ABSTRACT

Title: STUDY OF SOME CONNECTIONS ON ALMOST CONTACT, PARA CONTACT AND CONTACT MANIFOLDS

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The central objective of this thesis is to explore and analyze various types of manifolds admitting different solution structures with respect to semi-symmetric metric connections.

The thesis is organized into six Chapters.

Chapter one discusses the theoretical framework by presenting foundational concepts related to contact geometry, including contact, almost contact, para-contact. It also introduces various soliton structures such as Ricci soliton, η-Ricci soliton, η-Einstein soliton and conformal η-Einstein soliton, thereby setting the stage for the subsequent investigations carried out in this thesis.

The analysis conducted in Chapter two focuses on $(2n + 1)$-dimensional (κ, μ)-contact metric manifolds endowed with an η-Einstein, conformal η-Einstein soliton and conformal η-Ricci soliton. It is shown that for these solitons, the scalar curvature of (κ, μ)-contact metric manifolds is constant and the solitons represent shrinking, steady and expanding under same curvature conditions. It is established that a $(2n + 1)$-dimensional (κ, μ)-contact metric manifold satisfying an η-Einstein soliton is locally isometric to the Riemannian product $E^{n+1} \times R_t^n$ for $n > 1$ and flat for $n = 1$. Also we obtained that the Ricci tensor of a (κ, μ)-contact metric manifold with torse forming vector field takes the form of η-Einstein under η-Einstein soliton. Finally we give an example of a 3-dimensional (κ, μ)-contact metric manifold.

Chapter three addresses the concept of the characteristic vector fields in ϵ-Kenmotsu manifold. We have shown that, an ϵ-Kenmotsu manifold with conformal η-Einstein soliton becomes an η-Einstein manifold as well as Einstein manifold under some curvature conditions like Codazzi type, cyclic parallel, ξ-Ricci semi symmetric etc. Next we shall proof every contact vector field on a Kenmotsu manifold leaving the Ricci tensor with respect to semi-symmetric connection invariant is a strict contact vector field. Finally we have proved that an extended generalized ϕ-recurrent Kenmotsu manifold with respect to semi-symmetric metric connection is an Einstein manifold.

Chapter four investigates $*$-conformal η-Ricci soliton and $*$-conformal η-Einstein soliton on trans-Sasakian manifolds. It is demonstrated that a 3-dimensional trans-Sasakian manifold admitting $*$-conformal η-Ricci soliton is an η-Einstein manifold and the potential vector field is pointwise collinear with the characteristic vector field. Also, we obtained that a trans-Sasakian manifold with $*$-conformal η-Einstein soliton satisfies cyclic parallel Ricci tensor. Moreover, it is established that an Einstein semi-symmetric trans-Sasakian manifold admits a $*$-conformal η-Einstein soliton represents an Einstein manifold.

In Chapter five, we do a comprehensive study of Lorentzian para Sasakian manifolds with conformal η-Ricci soliton as well as conformal η-Einstein soliton. It is demonstrated that a Lorentzian para Sasakian manifold represent a dark-energy era. In addition we show that a 4-dimensional Ricci-recurrent Lorentzian para Sasakian manifold reduces to a Minkowski spacetime. Moreover, we obtained that an extended generalized ϕ-recurrent Lorentzian para Sasakian manifold with respect to semi-symmetric connection is an Einstein manifold. For contact vector field, we noticed that a Lorentzian para Sasakian manifold leaving the Ricci tensor with respect to semi-symmetric connection invariant is a strict-contact vector field. Lastly we give an example of a Lorentzian para Sasakian manifold.

Chapter six concludes with the examination of the study of -Ricci soliton on (κ, μ)-contact metric manifolds and the novel findings on Lorentzian para Sasakian manifolds. We present novel findings on Lorentzian para Sasakian manifolds and ϵ-Kenmotsu manifold under the influence of conformal transformations, and discuss several structure theorems that were derived, with validation examples for the theoretical models. We also provide an exploration guide into future research directions around additional classes of manifolds like cosymplectic, para-Sasakian and quasi-Sasakian geometrics, and potential applications in the context of general relativity that could lead to further theoretical developments and applications in differential geometry.

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