maker who deals with the site selection problem with some uncertainties in data.

8 Evaluation of the Treatment Options for COVID-19 Patients using Generalized Hesitant Fuzzy-MCDM Techniques

8.1 Introduction

Throughout history, coronaviruses have caused outbreaks in both humans and animals (Biscayart et al., 2020) such as the avian influenza outbreak in 1997, the Severe Acute Respiratory Syndrome (SARS) in 2003 (Rodriguez et al., 2020) and the severe fever with thrombocytopenia syndrome (SFTS) in China in 2010 (Rodriguez et al., 2018). December 2019 was another stark example where the zoonotic transmission caused a population of approximately 11.9 million people in Wuhan, China to come to a standstill (Ahmad et al., 2020). The coronavirus responsible for this was designated as Coronavirus Disease 2019 (COVID-19) by the World Health Organization [1]. This study seeks to identify, evaluate and rank the treatment options available for COVID-19 patients. The generalized hesitant fuzzy MCDM technique is applied for the same.

8.1.1 *Objectives of this study*

- Identification of different treatment alternatives. Preparation of questionnaires based on treatment and the criteria, collection of data from the experts.

This chapter is based on the work published in *Elsevier*, 2023, 88, 101614. https://doi.org/10.1016/j.seps.2023.101614

[1] https://www.who.int/health-topics/coronavirus.

- Identification of criteria for application of these treatment options, assigning proper weightage to them based on their importance.
- Development of GHPFN, the correspondence arithmetic operations, score functions and its application in our study.
- To use GHPFN-TOPSIS approach to rank the treatment alternatives.
- Ranking of treatment options using proposed model and checking the reliability and steadiness of the model through comparative analysis and sensitive analysis.

8.2 Overview of treatment options and the criteria

i.) Remdesivir

Remdesivir shows a broad spectrum anti viral activity against Ebola virus (Nature, 531 (2016), pp. 381–385), Nipah virus (SciTransl Med, 11(2019), p. eaau9242), respiratory syncitial virus and a large category of corona viruses including SARS CoV and MERS CoV (MBio, 9 (2018)e00221–e00218). Remdisivir, neucleotide analogue acts by blocking replication of viruses but its precise mechanism is still unknown.

Side effects: Nausea, constipation, pain, bleeding, bruising of the skin, soreness, or swelling near the place where the medication is injected. Yellow eyes or skin; dark urine; or pain or discomfort in right upper stomach area can cause liver damage.

ii.) Flavipiravir

Flavipiravir is a purine nucleic acid analog that has a similar mechanism of action to the antiherpesvirus drug acyclovir [2] and has the property of not producing a resistant virus (PharmacolTher.2020May;209:107512). The drug causing chain termination or viral mutagenesis. (PharmacolTher.2020May; 209:107512).

[2]doi:10.1016/j.mjafi.2020.08.004

Side effects: Hyperuricemia, diarrhoea, reduced neutrophil count, transa- minitis, rash, nausea, vomiting. Also has teratogenic potential and embryotoxicity.

iii.) Tocilizumab

Tocilizumab is a genetically engineered humanised monoclonal antibody that competitively inhibits the binding of interleukin-6 (IL-6) to its receptor (IL-6R) [3]. Inhibition of IL-6R prevents IL-6 signal transduction to inflammatory mediators leading to T cell population expansion and B cell differentiation (IntJ~Antimicrob~Agents. 2020 May; 55(5): 105954.).

Side effects: Upper respiratory tract infections, nasopharyngitis, headache, high blood pressure, elevated total cholesterol levels and in some cases elevated levels of alanine transaminase. Less common side effects include dizziness, mild rashes, gastritis and mouth ulcer.

iv.) Hydroxychloroquine

Chloroquine or hydroxycholoroqine in its unprotonated form diffuses through the cell membrane to the acidic vesicles in the cell called lysosomes. The drugs get trapped in these vesicles as a protonated form as it cannot diffuse out of the vesicles. The drug alters the acidic environment in the lysosome and, as a result, the cell cannot proceed with endocytosis, exosome release or phagolysosomal fusion [4]. The spike (S) protein of SARS-CoV-2 is cleaved in the phagosome by host cell proteases, which can be inhibited owing to the increased pH in the lysosome as a result of HCQ accumulation [5].

Side effects: Nausea, stomach cramps, diarrhoea, headache, itching, reduced apetite, vomiting, chronic use can result in retinopathy, altered eye pigmentation, acne, anemia, bleaching of hair, blisters in mouth and eyes, cardiomyopathy, convulsions, vision difficulties, diminished reflexes, muscle paralysis,weakness or atrophy,tinnitus, skin inflammation and scaling, skin rash, vertigo, weight loss, and occasionally urinary incontinence.

v.) Plasma exchange - Therapeutic Plasma exchange (TPE) is a procedure of removal of plasma from patient and replenishing it with plasma from healthy/cured

[3] https://doi.org/10.2146/ajhp070449
[4] doi:10.1016/j.ijantimicag.2020.106028
[5] https://doi.org/10.1016/j.ijantimicag.2020.105932

individual. Recently, convalescent plasma donated from survivors of COVID-19 infection, has been shown as a promising and safe treatment. The convalescent plasma is used to passively transfer antibodies to a diseased person from a previously infected individual. [6]

Side effects: Fever, chills, urticaria, muscle cramps, or paresthesias.

8.2.1 *GHPFN-TOPSIS approach*

For the ranking of treatment options in COVID-19, the MCDM tool TOPSIS introduced by Hwang and Yoon (1981), is applied. The GHPFN-TOPSIS procedure is described in the following:

Step 1. Determination of alternatives and their preferential ratings in terms of GHPFN 2nd type. Assuming a set of 'j' treatment options $T_1, T_2, \ldots, T_j$ and 'k' criteria $U_1, U_2, \ldots, U_k$. Let $D_1, D_2, \ldots, D_r$ be the number of decision experts. As there is no permanent specific medicine for COVID-19, the decision experts may be uncertain or imprecise about a particular treatment. Thus, the DMs assign HPFNs as his/her decision for the alternatives depending on different criteria.

Step 2. Calculation of score values using Eq. (1.73)

Step 3. Normalization of the score values, using the formula:

$$NZ_{ef} = \frac{n_{ef}}{\sqrt{\sum_{e=1}^{j} n_{ef}^2}} e = 1, 2, \ldots j; f = 1, 2, \ldots, k; \tag{8.1}$$

Step 4. Determination of the weighted normalized matrix:

$$WN_{ef} = W_e \times NZ_{ef} e = 1, 2, \ldots, j; f = 1, 2, \ldots, k; \tag{8.2}$$

Step 5. Calculation of the Positive Ideal Solution (PIS^{p+}) and the Negative Ideal Solution (NIS^{p-}).

[6] https://www.medrxiv.org

$$\begin{cases} PIS^{p+} = \{maxWN_{ef}(f = 1, 2, \ldots, k) \ for \ \ benefit \ \ criteria\} \\ \{minWV_{ef}(f = 1, 2, \ldots, k) \ for \ non \ \ benefit \ criteria\} \\ NIS^{p-} = \{minWN_{ef}(f = 1, 2, \ldots, k) \ for \ benefit \ \ criteria\} \\ \{maxWV_{ef}(f = 1, 2, \ldots, k) \ for \ non \ benefit \ \ criteria\} \end{cases} \tag{8.3}$$

Step 6. Computation of the distance measure between alternative T_p and the Positive Ideal Solution (PIS^{p+}).

$$D_e^+ = \sqrt{\sum_{f=1}^{k} (WN_{ef} - T_f^+)^2}, e = 1, 2, \ldots, j \tag{8.4}$$

Step 7. Computation of the distance measure between alternative T_p and the Negative Ideal Solution (NIS^{p-}).

$$D_e^- = \sqrt{\sum_{f=1}^{k} (WN_{ef} - T_f^-)^2}, e = 1, 2, \ldots, j \tag{8.5}$$

where D_e^+ and D_e^- denote the distance measure of the individual alternative from the positive ideal solution (PIS^{p+}) and negative ideal solution (NIS^{p-}), respectively.

Step 8. Calculation of the relative closeness (R_e) of each option.

$$R_e = \frac{D_e^-}{D_e^+ + D_e^-} \tag{8.6}$$

8.3 Numerical application

Score function of GHPFN (2nd type) is used for the calculation of criteria weights.

Table 8.1: Criteria of the COVID-19 treatment, their importance in linguistic terms and GHPFN (2nd type).

Linguistic scale for evaluation	**GHPFN (2nd type)**	**Criterion rating**
Highly important	⟨(3.3, 3.5, 3.7, 3.9, 4); 0.7, 0.8⟩	Plasma stability (C_1), Plasma turnover (C_2), Time of suppression (C_3), Drug-drug interaction (C_4), Compliance (C_5), Pneumonia (C_6), Intensive care (C_7), Organ failure (C_8), Macrophage activation syndrome (C_9), Hemophagocytic syndrome (C_{10})
Important	⟨(2.7, 2.9, 3, 3.1, 3.3); 0.8, 0.7⟩	Pregnancy (C_{11}), GFR (C_{12}), Age (C_{13}), Side Effect (C_{14}), Fever (C_{15})
Average	⟨(2.2, 2.3, 2.5, 2.7, 2.9); 0.5, 0.6⟩	Treatment duration (C_{16}), Ease of application (C_{17}), Regime cost (C_{18})

The treatment options for COVID-19 which are selected as alternatives are as follows:

1. Plasma exchange
2. Tocilizumab
3. Remdesivir
4. Favipravir
5. Hydroxychloroquine

Table 8.2: The linguistic terms represented as score functions of GHPFN (2nd type).

Treatment	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_8
Plasma Exchange	13.07	11	19.41	0	3.95	3.95	19.41	11	11
Tocilizumab	13.07	3.95	11	13.07	11	3.95	26.07	13.07	11
Remdesivir	13.07	13.07	26.07	11	19.41	19.41	26.07	3.95	11
Favipravir	13.07	19.41	11	13.07	13.07	13.07	3.95	3.95	11
Hydroxychloroquine	13.07	3.95	11	19.41	13.07	13.07	3.95	3.95	11

Table 8.3: The linguistic terms represented as score functions of GHPFN (2nd type).

Treatment	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}
Plasma Exchange	11	19.41	3.95	19.41	13.07	3.95	13.07	3.95	26.07
Tocilizumab	11	19.41	3.95	13.07	19.41	3.95	13.07	3.95	26.07
Remdesivir	11	13.07	3.95	13.07	11	19.41	19.41	13.07	26.07
Favipravir	11	19.41	3.95	11	11	13.07	13.07	13.07	19.41
Hydroxychloroquine	11	19.41	3.95	11	19.41	13.07	19.41	19.41	3.95

Table 8.4: Representation of normalized table.

Treatment	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_8
Plasma Exchange	0.447	0.416	0.515	0.000	0.135	0.144	0.462	0.598	0.447
Tocilizumab	0.447	0.149	0.292	0.451	0.376	0.144	0.620	0.710	0.447
Remdesivir	0.447	0.494	0.692	0.380	0.664	0.709	0.620	0.215	0.447
Favipravir	0.447	0.734	0.292	0.451	0.447	0.477	0.094	0.215	0.447
Hydroxychloroquine	0.447	0.149	0.292	0.670	0.447	0.477	0.094	0.215	0.447

Table 8.5: Representation of normalized table.

Treatment	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}
Plasma Exchange	0.447	0.474	0.447	0.626	0.383	0.144	0.367	0.144	0.529
Tocilizumab	0.447	0.474	0.447	0.422	0.568	0.144	0.367	0.144	0.529
Remdesivir	0.447	0.319	0.447	0.422	0.322	0.709	0.546	0.477	0.529
Favipravir	0.447	0.474	0.447	0.355	0.322	0.477	0.367	0.477	0.394
Hydroxychloroquine	0.447	0.474	0.447	0.355	0.568	0.477	0.546	0.709	0.080

Table 8.6: Representation of criteria weights

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_8
Weights of criteria	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496

Table 8.7: Representation of criteria weights

Criteria	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}
Weights of criteria	0.496	0.319	0.319	0.319	0.319	0.319	0.185	0.185	0.185

The GHPFN (Type 2) method combined with the TOPSIS model is used to rank the treatment alternatives. The calculation involves the following steps:

Step 1. A decision table is constructed by the team of experts for linguistic rating of the treatment alternatives with respect to the criteria. Table 8.8 represents the linguistic terms and its respective GHPFN (2nd type).

Table 8.8: Linguistic rating and its respective GHPFN (2nd type)

Linguistic Terms	GHPFN (2nd Type)
Very high	$\langle(7, 8, 9, 9, 10); 0.8, 0.9\rangle$
High	$\langle(5, 6, 7, 8, 9); 0.7, 0.9\rangle$
Average	$\langle(3, 6, 6, 7, 8); 0.6, 0.8\rangle$
Low	$\langle(3, 4, 5, 6, 7); 0.7, 0.8\rangle$
Very low	$\langle(1, 2, 2, 3, 5); 0.7, 0.9\rangle$

Step 2. The linguistic terms are converted to GHPFN (2nd type) using score function developed in Eq. (1.73), and shown in decision Tables 8.2 and 8.3.

Step 3. The normalized table is constructed using Eq. (8.1), and represented in Tables 8.4 and 8.5.

Step 4. The weighted normalized matrix is calculated using Eq. (8.2). For calculation of weighted normalized matrix, the weights of criteria obtained are represented in Tables 8.6 and 8.7.

Step 5. PIS^{P+} and NIS^{P-} are determined using Eq. (8.3).

Step 6. Distances of D_e^+ and D_e^- from (PIS^{P+}) and (NIS^{P-}) are calculated. The final ranking of the treatments are done by finding the relative closeness (R_e). Table 8.9 represents the distance measure, relative closeness and final ranking of the alternatives. Fig. 8.1 illustrates the distance measure, relative closeness and ranking obtained by GHPFN-TOPSIS methodology.

Table 8.9: Calculation of distance measure, relative closeness and final ranking.

Treatment	$D+$	$D-$	$R.C$	Rank
Plasma Exchange	0.44	0.52	0.54	2
Tocilizumab	0.48	0.52	0.52	4
Remdesivir	0.54	0.43	0.44	5
Favipravir	0.44	0.49	0.53	3
Hydroxychloroquine	0.41	0.57	0.58	1

Fig. 8.1 illustrates the distance measure, relative closeness and ranking obtained by GHPFN-TOPSIS methodology.

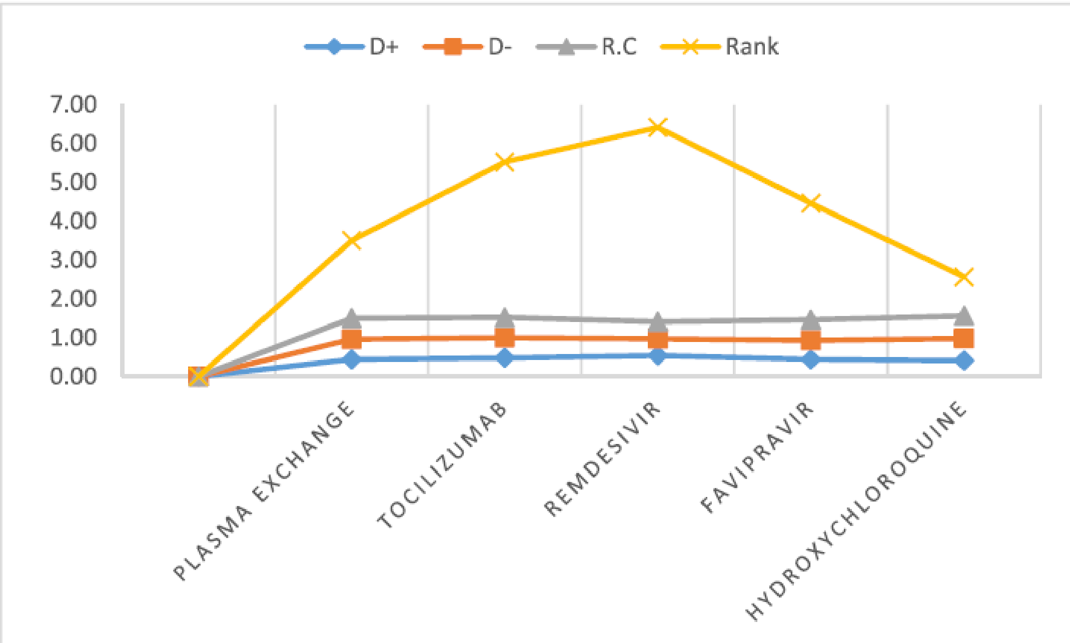

Figure 8.1: Distance measure, relative closeness and ranking of the treatment alternatives.

8.4 Comparative analysis

In this section, we apply different forms of generalized hesitant fuzzy number and notice the changes in the ranking.

Two different forms *GHTFN* and *GHTrFN* are taken and the ranking so obtained are illustrated in table 8.10 and Figure 7.2.

1. *GHTFN*- In this form, three numbers are taken in the order-low, medium, high i.e. the TFN to represent a linguistic term remains one but the degree of hesitancy/confidence are multiple depending on the DMs. The *GHTFN* score function is developed in the same way as *GHTrFN* (Deli and Karaaslan, 2021) and *GHPFN* are introduced in this chapter. *GHTFN*-TOPSIS approach is applied and ranks are obtained. The method ranked 'hydroxychloroquine' as the best treatment option, 'Plasma Exchange' as the second one, 'Favipravir' the third, 'Tocilizumab' the fourth and 'Remdesivir' the bottom, whereas GHTFN ranked 'tocilizumab' the first followed by 'plasma exchange, 'hydroxychloroquine', 'remdesivir' and 'favipravir'.

2. GHTrFN- In Table 8.10, for obtaining R3, the concept of GHTrFN (Deli and Karaaslan, 2021) is used with TOPSIS methodology to rank the alternatives. The score function defined for GHTrFN (Deli and Karaaslan, 2021) is used and the ranking so obtained under this method is depicted. This methodology ranked 'Plasma exchange' the first, followed by 'Tocilizumab', 'Remdesivir', 'Hydroxychloroquine' and 'favipravir'.

Table 8.10: Ranking obtained under different methods.

Treatment	GHPFN	GHTFN	GHTrFN
Plasma exchange	2	2	1
Tocilizumab	4	1	4
Remdesivir	5	4	5
Favipravir	3	3	3
Hydroxychloroquine	1	1	2

Fig. 8.2 depicts the different ranking obtained using different form of generalized hesitant fuzzy numbers.

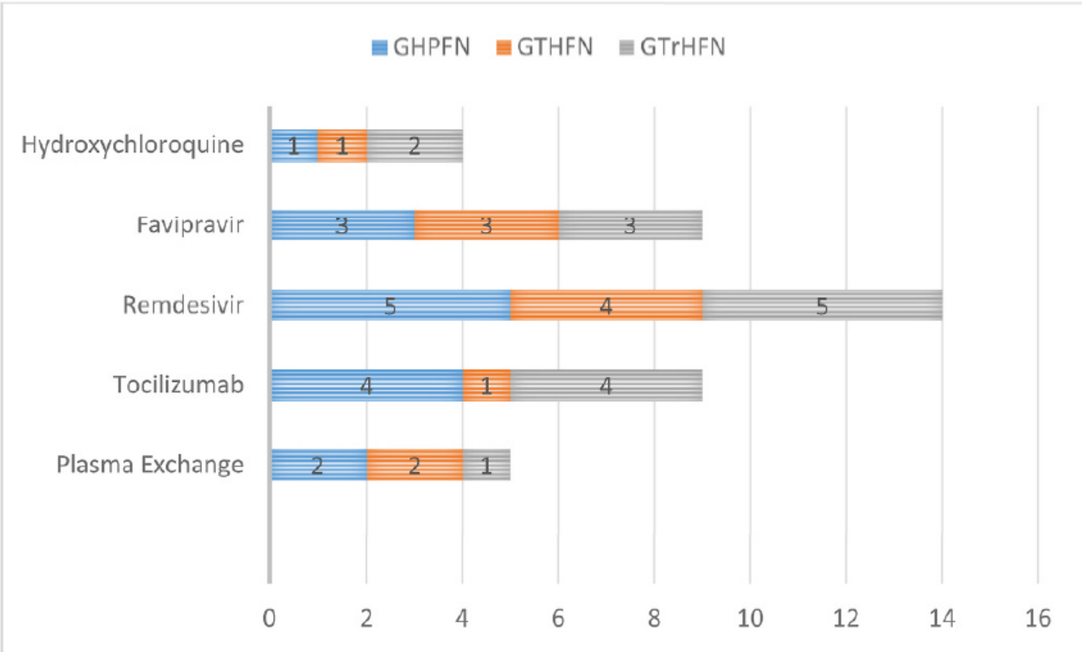

Figure 8.2: Representation of ranking obtained under GHPFN, GHTFN and GHTrFN.

8.5 Sensitivity analysis

The sensitivity analysis is conducted by interchanging of criteria weights. We consider three different cases and obtain ranking of treatment options. The ranking obtained under sensitivity analysis is illustrated in Table 8.11 and Fig. 8.3.

Table 8.11: Ranking obtained under sensitivity analysis.

Treatment	Rank (I)	Rank(II)	Rank (III)
Plasma Exchange	3	2	2
Tocilizumab	4	3	3
Remdesivir	5	4	4
Favipravir	1	1	1
Hydroxychloroquine	2	1	2

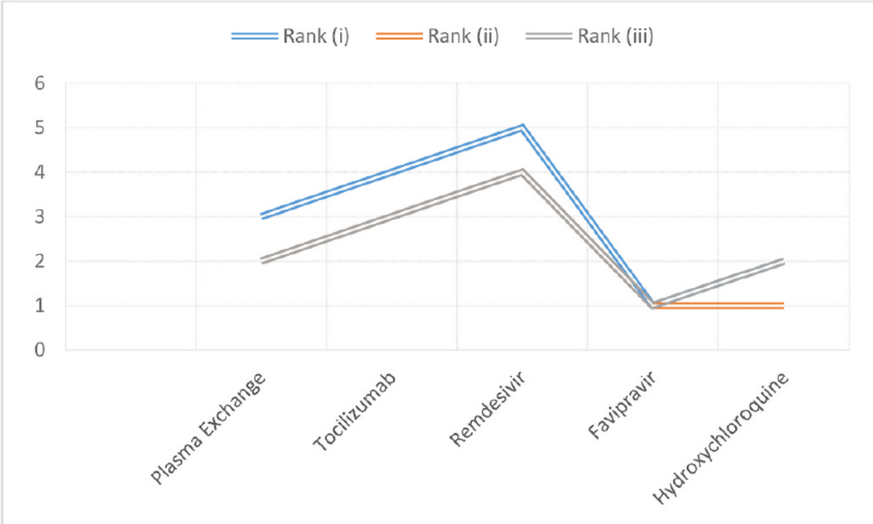

Figure 8.3: Representation of rankings under sensitivity analysis.

8.6 Results and discussions

In the present study, generalized pentagonal fuzzy number theory is applied for identification, application and ranking of five different treatment strategies of COVID-19 patients. It is found that, HCQs is ranked first, followed by Plasma Exchange, Favipravir, Tocilizumab and Remdisivir. The sensitivity analysis depicted in Table 8.3 and Fig. 8.1 clearly indicates that Favipravir is ranked 1 and the age old drug HCQS is ranked 1 or 2 consistently irrespective of different criteria weights. The Convalescent Plasma Exchange therapy is ranked 2 or 3 in treatment modality scale. The inflammatory response, in COVID patients, is indicated by the elevated immune-inflammatory biomarkers such as Procalcitonin, Ferritin, C-Reactive Protein (CRP), D-Dimer, Lactate Dehydrogenase and Interleukin-6 (IL-6), etc. These markers are used as severity indicators and even as prognostic factors. The use of Therapeutic Plasma Exchange has the theoretical ability to eliminate some of the pro-inflammatory substances and toxic substances of the sick individuals (Wan et al., 2020). The important findings from several Chinese-Korean studies (Yuki et al., 2020; Shen et al., 2020; Ye et al., 2020; Zhang et al., 2020) European (Barelli and Alberio, 2018) and American research (Ragab et al., 2020) works are in tandem with the results so obtained in this present work. There is convincing evidence in favor of Convalescent plasma exchange therapy, in its ability to reduce mortality in critically ill patients, by increase in neutralizing antibody titre and improvement of clinical symptoms and laboratory reports without jeopardizing the safety profile. However the need of further conclusive studies cannot be denied.

8.7 Conclusions

In this study, the $GHPFN$- TOPSIS methodology has been utilized to rank the treatment alternatives available for COVID-19. The results obtained have identified 'Hydroxychloroquine' in the first rank followed by, 'Plasma Exchange','Favipravir', 'Tocilizumab' and 'Remdesivir'. In this study, the HFS has been extended to $GHFS$ which is helpful in solving MCDM issues. Thus the integrated approach of $GHFS$ with MCDM technique provides efficiency to attain highest accuracy in the result. Comparative analysis has been conducted to check the validity of the proposed methodology. Sensitivity analysis has been discussed as how the ranking changes with the change in weight of the criteria. In future, the treatment options and criteria can be included depending on the availability.

9

DEMATEL and AHP Methods under Pythagorean Fuzzy Environment for Identification of Dominant Factor of Cardiac Arrest

9.1 Introduction

MCDM methodology has been applied in multiple areas such as transportation and logistics, water management, landfill management, medical diagnosis problem, energy management, etc (Toloie and Homayonfar, 2011). AHP and ANP (Satty, 1985) are MCDM methods which determine the priority weights. In AHP, the criteria are considered independent and arranged in a systematic hierarchy structure. However, in ANP, criteria dependency is taken into account and it is arranged in a network structure (Göncü & Çetin, 2022). Analytic Hierarchy Process (AHP) is one of the established MCDM tools used for evaluation of the criteria weights. AHP is based on construction of comparison matrices of various criteria. Decision Making Trial and Evaluation Laboratory (DEMATEL) methodology introduced by (Gabus and Fontela, 1973), is one of the most efficient and widely used tools in decision making to bring out the relations between the criteria in hesitant environment (Gabus and Fontela, 1973). This method is based on framework of structured model by clustering of knowledge by various decision makers and also bring out the causal relationships of factors through the causal diagram. DEMATEL method integrated with fuzzy logic is applied in various segments such as supply chain selection and management (Chang et al., 2011) , identification of hospital service quality (Shieh et al., 2010), safety management (Yazdi et al., 2020), and emergency management (Zhou et al., 2011). DEMATEL MCDM method (Gabus and Fontela, 1973) conceptualizes complex causal relationships and decomposes the criteria into cause and effect groups. Thus, the system is easily understood and analyzed by the decision experts.

From the literature survey, we observe that integrated work of AHP and DEMATEL with hexagonal pythagorean fuzzy number ($HPyFN$) has not been executed to solve medical problem. Thus, in order to fill this research gap, in this chapter, we propose MCDM tools AHP and DEMATEL under $HPyFN$ environment to solve a medical diafnosis problem.

The objectives of this research are as follows:

- To study in details the risk factors responsible for cardiac arrest.
- To apply two MCDM techniques AHP and DEMATEL to obtain the factor weights and understand the cause-effect relationship of the factors.
- To make a comparison between the criteria in linguistic ratings determined by the decision experts.

The remainder of the chapter is organized in the following way: Section 9.2 includes the classical MCDM technique AHP and DEMATEL. Extended AHP and DEMATEL methods with HPyFN are deliberated in this section. Description of criteria and numerical application are covered in section 9.3. Section 9.4 includes the results and discussion. Finally, conclusion and future research scopes are provided in section 9.5.

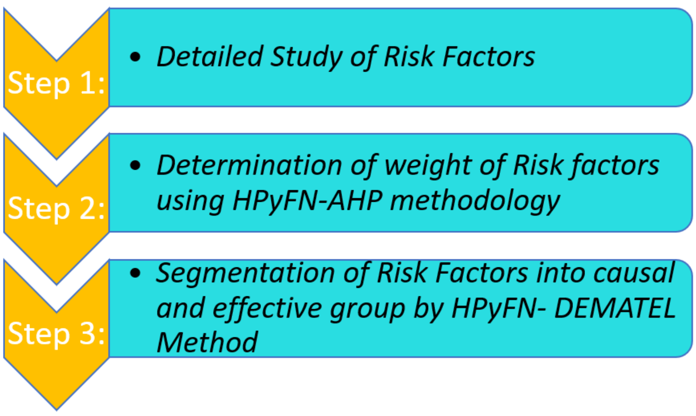

Figure 9.1: Flow chart depicting the steps involved in this study

9.2 HPyFN-based MCDM techniques

Pythagorean fuzzy AHP method

In this section, we introduce AHP method under Pythagorean fuzzy environment. To integrate this model, we use $PyFN$.

Step 1: Recognition and evaluation of the criteria taken for this research.

Step 2: Based on the group of decision experts' opinion, construction of pairwise comparison matrix in linguistic terms. The aggregated opinion of DMs in linguistic terms is then converted to hexagonal Pythagorean fuzzy sets. We assume that M number of DMs give their decisions and each decision maker expresses his/her opinion in the pairwise comparison matrix of criteria. Thus M set of matrices are obtained $T_e = \{t_{ije}\}$ where $e = 1, 2, \ldots, M$ and $i, j = 1, 2, \ldots, n$. Let us consider n criteria $f = \{f_1, f_2, \ldots\ldots, f_n\}$. Then the Pythagorean fuzzy comparison matrix can be obtained in the following form:

$$X = (x_{ij})_{n\times n} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_n \end{array} \begin{array}{c} \begin{array}{cccc} f_1 & f_2 & \cdots & f_n \end{array} \\ \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n1} & \beta_{n2} & \cdots & \beta_{nn} \end{pmatrix} \end{array} \tag{9.1}$$

where $\alpha_{ij} =< \mu_{ij}, \nu_{ij} >$ is a PFN presenting the rating value assigned by the decision experts for comparison of criteria c_i with respect to criteria C_j for $i = 1, 2, \ldots\ldots, n$ and $j = 1, 2, \ldots, n$. We consider the weight information of the criteria as $w = \{w_1, w_2, \ldots\ldots, w_n\}$ which is the normalized weight vector and it satisfies $0 \leq w_j \leq 1$ for $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$.

Step 3: De-fuzzification of $HPyFN$:

Definition 28. *(Grzegrorzewski, 2003) Let $D = \langle (d_1, d_2, d_3, d_4, d_5, d_6); \mu, \nu \rangle$ be a HPyFN where d_1, d_2, d_3, d_4, d_5 and d_6 are real numbers. Then the expected value of D is given by*

$$E(D) = \frac{(d_1 + d_2 + d_3 + d_4 + d_5 + d_6)}{6} \sqrt{\mu^2 + \nu^2} \tag{9.2}$$

It is easy to understand that if HPyFN is $\alpha = \langle(1,1,1,1,1,1);1,0\rangle$ then $E(D) = 1$, and if HPyFN is $\beta = \langle(0,0,0,0,0,0);0,1\rangle$ then $E(D) = 0$. The pairwise comparison matrix after de-fuzzification is of the form:

$$C_{n\times n} = [c_{ij}]_{n\times n} \tag{9.3}$$

where $i, j = 1, 2, \ldots, n$. The remaining steps of AHP i.e., standardization, evaluation of criteria weights, estimation of consistency index and verification of CR can be calculated using Eqs. (1.2), (1.3), (1.4) and (1.5), simultaneously.

HPyFN-DEMATEL method

Decision makers usually make judgement according to their experience and expertise. Exact evaluation of criteria for DEMATEL method or any other decision-making method is quite difficult in uncertain environment. Pythagorean fuzzy set effectively deals with uncertain environment in decision making process. In the proposed Pythagorean fuzzy DEMATEL method, assessment of evaluation criteria is done in terms of $HPyFN$. The proposed method is discussed below:

Step 1: Extracting the Pythagorean fuzzy direct relation

We consider Pythagorean fuzzy linguistic scale which is assigned to the corresponding HPyFN with the view point of the expert to deal with ambiguities of human assessment. We construct Pythagorean fuzzy direct relation matrix D for the criteria $C_1, C_2, ..., C_n$ as

$$D = [d_{ij}]_{n\times n} \tag{9.4}$$

where d_{ij} are TrPFNs. Govindan et al. (2015) calculated the expected value of trapezoidal intuitionistic fuzzy number(TrIFN). Here we determine the expected value of each d_{ij} and obtain the expected Pythagorean fuzzy direct relation matrix $\tilde{D}$ using equation (Grzegrorzewski, 2003):

$$\tilde{D} = [\tilde{d_{ij}}]_{n\times n} \tag{9.5}$$

where $\tilde{d_{ij}}$ is the expected value of TrPFN d_{ij} .

Step 2: Normalize the expected Pythagorean fuzzy direct relation matrix

Here various criteria dimensions are transformed into non-dimensional criteria. This allows comparison across criteria because various criteria are usually measured in different units. Hence the normalized expected Pythagorean fuzzy direct relation matrix with respect to Pythagorean fuzzy direct relation matrix is obtained as follows:

$$\begin{aligned} X &= k \times \tilde{D} \\ &= k \times [\tilde{d_{ij}}]_{n\times n} \end{aligned} \tag{9.6}$$

where

$$k = \frac{1}{\max\limits_{1\leq i\leq n} \sum_{j=1}^{n} \tilde{d_{ij}}}$$

Then X can be written as

$$X = [n_{ij}]_{n\times n} \tag{9.7}$$

Step 3: Construction of Pythagorean fuzzy total relation matrix

The Pythagorean fuzzy total relation matrix is calculated as

$$T = X(I - X)^{-1} \tag{9.8}$$

where T is a $(n \times n)$ Pythagorean fuzzy total relation matrix and I is a $(n \times n)$ identity matrix. Therefore

$$T = [t_{ij}]_{n\times n}. \tag{9.9}$$

Step 4: Generating causal diagram

Calculate the sum of rows and the sum of columns of the Pythagorean fuzzy total relation matrix $T = [t_{ij}]_{n\times n}$, denoted by D and R, respectively:

$$R = \Big[\sum_{j=1}^{n} t_{ij}\Big]_{1\times n} \tag{9.10}$$

$$D = \Big[\sum_{i=1}^{n} t_{ij}\Big]_{n\times 1} \tag{9.11}$$

Here $D + R$ denotes the impact strength index and $D - R$ represents the importance factor index. The important relation map can be drawn in cause and effect groups by putting the value in the form of $(D - R, D + R)$. The vertical axis, $D - R$ represents 'relation'. If the value of $D - R$ is positive then the criterion is grouped into the cause group and if the value of $D - R$ is negative then the criterion is grouped into the effect group.

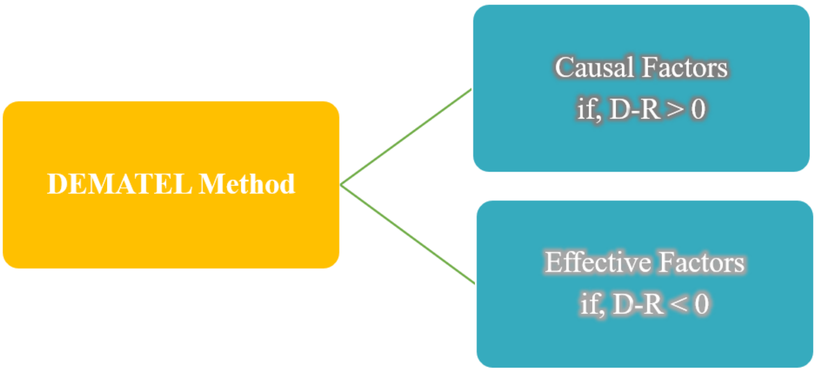

Figure 9.2: Categorization of factors in DEMATEL method

9.3 Numerical illustration: Cardiac arrest problem

Cardiac arrest problem has become quite common nowadays in young generation due to the number of risk factors which includes age, sedentary lifestyle, gender, genetics, diabetes, cholesterol, hypertension, obesity, pre-arrest history, stress. The exact reason for cardiac arrest is not completely known as it depends on multiple conflicting factors mentioned above. In this research, using Pythagorean fuzzy logic along with MCDM tool AHP, we calculate the factor weights and obtain ranking of factors. As all the factors seem to be equally important, we assigned 10 decision experts who gave their opinions in linguistic ratings. To capture the uncertainty and hesitancy of the decision experts, linguistic terms were then transformed to hexagonal Pythagorean fuzzy numbers ($HPyFN$). $HPyFN$ is an effective tool in handling the complexities of the problem. $HPyFN$ integrated with AHP and DEMATEL is proposed to solve the cardiac arrest problem.

9.3.1 *Risk factors associated with cardiac arrest*

The risk factors which are responsible for this catastrophic event are incompatible. We briefly explain the risk factors associated with cardiac arrest. To know the weights of these factors, MCDM technique AHP integrated with HPyFN is applied. Let the number of risk factors be m. The risk factors responsible for cardiac death are explained below:

- Age (f_1)- Growing age increases the risk of cardiac arrest.
- Gender (f_2)- Men are more expected to have risk of cardiac arrest compared to women as heart disease develop earlier in men. The rate of survival in hospital is lower in men compared to women.
- Genetics (f_3)- Persons having family history of blood pressure, blood glucose, cholesterol are at higher risk of heart problems. Inherited genetics risk factors such as heart failure, coronary artery disease influences the possibility of cardiac arrest. Thus, it can be said that genetics is an important factor in cardiac arrest.
- Diabetes (f_4)- Persons with diabetes over years are at risk of cardiac arrest as high blood sugar corresponds to damaging of blood vessels and nerves which control the heart.
- Hypertension (f_5)- High blood pressure results in damaging of the heart vessels as high BP indicates that the heart has to work more harder to supply blood to the different organs. So the unhealthy heart which has to do the extra work is more prone to cardiac arrest.
- Cholesterol (f_6)- High cholesterol means building up of cholesterol in the wall of the arteries. This makes the arteries narrow and thus reduces the rate of blood flow to the heart, brain, kidneys and other parts of the body. Thus, the diseased heart may cause sudden cardiac arrest.
- Sedentary lifestyle (f_7)- Bad lifestyle habits such as smoking and drinking acts as a toxin for the body and weakens the heart muscles and immune system. To lower the risk of cardiac arrest, one must quit smoking and reduce their alcohol intake. Alcohol and hypertension share direct relation. Greater alcohol intake increases blood pressure which ultimately impacts heart health.

- Stress (f_8)- High stress denotes high level of cortisol which means high blood sugar, blood pressure and high cholesterol. These medical conditions lead to greater probability of cardiac arrest. Also, the long term stress leads to depression and anxiety. Anxiety increases the heart rate. The increased heart rate interferes with the normal heart condition thus leading to greater chances of cardiac arrest.
- Obesity (f_9)- Obesity develops the risk of cardiovascular disease as obesity directly influence hypertension, diabetes and cholesterol.
- Pre-arrest history (f_{1_0})- Person with previous heart failure or heart disease is more prone to cardiac arrest.

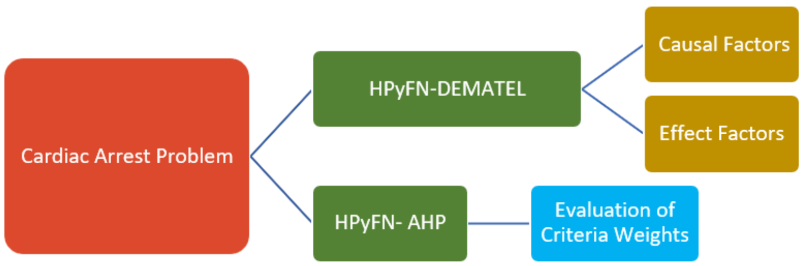

Figure 9.3: Potentiality of the MCDM techniques used

Table 9.1 illustrates the $HPyFN$ linguistic terms taken in this study for the calculation of weights of factors.

Table 9.1: Linguistic variables rating in terms of $HPyFN$

Linguistic Terms	**Hexagonal Pythagorean fuzzy number ($HPyFNs$)**
Absolutely Important (AI)	$<(1,1,1,1,1,1;1,0)>$
Very Strongly Important (VSI)	$<(0.7,0.8,0.9,1,1,1;0.8,0.3)>$
Strongly Important(SI)	$<(0.6,0.7,0.8,0.9,1,1;0.7,0.5)>$
Moderately Important (MI)	$<(0.4,0.5,0.6,0.7,0.8,0.9;0.6,0.5)>$
Less Important (AI)	$<(0.2,0.3,0.4,0.5,0.6,0.7;0.4,0.7)>$
Very Less Important (MUI)	$<(0,0.1,0.2,0.3,0.4,0.5;0.3,0.8)>$

In the first step of AHP, comparison matrix of the criteria is constructed in $HPyFN$. The following comparison matrix is expressed in linguistic term assigned by the DMs.

Table 9.2: Comparison matrix in linguistic terms

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
f_1	-	MI	LI	SI	MI	SI	MI	MI	SI	LI
f_2	MI	-	SI	LI	MI	MI	VSI	MI	SI	SI
f_3	LI	MI	-	SI	SI	MI	SI	MI	SI	SI
f_4	VSI	LI	VSI	-	VSI	VSI	SI	MI	MI	SI
f_5	SI	MI	VSI	SI	-	MI	SI	SI	SI	MI
f_6	VSI	SI	SI	SI	SI	-	SI	VSI	LI	SI
f_7	SI	MI	LI	VSI	MI	VSI	-	MI	VSI	MI
f_8	MI	LI	MI	SI	VSI	LI	SI	-	SI	MI
f_9	LI	LI	LI	SI	VSI	SI	SI	MI	-	LI
f_{10}	LI	MI	SI	MI	LI	SI	SI	MI	MI	-

Step 1: The linguistic ratings are now transformed to $HPyFN$. The $HPyFN$ value is then de-fuzzified using Eq. (9.2).
The de-fuzzified $HPyFN$ value is represented in Table 9.3.

Table 9.3: Criteria comparison matrix in de-fuzzified value

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
f_1	1	0.3965	0.2925	0.62	0.3965	0.62	0.3965	0.3965	0.62	0.2925
f_2	0.3965	1	0.62	0.2925	0.3965	0.3965	0.65	0.3965	0.62	0.62
f_3	0.2925	0.3965	1	0.62	0.62	0.3965	0.62	0.3965	0.62	0.62
f_4	0.657	0.2925	0.657	1	0.657	0.657	0.62	0.3965	0.3965	0.62
f_5	0.62	0.3965	0.657	0.62	1	0.3965	0.62	0.62	0.62	0.3965
f_6	0.657	0.62	0.62	0.62	0.62	1	0.62	0.657	0.2925	0.62
f_7	0.62	0.3965	0.2925	0.657	0.3965	0.657	1	0.3965	0.657	0.3965
f_8	0.3965	0.2925	0.3965	0.62	0.657	0.2925	0.62	1	0.62	0.3965
f_9	0.2925	0.2925	0.2925	0.62	0.657	0.62	0.62	0.3965	1	0.2925
f_{10}	0.2925	0.3965	0.62	0.3965	0.2925	0.62	0.62	0.3965	0.3965	1

Step 2: Normalization of the de-fuzzified matrix.
The de-fuzzified matrix is standardised using Eq. (1.2).

Table 9.4: Determination of standardized matrix

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
f_1	0.165	0.095	0.058	0.103	0.077	0.109	0.070	0.084	0.108	0.059
f_2	0.084	0.187	0.114	0.052	0.077	0.077	0.105	0.084	0.108	0.117
f_3	0.060	0.095	0.159	0.103	0.108	0.077	0.098	0.084	0.108	0.117
f_4	0.127	0.068	0.122	0.144	0.116	0.117	0.098	0.084	0.077	0.117
f_5	0.118	0.095	0.122	0.103	0,151	0.077	0.098	0.119	0.108	0.083
f_6	0.127	0.134	0.114	0.103	0.108	0.152	0.098	0.127	0.055	0.117
f_7	0.118	0.095	0.058	0.111	0.077	0.117	0.137	0.084	0.099	0.083
f_8	0.084	0.068	0.081	0.103	0.116	0.055	0.098	0.166	0.108	0.083
f_9	0.060	0.068	0.058	0.103	0.116	0.109	0.098	0.084	0.151	0.059
f_{10}	0.060	0.095	0.114	0.073	0.055	0.109	0.098	0.084	0.077	0.163

Step 3: Determination of criteria weights.
In this step, priority weights of the criteria are obtained.

Table 9.5: Determination of criteria weights using FAHP

Criteria	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
Criteria weight	0.092	0.099	0.102	0.108	0.110	0.114	0.099	0.096	0.089	0.092

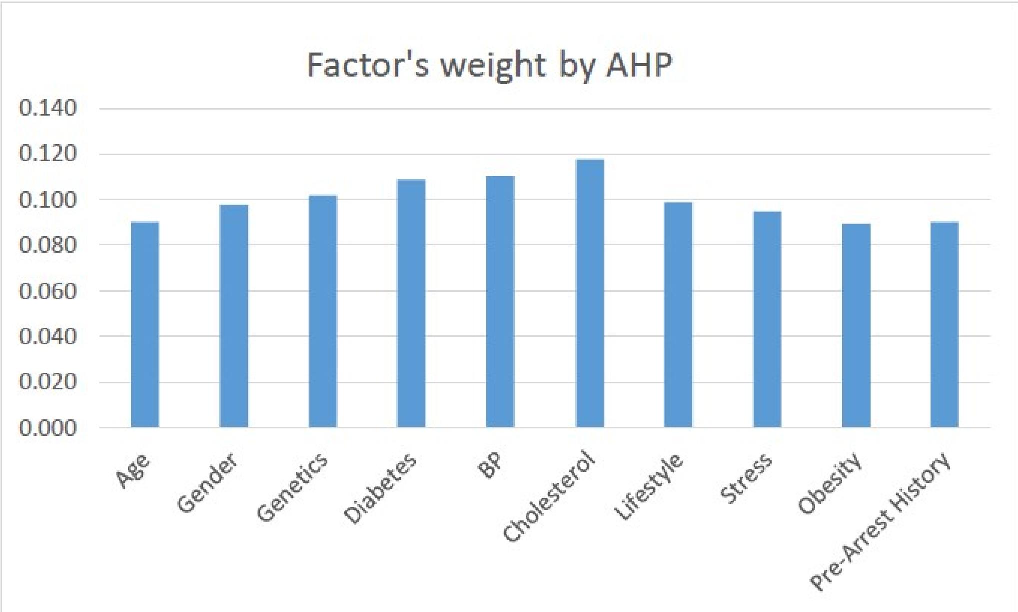

Figure 9.4: Weight of the factors obtained under $HPyFN$-AHP

Step 4: In this step, to check the consistency of the criteria weights obtained, the consistency ratio (CR) is estimated. The value of $(CR) < 0.1$ is considered to be consistent.
The criterion 'cholesterol(f_6)' scores the maximum weight of '0.114' followed by 'hypertension(f_5)', 'diabetes (f_4)', 'genetics (f_3)' in the second, third and fourth position, respectively. The criteria, 'gender (f_2)' and 'lifestyle (f_6)' acquire equal weights of '0.099' followed by the factor 'stress (f_8) with a value of '0.096'. The factors 'pre-arrest history (f_{10})' and 'age (f_1)' score equal weights of '0.092' followed by 'obesity (f_9)' scoring the least score value of '0.089'.

9.3.2 *Application of $HPyFN$-DEMATEL method*

Step 1: In the first step of DEMATEL, direct relation matrix is constructed with the assignment of linguistic ratings given by decision experts. Linguistic terms are transformed into $HPyFN$ and de-fuzzified using Eq. (9.2).

Table 9.6: Initial direct relation matrix in de-fuzzified value

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
f_1	0	0.3965	0.2925	0.62	0.3965	0.62	0.3965	0.3965	0.62	0.2925
f_2	0.3965	0	0.62	0.2925	0.3965	0.3965	0.65	0.3965	0.62	0.62
f_3	0.2925	0.3965	0	0.62	0.62	0.3965	0.62	0.3965	0.62	0.62
f_4	0.657	0.2925	0.657	0	0.657	0.657	0.62	0.3965	0.3965	0.62
f_5	0.62	0.3965	0.657	0.62	0	0.3965	0.62	0.62	0.62	0.3965
f_6	0.657	0.62	0.62	0.62	0.62	0	0.62	0.657	0.2925	0.62
f_7	0.62	0.3965	0.2925	0.657	0.3965	0.657	0	0.3965	0.657	0.3965
f_8	0.3965	0.2925	0.3965	0.62	0.657	0.2925	0.62	0	0.62	0.3965
f_9	0.2925	0.2925	0.2925	0.62	0.657	0.62	0.62	0.3965	0	0.2925
f_{10}	0.2925	0.3965	0.62	0.3965	0.2925	0.62	0.62	0.3965	0.3965	0

Step 2: The de-fuzzified direct relation matrix is normalized using Eq. 9.5.

Table 9.7: Determination of standardized matrix

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
f_1	0	0.08	0.06	0.11	0.08	0.11	0.08	0.08	0.11	0.06
f_2	0.08	0	0.11	0.06	0.08	0.08	0.12	0.08	0.11	0.12
f_3	0.06	0.08	0	0.11	0.11	0.08	0.11	0.08	0.11	0.11
f_4	0.12	0.06	0.12	0	0.12	0.12	0.11	0.08	0.08	0.11
f_5	0.11	0.08	0.12	0.11	0	0.08	0.11	0.11	0.11	0.08
f_6	0.12	0.11	0.11	0.11	0.11	0	0.11	0.12	0.06	0.11
f_7	0.11	0.08	0.06	0.12	0.08	0.12	0	0.08	0.10	0.08
f_8	0.08	0.06	0.08	0.11	0.12	0.06	0.11	0	0.11	0.08
f_9	0.06	0.06	0.06	0.11	0.12	0.11	0.11	0.08	0	0.06
f_{10}	0.06	0.08	0.11	0.08	0.06	0.11	0.11	0.08	0.08	0

Step 3: The Pythagorean fuzzy total relation matrix is obtained using Eq. (9.7).

Step 4: In this last step, causal relation is generated i.e., row sum D and column sum R are calculated. Finally, $D + R$ and $D - R$ are determined which represent the impact strength index and important factor index, respectively. The value of $D - R$, if positive, indicates the criteria to be grouped into the cause group and, if negative, it implies effect group. The following table depicts the values of D, R, $D + R$, $D - R$ and the identity. Thus, the table indicates that the factors 'age (f_1)', 'diabetes (f_4)', 'lifestyle (f_7)', 'obesity (f_9)', 'pre-arrest history (f_{10})' are classified into effect group or negatively affected factors. However, factors such as 'gender (f_2)', 'genetics (f_3)', 'hypertension (f_5)', 'cholesterol (f_6)', 'stress (f_8) are categorized into causal group or positively affected factors. Positive value of $D-R$ implies that these factors are affecting other factors while negative value signifies that these factors are being influenced by other factors. Maximum positive value of $D-R$ is associated with the factor 'gender (f_2)' followed by the factor 'cholesterol(f_6)'. This implies that the cardiac arrest problem is highly influenced by gender and cholesterol being one of the important key criterion.

Table 9.8: Identification of cause-effect group

	D	R	$D+R$	$D-R$	*Identity*
f_1	0.451	0.491	0.941	-0.040	Effect
f_2	0.520	0.353	0.873	0.167	Cause
f_3	0.552	0.516	1.067	0.036	Cause
f_4	0.629	0.639	1.268	-0.010	Effect
f_5	0.622	0.577	1.199	0.045	Cause
f_6	0.687	0.562	1.249	0.126	Cause
f_7	0.513	0.710	1.223	-0.197	Effect
f_8	0.500	0.471	0.971	0.028	Cause
f_9	0.454	0.571	1.025	-0.116	Effect
f_{10}	0.449	0.487	0.937	-0.038	Effect

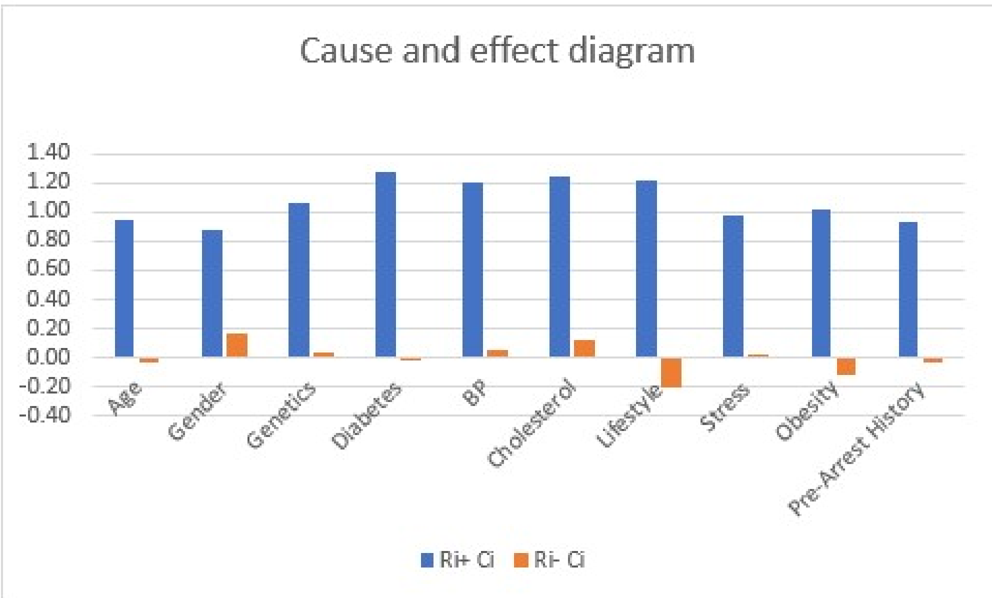

Figure 9.5: Causal and effective diagram of $R+C$,$R-C$

9.4 Results and discussion

In this research, factors responsible for cardiac arrest have been evaluated. The study is focused on root factors responsible for the cardiac arrest. Evaluation and obtaining weights of factors are of utmost priority to understand and analyze the problem. We have proposed the AHP and DEMATEL methodology under Hexagonal Pythagorean fuzzy environment and solved one specific medical diagnosis problem i.e. cardiac arrest problem. The weights of the factors are obtained through AHP and DEMATEL from which several deductions can be made, which are given below:

Firstly, MCDM tool AHP with $HPyFN$ helped in identifying the factor which is largely responsible for cardiac arrest among young generation. The calculation under $HPyFN$-AHP method depicted the factor 'cholesterol' scoring the maximum weight of '0.114'. Secondly, DEMATEL method is applied, which categorizes the factors into causal and effect groups. This division of factors into cause and effect help the researchers, DMs to understand which factors to focus on as primary and secondary. In order to suppress effect groups, one need to pay attention and control the causal groups. In this empirical study, factors 'gender (f_2)', 'genetics(f_3)', 'hypertension (f_5)', 'cholesterol(f_6)', 'stress (f_8)' are categorized into causal group whereas factors 'age (f_1)', 'diabetes (f_4)', 'lifestyle (f_7)', 'obesity (f_9)', 'pre-arrest history (f_{10})' are classified into effect group. This segmentation of factors into two categories under $HPyFN$- DEMATEL technique helps in achieving profound decision making. One can handle the issues easily and with less confusion.

9.5 Conclusions

In this research, we have considered cardiac arrest problem which has become quite vulnerable among youngsters due to number of factors. We have focused on the key factors responsible for progressive cardiac arrest. The study showed that the factor 'cholesterol' scored maximum weight under $HPyFN - AHP$ methodology. In $HPyFN - DEMATEL$ technique, it is noticed that the factors 'gender (f_2)', 'genetics (f_3)' , 'hypertension (f_5)', 'cholesterol(f_6)', 'stress (f_8) are classified into causal group factors. To understand and obtain the weights of the factors, MCDM techniques AHP and DEMATEL method have been applied. Initial step of AHP and DEMATEL comprises of construction of comparison matrices with the help of DMs. Opinions of DMs have been merged, and to capture the hesitancy and uncertainty of the DMs, $HPyFN$ has been utilized. Linguistic ratings assigned by DMs have been turned to $HPyFN$. Arithmetic operation and score value of $HPyFN$ have been defined. Two MCDM tools have been used because AHP determines the weights of the factors and DEMATEL method establishes the cause-effect group relation. $HPyFN$-AHP and $HPyFN$- DEMATEL have been proposed to solve and understand the root factor of cardiac arrest.

10 Conclusion

In this chapter, we discuss about research gap which reinforces the readers to understand the significance of the research, the contribution of the thesis, and future research directions.

i.) In ***chapter 2,*** a brief literature review has been discussed. A comprehensive overview of existing theories and knowledge has been presented in a structured framework.

ii.) In ***chapter 3,*** AHP-TOPSIS has been integrated with TFNs for selection of site for shopping mall in and around the city of Kolkata, West Bengal, India. Factors and sub-factors have been studied in a proper way. TFN-AHP has helped in obtaining weights of the factors and those of their respective sub-factors. Further, TFN-TOPSIS methodology has evaluated ranking of sites.

iv.) In ***chapter 4,*** HFS-AHP and HFS-TOPSIS approach have been used for the identification and ranking of risk factors associated with spreading of pandemic COVID-19 disease. Detailed research and data analytics will help in understanding the level of community spread. The strategy obtained in this study can be applied in the different sector problem. Anyone can also take the different uncertain parameters rather than fuzzy and hesitant fuzzy setting.

v.) In ***chapter 5,*** HFN defuzzified formulae has been introduced and used in the numerical application. Distance measure has been defined and included in TOPSIS method. GIS software has also been used in this chapter for graphical view of selected sites and distance measurement. AHP-TOPSIS and AHP-COPRAS have been utilized for selection of E-vehicle charging station. The other MCDM tools such as PROMETHEE, VIKOR, WASPAS can be used in future with intuitionistic, neutrosophic, hesitant fuzzy numbers to yield improved, robust and practical solutions.

vi.) In ***chapter 6,*** distance formulae between two PIFN has been proposed and used in this chapter. Ranking of CSPs has been done using PIFN-TOPSIS and PIFN-COPRAS techniques. PIFN-AHP has been used to evaluate the weights of factors considered in this chapter. Comparative and sensitivity analysis have helped in understanding the steadiness of the methods used.

vii.) In ***chapter 7,*** A model for university site selection has been proposed considering societal and national needs. De-neutrosophic technique has been introduced and utilized in the model. $TrNN$ with MCDM tools AHP, TOPSIS and COPRAS has been employed for ranking of sites preferable for women university. This methodology can be extended and applied in numerous fields. Some of the future scopes/extensions can be the following:

- The methodology can be used for setting up a private university, fully research-oriented institute, etc.
- Different sub-criteria may be taken for each criterion.
- Different de-neutrosophication techniques associated with different efficient MCDM methods, like MIVES, WASPAS, CoCOSo, PROMETHEE, VIKOR may be applied, different uncertain environments may be considered like hesitant neutrosophic environment, Pythagorean fuzzy, etc.
- Same methodology may be extended with more alternatives.
- Different new distance measures may be introduced.

viii.) In ***chapter 8,*** $GHPFN$, its arithmetic properties and score functions are introduced. Criteria and treatment options have been identified and studied in a detailed way. The $GHPFN$ (2nd type) score function has been used to determine the weight of the criteria. The $GHPFN$- TOPSIS methodology has been utilized to rank the treatment alternatives which were available for COVID-19. Comparative and sensitivity analysis have been performed to observe the transformation of ranking and validity of the developed methodology.

viii.) In ***chapter 9,*** we have considered cardiac arrest problem. The primary objectives of this study was to spread awareness in common people by identifying and evaluating weights of risk factors associated with cardiac arrest, $HyPFN$ with AHP has been applied for obtaining weights of the risk factors. DEMATEL method in extension has helped us to analyze the behavioral patterns of risk factors as it segments the factors into causal and effective groups.

Future Direction for the Readers

- Future research can consider involving a larger number of decision-makers from an administrative perspective. The methodology proposed in this study can also be applied to various fields, such as new vendor selection or treatment selection for emerging diseases, where higher levels of ambiguity and uncertainty are common.
- In the future study, the researchers can use different types of fuzzy numbers depending on the problem and availability of the data. Diverse $MCDM$ tools such as the weighted Aggregated Sum Product Assessment ($WASPAS$), Election et ChoixTraduisant La Realite ($ELECTRE$), Preference Ranking Organization Method for Enrichment Evaluation ($PROMETHEE$), Vlse Kriterijumska Optimizacija IKompromisno Resenje ($VIKOR$) , Combined Compromise Solution ($CoCOSo$), etc. can be used in future.
- ANN, ANP and DEMATEL methods can be applied for the evaluation of weights of factors in real life optimization problems.
- In future, different types of medical problems apart from COVID-19 and cardiac arrest can be considered with MCDM techniques to explore medical fields and contribute to the society.
- Extensions of various types of fuzzy numbers, along with alternative defuzzification formulas, score functions, and expected value approaches, can be developed and applied across different domains.
- Future researchers may apply our model across various domains. In this study, we have utilized $TFNs$, $HFNs$, HFS, $TrNNs$, $PIFNs$, $GHPFNs$ and $HPyFNs$ to effectively capture the hesitancy and uncertainty inherent in the problem.

Depending on the specific nature and requirements of a given problem, researchers are encouraged to explore other types or extensions of fuzzy numbers and criteria as needed.

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1. **AHP-TOPSIS inspired shopping mall site selection problem with fuzzy data.**
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2. **Identification of dominant risk factor involved in spread of COVID-19 using hesitant fuzzy MCDM methodology.**
   N. Ghorui, A. Ghosh, S. P. Mondal, Md Y. Bajuri, A, Ahmadian, S. Salahshour, M. Ferrara
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4. **Selection of cloud service providers using MCDM methodology under intuitionistic fuzzy uncertainty.**
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5. **Optimal site selection for women university using neutrosophic multi- criteria decision-making approach.**

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6. **Evaluation of the treatment options for COVID-19 patients using generalised hesitant fuzzy multi criteria decision making techniques.**
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7. **Identification of dominant factor of cardiac arrest by DEMATEL and AHP under Pythagorean fuzzy environment.**
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Article

AHP-TOPSIS Inspired Shopping Mall Site Selection Problem with Fuzzy Data

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Abstract: In the consumerist world, there is an ever-increasing demand for consumption in urban life. Thus, the demand for shopping malls is growing. For a developer, site selection is an important issue as the optimal selection involves several complex factors and sub-factors for a successful investment venture. Thus, these tangible and intangible factors can be best solved by the Multi Criteria Decision Making (MCDM) models. In this study, optimal site selection has been done out of multiple alternative locations in and around the city of Kolkata, West Bengal, India. The Fuzzy Analytic Hierarchy Process (FAHP) and Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (FTOPSIS) has been applied for shopping mall site selection. The AHP is used to obtain the crispified weight of factors. Imprecise linguistic terms used by the decision-maker are converted to Triangular Fuzzy Numbers (TFNs). This research used integrated sub-factors fuzzy weights using FAHP to FTOPSIS for ranking of the alternatives. Hardly any research is done with the use of sub-factors. In this study, seven factors and seventeen sub-factors are considered, the authors collected data from different locations with the help of municipal authorities and architects. This work further provides useful guidelines for shopping mall selection in different states and countries.

Keywords: site selection; shopping mall site selection; linguistic terms for fuzzy variable; fuzzy AHP; fuzzy TOPSIS

1. Introduction

Shopping malls can be considered as one of the most important growth points for Business strategies. There exist multiple factors which are responsible for a decision-maker (DM) to select the optimal site for shopping mall construction. Detailed study has been conducted to identify all the factors and sub-factors related to site selection. The weights are assigned to each of the factors and sub-factors with the help of an expert decision-maker (DM). The needs for shopping malls are increasing throughout the country. As several factors influence the selection of the best site, it can be considered as an application of Multi Criteria Decision Making (MCDM). MCDM is considered as the most significant branch of Operation Research, as it incorporates complex decisions of people's lives. There exists multiple MCDM models. The researcher uses MCDM techniques depending on the problem of the

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Identification of dominant risk factor involved in spread of COVID-19 using hesitant fuzzy MCDM methodology

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A R T I C L E I N F O

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A B S T R A C T

The outburst of the pandemic Coronavirus disease since December 2019, has severely impacted the health and economy worldwide. The epidemic is spreading fast through various means, as the virus is very infectious. Medical science is exploring a vaccine, only symptomatic treatment is possible at the moment. To contain the virus, it is required to categorize the risk factors and rank those in terms of contagion. This study aims to evaluate risk factors involved in the spread of COVID-19 and to rank them. In this work, we applied the methodology namely, Fuzzy Analytic Hierarchy Process (FAHP) to find out the weights and finally Hesitant Fuzzy Sets (HFS) with Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied to identify the major risk factor. The results showed that "long duration of contact with the infected person" the most significant risk factor, followed by "spread through hospitals and clinic" and "verbal spread". We showed the appliance of the Multi Criteria Decision Making (MCDM) tools in evaluation of the most significant risk factor. Moreover, we conducted sensitivity analysis.

Introduction

COVID-19 infectious diseases were earliest reported in the town Wuhan of China in the month of December 2019 [1]. Till date, the number of COVID patients has kept on increasing vigorously, thereby prompting WHO to say publicly COVID-19 to be a pandemic situation. As of August 7, 2020, there were approximately 18,902,735 cases in 216 countries and 709,511patients have lost their lives [1]. WHO has always been in a responsible mode to bring awareness and has updated the countries to take urgent necessary action since the origin of the pandemic. Several preventive measures have been taken by the people and government of different countries. As people are not immune to this disease, this prompted WHO to issue grave reminders regarding the severity of the disease and urged people to act responsibly seeing their neighbors who are suffering and fighting for their existence.

The COVID-19 infection has caused severe respiratory illness syndrome which led to the admission of critical patients largely to the ICU and high mortality among people with comorbidity has been observed. The data collected for the patients was from medical authorities who have made unsung sacrifices and proved to the world, their greatness when it comes for mankind [2]. The virus gets transmitted to other people when an infected person sneezes, speaks or coughs. Moreover, the infection can take place, if a person touches the facade which is contaminated with the virus and then touches his mouth, nose and eyes, findings obtained by the authors of [3–5]. The infection has been transmitted from human to human contact since the middle of December 2019. The mean incubation period was estimated to be 5.2 days after exposure according to the reference of [6]. The corona virus disease

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Article

Application of Hexagonal Fuzzy MCDM Methodology for Site Selection of Electric Vehicle Charging Station

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Abstract: In this paper, the application of hexagonal fuzzy multiple-criteria decision-making (MCDM) methodology for the site selection of electric vehicle charging stations is considered. In this regard, four factors and thirteen sub-factors have been taken into consideration for E-vehicle charging site selection. In this research, the geographic information system (GIS) has been incorporated with MCDM techniques. The fuzzy analytic hierarchy process (FAHP) is used to obtain a fuzzy weight of factors and sub-factors. MCDM tools fuzzy technique for order of preference by similarity to ideal solution (FTOPSIS) and fuzzy complex proportional assessment (FCOPRAS) have been used to rank the selected sites. A centroid-based method for defuzzification and distance measure between two hexagonal fuzzy numbers (HFN) has been developed for this paper. A practical example in Howrah, India, is considered to show the applicability and usefulness of the model. The results depict the suitability of the proposed research. Comparative and sensitivity analyses have been demonstrated to check the reliability, robustness and effectiveness of the proposed method.

Keywords: site selection; FAHP; FTOPSIS; FCOPRAS; hexagonal fuzzy number

1. Introduction

Electric vehicles play a very momentous role in addressing fossil fuel pollution and they are capable of making a paradigm shift in the entire transportation sector. Transportation is a significant contributor to urban air pollution and the reduction of urban city emission is the need of the hour. Electric vehicles make the world more liveable and provide a pollution-free mode of transportation in urban areas. The high level of pollution is degrading the environment and it has made the concept of sustainable development a fairy-tale phenomenon [1,2]. Sustainable consumption needs to be adopted by conducting timely environmental and sustainability assessments to prevent any large-scale ecological disaster from happening. This is where electrical vehicles come into the picture, with sustainability, environment-protecting and pocket-friendly being a few of their rewards. Electric vehicles use a minimum of one electric motor or traction motor for propulsion. They maybe self-contained with a generator or battery for converting the fuel into electricity or they may be power-drivenvia a collector scheme by using electricity from off-vehicle sources [3]. The problem of the energy crisis in the world can be tackled in the future by using this option. Ever-rising gas prices force people to look for alternative modes of

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FUZZY SYSTEMS AND THEIR MATHEMATICS

Selection of cloud service providers using MCDM methodology under intuitionistic fuzzy uncertainty

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Abstract

Cloud computing concept has taken prodigious growth over the last decade. With the vast options of Cloud Service Providers available nowadays and a variety of services and facilities to choose from, it is of paramount necessity to opt for the best cloud service provider based on multiple criteria and requirements ascertained by any organization or an individual. This study selects the cloud service provider based on various conflicting criteria. In this paper, pentagonal intuitionistic fuzzy number (PIFN) with MCDM tool analytic hierarchy process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methods have been used to rank the Cloud Service Providers (CSPs). Firstly, the criteria PIFN weights are calculated using comparison matrices with the help of decision-makers (DMs), and then, FTOPSIS is done to obtain the final ranking. Sensitivity and comparative analyses have been conducted to see the changes in ranking obtained. These analyses help analyze the most sensitive criteria and thus help the researchers mark and evaluate for future scope and further research.

Keywords Cloud Service Providers selection · Intuitionistic fuzzy number · Multi-criterion decision-making method · AHP-TOPSIS method

1 Introduction

We are living in the age of the digital era, where a massive amount of data is available online and offline. The requirement to store, analyze and access the necessary data securely and safely arises every day. We all need a tool that can help any individual or organization to store data online. This modern age requirement had thus necessitated the birth of "Cloud computing," which is basically storing the data on the web cloud, i.e., on shared data servers, and

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buildings

Article

Optimal Site Selection for Women University Using Neutrosophic Multi-Criteria Decision Making Approach

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Abstract: Site selection for an institute or a university is a challenging task. The selection of sites for setting up a new university depends on multiple criteria. In backward, under privileged area people's perception towards the co-educational universities and women universities are different. Poor families with their conservative mentality possess inhibitions while sending their girl child to co-educational universities as they have concerns about safety, security and family honor. Hence many attributes which are not so important for co-educational universities are more pertinent for women university. In this research paper, we have considered a model for selecting women's university sites in different backward locations in the state of West Bengal, India. This model incorporated different types of uncertainty related to site selection. Ten important criteria are chosen for the selection of sites. To capture the uncertainty of the problem, trapezoidal neutrosophic numbers are used along with the Multi-criteria Decision Making tool Analytic Hierarchy Process (AHP) for obtaining criteria weights. Finally, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and COmplex PRoportional ASsessment (COPRAS) are applied for ranking of the sites. Comparative and sensitivity analyses are conducted to check the steadiness of the techniques used.

Keywords: neutrosophic number; TRNNs; AHP; TOPSIS; COPRAS; university site selection

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1. Introduction

The site selection problem is quite significant in today's world. Location selection for setting up an industry, real estate, hospitality management, or other cases that require proper data, decision experts, future perspective, establishment cost, etc. Moreover, there exist several criteria which make the Decision Maker (DM) select the optimal alternative. Ranking the sites requires a mathematical understanding of the problem. In this context, multi-criteria decision-making (MCDM) can play an important role. Optimal selection or ranking of various disparate sites in decision-making is complicated as it depends on multiple conflicting criteria. Obtaining of criterion's weight is a major part of the DMs. The first step in decision-making is to integrate the opinion of decision experts in linguistic rating. The linguistic rating may not always be transformed to a fixed scale, as the decision experts may consider uncertainty, hesitancy, and vagueness. In this context, the researchers need to solve the MCDM techniques for the site selection problem in an uncertain, hesitant environment.

1.1. Motivation and Novelties of the Study

- MCDM tools in the neutrosophic environment have been applied in different areas, but the literature survey reflects that minimum work has been done on women's university site selection problem (any other type of university site selection also). So,

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Evaluation of the treatment options for COVID-19 patients using generalized hesitant fuzzy- multi criteria decision making techniques

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ABSTRACT

The breakout of the pandemic COVID-19 has affected numerous countries and territories worldwide. As COVID-19 specific medicines yet to be invented, at present the treatment is case specific, hence identification and evaluation of different prevalent treatment options based on various criteria and attributes are very important not only from the point of view of present pandemic but also for futuristic pandemic preparedness. The present study focuses on identifying, evaluation and ranking of treatment options using Multi Criteria Decision Making (MCDM). In this regard, the existing literature, doctors and scientist were interviewed to know the current treatment options in vogue and the scale of their importance with respect to the criteria. The criteria taken are side effect, regime cost, treatment duration, plasma stability, plasma turnover, time of suppression, ease of application, drug-drug interaction, compliance, fever, pneumonia, intensive care, organ failure, macrophage activation syndrome, hemophagocytic syndrome, pregnancy, kidney problem, age. This study extended Hesitant Fuzzy Set (HFS) to Generalized Hesitant Fuzzy Sets (GHFS). Generalized Hesitant Pentagonal Fuzzy Number (GHPFN) is developed. The properties of GHPFN are demonstrated. Two types of GHPFN has been described. The GHPFN (2nd type) along with MCDM tool Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has been applied to rank the treatment options. The result of the study ranked 'Hydroxychloroquine' as the first alternative followed by, 'Plasma Exchange', 'Tocilizumab', 'Remdesivir' and 'Favipravir'. To check the robustness and steadiness of the proposed methodology, comparative analysis and sensitivity analysis has been conducted.

1. Introduction

Human history has faced the onslaught of diseases since its inception. Most of these diseases have been caused by viruses which have been ever present in our environment. These viruses are ever mutating and every once in a while, when they come in contact with a large throng of individuals it affects the latter like wildfire. Even though medical science has progressed by heaps and bounds, it will never be able to keep pace with the ever changing nature of these viruses, mainly due to the various hosts it captures for procreating (see Table 1).

Throughout history Coronaviruses have been causing outbreaks in humans and animals [1] such as the avian influenza outbreak in 1997, the Severe Acute Respiratory Syndrome (SARS) in 2003 [2] and the severe fever with thrombocytopenia syndrome (SFTS) in China in 2010 [3].

December 2019 was another stark example where the zoonotic transmission caused a population of approximately 11.9 million people in Wuhan, China to come to a standstill [4]. The coronavirus responsible for this was designated as Coronavirus Disease 2019 (COVID-19) by the World Health Organization [5]. It spread in two stages-the Imported transmission stage wherein the individuals who had a foreign travelling history carried the virus from the foreign country to the host country.

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