JADAVPUR UNIVERSITY
FACULTY OF SCIENCE

DOCTORAL THESIS

Developing Efficient Pricing Strategies for Deterministic and Stochastic Inventory and Supply Chain Models

Author:
Indrani Modak

Supervisor:
Prof. Bibhas C. Giri

Co-supervisor:
Dr Sudarshan Bardhan

Thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Science)

in the

Department of Mathematics
Jadavpur University
Kolkata-700032, West Bengal, India

June, 2025

JADAVPUR UNIVERSITY
DEPARTMENT OF MATHEMATICS
KOLKATA - 700032

CERTIFICATE FROM THE SUPERVISORS

This is to certify that the thesis entitled "**Developing efficient pricing strategies for deterministic and stochastic inventory and supply chain models**"submitted by Ms **Indrani Modak** who got her name registered on 10th February, 2022 (Index No : 14/22/Maths./27) for the award of Ph. D. (Science) degree of Jadavpur University, is absolutely based upon her own work under the supervision of **Prof. Bibhas C. Giri** and **Dr Sudarshan Bardhan**, and that neither this thesis nor any part of it has been submitted for either any degree/diploma or any other academic award anywhere before.

25/06/25

(Prof. Bibhas C. Giri)
Supervisor

Professor
Department of Mathematics
Jadavpur University
Kolkata-700032

Sudarshan Bardhan 25/6/25

(Dr Sudarshan Bardhan)
Co-supervisor

Assistant Professor
Department of Mathematics
Kanchrapara College
Kanchrapara, 24 Pgs.(N), 7431

Dedicated to

the wonderful family I am truly blessed to have...

Acknowledgements

I remember failing my third-grade Mathematics test. As a little girl, I didn't understand the significance of the red marks on my report card. Excitedly, I ran to my father and showed him the marksheet, pointing out the red marks with glee. He looked at it and simply smiled. Today, as I submit my doctoral thesis in Mathematics, I finally understand the true meaning of that smile. Despite countless failures, my father ensured I never truly felt what it means to fail. Despite all the odds, he always ensured I had everything I needed and also taught me the true meaning of humanity, success and feminism. I could write another thesis on his contributions to my life and my achievements, but it would still fall short of capturing his impact. Along with my father, the other one who is to get acknowledged, the opposite tempered one, my mother, whom I have offered each kind of tensions from every aspect. Yet she has always been there, steadfastly correcting my mistakes, from my ABCs to handling life's complexities. Besides my parents, my elder brother has been a constant source of support, whether in my studies or in other aspects of life and not to mention my sweet grandmother who have always portrayed each of my success as her own. My sister-in-law Bashita Biswas also deserve a special thanks for making the environment sweeter with her presence.

Overcoming my weaknesses in Mathematics and developing a love for the subject was only possible due to the continuous support from my family and teachers. On this special day, I want to express my heartfelt gratitude to all of them, with a few deserving special mention. To illustrate my journey, I would like to move in a sequential manner. First of all, I want to give special thanks to Krishna Banerjee and Arjun Paul, my Maths teachers during my school days. It was their guidance that sparked my interest in Mathematics and inspired me to pursue it further, ultimately choosing it as my subject for graduation. This decision introduced me to the next significant chapter of my academic journey under the mentorship of Dr Sudarshan Bardhan. This was the period when my interest grew further in my subject and started finding some more of it. He helped me understand the subject more profoundly, encouraged me to delve deeper, shaped my research interests and have provided resolute attention in my research during my Ph.D. journey. This journey is ongoing, and I hope to continue it for many more years to come. It was my teacher Dr Bardhan who introduced me to my research guide, Professor Bibhas C. Giri, who has had a thorough impact on my research journey. I am deeply grateful to my guide, Professor Bibhas C. Giri for being the epitome of patience and providing unwavering motivation in every situation. He has consistently nurtured my research interests and provided continuous guidance, while also giving me the freedom to explore my ideas. I will forever remember and remain indebted to his contributions to me until my last breath. My guide not only ensured my thorough theoretical understanding but also provided me with a research environment that would be a dream for any aspiring scholar. When I first entered the lab, I was welcomed with open arms. All my seniors Chirantan da, Mahetab da, Sushil da, Jayanta da, Sumon da, Monalisa di, Anamika di, Debapriya da, Satakshi di and Saddam da – provided me with a warm and cozy environment. I am immensely grateful for such a beautiful workplace. They supported me through my difficult times, patiently corrected my mistakes, and endured my childish acts with care and kindness. And then there were friends like Subhomay da, Doli di, Shuvendu da, Runu, Saikat, Brati di, Abhishek, and finally my sweet juniors like Nupur, Pritam and Purnendu. Together, we shared unforgettable bonds and cherished moments that a lifetime would scarcely suffice to nurture. Beyond being my labmates, Runu has also been my roommate, alongside Sayani Di, and both of them have played a profound role in shaping the person I am today. These cherished memories will always bring a smile to my face whenever they come to mind. Thanks to Jadavpur University for providing and preserving such wonderful research environment.

Apart from the straight story, there are many more individuals who deserve special recognition. All my teachers have consistently strived to try to bring out the best version of myself. My friends like Akash, Sujata, Priya, Ananya, and my extended family including uncles, aunts, brothers, and sisters, especially my dear sister Anusuya who has continuously inspired me to strive for my goals, aiming to be a role model for her. I must also mention of my nieces Joyeeta, Ankita, and Sudipta, who have always reminded me to balance life's sweetness and spice. They have all been there for me whenever I needed their support.

With a limitation in words, it is impossible to name each and every person separately to acknowledge them properly. Every individual you interact with and every circumstance you encounter leaves an indelible mark on you, fostering your personal growth and helping you evolve as a person. On this special day, I want to express my gratitude to everyone in my personal and professional life who has contributed to my growth and helped shape the person I am today.

Finally, I would like to thank myself for persevering until the end and the Almighty for always being by my side, smoothing the path for me to move forward steadily.

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(**Indrani Modak**)

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CHAPTER 1

Introduction

"Price is what you pay. Value is what you get."- Warren Buffett

In today's interconnected and everchanging global market, supply chain systems have become increasingly sophisticated, responding to the rapid evolution of industries, technological advancements, and consumer preferences. These developments have transformed traditional business practices with a greater emphasis on efficiency, sustainability, and adaptability. Modern supply chain networks no longer operate in isolation but are integrated ecosystems where each component significantly impacts the overall performance and competitiveness of a business. Companies now face the challenge of optimizing diverse operational strategies to remain relevant in an environment where both market demands and environmental concerns are rapidly evolving.

One critical area of transformation has been the incorporation of innovative strategies into supply chain management to address key issues such as enhancing demands, resource scarcity, environmental degradation, and economic instability. The introduction of concepts such as closed-loop supply chains, lean manufacturing, and sustainable practices highlights the growing need for businesses to not only focus on profitability but also to address long-term sustainability and consumer expectations. These approaches have reshaped industries, allowing firms to reduce costs, improve resource utilization, and align with global environmental goals. Within this evolving landscape, inventory modeling has emerged as a vital tool for optimizing supply chain operations. Inventory models provide structured frameworks to manage stock levels, minimize costs, and meet fluctuating demand effectively. From perishable goods requiring preservation strategies to durable items with long replenishment cycles, inventory models ensure businesses maintain an equilibrium between supply and demand. These models also incorporate advanced elements such as learning effects, dynamic demand patterns, and preservation investments, enabling companies to manage their resources more effectively while addressing environmental concerns. In addition to inventory modeling, various contractual agreements within supply chains play a crucial role in shaping operational outcomes. Contracts such as buyback agreements, revenue-sharing arrangements, and cost-sharing strategies directly influence the efficiency and profitability of supply chain models. For example, buyback contracts reduce retailer risk by allowing unsold inventory to be returned, while revenue-sharing contracts align incentives between suppliers and retailers. These mechanisms not only ensure smoother collaboration but also impact pricing and production decisions, making them integral to supply chain optimization. The role of decision-making at different levels of the

Figure 1.1: Factors affecting the customers while purchasing a product

supply chain is further magnified when considering the integration of pricing strategies. Every cost incurred in production, logistics, and inventory management must ultimately be offset by revenue, which is determined by pricing decisions. Pricing not only serves as a tool to recover costs but also directly influences consumer behavior, demand, and competitive positioning. As a result, companies must develop pricing strategies that are aligned with broader supply chain goals, ensuring profitability while addressing customer expectations. From dynamic pricing models to green-sensitive strategies, pricing decisions are increasingly intertwined with supply chain dynamics. They are influenced by factors such as demand elasticity, product life-cycle stages, and even randomness introduced by external market forces like seasonal trends and promotions. The randomness in pricing, often employed as part of strategic models, allows companies to navigate volatile market conditions while maintaining competitiveness.

This thesis delves into the intricate relationship between inventory and supply chain management, and pricing strategies. By exploring the interplay between these elements, it examines how businesses can optimize their operations to achieve both profitability and sustainability. Before delving into the specific focus on pricing, the foundational elements of inventory and supply chain management, and the influence of contracts will be discussed to provide a comprehensive understanding of the broader context. The discussion of pricing will follow these insights, emphasizing its role as a strategic component for balancing costs, aligning with consumer expectations, and achieving competitive success.

The following sections will briefly discuss key topics related to inventory and supply chain management, exploring their fundamental principles, challenges, and strategies. Each section will discuss how pricing plays a pivotal role in addressing these aspects, serving as a critical tool to balance costs, influence demand, and enhance operational efficiency. By highlighting the interplay between price and these relevant topics, the discussion will gradually build toward an in-depth exploration of pricing strategies. The final section will focus exclusively on pricing, analyzing its nuances, methodologies, and impact, offering a comprehensive understanding of its role as the cornerstone of successful inventory and supply chain management.

1.1 Inventory and supply chain issues

This section delves into key topics related to inventory and supply chain management, building upon the issues highlighted in the previous section. The issues will be explored in greater depth, emphasizing their interconnections and mutual influences. This discussion will provide a foundation for understanding how strategic approaches can address these complexities effectively.

1.1.1 Deterioration and preservation

Deterioration refers to the continuous changes in an item that render it unsuitable for use. While all items undergo some degree of depreciative changes over time, certain items such as food products and electronics, experience these changes more visibly and rapidly than others. The process of deterioration impacts both the quality and quantity of items. When deterioration affects the quality without rendering the item unusable, it is known as qualitative deterioration. On the other hand, when deterioration makes the item unusable, it is termed quantitative deterioration. To illustrate through real-life examples, fresh produce initially experiences qualitative deterioration as its quality degrades over time. Eventually, it becomes unsuitable for consumption, marking the shift to quantitative deterioration. In contrast, electronic items may not show noticeable quality changes until they become obsolete, indicating purely quantitative deterioration. The deterioration process significantly impacts both customer demand and business profitability. Freshness plays a crucial role in attracting customers, as products in optimal condition are more appealing. When items deteriorate, it not only diminishes their quality but also reduces their marketability. This directly affects inventory levels, leading to financial losses due to decreased sellable stock and the associated disposal costs. Managing inventory effectively is therefore essential to maintain product quality, customer satisfaction, and overall business profitability. To optimize the profit, the business manager should thus invest in preservation technologies to reduce the deterioration to some extent. Preservation refers to the act or process of maintaining something in its existing state or preventing it from decaying, deteriorating, or being damaged. It involves various techniques and strategies aimed at prolonging the lifespan, usability, or value of objects, resources, or materials. The goal of preservation is typically to retain the original quality, functionality, or cultural significance of the preserved item for as long as possible. Preservation, like all other parameters, embodies a delicate push-pull dynamics. While increased investment in preservation can mitigate deterioration losses, excessive allocation to it can impose a significant strain on a company's fiscal policy. In this context, pricing plays as a pivotal lever. Beyond merely addressing preservation costs, pricing can serve as a strategic tool to balance all influencing factors, ensuring that preservation efforts are sustainable and fiscally prudent. By judiciously aligning preservation investment with pricing strategies, companies can navigate the intricate interplay between protecting assets and maintaining financial health, achieving an optimal equilibrium that supports long-term growth.

1.1.2 Lead time reduction

Delivering the right items to the right location at the correct time in the appropriate quantity under the right circumstances are the key objectives of logistics. In this regard, the lead time between the initiation of an order and the final delivery is extremely important for both customer satisfaction as well as inventory levels. Factors that may affect lead times include a scarcity of raw materials, equipment malfunction, transportation problems, a personnel shortage, natural calamities, and human errors. In the bustling world of today, customers desire goods or services as quickly as possible and with a minimal effort. Longer lead times can thus have a negative impact on customer perception and result in lower sales volume. Companies should therefore reduce lead times in order to deliver products or services to the customer at the anticipated time. However, there is a risk of understocking or overstocking while doing so; whereas overstocking might place a burden on the budget, understocking can result in a loss of prospective revenue. In a word, although difficult, reducing and efficiently managing lead time and optimizing inventory level boost productivity and increases profitability. For perishable items, lead time is intricately linked to freshness, as prolonged waiting periods contribute to quality degradation. This connection naturally extends to preservation investment as well, as efforts to mitigate freshness loss during lead time require strategic resource allocation. Consequently, price becomes a critical factor, integrating lead time, preservation efforts, and the value offered to customers into a cohesive framework. Furthermore, during stock-out scenarios, when inventory falls short of customer expectations, pricing once again demonstrates its versatility. Strategic price discounts can serve as a compensatory measure, mitigating customer dissatisfaction and retaining loyalty despite inventory shortfalls. Thus, pricing not only addresses challenges arising from inventory losses but also emerges as a dynamic tool for managing lead time complexities and ensuring a seamless customer experience.

1.1.3 Warranty

Product's warranty policy plays a pivotal role in shaping its pricing strategy, as it directly influences customer perception of value and risk. Warranties serve as a guarantee of quality, providing consumers with the assurance that defective items will be repaired or replaced within a specified period. This perceived reduction in risk often allows businesses to justify higher price points, as customers are willing to pay a premium for reliability and post-purchase support. However, the cost implications for manufacturers and retailers must also be factored into pricing decisions. Longer or more comprehensive warranties increase the likelihood of returns or replacements, which can escalate costs. To offset these expenses, pricing policies may incorporate higher margins or tiered pricing models based on warranty duration. Warranties also contribute to sustainability by facilitating the return, refurbishment, and resale of products, creating opportunities for additional revenue streams. The interplay between warranty policies and pricing is further shaped by consumer behavior, competition, and market conditions, requiring businesses to strike a delicate balance between offering competitive prices and maintaining profitability while delivering value through warranty commitments.

1.1.4 Learning effect

Learning is an essential aspect of daily life, where individuals improve through repeated tasks, leading to increased efficiency, reduced errors, and enhanced work quality. For instance, in manufacturing, workers assembling products repeatedly become faster and more proficient, resulting in higher output and fewer mistakes.

This phenomenon underscores the value of experience across industries, as skilled workers require less supervision, complete tasks efficiently, and deliver superior results, whether in healthcare, construction, or production. In business, the learning effect significantly reduces labor costs by decreasing production time as workers master their tasks, enabling companies to lower operational expenses. This cost efficiency supports competitive pricing strategies, allowing businesses to maintain profitability while appealing to price-sensitive customers, thereby driving both success and sustainability.

1.1.5 Shortages and backlog

In inventory modeling, shortages occur when a business lacks sufficient stock to meet consumer demand or production needs, often caused by sudden demand surges, supply chain disruptions, inaccurate forecasting, or replenishment delays. Effective inventory management is essential to mitigate the negative impacts of shortages, such as lost sales, reduced customer satisfaction, and production delays. Shortages are sometimes planned strategically to minimize costs or improve efficiency, such as reducing holding costs, adhering to lean manufacturing principles, managing seasonal variations, or creating urgency during promotions. However, planned shortages require careful monitoring to avoid adverse outcomes like customer dissatisfaction and lost sales. When shortages lead to backlogs, three scenarios may arise: full backlog which is an unrealistic one to accept in all situations, no backlog (complete demand loss) which is unfavorable for businesses, or partial backlog with partial demand loss which is most realistic as well as acceptable to business managers to some extent. The extent of backlogging often depends on customer patience, influenced by waiting times. To optimize shortages and manage backlogs, businesses can employ strategies such as discounting where price reductions incentivize customers to wait, time-varying pricing where prices vary based on waiting periods, or reducing lead times to ensure faster delivery and better product quality. These measures, when combined with effective planning, help balance costs, customer satisfaction, and operational efficiency during inventory shortages.

1.1.6 Green supply chain

Figure 1.2: Components of green supply chain

The concept of green supply chain has emerged as a strategic approach towards integrating environmental considerations into traditional supply chain operations. This shift is driven by the need to address global issues like resource depletion, climate change, and pollution while maintaining economic growth. Businesses are increasingly recognizing that sustainable supply chains not only reduce environmental harm but also offer significant cost benefits, which directly influence product pricing. By adopting green practices such as renewable energy usage, waste minimization, and circular economies, companies can reduce production costs, enabling competitive pricing strategies that appeal to eco-conscious consumers. Green supply chain management optimizes resource efficiency, reduces reliance on virgin materials, and mitigates long-term risks associated with environmental regulations and resource scarcity. These cost-saving measures create a direct link between sustainability and pricing, allowing businesses to maintain profitability while offering value-driven prices. Despite challenges like technological gaps, high initial investments, and consumer awareness deficits, businesses adopting green supply chains can achieve a competitive edge through innovative pricing strategies that align with both market demand and ecological responsibility.

1.1.7 Closed-loop supply chain

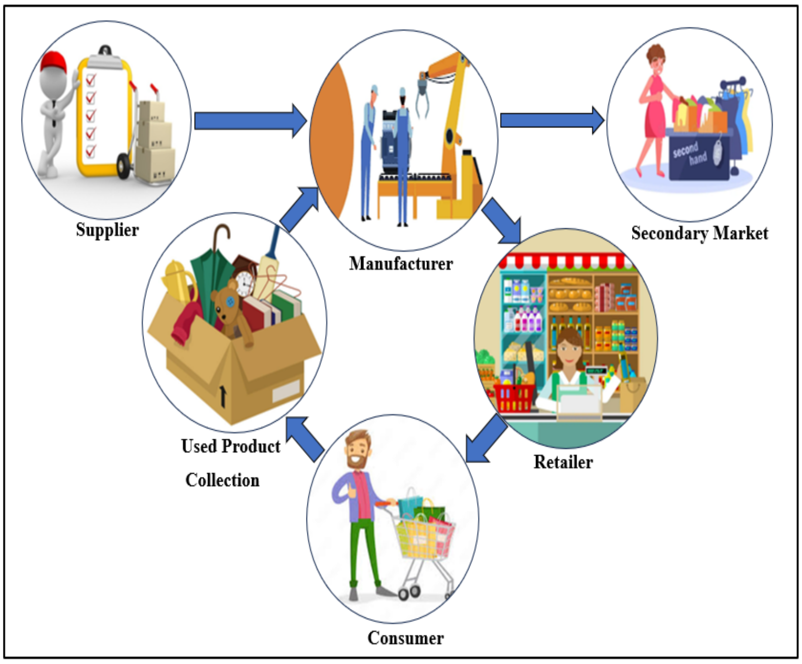


Figure 1.3: Schematic diagram of a closed loop supply chain

A closed-loop supply chain (CLSC) integrates forward and reverse logistics to recover, reuse, and recycle materials, creating a sustainable system that minimizes environmental impact while optimizing operational costs. This approach directly influences product pricing by reducing reliance on virgin materials, lowering production costs, and enabling competitive pricing strategies. Unlike traditional linear models, CLSCs reintroduce returned goods and waste into the production cycle, supporting a circular economy. By leveraging CLSCs, companies like Dell and H&M (Hennes & Mauritz) achieve dual objectives of sustainability and profitability, meeting regulatory compliance and customer demand for eco-friendly products while maintaining price competitiveness. Despite challenges like high implementation costs and complex logistics, advancements in technology and incentives are driving the adoption of CLSCs, ensuring both affordability and sustainable growth.

1.1.8 Coordinating supply chains

The evolution of supply chain networks in the twenty-first century has been profoundly influenced by globalization, technological advancements, and the rise of multinational corporations, joint ventures, and strategic partnerships. These dynamics have introduced opportunities for efficiency and growth but have also created complexities requiring greater coordination. Lack of coordination occurs when different stages or entities within a supply chain focus on conflicting objectives, or when information is delayed and distorted as it flows through the chain. This misalignment negatively affects production costs, replenishment times, product availability, pricing, transportation, and labor costs, ultimately hindering overall performance.

Supply chain coordination involves aligning the objectives and actions of all participants to manage interdependencies and reduce uncertainties effectively. It ensures that the supply chain operates as an integrated system rather than as disjointed parts. For example, *buyback contract* is a coordination mechanism where suppliers agree to repurchase unsold inventory from retailers. This arrangement reduces the retailer's risk of overstocking, encourages larger orders, and ensures better product availability for end customers. This contract is commonly used in industries dealing with seasonal or perishable goods, such as fashion or consumer electronics. However, it can lead to overproduction by suppliers and higher handling costs for returned goods, making inventory management more complex. Another effective coordination tool is the *revenue-sharing contract*, which allows supply chain partners to share profits in a mutually beneficial way. For instance, manufacturers may reduce wholesale prices in exchange for a share of the retailer's revenue, motivating both parties to align their decisions for maximizing joint profits. Such a contract is widely used in the film industries (Palsule-Desai, 2013), in apple stores and amazon marketplace (Bart et al., 2021), agro-business industries (Cui et al., 2020, Rajput and Venkataraman, 2024), and video rental and newspaper industries (Liu and Yan, 2024). The drawback lies in the need for accurate revenue tracking and trust, as discrepancies in profit reporting can lead to disputes. The *option contract* allows retailers to reserve additional inventory by paying a premium, which they can later purchase if demand exceeds forecasts. This is widely used in industries with demand volatility, such as energy or luxury goods. However, high reservation costs may deter retailers, particularly if demand remains below expectations, leading to unnecessary expenses. *Quantity flexibility contract* requires the retailers to commit to a minimum purchase while suppliers provide flexibility for additional orders to manage demand fluctuations. This is particularly useful in industries with unpredictable demand, such as pharmaceuticals or technology. However, suppliers may face risks of unused capacity or overstocking if demand estimates prove inaccurate. A *cost-sharing contract* involves suppliers and retailers splitting costs for joint activities, such as marketing campaigns or sustainability initiatives, to achieve mutual benefits. These contracts are commonly used in branding efforts or eco-friendly supply chains. The drawback is potential disagreements over cost allocation, which can strain business relationships if not clearly defined. Apart from applying the contracts straightway, complex business scenarios often require implementation of the combination of two or more contracts- termed as composite contract- to enhance profits. Contracts not only mitigate risks but also promote collaboration, reduce inefficiencies, and enhance overall supply chain resilience. By fostering alignment and trust among participants, supply chain coordination transforms potential disruptions into opportunities for streamlined operations, ensuring competitiveness in a complex and interconnected global market.

1.1.9 Pricing

Pricing is one of the most critical components of a business's overall strategy, directly influencing both profitability and market competitiveness. It serves as a key signal of a product's value, quality, and market positioning while acting as a driver of consumer behavior. Effective pricing strategies align closely with market dynamics, consumer preferences, and supply chain efficiencies to maximize revenue, enhance value perception, and achieve competitive advantages. A pricing strategy refers to the method businesses use to set the prices for their products or services. It is a multidimensional concept that integrates costs, market demand, competitive positioning, and long-term business goals. Kotler and Keller, 2009 describes pricing as the art and science of translating the value of a product into monetary terms to optimize profits while meeting consumer expectations:
"A pricing strategy is the critical linkage between a company's value proposition and its

revenue-generation process. It reflects not only the company's costs and profit objectives but also its brand image and market positioning."

1.1.9.1 Role of pricing in modern businesses

In today's highly competitive global markets, a well-defined pricing strategy is indispensable for sustainable success. As Figure 1.1 exhibits, several factors influence customer behavior and purchasing decisions. While these factors may vary depending on the type of product or service, price consistently emerges as a pivotal and universal determinant. Regardless of the industry or product category, pricing plays a critical role in shaping customer perceptions, balancing quality expectations, and driving value propositions. It serves as the connecting thread that aligns other business aspects such as product positioning, market demand, and competition—creating a harmonious balance in the overall business landscape. Pricing impacts every aspect of the value chain, from supply chain decisions to customer satisfaction. Key roles of pricing are the following:

- ✍ **Revenue Maximization:** Pricing directly determines the revenue a business generates. Properly calibrated pricing strategies can balance volume with profit margins to optimize overall financial outcomes.
- ✍ **Value Communication:** The price of a product is often a consumer's first indicator of its value. Premium pricing can signal exclusivity and quality, while competitive pricing attracts cost-conscious buyers.
- ✍ **Market Positioning:** Pricing defines where a product or service stands relative to competitors. Luxury brands rely on high pricing to convey prestige, while budget brands use lower prices to capture price-sensitive markets.
- ✍ **Driving Consumer Behavior:** Behavioral pricing strategies like discounts, bundling, or dynamic pricing can influence purchase decisions and enhance customer retention.
- ✍ **Strategic Alignment:** Pricing strategies align with broader business goals, whether targeting growth, profitability, or market penetration.

1.1.9.2 Key components of pricing strategy

Effective pricing strategies are built on the following elements:

- ✍ **Cost Analysis**: Ensuring prices cover costs while achieving desired profit margins.
- ✍ **Market Dynamics**: Considering competition, demand elasticity, and consumer expectations.
- ✍ **Perceived Value**: Pricing based on what consumers are willing to pay relative to their perception of the product's worth.
- ✍ **Profit Maximization**: Balancing short-term revenue with long-term growth objectives.
- ✍ **Supply Chain Consideration**: Efficient supply chains reduce costs, enabling more competitive pricing.

Supply chain efficiencies play a pivotal role in determining a company's pricing strategy. Cost reductions through optimized logistics, inventory management, and supplier negotiations allow businesses to offer competitive prices while maintaining profitability. Christopher, 1998 notes that integrating supply chain strategies with pricing decisions enhances customer satisfaction and market adaptability:
"A well-integrated pricing and supply chain strategy helps companies meet customer expectations in price, quality, and delivery, reinforcing brand loyalty."

1.1.9.3 Types of pricing strategies

Pricing strategies can be broadly categorized into three main types: static, dynamic, and random. Each type has its unique characteristics and applicability depending on business models, market conditions, and consumer behavior.

- **Static Pricing Strategy**
  Static pricing refers to a fixed pricing model where the price of a product or service remains unchanged over time, irrespective of market conditions

or demand fluctuations. This strategy is straightforward and easy to implement, often employed by businesses offering standardized products with predictable costs, such as fast-moving consumer goods (FMCG). Static pricing fosters transparency and consistency, making it appealing to consumers who prefer stability and trust. However, it lacks flexibility, which can be a disadvantage in volatile markets where costs and demand frequently shift.

- **Dynamic Pricing Strategy**

  Dynamic pricing is a flexible approach where prices fluctuate based on factors like market demand, competition, and time of purchase. This strategy is widely used in industries such as travel, e-commerce, and entertainment. Airlines and ride-hailing services, for instance, use sophisticated algorithms to adjust prices in real time, maximizing revenue by capitalizing on high-demand periods while attracting price-sensitive customers during low-demand times. A robust dynamic pricing model must account for inventory changes, product decay, and demand elasticity. By lowering prices to clear excess inventory or raising them during low-stock scenarios, businesses can effectively balance supply and demand. Additionally, integrating supply chain factors, such as replenishment lead times and variability, ensures efficient inventory management. While dynamic pricing allows for profit optimization, it requires advanced technology and data analytics to monitor market trends, which can be challenging for smaller businesses. Additionally, it may risk alienating consumers if perceived as unfair or inconsistent. Dynamic pricing can further be subcategorized in the following way:

  ✍ **Continuous Dynamic Pricing**: Continuous dynamic pricing is a real-time pricing strategy where prices are adjusted continuously based on factors such as demand, inventory levels, and market conditions. Leveraging sophisticated algorithms and real-time data analytics, this approach enables immediate responses to competitor pricing, stock fluctuations, and customer demand. For perishable items, where quality declines over time, dynamic pricing optimizes revenue by reflecting the diminishing value of aging products and varying consumer demand for freshness. The e-commerce platforms mostly use this kind of pricing strategy. This strategy reduces waste, maximizes profitability, and enhances supply chain efficiency, making it indispensable for managing perishable goods in competitive markets. Ride-Sharing Apps (e.g., Uber, Lyft) also use continuous dynamic pricing to adjust fares in real-time based on demand, traffic conditions, and availability of drivers. This ensures that prices fluctuate continuously rather than at fixed intervals.

✍ **Discrete Dynamic Pricing**: While continuous dynamic pricing is highly effective in addressing real-time market dynamics, implementing it requires advanced technological infrastructure, including real-time data analytics, automated algorithms, and integrated inventory systems. However, many retail stores lack the technological capabilities to implement continuous dynamic pricing. For such businesses, discrete dynamic pricing offers a practical alternative. In this approach, prices are adjusted at specific intervals—daily, weekly, or after significant events like restocking or sales period completion. Unlike continuous pricing, updates occur in batches rather than in real-time, making the model simpler and more computationally manageable. Discrete dynamic pricing balances flexibility and practicality, allowing stores to regularly adjust prices based on inventory levels, market trends, and competitor pricing without requiring continuous monitoring. Retailers can use this strategy to optimize stock turnover and profitability within their technological constraints, making it a viable and efficient option for adapting to changing conditions. The airlines (Indigo, Singapore Airlines, *etc*) adjust ticket prices at discrete intervals based on factors like booking windows, demand, and competition. For instance, prices often increase closer to the departure date or during holiday seasons, reflecting time-based price changes in steps.

- **Random Pricing Strategy**

  Random pricing involves unpredictable price changes that do not follow a specific pattern or strategy. This unconventional approach is often used in experimental marketing campaigns or to create buzz around a product. For instance, companies may randomly offer discounts or flash sales to surprise and delight customers. While random pricing can generate excitement and attract attention, it carries the risk of confusing or frustrating consumers who may struggle to understand the rationale behind price changes. Businesses employing this strategy must carefully balance unpredictability with clear communication to maintain trust and engagement. The e-commerce platforms like Amazon or eBay occasionally implement random discounts or flash sales, offering sudden, unannounced price reductions to incentivize purchases. Customers cannot predict these price drops, creating an element of randomness.

By understanding and leveraging these pricing strategies, businesses can adapt to different market scenarios, enhance consumer engagement, and achieve financial objectives effectively. Each approach offers unique advantages and limitations, making it essential for companies to align their pricing strategy with their goals and market dynamics.

1.1.9.4 Challenges in pricing strategy

The challenges business managers face while adhering to a pricing strategy often includes, but not limited to

- ✍ **Market Volatility**: Sudden changes in demand or costs can disrupt pricing strategies.
- ✍ **Consumer Perception**: Misaligned pricing can harm brand image and sales.
- ✍ **Global Competition**: Competing in international markets requires localized pricing strategies.

Besides the challenges, several critical factors influence the development and execution of a pricing strategy. These factors ensure that pricing decisions align with business objectives, market conditions, and customer expectations. Some of the key factors include the following:

- ✍ **Cost structure**: The fixed and variable costs of production, distribution, and marketing significantly shape pricing decisions to ensure profitability.

- ✍ **Market demand**: Understanding demand elasticity helps businesses to set prices that optimize sales volume and revenue.
- ✍ **Competition**: Competitor pricing strategies and market positioning impact how businesses price their offerings to remain competitive.
- ✍ **Customer perception**: Prices must reflect the perceived value of a product or service to the target audience.
- ✍ **Economic conditions**: Inflation, currency fluctuations, and purchasing power influence consumer behavior and pricing flexibility.
- ✍ **Regulatory environment**: Legal constraints such as price caps and anti-competitive regulations can limit pricing strategies.
- ✍ **Product life-cycle**: Prices often evolve based on whether a product is in its introduction, growth, maturity, or decline phase.
- ✍ **Channel strategy**: The distribution channels used—online, retail, or direct-to-consumer—affect pricing dynamics due to varying costs and customer reach.
- ✍ **Technological advancements:** Pricing algorithms and real-time data analysis enable sophisticated and adaptable pricing mechanisms.

By carefully analyzing these factors, businesses can develop pricing strategies that are competitive, sustainable, and responsive to market demands.

1.2 Significance of the study

The study of pricing policy is crucial for businesses, economies, and consumers alike, as it directly influences profitability, competitiveness, and market dynamics. Pricing policies serve as a strategic tool that impacts consumer behavior, cost management, and overall business sustainability. Understanding their significance allows businesses to make informed decisions and achieve a balance between value creation and financial performance. Key reasons for studying pricing policy include the following:

- ✍ **Profit maximization**: Pricing directly determines revenue and profit margins. A well-designed pricing policy enables businesses to capture maximum value from their products or services while staying competitive in the market.

- ✍ **Survive market competitiveness**: Analyzing pricing policies helps businesses position themselves effectively against competitors. By understanding the nuances of price elasticity, market demand, and competitive pricing, companies can differentiate their offerings and attract target customers.

- ✍ **Influencing consumer behavior**: Pricing shapes consumer perceptions of value, quality, and affordability. A strategic pricing policy can enhance brand loyalty and influence purchasing decisions by aligning with customer expectations.

- ✍ **Cost management and efficiency**: Pricing policies account for production costs, operational expenses, and market trends. They ensure that prices cover costs while optimizing resource allocation and minimizing waste.

- ✍ **Adaptation to market changes**: Studying pricing policies equips businesses to respond effectively to market dynamics such as demand fluctuations, economic shifts, and competitor actions, For instance, dynamic pricing strategies allow for real-time adjustments to maintain profitability, while static pricing is helpful in scenarios with stable demand and predictable market conditions, providing consistency and simplicity for both businesses and customers.

- ✍ **Enhancing customer retention and loyalty**: Consistent and fair pricing fosters trust and loyalty among customers. Strategic discounts, bundled pricing, or loyalty-based pricing policies can enhance customer satisfaction and long-term retention.

- ✍ **Economic and regulatory compliance**: Understanding pricing policies ensures adherence to market regulations, preventing practices like price-fixing, unfair competition, or predatory pricing, which can result in legal penalties.

- ✍ **Sustainability and ethical considerations**: Pricing policies increasingly incorporate sustainability goals such as incentivizing eco-friendly products or reducing waste. This aligns businesses with global sustainability standards and consumer preferences for ethical practices.

In conclusion, the study of pricing policy is vital for optimizing business performance, fostering customer satisfaction, and navigating competitive markets effectively. It serves as a cornerstone for strategic decision-making, ensuring that businesses remain agile, profitable, and aligned with both market demands and broader economic objectives.

1.3 Organization of the thesis

This thesis is organized into eight chapters. The first two chapters present the introduction and literature review, followed by five chapters that form the core of the thesis, each focusing on a distinct model developed during the research. These models vividly illustrate how pricing strategies evolve with shifting scenarios, offering valuable insights into their practical applications. The thesis concludes by integrating the main findings and highlighting potential directions for future study. The details elaboration of the chapters are as follows:

Chapter 1: Introduction
This chapter provides an introduction to the key topics explored throughout the thesis, laying the groundwork for understanding the diverse pricing strategies examined in various scenarios. By offering a concise overview of the fundamental concepts and approaches, this chapter serves as a guide to navigate the research presented. It offers valuable context for comprehending the intricacies of pricing strategies and their applications, making it an essential resource for appreciating the contributions and findings of the work undertaken in this thesis.

Chapter 2: Literature review
This chapter provides a comprehensive overview of the existing literature relevant to the research presented in this thesis. By tracing the evolution of key topics and methodologies, it highlights the foundational works that have shaped the field and situates the contributions of this thesis within the broader scholarly context. Studying this chapter offers valuable insights into how the explored topics have developed over time and clarifies the unique position and significance of the works presented here in advancing the existing body of knowledge.

Chapter 3: Pricing strategies in perishable inventory management: leveraging learning effects and preservation investments

Managing inventory for perishable items presents unique challenges, as excessive stock leads to high deterioration costs. To address this, it is crucial for business managers to optimize both the order quantity and the timing of orders. Considering the standard life cycle of product demand, other key factors such as preservation efforts and operational efficiency must also be carefully calibrated. Pricing strategy plays a pivotal role in this context, serving as a tool to balance demand, minimize wastage, and enhance profitability. This chapter delves into the interplay of these factors, offering insights into strategies that ensure effective inventory management for perishable goods.

Chapter 3.1 Pricing strategy for a perishable inventory model with shortages

Price and time significantly influence market demand, particularly for fashion items, newly launched electronics, and similar products. After-sale service facilities boost demand, while investments in preservation technology reduce spoilage. This subchapter develops a multi-period inventory model incorporating these factors, where demand depends on price, time, and service quality. Equal-length replenishment cycles are adopted, allowing varying stock-in and stock-out periods due to time-dependent demand. A planned shortage policy, followed by replenishment, proves effective, while the learning effect in holding and ordering costs is also considered. Limited capital and warehousing constraints are addressed, and numerical examples illustrate the model's practicality and offer managerial insights.

Chapter 3.2 Dynamic pricing and discounting policy in multi-period perishable inventory models

This subchapter explores a multi-period inventory model where demand is influenced by time, price, and service levels. Discrete dynamic pricing across periods-including offering discounts during shortages- is analyzed as a strategy to optimize demand and revenue. The model considers fixed time horizons with equal replenishment cycles but varying shortage periods, allowing retailers to adjust prices and incentivize customers during stockouts. Learning effects in holding and ordering costs are incorporated to improve service and operational efficiency. Numerical experiments highlight that discounts effectively boost revenue when back-order rates are low, while maintaining a high service level is crucial during extended stockouts. Key managerial insights are derived to guide optimal pricing and replenishment policies.

Chapter 4: Developing pricing and lead time strategies in a perishable inventory model facing price, quality, and stock-dependent demand
This chapter examines a continuous dynamic pricing problem where demand depends on price, stock levels, and freshness. The business period is divided into two phases: a stock-out period, where investments in lead time reduction and freshness preservation are prioritized, and a stock-in period. Using Pontryagin's maximum principle, the optimal dynamic pricing strategy is derived. Numerical results show that, even with inventory-dependent demand, stock-in prices may be lower than stock-out prices, with the price slope influenced by stock sensitivity. Key insights into pricing and investment strategies are provided for managing perishable inventory effectively.

Chapter 5: Interplay of greenness and preservation investments: pricing strategies for freshness- and green-conscious customers
For perishable items that continuously lose freshness, pricing strategies must align with the declining value of the product over time. Preservation efforts become critical to prolong freshness, while the environmental impact of disposal underscores the importance of greening initiatives. For items with instantaneous freshness loss, lead time plays a pivotal role in ensuring timely delivery and minimizing waste. This chapter addresses these interconnected challenges, illustrating how pricing, preservation, greening, and lead time management collectively contribute to optimizing the supply chain for perishable goods.
Chapter 5.1: Optimal dynamic pricing, preservation, and green strategies for green-sensitive customers
This subchapter addresses demand regulation by considering price, product greenness, and preservation investments for perishable inventory. With growing environmental awareness, customers increasingly value greenness alongside quality, making preservation technology and sustainability crucial. The study proposes an optimal pricing strategy tailored to specific demand patterns while incorporating preservation and greening investments. Analytical and numerical insights highlight the model's versatility and provide actionable strategies for various business scenarios. Managerial implications are also explored, demonstrating the model's

applicability across diverse contexts.

Chapter 5.2: Role of lead time in a perishable inventory model where the quality is an issue
This subchapter develops an inventory model for perishable products with demand influenced by price, freshness, and greening levels. Both instantaneous qualitative and non-instantaneous quantitative deterioration are considered alongside investments in preservation and lead time reduction. A hybrid pricing strategy is proposed for stock-in and stock-out periods, complemented by optimal cycle length, greening, preservation, and lead time investments. Sensitivity analysis confirms the model's robustness and highlights a complementary relationship between greening and freshness. Particularly suitable for freshly packaged food and e-commerce sales, the model also aligns with strategies like Dell's customized marketing. Numerical examples provide insights into its practical applications.

Chapter 6: Production and preservation strategies in a two-echelon supply chain under revenue sharing contract
This chapter examines a manufacturer-retailer supply chain under a multi-period business model with price- and time-dependent demand. A revenue-sharing contract is proposed to enhance profits for both parties, introducing a flexible policy where pricing and revenue shares are dynamically adjusted across intervals. Using differential calculus, optimal pricing strategies and the win-win range for revenue-sharing fractions are derived. The analysis reveals that profits decline in later intervals under a price-only contract, advocating for reduced shared revenue portions in subsequent periods. Rising costs are shown to negatively impact profits, emphasizing the importance of preservation technology investments to sustain profitability. The study offers valuable insights into dynamic pricing, supply chain coordination, and contract optimization for perishable goods management.

Chapter 7: Exploring random pricing strategies in a closed-loop supply chain with greening investment
While previous chapters assumed rational and predictable factors, real-world supply chains often grapple with uncertainties. This chapter delves into the inherent randomness associated with product pricing, returns, and production within the framework of a closed-loop supply chain. The discussion highlights how such uncertainties influence decision-making and operational efficiency. A key focus is

placed on pricing strategies under these conditions, exploring their characteristics and importance in managing demand and profitability. Additionally, the role of buyback contracts in mitigating risks and enhancing coordination between supply chain partners is thoroughly examined, offering valuable insights into managing uncertainty in dynamic business environments.

Chapter 7.1: Optimal pricing, greening and warranty investments under random yield

This subchapter explores random pricing in a closed-loop supply chain with demand influenced by price, warranty periods, and product greening levels, reflecting consumers' environmental awareness. During the warranty period, manufacturers replace defective items and remanufacture or refurbish returns for resale. The model shows that buyback contracts improve profits in decentralized settings, while higher salvage values enable retailers to reduce prices. Analytical insights highlight the profitability of green products, as the absence of green sensitivity leads to diminished profits. Relevant to industries like electronics and supermarkets, the study emphasizes the importance of contracts and pricing strategies in managing customer expectations and demand fluctuations.

Chapter 7.2: Optimal pricing and remanufacturing strategies under carbon emissions and government regulations

This chapter examines a two-period closed-loop integrated supply chain model with randomness in pricing, demand, and production. Incorporating environmental considerations, the study extends the traditional model by including carbon emission constraints and cap-and-trade policies alongside greening investments. Numerical examples highlight that green initiatives and remanufacturing enhance profits while benefiting the environment. Sensitivity analysis provides managerial insights on responding to parameter changes, reinforcing the profitability of environmentally conscious practices.

Chapter 8: Conclusion and future research prospects

This chapter encapsulates the key findings and conclusions derived from the study, offering a comprehensive understanding of the research conducted. Additionally, it outlines promising directions for future research, encouraging the exploration of new ideas and advancements in the field.

CHAPTER 2

Literature Review

This chapter provides a focused review of the literature on inventory and supply chain management, examining various facets of these fields and addressing key issues pertinent to this thesis. It further highlights the gaps and limitations in prior studies, establishing a strong foundation for the novel contributions and insights presented in this research.

2.1 Preservation investment

Deterioration has managed to maintain the attention of many researchers for decades after the notion was initially introduced in inventory modelling by Whitin, 1957. There are a huge amount of related research works, details of which can be found in Goyal and Giri, 2001 and Bakker et al., 2012. At the beginning, the deterioration process was considered to be of instantaneous type, *i.e.* items are subject to deterioration from the moment they are produced. Wu et al., 2006 first introduced the idea of 'non-instantaneous' deterioration. Hsu et al., 2010 assumed a constant cost estimate for preservation technology which was independent of the cycle-length. Dye and Hsieh, 2012 proposed it to be dependent on the length of the replenishment cycle.

At the earlier stages, it was assumed that the preservation technology investment controls only the rate of deterioration and not the non-deterioration period, which may be found in Hsieh and Dye, 2013 and Mishra et al., 2017, to name a few. It was Li et al., 2019 who introduced the idea of delaying the non-deterioration period with the preservation technology investment and maximized the profit function. Bardhan et al., 2019 developed a model on deteriorating inventory with preservation technology investment. Khanna et al., 2020 proposed a cost minimization model by finding the optimal investment in preservation technology and optimal cycle length. Ahmad and Benkherouf, 2020 discussed how the deterioration rate impacts the optimal policies. Mishra et al., 2021 considered controllable deterioration in production inventory and developed their work to find an optimal production policy. Mahata and Debnath, 2022 worked on maximizing the profit level considering the deterioration to be an increasing function of preservation technology investment.

2.2 Lead time reduction

Gross and Soriano, 1969 examined lead time factors in inventory management. Since then, several notable contributions have been made in the field (Tatsiopoulos and Kingsman, 1983, Liao and Shyu, 1991, Pan and Yang, 2002, Tang et al., 2007, Jha and Shanker, 2013, Bandaly et al., 2016). Tiwari et al., 2018 studied a production-inventory model assuming order processing costs reduction to be of logarithmic type. They developed a joint optimization model with stochastic demand and controllable lead time by reducing ordering and setup costs. Chang and Lin, 2019 analyzed the effect of lead time on supply chain resilience performance. Tharani and Uthayakumar, 2020 evaluated the effect of lead time reduction within an integrated vendor-buyer supply chain. Sarkar et al., 2021 discussed a sustainable online-to-offline retailing strategy for supply chain management with controllable lead time. Dey et al., 2021 worked on smart manufacturing systems and discussed the characteristics of marginal lead time for both deterministic and variable cases, and also discussed how the quality improvement process helps to reduce defective production. Yadav et al., 2022 compared centralized and decentralized models with several constraints, and optimized the order quantity, reorder point, lead time, and number of shipments taking the learning-forgetting phenomenon into consideration. Barman and Mahata, 2022 studied a two-echelon integrated supply chain model with stochastic demand where they considered controllable lead time to depend on order quantity. Barron, 2023 elaborated an inventory control problem with two storage facilities: a primary warehouse (PW) with limited capacity and a subsidiary warehouse (SW) with larger capacity. The PW was managed using a triple-parameter band policy (M, S, s) under the assumption of lost sales and random lead time. They considered the refilling of inventory whenever the stock fell below s. However, although most of the literature with controllable lead time discussed the uncertainty of the demand factor, none of them preferred to opt the strategies in dynamic form.

2.3 Warranty policy

After the introduction of warranty policy for demand improvement in Glickman and Berger, 1976, it has been widely studied by many researchers in various inventory models. Chen et al., 2012 were the first to introduce the idea in a supply chain framework to derive the optimal pricing strategy for the manufacturer where the retailer's demand competitively depends on the warranty period which includes free sales service. Yazdian et al., 2016 developed a supply chain model considering a warranty period dependent demand with free repairing during warranty section. They assumed that the products are returned to the manufacturer after expiry- the return rate being random- and are remanufactured. Alqahtani and Gupta, 2017 analyzed how the renewing warranty policy would impact the remanufactured inventory items. Taleizadeh et al., 2019 and Sarada and Sangeetha, 2022 developed reverse supply chain models with price and warranty dependent demand where the return rate of defective products are stochastic in nature, and remanufacturing is subject to random yield. Keshavarz-Ghorbani and Arshadi Khamseh, 2022 developed a multi-period closed-loop supply chain model with price and warranty dependent demand where they provided free repairing service during warranty period and remanufacturing for the collected used items.

2.4 Learning effect

Learning may be defined as an improvement in quality or performance over time without any change in investment (Wright, 1936). Jaber et al., 2008 assumed 'S' shaped learning curve that reduces number of defective items per lot. Khan et al., 2010 constructed an inventory model with a screening rate in which the effect of learning determined the production cost. Konstantaras et al., 2012 applied learning in the screening process of an inventory model. Kumar et al., 2015 used learning effect to reduce ordering cost. Aggarwal et al., 2017 used learning to reduce holding costs in a perishable inventory model that allows for shortages with partial backlog. Shah and Naik, 2018 extended the learning application to reduce both ordering and holding cost for the inventory. Afshari et al., 2019 studied a supply chain model to investigate the effect of learning and forgetting on the feasibility of adopting additive manufacturing. Jayaswal et al., 2019 discussed an EOQ (economic order quantity) model with perishable items considering learning to reduce holding and ordering costs as well as to screen out defective items, whereas Jayaswal et al., 2021b considered demand to depend on credit period and learning effect for deterioration cost. Jayaswal et al., 2021a discussed the effect of learning and credit financing on lot size and the corresponding costs in a fuzzy environment. Considering learning effect and carbon emission, Alamri et al., 2022 developed an EOQ model under inflationary circumstances. Ye et al., 2022 analyzed a PC supply chain and applied demand learning to find out the optimal ordering quantity and established integrated decision-making model for dynamic pricing and inventory control. Yadav et al., 2022 applied learning-forgetting phenomena on setup cost in a two-echelon supply chain and found out the ideal order quantity, lead time, reorder point, and shipment quantity values that maximize the profit level.

2.5 Shortages and backlog

There are plenty of research elaborating the shortage and backlogging. At the beginning, the researchers like Park, 1982, Hollier and Mak, 1983, Padmanabhan and Vrat, 1995 considered the backlogging to be constant only. Chang and Dye, 1999 were the first to consider the partial backlogging to depend on the waiting time of the customers. Recent research has focused on applying shortage and backlog models to real-world scenarios, including multi-echelon supply chains and closed-loop systems. Singh and Saxena, 2013 developed a closed-loop supply chain model incorporating flexible manufacturing and reverse logistics for deteriorating items. It considers two quality standards where shortages are fully backlogged, and remanufactured products are sold in a secondary market at lower prices. Lin, 2017 developed a production–inventory model with partial backlogging, in which a reflected Brownian motion governs the inventory level variation. Pervin et al., 2018 developed a deterministic inventory model for deteriorating items, incorporating stochastic Weibull distribution for deterioration and time-dependent demand. The model was formulated to optimize replenishment decisions while considering time-dependent holding costs and shortages. Lai et al., 2019 developed an economic production quantity (EPQ) model for imperfect manufacturing systems, incorporating hybrid maintenance policies that combined emergency and preventive maintenance. It optimized production quantity and maintenance inspection frequency to minimize the expected average cost, considering machine reliability, quality uncertainty, and shortages with partial backlogging. Das et al., 2020 developed an non-instantaneous deteriorating inventory model with price dependent demand, allowing shortages with two different partial backlogging rates. Khan et al., 2021 developed a profit-maximizing EOQ model under hybrid prepayment and delay payment schemes, addressing limited retailer warehouse capacity. Considering the shortage with partial backlogging during stock-out periods, they determined the optimal ordering and replenishment policies. Adak and Mahapatra, 2022 developed a multi-item EOQ model where deterioration depends on time and reliability, and demand is influenced by advertisement, time, and reliability. It incorporates reliability-dependent ordering costs and partially backlogged shortages. Mondal et al., 2023 analyzed two cases involving advance payments with and without discounts in a partially backlogged inventory model under interval uncertainty, where products deteriorated during stock-in periods. Using the centre-radius optimization technique and Quantum-behaved Particle Swarm Optimization (QPSO) variants, the study optimized inventory policies under imprecise inventory parameters. Modak et al., 2024a applied dynamic discount policy to optimize the shortage period of an inventory model based on dynamic pricing strategy with the consideration of shortages with partial backlogging. Bardhan et al., 2025 established the benefit of planned shortage in a multi-period inventory model with price, time and service dependent demand with the consideration of waiting time dependent partial backlogging.

2.6 Green supply chain

Murphy and Poist, 2003 were the first to identify the lack of consistent framework for greening practices. Zhu et al., 2005 identified that the green method include selling leftover stock, scrap and used materials, conducting environmental audits, obtaining commitment from top management, and implementing complete quality environment management. A detailed review in this topic could be found in Srivastava, 2007. Focusing on literature which are relevant to the thesis, Wangsa, 2017 applied the greening cost to a stochastic demand model and minimized the overall cost of the system while taking into account both the transportation and industrial greenhouse gas emissions using both the penalty and incentive policies. Saga et al., 2019 further extended the model by including imperfect production with inspection policy and service level constraint. Rout et al., 2020 considered the carbon cap-and-offset policy along with the carbon cap and trade & carbon tax policy while developing a single-manufacturer single-buyer integrated model with simultaneous deterioration and imperfect production. Ramandi and Bafruei, 2020 studied the effect of government policies on greenhouse gas emission in a two-echelon supply chain model with stochastic demand. Halat et al., 2021 illustrated the role of carbon tax policy to reduce the coalition cost and to improve the emission savings in a multi-echelon supply chain model. Karim and Nakade, 2021 studied the effect of carbon emission restriction on a decentralized supply chain with defective production model where both the demand and production are stochastic in nature. Asadkhani et al., 2022 implemented the carbon tax and carbon cap-and-trade policy in a vendor managed inventory model with withdrawal and consignment policy. Lu et al., 2022 discussed a global supply chain model for deteriorating items where they applied carbon tax regulation for the retailer and cap-and-trade policy for the manufacturer and vice versa. Malleeswaran and Uthayakumar, 2022 compared various carbon policies and established the superiority of the limited carbon emission policy over carbon cap-and-trade or carbon taxation policy. They further exhibited the superiority of consignment stock in terms of cost reduction over the traditional ones. Focusing on the textiles and fashion industries, John and Mishra, 2023a applied the carbon emission along with the water purification technology and green investment to discuss the environmental issue in more detailed manner. John and Mishra, 2023b used the carbon cap-and-trade policy and green technology to control the carbon emission rate and resource depletion in a circular economy concept for LED industries. Jauhari et al., 2023 developed an inventory model elaborating how the greening investment is necessary in conventional inventory model in presence of carbon tax regulation and showed how the hybrid model is superior to maximize the profit level. Li et al., 2024 explored closed-loop supply chains under cap-and-trade and carbon tax policies. Proposing a low-carbon circular system to balance supply chain costs and emissions, the study finds that combining remanufacturing and low-carbon investments can optimize costs and emissions, with a non-linear complementary relationship between carbon price and tax.

2.7 Closed-loop supply chain

After the pioneer work of Ayres et al., 1997, many researchers have worked on remanufactured inventory models and listed essential findings. Shekarian, 2020 reviewed various factors affecting the closed-loop supply chain (CLSC) policy while MahmoumGonbadi et al., 2021 comprehensively analyzed 254 articles on closed-loop supply hain design, assessing their alignment with circular economy principles and sustainability dimensions. Considering the manufacturer's fairness concerns and retailer's sales efforts, Jian et al., 2021 studied a green closed-loop supply chain under a Stackelberg game structure to find that fairness concerns impacts the environmental performance, sales efforts, and profits, while a profit-sharing contract improves coordination, fostering sustainable economic and environmental development. Luo et al., 2022 used game-theoretic models to evaluate the impact of carbon tax policies on manufacturing and remanufacturing in CLSCs. They found that carbon taxes encourage carbon reduction investments and remanufacturing, but poorly designed taxes can demotivate such efforts. They showed that the centralized CLSCs yield higher profits but may increase emissions under low taxes, while decentralized decisions depend on wholesale prices. Xu et al., 2023 proposed a two-stage stochastic model for designing a CLSC under a carbon trading scheme, addressing uncertain demands and carbon prices in a multi-period context. Applied to the aluminum industry, the model incorporated scenario reduction to optimize cost and emissions efficiently, achieving a near-optimal network configuration with significantly reduced computational effort. Abbasi and Erdebilli, 2023 optimized COVID-19 logistics management and green closed-loop supply chain design (GCLSCD) under three types of CO_2 regulatory restrictions. By balancing costs, emissions, location selection, and shipment options, the proposed models analyzed the impact of policies on supply chain efficiency and provide insights for managers to predict regulatory effects on emissions and costs. Yu et al., 2024 developed differential game models for a dual-channel closed-loop supply chain (DCCLSC) under non-cooperation, partial cooperation, and full cooperation scenarios. It analyzed recycling and pricing decisions dynamically, proposed a cost-sharing contract for coordination, and examined the effects of government intervention and inventory management. Gaula and Jha, 2024 examined a two-period CLSC model with dual collection channels under uncertainty in return quantities. Using a newsvendor framework and backward induction, it analyzed how acquisition and transfer prices impact the collection of used products, revealing the effects of competition between the manufacturer and a third party on pricing strategies and return quantities. Modak et al., 2025 developed an integrated CLSC model with both the manufacturing and remanufacturing to be random in nature. With the consideration of random pricing and different carbon emission level for manufacturing and remanufacturing, they optimized the pricing, production, remanufacturing and green policies to make the most out of the scenario.

2.8 Supply chain coordination

Supply chain coordination is essential for optimizing the performance of supply chain networks, where independent entities such as manufacturers, suppliers, and retailers work together to achieve shared objectives. Among the numerous mechanisms developed to enhance coordination, contracts play a pivotal role in aligning the incentives of supply chain participants. The supply chain contract issues thus have always been a key focus for the researchers. Two widely studied and applied contracts are the buyback contract and the revenue-sharing contract, both of which address specific challenges in supply chain coordination.

2.8.1 Buyback Contract

Padmanabhan and Png, 1995 demonstrated that a manufacturer can offer full credit for a partial return of goods to achieve channel coordination. Emmos and Gilbert, 1998 studied buyback contracts in a price sensitive market and found that they can improve supply chain performance under certain conditions. Giri and Bardhan, 2014 applied buyback contract in a supply chain model with a single retailer and a disruption-prone supplier to enhance profits, and coordinated the chain despite disruptions. Additionally, a backup supplier was introduced to mitigate disruption effects. Shi et al., 2020 examined the capital-constrained newsvendor problem in a supply chain finance (SCF) system where a manufacturer offers a buyback contract to mitigate lender risks from retailer defaults. They analyzed a three-level Stackelberg game under monopolistic and competitive bank markets and showed that the combination of buyback and wholesale price contracts fully coordinates the SCF system. The study also evaluates conditional buyback contracts and compares them with partial credit guarantee contracts, highlighting their substitutability. Salami et al., 2022 analyzed a two-echelon reverse supply chain involving a remanufacturer and a collector, focusing on buyback contracts to coordinate the chain under uncertainties in remanufacturing capacity and product quality. Momeni et al., 2022 proposed a buy-back mechanism to incentivize cooperation between a retailer and manufacturer with the circular economy's zero-waste concept. They suggested strategy integrating technological capabilities and supply chain coordination to reuse expired products in new applications. Adnan and Özelkan, 2023 studied how the buyback contract impacts the price variability of a supply chain model. Tian et al., 2024 combined buyback contract with partial credit guarantees and trade credit to propose two financing schemes- a partial credit guaranteed buyback (PCGB) and trade credit buyback (TCB)- for supply chains with high salvage values. It analyzed supply chain coordination, identified optimal strategies for each scheme, and highlighted the manufacturer's choice based on credit guarantee coefficients, buyback prices, and financing rates emphasizing the risk-sharing mechanisms for flexible supply chain coordination.

2.8.2 Revenue Sharing Contract

Researchers have studied revenue sharing contract in great detail since its introduction in the literature by Cachon and Lariviere, 2005. Chen et al., 2022 developed a multi-channel supply chain model with revenue sharing contract, dynamic retail price and after sales service where the customer's trust was the focusing issue. Liu et al., 2022 analyzed a dual channel supply chain model with a service factor in demand. Other than the servicing factor, they considered the price of both the traditional and e-commerce supply chain model to impact each other. Samanta et al., 2023 discussed a multi-period vendor-buyer supply chain model with price, greening level, and warranty period-dependent demand. Following an in-depth analysis of the centralized and decentralized scenarios, a hybrid greening cost and revenue sharing contract was implemented to achieve mutually beneficial outcomes for both vendor and buyer.

2.9 Discrete dynamic pricing

Although static pricing policy has been widely accepted by researchers for decades, after Rajan et al., 1992, many researchers have so far worked on dynamic pricing policy as well. Two possible choices of dynamic pricing have been adopted by the researchers - one being continuously changing price over time, while the other one being adopted in multi-period models where price is different in different periods but static during one particular period. Monahan et al., 2004, Lin Lin, 2006, Yang and Zhang, 2014, and Huang et al., 2014 are some of the early researchers to apply discrete dynamic pricing policy in inventory models. Pang, 2011 studied the optimal discrete dynamic pricing and inventory policies with deteriorating inventory and stock dependent demand. Dye and Yang, 2016 addressed joint discrete dynamic pricing, replenishment, and preservation investment problems for a deteriorating item considering time and price dependent demand. Hsieh and Dye, 2017 and Chen et al., 2018 studied discrete dynamic pricing models with the effect of menu cost and reference price. Dye, 2020 studied a multi-period inventory model with price, advertising and psychic stock effects to find the most suitable discrete pricing, advertising, and psychic stock strategies. Prakash and Spann, 2022 studied a discrete dynamic pricing model with reference price effect, and found that customers react to price rises more strongly than price decreases. Li and Mizuno, 2022 developed a two-echelon multi-period supply chain model that incorporates stochastic demand. They examined the efficacy of static and dynamic pricing policies adopted by the retailer and manufacturer, respectively. They also analyzed various scenarios where the two entities adopted distinct strategies. Sana, 2020 conducted a study on the newsvendor inventory model within the framework of corporate social responsibility and examined the impact of a tax subsidy provided by the government on green products as a motivational factor. The demand function was postulated to be dependent upon the price, level of environmental sustainability, and corporate social responsibility index of the commodity. Given the possible scenario of price competition between green and non-green products, the author aimed to optimize the overall profit of the system.

2.10 Continuous dynamic pricing

To the best of the authors' knowledge, Evans, 1924 was the first to introduce the concept of dynamic pricing. Rajan et al., 1992, Petruzzi and Dada, 2002, and Adenso-Díaz et al., 2017 are some of the researchers who considered the continuous pricing model. Keller et al., 2022 studied the effect of dynamic pricing on customers, and discussed the importance of discounting to alleviate the negative effect of it. Zhang et al., 2015 applied the concept in inventory modeling and explored a model of perishable items with a constrained capacity for replenishment, and developed optimal joint dynamic pricing and replenishment strategy using Pontryagin's maximum principle to solve the optimization problem. Liu et al., 2015 determined the optimal dynamic pricing and preservation investment strategy for a perishable inventory model where the demand depends both on price and quality, and the preservation not only reduces the deterioration rate but also keeps the product fresher for a longer time. Feng, 2019 considered both quantitative as well as qualitative deteriorations to find out the optimal dynamic pricing strategy for the inventory model where the demand is both price and quality-sensitive. Keeping the business dynamics of agricultural products in mind, Lu et al., 2019 examined the optimal dynamic pricing and other strategies such as replenishment cycle length, replenishment quantity, and preservation investment that would maximize the profit of the firm. Fan et al., 2020 considered a multi-batch model for freshly produced items and established that order quantity depends solely on the freshness level with the higher freshness of the product, while it depends both on the freshness and inventory level when the freshness is lower than a certain threshold. Zhang et al., 2023 considered a firm with multiple stores and limited inventory for a perishable item under optimal dynamic pricing strategy together with the shipment consolidation policies, and established the non-monotonicity of optimal dynamic price with respect to deterioration rate and holding cost. Li et al., 2023 elaborated the correlation of demand of two items with a dedicated and a shared flexible production source for each with the consideration of lost sales. Modak et al., 2024b developed a continuous dynamic pricing model with price, freshness, and greening sensitive demand with the consideration of both the qualitative and quantitative deterioration.

2.11 Random pricing

Despite the fact that stochastic demand patterns have been extensively researched to far, there aren't many articles about random selling prices. Sana, 2011 was the first to estimate the optimal number of orders in the event of a shortage and to consider randomness in price in inventory modelling. The ideal sales price and expected ordering quantity were obtained by Sana, 2012 where he extended the newsboy problem by considering random sales price. Sodhi et al., 2014 exhibited that stochastic price variation in an EOQ model has a bullwhip effect on the manufacturer. In order to determine the ideal selling price and production quantity, Das Roy and Sana, 2017 considered an imperfect production inventory model with price-sensitive demand, taking into account a uniformly distributed price and a production rate change from an in-control to an out-of-control state.

Concluding remarks

In conclusion, the literature review in this chapter highlights the significant advancements in inventory and supply chain management, particularly in areas such as dynamic pricing, lead time optimization, preservation strategies, and sustainability initiatives—topics directly relevant to the focus of this thesis. Research on closed-loop supply chains and reverse logistics has also underscored their critical role in addressing environmental challenges and fostering circular economies. However, despite these advancements, key challenges such as managing uncertainty, integrating advanced technologies, and coordinating reverse logistics with forward operations remain insufficiently explored. This thesis aims to address these gaps by presenting comprehensive models that integrate pricing strategies, inventory policies, and sustainable practices. By examining these elements within both traditional and closed-loop supply chain contexts, the research seeks to provide actionable insights for enhancing operational efficiency and fostering sustainable development in both academic and industrial settings.

CHAPTER 3

Pricing Strategies in Perishable Inventory Management: Leveraging Learning Effects and Preservation Investments

This chapter highlights the critical role of pricing strategies in managing perishable inventory, building on the concepts of deterioration and preservation discussed earlier. While preservation efforts reduce deterioration and extend product life, effective pricing strategies address the dual challenges of inventory losses and fluctuating consumer demand in a competitive market. Aligning pricing with preservation investments ensures optimal revenue, minimizes waste, and balances supply and demand dynamics.

Pricing significantly influences demand by shaping affordability and perceived value, while interacting with factors like product quality, time sensitivity, innovation, and customer service. Mispricing can lead to overstocking or understocking, compounding inventory challenges. Time is another significant factor, even when other parameters remain constant, prompting business managers to realign their strategies with time in a long-term business context, making the scenario inherently dynamic. When a new product is introduced, it is seen that its market share grows first, eventually reaching a peak, and then the demand gradually declines as similar products with improved features arrive. Ghosh and Chaudhuri, 2006 categorized how quadratic time dependent form represents the standard life cycle of wide range of products. To remain competitive, firms must also incorporate demand-enhancing measures such as promotions, discounts, and after-sales services to boost customer satisfaction and loyalty. Additionally, modern consumers are increasingly drawn to discounts, and the importance of pre-sale promotions and after-sales services has become pivotal in sustaining demand and building customer trust.

The main concept of this chapter is a multi-period perishable inventory model where demand is influenced by price, time, and service level. The model features equal-length replenishment cycles with varying stock-in and stock-out periods to account for the time-dependent nature of demand. A planned shortage policy followed by replenishment is adopted, demonstrating its effectiveness. It also incorporates investments in preservation technology to reduce spoilage and emphasizes the role of after-sales service in boosting demand. Additionally, the learning effect is considered, reducing holding and ordering costs over time and improving operational efficiency. This integrated approach provides a robust framework for optimizing perishable inventory management in dynamic market conditions. The basic schematic diagram of the models is given by the Figure 3.1.

This chapter is divided into two parts. The first part addresses static pricing, focusing on fixed-price strategies for stock-in and stock-out scenarios. The second part explores discrete dynamic pricing and discounting, incorporating time-sensitive adjustments to stimulate demand and reduce waste. Together, these models provide a comprehensive framework for balancing profitability, demand, and effective inventory management, forming a vital part of this thesis. A key factor underlying in both parts of this chapter is the improvement in service level driven by the learning

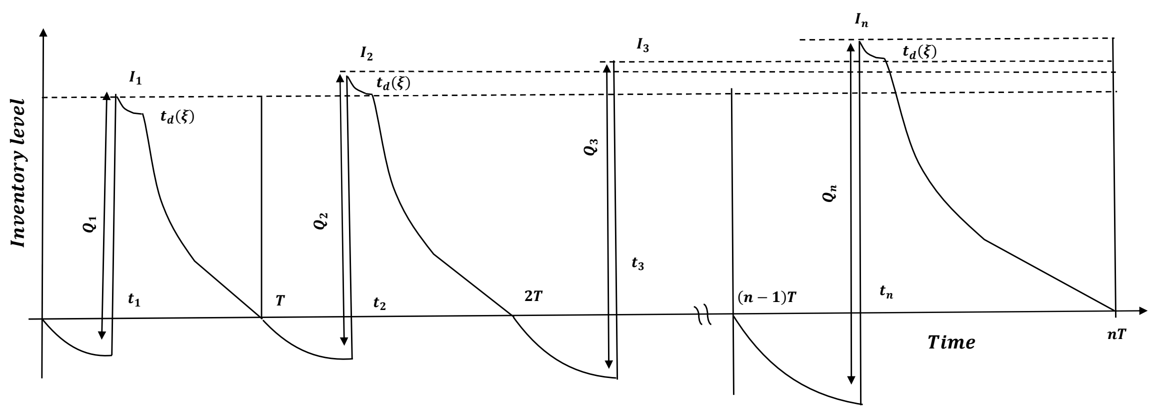


Figure 3.1: Schematic diagram of the inventory system

effect, which subsequently leads to an increase in demand, as illustrated in the Figure 3.2.

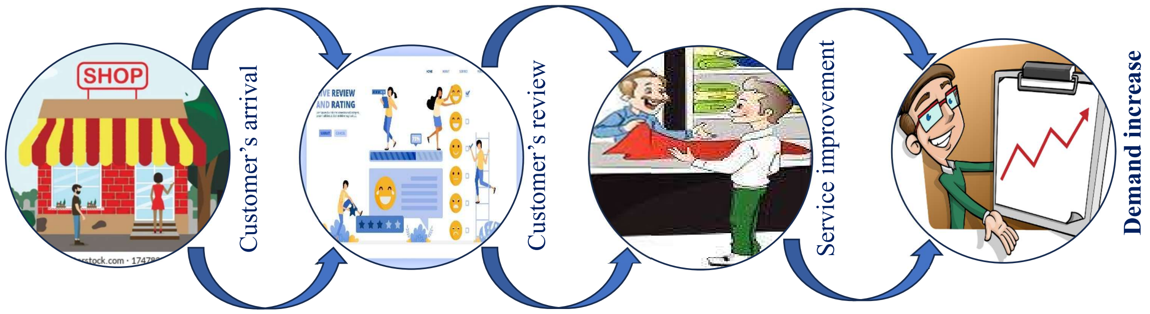


Figure 3.2: Service improvement and demand improvement

The common notations used in this chapter are tabulated below.

Table 3.1: Notations

n	:	number of replenishment periods (decision variable)
p	:	unit selling price (decision variable)
s_0	:	service infrastructure at initial level, $0 \leqslant s_0 \leqslant 1$, depending directly on service investment (decision variable)
ξ	:	preservation technology investment per unit time (decision variable)
t_i	:	replenishment time point in i^{th} cycle, $i = 1, 2, \cdots, n$ (decision variable)
T	:	length of one cycle
H	:	time horizon, $H = nT$
s_i	:	service level at i^{th} cycle, $i = 1, 2, \cdots, n$, $(0 \leqslant s_i \leqslant 1)$
t	:	time
$D(p, s_i, t)$	:	demand rate during i^{th} replenishment period, $i = 1, 2, \cdots, n$
c_1	:	per unit purchasing cost
$t_d(\xi)$	:	non-deterioration period at preservation technology investment ξ

$B(x)$	:	backlogging rate, where x is the waiting time to the next replenishment
θ	:	constant deterioration rate per unit time, $0 \leq \theta < 1$
$m(\xi)$	:	proportion of reduced deterioration rate with preservation investment, $0 \leq m(\xi) < 1$
$I(t)$	:	inventory level at any time t, $t \in [0, nT]$
Q_i	:	ordering quantity at the i^{th} replenishment, $i = 1, 2, \cdots, n$
A_1	:	constant component of the ordering cost
$\frac{A_2}{i^{\alpha_1}}$	:	variable component of the ordering cost, decreasing in each cycle due to learning effect
h_1	:	constant component of holding cost
$\frac{h_2}{i^{\alpha_2}}$	:	variable component of holding cost which decreases in each cycle due to learning effect
c_s	:	shortage cost
c_d	:	disposal cost of deteriorated items
α	:	operating efficiency of the staffs
B_i	:	total sold item in i^{th} period, $i = 1, 2, \cdots, n$
S_i	:	Total backlog in i^{th} period, $i = 1, 2, \cdots, n$
k	:	parameter associated with the cost per unit service level

The common assumptions of this chapter are listed as follows.

- The entire business horizon H is divided into n equal shipment intervals, each being of time length T. Each interval starts with planned shortages; the backlog is mitigated and storage is filled at time points t_i, $i = 1, 2, \cdots, n$.
- Combining the demand patterns proposed by Khanra et al., 2011, Shah et al., 2016, Li et al., 2019, and Sarkar and Pal, 2022, the demand rate in the i^{th} interval is assumed to be of the form $D(p, s_i, t) = a + bt - ct^2 - \beta p + \delta s_i$, which is a function of time, selling price and service level; $a > 0$ denotes the total market potential demand, $b > 0$ and $c > 0$ denote respectively the linear and quadratic rates of change of demand with respect to time. As pointed out by

Sarkar et al., 2012, the distinct advantage of time-dependent quadratic demand function is that it accommodates increasing, decreasing and constant patterns depending upon its parameter values. The parameter-values chosen here indicate that the demand increases with time at initial levels, but subsequently falls down. The condition $a + bH - cH^2 > 0$ is imposed to ensure that business does not collapse within its horizon. It is worth mentioning that demand eventually reaches zero at some point of time, indicating that the product has become completely obsolete or useless thereafter. The parameters $\beta > 0$ and $\delta > 0$ indicate sensitivity towards price and service level, respectively.

- Following Li et al., 2019, this chapter assumes that the preservation technology investment not only mitigates the deterioration rate but also delays deterioration. The inventory system involves single non-instantaneous deteriorating items. $t_d(\xi)$ is assumed to be an increasing function of preservation technology investment ξ. There is no repair or replacement of the deteriorated inventory. The authors use $t_d(\xi)$ and t_d interchangeably, when no confusion arises.

- As is seen, a higher investment in preservation ensures more reduction in deterioration rate. However, deterioration can not be completely eliminated in any way, *i.e.* no finite amount of investment is able to wipe out the effect of deterioration completely. Hence, the relationship between preservation investment and reduction in deterioration has to be such that when the investment tends to be very large, the deterioration rate tends to be very small, near zero. A concave pattern of the reduction function justifies this behavior. Further, the concave pattern ensures the existence of an optimal level of preservation investment. Hence, the proportion of reduced deterioration rate $m(\xi)$ is assumed to be a continuous, concave, increasing function of preservation investment ξ, with $m(0) = 0$ and $\lim_{\xi \to \infty} m(\xi) = 1$.

- The warehouse is of infinite capacity, and lead time is constant, so that without any loss of generality it is assumed to be zero. If the lead time is not zero, the retailer just needs to place an order in advance by a period of the lead time and the result holds.

- Shortages are allowed and partially backlogged. The fraction of shortages back-ordered is a decreasing function $B(x)$, where x is the waiting time up to the next replenishment, $0 \leqslant B(x) \leqslant 1$ with $B(0) = 1$. To guarantee the existence of an optimal solution, it is assumed that $B(x) + HB'(x) \geq 0$, where $B'(x)$ is the first derivative of $B(x)$. This chapter follows Pal et al., 2017 and Pal and Adhikari, 2019 to consider the specific form as $B(t_i - t) = e^{-\eta(t_i - t)}$.

- In an everchanging business world, no matter how much one learn about service management that can never be considered as complete learning. The market, customers and even the concept of 'best service' is always transforming. But with time, one get accustomed to those changes somewhat. Keeping all the arguments into consideration, the service level at i^{th} cycle is defined as $s_i = (s_0)^{(\alpha+1)/i}$ (Jaber, 2006), α being a parameter. Learning effect also helps to reduce the ordering and holding costs significantly.

3.1 Pricing Strategy for a Perishable Inventory Model with Shortages

3.1.1 Introduction

This part focuses on developing a model to address the pricing and inventory management challenges for newly launched products, particularly edible and electronic goods commonly sold in supermarkets. These products exhibit a unique demand pattern: demand initially increases after launch due to market excitement but eventually declines as newer models with improved features become available. This demand behavior is best captured using a time-quadratic pattern.

Service levels, encompassing customer handling, feedback implementation, and employee training, improve over time due to the learning effect. This effect, depicted in Figure 3.2, enhances demand by increasing customer satisfaction and trust in the product. Customers often favor products that include after-sales support, especially for technical goods. Recognizing the interplay of these factors, the model in this part considers demand as dependent on time, price, and service level. While the demand structure shares similarities with Shah et al., 2016, this work introduces key extensions, such as linear price dependency and the inclusion of service-level effects, allowing the model to capture the dynamics of a broader range of products like fashion goods and high-tech accessories. These goods often require preservation for quality maintenance and benefit maximization. Additionally, the proposed model reflects realistic business scenarios, such as advance bookings prior to product availability and multi-periodic inventory cycles. Unlike single-period models, a multi-period approach considers the economic implications of spreading inventory replenishment across cycles to avoid excessive deterioration costs. The learning effect, which enhances service levels and reduces costs with repeated cycles, is a central feature of this approach. Improved service levels in consecutive periods stimulate demand, counteracting the negative impacts of price sensitivity and time decay. This part also considers how time-varying demand influences replenishment timing and ordering quantity, demonstrating the importance of adaptive strategies for inventory management. This chapter tries to answer the following research questions:

*This part of the chapter is based on the work published in *European Journal of Industrial Engineering* (2025), volume 19, issue 1, pages 18-44.

RQ1: How does the integration of preservation technology investment impact the optimal pricing and replenishment policies in a multi-period inventory system with time-dependent demand?
RQ2: What is the role of the learning effect in reducing ordering and holding costs, and how does it influence service level improvement over multiple replenishment cycles?
RQ3: How do capital and warehouse space constraints affect managerial decisions regarding price, service level investment, and replenishment strategies in a dynamic inventory environment?
To the best of our knowledge, this is the first attempt to optimize a multi-period pricing, service, and preservation investment problem under learning effects and constraints. Furthermore, two realistic extensions, incorporating space and capital constraints-are presented to guide managerial strategies. Given the limited shelf space in supermarkets, space constraints are critical, and capital limitations further affect inventory decisions. By addressing these elements, the model provides a comprehensive framework for effective decision-making in dynamic market environments.

3.1.2 Model Formulation

We now develop the proposed model in accordance with the assumptions stated in the previous section. A schematic diagram of the inventory system is depicted in Figure 3.1. The changes in inventory level at any time point t during the interval $[(i-1)T, iT]$ is given by

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\eta(t_i-t)}D(p,s_i,t) & \text{with } I((i-1)T)=0 \text{ and } I(t_i)=-S_i, \\ & (i-1)T \leqslant t \leqslant t_i \\ -D(p,s_i,t) & \text{with } I(t_i)=Q_i-S_i, \quad t_i < t \leqslant t_i+t_d \\ -D(p,s_i,t)-(1-m(\xi))\theta I(t) & \text{with } I(iT)=0, \ \ t_i+t_d < t \leqslant iT. \end{cases} \tag{3.1}$$

Solving the equations with the given initial and continuity conditions on inventory and the fact that Q_i amount is replenished at time point t_i, the inventory level during the entire period may be obtained.
Solving the first equation, we get

$I(t) = -\frac{e^{-\eta(t_i-t)}}{\eta}\left((a+bt-ct^2-\beta p+\delta s_i)-\frac{(b-2ct)}{\eta}-\frac{2c}{\eta^2}\right)+k_1$, where the value of k_1 may be derived from the condition $I((i-1)T) = 0$ as $k_1 = \frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\left(\left(a+b(i-1)T-c((i-1)T)^2-\beta p+\delta s_i\right)-\frac{(b-2c(i-1)T)}{\eta}-\frac{2c}{\eta^2}\right)$. The inventory level can thus be written as

$$\begin{aligned} I(t) &= -\frac{e^{-\eta(t_i-t)}}{\eta}\left\{(a+bt-ct^2-\beta p+\delta s_i)-\frac{(b-2ct)}{\eta}-\frac{2c}{\eta^2}\right\}+\frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\times \\ &\quad \left\{a+b(i-1)T-c((i-1)T)^2-\beta p+\delta s_i-\frac{b-2c(i-1)T}{\eta}-\frac{2c}{\eta^2}\right\}. \end{aligned}$$

Putting $I(t_i) = -S_i$, the total backlogged amount is obtained and is given in the equation 3.3.

The replenishment occurs at time $t = t_i$, so the inventory level at time t_i would be $Q_i - S_i$.

The second equation is now solved using $Q_i - S_i$ as the initial inventory level to obtain
$I(t) = Q_i - S_i + at_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - \beta pt_i + \delta s_i t_i - at - \frac{bt^2}{2} + \frac{ct^3}{3} + \beta pt - \delta s_i t.$
Using this, the inventory level at time point $t_i + t_d$ is obtained as

$$\begin{aligned} I(t_i+t_d) &= Q_i - S_i + at_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - \beta pt_i + \delta s_i t_i - a(t_i+t_d) - \frac{b(t_i+t_d)^2}{2} \\ &\quad + \frac{c(t_i+t_d)^3}{3} + \beta p(t_i+t_d) - \delta s_i(t_i+t_d), \end{aligned}$$

which is further used as initial condition for the third differential equation, from which direct calculation yields the inventory level

$$I(t) = -\frac{(a+bt-ct^2-\beta p+\delta s_i)}{(1-m)\theta}+\frac{(b-2ct)}{(1-m)^2\theta^2}+\frac{2c}{(1-m)^3\theta^3}+c_3 e^{-(1-m)\theta t},$$

and the initial condition $I(t_i + t_d)$ specifies the value of c_3 as

$$c_3 = \left(Q_i - S_i + at_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - \beta pt_i + \delta s_i t_i - a(t_i+t_d) - \frac{b(t_i+t_d)^2}{2} + \frac{c(t_i+t_d)^3}{3}\right.$$

$$+\beta p(t_i+t_d)-\delta s_i(t_i+t_d)+\frac{(a+b(t_i+t_d)-c(t_i+t_d)^2-\beta p+\delta s_i)}{(1-m)\theta}$$
$$-\frac{(b-2c(t_i+t_d))}{(1-m)^2\theta^2}-\frac{2c}{(1-m)^3\theta^3}\bigg)e^{(1-m)\theta(t_i+t_d)}.$$

The inventory level during the period $[t_d, T]$ can now be written as

$$\begin{aligned}I(t) &= \bigg(Q_i-S_i+at_i+\frac{bt_i^2}{2}-\frac{ct_i^3}{3}-\beta pt_i+\delta s_it_i-a(t_i+t_d)-\frac{b(t_i+t_d)^2}{2}\\&+\frac{c(t_i+t_d)^3}{3}+\beta p(t_i+t_d)-\delta s_i(t_i+t_d)-\frac{(b-2c(t_i+t_d))}{(1-m)^2\theta^2}\\&+\frac{(a+b(t_i+t_d)-c(t_i+t_d)^2-\beta p+\delta s_i)}{(1-m)\theta}-\frac{2c}{(1-m)^3\theta^3}\bigg)e^{-(1-m)\theta(t-(t_i+t_d))}\\&-\frac{(a+bt-ct^2-\beta p+\delta s_i)}{(1-m)\theta}+\frac{(b-2ct)}{(1-m)^2\theta^2}+\frac{2c}{(1-m)^3\theta^3}.\end{aligned}$$

The inventory level during the period $[(i-1)T, iT]$ is given by

$$I(t)=\begin{cases}-\frac{e^{-\eta(t_i-t)}}{\eta}\left\{a+bt-ct^2-\beta p+\delta s_i-\frac{(b-2ct)}{\eta}-\frac{2c}{\eta^2}\right\}+\frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\times\\ \left\{a+b(i-1)T-c((i-1)T)^2-\beta p+\delta s_i-\frac{b-2c(i-1)T}{\eta}-\frac{2c}{\eta^2}\right\}, \\ \qquad (i-1)T\leqslant t\leqslant t_i,\\ Q_i-S_i+at_i+\frac{bt_i^2}{2}-\frac{ct_i^3}{3}-\beta pt_i+\delta s_it_i-at-\frac{bt^2}{2}+\frac{ct^3}{3}+\beta pt-\delta s_it,\\ \qquad t_i<t\le(t_i+t_d),\\ \bigg(Q_i-S_i+\frac{bt_i^2}{2}-\frac{ct_i^3}{3}-at_d-\frac{b(t_i+t_d)^2}{2}+\frac{c(t_i+t_d)^3}{3}+\beta pt_d-\delta s_it_d\\ +\frac{(a+b(t_i+t_d)-c(t_i+t_d)^2-\beta p+\delta s_i)}{(1-m)\theta}-\frac{(b-2c(t_i+t_d))}{(1-m)^2\theta^2}-\frac{2c}{(1-m)^3\theta^3}\bigg)e^{-(1-m)\theta(t-(t_i+t_d))}\\ -\frac{(a+bt-ct^2-\beta p+\delta s_i)}{(1-m)\theta}+\frac{(b-2ct)}{(1-m)^2\theta^2}+\frac{2c}{(1-m)^3\theta^3}, \qquad t_i+t_d<t\leqslant iT.\end{cases} \tag{3.2}$$

We further derive the following relations:

$$\begin{aligned} S_i &= \frac{1}{\eta}\left\{a + bt_i - ct_i^2 - \beta p + \delta s_i - \frac{(b-2ct_i)}{\eta} - \frac{2c}{\eta^2}\right\} - \frac{e^{-\eta(t_i-(i-1)T)}}{\eta} \times \\ &\quad \left\{a + b(i-1)T - c((i-1)T)^2 - \beta p + \delta s_i - \frac{b-2c(i-1)T}{\eta} - \frac{2c}{\eta^2}\right\}, \quad (3.3) \end{aligned}$$

for $i = 1, 2, \cdots, n$, and from $I(iT) = 0$ and $H = nT$,

$$\begin{aligned} Q_i &= S_i - \frac{bt_i^2}{2} + \frac{ct_i^3}{3} + at_d + \frac{b(t_i+t_d)^2}{2} - \frac{c(t_i+t_d)^3}{3} - \beta pt_d + \delta s_i t_d + \frac{2c}{(1-m)^3\theta^3} \\ &\quad - \frac{(a + b(t_i+t_d) - c(t_i+t_d)^2 - \beta p + \delta s_i)}{(1-m)\theta} + \frac{(b - 2c(t_i+t_d))}{(1-m)^2\theta^2} \\ &\quad + \left(\frac{(a + biT - ci^2T^2 - \beta p + \delta s_i)}{(1-m)\theta} - \frac{(b-2ciT)}{(1-m)^2\theta^2} - \frac{2c}{(1-m)^3\theta^3}\right) \times \\ &\quad e^{(1-m)\theta(iT-(t_i+t_d))}. \quad (3.4) \end{aligned}$$

Now we derive the cost function in each interval. The total number of items sold during the period (t_i, iT) is given by $B_i = \int_{t_i}^{iT} D(p, s_i, t)dt = a(iT - t_i) + \frac{b}{2}(i^2T^2 - t_i^2) - \frac{c}{3}(i^3T^3 - t_i^3) - \beta p(iT - t_i) + \delta s_i(iT - t_i)$, which, combined with total backlog S_i generates total revenue (SR_i) in the i^{th} interval as $p(S_i + B_i)$.
The ordering cost is $(OC_i) = \left(A_1 + \frac{A_2}{i^{\alpha_1}}\right)$.
The holding cost in each interval is given by $(HC_i) = \left(h_1 + \frac{h_2}{i^{\alpha_2}}\right)\int_{t_i}^{iT} I(t)dt$

$$\begin{aligned} &= \left(h_1 + \frac{h_2}{i^{\alpha_2}}\right)\Bigg[\left(Q_i - S_i + at_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - \beta pt_i + \delta s_i t_i\right)t_d - \Bigg(\frac{a((t_i+t_d)^2 - t_i^2)}{2} \\ &\quad + \frac{b((t_i+t_d)^3 - t_i^3)}{6} - \frac{c((t_i+t_d)^4 - t_i^4)}{12} - \frac{\beta p((t_i+t_d)^2 - t_i^2)}{2} + \frac{\delta s_i((t_i+t_d)^2 - t_i^2)}{2}\Bigg) \\ &\quad + \Bigg(Q_i - S_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - at_d - \frac{b(t_i+t_d)^2}{2} + \frac{c(t_i+t_d)^3}{3} + \beta pt_d - \delta s_i t_d \\ &\quad - \frac{(b - 2c(t_i+t_d))}{(1-m)^2\theta^2} + \frac{(a + b(t_i+t_d) - c(t_i+t_d)^2 - \beta p + \delta s_i)}{(1-m)\theta} - \frac{2c}{(1-m)^3\theta^3}\Bigg) \times \\ &\quad \left(\frac{1 - e^{-(1-m)\theta(iT-(t_i+t_d))}}{(1-m)\theta}\right) - \frac{a(iT - (t_i+t_d))}{(1-m)\theta} - \frac{b(i^2T^2 - (t_i+t_d)^2)}{2(1-m)\theta} \end{aligned}$$

$$+\frac{c(i^3T^3-(t_i+t_d)^3)}{3(1-m)\theta}+\frac{\beta p(iT-(t_i+t_d))}{(1-m)\theta}-\frac{\delta s_i(iT-(t_i+t_d))}{(1-m)\theta}$$
$$+\frac{b(iT-(t_i+t_d))}{(1-m)^2\theta^2}-\frac{c(i^2T^2-(t_i+t_d)^2)}{(1-m)^2\theta^2}+\frac{2c}{(1-m)^3\theta^3}(iT-(t_i+t_d))\Bigg].$$

The costs related to preservation technology investment (PTC_i) and purchase (PC_i) are $\xi(iT-t_i)$ and c_1Q_i, respectively.
The deterioration cost (DC_i) may be obtained as

$$(c_d)(1-m)\theta\Bigg[\Bigg(Q_i-S_i+\frac{bt_i^2}{2}-\frac{ct_i^3}{3}-at_d-\frac{b(t_i+t_d)^2}{2}+\frac{c(t_i+t_d)^3}{3}+\beta pt_d-\delta s_it_d$$
$$-\frac{2c}{(1-m)^3\theta^3}+\frac{(a+b(t_i+t_d)-c(t_i+t_d)^2-\beta p+\delta s_i)}{(1-m)\theta}-\frac{(b-2c(t_i+t_d))}{(1-m)^2\theta^2}\Bigg)\times$$
$$\left(\frac{1-e^{-(1-m)\theta(iT-t_i-t_d)}}{(1-m)\theta}\right)-\frac{a(iT-(t_i+t_d))}{(1-m)\theta}-\frac{b(i^2T^2-(t_i+t_d)^2)}{2(1-m)\theta}$$
$$+\frac{c(i^3T^3-(t_i+t_d)^3)}{3(1-m)\theta}+\frac{\beta p(iT-(t_i+t_d))}{(1-m)\theta}-\frac{\delta s_i(iT-(t_i+t_d))}{(1-m)\theta}+\frac{b(iT-(t_i+t_d))}{(1-m)^2\theta^2}$$
$$-\frac{c(i^2T^2-(t_i+t_d)^2)}{(1-m)^2\theta^2}+\frac{2c}{(1-m)^3\theta^3}(iT-(t_i+t_d))\Bigg].$$

The shortage cost is derived as

$$\begin{aligned}SC_i &= c_s\Big\{a(t_i-(i-1)T)+\frac{b(t_i^2-((i-1)T)^2)}{2}-\frac{c(t_i^3-((i-1)T)^3)}{3}\\&\quad-\beta p(t_i-(i-1)T)+\delta s_i(t_i-(i-1)T)-S_i\Big\}.\end{aligned}$$

Noting that the service cost, *i.e.* investment in maintaining service level is an one time investment for the entire business period, and the service level will automatically be improved as the effect of learning, the total profit during the entire time horizon is given by

$$\Pi=\sum_{i=1}^{n}\Big(SR_i-(OC_i+HC_i+PTC_i+PC_i+DC_i+SC_i)\Big)-\frac{ks_0^2}{2}. \tag{3.5}$$

The aim is now to maximize the total profit with respect to the decision variables p, ξ, n, s_0, and $t_i, i = 1, 2, \cdots, n$, n being a positive integer, subject to the set of equations 3.3 and 3.4, $0 \leq t_i < iT$ and $t_d < iT - t_i$, $i = 1, 2, \cdots, n$. For each n, the necessary and sufficient condition for existence of optimal maxima is that the Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2\Pi}{\partial t_1^2} & \frac{\partial^2\Pi}{\partial t_1\partial t_2} & \cdots & \frac{\partial^2\Pi}{\partial t_1\partial t_n} & \frac{\partial^2\Pi}{\partial t_1\partial p} & \frac{\partial^2\Pi}{\partial t_1\partial s_0} & \frac{\partial^2\Pi}{\partial t_1\partial \xi} \\ \frac{\partial^2\Pi}{\partial t_1\partial t_2} & \frac{\partial^2\Pi}{\partial t_2^2} & \cdots & \frac{\partial^2\Pi}{\partial t_2\partial t_n} & \frac{\partial^2\Pi}{\partial t_2\partial p} & \frac{\partial^2\Pi}{\partial t_2\partial s_0} & \frac{\partial^2\Pi}{\partial t_2\partial \xi} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2\Pi}{\partial t_1\partial t_n} & \frac{\partial^2\Pi}{\partial t_1\partial t_n} & \cdots & \frac{\partial^2\Pi}{\partial t_n^2} & \frac{\partial^2\Pi}{\partial t_n\partial p} & \frac{\partial^2\Pi}{\partial t_n\partial s_0} & \frac{\partial^2\Pi}{\partial t_n\partial \xi} \\ \frac{\partial^2\Pi}{\partial t_1\partial p} & \frac{\partial^2\Pi}{\partial t_2\partial p} & \cdots & \frac{\partial^2\Pi}{\partial t_n\partial p} & \frac{\partial^2\Pi}{\partial p^2} & \frac{\partial^2\Pi}{\partial s_0\partial p} & \frac{\partial^2\Pi}{\partial p\partial \xi} \\ \frac{\partial^2\Pi}{\partial t_1\partial s_0} & \frac{\partial^2\Pi}{\partial t_2\partial s_0} & \cdots & \frac{\partial^2\Pi}{\partial t_n\partial s_0} & \frac{\partial^2\Pi}{\partial p\partial s_0} & \frac{\partial^2\Pi}{\partial s_0^2} & \frac{\partial^2\Pi}{\partial s_0\partial \xi} \\ \frac{\partial^2\Pi}{\partial t_1\partial \xi} & \frac{\partial^2\Pi}{\partial t_2\partial \xi} & \cdots & \frac{\partial^2\Pi}{\partial t_n\partial \xi} & \frac{\partial^2\Pi}{\partial p\partial \xi} & \frac{\partial^2\Pi}{\partial s_0\partial \xi} & \frac{\partial^2\Pi}{\partial \xi^2} \end{bmatrix}$$

should have the following properties: $|H_1| < 0$, $|H_2| > 0$, $\cdots$, $(-1)^{n+3}|H_{n+3}| > 0$, where $|H_i|$ denotes ith order minor. Although we are unable to derive the optimal conditions explicitly due to the profit function's complex form, we deduce the concavity criterion justifying $|H_1| < 0$ in the following proposition.

Proposition 3.1. *The total profit function is concave in p for each n.*

Proof: Noting that Q_is and S_is are linear in p for all i, we have

$$\begin{aligned} \frac{\partial \Pi_i}{\partial p} &= \frac{SR_i}{p} + p\left[-\frac{\beta}{\eta} + \frac{e^{-\eta(t_i-(i-1)T}\beta}{\eta} - \beta(iT-t_i)\right] - \left(h_1 + \frac{h_2}{\alpha_2}\right)\left\{\left(\frac{\partial Q_i}{\partial p} - \frac{\partial S_i}{\partial p}\right.\right. \\ &\quad \left.-\beta\right)t_d - \frac{\beta(2t_i+t_d)t_d}{2} + \left(\frac{\partial Q_i}{\partial p} - \frac{\partial S_i}{\partial p} + \beta t_d - \frac{\beta}{(1-m)\theta}\right) \times \\ &\quad \left.\left(\frac{1-e^{-(1-m)\theta(iT-(t_i+t_d)}}{(1-m)\theta}\right) + \frac{\beta(iT-(t_i+t_d))}{(1-m)\theta}\right\} + \left(\beta(t_i-(i-1)T) + \frac{\partial S_i}{\partial p}\right)c_s \end{aligned}$$

$$-c_d(1-m)\theta\Bigg[\left(\frac{\partial Q_i}{\partial p}-\frac{\partial S_i}{\partial p}+\beta t_d-\frac{\beta}{(1-m)\theta}\right)\left(\frac{1-e^{-(1-m)\theta(iT-(t_i+t_d)}}{(1-m)\theta}\right)$$
$$+\frac{\beta(iT-(t_i+t_d))}{(1-m)\theta}\Bigg],$$

where Π_i denotes the profit in the ith interval, excluding the cost of providing service which is independent of retail price. It is straightforward to deduce that

$$\begin{aligned}\frac{\partial^2\Pi_i}{\partial p^2} &= 2\left[-\frac{\beta}{\eta}+\frac{e^{-\eta(t_i-(i-1)T}\beta}{\eta}-\beta(iT-t_i)\right]\\ &= -2\beta\left[iT-t_i+\frac{1-e^{-\eta(t_i-(i-1)T}}{\eta}\right]<0;\end{aligned}$$

summing up over i completes the proof.

The subsequent analytical observations are combined in the form of a proposition.

Proposition 3.2. *(a)* $\frac{\partial S_i}{\partial a}>0$, $\frac{\partial Q_i}{\partial a}>0$, $\frac{\partial S_i}{\partial \beta}<0$, $\frac{\partial Q_i}{\partial \beta}<0$, $\frac{\partial S_i}{\partial \delta}>0$, $\frac{\partial Q_i}{\partial \delta}>0$, $\frac{\partial S_i}{\partial c}<0$, $\frac{\partial Q_i}{\partial c}<0$ *(b) If* $t_i-(i-1)T>\frac{1}{1+\eta(i-1)T}$, $\frac{\partial S_i}{\partial b}>0$ *and* $\frac{\partial Q_i}{\partial b}>0$ *(c) There is a time point* t^* *such that* $\frac{\partial S_i}{\partial t_i}>$ *or* <0 *according as* $t_i<$ *or* $>t^*$.

Proof: We have

$$\begin{aligned}\frac{\partial S_i}{\partial c} &= -\frac{t_i^2}{\eta}+\frac{2t_i}{\eta}-\frac{2}{\eta^2}-\frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\left\{-(i-1)^2T^2+\frac{2(i-1)T}{\eta}-\frac{2}{\eta^2}\right\}\\ &= -(t_i+\frac{1}{\eta})^2-\frac{1}{\eta^2}-\frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\left\{-\left((i-1)T-\frac{1}{\eta}\right)^2-\frac{1}{\eta^2}\right\}\\ &< -\left((i-1)T+\frac{1}{\eta}\right)^2\left(1-\frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\right),\end{aligned}$$

since $t_i>(i-1)T \implies \left(t_i+\frac{1}{\eta}\right)^2>\left((i-1)T+\frac{1}{\eta}\right)^2$, the proof is completed. Further,

$$\begin{aligned}\frac{\partial Q_i}{\partial c} &= \frac{\partial S_i}{\partial c}+\frac{t_i^3}{3}-\frac{(t_i+t_d)^3}{3}+\frac{(t_i+t_d)^2}{(1-m)\theta}-\frac{2(t_i+t_d)}{(1-m)^2\theta^2}+\frac{2}{(1-m)^3\theta^3}\\ &\quad-\left(\frac{2}{(1-m)^3\theta^3}+\frac{i^2T^2}{(1-m)\theta}-\frac{2iT}{(1-m)^2\theta^2}\right)e^{(1-m)\theta(iT-(t_i+t_d))}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{(1-m)\theta}\left(t_i+t_d-\frac{1}{(1-m)\theta}\right)^2+\frac{1}{(1-m)^3\theta^3}\left(1-e^{(1-m)\theta(iT-(t_i+t_d))}\right)\\
&\quad -\frac{1}{(1-m)\theta}\left(iT-\frac{1}{(1-m)\theta}\right)^2 e^{(1-m)\theta(iT-(t_i+t_d))}\\
&< -\left((i-1)T+\frac{1}{\eta}\right)^2\left(1-\frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\right)+\frac{1}{(1-m)\theta}\left(iT-\frac{1}{(1-m)\theta}\right)^2\\
&\quad -\frac{1}{(1-m)\theta}\left(iT-\frac{1}{(1-m)\theta}\right)^2 e^{(1-m)\theta(iT-(t_i+t_d))}+\frac{1}{(1-m)^3\theta^3}\times\\
&\quad \left(1-e^{(1-m)\theta(iT-(t_i+t_d))}\right)\\
&= -\left((i-1)T+\frac{1}{\eta}\right)^2\left(1-\frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\right)-\frac{1}{(1-m)^3\theta^3}\left(e^{(1-m)\theta(iT-(t_i+t_d))}\right.\\
&\quad \left.-1\right)-\frac{1}{(1-m)\theta}\left(iT-\frac{1}{(1-m)\theta}\right)^2\left(e^{(1-m)\theta(iT-(t_i+t_d))}-1\right)\times\\
&\quad \left(iT-\frac{1}{(1-m)\theta}\right)^2<0,\ \text{completing the proof.}
\end{aligned}
$$

For (b), it is observed that

$$
\frac{\partial S_i}{\partial b} = \frac{t_i-\{1+e^{-\eta(t_i-(i-1)T)}(i-1)T\}}{\eta}.
$$

Since η is very small in magnitude, approximating $e^{-\eta(t_i-(i-1)T)}$ linearly, straightforward calculation reveals that whenever $[t_i-(i-1)T][1+\eta(i-1)T] > 1$, or $t_i-(i-1)T > \frac{1}{1+\eta(i-1)T}$, $\frac{\partial S_i}{\partial b} > 0$. For the second part, it is observed that $\frac{\partial Q_i}{\partial b} = \frac{\partial S_i}{\partial b} + A_i$, where $A_i = \left(\frac{(t_i+t_d)^2}{2}-\frac{t_i^2}{2}\right)+\frac{1}{(1-m)\theta}\left(iTe^{(1-m)\theta(iT-(t_i+t_d))}-(t_i+t_d)\right)+\frac{1}{(1-m)^2\theta^2}\left(e^{(1-m)\theta(iT-(t_i+t_d))}-1\right)$, so that $\frac{\partial S_i}{\partial b} > 0$ implies $\frac{\partial Q_i}{\partial b} > 0$.

(c) We may write $\frac{\partial S_i}{\partial t_i} = \frac{1}{\eta}\left(b-2ct_i+\frac{2c}{\eta}\right)+e^{-\eta(t_i-(i-1)T)}R$, where $R = a+b(i-1)T-c((i-1)T)^2-\beta p+\delta s_i-\frac{(b-2c(i-1)T)}{\eta}-\frac{2c}{\eta^2}$, and subsequently $\frac{\partial^2 S_i}{\partial t_i^2} = -\left[\frac{2c}{\eta}+\eta Re^{-\eta(t_i-(i-1)T)}\right] < 0$, establishing unique maxima. As t_i's are time points in the ith interval on the time horizon, t_is increase with i, so there is a value t^* to be obtained from first order condition $\frac{\partial S_i}{\partial t_i} = 0$ such that whenever t_i's are lesser than t^*, S_i increases, and decreases when t_i's are higher in magnitude than t^*.

The proposition provides some valuable managerial insights as given below.

- The demand parameters significantly influence total backlog and order quantities in each interval. Higher base demand or greater sensitivity to service level naturally increases demand, leading to higher backlog amounts and order quantities. Conversely, greater price sensitivity compels managers to lower prices, reducing per-unit and total profits. To mitigate this loss, managers may reduce business volume, thereby lowering demand. Similarly, when demand is highly sensitive to the quadratic time parameter, which accelerates demand decline over time, managers should decrease both backlog and order quantities in each interval to adapt to the faster demand reduction.

- The impact of time dependency parameters is more complex. An increase in the linear time sensitivity parameter b leads to higher backlog and demand only if the shortage period $t_i - (i-1)T$ exceeds a pre-determined threshold. If the shortage period is small, the effect of time on generating demand becomes negligible, resulting in smaller backlogs and lower order quantities. As this threshold decreases with increasing i, later intervals tend to show higher backlog and replenishment for greater b. Interestingly, there are cases where $\frac{\partial Q_i}{\partial b} > 0$ even when $\frac{\partial S_i}{\partial b} < 0$, indicating that in some later intervals, higher b can lead to reduced backlog but increased replenishment, primarily due to higher demand generated in later periods (at larger t values).

- The backlogged amount initially increases over the first few intervals as demand rises with time during the early periods but begins to decrease as the demand trend reverses in subsequent intervals. Reduced demand at the retailer ultimately impacts storage levels and negatively affects overall business performance.

3.1.3 Numerical illustration

3.1.3.1 The unconstrained model

We now examine the developed model using a numerical example. The parameter-values are set as follows: $a = 500$ units/month, $b = 5$, $c = 0.001$, $\alpha_1 = 2$, $\alpha_2 = 2$, $h_1 = \$3$/unit/month, $h_2 = \$1$/unit/month, $A_1 = \$3000$/order, $A_2 = \$100$/order, $\theta = 0.05$/month, $c_s = \$7$/unit, $\delta = 100$ units/month, $k = 1$, $\beta = 10$ units/\$/month,

$\eta = 0.2$, $H = 10$ months, $c_1 = \$10$/unit, $c_d = \$3$/unit, $\alpha = 2$, $\omega = 3$, $m(\xi) = 1 - e^{-\gamma\xi}$ with $\gamma = 0.1$, $t_d(\xi) = t_0 + \omega(1 - e^{-\psi\xi})$ with $t_0 = 0$ months, $\psi = 0.005$, and $\omega = 2$. Since the concavity of the profit function with respect to T (or n) could not be established analytically, a line search method is applied on n to find the optimal decisions. The model is solved by built-in multi-objective optimization function of genetic algorithm in MatLab 2018b. Table 3.2 establishes that the model indeed attains its maxima for some value of n.

Table 3.2: Optimal profit for different values of n

n	1	2	3	4	5	6	7	8	9
Total profit ($)	21558	28219	30175	**30581***	29978	28784	27217	25394	23382

It is observed that for the set of chosen parameter-values, the optimal number of shipments is 4. We further see that the optimal values of t_i's are as follows: $t_1 = 1.086$ months, $t_2 = 3.451$ months, $t_3 = 5.921$ months, and $t_4 = 8.409$ months, indicating that it is indeed beneficiary for the retailer to start with planned shortage,

$$H = \begin{bmatrix} -1606 & 0 & 0 & 0 & -30.4521 & -24.9159 & -0.7681 \\ 0 & -1790 & 0 & 0 & -33.3963 & -28.0727 & -1.2887 \\ 0 & 0 & -1993.4 & 0 & -37.2661 & -23.09 & -1.6218 \\ 0 & 0 & 0 & -2167.6 & -40.6749 & -18.5852 & -1.8798 \\ -30.4521 & -33.3963 & -37.2661 & -40.6749 & -192.95 & 889.1556 & -0.2672 \\ -24.9159 & -28.0727 & -23.09 & -18.5852 & 889.1556 & -20985 & 2.4845 \\ -0.7681 & -1.2887 & -1.6218 & -1.8798 & -0.2672 & 2.4845 & -159040 \end{bmatrix}$$

and the shortage period gets reduced in subsequent intervals. Optimal values of other decision variables are obtained as $p = \$34.16$, $s_0 = 0.495$, and $\xi = \$23.17/\text{month}$, in appropriate units. The Hessian matrix for the optimal case is a 7×7 matrix specified above, from which it is straightforward to derive that $|H_1| = -1606 < 0$, $|H_2| = 2874740 > 0$, $|H_3| = -5730506716 < 0$, $|H_4| = 12421446357601.6 > 0$, $|H_5| = -23636715805746 73 < 0$, $|H_6| = 397390351726041788 03 > 0$, and $|H_7| = -6.32009597217002578 54e + 24 < 0$, thereby establishing the concavity numerically. A sensitivity analysis is further carried out with respect to the key parameters, and

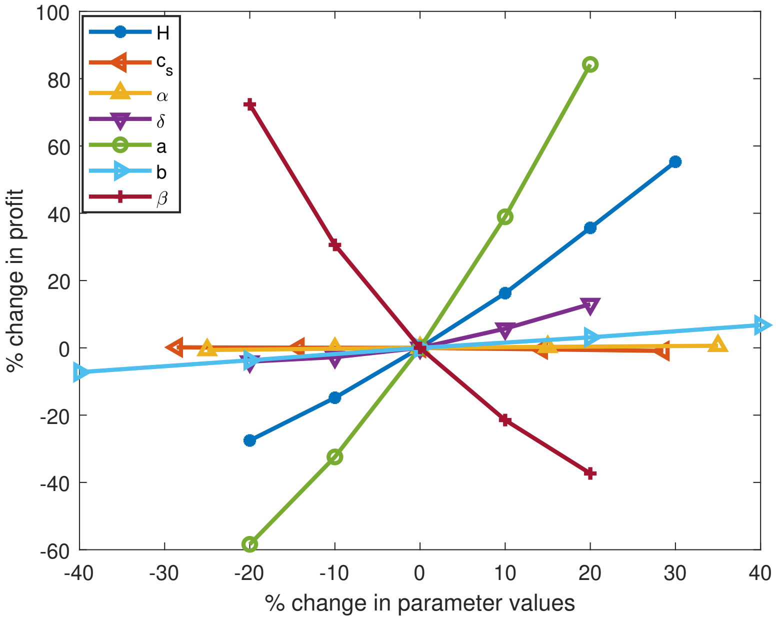


Figure 3.3: Sensitivity of profit with parameters

the results are summarized in Table 3.3 and Figure 3.3.

Managerial insights

From the table, the following managerial insights can be drawn.

- The demand parameter a positively impacts profit, as a higher base demand enables the manager to charge a higher price, generating more revenue from the increased demand. Additionally, the manager reduces the shortage period to reduce lost sales and capture more demand, further boosting revenue. Both the optimal service level and preservation investment also increase with a, suggesting that the retailer strategically reinvests a portion of the additional earnings into the business to drive further growth. A similar trend is observed

Table 3.3: Sensitivity Analysis

parameter	value	p	s_0	ξ	t_i
	400	28.15	0.234	7.62	1.76; 4.9; 8.2
	450	30.85	0.306	9.78	1.645; 4.783; 8.079
a	500	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	550	40.17	0.98	29	1.31; 4.47; 7.77
	600	42.34	0.999	43.413	0.94; 3.309; 5.778; 8.264
	3	23.555	0.467	10.76	1.127; 3.488; 5.961; 8.449
	4	33.969	0.484	12.62	1.106; 3.466; 5.948; 8.445
b	5	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	6	34.479	0.509	29.042	1.083; 3.447; 5.915; 8.402
	7	34.8	0.524	26.128	1.082; 3.444; 5.913; 8.399
	0.001	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	0.002	34.16	0.49	23.49	1.086; 3.45; 5.921; 8.409
c	0.003	34.16	0.495	24.476	1;086; 3.449; 5.921; 8.409
	0.004	34.164	0.494	27.524	1.086 ; 3.45; 5.919; 8.408
	0.005	34.155	0.495	25.97	1.085; 3.45; 5.92; 8.407
	8	45.2586	0.999	14.2518	1.22; 4.391; 7.698
	9	41.163	0.999	15.004	1.314; 4.484; 7.784
β	10	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	11	30.844	0.306	25.373	1.587; 4.731; 8.02
	12	28.6	0.246	25.383	1.667; 4.81; 8.098
	80	33.091	0.254	12.135	1.543; 4.705; 7.991
	90	33.567	0.392	19.133	1.125; 3.503; 5.98; 8.452
δ	100	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409

	110	38.185	0.999	31.575	1.386; 4.568; 7.875
	120	38.639	0.999	31.568	1.34; 4.511; 7.803
α_1	1.5	34.233	0.495	24.721	1.086; 3.45; 5.92; 8.409
	1.8	34.153	0.495	33.579	1.088; 3.451; 5.919; 8.407
	2	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	2.3	34.151	0.494	23.504	1.086; 3.451; 5.992; 8.41
	2.7	34.172	0.495	28.411	1.084; 3.449; 5.919; 8.407
θ	0.01	34.175	0.495	6.721	1.079; 3.443; 5.917; 8.405
	0.03	34.159	0.494	14.926	1.088; 3.45; 5.926; 8.41
	0.05	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	0.075	34.144	0.487	30.22	1.16; 3.521; 5.99; 8.485
	0.1	34.184	0.494	41.845	1.168; 3.522; 6; 8.467
c_s	5	34.067	0.494	22.849	1.131; 3.495; 5.964; 8.452
	6	34.128	0.496	23.601	1.108; 3.471; 5.942; 8.432
	7	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	8	34.192	0.495	23.644	1.064; 3.43; 5.9; 8.408
	9	34.221	0.495	29.486	1.044; 3.411; 5.882; 8.37
H	8	32.706	0.26	23.663	1.197; 3.718; 6.352
	9	33.19	0.328	24.491	1.347; 4.181; 7.144
	10	34.16	0.495	23.17	1.086; 3.451; 5.921; 8.409
	11	37.762	0.999	28.645	1.538; 5.002; 8.625
	12	37.701	0.999	26.977	1.223; 4.; 7.067; 10.06
	13	37.93	0.999	42.836	1.334; 4.41; 7.665; 10.86
	15	38.07	0.999	28.203	1.22; 4.066; 7.031; 10.021; 13.012

with the parameter b, which also positively affects profit. In contrast, an increase in the parameter c, representing a faster decline in demand over time, leads to a reduction in profit. However, since smaller values of c were considered, the variation had minimal impact on the optimal decisions, indicating that c played a less significant role in influencing outcomes under these conditions.

- The demand parameter β, associated with price, negatively impacts both demand and profit. A one-unit increase in β results in a reduction of p units in demand, causing a sharp decline in profit levels. To counter this, the manager must lower prices to stimulate demand, invest more in preservation to minimize spoilage, and extend the shortage period to mitigate further losses. However, to partially offset the impact of the price reduction, the manager reduces investment in servicing, as reflected in the lower values of s_0. This suggests that *pricing* should take precedence over *servicing*, as demand is more sensitive to price than to service level.

- An increase in δ leads to higher demand and, consequently, greater profit, as illustrated in the figure. The table further shows that a higher δ, coupled with increased investment in preservation, enhances the manager's confidence in allocating more resources to service level improvements. This, in turn, allows the manager to generate additional revenue by charging a higher price.

- An increase in the deterioration rate θ reduces profit levels, though the impact is mitigated by the presence of preservation technology investment. As shown in Table 3.3, a rise in θ prompts greater investment in preservation technology and a reduction in service level compared to other parameters, aiming to maintain demand as consistently as possible. Additionally, the manager extends shortage periods to further minimize spoilage.

- Higher shortage costs, whether due to increased holding costs or lost sales, directly reduce profit. Thus, it is unsurprising that profits decline as c_s rises. Table 3.3 supports this observation, showing shorter shortage periods with higher shortage costs. To mitigate the impact, the retailer opts to incur higher holding costs rather than risk losing goodwill and revenue opportunities, thereby reducing the shortage period. Prolonged holding of inventory also necessitates increased investment in preservation.

- The profit level increases with a longer business horizon H, as demand continues to grow within the sensitivity range of H, indicating rising demand with higher values of H. However, once H exceeds a certain threshold, profits begin to decline, eventually reaching zero when H becomes excessively large and the product becomes obsolete. Notably, as the time horizon lengthens, the retailer divides the total horizon into more cycles, leveraging the learning effect to reduce holding and ordering costs, which partially offsets the challenges of a longer horizon.

3.1.3.2 Space limitation

This subsection aims to study the effect of space limitation on the optimal decisions. Space limitation is a real life constraint for the business managers, since holding a large warehouse often involves large capitals, or sometimes it becomes impossible

Table 3.4: Effect of space constraint on optimal decisions

W	n	p	t_i	ξ	s_0	Total profit
200	4	38	1.2; 3.9; 6.54; 9.35	21	0.547	27953
250	4	36.6	1.117; 3.755; 6.4; 8.993	42.97	0.591	29256
300	4	34.1	1.094; 3.597; 6.256; 8.863	36.37	0.467	30190
350	4	34.15	1.091; 3.455; 6.053; 8.675	27.19	0.493	30481
400	4	34.19	1.09; 3.46; 5.92; 8.486	26.2	0.499	30571

to arrange larger warehouse due to limited resource, manpower or other issues. For the above-mentioned numerical example, optimal value of n is obtained as 4 and the highest inventory level is derived as 419 units. If the retailer does not have enough space to hold 419 units at a time, he has to redesign his optimal strategy. Table 3.4 summarizes the optimal values of the decision variables under different space limitations at the buyer. It is reasonable to believe that the more space a warehouse has, the faster its rent rises (the owner is assumed to pay the rent for owned warehouse to himself). According to the table, profit rises as inventory space increases. Space limitation has its effect mostly on the shortage period; the manage is unable to mitigate demand under space limitation, so he has to incur shortage. Optimal shortage periods continuously decrease with the increasing inventory space and so does the preservation technology investment. The preservation technology is assumed to be independent of inventory level, which signifies the event that the manager can afford more deterioration when he possesses larger quantity of items.

3.1.3.3 Capital constraint

This subsection addresses a crucial topic that business managers must deal with:

Table 3.5: Effect of capital constraint on optimal decisions

R	n	p	t_i	ξ	s_0	Total profit
20000	2	42.96	2.75; 7.119	77	0.223	19547
25000	3	39	1.981; 5.171; 8.533	25	0.146	25157
30000	3	35.64	1.841; 4.972; 8.239	23	0.192	28794
35000	3	34	1.565; 4.728; 7.997	33.97	0.31	30032
40000	4	34.69	1.113; 3.506; 5.948; 8.479	46.871	0.424	30356

the issue of capital constraint. It is reasonable to take resource limitations into consideration because businesses frequently have a restricted or pre-specified budget to spend on particular streams. Overspending in one or more areas puts other decisions in jeopardy. Note that the total cost incurred in the unconstrained model is \$42767, the total capital R varies ranging from \$20000 to \$40000 to examine the effect on optimal decisions. It is revealed that financial constraint impacts every aspects of business decision- pricing, replenishment timings, servicing, and preservation investment. With lower capital at hand, the manager even reduces the number of shipments to reduce ordering cost. Shortage period is also extended as the manager faces hardship to store more amount; preservation investment is reduced, and so is service investment. All the decisions together reduce the total profit.

3.1.4 Conclusion

This study develops and analyzes a multi-period inventory model that accounts for key factors such as demand, service levels, replenishment schedules, preservation investment, and the learning effect. The proposed model allows managers to adjust stock-in periods, pricing, service quality, and preservation investment dynamically across different periods, considering the time-dependent nature of demand within a defined business horizon. The unconstrained model is further extended to include (a) warehouse space limitations and (b) capital constraints. Analytical findings confirm the existence of an optimal pricing strategy, while numerical results validate the model's stability and practical applicability. The results highlight the significant influence of parameters such as price sensitivity, time dependency, and business horizon on optimal decisions. These insights provide valuable guidance for managers on how to adjust pricing and related strategies to remain competitive in evolving market conditions.

Future extensions of the model could explore several directions. First, the assumption of a constant deterioration rate could be replaced with a time-dependent deterioration function. Similarly, static pricing could be further refined into a fully dynamic pricing model. While this study assumes an exponential learning curve, alternative learning models could be investigated. Lastly, the inventory model could be extended to a supply chain framework, enabling the development of new coordination contracts to optimize and align the supply chain effectively.

3.2 Dynamic Pricing and Discounting Policy in Multi-period Perishable Inventory Models

3.2.1 Introduction

In this study, we introduce the concept of dynamic pricing across different cycles and a dynamic discounting strategy during shortage periods. While the earlier model considered a constant pricing strategy throughout the business cycle, such an approach may not align with the practical realities of varying demand patterns. Fluctuating demand over time makes it essential for managers to adopt pricing strategies that reflect these changes. By implementing dynamic pricing, managers can respond effectively to shifting market conditions, maximizing revenue and improving inventory efficiency.

In the context of a multi-period inventory model, discrete dynamic pricing simplifies decision-making by allowing prices to be adjusted at the start of each cycle. This flexibility ensures that the pricing strategy remains aligned with the demand behavior of each period. Moreover, segmenting a longer business horizon into multiple cycles prevents overstocking and minimizes the risk of depreciation. A long-term perspective, combined with dynamic pricing, offers a balanced approach to managing inventory while optimizing profitability.

Shortages, though often perceived as unfavorable due to lost sales, can also reduce holding and preservation costs, presenting a nuanced impact on business operations. To mitigate the adverse effects of shortages, firms can introduce dynamic discounting during stock-out periods. This strategy incentivizes customers to wait for replenishment by offering discounts for pre-booked items, thus retaining demand while managing inventory constraints.

In this study, our aim is to find answer of the following research questions:

RQ1: How do dynamic pricing and discounting strategies influence demand and profitability in a multi-period inventory system with service level and preservation

*This part of the chapter is based on the work published in *Journal of Management Analytics* (2024), volume 11, issue 2, pages 272-301.

technology investment?

RQ2: What is the optimal balance between service investment and discount incentives to maximize revenue while minimizing stockouts and deterioration losses?

RQ3: How does the relationship between price sensitivity, backlogging rate, and preservation investment affect the retailer's optimal pricing and replenishment decisions over multiple periods?

This extended model addresses the limitations of static pricing by integrating dynamic pricing strategies across cycles. Additionally, the introduction of dynamic discounts during shortages enhances the firm's ability to manage demand fluctuations and maintain competitiveness. Together, these concepts offer a comprehensive framework for handling the challenges of inventory management in a multi-period, dynamic pricing environment.

3.2.2 Notations and assumptions

Notations are same as given in section 3.1. The additional notations are used for the dynamic price and dynamic discounting. Retail price in the i^{th} interval is denoted by p_i, $i = 1, 2, \cdots, n$. To encourage customers to wait until replenishment, a price discount r_i (*i.e.* $100r_i$% discounts) is offered for the orders placed during the shortage period in the i^{th} period. Considering the time-varying nature of the demand, different rates of discounts are set in different periods.

3.2.3 The Model

The changes in inventory level at any time point t during the interval $[(i-1)T, iT]$ are given by

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\eta(t_i - t)} D((1-r_i)p_i, s_i, t) & \text{with } I((i-1)T) = 0 \text{ and } I(t_i) = -S_i, \\ & (i-1)T \leqslant t \leqslant t_i \\ -D(p_i, s_i, t) & \text{with } I(t_i) = Q_i - S_i, \quad t_i < t \leqslant t_i + t_d \\ -D(p_i, s_i, t) - (1 - m(\xi))\theta I(t) & \text{with } I(iT) = 0, t_i + t_d < t \leqslant iT. \end{cases} \tag{3.6}$$

Solving the equations with the given initial and continuity conditions on inventory and the fact that Q_i amount is replenished at time point t_i, the inventory level during the entire period can be obtained.

Solving the first equation, we get $I(t) = -\frac{e^{-\eta(t_i-t)}}{\eta}\left((a+bt-ct^2-\beta(1-r_i)p_i+\delta s_i) - \frac{(b-2ct)}{\eta} - \frac{2c}{\eta^2}\right) + c_1$, where the value of c_1 is derived from the condition $I((i-1)T) = 0$ as $c_1 = \frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\left(\left(a+b(i-1)T-c((i-1)T)^2-\beta(1-r_i)p_i+\delta s_i\right) - \frac{(b-2c(i-1)T)}{\eta} - \frac{2c}{\eta^2}\right)$, so that we may write

$$\begin{aligned} I(t) &= -\frac{e^{-\eta(t_i-t)}}{\eta}\left\{(a+bt-ct^2-\beta(1-r_i)p_i+\delta s_i) - \frac{(b-2ct)}{\eta} - \frac{2c}{\eta^2}\right\} \\ &\quad + \frac{e^{-\eta(t_i-(i-1)T)}}{\eta}\left\{a+b(i-1)T-c((i-1)T)^2-\beta(1-r_i)p_i+\delta s_i \right. \\ &\quad \left. - \frac{b-2c(i-1)T}{\eta} - \frac{2c}{\eta^2}\right\}. \end{aligned}$$

Further, putting $I(t_i) = -S_i$, we obtain equation 3.10. Next we solve the second equation using the given initial condition to obtain inventory level as

$$I(t) = Q_i - S_i + (a-\beta p_i+\delta s_i)(t_i-t) + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - \frac{bt^2}{2} + \frac{ct^3}{3}.$$

Using this, we obtain

$$I(t_i+t_d) = Q_i - S_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - at_d - \frac{b(t_i+t_d)^2}{2} + \frac{c(t_i+t_d)^3}{3} + \beta p_i t_d - \delta s_i t_d,$$

which is further used as initial condition for the third differential equation. Solving it, the inventory level is obtained as

$$I(t) = -\frac{(a+bt-ct^2-\beta p_i+\delta s_i)}{(1-m)\theta} + \frac{(b-2ct)}{(1-m)^2\theta^2} + \frac{2c}{(1-m)^3\theta^3} + c_3 e^{-(1-m)\theta t},$$

where the continuity condition at $t = t_i + t_d$ specifies the value of c_3 as

$$
\begin{aligned}
c_3 \;=\; & \left(Q_i - S_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - at_d - \frac{b(t_i+t_d)^2}{2} + \frac{c(t_i+t_d)^3}{3} + \beta p_i t_d - \delta s_i t_d \right. \\
& \left. + \frac{(a + b(t_i+t_d) - c(t_i+t_d)^2 - \beta p_i + \delta s_i)}{(1-m)\theta} - \frac{(b - 2c(t_i+t_d))}{(1-m)^2\theta^2} - \frac{2c}{(1-m)^3\theta^3}\right) \times \\
& e^{(1-m)\theta(t_i+t_d)}.
\end{aligned}
$$

Putting the value we obtain equations 3.7-3.9.

$$
\begin{aligned}
I(t) \;=\; & -\frac{e^{-\eta(t_i-t)}}{\eta}\left\{a + bt - ct^2 - \beta(1-r_i)p_i + \delta s_i - \frac{(b-2ct)}{\eta} - \frac{2c}{\eta^2}\right\} \\
& + \frac{e^{-\eta(t_i-(i-1)T)}}{\eta} \times \left\{a + b(i-1)T - c((i-1)T)^2 - \beta(1-r_i)p_i + \delta s_i \right. \\
& \left. - \frac{b - 2c(i-1)T}{\eta} - \frac{2c}{\eta^2}\right\} \qquad (i-1)T \leqslant t \leqslant t_i, \qquad (3.7) \\
I(t) \;=\; & Q_i - S_i + (a - \beta p_i + \delta s_i)(t_i - t) + \frac{b(t_i^2 - t^2)}{2} - \frac{c(t_i^3 - t^3)}{3}, \\
& t_i < t \le (t_i + t_d), \qquad (3.8) \\
I(t) \;=\; & \left(Q_i - S_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - at_d - \frac{b(t_i+t_d)^2}{2} + \frac{c(t_i+t_d)^3}{3} + \beta p_i t_d - \delta s_i t_d \right. \\
& - \frac{(b - 2c(t_i+t_d))}{(1-m)^2\theta^2} + \frac{(a + b(t_i+t_d) - c(t_i+t_d)^2 - \beta p_i + \delta s_i)}{(1-m)\theta} \\
& \left. - \frac{2c}{(1-m)^3\theta^3}\right) e^{-(1-m)\theta(t-(t_i+t_d))} - \frac{(a + bt - ct^2 - \beta p_i + \delta s_i)}{(1-m)\theta} + \frac{(b-2ct)}{(1-m)^2\theta^2} \\
& + \frac{2c}{(1-m)^3\theta^3}, \qquad t_i + t_d < t \leqslant iT. \qquad (3.9)
\end{aligned}
$$

We further derive the following relations for $i = 1, 2, \cdots, n$:

$$
\begin{aligned}
S_i \;=\; & \frac{1}{\eta}\left\{a + bt_i - ct_i^2 - \beta(1-r_i)p_i + \delta s_i - \frac{(b-2ct_i)}{\eta} - \frac{2c}{\eta^2}\right\} - \frac{e^{-\eta(t_i-(i-1)T)}}{\eta} \times \\
& \left\{a + b(i-1)T - c((i-1)T)^2 - \beta(1-r_i)p_i + \delta s_i - \frac{b - 2c(i-1)T}{\eta} - \frac{2c}{\eta^2}\right\}, \qquad (3.10)
\end{aligned}
$$

and from $I(iT) = 0$,

$$\begin{aligned}
Q_i &= e^{(1-m)\theta(iT-(t_i+t_d))}\left(\frac{a+biT-ci^2T^2-\beta p_i+\delta s_i}{(1-m)\theta}-\frac{(b-2ciT)}{(1-m)^2\theta^2}-\frac{2c}{(1-m)^3\theta^3}\right)\\
&+S_i+\frac{ct_i^3}{3}+(a-\beta p_i+\delta s_i)t_d+\frac{b(2t_it_d+t_d^2)}{2}-\frac{c(t_i+t_d)^3}{3}+\frac{b-2c(t_i+t_d)}{(1-m)^2\theta^2}\\
&+\frac{2c}{(1-m)^3\theta^3}-\frac{a+b(t_i+t_d)-c(t_i+t_d)^2-\beta p_i+\delta s_i}{(1-m)\theta}. \qquad (3.11)
\end{aligned}$$

Now we aim at deriving the cost functions in each interval. The total item sold during the period (t_i, iT) is given by $B_i = \int_{t_i}^{iT} D(p_i, s_i, t)dt = a(iT - t_i) + \frac{b}{2}(i^2T^2 - t_i^2) - \frac{c}{3}(i^3T^3 - t_i^3) - \beta p_i(iT - t_i) + \delta s_i(iT - t_i)$, which, combined with the total backlog S_i generates total revenue in the i^{th} interval as $p_i\{(1 - r_i)S_i + B_i\}$.
The ordering cost is $OC_i = A_1 + \frac{A_2}{i^{\alpha_1}}$.
The holding cost in each interval is given by

$$\begin{aligned}
HC_i &= \left(h_1+\frac{h_2}{i^{\alpha_2}}\right)\int_{t_i}^{iT} I(t)dt\\
&= \left(h_1+\frac{h_2}{i^{\alpha_2}}\right)\Bigg[\left(Q_i-S_i+at_i+\frac{bt_i^2}{2}-\frac{ct_i^3}{3}-\beta p_it_i+\delta s_it_i\right)t_d\\
&-\Bigg(\frac{a((t_i+t_d)^2-t_i^2)}{2}+\frac{b((t_i+t_d)^3-t_i^3)}{6}-\frac{c((t_i+t_d)^4-t_i^4)}{12}\\
&-\frac{\beta p_i((t_i+t_d)^2-t_i^2)}{2}+\frac{\delta s_i((t_i+t_d)^2-t_i^2)}{2}\Bigg)+\Bigg(Q_i-S_i+at_i+\frac{bt_i^2}{2}-\frac{ct_i^3}{3}\\
&-\beta p_it_i+\delta s_it_i-a(t_i+t_d)-\frac{b(t_i+t_d)^2}{2}+\frac{c(t_i+t_d)^3}{3}+\beta p_i(t_i+t_d)\\
&-\delta s_i(t_i+t_d)+\frac{(a+b(t_i+t_d)-c(t_i+t_d)^2-\beta p_i+\delta s_i)}{(1-m)\theta}-\frac{(b-2c(t_i+t_d))}{(1-m)^2\theta^2}\\
&-\frac{2c}{(1-m)^3\theta^3}\Bigg)\left(\frac{1-e^{-(1-m)\theta(iT-(t_i+t_d)}}{(1-m)\theta}\right)-\frac{a(iT-(t_i+t_d))}{(1-m)\theta}\\
&-\frac{b(i^2T^2-(t_i+t_d)^2)}{2(1-m)\theta}+\frac{c(i^3T^3-(t_i+t_d)^3)}{3(1-m)\theta}+\frac{\beta p_i(iT-(t_i+t_d))}{(1-m)\theta}\\
&-\frac{\delta s_i(iT-(t_i+t_d))}{(1-m)\theta}+\frac{b(iT-(t_i+t_d))}{(1-m)^2\theta^2}-\frac{c(i^2T^2-(t_i+t_d)^2)}{(1-m)^2\theta^2}\\
&+\frac{2c}{(1-m)^3\theta^3}(iT-(t_i+t_d))\Bigg].
\end{aligned}$$

The costs related to preservation technology investment PTC_i and purchase PC_i are

$\xi(iT-t_i)$ and c_1Q_i, respectively.
The deterioration cost (DC_i) can be obtained as

$$c_d(1-m)\theta\Bigg[\Bigg(Q_i - S_i + \frac{bt_i^2}{2} - \frac{ct_i^3}{3} - (a-\beta p_i+\delta s_i)t_d - \frac{b(t_i+t_d)^2}{2} + \frac{c(t_i+t_d)^3}{3}$$
$$+\frac{a+b(t_i+t_d)-c(t_i+t_d)^2-\beta p_i+\delta s_i}{(1-m)\theta} - \frac{(b-2c(t_i+t_d))}{(1-m)^2\theta^2} - \frac{2c}{(1-m)^3\theta^3}\Bigg)\times$$
$$\left(\frac{1-e^{-(1-m)\theta(iT-t_i-t_d)}}{(1-m)\theta}\right) - \frac{a(iT-(t_i+t_d))}{(1-m)\theta} - \frac{b(i^2T^2-(t_i+t_d)^2)}{2(1-m)\theta}$$
$$+\frac{c(i^3T^3-(t_i+t_d)^3)}{3(1-m)\theta} + \frac{\beta p_i(iT-(t_i+t_d))}{(1-m)\theta} - \frac{\delta s_i(iT-(t_i+t_d))}{(1-m)\theta}$$
$$+\frac{b(iT-(t_i+t_d))}{(1-m)^2\theta^2} - \frac{c(i^2T^2-(t_i+t_d)^2)}{(1-m)^2\theta^2} + \frac{2c}{(1-m)^3\theta^3}(iT-(t_i+t_d))\Bigg].$$

The shortage cost is derived as

$$\begin{aligned} SC_i &= c_s\Big\{a(t_i-(i-1)T) + \frac{b(t_i^2-((i-1)T)^2)}{2} - \frac{c(t_i^3-((i-1)T)^3)}{3} \\ &\quad -\beta(1-r_i)p_i(t_i-(i-1)T) + \delta s_i(t_i-(i-1)T) - S_i\Big\}. \end{aligned}$$

Noting that the service cost, *i.e.* investment in maintaining service level is the same in each replenishment period, and the service level will automatically be improved as the effect of learning, the total profit during the entire business horizon can be summed up as

$$\Pi = -\frac{nks_0^2}{2} + \sum_{i=1}^{n}\Big(SR_i - (OC_i + HC_i + PTC_i + PC_i + DC_i + SC_i)\Big). \tag{3.12}$$

The aim is now to maximize the total profit with respect to the decision variables ξ, n, s_0, p_i, r_i, and $t_i, i = 1, 2, \cdots, n$; n is a positive integer, subject to the set of equations 3.11 and 3.12, $0 \le t_i < iT$ and $t_d < iT - t_i$, $i = 1, 2, \cdots, n$, $T = \frac{H}{n}$. Although the complex form of the profit function prevents us from deriving the optimal conditions explicitly, we derive the following concavity criterion in the form of a proposition:

Proposition 3.3. *The total profit function is individually concave in p_i and r_i for all i, and optimal pricing and discounting in the i^{th} interval is given by*

$$\begin{aligned}
p_i^* &= \frac{1}{\beta Z_1}\Bigg[a\left\{\frac{(1-r_i)}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right)+(iT-t_i)\right\}\\
&+b\Bigg\{\frac{(1-r_i)(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)})}{\eta}+\frac{(1-r_i)}{\eta^2}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\\
&+\frac{i^2T^2-t_i^2}{2}\Bigg\}-c\Bigg\{(1-r_i)\left(t_i^2-(i-1)^2T^2e^{-\eta(t_i-(i-1)T)}\right)\\
&-\frac{2(1-r_i)}{\eta^2}(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)})+\frac{2(1-r_i)}{\eta^3}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\\
&+\frac{i^3T^3-t_i^3}{3}\Bigg\}+\beta\Bigg\{\left(h_1+\frac{h_2}{i^{\alpha_2}}\right)\left(\left(t_d+1+\frac{e^{(1-m)\theta(iT-(t_i+t_d))}-1}{(1-m)\theta}\right)t_d\right.\\
&\left.+\frac{(2t_i+t_d)t_d}{2}+Z_2\right)+c_s\Big((1-r_i)(t_i-(i-1)T)\Big)+c_d(1-m)\theta Z_2\\
&+c_1\left(\frac{(1-r_i)\left(1-e^{-\eta(t_i-(i-1)T)}\right)}{\eta}+t_d+\frac{e^{(1-m)\theta(iT-(t_i+t_d))}-1}{(1-m)\theta}\right)\Bigg\}\\
&+\delta\left\{(1-r_i)s_i\left(1-e^{-\eta(t_i-(i-1)T)}\right)+s_i(iT-t_i)\right\}\Bigg],
\end{aligned}$$

$$\begin{aligned}
\text{and}\;\; r_i^* &= \frac{1}{\beta Z_3}\Bigg[-\frac{a}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right)-b\Bigg\{\frac{1}{\eta}\left(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right)\\
&-\frac{1}{\eta^2}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\Bigg\}+\frac{c}{\eta}\Bigg\{\left(t_i^2-(i-1)^2T^2e^{-\eta(t_i-(i-1)T)}\right)\\
&-\frac{2}{\eta}\left(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right)+\frac{2}{\eta^2}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\Bigg\}\\
&+\beta\Bigg\{\frac{2p_i\left(1-e^{-\eta(t_i-(i-1)T)}\right)}{\eta}-c_sp_i(t_i-(i-1)T)+\frac{c_sp_i}{\eta}\Big(1\\
&-e^{-\eta(t_i-(i-1)T)}\Big)-\frac{c_1}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\Bigg\}-\frac{\delta s_i}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\Bigg],
\end{aligned}$$

where $Z_1=2(iT-t_i)+\frac{(1-r_i)^2}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right)+\frac{(1-r_i)}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right)$,
$Z_2=\frac{1}{(1-m)\theta}\left(\frac{e^{(1-m)\theta(iT-t_i-t_d)}-1}{(1-m)\theta}\right)-\frac{(iT-(t_i+t_d))}{(1-m)\theta}$, *and* $Z_3=2\frac{p_i^2}{\eta}\left(1-e^{-\eta(t_i-(i-1)T}\right)$.

Proof: Note first that p_i affects the profit generated in the i^{th} interval only. Noting that Q_is and S_is are linear in p_i for all i, and $\frac{\partial S_i}{\partial r_i} = \frac{\beta p_i}{\eta}\left(1 - e^{-\eta(t_i-(i-1)T)}\right)$ and $\frac{\partial S_i}{\partial p_i} = -\frac{\beta(1-r_i)}{\eta}\left(1 - e^{-\eta(t_i-(i-1)T)}\right)$, we have

$$\begin{aligned}\frac{\partial \Pi}{\partial p_i} &= \{B_i + (1-r_i)S_i\} + \beta p_i\left[-\frac{1}{\eta}(1-r_i) + \frac{e^{-\eta(t_i-(i-1)T}}{\eta}(1-r_i) - (iT-t_i)\right] \\ &\quad - \left(h_1 + \frac{h_2}{i^{\alpha_2}}\right)\left\{\left(\frac{\partial Q_i}{\partial p_i} - \frac{\partial S_i}{\partial p_i} - \beta\right)t_d - \frac{\beta(2t_i+t_d)t_d}{2} + \left(\frac{\partial Q_i}{\partial p_i} - \frac{\partial S_i}{\partial p_i} + \beta t_d\right.\right. \\ &\quad \left.\left. - \frac{\beta}{(1-m)\theta}\right)\left(\frac{1-e^{-(1-m)\theta(iT-(t_i+t_d)}}{(1-m)\theta}\right) + \frac{\beta(iT-(t_i+t_d))}{(1-m)\theta}\right\} + \left(\beta(1-r_i)\times\right. \\ &\quad \left.(t_i-(i-1)T) + \frac{\partial S_i}{\partial p_i}\right)c_s - c_d(1-m)\theta\left[\left(\frac{\partial Q_i}{\partial p_i} - \frac{\partial S_i}{\partial p_i} + \beta t_d - \frac{\beta}{(1-m)\theta}\right)\times\right. \\ &\quad \left.\left(\frac{1-e^{-(1-m)\theta(iT-(t_i+t_d)}}{(1-m)\theta}\right) + \frac{\beta(iT-(t_i+t_d))}{(1-m)\theta}\right] - c_1\frac{\partial Q_i}{\partial p_i},\end{aligned}$$

from which it is straightforward to deduce that

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = -2\beta\left[\frac{(1-r_i)}{\eta}\{1-e^{-\eta(t_i-(i-1)T}\} + iT - t_i\right] < 0,$$

proving the concavity with respect to p_i. For second part, we have

$$\frac{\partial \Pi}{\partial r_i} = -p_i S_i + (1-r_i)p_i\frac{\partial S_i}{\partial r_i} - c_1\frac{\partial Q_i}{\partial r_i} + c_s\left(\frac{\partial S_i}{\partial r_i} - \beta p_i(t_i-(i-1)T)\right),$$

so that, $$\frac{\partial^2 \Pi}{\partial r_i^2} = -2p_i\frac{\partial S_i}{\partial r_i} < 0, \quad \text{completing the proof.}$$

Optimal decision variables can now be found by solving first order conditions.

Proposition 3.4. *(i)* $\frac{\partial^2 S_i}{\partial p_i \partial r_i} > 0$, $\frac{\partial^2 Q_i}{\partial p_i \partial r_i} > 0$ *(ii)* $\frac{\partial p_i}{\partial a} > 0$, $\frac{\partial r_i}{\partial a} < 0$ *(iii)* $\frac{\partial p_i}{\partial b} > 0$ *(iv)* $\frac{\partial p_i}{\partial \delta} > 0$, $\frac{\partial r_i}{\partial \delta} < 0$ *(v)* $\frac{\partial p_i}{\partial c_s} < 0$ *(vi)* $\frac{\partial^2 S_i}{\partial t_i \partial s_i} > 0$ *(vii)* $\frac{\partial^2 Q_i}{\partial \theta \partial \xi} > 0$.

Proof:
(i) $\frac{\partial S_i}{\partial r_i} = \frac{\beta p_i}{\eta}\left(1 - e^{-\eta(t_i-(i-1)T)}\right)$, so that $\frac{\partial^2 S_i}{\partial p_i \partial r_i} = \frac{\beta}{\eta}\left(1 - e^{-\eta(t_i-(i-1)T)}\right) > 0$. Similarly, $\frac{\partial^2 Q_i}{\partial p_i \partial r_i} = \frac{\beta}{\eta}\left(1 - e^{-\eta(t_i-(i-1)T)}\right) > 0$.
(ii) Noting that $Z_1 > 0$, $Z_2 < 0$, and $Z_3 > 0$ as given in Proposition 3.3, it is easy to deduce that

$\frac{\partial p_i^*}{\partial a} = \frac{1}{\beta Z_1}\left\{\frac{(1-r_i)}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right)+(iT-t_i)\right\} > 0$ and
$\frac{\partial r_i^*}{\partial a} = -\frac{1}{\beta Z_3}\frac{1}{\eta}\left(1-e^{-\eta(t_i-(i-1)T)}\right) < 0.$
(iii) $\frac{\partial p_i}{\partial b} = \frac{1}{\beta Z_1}\left[\frac{(1-r_i)\{t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\}}{\eta}+\frac{(1-r_i)}{\eta^2}\left(1-e^{-\eta(t_i-(i-1)T)}\right)+\frac{i^2T^2-t_i^2}{2}\right] > 0.$
(iv) $\frac{\partial p_i}{\partial \delta} = \frac{1}{\beta Z_1}\left\{(1-r_i)s_i\left(1-e^{-\eta(t_i-(i-1)T)}\right)+s_i(iT-t_i)\right\} > 0$, and
$\frac{\partial r_i}{\partial \delta} = -\frac{1}{\beta Z_3}s_i\left(1-e^{-\eta(t_i-(i-1)T)}\right) < 0.$
(v)-(vii) The proofs are straightforward, hence omitted.

Proposition 3.5. *(i) If* $\eta < \frac{\left(1-e^{-\eta(t_i-(i-1)T)}\right)}{\left(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right)}$, *then* $\frac{\partial p_i}{\partial c} < 0$, $\frac{\partial r_i}{\partial c} > 0$ *(ii)* $\frac{\partial r_i}{\partial b} > 0$ *if and only if* $\eta < \frac{\left(1-e^{-\eta(t_i-(i-1)T)}\right)}{\left(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right)}$.

Proof:
(i) When $\eta < \frac{\left(1-e^{-\eta(t_i-(i-1)T)}\right)}{\left(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right)}$, it is seen that $\frac{\partial p_i}{\partial c} = -\frac{1}{\beta Z_1}\Bigg\{(1-r_i)\Big\{t_i^2 - (i-1)^2T^2e^{-\eta(t_i-(i-1)T)}\Big\} - \frac{2(1-r_i)}{\eta^2}\left\{t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right\}+\frac{2(1-r_i)}{\eta^3}\times \left(1-e^{-\eta(t_i-(i-1)T)}\right)+\frac{i^3T^3-t_i^3}{3}\Bigg\} < 0$. In a similar manner, it can be ensured that

$$\frac{\partial r_i}{\partial c} = \frac{1}{\eta\beta Z_3}\Bigg\{t_i^2-(i-1)^2T^2e^{-\eta(t_i-(i-1)T)}-\frac{2}{\eta}\left(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right) + \frac{2}{\eta^2}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\Bigg\} > 0.$$

(ii) Noting that $\frac{\partial r_i}{\partial b} = -\frac{1}{\beta Z_3}\left\{\frac{1}{\eta}\left(t_i-(i-1)Te^{-\eta(t_i-(i-1)T)}\right)-\frac{1}{\eta^2}\left(1-e^{-\eta(t_i-(i-1)T)}\right)\right\}$, the proof is straightforward.

3.2.4 Numerical illustration

We now illustrate the developed model through a numerical example. We choose the parameter values as follows: $\beta = 10$ units/\$/month, $\delta = 100$ units/month, $h_1 = \$3$/unit/month, $h_2 = \$1$/unit/month, $A_1 = \$3000$/order, $A_2 = \$90$/order, $\alpha_1 = \alpha_2 = 2$, $\theta = 0.05$/month, $k = 40000$, $\eta = 0.2$, $c_s = \$7$/unit, $H = 10$ months, $c_1 = \$10$/unit, $c_d = \$3$/unit, $\alpha = 0.7$, $m(\xi) = 1-e^{-\gamma\xi}$ with $\gamma = 0.1$, and $t_d(\xi) = t_0+\omega(1-e^{-\psi\xi})$ with $t_0 = 0$ months, $\psi = 0.005$, and $\omega = 3$. To encompass both the increasing and decreasing nature of the demand during business horizon, we set

$a = 500$ units/month, $b = 5$, and $c = 1$. Since the concavity of the profit function with respect to n is not established analytically, a line search method is applied on n

Table 3.6: Optimal profit for different values of n

n	1	2	3	4	5	6
Total profit (\$)	18663	23644	**24750**	23712	23022	21357

to find the optimal decisions. Computational results are shown in Table 3.6. The in-built multi-objective optimization function through the genetic algorithm provided by MatLab 2018b is used to derive numerical results. It is seen that the maximum profit is attained for $n = 3$, where optimal values of other decision variables are

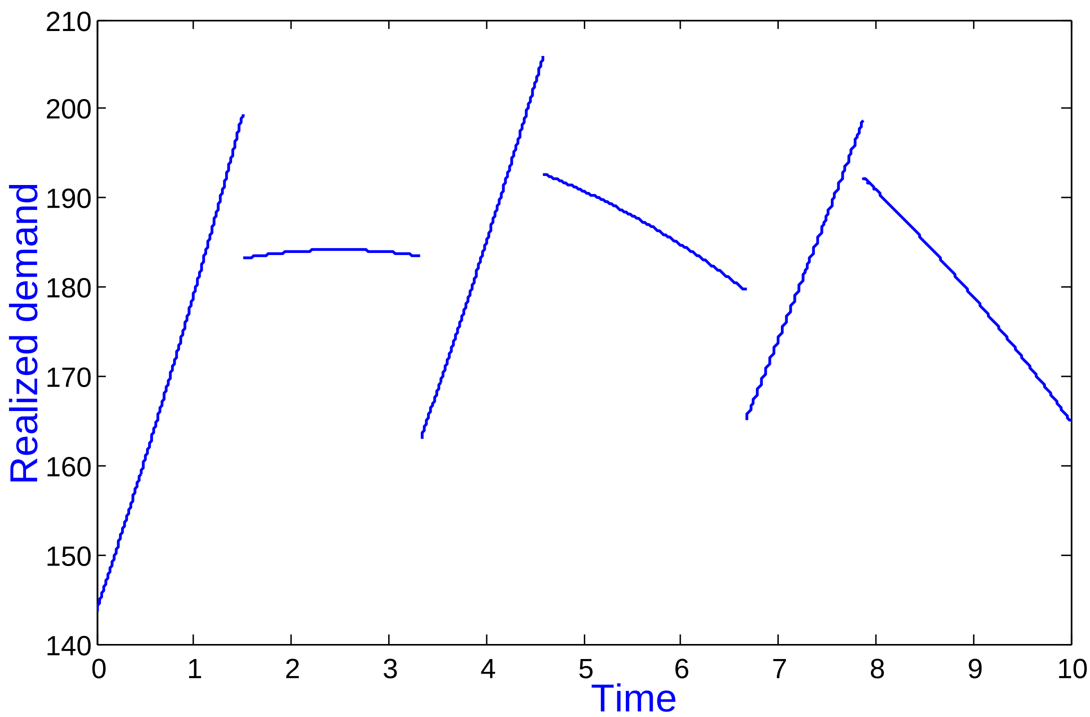


Figure 3.4: Realized demand over the business period under optimal decision variables

obtained as $t_1 = 1.507$ months, $t_2 = 4.589$ months, $t_3 = 7.875$ months, $p_1 = \$32.6$, $p_2 = \$32.9$, $p_3 = \$31.9$, $r_1 = 0.05$ (*i.e.* 5% discount), $r_2 = 0.04$ (4% discount), $r_3 = 0.02$ (2% discount), $s_0 = 0.149$ and $\xi = \$34$/month. Figure 3.4 illustrates the behavior of the realized demand pattern over the entire business horizon, where it may be observed that realized demand placed during the shortage period when replenishment is about to take place is even higher than the demand placed during the stock-in period itself, which is the result of discounting policy. However, this extra demand does not generate enough revenue, so that the manager can't afford to continue with the planned shortage.

3.2.5 Sensitivity analysis and managerial insights

Here we perform the sensitivity analysis of key model parameters. Table 3.7 illustrates how the optimal decision variables are adjusted with changing business situations. The following managerial insights may be drawn from the sensitivity analysis.

- *Pricing policy:* Although demand explicitly depends on price, other factors influencing demand also indirectly shape pricing strategies. An increase in the base demand (a) or the linear time component (b) leads to a significant positive impact on profitability, as highlighted in Proposition 3.4 and Table 3.7. Higher demand levels not only enhance profitability but also enable price increases, reflecting the benefits of operating in a high-demand environment. Conversely, higher price sensitivity among customers necessitates lower retail prices, as even a small price hike would result in a greater loss in sales volume. Customer sensitivity to service quality allows for price increases, as additional revenue compensates for the increased investment in service. However, higher shortage costs push managers to lower prices during both stock-in and shortage periods to minimize losses. A longer business horizon provides managers with the opportunity to allocate resources more efficiently, enabling them to generate higher demand even with reduced prices. Interestingly, changes in service investment costs or deterioration rates have little direct effect on pricing. However, when deterioration rates are sufficiently high, managers are encouraged to charge higher prices during periods of increasing demand to offset losses due to spoilage.
  Proposition 3.4 (i) establishes a positive relationship between pricing and the effect of discounting on backlogged amounts. Customers are more responsive

Table 3.7: Sensitivity Analysis

Parameter	% change	n	t_i	p_i	r_i	s_0	ξ	Profit
	-20	2	2.544; 7.097	28; 26.8	0.068; 0.022	0.117	23.862	9241
a	-10	3	1.574;4.672;7.951	30.3; 30; 29.3	0.065; 0.035; 0.022	0.13	29	16214
$(units$	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
$/month)$	10	3	1.43; 4.539; 7.811	35.3; 35.3; 34.47	0.045; 0.032; 0.018	0.166	36.85	34526
	20	4	1; 3.328; 5.839; 8.307	38.1; 38.4; 38.4; 37.4	0.056; 0.048; 0.04; 0.029	0.169	38.07	45845
	-20	3	1.52; 4.6; 7.868	33.1; 32.7; 31.5	0.47; 0.046; 0.032	0.144	32.4	22887
b	-10	3	1.51; 4.6; 7.88	32.3; 32.8; 31.7	0.049; 0.032; 0.014	0.146	33.85	23847
$(units/$	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
$\$/month^2)$	10	3	1.5; 4.59; 7.856	33.68; 33.7; 32.8	0.062; 0.05; 0.038	0.154	35.5	25623
	20	3	1.5; 4.59; 7.87	34; 33.53; 32.9	0.063; 0.043; 0.024	0.157	39.4	26595
	-20	3	1.51; 4.628; 7.9	32.9; 33.3; 32.9	0.043; 0.047; 0.033	0.152	44.38	25925
c	-10	3	1.5; 4.598; 7.872	32.85; 33.4; 32.4	0.048; 0.045; 0.028	0.151	36	25340
$(units$	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
$/month^3)$	10	3	1.51; 4.6; 7.87	32.59; 32.67; 31.45	0.053; 0.035 ; 0.015	0.147	32.84	24133

	20	3	1.52; 4.6; 7.88	32; 32; 30.9	0.123; 0.1; 0.1	0.145	30.38	23008
	-20	3	1.29; 4.47; 7.65	40.2; 40.5; 39.77	0.045; 0.042; 0.047	0.199	35.1	39623
	-10	3	1.433; 4.54; 7.8	36; 36.18; 35.17	0.047; 0.045; 0.023	0.173	35	31278
$\beta(units$	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
$/\$)$	10	3	1.57; 4.68; 7.955	30.4; 30.3; 29.46	0.06; 0.038; 0.021	0.133	29	19538
$/month$	20	3	1.63; 4.74; 8	28.9; 29.78; 28.57	0.079; 0.089; 0.063	0.125	28.9	15139
	-20	3	1.52; 4.6; 7.9	31.67; 31.6; 30.39	0.08; 0.061; 0.041	0.121	38.14	24086
	-10	3	1.522; 4.54; 7.89	32.5; 32.6; 31.3	0.08; 0.066; 0.041	0.138	37	24347
$\delta(units)$	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
$/month$	10	3	1.5; 4.6; 7.87	33.35; 34; 33.6	0.044; 0.039; 0.021	0.169	31.9	25030
	20	3	1.5; 4.55; 7.8	34.2; 34.6; 33.6	0.048; 0.032; 0.026	0.185	31.5	25362
	-20	3	1.52; 4.6; 7.89	32.8; 32.96; 32.1	0.07; 0.046; 0.004	0.185	35.2	25083
	-10	3	1.51; 4.58; 7.844	33.1; 33.34; 32.39	0.068; 0.051; 0.036	0.165	34.4	24889
k	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
	10	3	1.485; 4.586; 7.87	32.9; 32.7; 32	0.086; 0.046; 0.048	0.143	31.66	24569
	20	3	1.459; 4.58; 7.83	33.33; 33.7; 32.9	0.077; 0.07; 0.054	0.13	28	24451
	-20	3	1.54; 4.64; 7.9	33.1; 33.87; 32.55	0.57; 0.07; 0.042	0.152	39.2	24845

$c_s(\$/unit)$	-10	3	1.53; 4.62; 7.9	32.5; 32.94; 31.9	0.05; 0.032; 0.018	0.148	35.8	24819
	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
	10	3	1.48; 4.58; 7.84	32.2; 32.17; 31.5	0.047; 0.048; 0.041	0.151	33.24	24652
	20	3	1.41; 4.55; 7.79	32.18; 32; 31.1	0.048; 0.043; 0.029	0.149	32.33	24602
$\theta(/month)$	-40	3	1.486; 4.602; 7.856	32.9; 32.8; 31.9	0.041; 0.035; 0.022	0.149	31.9	24763
	0	3	1.507; 4.6; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
	40	3	1.51; 4.6; 7.883	32.5; 32.6; 31.7	0.05; 0.031; 0.015	0.149	38.9	24731
	100	3	1.51; 4.61; 7.89	32.9; 32.8; 32.1	0.062; 0.039; 0.028	0.149	46.7	24676
	500	3	1.54; 4.61; 7.9	35.9; 35.2; 31.8	0.14; 0.1; 0.015	0.156	47	24285
$H(months)$	-20	2	1.821; 5.507	32.9; 32.6	0.067; 0.043	0.144	46.3	19941
	-10	3	1.34; 4.16; 7.08	32.8; 32.97; 32.65	0.07; 0.042; 0.037	0.14	42.9	22395
	0	3	1.507; 4.589; 7.875	32.6; 32.9; 31.9	0.05; 0.04; 0.02	0.149	34	24750
	10	3	1.638; 5.06; 8.637	32.5; 32.8; 31.5	0.072; 0.034; 0.018	0.165	32.6	26698
	20	4	1.347; 4.131; 7.09; 10.05	32.7; 32.98; 32.56; 30.9	0.058; 0.042; 0.034; 0.019	0.147	27	28540

to discounts during shortage periods when prices are higher. Therefore, managers should pair higher pricing with increased discount rates to secure more early-bird bookings during shortages. A similar logic applies to order quantity adjustments.

When demand declines rapidly over time and the back-order rate is low, managers should lower retail prices to attract customers. Conversely, during periods of growing demand, higher prices in consecutive intervals are recommended to maximize revenue. However, during periods of declining demand, managers should strategically reduce prices to maintain sales. Balancing pricing adjustments with fluctuations in demand ensures profitability while addressing customer behavior effectively.

- *Discounting policy:* Table 3.7 highlights that an early-bird discounting policy significantly boosts revenue, particularly during a product's launch phase. As shown, discount rates decrease in successive periods, illustrating their alignment with demand patterns. The primary advantage of early-bird discounts lies in attracting customers during the initial stages of a product's life cycle. Offering a fixed discount rate throughout the business period, however, is an imprudent strategy. When base demand is high, discounts can be reduced as there is less need to stimulate demand. Conversely, discounts should be higher when demand decreases rapidly, as indicated by higher values of c.

  Proposition 3.5 indicates that the time parameter b plays a pivotal role in determining the discount rate. Higher discounts are effective in encouraging customers to wait, but only if the back-order rate is sufficiently low. This sufficiency depends on the shortage duration, $t_i - (i-1)T$. If the back-order rate is high, the managers are confident that higher values of b will sustain demand. When customer demand is highly sensitive to service quality, reduced discounts are justified since demand can be driven by improved service instead. On the other hand, higher price sensitivity necessitates greater discounts to attract customers and leverage their responsiveness to price changes. For products with high shortage costs, the stock-out period is minimized, reducing the need for aggressive discounting during shortages.

  Under high deterioration rates, the manager must prioritize clearing inventory quickly. In this scenario, offering higher discounts is an effective strategy to accelerate stock turnover before spoilage occurs. By dynamically adjusting discounts based on demand patterns, service sensitivity, and product deterioration, managers can optimize profitability while meeting customer expectations.

- *Replenishment policy:* Starting with a planned shortage can be advantageous for business managers. Such a strategy not only reduces holding costs and spoilage but also provides an opportunity to strategically control demand through well-informed decisions. Over successive periods, shortages typically decrease as demand stabilizes. The manager should shorten shortage periods when demand-enhancing factors like a, b, and δ are high, as a sufficiently high demand rate diminishes the benefits of planned stock-outs. In such cases, it is more prudent to incur higher holding or deterioration costs, as the additional demand ensures greater profitability.

  When shortage costs are high, the manager is compelled to significantly reduce the shortage period to minimize associated losses. Similarly, heightened sensitivity to service parameters encourages holding inventory longer, further reducing shortage durations. Conversely, a higher deterioration rate not only accelerates product spoilage but also extends the shortage period, as managers may struggle to maintain optimal stock levels.

  Price sensitivity also influences shortage durations. With higher price dependency, managers may prolong the shortage period to avoid excessive spoilage of costlier items. However, managing high per-unit deterioration costs by extending the shortage period becomes challenging when shortage costs are equally high. As the sensitivity analysis suggests, prolonged shortages in such scenarios can result in substantial costs.

  In these cases, managers must carefully assess the relative weight of influencing factors—such as demand, sensitivity, and cost parameters—and adjust their decisions accordingly. A nuanced approach ensures that planned shortages are strategically leveraged while minimizing adverse impacts on profitability.

- *Preservation investment:* As this current chapter focuses on the interplay between deterioration and preservation, the sensitivity of the deterioration parameter is of paramount importance. Preservation investment directly addresses the impact of deterioration, making it closely tied to this parameter. Preservation spending tends to increase with demand parameters a and b, as higher demand necessitates protecting larger inventories from spoilage. Conversely, as the demand-reducing parameter c rises, preservation investment decreases due to lower product allocation and reduced need for preservation efforts.
  When service sensitivity is high, managers prioritize service over preservation, leading to a reduction in preservation investment. Similarly, as the θ value (reflecting deterioration sensitivity) increases, preservation investment rises to counteract the increased risk of spoilage. However, higher shortage costs prompt managers to allocate fewer resources to preservation, as the focus shifts toward minimizing stock-out losses.
  Proposition 3.4(vii) highlights a positive relationship between the marginal effect of deterioration and the impact of preservation investment on order quantity. This indicates that managers must invest more in preservation technologies to maintain demand at acceptable levels when deterioration rates are high. On the other hand, greater price sensitivity (β), which shortens the stock-in period, limits the manager's ability to raise prices. In such cases, reducing preservation costs becomes a practical strategy for maintaining profitability. By carefully balancing preservation investments with these influencing factors, managers can effectively mitigate the effects of deterioration while optimizing overall business performance.

- *Service investment:* Higher demand parameters, such as a, b, and δ, encourage managers to allocate more resources toward servicing, with two notable exceptions: the price parameter β, where increased price sensitivity reduces service investment, and c, which negatively impacts demand and similarly lowers service investment. When customers are highly sensitive to price, the focus shifts from servicing to pricing strategies. Additionally, an increase in service-related costs raises overall expenses, prompting a reduction in the service level to balance total investment.

While shortage costs and deterioration rates have minimal direct impact on service investment, extremely high deterioration rates may lead to an increase in service spending. Proposition 3.4(vi) establishes that longer stock-out periods positively influence the relationship between service level and backlogged demand. In other words, customers become more discerning about service quality when faced with longer wait times, compelling managers to enhance service levels to retain customer satisfaction and make waiting worthwhile.
The sensitivity analysis of the service parameter δ reveals that businesses can increase prices or reduce discounts without negatively affecting profitability. For example, in industries like electronics, where after-sales support is crucial, higher pricing can be justified due to the emphasis on service quality. However, service patterns and associated costs vary by product and industry, making it impractical to adopt a uniform service strategy across all businesses.
For businesses with high service costs, an improved service level may impose a greater financial burden than the revenue it generates. In such scenarios, strategies must be realigned to optimize the trade-off between service quality and profitability. Managers must carefully tailor their approach to the specific demands and cost structures of their business to ensure sustainable growth.

- *Impact on profit:* The profit level increases with demand parameters that positively influence demand (a, b, and δ) and decreases with those that have a negative impact (c and β). This aligns with expectations, as higher demand naturally boosts profits without requiring additional investment. Conversely, higher price sensitivity (β) significantly reduces both demand and revenue, as companies are forced to lower prices to remain competitive, making β the most influential factor in reducing profit levels. Similarly, increased servicing costs (k) and higher shortage costs also negatively affect profitability.
  An extended business horizon generally provides managers with more opportunities to generate profit. However, once a product reaches its obsolescence—due to factors such as the launch of new products, technological advancements, or shifting market preferences—further extending the planning horizon becomes counterproductive. Additionally, average profit per cycle tends to decrease as the planning horizon grows, highlighting the importance of maintaining an optimal horizon length to avoid the inefficiencies of overly

short or long planning periods.

Deterioration rates negatively impact profit levels, though the effect is mitigated by investments in preservation technologies. However, while these investments help reduce losses from spoilage, they also increase total costs. To address this, managers should focus on optimizing shortage periods and increasing preservation investments. Extending the shortage period, coupled with higher discount rates, can boost sales and reduce deterioration costs by clearing inventory more effectively.

Ultimately, maintaining an appropriate balance between preservation efforts and other strategies, such as dynamic pricing and discounting during shortages, ensures that profit levels remain stable. By carefully managing these factors, businesses can mitigate risks and sustain profitability over time.

3.2.6 Conclusion

A multi-period inventory model incorporating learning effects is discussed in this study. Planned shortages are allowed, with rebates offered on selling prices to offset lost sales during extended waiting periods, optimizing overall profitability. Selling prices are adjusted at the start of each cycle based on market conditions, while service levels improve progressively due to learning effects. Preservation technology investment is also optimized, providing insights into adjusting decisions under changing business scenarios. The analysis highlights how key decision variables-such as discount rates during shortages- evolve with each cycle, helping managers allocate resources effectively. The study establishes that planned shortages can be advantageous within a controlled demand range, and offering discounts to patient customers is a profitable strategy when customer reluctance to wait is high. Furthermore, discounts should align positively with pricing strategies.

Potential extensions to the model include updating the discrete dynamic pricing and discounting approach to a continuous framework. The current assumption of a constant deterioration rate could be replaced with a time-varying rate for more realistic applications. Dynamic preservation technology investment and incorporating inventory levels into the demand function could further broaden the model's scope. Additionally, the deterministic nature of the current model could be expanded to consider stochastic demand, reflecting real-world uncertainties. The model could also be extended to a two-level supply chain framework, examining the interactions between channel members under profit-enhancing policies such as trade credit, sales rebates, or buyback agreements. These extensions would provide valuable insights into managing inventory and pricing strategies across a wider range of practical business scenarios.

CHAPTER 4

Developing Pricing and Lead Time Strategies in a Perishable Inventory Model Facing Price, Quality, and Stock-dependent Demand

4.1 Introduction

"The moment you make a mistake in pricing, you're eating into your reputation or your profits."- Katharine Paine

For perishable goods like fruits and vegetables, freshness is of paramount importance. Consumers are willing to pay a premium for healthier, fresher options. However, all products degrade over time—whether it's freshness loss in food or evaporation in gasoline—leading to reduced sales and increased management costs. While preservation technology can slow deterioration, this chapter assumes the business cycle is shorter than the product's expiration date. The chapter explores an inventory model where price adjusts dynamically during the stock-in period to address varying demand. Demand depends on both freshness and inventory level, as customers are drawn to well-stocked shelves. Yet, overstocking ties up cash flow, raises carrying costs, and risks obsolescence. Shortages are managed with waiting-time-dependent partial backlogging, where discounts incentivize customers to wait for

*This chapter is based on the work published in *International Journal of Systems Science: Operations and Logistics*, volume 12, issue 1, article id 2462098.

replenishment. However, not all customers will wait, making this a practical consideration. Key insights reveal that dynamic pricing, based on freshness and inventory levels, balances demand while mitigating deterioration effects. Discounts during shortages encourage customer patience, reducing stock-out impacts. The proposed model integrates preservation investment, inventory-dependent demand, and partial backlogging to offer actionable strategies for managing perishable products in competitive markets.

Motivation of the current work

To enhance the effectiveness of pricing strategies, it is crucial to simultaneously analyze other factors that influence demand. It is interesting to note that for products where freshness is a concern, higher inventory levels often attract more buyers due to the perception of greater freshness. Since both freshness and inventory levels vary over time, it is imperative for business managers to adopt dynamic pricing strategies that adjust with these changes. While previous studies, such as Zhang et al., 2015, have explored demand as a function of price and inventory levels, they did not account for freshness or incorporate planned shortages into their models. From a practical perspective, however, freshness is a critical determinant of demand for perishable goods. Furthermore, the impact of freshness loss during transportation must also be considered for a more comprehensive approach. This chapter seeks to address the following research questions:
RQ1: How does a dynamic pricing strategy impact the market demand for perishable products over time when demand depends on freshness, inventory level, and price?
RQ2: How does investing in preservation to reduce freshness loss during transportation influence overall inventory management and profitability? Is the investment beneficial at all?
RQ3: What is the effect of price discount on lost sale reduction? How does this strategy influence customer behavior and demand during shortages?
RQ4: What is the optimal strategy for balancing dynamic pricing, preservation investment, and shortage management in order to maximize profitability and customer satisfaction?
To make the proposed model realistic, the work considers waiting time-dependent

partial backlogging. To encourage the customer to wait, a waiting time-dependent price discount is adopted during the shortage. The customers who are patient enough to wait for the next replenishment will get the freshest item compared to others, that too at a reduced price. Following Modak et al., 2024b, the investment to improve the freshness loss is adopted in this work. Following the solution methods from Sethi and Thompson, 2000 for the pricing part, the shortage period is not only included but every aspect related to that is analyzed properly.

4.2 Notations and Assumptions

The following notations are used throughout the chapter.

Table 4.1: Notations

$p(t)$	:	unit retail price of the item at time t during stock-in period (dynamic decision variable)
p_0	:	unit retail price during shortage period (static decision variable)
ξ_1	:	preservation investment per unit time during transportation (static decision variable)
ξ_2	:	preservation investment per unit time in warehouse (static decision variable)
T	:	complete length of business cycle (static decision variable)
c_l	:	investment to reduce the lead time (static decision variable)
t_0	:	replenishment time point
$m(\xi)$	:	reduced deterioration rate per unit preservation investment
I_0	:	inventory level just after replenishment
$g(t)$	:	function denoting freshness index
$I_1(t)$	:	inventory level at time t during $(0, t_l)$, *i.e.* stock-out period
$I_2(t)$	:	inventory level at time t during (t_l, T), *i.e.* stock-in period

$B(t)$	:	time dependent backlogging rate
Q	:	replenishment size
c_m	:	unit purchasing cost
c_d	:	unit deterioration cost
h	:	unit holding cost per unit time
A	:	ordering cost per order
c_s	:	per unit shortage cost
S	:	total backlog

The following assumptions are made for development of the proposed model.

- The deterioration is assumed to be instantaneous in nature and qualitative only (Li et al., 2019). The preservation investment helps to slow down freshness losing process. The actual deterioration rate θ is assumed to be reduced to $\{1 - m(x)\}\theta$ with x amount of investment, where $m(x)$ is an increasing function of x with $m''(x) < 0$ to ensure diminishing return on investment, and $0 \leq m(x) \leq 1$. The initial freshness of a product is assumed to be 1. Since the preservation infrastructure is different when the product is being transported or is in the warehouse, so should be the corresponding investments too.

- An investment c_l is made to reduce the deterministic lead time t_l. We assume $\frac{dt_l}{dc_l} < 0$ to justify such investment, and $\frac{d^2t_l}{dc_l^2} < 0$ to ensure diminishing return of investment. In consistent with the assumption and following Sarkar et al., 2022, the specific pattern of the transportation lead time is assumed as $t_l = e^{-\rho c_l}t_0$, where t_0 is the basic lead time in absence of any investment, and ρ indicates the efficiency of the system in the sense that, with higher values of it, a fixed investment would result in lesser lead time.

- Shortages are partially backlogged, where the backlog rate depends on waiting time until next replenishment. The precise functional form is $B(t) = e^{-\eta t}$, η being a non-negative parameter depicting impatience of the customers to wait (Modak et al., 2024a). Since higher value of η indicates lower backlog rate, η may be considered to represent a measure of lost sale. All the customers arriving during shortage period will get the same quality product. To mitigate the effect of waiting time to some extend, customers are offered discounted price which is directly proportional to the waiting time. Following Rabbani et al., 2016, the specific pattern of pricing during shortage period is assumed as $p_0 e^{-\sigma(t_l - t)}$, where p_0 is the price when the shipment arrives at the inventory, and σ is the discounting factor.

- The business period consists of two parts: the first part is shortage period, followed by a stock-in period. Following Macías-López et al., 2021, the demand is assumed to depend on price, on-hand stock, and freshness of the product simultaneously during stock-in period, and only on the price and the freshness level during stock-out period. The price during stock-in period is dynamic in nature, and it varies continuously with time. The demand pattern is specified as $D = \{a - bp(t) + \alpha I(t)\}g(t)$ where $g(t) = ke^{-(1-m)\theta t}$ represents freshness of the product at time t. It is to be noted that while placing an order during shortage period, the customers know at what time it will be delivered, and what the freshness level of the product will be at that time. During shortage, the demand rate is thus justified to be considered as $D(p_0, t, t_l) = \left(a - bp_0 e^{-\sigma(t_l - t)}\right) g(t_l)$.

- There is no goodwill cost for lost sale.

4.3 Model Formulation

The sequence of events for the proposed model is as follows. The retailer places an order at the beginning of the business cycle. The products leave supplier's inventory instantaneously, but needs t_l time period to reach retailer's inventory. The retailer invests ξ_1 in preserving the quality of the products during transportation, and an amount c_l to reduce lead time. He offers a price discount for the customers arriving during shortage period. The products reach retailer's inventory at time point t_l; payment to supplier and the backlogged demand is met. The retailer further invests in preservation for the stock-in period, and decides the duration of the business cycle.

4.3.1 Total profit in stock-out period ($0 \leqslant t \leqslant t_l$)

The change in inventory level during shortage period is due to partially backlogged demand only. Considering that the demand is further influenced by discounted price, the rate of change of inventory level is governed by the differential equation

$$\frac{dI_1(t)}{dt} = -ke^{-\eta(t_l-t)}\left(a - bp_0e^{-\sigma(t_l-t)}\right)e^{-(1-m_1)\theta t_l}, \quad 0 < t < t_l.$$

Solving the differential equation with the initial condition $I(0) = 0$, the inventory level at any time t can be obtained as

$$I_1(t) = -ke^{-(1-m_1)\theta t_l}\left\{\frac{ae^{-\eta t_l}\left(e^{\eta t}-1\right)}{\eta} - \frac{bp_0e^{-(\eta+\sigma)t_l}\left(e^{(\eta+\sigma)t}-1\right)}{(\eta+\sigma)}\right\}.$$

The total backlogged item S is obtained as

$$S = I_1(t_l) = ke^{-(1-m_1)\theta t_l}\left\{\frac{a\left(1-e^{-\eta t_l}\right)}{\eta} - \frac{bp_0\left(1-e^{-(\eta+\sigma)t_l}\right)}{(\eta+\sigma)}\right\}. \qquad (4.1)$$

The total revenue for the first period is obtained as

$$\begin{aligned} TR_1 &= \int_0^{t_l} p_0e^{-\sigma(t_l-t)}D(t)dt \\ &= ke^{-(1-m_1)\theta t_l}\left[\frac{ap_0\left(1-e^{-(\sigma+\eta)t_l}\right)}{\sigma+\eta} - \frac{bp_0^2}{2\sigma+\eta}\left(1-e^{-(2\sigma+\eta)t_l}\right)\right]. \end{aligned}$$

The shortage cost during the stock-out period is obtained as

$$SC = kc_se^{-(1-m_1)\theta t_l}\left[a\left(t_l - \frac{1-e^{-\eta t_l}}{\eta}\right) - bp_0\left\{\frac{\eta}{\sigma(\sigma+\eta)} - e^{-\sigma t_l}\left(\frac{1}{\sigma} - \frac{e^{-\eta t_l}}{(\sigma+\eta)}\right)\right\}\right].$$

Noting that the costs associated with the first period are ordering cost, purchasing cost, shortage cost, cost to reduce lead time, and preservation investment for transportation period, the total profit for the first period is given by

$$\begin{aligned} TP_1(\xi_1, p_0, c_l) &= TR_1 - A - SC - c_l - \xi_1 t_l \\ &= ke^{-(1-m_1)\theta t_l}\left[\frac{ap_0\left(1-e^{-(\sigma+\eta)t_l}\right)}{\sigma+\eta} - \frac{bp_0^2}{2\sigma+\eta}\left(1-e^{-(2\sigma+\eta)t_l}\right)\right] \\ &\quad -A - c_m Q - c_l - \xi_1 t_l - kc_s e^{-(1-m_1)\theta t_l}\left[a\left(t_l - \frac{1-e^{-\eta t_l}}{\eta}\right)\right. \\ &\quad \left. -bp_0\left\{\frac{\eta}{\sigma(\sigma+\eta)} - e^{-\sigma t_l}\left(\frac{1}{\sigma} - \frac{e^{-\eta t_l}}{(\sigma+\eta)}\right)\right\}\right]. \end{aligned} \tag{4.2}$$

4.3.2 Total profit in stock-in period $(t_l \leqslant t \leqslant T)$

Note that the product reaches the inventory with freshness level $e^{-(1-m_1)\theta t_l}$. With the preservation investment ξ_2 in the warehouse, the freshness level at the warehouse would be $e^{-(1-m_1)\theta t_l}e^{-(1-m_2)\theta(t-t_l)}$ which is equal to $e^{-(m_2-m_1)\theta t_l}e^{-(1-m_2)\theta t}$. Denoting $k_1 = ke^{-(m_2-m_1)\theta t_l}$, we now derive the optimal dynamic pricing policy during the stock-in period using Pontryagin's maximum principle. The objective function is rewritten as

$$\max_{p(.)} J_L = \frac{1}{T}\left(\int_{t_l}^{T} p(t)k_1(a - bp(t) + \alpha I(t))e^{-(1-m_2)\theta t}dt - \int_{t_l}^{T} \xi_2 dt - h\int_{t_l}^{T} I(t)dt\right), \tag{4.3}$$

subject to the stationary condition

$$\frac{dI(t)}{dt} = -k_1\{a - bp(t) + \alpha I(t)\}e^{-(1-m_2)\theta t}. \tag{4.4}$$

The associated Hamiltonian function is given by

$$\begin{aligned} H(I, p, \lambda, t) &= p(t)k_1(\{a - bp(t) + \alpha I(t)\}e^{-(1-m_2)\theta t} - hI(t) - \xi \\ &\quad -\lambda k_1\{a - bp(t) + \alpha I(t)\}e^{-(1-m_2)\theta t}, \quad (t_l \leqslant t \leqslant T), \end{aligned} \tag{4.5}$$

λ being the costate variable. According to the principle mentioned above, for the existence of optimal pricing policy, there must exist a continuous and piecewise continuously differentiable function $\lambda(\cdot)$ satisfying costate equation $\frac{\partial \lambda}{\partial t} = -\frac{\partial H}{\partial I}$, *i.e.*

$$\frac{d\lambda}{dt} = h + \lambda k_1 \alpha e^{-(1-m_2)\theta t} - p(t) k_1 \alpha e^{-(1-m_2)\theta t}, \qquad (t_l \leqslant t \leqslant T) \quad (4.6)$$

with the transversality condition $\lambda(T)I(T) = 0$.
The Hamiltonian maximizing condition thus becomes

$$H(I^*(t), p^*(t), \lambda(t), t) = \max_{0 \leqslant p(t) \leqslant \frac{a}{b}} H(I^*(t), p(t), \lambda(t), t).$$

Since the state variable $I(t)$ is always non-negative, the Hamiltonian function is concave in p. From equation 4.5, the optimal pricing policy can be derived as

$$p^*(t) = \begin{cases} c_m & \lambda \leqslant -\frac{a+\alpha I(t)-2bc_m}{b} \\ \frac{a+\lambda b+\alpha I(t)}{2b} & -\frac{a+\alpha I(t)-2bc_m}{b} \leqslant \lambda \leqslant \frac{a-\alpha I(t)}{b} \\ \frac{a}{b} & \lambda \geqslant \frac{a-\alpha I(t)}{b} \end{cases}. \quad (4.7)$$

Due to the nature of the pricing strategy, the model becomes a non-linear one. Substituting in 4.4 the expression of the price obtained in 4.7 and then differentiating, we get

$$\frac{d^2 I}{dt^2} + (1-m_2)\theta \frac{dI}{dt} = \frac{bhk_1}{2} e^{-(1-m_2)\theta t}, \quad (4.8)$$

solving which we get

$$I(t) = c_1 + c_2 e^{-(1-m_2)\theta t} - \frac{bhk_1 t}{2(1-m_2)\theta} e^{-(1-m_2)\theta t},$$

where c_1 and c_2 are constants of integration. Using $I(t_l) = Q - S$, the value of c_1 is obtained as

$$c_1 = (Q-S) + \frac{bhk_1 t_l}{2(1-m_2)\theta} e^{-(1-m_2)\theta t_l} - c_2 e^{-(1-m_2)\theta t_l}. \tag{4.9}$$

Substitution of $p(t)$ (given in 4.7) into 4.4 yields

$$\begin{aligned}
\frac{dI(t)}{dt} &= -k_1 \left(\frac{a - \lambda b + \alpha I(t)}{2}\right) e^{-(1-m_2)\theta t}, \\
\text{so that } \lambda(t) &= \frac{2}{bk_1} e^{(1-m_2)\theta t} \frac{dI(t)}{dt} + \frac{a}{b} + \frac{\alpha}{b} I(t), \\
\text{or } \lambda(t) &= \frac{2}{bk_1} \left\{ \frac{bhk_1 t}{2} - \frac{bhk_1}{2(1-m_2)\theta} - c_2(1-m_2)\theta \right\} + \frac{a}{b} \\
&\quad + \frac{\alpha}{b} \left(c_1 + c_2 e^{-(1-m_2)\theta t} - \frac{bhk_1 t}{2(1-m_2)\theta} e^{-(1-m_2)\theta t} \right). \qquad (4.10)
\end{aligned}$$

Finally, using $\lambda(T)I(T) = 0$, the value of c_2 is obtained. Equation 4.10 further yields

$$\begin{aligned}
\lambda(t_l) &= \frac{2}{bk_1} \left(\frac{bhk_1 t_l}{2} - \frac{bhk_1}{2(1-m_2)\theta} - c_2(1-m_2)\theta \right) + \frac{a}{b} \\
&\quad + \frac{\alpha}{b} \left(c_1 + c_2 e^{-(1-m_2)\theta t_l} - \frac{bhk_1 t_l}{2(1-m_2)\theta} e^{-(1-m_2)\theta t_l} \right), \\
\text{and } \dot{\lambda}(t) &= h - \frac{\alpha}{b} \left\{ (1-m_2)\theta c_2 + \frac{bhk_1}{2(1-m_2)\theta} - \frac{bhk_1}{2} \right\} e^{-(1-m_2)\theta t}.
\end{aligned}$$

Note that $\lambda(t)$ is strictly increasing or decreasing throughout the interval (t_l, T) according as $h > \frac{\alpha}{b}\left((1-m_2)\theta c_2 + \frac{bhk_1}{2(1-m_2)\theta} - \frac{bhk_1}{2}\right)$ or $h < \frac{\alpha}{b}\left((1-m_2)\theta c_2 + \frac{bhk_1}{2(1-m_2)\theta} - \frac{bhk_1}{2}\right)e^{-(1-m_2)\theta T}$, respectively. The pricing strategy is summarized in the the following proposition.

Proposition 4.1. *(i) If* $\frac{\alpha}{b}\left((1-m_2)\theta c_2 + \frac{bhk_1}{2(1-m_2)\theta} - \frac{bhk_1}{2}\right) e^{-(1-m_2)\theta t_l} < h < \frac{2(1-m_2)\theta(ak_1+\alpha k_1(Q-S)-bk_1c_m-c_2(1-m_2)\theta)}{bk_1(1-(1-m_2)\theta t_l)}$ *or* $\frac{(\alpha(Q-S)k_1-c_2(1-m_2)\theta)(1-m_2)\theta}{bk_1(1-(1-m_2)\theta t_l)} < h <$

$\frac{\alpha}{b}\left((1-m_2)\theta c_2+\frac{bhk_1}{2(1-m_2)\theta}-\frac{bhk_1}{2}\right)e^{-(1-m_2)\theta T}$, *the pricing strategy will be*

$$\begin{aligned} p^*(t) &= \frac{a}{b}+\frac{\alpha}{b}\left(c_1+c_2e^{-(1-m_2)\theta t}-\frac{bhk_1t}{2(1-m_2)\theta}e^{-(1-m_2)\theta t}\right) \\ &\quad +\left(\frac{ht}{2}-\frac{h}{2(1-m_2)\theta}-\frac{c_2}{bk_1}(1-m_2)\theta\right), \qquad t_l \leqslant t \leqslant T; \end{aligned}$$

(ii) If $h > \max\left\{\frac{2(1-m_2)\theta(ak_1+\alpha k_1(Q-S)-bk_1c_m-c_2(1-m_2)\theta)}{bk_1(1-(1-m_2)\theta t_l)}, \frac{\alpha}{b}\left((1-m_2)\theta c_2+\frac{bhk_1}{2(1-m_2)\theta}-\frac{bhk_1}{2}\right)e^{-(1-m_2)\theta t_l}\right\}$, *the pricing strategy will be*

$$p^*(t)=\begin{cases} c_m & t_l \leqslant t \leqslant t_1 \\ \frac{a}{b}+\frac{\alpha}{b}\left(c_1+c_2e^{-(1-m_2)\theta t}-\frac{bhk_1t}{2(1-m_2)\theta}e^{-(1-m_2)\theta t}\right) & \\ +\left(\frac{ht}{2}-\frac{h}{2(1-m_2)\theta}-\frac{c_2}{bk_1}(1-m_2)\theta\right) & t_1 \leqslant t \leqslant T \end{cases},$$

where t_1 *is the solution of the equation* $\lambda(t)=-\frac{a+\alpha I(t)-2bc_p}{b}$;

(iii) If $h < \min\left\{\frac{(\alpha(Q-S)k_1-c_2(1-m_2)\theta)(1-m_2)\theta}{bk_1(1-(1-m_2)\theta t_l)}, \frac{\alpha}{b}\left((1-m_2)\theta c_2+\frac{bhk_1}{2(1-m_2)\theta}-\frac{bhk_1}{2}\right)\times e^{-(1-m_2)\theta T}\right\}$, *the pricing strategy will be*

$$p^*(t)=\begin{cases} \frac{a}{b} & t_l \leqslant t \leqslant t_2 \\ \frac{a}{b}+\frac{\alpha}{b}\left(c_1+c_2e^{-(1-m_2)\theta t}-\frac{bhk_1t}{2(1-m_2)\theta}e^{-(1-m_2)\theta t}\right) & \\ +\left(\frac{ht}{2}-\frac{h}{2(1-m_2)\theta}-\frac{c_2}{bk_1}(1-m_2)\theta\right) & t_2 \leqslant t \leqslant T \end{cases},$$

where t_2 *is the solution of the equation* $\lambda(t)=-\frac{a-\alpha I(t)}{b}$.

Proof: For cases when the end level inventory is not zero, during (t_l, T), the optimal price as given in 4.7 is substituted in the costate equation 4.6 to obtain the following four regimes:

$$R_1 \;:\; \lambda(t) \leqslant -\frac{a+\alpha I(t)-2bc_m}{b},\; p^*(t)=c_m,\; \dot{I}(t)=-k_1(a-bc_m+\alpha I(t))e^{-(1-m_2)\theta t},$$

$$\dot{\lambda}(t)=h-\frac{\alpha}{b}\left\{(1-m_2)\theta c_2+\frac{bhk_1}{2(1-m_2)\theta}-\frac{bhk_1}{2}\right\}e^{-(1-m_2)\theta t}$$

$$R_2 \;:\; -\frac{a+\alpha I(t)-2bc_m}{b} < \lambda(t) \leqslant 0,\; p^*(t)=\frac{a+\lambda b+\alpha I(t)}{2b},$$

$$\dot{I}(t)=-k_1\frac{(a-\lambda b+\alpha I(t))}{2}e^{-(1-m_2)\theta t}$$

$$,\; \dot{\lambda}(t)=h-\frac{\alpha}{b}\left\{(1-m_2)\theta c_2+\frac{bhk_1}{2(1-m_2)\theta}-\frac{bhk_1}{2}\right\}e^{-(1-m_2)\theta t}$$

$$R_3 \;:\; 0<\lambda(t)<\frac{a-\alpha I(t)}{b},\; p^*(t)=\frac{a+\lambda b+\alpha I(t)}{2b},$$

$$\dot{I}(t)=-k_1\frac{(a-\lambda b+\alpha I(t))}{2}e^{-(1-m_2)\theta t}$$

$$,\; \dot{\lambda}(t)=h-\frac{\alpha}{b}\left\{(1-m_2)\theta c_2+\frac{bhk_1}{2(1-m_2)\theta}-\frac{bhk_1}{2}\right\}e^{-(1-m_2)\theta t}$$

$$R_4 \;:\; \lambda(t) \geqslant \frac{a-\alpha I(t)}{b},\; p^*(t)=\frac{a}{b},\; \dot{I}(t)=-\alpha k_1 I(t)e^{-(1-m_2)\theta t},$$

$$\dot{\lambda}(t)=h-\frac{\alpha}{b}\left\{(1-m_2)\theta c_2+\frac{bhk_1}{2(1-m_2)\theta}-\frac{bhk_1}{2}\right\}e^{-(1-m_2)\theta t},$$

on the basis of which it is seen that

$$\dot{\lambda}_{R_2}(t)\Big|_{\lambda_{R_2}(t)\to-\frac{a+\alpha I(t)-2bc_m}{b}}=\dot{\lambda}_{R_1}(t)\Big|_{\lambda_{R_1}(t)\to-\frac{a+\alpha I(t)-2bc_m}{b}}$$

$$=h-\frac{a+\alpha I(t)-bc_m}{b}k_1\alpha e^{-(1-m)\theta t},$$

$$\dot{\lambda}_{R_3}(t)\Big|_{\lambda_{R_3}(t)\to 0}=\dot{\lambda}_{R_2}(t)\Big|_{\lambda_{R_2}(t)\to 0}=h-\frac{a+\alpha I(t)}{2b}k_1\alpha e^{-(1-m)\theta t},$$

$$\dot{\lambda}_{R_3}(t)\Big|_{\lambda_{R_3}(t)\to\frac{a-\alpha I(t)}{b}}=\dot{\lambda}_{R_4}(t)\Big|_{\lambda_{R_4}(t)\to\frac{a-\alpha I(t)}{b}}=h-\frac{\alpha I(t)}{b}k_1\alpha e^{-(1-m)\theta t}.$$

The ending value of λ is 0. So the possible sequences are $R_1 \to R_2$, $R_3 \to R_2$, and $R_4 \to R_3 \to R_2$. The pricing strategy for the case $R_1 \to R_2$ is

$$p^*(t) = \begin{cases} c_m & t_l \leqslant t \leqslant t_1 \\ \frac{a+b\lambda(t)+\alpha I(t)}{2b} & t_1 \leqslant t \leqslant T \end{cases},$$

the pricing strategy for the case $R_3 \to R_2$ is

$$p^*(t) = \frac{a + b\lambda(t) + \alpha I(t)}{2b} \quad t_l \leqslant t \leqslant T,$$

and the pricing strategy for the case $R_4 \to R_3 \to R_2$ is

$$p^*(t) = \begin{cases} \frac{a}{b} & t_l \leqslant t \leqslant t_2 \\ \frac{a+b\lambda(t)+\alpha I(t)}{2b} & t_2 \leqslant t \leqslant T\,. \end{cases}$$

Solving $\dot{\lambda}(t) = h - \frac{\alpha}{b}\left\{(1-m_2)\theta c_2 + \frac{bhk_1}{2(1-m_2)\theta} - \frac{bhk_1}{2}\right\} e^{-(1-m_2)\theta t}$ with $\lambda(T) = 0$, we get

$$\begin{aligned} \lambda(t) \quad &= \quad \frac{2}{bk_1}\left\{\frac{bhk_1 t}{2} - \frac{bhk_1}{2(1-m_2)\theta} - c_2(1-m_2)\theta\right\} + \frac{a}{b} \\ &\quad + \frac{\alpha}{b}\left(c_1 + c_2 e^{-(1-m_2)\theta t} - \frac{bhk_1 t}{2(1-m_2)\theta} e^{-(1-m_2)\theta t}\right), \end{aligned}$$

so that $\lambda(t_l) = \frac{2}{bk_1}\left(\frac{bhk_1 t_l}{2} - \frac{bhk_1}{2(1-m_2)\theta} - c_2(1-m_2)\theta\right) + \frac{a}{b}$
$+ \frac{\alpha}{b}\left(c_1 + c_2 e^{-(1-m_2)\theta t_l} - \frac{bhk_1 t_l}{2(1-m_2)\theta} e^{-(1-m_2)\theta t_l}\right).$

Putting the derived expression of λ in 4.7, the results are obtained.

To approximate the values of t_is, the exponential function $e^{-(1-m_2)\theta t}$ is further expanded in Maclaurin's series up to second order. The equations $\lambda(t) = -\frac{a+\alpha I(t)-2bc_p}{b}$, $\lambda(t) = \frac{a-\alpha I(t)}{b}$ and $\lambda(t) = \frac{2bp_0-a-\alpha I(t)}{b}$ can respectively be elaborated as

$$\begin{aligned} &\frac{hk_1(1-m_2)\theta\alpha}{4} t^3 - \left(\frac{(1-m_2)^2\theta^2 c_2\alpha}{2b} + \frac{hk_1\alpha}{2}\right) t^2 + \left(\frac{(1-m_2)\theta\alpha c_2}{b} + \frac{hk_1 t\alpha}{2(1-m_2)\theta}\right. \\ &\left. -\frac{h}{2}\right) t - \left(\frac{a}{b} + \frac{\alpha}{b}(c_1+c_2) - \frac{h}{2(1-m_2)\theta} + 2c_m - \frac{c_2(1-m_2)\theta}{bk_1}\right) = 0, \end{aligned} \tag{4.11}$$

$$\frac{\alpha hk_1(1-m_2)\theta}{4}t^3 - \left(\frac{\alpha hk_1}{2} + \frac{(1-m_2)^2\theta^2\alpha c_2}{2b}\right)t^2 + \left(\frac{\alpha hk_1}{2(1-m_2)\theta} + \frac{(1-m_2)\theta\alpha c_2}{b}\right.$$
$$\left.-\frac{h}{2}\right)t - \left(\frac{\alpha}{b}(c_1+c_2) - \frac{h}{2(1-m_2)\theta} - \frac{c_2(1-m_2)\theta}{bk_1}\right) = 0, \text{and} \tag{4.12}$$

$$\frac{hk_1(1-m_2)\theta\alpha}{4}t^3 - \left(\frac{(1-m_2)^2\theta^2c_2\alpha}{2b} + \frac{hk_1\alpha}{2}\right)t^2 + \left(\frac{(1-m_2)\theta\alpha c_2}{b} + \frac{hk_1t\alpha}{2(1-m_2)\theta}\right.$$
$$\left.-\frac{h}{2}\right)t - \left(\frac{a}{b} + \frac{\alpha}{b}(c_1+c_2) - \frac{h}{2(1-m_2)\theta} + 2p_0 - \frac{c_2(1-m_2)\theta}{bk_1}\right) = 0. \tag{4.13}$$

As evident from the nature of $\lambda(t)$, it can have at most one real root. Let us denote the real roots of the equations 4.11, 4.12 and 4.13 by t_1, t_2 and t_3, respectively.

Based on the pricing strategy, the total holding cost can be derived as

$$HC = h\left[c_1(T-t_l) + \frac{c_2}{(1-m_2)\theta}\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta T}\right) - \frac{bhk_1}{2(1-m_2)\theta} \times \right.$$
$$\left.\left\{\frac{\left(t_le^{-(1-m_2)\theta t_l} - Te^{-(1-m_2)\theta T}\right)}{(1-m_2)\theta} + \frac{\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta T}\right)}{(1-m_2)^2\theta^2}\right\}\right].$$

The demand at any particular time t during stock-in period may now be determined as $D(t) = k_1e^{-(1-m_2)\theta t}\left(\frac{h}{2(1-m_2)\theta} + \frac{c_2}{bk_1}(1-m_2)\theta - \frac{ht}{2}\right)$, and the revenue at time t is obtained as

$$Rev(t) = \frac{e^{-(1-m_2)\theta t}}{b}\left(\frac{bhk_1t}{2} - \frac{bhk_1}{2(1-m_2)\theta} - c_2(1-m_2)\theta\right)\left[a + \alpha\left(c_1 + c_2e^{-(1-m_2)\theta t}\right.\right.$$
$$\left.\left.-\frac{bhk_1t}{2(1-m_2)\theta}e^{-(1-m_2)\theta t}\right) - \frac{1}{k_1}\left(\frac{bhk_1t}{2} - \frac{bhk_1}{2(1-m_2)\theta} - c_2(1-m_2)\theta\right)\right].$$

The total revenue is $TR = \int_{t_l}^{T} Rev(t)dt$. Since the costs associated with the stock-in period are holding cost, purchase cost, and preservation investment, the total profit in the second period is given by $TP_2 = TR - HC - c_mQ - \xi_2(T-t_l)$. Therefore, the average profit in a business cycle is given by

$$\Pi(p_0, \xi_1, \xi_2, c_l, T, p(t)) = \frac{TP_1(\xi_1, p_0, c_l) + TP_2(\xi_1, \xi_2, T, c_l)}{T}. \tag{4.14}$$

Our aim is to maximize the average profit function Π with respect to its decision variables. Next proposition throws some light in this regard.

Proposition 4.2. *(i) The average profit function Π is concave in p_0; (ii) The profit during stock-out period is concave in ξ_1.*

(i) Since the profit of the stock-in period does not involve p_0, we are to consider TP_1 only. Since

$$\begin{aligned}\frac{\partial TP_1}{\partial p_0} &= ke^{-(1-m_1)\theta t_l}\left[\frac{a\left(1-e^{-(\sigma+\eta)t_l}\right)}{(\sigma+\eta)} - \frac{2bp_0}{(2\sigma+\eta)}\left(1-e^{-(2\sigma+\eta)t_l}\right)\right] \\ &\quad +bkc_s e^{-(1-m_1)\theta t_l}\left\{\frac{\eta}{\sigma(\sigma+\eta)} - e^{-\sigma t_l}\left(\frac{1}{\sigma}-\frac{e^{-\eta t_l}}{(\sigma+\eta)}\right)\right\},\end{aligned}$$

we have
$$\frac{\partial^2 TP_1}{\partial p_0^2} = -\frac{2bke^{-(1-m_1)\theta t_l}}{(2\sigma+\eta)}\left(1-e^{-(2\sigma+\eta)t_l}\right) < 0,$$

Hence the proof is completed.

$$\begin{aligned}(ii)\frac{\partial TP_1}{\partial \xi_1} &= k\delta\theta t_l e^{-(1-m_1)\theta t_l}e^{-\delta\xi_1}\left[\left\{\frac{ap_0\left(1-e^{-(\sigma+\eta)t_l}\right)}{\sigma+\eta} - \frac{bp_0^2}{2\sigma+\eta}\left(1-e^{-(2\sigma+\eta)t_l}\right)\right\}\right. \\ &\quad \left. -c_s\left\{a\left(t_l-\frac{1-e^{-\eta t_l}}{\eta}\right) - bp_0\left\{\frac{\eta}{\sigma(\sigma+\eta)} - e^{-\sigma t_l}\left(\frac{1}{\sigma}-\frac{e^{-\eta t_l}}{(\sigma+\eta)}\right)\right\}\right\}\right] \\ &\quad -t_l, \\ \frac{\partial^2 TP_1}{\partial \xi_1^2} &= -k\delta^2\theta t_l\left(1-\theta t_l e^{-\delta\xi_1}\right)e^{-(1-m_1)\theta t_l}e^{-\delta\xi_1}\left[\left\{\frac{ap_0\left(1-e^{-(\sigma+\eta)t_l}\right)}{\sigma+\eta}\right.\right. \\ &\quad \left. -\frac{bp_0^2}{2\sigma+\eta}\left(1-e^{-(2\sigma+\eta)t_l}\right)\right\} - c_s\left\{a\left(t_l-\frac{1-e^{-\eta t_l}}{\eta}\right)\right. \\ &\quad \left.\left. -bp_0\left\{\frac{\eta}{\sigma(\sigma+\eta)} - e^{-\sigma t_l}\left(\frac{1}{\sigma}-\frac{e^{-\eta t_l}}{(\sigma+\eta)}\right)\right\}\right\}\right] < 0,\end{aligned}$$

completing the proof.

The complexity of the model restricted us from deriving further analytical results. In the next section, the model is illustrated numerically to gain managerial insights from it. The following algorithm is used to find the optimal values of the decision variables of the developed model.

Algorithm

Step 1: Set $\epsilon = 0.01$, $\xi_1 = 0$, $c_l = 0$, $p_0 = c_m$, $\xi_2 = 0$, $T = t_l$, $\Pi^* = 0$, and go to step 2.
Step 2: Set predetermined terminal values of Ξ_1, C_l, P_0, Ξ_2, and T_r, then go to step 3.
Step 3: Set $\xi_1 = \xi_1 + \epsilon$, then go to step 4.
Step 4: Set $c_l = c_l + \epsilon$, then go to step 5.
Step 5: Set $\xi_2 = \xi_2 + \epsilon$, then go to step 6.
Step 6: Set $T = T + \epsilon$, then go to step 7.
Step 7: Obtain unique p_0 (by virtue of proposition 2), and calculate Π. If $\Pi^* < \Pi$, set $\Pi^* = \Pi$, $\xi_1^* = \xi_1$, $c_l^* = c_l$, $\xi_2^* = \xi_2$, $T^* = T$ and $p_0^* = p_0$, then go to step 8.
Step 8: If $\xi_1 < \Xi_1$, $c_l < C_l$, $\xi_2 < \Xi_2$, and $T_2 < T_r$, go to step 6; if $\xi_1 < \Xi_1$ and $c_l < C_l$, $\xi_2 < \Xi_2$ and $T \geq T_r$, set $T = t_l$ and go to step 5; if $\xi_1 < \Xi_1$ and $c_l < C_l$ and $\xi_2 \geq \Xi_2$, set $\Xi_2 = 0$, $T = t_l$ and go to step 4; if $\xi_1 < \Xi_1$ and $c_l \geq C_l$, set $c_l = 0$, $\Xi_2 = 0$, $T = t_l$ and go to step 3; else print ξ_1^*, c_l^*, p_0^*, ξ_2^*, and T^* as optimal values of the decision variables, and Π^* as optimal profit.

The model can also be solved by using a suitable in-built optimization function, or in-built metaheuristic algorithms such as genetic algorithm which is available in the Matlab 2018b software.

4.4 Numerical illustration

To illustrate the developed model numerically, we consider three numerical examples.
Example 1: The parameter values are taken as $a = 300$ units/week, $b = 4$ units/\$/week, $\theta = 0.1$/week, $A = \$100$/order, $t_0 = 2$ week, $c_m = \$10$/unit, $h = \$1$/unit/week, $c_s = \$2$/unit, $Q = 900$ units, $\alpha = 0.1$/week, $k = 0.9$, $\eta = 0.5$, $\sigma = 0.2$, $\rho = 0.003$. The reduced deterioration rate is assumed to be of the form $1 - e^{-\gamma\xi}$ with $\gamma = 0.005$.
Example 2: This example is framed to consider the scenario where demand rate is relatively low and customers are more impatient. The parameter values are taken as $a = 100$ units/week, $b = 3$ units/\$/week, $k = 0.7$, $\alpha = 0.05$/week, and $Q = 300$ units, and the other parameter values are same as Example 1.
Example 3: This example is designed to represent an unfavorable situation where deterioration rate is higher while preservation efficiency γ is low. The parameter

values are $\theta = 0.3/\text{week}$, and $\gamma = 0.0005$, $Q = 300$ units, and the other parameter values remain same as Example 1.
The obtained results are shown in the following table. For **Example 1**, the dynamic

Table 4.2: Optimal results

Variables	ξ_1 (\$/week)	ξ_2 (\$/week)	c_l (\$)	p_0 (\$)	T (weeks)	t_l (weeks)	Π (\$)
Example 1	500.6	362.3	664.7	44.3	5.78	0.27	3689.04
Example 2	117.9	0	231.6	18.9	10	0.1	141.65
Example 3	1104.3	0	684.2	40.4	3.2968	0.26	2007.56

pricing during $(0, T)$ takes the form

$$p(t) = \begin{cases} 44.3e^{0.2(t-0.27)} & 0 \leqslant t \leqslant 0.27 \\ 15.09 + \frac{t}{2} + (33.24 - 2.81t)e^{-0.016t} & 0.27 \leqslant t \leqslant 5.78 \end{cases},$$

where the first part denotes the pricing during stock-out period whereas the second part denotes pricing strategy during stock-in period. This form taken as per proposition 1(i). It is further seen that after 2.33 weeks, the price drops even lower than that during stock-out period. For **Example 2**, the pricing strategy is

$$p(t) = \begin{cases} 18.9e^{0.2(t-0.1)} & 0 \leqslant t \leqslant 0.1 \\ 6.69 + (6.66 - 0.18t)e^{-0.1t} + \frac{t}{2} & 0.1 \leqslant t \leqslant 10 \end{cases}.$$

The stock-in price always stays lower than the stock-out price. For **Example 3**, the pricing strategy becomes

$$p(t) = \begin{cases} 40.4e^{0.2(t-0.26)} & 0 \leqslant t \leqslant 0.26 \\ 36 + (11.23 - 0.15t)e^{-0.3t} + \frac{t}{2} & 0.26 \leqslant t \leqslant 3.3 \end{cases}.$$

For this example the stock-in price is always higher than the stock-out price. The total realized demands in examples 1, 2 and 3 are obtained as 891, 285, and 256 units, respectively. The price functions are depicted in Figure 4.1 for ready reference. Corresponding optimal inventory dynamics and optimal realized demand pattern are also provided in Figures 4.2 and 4.3. The sensitivity analysis has been performed to

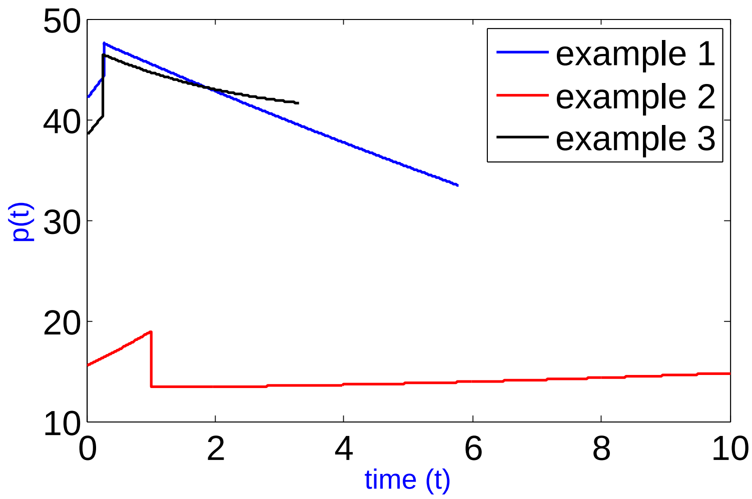
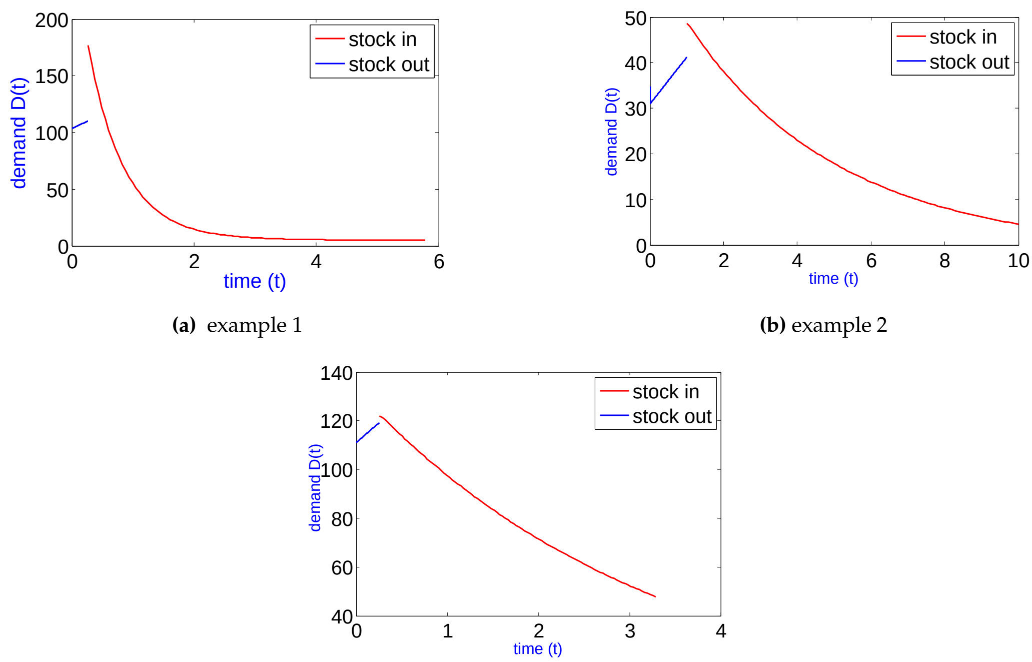


Figure 4.1: Optimal pricing strategies

establish the robustness of the solution as well as to gain managerial insight so as to cope up with changing business environment. The important findings from the



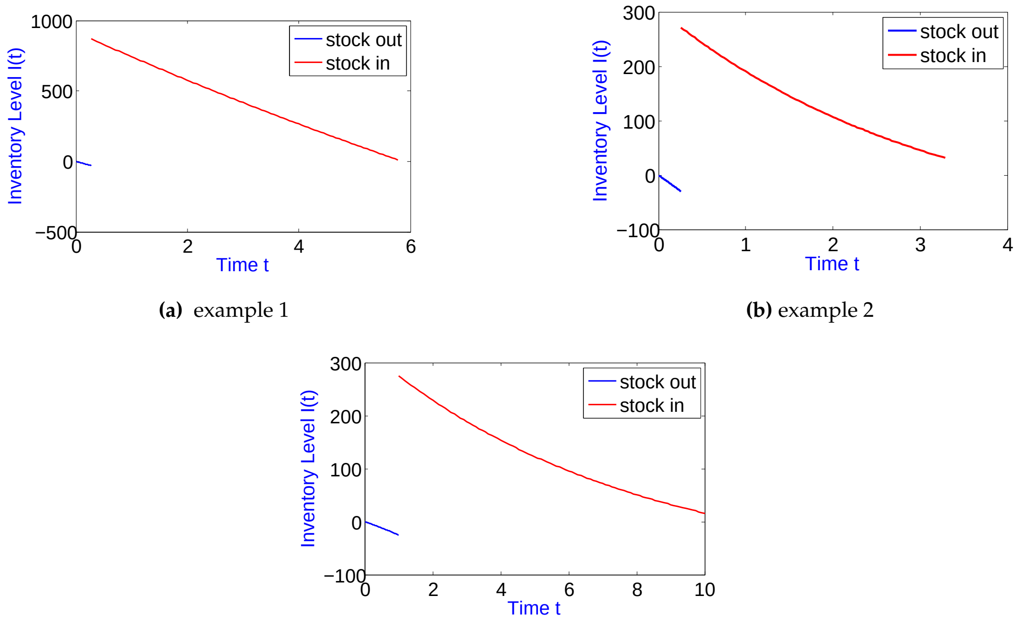
(a) example 1

(b) example 2

(c) example 3

Figure 4.2: Optimal demand functions

three examples and sensitivity analysis are summarized below.



(a) example 1

(b) example 2

(c) example 3

Figure 4.3: Optimal inventory level

Managerial insights

- In moderate business scenarios, managers may lower prices at the end of the cycle to clear remaining stock, even below stock-out discounts. To manage impatient customers during low-demand periods, pre-booking discounts should be avoided; instead, discounts should be offered when stock is available to encourage waiting. Preservation investments during the stock-in period can be minimized, as smaller order quantities reduce the need for such measures, and profits remain limited. For high deterioration rates and inefficient preservation, managers should shorten the stock-in period and overall cycle length while focusing on maintaining freshness during transportation and reducing lead times. Preservation during stock-in should be avoided in these cases. Crucially, stock-in prices should not drop below earlier discounted rates to avoid discouraging customers from waiting.

- Customer price sensitivity is crucial for any business aiming to maximize profit.

Table 4.3: Sensitivity Analysis

parameter	values	ξ_1	ξ_2	c_l	p_0	T	price	Π
	2	575.1	524	920	85.9	3.69	$p(t)=\begin{cases}85.9e^{0.2(t-0.13)} & 0\leqslant t\leqslant 0.13\\ 22.69+\frac{t}{2}+(70.5-6.43t)e^{-0.007t} & 0.13\leqslant t\leqslant 3.69\end{cases}$	8903
	3	529.2	431	766.3	58.4	4.74	$p(t)=\begin{cases}58.4e^{0.2(t-0.2)} & 0\leqslant t\leqslant 0.2\\ 20.8+\frac{t}{2}+(44.85-3.71t)e^{-0.012t} & 0.2\leqslant t\leqslant 4.74\end{cases}$	5366.66
b	4	500.6	362.3	664.7	44.3	5.78	$p(t)=\begin{cases}44.3e^{0.2(t-0.27)} & 0\leqslant t\leqslant 0.27\\ 15.09+\frac{t}{2}+(33.24-2.81t)e^{-0.016t} & 0.27\leqslant t\leqslant 5.78\end{cases}$	3689.04
	5	470	310	656	35.1	6.28	$p(t)=\begin{cases}35.1e^{0.2(t-0.28)} & 0\leqslant t\leqslant 0.28\\ 11.82+\frac{t}{2}+(26.36-2.15t)e^{-0.021t} & 0.28\leqslant t\leqslant 6.28\end{cases}$	2727.76
	6	469.8	262.3	522.5	29.6	8.11	$p(t)=\begin{cases}29.6e^{0.2(t-0.42)} & 0\leqslant t\leqslant 0.42\\ 9.27+\frac{t}{2}+(22.64-1.71t)e^{-0.027t} & 0.42\leqslant t\leqslant 8.11\end{cases}$	2129.85

	0.005	546.7	125.5	290.4	39	4.89	$p(t) = \begin{cases} 39e^{0.2(t-0.84)} & 0 \leqslant t \leqslant 0.84 \\ 26.07 + \frac{t}{2} + (3.23 - 0.044t)e^{-0.05t} & 0.84 \leqslant t \leqslant 4.89 \end{cases}$	3208.11
	0.05	526.4	313.2	647.8	43.8	6.25	$p(t) = \begin{cases} 43.8e^{0.2(t-0.29)} & 0 \leqslant t \leqslant 0.29 \\ 14.65 + \frac{t}{2} + (28.38 - 1.08t)e^{-0.021t} & 0.29 \leqslant t \leqslant 6.25 \end{cases}$	36610.08
α	0.1	500.6	362.3	664.7	44.3	5.78	$p(t) = \begin{cases} 44.3e^{0.2(t-0.27)} & 0 \leqslant t \leqslant 0.27 \\ 15.09 + \frac{t}{2} + (33.24 - 2.81t)e^{-0.016t} & 0.264 \leqslant t \leqslant 5.78 \end{cases}$	3689.04
	0.15	467.3	377.5	747.8	46.2	4.51	$p(t) = \begin{cases} 46.2e^{0.2(t-0.21)} & 0 \leqslant t \leqslant 0.21 \\ 18.57 + \frac{t}{2} + (34.77 - 4.46t)e^{-0.015t} & 0.21 \leqslant t \leqslant 4.51 \end{cases}$	3802.1
	0.25	410.3	394.3	781.7	48	3.31	$p(t) = \begin{cases} 48e^{0.2(t-0.19)} & 0 \leqslant t \leqslant 0.19 \\ 25.09 + \frac{t}{2} + (37.19 - 8.08t)e^{-0.014t} & 0.19 \leqslant t \leqslant 3.31 \end{cases}$	4249.15
	1	488.2	364.4	424.4	43.1	5.59	$p(t) = \begin{cases} 43.1e^{0.2(t-0.28)} & 0 \leqslant t \leqslant 0.28 \\ 15 + \frac{t}{2} + (33.26 - 2.79t)e^{-0.016t} & 0.28 \leqslant t \leqslant 5.59 \end{cases}$	3729.48

	1.5	490.5	362.8	556.8	43.8	5.7	$p(t) = \begin{cases} 43.8e^{0.2(t-0.28)} & 0 \leqslant t \leqslant 0.28 \\ 15.22 + \frac{t}{2} \\ +(33.26 - 2.77t)e^{-0.0163t} & 0.28 \leqslant t \leqslant 5.7 \end{cases}$	3705.64
t_0	2	500.6	362.3	664.7	44.3	5.78	$p(t) = \begin{cases} 44.3e^{0.2(t-0.27)} & 0 \leqslant t \leqslant 0.27 \\ 15.09 + \frac{t}{2} \\ +(33.24 - 2.81t)e^{-0.016t} & 0.27 \leqslant t \leqslant 5.78 \end{cases}$	3689.04
	3	505	361.9	799.4	44.3	5.87	$p(t) = \begin{cases} 44.3e^{0.2(t-0.27)} & 0 \leqslant t0.27 \leqslant \\ 15.39 + \frac{t}{2} \\ +(33.21 - 2.75t)e^{-0.016t} & 0.27 \leqslant t \leqslant 5.87 \end{cases}$	3665.83
	3.5	509	361.8	848.5	44.4	5.91	$p(t) = \begin{cases} 44.4e^{0.2(t-0.27)} & 0 \leqslant t \leqslant 0.27 \\ 15.42 + \frac{t}{2} \\ +(33.2 - 2.75t)e^{-0.016t} & 0.27 \leqslant t \leqslant 5.91 \end{cases}$	3657.11
	700	390.5	341.8	615.8	45.8	3.22	$p(t) = \begin{cases} 45.8e^{0.2(t-0.32)} & 0 \leqslant t \leqslant 00.32 \\ 14.2 + \frac{t}{2} \\ +(31.93 - 2.49t)e^{-0.018t} & 0.32 \leqslant t \leqslant 3.22 \end{cases}$	3614.5
	800	448.5	361	624	44.4	4.59	$p(t) = \begin{cases} 44.4e^{0.2(t-0.31)} & 0 \leqslant t \leqslant 0.31 \\ 13.76 + \frac{t}{2} \\ +(32.58 - 2.74t)e^{-0.016t} & 0.31 \leqslant t \leqslant 4.59 \end{cases}$	3671.91

Q	900	500.6	362.3	664.7	44.3	5.78	$p(t)=\begin{cases}44.3e^{0.2(t-0.27)} & 0\leqslant t\leqslant 0.27\\ 15.09+\frac{t}{2}+(33.24-2.81t)e^{-0.016t} & 0.27\leqslant t\leqslant 5.78\end{cases}$	3689.04
	1000	519.6	367	698.9	44.2	6.26	$p(t)=\begin{cases}44.2e^{0.2(t-0.24)} & 0\leqslant t\leqslant 0.24\\ 16.21+\frac{t}{2}+(34.09-2.83t)e^{-0.016t} & 0.24\leqslant t\leqslant 6.26\end{cases}$	3718.17
	1100	532	373.4	718.5	44.1	6.64	$p(t)=\begin{cases}44.1e^{0.2(t-0.23)} & 0\leqslant t\leqslant 0.23\\ 16.91+\frac{t}{2}+(34.93-2.92t)e^{-0.015t} & 0.23\leqslant t\leqslant 6.64\end{cases}$	3764.34
	0.002	801.9	869.5	695.8	43.4	5.92	$p(t)=\begin{cases}43.4e^{0.2(t-0.25)} & 0\leqslant t\leqslant 0.25\\ 17.29+\frac{t}{2}+(33.36-2.5t)e^{-0.018t} & 0.25\leqslant t\leqslant 5.92\end{cases}$	3152.72
	0.003	678.9	592.1	689	44.1	5.85	$p(t)=\begin{cases}44.1e^{0.2(t-0.25)} & 0\leqslant t\leqslant 0.25\\ 16.13+\frac{t}{2}+(33.3-2.65t)e^{-0.017t} & 0.25\leqslant t\leqslant 5.85\end{cases}$	3446.55
γ	0.005	500.6	362.3	664.7	44.3	5.78	$p(t)=\begin{cases}44.3e^{0.2(t-0.27)} & 0\leqslant t\leqslant 0.27\\ 15.09+\frac{t}{2}+(33.24-2.81t)e^{-0.016t} & 0.27\leqslant t\leqslant 5.78\end{cases}$	3689.04

	0.01	316.3	183.4	649.6	44.4	5.72	$p(t) = \begin{cases} 44.4e^{0.2(t-0.28)} & 0 \leqslant t \leqslant 0.28 \\ 14.87 + \frac{t}{2} + (33.18 - 2.82t)e^{-0.016t} & 0.28 \leqslant t \leqslant 5.72 \end{cases}$	3875.96
c_m	5	456.4	369.5	675.7	43	4.24	$p(t) = \begin{cases} 43e^{0.2(t-.26)} & 0 \leqslant t \leqslant .26 \\ 13.78 + \frac{t}{2} + (33.75 - 2.869t)e^{-0.016t} & 0.26 \leqslant t \leqslant 4.24 \end{cases}$	4090.44
	7	488.5	365.3	669.4	43.5	4.89	$p(t) = \begin{cases} 43.5e^{0.2(t-.265)} & 0 \leqslant t \leqslant .265 \\ 14.60 + \frac{t}{2} + (33.52 - 2.8t)e^{-0.016t} & 0.265 \leqslant t \leqslant 4.89 \end{cases}$	3914
	10	500.6	362.3	664.7	44.3	5.78	$p(t) = \begin{cases} 44.3e^{0.2(t-.27)} & 0 \leqslant t \leqslant .27 \\ 15.09 + \frac{t}{2} + (33.24 - 2.81t)e^{-0.016t} & 0.27 \leqslant t \leqslant 5.78 \end{cases}$	3689.04
	12	505.6	361	660.8	44.6	6.31	$p(t) = \begin{cases} 44.6e^{0.2(t-.28)} & 0 \leqslant t \leqslant .28 \\ 15.65 + \frac{t}{2} + (33.1 - 2.74t)e^{-0.016t} & 0.28 \leqslant t \leqslant 6.31 \end{cases}$	3556.92
	15	528.5	356.2	653	45.3	7.197	$p(t) = \begin{cases} 45.3e^{0.2(t-.28)} & 0 \leqslant t \leqslant .28 \\ 16.41 + \frac{t}{2} + (32.94 - 2.68t)e^{-0.017t} & 0.28 \leqslant t \leqslant 7.197 \end{cases}$	3379.63

ρ	0.001	512.1	350.4	1338.8	48	6.49	$p(t) = \begin{cases} 48e^{0.2(t-.52)} & 0 \leqslant t \leqslant .52 \\ 16.88 + \frac{t}{2} + (33.34 - 2.61t)e^{-0.017t} & 0.52 \leqslant t \leqslant 6.49 \end{cases}$	3516.76
	0.002	509	360.1	847.1	44.7	6.04	$p(t) = \begin{cases} 44.7e^{0.2(t-.37)} & 0 \leqslant t \leqslant .37 \\ 15.68 + \frac{t}{2} + (33.21 - 2.73t)e^{-0.016t} & 0.37 \leqslant t \leqslant 6.04 \end{cases}$	3637.17
	0.003	500.6	362.3	664.7	44.3	5.78	$p(t) = \begin{cases} 44.3e^{0.2(t-.27)} & 0 \leqslant t \leqslant .27 \\ 15.09 + \frac{t}{2} + (33.24 - 2.81t)e^{-0.016t} & 0.27 \leqslant t \leqslant 5.78 \end{cases}$	3689.04
	0.005	486.8	364.3	469.2	43.9	5.52	$p(t) = \begin{cases} 43.9e^{0.2(t-.19)} & 0 \leqslant t \leqslant .19 \\ 14.96 + \frac{t}{2} + (33.28 - 2.78t)e^{-.0162t} & 0.19 \leqslant t \leqslant 5.52 \end{cases}$	3740.14
	0.01	485.6	364.8	296.6	43.4	5.32	$p(t) = \begin{cases} 43.4e^{0.2(t-.1)} & 0 \leqslant t \leqslant .1 \\ 14.78 + \frac{t}{2} + (33.21 - 2.791t)e^{-.0161t} & 0.1 \leqslant t \leqslant 5.32 \end{cases}$	3789.04
	0.1	486.6	839.3	689.7	46.2	5.34	$p(t) = \begin{cases} 46.2e^{0.2(t-.25)} & 0 \leqslant t \leqslant .25 \\ 16.89 + .05t + (34.44 - 2.98t)e^{-0.002t} & 0.25 \leqslant t \leqslant 5.34 \end{cases}$	3756.77

	0.5	486.9	511	681.6	45.3	5.51	$p(t) = \begin{cases} 45.3e^{0.2(t-.26)} & 0 \leqslant t \leqslant .26 \\ 16.19 + .25t & \\ +(33.85 - 2.9t)e^{-0.008t} & 0.26 \leqslant t \leqslant 5.51 \end{cases}$	3727.24
h	1	500.6	362.3	664.7	44.3	5.78	$p(t) = \begin{cases} 44.3e^{0.2(t-.27)} & 0 \leqslant t \leqslant .27 \\ 15.09 + \frac{t}{2} & \\ +(33.24 - 2.81t)e^{-0.016t} & 0.27 \leqslant t \leqslant 5.78 \end{cases}$	3689.04
	2	501	210.2	524.2	42.1	5.8	$p(t) = \begin{cases} 42.1e^{0.2(t-.41)} & 0 \leqslant t \leqslant .41 \\ 12.3 + t & \\ +(32.11 - 2.60t)e^{-0.035t} & 0.41 \leqslant t \leqslant 5.8 \end{cases}$	3610.9
	3	507.1	121	455.9	40.4	5.84	$p(t) = \begin{cases} 40.4e^{0.2(t-.51)} & 0 \leqslant t \leqslant .51 \\ 8.82 + 1.5t & \\ +(30.98 - 2.53t)e^{-0.055t} & 0.51 \leqslant t \leqslant 5.84 \end{cases}$	3438.33

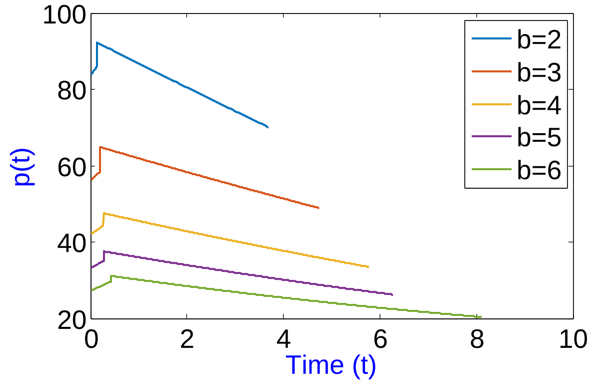


Figure 4.4: Pricing strategies with varying b

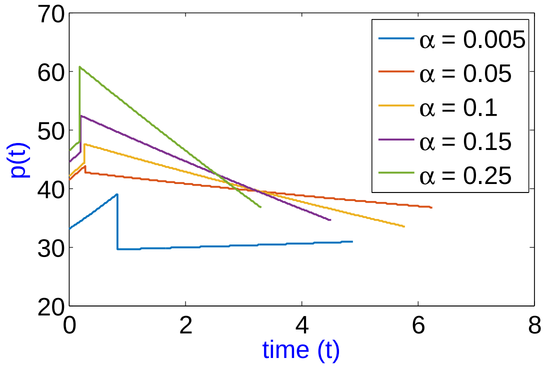


Figure 4.5: Pricing strategies with varying α

A firm's strategies must align with customers' willingness to pay. When customers are highly price-sensitive, the model naturally adjusts to lower price levels. To minimize holding and lead time reduction costs, the business owner can extend the shortage period and reduce preservation investments during transportation and storage. However, longer lead times and reduced preservation efforts lead to lower product quality, negatively impacting demand. With unchanged order quantities, this results in an extended overall business cycle.

- With a fixed order quantity, varying stock sensitivity produces notable effects. For highly stock-sensitive customers, the business owner should set high prices when inventory levels are high, then reduce prices sharply as inventory decreases—the greater the stock sensitivity, the steeper the price decline. To leverage stock display benefits, the manager should shorten the stock-out period and invest more in lead time reduction. At the start of the cycle, when inventory levels are high and freshness is less influential, preservation investments during transportation can be reduced. However, preservation investments during the stock-in period should be increased. Higher stock sensitivity also leads to shorter cycle lengths and a steeper pricing curve. Additionally, as high stock sensitivity boosts demand, there is a notable increase in average profit.

- A key goal of lead time reduction is to preserve product freshness throughout the business cycle. When the initial lead time is low, preservation investments during transportation can be minimized, keeping the optimal lead time unchanged. Variations in initial lead time have little impact on decision variables, with only a slight profit reduction due to increased lead time reduction costs.

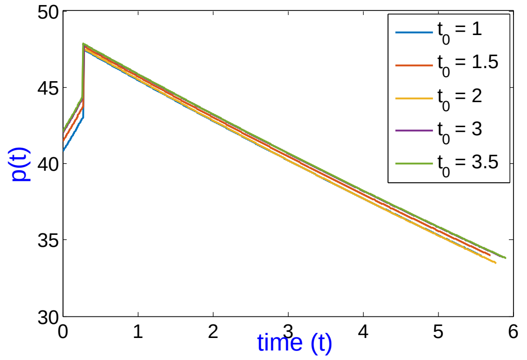


Figure 4.6: Pricing strategies with varying t_0

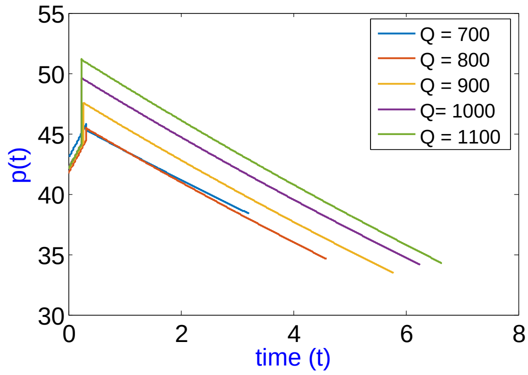


Figure 4.7: Pricing strategies with varying Q

- In a stock-dependent demand scenario, order quantity plays a crucial role. With a higher order quantity, the manager should extend the business period, invest more in lead time reduction, and lengthen the stock-in period to capitalize on stock dependence. While the initial price can be lower, maintaining higher prices during the stock-in period is recommended. Preservation investments during both periods should also increase to maintain freshness. A higher order quantity generally boosts demand and can sustain profit levels, even with warehouse space constraints up to a certain limit.
- With higher values of γ, representing advanced preservation technology, lower preservation investment suffices. In such cases, the manager can reduce preser-

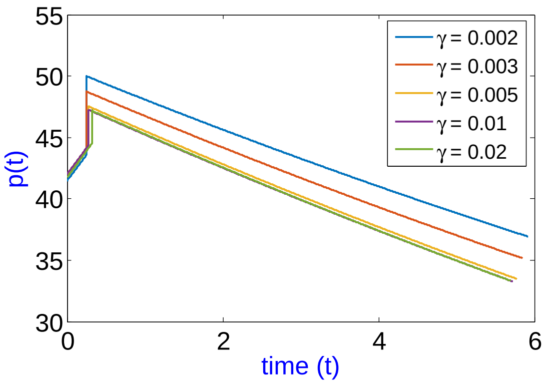


Figure 4.8: Sensitivity of price wrt γ

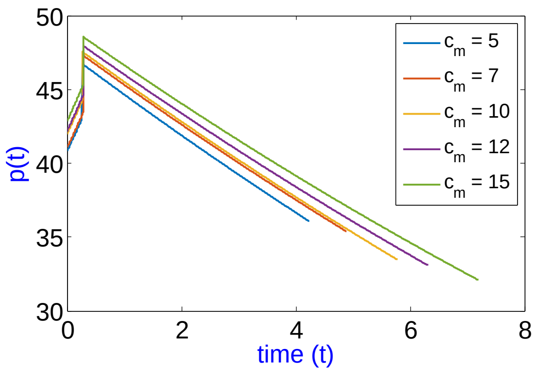


Figure 4.9: Pricing strategies with varying c_m

vation costs and focus on shortening lead time to obtain fresher items. The cycle length may be slightly reduced, and lower prices can be set during the stock-in period. However, the use of advanced technology allows the manager to achieve higher profits despite these adjustments.

- A higher purchasing cost compels the manager to set a higher retail price. A lower demand at higher price indicates a longer business cycle. In order to charge higher price, the manager must maintain high quality for the products, thereby investing more in preservation during transportation. The business manager should adjust the cost to some extend by investing lesser in lead time reduction. Increased costs at various stages ultimately result in lower profit.

- The parameter ρ, representing system efficiency in lead time reduction, significantly influences costs and profitability. Lower values of ρ require higher investment in lead time reduction, leading to reduced profits. To offset these additional costs, the manager should set higher prices. In contrast, a more efficient system (higher ρ) allows for slightly lower preservation investment during the stock-out period, resulting in shorter lead times and a reduced business cycle. However, system efficiency does not appear to affect preservation investment during the stock-in period.

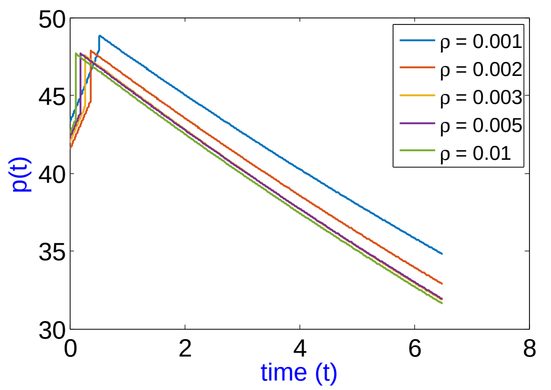


Figure 4.10: Pricing strategies with varying ρ

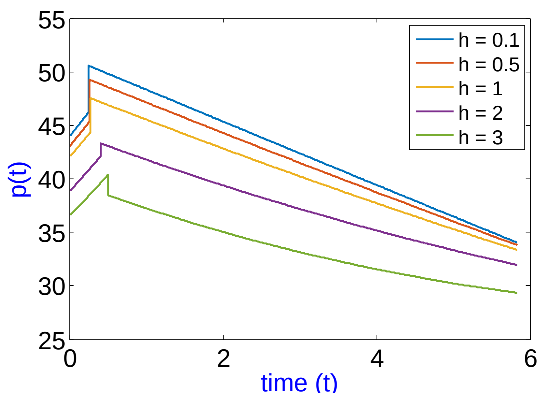


Figure 4.11: Pricing strategies with varying h

- The manager should minimize investment in lead time reduction to maximize benefits while increasing preservation investment during transportation to maintain product freshness. To cut costs, preservation investment during the stock-in period can be reduced. With higher holding costs and a lower freshness index, the manager is inclined to clear stock more quickly, adopting a lower pricing strategy.

4.5 Conclusion

This chapter presents a perishable inventory model where demand depends on price, stock levels, and product freshness. The model optimizes dynamic pricing and inventory policies by incorporating lead time reduction and preservation investments. Starting the business period with an initial shortage and offering price discounts to waiting customers is shown to benefit managers. A key insight is that as inventory levels and freshness decline, prices during stock-in periods may drop below those in stock-out periods, reflecting how consumers value freshness over availability. The model introduces strategies like lead time reduction and preservation investments to extend product freshness, enhancing both customer satisfaction and profitability. Additionally, it incorporates partial backlogging with waiting-time-based price discounts, allowing patient customers to purchase fresher products at reduced prices during the next replenishment cycle. The study contributes to the literature in three significant ways: (1) it is the first to model demand as dependent on stock and freshness under dynamic pricing; (2) it addresses controllable lead times in this context; and (3) it introduces shortage management through discounted pricing for patient customers.

Future research could address limitations and explore extensions, such as incorporating quantitative deterioration (e.g., product loss due to evaporation or spoilage), which would align more closely with real-world scenarios. Examining interactions with supply chain partners or extending the model to a two-tier supply chain could provide deeper insights. Considering stochastic demand instead of deterministic approximations would account for market uncertainties. Risk-averse contracts and alternative inventory policies, such as (S, s, ℓ) systems, Markov chain models for omni-channel retail, customer product returns, and the use of transaction data for planning during disruptions, could further enrich the model's applications.

CHAPTER 5

Interplay of Greenness and Preservation Investments: Pricing Strategies for Freshness- and Green-conscious Customers

In the management of perishable inventory, pricing strategies must account for the continuous decline in product freshness over time. Unlike standard inventory models where demand remains relatively stable, perishables face unique challenges due to quality deterioration, environmental concerns, and the increasing importance of sustainable business practices. To address these complexities, this chapter explores how pricing, preservation investments, greening initiatives, and lead time management collectively optimize supply chain performance for perishable goods.

Consumers today are becoming more environmentally conscious, valuing product greenness alongside quality. Businesses must, therefore, integrate sustainability into their inventory and pricing decisions, balancing profitability with eco-friendly practices. In this context, preservation efforts play a dual role—not only extending product usability but also reducing waste, aligning with both economic and environmental objectives. Similarly, lead time management is critical- particularly for products with instantaneous freshness loss- where timely replenishment is essential to maintaining quality and minimizing disposal costs.

This chapter is divided into two parts. The first part develops an optimal pricing strategy considering price, greenness, and preservation investments, particularly for environmentally aware consumers. By incorporating both analytical and numerical insights, this section provides a strategic framework for businesses aiming to enhance demand through green-conscious pricing and sustainability initiatives. The second part focuses on the role of lead time in managing perishable inventory, particularly in scenarios where quality loss is a significant concern. It presents a hybrid pricing strategy that adapts across stock-in and stock-out periods while optimizing greening, preservation, and lead time investments. Sensitivity analysis further demonstrates the interplay between freshness and sustainability, offering insights into practical applications, especially for sectors like packaged food and e-commerce. By integrating pricing, preservation, greening, and lead time management, this chapter provides a comprehensive approach to tackling the challenges of perishable inventory while ensuring economic viability and environmental responsibility.

The common notations used in the chapter are given in the following table.

Table 5.1: Notations

T	:	cycle length (decision variable)
g	:	greening level (decision variable)
ξ	:	preservation technology investment per unit time (decision variable)
$p(t)$	:	dynamic selling price at time t (decision variable)
$I(t)$	:	inventory level at any time t, $t \in [0, T]$
$\lambda(t)$	:	costate variable associated with the state variable $I(t)$
$q(t, \xi)$	:	quality of the product at preservation investment ξ at time t, $0 \leq q(t, \xi) \leq 1$
$D(p(t), g, q(\xi))$	:	demand rate
c_p	:	per unit purchasing cost
θ	:	constant deterioration rate per unit time
Q	:	ordering quantity
A	:	ordering cost

h	:	per unit holding cost per unit time
t_d	:	non-deterioration period (in time length)
c_d	:	per unit disposal cost
$m(\xi)$	:	proportion of reduced deterioration rate with preservation investment ξ

The common assumptions used in this chapter are regarding pricing and green investment. The price of the product is assumed to change as a result of fluctuating business dynamics of the company. A continuous dynamic pricing technique is adopted here to cope up with the changing demand. To achieve g level of greenness, the associated cost is assumed to be quadratic with the level itself, *i.e.* $\frac{1}{2}kg^2$ ($k > 0$), ensuring diminishing return on investment.

5.1 Optimal Dynamic Pricing, Preservation, and Green Strategies for Green-Sensitive Customers

5.1.1 Introduction

Although price has been regarded as the primary criterion in inventory research, the dynamic nature of pricing has not been extensively deliberated upon. The discrete dynamic pricing approach as adopted by Dye, 2020 is a promising strategy for modeling multi-period inventory systems. However, in case of certain items such as perishable food products exhibiting dynamic variations in their nature over a given period, maintaining an unchanged pricing approach over the entire duration may appear impractical. This encourages us to implement continuous dynamic pricing policy exhibiting continuous variation. While discussing perishable goods, majority of the research articles such as Zhang et al., 2014 paid attention only to the quantitative deterioration issue, whereas the qualitative deterioration issue is frequently ignored. However, for the vast majority of perishable goods, qualitative loss marks the beginning of quantitative deterioration. The qualitative degradation of an item has a discernible impact on its overall quality, yet it does not render the product inoperable, as opposed to the quantitative degradation which does. Nonetheless, it is appropriate to regulate both entities through the implementation of preservation technology funding. Hence, it is imperative to evaluate them individually for a comprehensive analysis. Although the research articles on perishable items are not scarce (*e.g.* Zhang et al., 2014, Liu et al., 2015), there are hardly any paper that considers both issues simultaneously. Keeping this in mind, both occurrences are thoroughly discussed in the current work. In addition to the investment in preservation technology to retain quality and prevent spoilage, it is also important to note that while the rate of deterioration is reduced, there is a concomitant environmental pollution resulting from its application, which is often overlooked. It is thus important that one conducts an analysis of the environmental impact while discussing on preservation investment, an issue that has been duly considered in the present work. Finally, it is worth noting that prior literature on dynamic pricing has often

*This part of the chapter is based on the work published in *Journal of Industrial & Management Optimization* (2024), volume 20, issue 7, pages 2260-2281.

assumed a theoretical minimum price of zero, which is arguably unrealistic. In contrast, the present study adopts a more pragmatic approach by setting the minimum price at the actual purchasing cost of the product, thereby enhancing the model's real-world applicability. A perishable product facing demand influenced by price, freshness and greening level of the product has been considered in the present work. The greening level is determined at the time of production. The freshness of the product gets reduced with time. The investment in preservation technology serves the dual purpose of mitigating the rate of deterioration as well as preserving the quality level. To counteract the deleterious effects of this deterioration, a substantial allocation of resources towards preservation technology investment is also being factored in. Investing in preservation technology facilitates the maintenance of quality standards and reduces the rate of deterioration in a declining manner. In order to optimize business outcomes, it is advisable to adjust pricing in accordance with prevailing market conditions. The implementation of this strategy is performed in order to enhance the responsiveness of the customer service system and optimize the management of inventory in the warehouse. In this current chapter, the focus is on finding the pricing strategy of an inventory model that contemplates the perishable items. This chapter tries to answer the following research questions:
RQ1: How do pricing and preservation investment strategies interact to maximize profitability for perishable products with demand influenced by quality, price, and environmental sustainability?
RQ2: What is the optimal trade-off between investing in product greenness and preservation technology to balance consumer preferences, cost constraints, and inventory management efficiency?
RQ3: How does the incorporation of both qualitative and quantitative deterioration impact dynamic pricing decisions, and what managerial insights can be derived for sustainable inventory practices?

5.1.2 Assumptions

Apart from the general assumptions mentioned already, the particular assumptions for this chapter are as follows:

- A single period inventory model where demand depends simultaneously on retail price, green level, and quality of the product is considered here. The specific pattern of demand is expressed as $D(p(t), g, q(t,\xi)) = \alpha q(t,\xi)\Big(a - bp(t) + \beta g\Big)$, where a b, β, and α are positive parameters denoting base demand, price sensitivity, green sensitivity and quality sensitivity, respectively.

- The lead time is deterministic, so that without any loss of generality, it may be assumed to be zero. In the context of quality management, it is postulated that the initial quality of the item, upon its arrival at the warehouse, is presumed to be of a magnitude equivalent to q_0 $(0 < q_0 \leq 1)$.

- Both quantitative and qualitative deterioration are mitigated by preservation technology. For every ξ amount of money invested in preservation, both qualitative and quantitative deterioration are reduced by $m(\xi)$, $0 < m(\xi) < 1$. $m(\xi)$ is assumed to be of concave pattern which guarantees the profitability of preservation investments and the achievement of an optimal investment.

- The quality degradation over time is assumed to be of the form $\dot{q}(t) = -(1 - m)\theta q(t)$, $q(0) = q_0$ (Liu et al., 2015). For the quantitative deterioration, the rate θ is assumed to be constant.

5.1.3 The Pricing Strategy

Based on the assumptions, two different situations may emerge, depending on whether the business cycle ends before deterioration occurs or not. We now develop dynamic pricing strategies for both the models one by one.

5.1.3.1 Case 1: $t_d \leq T$

The Hamiltonian function in this case is given by

$$H(p, I, \lambda, t) = \begin{cases} \alpha pq(a - bp + \beta g) - \frac{1}{2}kg^2 - hI(t) - \lambda \alpha q(a - bp + \beta g), & 0 \leqslant t \leqslant t_d, \\ \alpha pq(a - bp + \beta g) - \frac{1}{2}kg^2 - hI(t) - c_d(1 - m)\theta I(t) & \\ -\lambda\Big(\alpha q(a - bp + \beta g) + (1 - m)\theta I(t)\Big), & t_d \leqslant t \leqslant T \end{cases}.$$

The pricing strategy is

$$p^*(t) = \begin{cases} c_p, & \lambda < \frac{2bc_p - a - \beta g}{b} \\ \frac{a+\lambda b+\beta g}{2b}, & \frac{2bc_p - a - \beta g}{b} \leqslant \lambda \leqslant \frac{a-\beta g}{b} \\ \frac{a}{b}, & \lambda > \frac{a-\beta g}{b} \end{cases}. \tag{5.1}$$

The costate equation is given by, $\frac{d\lambda}{dt} = -\frac{\partial H}{\partial I}$, *i.e.*

$$\frac{d\lambda}{dt} = \begin{cases} h, & 0 \leqslant t \leqslant t_d \\ h + c_d(1-m)\theta + \lambda(1-m)\theta, & t_d \leqslant t \leqslant T \end{cases}.$$

Solving the above equation with the transversality condition $\lambda(0) = c_p$, we get the expression for the costate variable as

$$\lambda(t) = \begin{cases} ht + c_p, & 0 \leqslant t \leqslant t_d \\ \left[ht_d + c_p + \frac{h+c_d(1-m)\theta}{(1-m)\theta}\right] e^{(1-m)\theta(t-t_d)} - \frac{h+c_d(1-m)\theta}{(1-m)\theta}, & t_d \leqslant t \leqslant T \end{cases}.$$

Putting the value of $\lambda(t)$ in equation 5.1, we get the optimal pricing strategy as follows.

Proposition 5.1. *When* $c_p < \frac{a-\beta g - X_1 e^{(1-m)\theta(T-t_d)}}{b\left\{e^{(1-m)\theta(T-t_d)}-1\right\}} - ht_d$, *the pricing strategy is given by*

$$p^*(t) = \begin{cases} \frac{a+bc_p+\beta g+bht}{2b}, & 0 \leqslant t \leqslant t_d \\ \frac{a+X_2 e^{(1-m)\theta(t-t_d)} - X_1 + \beta g}{2b}, & t_d \leqslant t \leqslant T \end{cases}, \tag{5.2}$$

where $X_1 = b\frac{\{h+c_d(1-m)\theta\}}{(1-m)\theta}$ *and* $X_2 = b\left\{\frac{h+c_d(1-m)\theta}{(1-m)\theta} + ht_d + c_p\right\}$.

Proposition 5.2. *When* $\frac{a-\beta g - X_1 e^{(1-m)\theta(T-t_d)}}{b\left\{e^{(1-m)\theta(T-t_d)}-1\right\}} - ht_d < c_p$, *the pricing strategy is given by*

$$p^*(t) = \begin{cases} \frac{a+bc_p+\beta g+bht}{2b}, & 0 \leqslant t \leqslant t_d \\ \frac{a+X_2e^{(1-m)\theta(t-t_d)}-X_1+\beta g}{2b}, & t_d \leqslant t \leqslant t_1\,, \\ \frac{a}{b}, & t_1 \leqslant t \leqslant T \end{cases} \tag{5.3}$$

where $t_1 = t_d + \frac{1}{(1-m)\theta}\ln\left(\frac{X_1+a-\beta g}{X_2}\right)$.

Propositions 5.1 and 5.2 exhibit that when the production cost is *sufficiently high*, the retailer opts for static pricing during later period of the cycle.

5.1.3.2 Case 2: $T \leq t_d$

Here the Hamiltonian function is given by

$$H(p, I, \lambda, t) = \alpha pq(a - bp + \beta g) - \frac{1}{2}kg^2 - hI(t) - \lambda\alpha q(a - bp + \beta g), \qquad 0 \leqslant t \leqslant T.$$

Under the general pricing strategy given by 5.1, solving the costate equation $\frac{d\lambda}{dt} = -\frac{\partial H}{\partial I} = h$ with the transversality condition $\lambda(0) = c_p$, we get the costate variable as $\lambda(t) = ht + c_p$ which gives the following specific pricing strategy:

Proposition 5.3. *If* $c_p < \frac{a-\beta g}{b} - hT$*, the pricing strategy is*

$$p^*(t) = \frac{a + bc_p + \beta g + bht}{2b} \qquad 0 \leqslant t \leqslant T, \tag{5.4}$$

otherwise, the pricing strategy is

$$p^*(t) = \begin{cases} \frac{a+bc_p+\beta g+bht}{2b} & 0 \leqslant t \leqslant \frac{1}{h}\left(\frac{a-\beta g}{b} - c_p\right) \\ \frac{a}{b} & \frac{1}{h}\left(\frac{a-\beta g}{b} - c_p\right) \leqslant t \leqslant T \end{cases}. \tag{5.5}$$

We now state the following properties which characterize the pricing policies in both the cases in general.

Property 1: *(i)* $\frac{\partial p(t)}{\partial a} > 0$ *(ii)* $\frac{\partial p(t)}{\partial b} < 0$ *(iii)* $\frac{\partial p(t)}{\partial \beta} > 0$ *(iv)* $\frac{\partial p(t)}{\partial g} > 0$ *(v)* $\frac{\partial p(t)}{\partial h} > 0$ (vi) $\frac{\partial p(t)}{\partial c_d} \geq 0$ (vii) $\frac{\partial p(t)}{\partial t_d} \leq 0$.

Proof:

Proofs of (i)-(iv) are straightforward, hence omitted. For (v), note that

$$\frac{\partial p(t)}{\partial h} = \begin{cases} \frac{t}{2}, & 0 \leqslant t \leqslant t_d \\ \frac{1}{2(1-m)\theta}\left(\frac{e^{(1-m)\theta(t-t_d)}}{(1-m)\theta} - 1\right) + \frac{t_d e^{(1-m)\theta(t-t_d)}}{2}, & t_d \leqslant t \leqslant T \end{cases},$$

where $(1-m)\theta < 1$. Expanding $e^{(1-m)\theta(t-t_d)}$, we have the first term $\frac{1}{(1-m)\theta} > 1$, and hence the proof follows. Similar argument holds good for (vi), since

$$\frac{\partial p(t)}{\partial c_d} = \begin{cases} 0, & 0 \leqslant t \leqslant t_d \\ \frac{1}{2}\left[\frac{e^{(1-m)\theta(t-t_d)}}{(1-m)\theta} - 1\right], & t_d \leqslant t \leqslant T \end{cases}.$$

Proof of (vii) is also straightforward in view of

$$\frac{\partial p(t)}{\partial t_d} = \begin{cases} 0 & (0 \leqslant t \leqslant t_d) \\ -\frac{(ht_d+c_p+c_d)((1-m)\theta)e^{(1-m)\theta(t-t_d)}}{2} & (t_d \leqslant t \leqslant T) \end{cases}.$$

A positive correlation between the price and the base demand is observed in (i). A high demand affords business managers the opportunity to increase price. An increase in price sensitivity b affects the demand more negatively with price, so if the demand is highly sensitive with price, it would be prudent to maintain a lower price. On the contrary, when the customers are not so bothered about price, the manager uses the opportunity to generate more revenue by charging high price. The greening parameter exerts a favourable influence on consumer demand, and a higher green sensitivity reduces relative sensitivity on price as well, thereby affording the business manager the opportunity to set a higher price for the product. Since the retailer finds it worthy investing more in greenness only if customers are sensitive enough to value it, such customers will eventually be less sensitive to price. A higher investment in greening thus results in higher retail price. A higher holding cost leads to an increase in the total cost of the system, thereby necessitating an increase in price to achieve a commensurate profit, as demonstrated by the aforementioned relationship in (v). An elevated cost of disposal results in an added overall cost of the system. Consequently, it becomes imperative to establish a higher price in order to achieve a favourable return on investment. A postponement of the initiation of deterioration has a favourable effect on pricing policies. This is due to the fact that a higher initiation point of deterioration results in reduced product spoilage over a longer period of time. This, in turn, provides a more favourable environment for business managers, allowing for a slight reduction in demand with maintaining a lower spoilage factor.

5.1.4 The Model

Having the pricing strategies at hand, we are now in a position to derive cost and revenue functions in each case.

5.1.4.1 Case 1: $t_d < T$

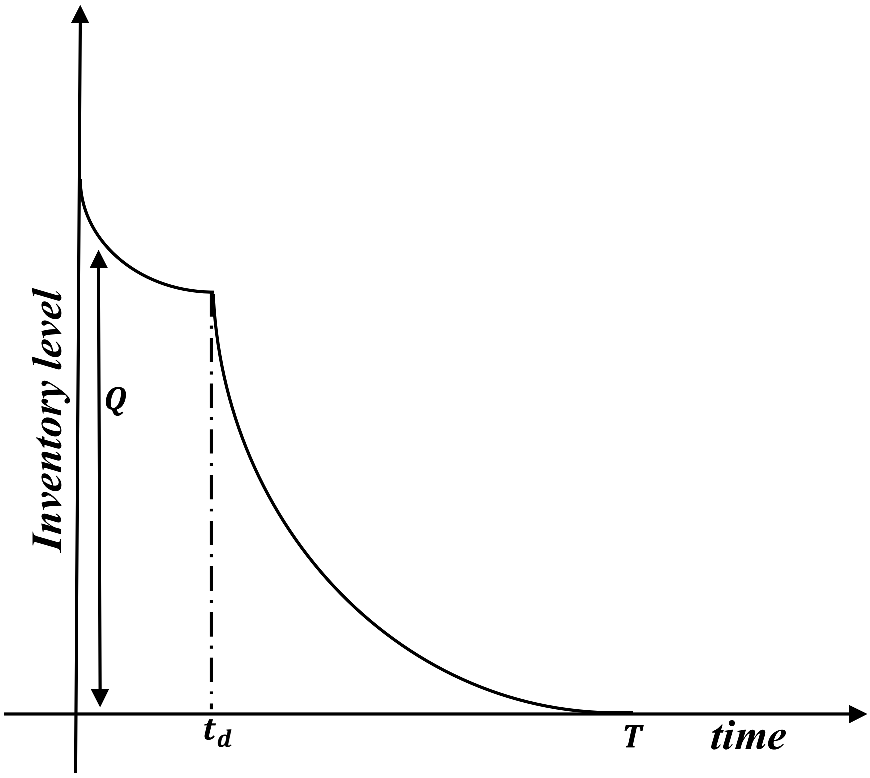


Figure 5.1: Schematic diagram of inventory level for model 1

For the pricing strategy given in Proposition 5.1, we start with the inventory dynamics first. The changes in inventory level during non-deterioration period is due to demand only, while the effect of deterioration comes into play as well during later period. The inventory level at any time t is thus governed by the differential equation

$$\frac{dI(t)}{dt} = \begin{cases} -\alpha q\{a - bp(t) + \beta g\} & \text{with } I(0) = Q, \quad 0 < t \leqslant t_d \\ -\alpha q\{a - bp(t) + \beta g\} - (1-m)\theta I(t) & \text{with } I(T) = 0, \quad t_d < t \leqslant T \end{cases}. \tag{5.6}$$

Using the continuity condition of the inventory level at $t = t_d$ and solving the above differential equation, we get

$$I(t) = \begin{cases} Q - \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)\theta}\left(1 - e^{-(1-m)\theta t}\right) + \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1 - e^{-(1-m)\theta t}\right) \\ -\frac{\alpha q_0 bht}{2(1-m)\theta}e^{-(1-m)\theta t}, & 0 \leqslant t \leqslant t_d \\ \frac{\alpha q_0 X_2}{2(1-m)\theta}e^{-(1-m)\theta t_d} - \frac{\alpha q_0}{2}(a+\beta g + X_1)te^{-(1-m)\theta t} + K_1 e^{-(1-m)\theta t}, & t_d \leqslant t \leqslant T \end{cases}, \tag{5.7}$$

where
$$K_1 = \left[Q - \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)\theta}\left(1 - e^{-(1-m)\theta t_d}\right) + \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1 - e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0 bht_d}{2(1-m)^2\theta^2}e^{-(1-m)\theta t_d}\right]e^{(1-m)\theta t_d} - \frac{\alpha q_0}{2}\left\{\frac{X_2}{(1-m)\theta} - (a+\beta g + X_1)t_d\right\},$$

and X_1 and X_2 are as defined in Proposition 5.1. Putting $I(T) = 0$, we get the order quantity Q as

$$\begin{aligned} Q = {} & \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)\theta}\left(1 - e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1 - e^{-(1-m)\theta t_d}\right) \\ & + \frac{\alpha q_0 bht_d}{2(1-m)^2\theta^2}e^{-(1-m)\theta t_d} + \left\{\frac{\alpha q_0}{2}(a+\beta g + X_1)T - \frac{\alpha q_0 X_2}{2(1-m)\theta}e^{(1-m)\theta(T-t_d)}\right. \\ & \left. + \frac{\alpha q_0}{2}\left\{\frac{X_2}{(1-m)\theta} - (a+\beta g + X_1)t_d\right\}\right\}e^{-(1-m)\theta t_d}. \end{aligned} \tag{5.8}$$

The total holding cost during a cycle is derived as

$$
\begin{aligned}
HC &= \int_0^{t_d} I(t)dt + \int_{t_d}^{T} I(t)dt \\
&= h\Bigg\{ \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_d}\right) + \frac{\alpha q_0 bht_d}{2(1-m)^2\theta^2} e^{-(1-m)\theta t_d} \\
&\quad - \frac{\alpha q_0 bh}{2(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0 bh}{2(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_d}\right) + K_2 t_d \\
&\quad + \frac{\alpha q_0 X_2 e^{-(1-m)\theta t_d}}{2(1-m)\theta}(T-t_d) + \frac{K_1}{(1-m)\theta}\left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta T}\right) \\
&\quad + \frac{\alpha q_0(a+\beta g + X_1)}{2(1-m)\theta}\left(Te^{-(1-m)\theta T} - t_d e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0(a+\beta g + X_1)}{2(1-m)^2\theta^2} \times \\
&\quad \left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta T}\right)\Bigg\},
\end{aligned}
$$

$$
\text{where} \quad K_2 = Q - \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)\theta} + \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}.
$$

The total deterioration cost is given by

$$
\begin{aligned}
DC &= c_d(1-m)\theta \int_{t_d}^{T} I(t)dt \\
&= c_d(1-m)\theta\Bigg[\frac{\alpha q_0 X_2 e^{-(1-m)\theta t_d}}{2(1-m)\theta}(T-t_d) + \frac{K_1}{(1-m)\theta}\left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta T}\right) \\
&\quad + \frac{\alpha q_0(a+\beta g + X_1)}{2(1-m)\theta}\left(Te^{-(1-m)\theta T} - t_d e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0(a+\beta g + X_1)}{2(1-m)^2\theta^2} \times \\
&\quad \left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta T}\right)\Bigg].
\end{aligned}
$$

The greening cost is $GC = \frac{1}{2}kg^2$, total production cost is $PC = c_pQ$, and investment in preservation technology is $PTC = \xi T$. Therefore, the total revenue for the entire period is derived as

$$
\begin{aligned}
TR &= \int_0^T p(t)D(p(t), g, q(\xi))dt \\
&= \frac{\alpha q_0}{4b}\Bigg[\frac{(a+\beta g)^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta t_d}\right) - \frac{b^2c_p^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta t_d}\right) + b^2h^2 \times
\end{aligned}
$$

$$\left\{\frac{t_d^2 e^{-(1-m)\theta t_d}}{(1-m)\theta} + \frac{2t_d e^{-(1-m)\theta t_d}}{(1-m)^2\theta^2} - \frac{2}{(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_d}\right)\right\} + 2b^2hc_p \times$$
$$\left\{\frac{t_d e^{-(1-m)\theta t_d}}{(1-m)\theta} - \frac{1}{(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_d}\right)\right\} + \frac{\{(a+\beta g)^2 - X_1^2\}}{(1-m)\theta} \times$$
$$\left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta T}\right) - \frac{X_2^2}{(1-m)\theta}\left(e^{(1-m)\theta(T-2t_d)} - e^{-(1-m)\theta t_d}\right)$$
$$-2X_1X_2e^{-(1-m)\theta t_d}(T-t_d)\Bigg].$$

The total profit is thus $TP_{11} = TR - HC - DC - PTC - GC - PC$, and average profit is $AP_{11}(p(t), g, \xi, T) = \frac{TP_{11}}{T}$. The aim is to maximize the average profit subject to $t_d \leq T$. Although establishing the joint concavity of the average profit function analytically is not possible, the following proposition ensures the existence of an optimal green investment amount g.

Proposition 5.4. *The average profit is concave in g when $k > \frac{\alpha q_0 \beta^2}{2b(1-m)\theta}\left(1-e^{-(1-m)\theta T}\right)$.*

Proof: Since, $\frac{\partial^2 AP_{11}}{\partial g^2} = \frac{1}{T}\frac{\partial^2 TP_{11}}{\partial g^2} = \frac{1}{T}\left\{\frac{\alpha q_0 \beta^2}{2b(1-m)\theta}\left(1-e^{-(1-m)\theta T}\right) - k\right\}$, the proof is straightforward.

The following lemma ensures the existence of an optimal order quantity in a cycle.

Lemma 1: *The order quantity Q is concave with respect to the cycle length T.*

The proof is straightforward from equation 5.8, since $\frac{\partial^2 Q}{\partial T^2} = -\frac{\alpha q_0 X_2(1-m)\theta}{2}e^{(1-m)\theta(t-t_d)}$ < 0. The lemma ensures that there is an optimal order quantity to be decided by the retailer. Since the order quantity is directly linked with the profit function, the lemma ensures existence of an optimal cycle length.

In a similar manner, the cost and revenue components for the pricing strategy given in Proposition 5.2 can be derived. The inventory level at any time t is

$$I(t) = \begin{cases} Q - \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)\theta}\left(1-e^{-(1-m)\theta t}\right) & \\ +\frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t}\right) - \frac{\alpha q_0 bht}{2(1-m)\theta}e^{-(1-m)\theta t}, & 0 \leqslant t \leqslant t_d \\ \frac{\alpha q_0 X_2}{2(1-m)\theta}e^{-(1-m)\theta t_d} - \frac{\alpha q_0}{2}(a+\beta g + X_1)te^{-(1-m)\theta t} & \\ +K_1 e^{-(1-m)\theta t}, & t_d \leqslant t \leqslant t_1 \\ Ce^{-(1-m)\theta t_1} - \alpha q_0 t e^{-(1-m)\theta t}, & t_1 \leqslant t \leqslant T \end{cases}, \quad (5.9)$$

where $C = I(t_1)e^{(1-m)\theta t_1} + \alpha q_0 t_1$. Putting $I(T) = 0$, we get the ordering quantity Q as

$$\begin{aligned} Q &= \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)\theta}\left(1-e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_d}\right) \\ &+ \frac{\alpha q_0 bht_d}{2(1-m)^2\theta^2}e^{-(1-m)\theta t_d} + \left\{\frac{\alpha q_0}{2}\left(a+\beta g + X_1\right)t_1 - \frac{\alpha q_0 X_2}{2(1-m)\theta}e^{(1-m)\theta(t_1-t_d)}\right. \\ &\left. + \frac{\alpha q_0}{2}\left(\frac{X_2}{(1-m)\theta} - (a+\beta g + X_1)t_d\right)\right\}e^{-(1-m)\theta t_d} + \alpha q_0(T-t_1)e^{-(1-m)\theta t_1}. \end{aligned} \tag{5.10}$$

The total holding cost in cycle is

$$\begin{aligned} HC &= \int_0^{t_d} I(t)dt + \int_{t_d}^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt \\ &= h\left\{\frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_d}\right) + \frac{\alpha q_0 bht_d}{2(1-m)^2\theta^2}e^{-(1-m)\theta t_d}\right. \\ &- \frac{\alpha q_0 bh}{2(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0 bh}{2(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_d}\right) \\ &+ K_2 t_d + \frac{\alpha q_0 X_3 e^{-(1-m)\theta t_d}}{2(1-m)\theta}(t_1-t_d) + \frac{K_1}{(1-m)\theta}\left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta t_1}\right) \\ &+ \frac{\alpha q_0(a+\beta g + X_1)}{2(1-m)\theta}\left(t_1 e^{-(1-m)\theta t_1} - t_d e^{-(1-m)\theta t_d}\right) - \frac{\alpha q_0(a+\beta g+X_1)}{2(1-m)^2\theta^2} \times \\ &\left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta t_1}\right) + \frac{C\left(e^{-(1-m)\theta t_1} - e^{-(1-m)\theta T}\right)}{(1-m)\theta} + \\ &\left.\frac{\alpha q_0}{2}\left(\frac{\left(Te^{-(1-m)\theta T} - t_1 e^{-(1-m)\theta t_1}\right)}{(1-m)\theta} - \frac{\left(e^{-(1-m)\theta t_1} - e^{(1-m)\theta T}\right)}{(1-m)^2\theta^2}\right)\right\}. \end{aligned}$$

The total deterioration cost is

$$\begin{aligned} DC &= c_d(1-m)\theta\int_{t_d}^{T} I(t)dt \\ &= c_d(1-m)\theta\left\{\frac{\alpha q_0 X_2 e^{-(1-m)\theta t_d}}{2(1-m)\theta}(t_1-t_d) + \frac{K_1}{(1-m)\theta}\left(e^{-(1-m)\theta t_d} - e^{-(1-m)\theta t_1}\right)\right. \end{aligned}$$

$$+\frac{\alpha q_0(a+\beta g+X_1)}{2(1-m)\theta}\left(t_1e^{-(1-m)\theta t_1}-t_de^{-(1-m)\theta t_d}\right)-\frac{\alpha q_0(a+\beta g+X_1)}{2(1-m)^2\theta^2}\times$$
$$\left(e^{-(1-m)\theta t_d}-e^{-(1-m)\theta t_1}\right)+\frac{C\left(e^{-(1-m)\theta t_1}-e^{-(1-m)\theta T}\right)}{(1-m)\theta}+$$
$$\frac{\alpha q_0}{2}\left(\frac{\left(Te^{-(1-m)\theta T}-t_1e^{-(1-m)\theta t_1}\right)}{(1-m)\theta}-\frac{\left(e^{-(1-m)\theta t_1}-e^{(1-m)\theta T}\right)}{(1-m)^2\theta^2}\right)\Bigg\}.$$

The greening cost $GC=\frac{1}{2}kg^2$, total production cost $PC=c_pQ$, and investment in preservation technology $PTC=\xi T$. Therefore, the total revenue for the entire period is given by

$$\begin{aligned}
TR &= \int_0^T p(t)D(p(t),g,q(\xi))dt\\
&= \frac{\alpha q_0}{4b}\Bigg[\frac{(a+\beta g)^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta t_d}\right)-\frac{b^2c_p^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta t_d}\right)+\\
&\quad b^2h^2\left\{\frac{t_d^2e^{-(1-m)\theta t_d}}{(1-m)\theta}+\frac{2t_de^{-(1-m)\theta t_d}}{(1-m)^2\theta^2}-\frac{2}{(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_d}\right)\right\}+\\
&\quad 2b^2hc_p\left\{\frac{t_de^{-(1-m)\theta t_d}}{(1-m)\theta}-\frac{1}{(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_d}\right)\right\}+\\
&\quad \frac{((a+\beta g)^2-X_1^2)\left(e^{-(1-m)\theta t_d}-e^{-(1-m)\theta t_1}\right)}{(1-m)\theta}-\frac{X_2^2}{(1-m)\theta}\times\\
&\quad \left(e^{(1-m)\theta(t_1-2t_d)}-e^{-(1-m)\theta t_d}\right)-2X_1X_2e^{-(1-m)\theta t_d}(t_1-t_d)\Bigg]\\
&\quad +\frac{a\alpha q_0\beta g}{(1-m)\theta}\left(e^{-(1-m)\theta t_1}-e^{-(1-m)\theta T}\right).
\end{aligned}$$

The total profit in this case is $TP_{12}=TR-HC-DC-PTC-GC-PC$, and hence the average profit is $AP_{12}(p(t),g,\xi,T)=\frac{TP_{12}}{T}$.

5.1.4.2 Case 2: $t_d>T$

Now we model the scenario where the business manager wraps up business cycle before the products get subjected to physical spoilage. It is to be noted that the manager still invests in preservation to prevent quality deterioration. If the pricing strategy is the one given in the first part of Proposition 5.3, the rate of change in the

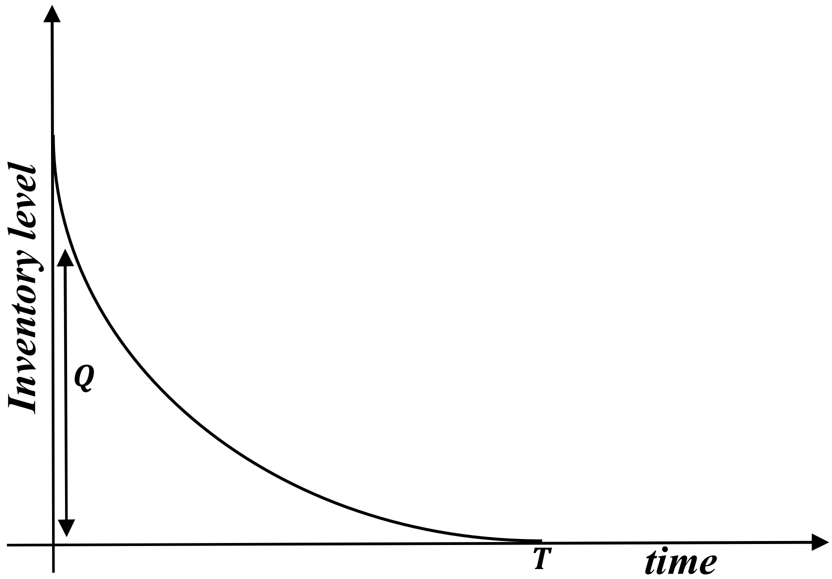


Figure 5.2: Schematic diagram of inventory level for model 2

inventory level is due to demand only, the mathematical expression being governed by

$$\frac{dI(t)}{dt} = -\alpha q\{a - bp(t) + \beta g\}, \qquad (0 \leqslant t \leqslant T) \text{ with } I(0) = Q, \tag{5.11}$$

solving which we get the explicit form of the inventory level as

$$\begin{aligned} I(t) &= Q - \frac{\alpha q_0(a + \beta g - bc_p)}{2(1-m)\theta}\left(1 - e^{-(1-m)\theta t}\right) + \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1 - e^{-(1-m)\theta t}\right) \\ &\quad - \frac{\alpha q_0 bht}{2(1-m)^2\theta^2} e^{-(1-m)\theta t}. \end{aligned} \tag{5.12}$$

Putting $I(T) = 0$, we further get the order quantity Q as

$$\begin{aligned} Q &= \frac{\alpha q_0(a + \beta g - bc_p)}{2(1-m)\theta}\left(1 - e^{-(1-m)\theta T}\right) - \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1 - e^{-(1-m)\theta T}\right) \\ &\quad + \frac{\alpha q_0 bhT}{2(1-m)^2\theta^2} e^{-(1-m)\theta T}. \end{aligned} \tag{5.13}$$

The total holding cost during a cycle is given by

$$\begin{aligned} HC &= \int_0^T I(t)dt \\ &= h\Bigg\{ \frac{\alpha q_0(a+\beta g - bc_p)}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta T}\right) + \frac{\alpha q_0 bhT}{2(1-m)^2\theta^2}e^{-(1-m)\theta T} \\ &\quad - \frac{\alpha q_0 bh}{2(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta T}\right) - \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta T}\right) + K_2 T\Bigg\}. \end{aligned}$$

The total revenue in this case is given by

$$\begin{aligned} TR &= \int_0^T p(t)D(p(t),g,q(\xi))dt \\ &= \frac{\alpha q_0}{4b}\Bigg[\frac{(a+\beta g)^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta T}\right) - \frac{b^2c_p^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta T}\right) + b^2h^2 \times \\ &\quad \left\{\frac{T^2e^{-(1-m)\theta T}}{(1-m)\theta} + \frac{2Te^{-(1-m)\theta T}}{(1-m)^2\theta^2} - \frac{2}{(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta T}\right)\right\} + 2b^2hc_p \times \\ &\quad \left\{\frac{Te^{-(1-m)\theta T}}{(1-m)\theta} - \frac{1}{(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta T}\right)\right\}\Bigg]. \end{aligned}$$

Since greening investment, preservation investment and production cost are similar to those obtained in Case 1, the total profit is thus $TP_{21} = TR - HC - PTC - GC - PC$, and the average profit is $AP_{21}(p(t), g, \xi, T) = \frac{TP_{22}}{T}$. The aim is to maximize the average profit subject to $T \leq t_d$. The existence of optimal green investment is guaranteed by the following proposition:

Proposition 5.5. *The average profit function is concave in g when $k > \frac{\alpha q_0 \beta^2}{2b(1-m)\theta} \times \left(1-e^{-(1-m)\theta T}\right)$.*

The proof is omitted since it is a verbatim copy of the proof of Proposition 5.4. We have an analogous lemma in this case as well, however it is only valid under certain conditions:

Lemma 2: *The order quantity Q is concave with respect to the cycle length T if $T < \frac{2}{(1-m)\theta} - 1$.*

The proof is straightforward since $\frac{\partial^2 Q}{\partial T^2} = -\frac{\alpha q_0(a+\beta g-bc_p)(1-m)\theta}{2}e^{-(1-m)\theta T} - \frac{\alpha q_0 bh}{2(1-m)\theta}\{2-(1+T)(1-m)\theta\}e^{-(1-m)\theta T}$. A sufficient condition for concavity is thus derived. If the cycle length is sufficiently small, case 5.1.4.2 ensures existence of optimal order quantity, thereby establishing the optimal result. However, it also indicates that this case may not converge to any results at all in certain circumstances.

Finally, if the pricing strategy is as per the second part of Proposition 5.3, the inventory level can be derived as

$$I(t) = \begin{cases} Q - \frac{\alpha q_0(a+\beta g-bc_p)}{2(1-m)\theta}\left(1-e^{-(1-m)\theta t}\right) \\ +\frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t}\right) - \frac{\alpha q_0 bht}{2(1-m)^2\theta^2}e^{-(1-m)\theta t}, & 0 \leqslant t \leqslant t_2\ , \\ A + \frac{\alpha q_0 \beta g}{(1-m)\theta}e^{-(1-m)\theta t}, & t_2 \leqslant t \leqslant T \end{cases} \tag{5.14}$$

where $t_2 = \frac{1}{h}\left(\frac{a-\beta g}{b} - c_p\right)$ and $A = I(t_2) - \frac{\alpha q_0 \beta g}{(1-m)\theta}e^{-(1-m)\theta t_2}$, from which the order quantity Q is derived as

$$\begin{aligned} Q \;=\; & \frac{\alpha q_0(a+\beta g-bc_p)}{2(1-m)\theta}\left(1-e^{-(1-m)\theta t_2}\right) - \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_2}\right) + \\ & \frac{\alpha q_0 bht_2}{2(1-m)^2\theta^2}e^{-(1-m)\theta t_2} + \frac{\alpha q_0\beta g}{(1-m)\theta}\left(e^{-(1-m)\theta t_2} - e^{-(1-m)\theta T}\right). \end{aligned} \tag{5.15}$$

The total holding cost in a cycle is given by

$$\begin{aligned} HC \;=\; & \int_0^T I(t)dt \\ =\; & h\Bigg\{\frac{\alpha q_0(a+\beta g-bc_p)}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_2}\right) + \frac{\alpha q_0 bht_2}{2(1-m)^2\theta^2}e^{-(1-m)\theta t_2} \\ & -\frac{\alpha q_0 bh}{2(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_2}\right) - \frac{\alpha q_0 bh}{2(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_2}\right) \\ & +K_2 t_2 + A(T-t_2) + \frac{\alpha q_0\beta g}{(1-m)^2\theta^2}\left(e^{-(1-m)\theta t_2} - e^{-(1-m)\theta T}\right)\Bigg\}. \end{aligned}$$

Finally, the total revenue is

$$\begin{aligned}
TR &= \int_0^T p(t)D(p(t),g,q(\xi))dt \\
&= \frac{\alpha q_0}{4b}\left[\frac{(a+\beta g)^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta t_2}\right)-\frac{b^2c_p^2}{(1-m)\theta}\left(1-e^{-(1-m)\theta t_2}\right)\right. \\
&+b^2h^2\left\{\frac{t_2^2e^{-(1-m)\theta t_2}}{(1-m)\theta}+\frac{2t_2e^{-(1-m)\theta t_2}}{(1-m)^2\theta^2}-\frac{2}{(1-m)^3\theta^3}\left(1-e^{-(1-m)\theta t_2}\right)\right\} \\
&\left.+2b^2hc_p\left\{\frac{t_2e^{-(1-m)\theta t_2}}{(1-m)\theta}-\frac{1}{(1-m)^2\theta^2}\left(1-e^{-(1-m)\theta t_2}\right)\right\}\right] \\
&+\frac{a\alpha q_0\beta g}{(1-m)\theta}\left(e^{-(1-m)\theta t_2}-e^{-(1-m)\theta T}\right),
\end{aligned}$$

and the average profit in this sub-case is derived as $AP_{22}=\frac{TP_{22}}{T}$ where $TP_{22}=TR-HC-PTC-GC-PC$. The retailer is to derive optimal results for the four cases and adopt the best one among them.

5.1.5 Numerical illustration

To illustrate the developed model, the parameter values are chosen as follows: $a=500$ units/week, $b=10$ units/\$/week, $\gamma=0.001$, $\beta=3$ units/week, $\alpha=10$, $\theta=0.1$/week, $q=1$, $c_d=\$3$/unit, $t_d=1$ week, $k=100$, $h=\$1$/unit/week, and $c_p=\$10$/unit. Both the sub-cases under cases 5.1.4.1 and 5.1.4.2 are run separately under corresponding constraint functions in Matlab 2018b software using in-built multi-objective optimization function to get the desired result. It is seen that Case 5.1.4.1 provides better result with an average profit \$1008 under the condition of Proposition 5.3, and optimal values are $g=\$14.7$, $T=2.663$ weeks, $\xi=\$0$/week, and pricing strategy is $p^*(t)=\begin{cases}\frac{644.1+10t}{20}, & 0\leqslant t\leqslant 1\\ \frac{414.1+240e^{0.1(t-1)}}{20} & 1\leqslant t\leqslant 2.663\end{cases}.$

Table 5.2: Sensitivity analysis

Parameter	value	Optimal case	Proposition	ξ	T	g	price	average profit
β	3	1	1	0	2.66	14.7	$p^*(t) = \begin{cases} \frac{644.1+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{414.1+240e^{0.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	1008
	5	1	1	1.77	4.79	31.37		15410
	7	1	1	2	3.66	42		35442
	10	1	1	4.2	2.67	52.5	$p^*(t) = \begin{cases} \frac{225+2t}{4} & 0 \leqslant t \leqslant 1 \\ \frac{179+48e^{.1(t-1)}}{4} & 1 \leqslant t \leqslant 2.66 \end{cases}$	54672
b	5	1	2	647	8.57	103	$p^*(t) = \begin{cases} \frac{110+t}{2} & 0 \leqslant t \leqslant 1 \\ \frac{591.23+123.19e^{.094(t-1)}}{10} & 1 \leqslant t \leqslant 4.44 \\ 100 & 4.44 \leqslant t \leqslant 8.57 \end{cases}$	146356
	7	1	1	0.88	2.734	25		39728
	8	1	1	0	2.54	19		24039
	9	1	1	0	2.58	16.77		11467
	10	1	1	0	2.66	14.7	$p^*(t) = \begin{cases} \frac{644.1+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{414.1+240e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	1008
θ	0.1	1	1	0	2.66	14.7	$p^*(t) = \begin{cases} \frac{644.1+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{414.1+240e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	1008
	0.15	1	2	35.3	5.14	34.7		621

	0.2	1	2	181	6.39	39.44	$p^*(t) = \begin{cases} \frac{718.335+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{529.17+199.17e^{.169(t-1)}}{20} & 1 \leqslant t \leqslant 6.09 \\ 50 & 6.09 \leqslant t \leqslant 6.39 \end{cases}$	273
c_d	0.01	1	1	0	3.45	18.63	$p^*(t) = \begin{cases} \frac{655.89+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{455.79+210e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	7564
	0.1	1	1	0	3.46	19		7370
	1	1	1	0	3.24	17.79		5415
	2	1	1	0	2.97	16.584		3212
	3	1	1	0	2.66	14.7	$p^*(t) = \begin{cases} \frac{644.1+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{414.1+240e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	1008
h	0.05	2	3 (a)	1.43	0.65	3.91	$p^*(t) = \frac{611.73+0.5t}{20} \quad 0 \leqslant t \leqslant 0.65$	37691
	0.1	2	3 (a)	.023	0.91	5.3		35513
	0.3	1	1	0	1.64	9.62		27677
	0.5	1	1	0	2.12	12		20530
	1	1	1	0	2.66	14.7	$p^*(t) = \begin{cases} \frac{644.1+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{414.1+240e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	1008
	0.1	2	3 (a)	73.55	0.656	8.59	$p^*(t) = \frac{526.767+10t}{20} \quad 0 \leqslant t \leqslant 0.66$	52060
	1	1	1	10	1	10.57		49637

c_p	5	1	1	0.64	1.10	7		24814
	8	1	1	0	2.2	13		9748
	10	1	1	0	2.66	14.7	$p^*(t) = \begin{cases} \frac{644.1+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{414.1+240e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	1008
q	0.8	1	1	0	2.55	11.11	$p^*(t) = \begin{cases} \frac{633.33+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{403.33+240e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.55 \end{cases}$	165
	0.9	1	1	0	2.617	13		539
	1	1	1	0	2.663	14.7	$p^*(t) = \begin{cases} \frac{644.1+10t}{20} & 0 \leqslant t \leqslant 1 \\ \frac{414.1+240e^{.1(t-1)}}{20} & 1 \leqslant t \leqslant 2.66 \end{cases}$	1008

We now perform the sensitivity analysis of the proposed model. Table 5.2 exhibits some managerial insights which are summarized below.

- The sensitivity analysis reveals that the green sensitivity significantly impacts the profitability. The observed trend indicates that a rise in the value of β leads to a corresponding increase in the level of profitability since a positive correlation exists between the β value and the demand. The analysis of price sensitivity reveals that an enhanced value of the parameter β confers greater flexibility to the business manager to raise the price, thereby resulting in a corresponding increase in revenue as well. The sensitivity table shows a clear positive impact on the profit level resulting from both factors. Regarding the investment in greening, it can be observed that an augmented β value corresponds to an elevated green sensitivity. This, in turn, incentivises business managers to enhance their investments in greening. Moreover, as the level of environmental consciousness among consumers continues to rise, the market demand for eco-friendly products also increases. Consequently, the marginal value of each unit of such products escalates, thereby exacerbating the financial burden of the degradation process on the company. This is indicative of a greater degree of significance in the preservation investment, as demonstrated by the sensitivity table.

- Analogously, as the price sensitivity experiences an upsurge in value, a marked decline in the profit level is observed. The business manager's price sensitivity necessitates maintaining a price point that is sufficiently low to entice customers, thereby directly impacting the level of profit, as evidenced by the sensitivity table.

- The expense linked to inventory storage is a pivotal consideration in the realm of inventory control. An increase in the per unit holding cost has a significant effect on the overall inventory expenses. The business manager faces a dilemma regarding the rising costs and its impact on demand and revenue. In response to rising costs, the business manager is shiftless to implement a price increase beyond a certain threshold. This decision has a direct impact on both demand and revenue. Thus, the exacerbation of carrying expenses has an adverse impact on both revenues and expenditure, resulting in an unfavourable reduction in profitability.

- It is seen that with an increase in purchasing cost the profit level simply declines. The reason follows similar analysis as before. As we can see, raising c_p has a favourable impact on price, *i.e.*, as c_p rises, the price level likewise rises, which has a detrimental impact on demand. Also, as the c_p value falls, there may be seen an increase in the preservation technology investment and the model may be noticed to shift from 5.1.4.1 to 5.1.4.2, shrinking the cycle length.

- With the increase in deterioration cost, the profit level may be seen to fall sharply. On the other hand, from the price function, one can see that an increase in deterioration cost causes the price to rise. Though a higher price generates more revenue but it affects the demand negatively. Also, a higher per unit deterioration cost makes the deterioration more costlier, thus increasing the total cost of the system. The effect of it may also be noticed in the total cycle length which is continuously diminishing with its arise. All these phenomena ultimately affect the profit level negatively, which is noticeable in the sensitivity table.

- The sensitivity analysis reveals a negative correlation between the deterioration rate and the total profit of the system. As the deterioration rate increases, the total profit level experiences a decline, though the pricing may be observed to increase with it evidently. This phenomenon can be attributed to the fact that the process of deterioration results in only the loss of the item's purchasing value but also the imposition of disposal costs. As the rate of deterioration increases, there appears to be a corresponding increase in investment in preservation technology in order to maintain control. All these ultimately results in an increase of the total cost of the system. Thus the only alternative left to the business managers is to raise the pricing level. The increase in investment towards greening initiatives is indicative of a concerted effort to bolster demand, thereby reducing inventory levels at a faster pace while simultaneously generating revenue. The elongation of the business cycle may be noted as a concurrent effect of escalating deterioration.

5.1.6 Conclusion

An inventory model with price, quality and green investment dependent demand under dynamic pricing strategy is developed and studied in this part. This study provides a detailed illustration of the evolution of pricing strategy conditioned upon various factors. It addresses the dynamic nature of pricing exclusively, thereby rendering the treatment of other attributes such as preservation investment a formidable yet intellectually stimulating task. The propositions under consideration explain the relationship between the retail price and the purchasing cost of the commodity. To maximize the profit level, further inventory management factors such as cycle length, preservation technology investment, and greening investment are also optimized. The numerical illustration establishes the stability and robustness of the solution, and provides valuable managerial insights out of it. However, the consideration of the shortage aspect is omitted here. Incorporating a model featuring planned shortages would introduce a novel facet to this investigation. One can encompass the impact of waiting time on pricing. An investigation of the model under conditions where the rate of deterioration varies over time would be a worthy contribution.

5.2 Role of Lead Time in a Perishable Inventory Model where Quality is an Issue

5.2.1 Introduction

In the perishable food industry, lead time plays a critical role in maintaining product quality and maximizing profitability. A perishable item usually begins to deteriorate as soon as it leaves the warehouse, making efficient transportation and storage essential to preserving freshness. Longer lead times increase the risk of spoilage, reducing the number of high-quality products reaching consumers. To counteract this, businesses must strategically invest in lead time reduction, balancing costs with the benefits of faster delivery and improved product quality. This part examines how lead time, alongside pricing, preservation investment, and sustainability initiatives, influences supply chain performance in a perishable goods market.

One of the major challenges in preserving perishable items with long lead time is the differing preservation needs during transportation versus storage. In transit, products are exposed to fluctuating temperatures, humidity, and handling conditions, necessitating costly refrigeration technologies, insulated packaging, and continuous monitoring systems. Warehouses, in contrast, offer controlled environments where storage conditions can be maintained more efficiently, requiring lower preservation costs. Businesses must therefore allocate preservation investments strategically across these stages to minimize overall deterioration. This part develops a model that integrates lead time reduction investments with dynamic pricing strategies, preservation efforts, and green initiatives. The demand function considers the interplay between price, quality, and environmental factors, reflecting evolving consumer preferences. Additionally, the model introduces a dynamic pricing approach during stock-in periods and a static pricing strategy for planned shortages, providing insights into how businesses can optimize their decisions under different market conditions. By addressing the complexities of lead time, this study offers valuable managerial strategies for improving supply chain efficiency and ensuring sustained profitability in the perishable goods sector.

Motivation of the current work

Pricing alone is not sufficient for effective demand management, especially for perishable and environment-sensitive products such as food, vegetables, and electronics. These items experience both qualitative and quantitative deterioration, with freshness playing a crucial role in consumer demand. While preservation investments can delay deterioration (Li et al., 2019), they often neglect the early-stage quality loss that occurs before measurable spoilage. Additionally, the transportation of perishable goods introduces freshness loss, making lead time a critical factor. For modern, environmentally conscious consumers, sustainability is another key consideration. While Modak et al., 2024b accounted for demand dependencies on price, quality, and greening, their study did not incorporate planned shortages, which can lower costs (Dye et al., 2006) but also lead to potential revenue losses. Furthermore, the adulteration of products to artificially extend freshness raises ethical and environmental concerns, highlighting the need for effective green investment strategies.

To address these gaps, this study develops a dynamic pricing model where demand depends on price, freshness, and greening levels. A hybrid pricing approach is proposed: during shortage periods, a static price ensures uniform quality for all customers, while during stock-in periods, a higher price incentivizes early bookings and encourages customers to wait for fresher products at a lower cost. Investments in preservation and lead time reduction are explored to optimize profitability and sustainability. In this study, we try to answer the following research questions:

RQ 1: What pricing strategy should be adopted for waiting customers when all receive the same quality items?

RQ 2: How should investments be allocated to minimize spoilage, considering different deterioration rates during transportation and storage?

RQ 3: What is the optimal investment to reduce lead time, and is it worth minimizing at all?

RQ 4: Given the rise in product adulteration to extend freshness at the cost of sustainability, can green investments improve demand?

This study is the first to consider both shortages and continuous dynamic pricing, offering a more realistic and practical approach to pricing strategy for perishable and environmentally sensitive products.

5.2.2 Notations and Assumptions

The following notations are used throughout this part of the chapter.

Table 5.3: Notations

p_0	:	static selling price during shortage period (decision variable)
ξ_1	:	preservation technology investment during the transportation period per unit time (decision variable)
ξ_2	:	preservation technology investment during stock-in period per unit time (decision variable)
ϕ	:	investment to reduce the lead time (decision variable)
m_1	:	reduced deterioration rate during the transportation period
m_2	:	reduced deterioration rate during the stock in period
$t_l(\phi)$	:	controllable lead time
$B(x)$	:	backlogging rate, where x is the waiting time to the next replenishment
Q	:	ordering quantity that reaches the warehouse
Q_2	:	ordering quantity when the non-deterioration period is less than the lead time
B	:	total sold items
S	:	total backlogged quantity
α	:	green sensitivity of the customers

Apart from these, the following terms are used to represent the expressions:

$$R = \frac{h + c_d(1-m_2)\theta}{(1-m_2)\theta};$$

$$t_1 = t_d + \frac{2p_0}{h} + \frac{R}{h}\left(1 - e^{-(1-m_2)\theta(T-t_d)}\right) - \frac{(a+\alpha g)}{bh}\left(1 + e^{-(1-m_2)\theta(T-t_d)}\right);$$

$$t_2 = t_1' = T - \frac{1}{(1-m_2)\theta}\log\left(\frac{a+\alpha g + Rb}{Rb + 2bp_0 - a - \alpha g}\right);$$

$$\begin{aligned} t_3 &= T - \frac{2(a + \alpha g - bp_0)}{bh}; \\ X_1 &= (a + \alpha g + Rb)e^{-(1-m_2)\theta(T-t_d)} - Rb - hbt_d. \end{aligned}$$

The following assumptions are made for developing the proposed models.

- Demand is influenced by product price, quality, and greening efforts concurrently. The price is static during the shortage period but varies with time during the stock-in period. Since the customers arriving during shortage period and waiting until next replenishment are aware that the quality starts decaying right after the product leaves the production inventory, and the item will be received at time t_l, the demand during shortage period is thus justified to be assumed as $D(p_0, g, q) = ye^{-(1-m_1)\theta t_l}\left(a - bp_0 + \alpha g\right)$ where $y = y_0 q_0$, q_0 being the initial freshness level of the product. The product reaches the warehouse with a freshness level of $q_0 e^{-(1-m_1)\theta t_l}$. During stock-in period, the freshness level is $q(t_l)e^{-(1-m_2)\theta(t-t_l)}$, the demand during stock-in period can therefore be expresses as $D(p(t), g, q(t, \xi)) = we^{-(1-m_2)\theta(t-t_l)}\left(a - bp(t) + \alpha g\right)$ (Modak et al., 2024a), where $(w = ye^{-(m_2-m_1)\theta t_l})$. To keep the demand always positive even in the absence of green investment, the sufficient conditions $p(t) \leq \frac{a}{b}$ and $p_0 \leq \frac{a}{b}$ are assumed.

- An investment ϕ is made to reduce the deterministic lead time t_l. We assume $\frac{\partial t_l}{\partial \phi} < 0$ to justify such investment, and $\frac{\partial^2 t_l}{\partial \phi^2} > 0$ to ensure diminishing return of investment. Following Sarkar et al., 2022, the specific pattern of the lead time is assumed as $t_l = e^{-\rho\phi}t_0$, where t_0 is the original lead time in the absence of any investment, and ρ is the measure of the efficiency of the reduction system.

- The quality $q(t, \xi_i)$ of the product decreases continuously with time from the very moment it is dispatched, so that $\frac{\partial q(t,\xi_i)}{\partial t} < 0$. The quality degradation may be reduced by preservation investment ξ_i so that $\frac{\partial q(t,\xi_i)}{\partial \xi_i} > 0$, and $\frac{\partial^2 q(t,\xi_i)}{\partial \xi_i^2} < 0$ which ensures diminishing return on such investment. Following Modak et al., 2024a, the quality change during transportation period would be $\dot{q}(t) = -[1 - m_1(\xi_1)]\theta q(t)$ with initial quality $q(0) = 1$, and that during stock-in period would be $\dot{q}(t) = -[1 - m_2(\xi_2)]\theta q(t)$.

- The qualitative deterioration is assumed to be instantaneous, so the product reaches the inventory with a freshness level $q_0 e^{-(1-m_1)\theta t_l}$.

- A cycle starts with a planned shortage. The backlogging rate depends on the waiting time of the customers, the specific form being $B(t_l - t) = e^{-\eta(t_l - t)}$ (Pervin et al., 2018), η denoting unwillingness of the customers to wait.

5.2.3 Model Formulation

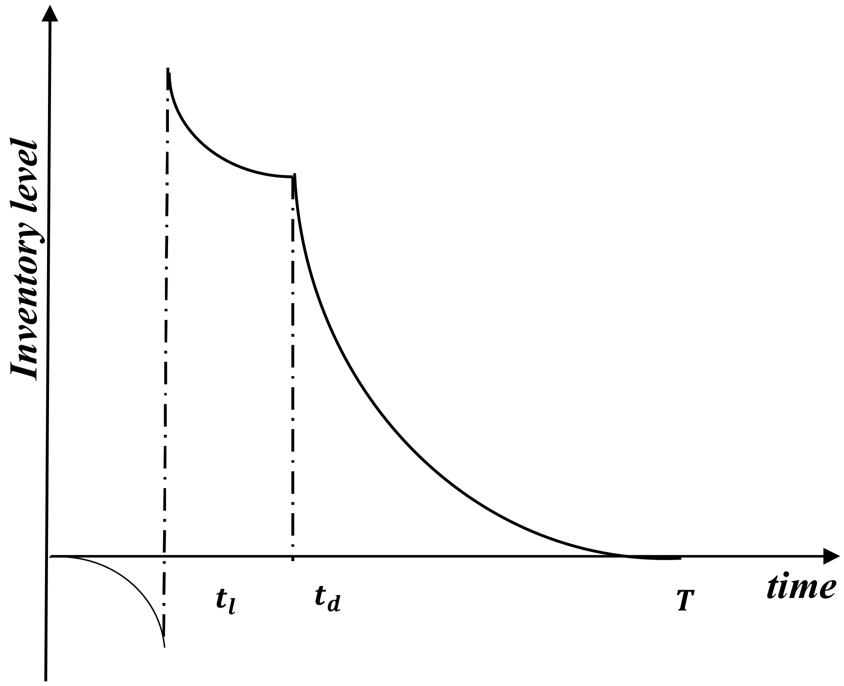


Figure 5.3: Schematic diagram of the inventory level for model 1

Based on the assumptions listed above, let us now develop the model. Depending on the controllable lead time (t_l), non-deterioration period (t_d), and cycle length (T), three different cases may arise, which need separate derivation and analysis. The cases are discussed in the consecutive subsections.

5.2.3.1 Case 1: $t_l \leqslant t_d \leqslant T$

Figure 5.3 represents the inventory level of the case. The condition represents the scenario when the items are not subject to quantitative deterioration while being transported. The inventory level at any time t is governed by

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\eta(t_l - t)} D(p_0, g, q_{t_l}) & \text{with } I(0) = 0 \quad 0 \leqslant t \leqslant t_l \\ -D(p, g, q) & \text{with } I(t_l) = Q - S, \quad t_l < t \leqslant t_d \\ -D(p, g, q) - (1 - m_2)\theta I(t), & t_d < t \leqslant T. \end{cases} \tag{5.16}$$

Solving the above differential equation under the continuity of the inventory level at $t = t_d$, we get

$$I(t) = \begin{cases} -\frac{y(a - bp_0 + \alpha g)e^{-(1-m_1)\theta t_l}}{\eta} e^{-\eta t_l}\left(e^{\eta t} - 1\right), & (0 \leqslant t \leqslant t_l) \\ (Q - S) - \frac{w(a - bp_0 + \alpha g)}{(1 - m_2)\theta}\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta t}\right), & (t_l \leqslant t \leqslant t_1) \\ I(t_1) - \frac{w(a + \alpha g - X_1)}{2b(1 - m_2)\theta}\left(e^{-(1-m_2)\theta t_1} - e^{-(1-m_2)\theta t}\right) & \\ -\frac{hbw}{2(1 - m_2)\theta}\left(te^{-(1-m_2)\theta t} - t_1 e^{-(1-m_2)\theta t_1}\right) & \\ -\frac{hbw}{2(1 - m_2)^2\theta^2}\left(e^{-(1-m_2)\theta t} - e^{-(1-m_2)\theta t_1}\right), & (t_1 \leqslant t \leqslant t_d) \\ I(t_d)e^{-(1-m_2)\theta(t - t_d)} - \frac{w(a + \alpha g + Rb)}{2b}\Big\{e^{-(1-m_2)\theta t}(t - t_d) & \\ -\frac{e^{-(1-m_2)\theta T}}{(1 - m_2)\theta}\left(1 - e^{-(1-m_2)\theta(t - t_d)}\right)\Big\}, & (t_d \leqslant t \leqslant T), \end{cases} \tag{5.17}$$

where $I(t_1) = (Q - S) - \frac{w(a - bp_0 + \alpha g)}{(1 - m_2)\theta}\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta t_1}\right)$ and $I(t_d) = I(t_1)$ $- \frac{w(a + \alpha g - X_1)}{2b(1 - m_2)\theta}\left(e^{-(1-m_2)\theta t_1} - e^{-(1-m_2)\theta t_d}\right) - \frac{hw}{2(1 - m_2)\theta}\left(t_d e^{-(1-m_2)\theta t_d} - t_1 e^{-(1-m_2)\theta t_1}\right)$ $- \frac{hbw}{2(1 - m_2)^2\theta^2}\left(e^{-(1-m_2)\theta t_d} - e^{-(1-m_2)\theta t_1}\right)$. The origin of t_1 is mentioned in later section of this chapter while its expression along with that of R have already been provided in section 5.2.2. Total amount of backlog is obtained as the (negative) inventory at the endpoint of the shortage period (*i.e.* at $t = t_l$):

$$S = \frac{y(a - bp_0 + \alpha g)e^{-(1-m_1)\theta t_l}}{\eta}\left(1 - e^{-\eta t_l}\right). \tag{5.18}$$

Since inventory at the end of the period reaches zero, *i.e.* $I(T) = 0$, the ordering quantity may further be derived as

$$Q = S + \frac{(a - bp_0 + \alpha g)w}{(1 - m_2)\theta}\left(1 - e^{-(1-m_2)\theta t_1}\right) + \frac{w(a + \alpha g - X_1)}{2b(1 - m_2)\theta}\left(1 - e^{-(1-m_2)\theta t_d}\right)$$

$$+\frac{hbw}{2(1-m_2)\theta}\left(t_d e^{-(1-m_2)\theta t_d} - t_1 e^{-(1-m_2)\theta t_1}\right) + \frac{hbw}{2(1-m_2)^2\theta^2}\Big(e^{-(1-m_2)\theta t_d}$$
$$-e^{-(1-m_2)\theta t_1}\Big) + \frac{w(a+\alpha g+Rb)}{2b}\Big\{e^{-(1-m_2)\theta t_d}(T-t_d)$$
$$-\frac{1}{(1-m_2)\theta}\left(e^{-(1-m_2)\theta t_d} - t_1 e^{-(1-m_2)\theta T}\right)\Big\}. \tag{5.19}$$

Total holding cost during the entire period is given by $HC = h\left(\int_{t_l}^{T} I(t)dt\right)$, where

$$\int_{t_l}^{T} I(t)dt$$
$$= (Q-S)(t_1-t_l) - \frac{(a-bp_0+\alpha g)we^{-(1-m_2)\theta t_l}}{(1-m_2)\theta}\left\{(t_1-t_l) - \frac{1-e^{-(1-m_2)\theta t_1}}{(1-m_2)\theta}\right\}$$
$$+I(t_1)(t_d-t_1) - w\Bigg[\frac{(a+\alpha g-X_1)}{2(1-m_2)\theta}\Big\{e^{-(1-m_2)\theta t_1}(t_d-t_1)$$
$$-\frac{e^{-(1-m_2)\theta t_1}-e^{-(1-m_2)\theta t_d}}{(1-m_2)\theta}\Big\} + \frac{hb}{2(1-m_2)^2\theta^2}\Big\{\frac{e^{-(1-m_2)\theta t_1}-e^{-(1-m_2)\theta t_d}}{(1-m_2)\theta}$$
$$+\frac{1}{(1-m_2)\theta}\Big((1+(1-m_2)\theta t_1)e^{-(1-m_2)\theta t_1} - (1+(1-m_2)\theta t_d)e^{-(1-m_2)\theta t_d}\Big)$$
$$-(1+(1-m_2)\theta t_1)e^{-(1-m_2)\theta t_1}(t_d-t_1)\Big\}\Bigg] + I(t_d)\frac{\left(1-e^{-(1-m_2)\theta(T-t_d)}\right)}{(1-m_2)\theta}$$
$$-\frac{w(a+\alpha g+Rb)}{2}\Bigg[\Big\{\frac{(1+(1-m_2)\theta t_d)e^{-(1-m_2)\theta t_d}}{((1-m_2)^2\theta^2)}$$
$$-\frac{(1+(1-m_2)\theta T)e^{-(1-m_2)\theta T}}{((1-m_2)^2\theta^2)}\Big\} - \frac{t_d\left(e^{-(1-m_2)\theta t_d}-e^{-(1-m_2)\theta T}\right)}{(1-m_2)\theta}$$
$$-\frac{e^{-(1-m_2)\theta T}}{(1-m_2)\theta}(T-t_d) + \frac{e^{-(1-m_2)\theta T}}{(1-m_2)^2\theta^2}\left(1-e^{-(1-m_2)\theta(T-t_d)}\right)\Bigg].$$

The shortage cost is

$$SC = c_s y e^{-(1-m_1)\theta t_l}\left\{(a-bp_0+\alpha g)t_l - \frac{(a-bp_0+\alpha g)\left(1-e^{-\eta t_l}\right)}{\eta}\right\}.$$

The deterioration cost is given by $DC = c_d(1-m_2)\theta\int_{t_d}^{T} I(t)dt$

$$= c_d(1-m_2)\theta\Bigg(I(t_d)\frac{\left(1-e^{-(1-m_2)\theta(T-t_d)}\right)}{(1-m_2)\theta} - \frac{w(a+\alpha g+Rb)}{2}\times$$
$$\Big\{\Big((1+(1-m_2)\theta t_d)e^{-(1-m_2)\theta t_d} - (1+(1-m_2)\theta T)e^{-(1-m_2)\theta T}\Big)$$
$$-\frac{t_d\left(e^{-(1-m_2)\theta t_d} - e^{-(1-m_2)\theta T}\right)}{(1-m_2)\theta} - \frac{e^{-(1-m_2)\theta T}}{(1-m_2)\theta}(T-t_d)$$
$$+\frac{e^{-(1-m_2)\theta T}}{(1-m_2)^2\theta^2}\left(1-e^{-(1-m_2)\theta(T-t_d)}\right)\Big\}\Bigg).$$

The total revenue accumulated during the entire period is $TR(p_0,\xi_1,\xi_2,\phi,T,g)$

$$= p_0S + \int_{t_l}^{T} wp(t)\{a-bp(t)+\alpha g\}q(t)dt$$
$$= p_0S + wp_0(a-bp_0+\alpha g)\frac{\left(e^{-(1-m_2)\theta t_1} - e^{-(1-m_2)\theta t_l}\right)}{(1-m_2)\theta} + \frac{w\left((a+\alpha g)^2 - X_1^2\right)}{4b(1-m_2)\theta}\times$$
$$\left(e^{-(1-m_2)\theta t_1} - e^{-(1-m_2)\theta t_d}\right) - \frac{X_1hw}{2(1-m_2)^2\theta^2}\Big((1+(1-m_2)\theta t_1)\,e^{-(1-m_2)\theta t_1}$$
$$-(1+(1-m_2)\theta t_d)\,e^{-(1-m_2)\theta t_d}\Big) - \frac{h^2wb}{4(1-m_2)^3\theta^3}\Big[\Big((1-m_2)^2\theta^2t_1^2$$
$$+2(1-m_2)\theta t_1+2\Big)e^{-(1-m_2)\theta t_1} - \Big((1-m_2)^2\theta^2t_d^2+2(1-m_2)\theta t_d+2\Big)\times$$
$$e^{-(1-m_2)\theta t_d}\Big] + \frac{w\left((a+\alpha g)^2-R^2b^2\right)}{4(1-m_2)b\theta}\left(e^{-(1-m_2)\theta t_d} - e^{-(1-m_2)\theta T}\right)$$
$$+\frac{(a+\alpha g+Rb)wR}{2}e^{-(1-m_2)\theta T}(T-t_d) + \frac{w(a+\alpha g+Rb)^2}{4(1-m_2)\theta b}\times$$
$$\left(e^{-(1-m_2)\theta(2T-t_d)} - e^{-(1-m_2)\theta T}\right),$$

where X_1 has been explained in section 5.2.2. The total greening cost GC, lead time reduction cost LC, and preservation technology investment cost PTC are given by $GC = \frac{1}{2}kg^2$, $LC = \phi$, and $PTC = \xi_1 t_l + \xi_2(T-t_l)$, respectively. The total and average profits are respectively given by

$$\begin{aligned} TP_1(p_0,\xi_1,\xi_2,\phi,T,g) &= TR(p_0,\xi_1,\xi_2,\phi,T,g) - PC(\xi_1,\xi_2,\phi,g,T) \\ &\quad -DC(p_0,\xi_1,\xi_2,\phi,T,g) - HC(p_0,\xi_1,\xi_2,\phi,T,g) - GC(g) \\ &\quad -PTC(\xi_1,\xi_2) - LC(\phi) - SC(p_0,\xi_1,\phi,g), \end{aligned}$$

and $AP_1 = \frac{TP_1}{T}$. AP_1 is now to be maximized subject to $0 < t_l \leqslant t_d \leqslant T$.

5.2.3.2 Case 2: $t_d \leqslant t_l \leqslant T$

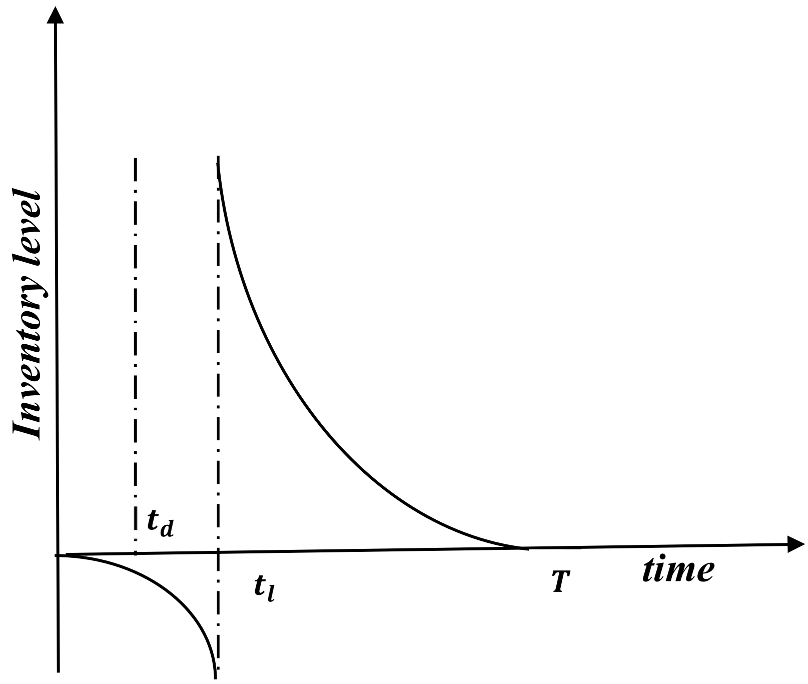


Figure 5.4: Schematic diagram of the inventory level for model 2

Figure 5.4 represents inventory dynamics of this case. This case needs attention to two different inventory dynamics. The inventory level in the cargo during shipment is governed by

$$\frac{dI(t)}{dt} = \begin{cases} 0 & \text{with } I(0) = Q_2, \quad 0 \leqslant t \leqslant t_d \\ -(1-m_1)\theta I(t) & \text{with } I(t_l) = Q, \quad t_d < t \leqslant t_l, \end{cases}$$

from which the continuity of inventory level at $t = td$ yields the following proposition:

Proposition 5.6. $Q_2 = Qe^{(1-m_1)\theta(t_l - t_d)}$.

The rate of change of inventory level at any time t in retailer's inventory however continues to follow the following differential equation:

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\eta(t_l - t)}D(p_0, g, q_{t_l}) \quad \text{with } I(0) = 0, I(t_l) = -S & 0 \leqslant t \leqslant t_l \\ -D(p, g, q) - (1 - m_2)\theta I(t) \text{ with } I(t_l) = Q - S, I(T) = 0, & t_l < t \leqslant T. \end{cases} \tag{5.20}$$

Solving 5.20, we get

$$I(t) = \begin{cases} -\frac{(a - bp_0 + \alpha g)ye^{-(1-m_1)\theta t_l}}{\eta} e^{-\eta t_l}\left(e^{\eta t} - 1\right), & (0 \leqslant t \leqslant t_l) \\ (Q - S)e^{-(1-m_2)\theta(t - t_l)} - w(a - bp_0 + \alpha g)\times & \\ (t - t_l)e^{-(1-m_2)\theta t}, & (t_l \leqslant t \leqslant t_2) \\ I(t_2)e^{-(1-m_2)\theta(t - t_2)} - \frac{(a + \alpha g + Rb)w}{2}\Big\{(t - t_2)e^{-(1-m_2)\theta t} & \\ -\frac{e^{-(1-m_2)\theta T}}{(1-m_2)\theta}\left(1 - e^{-(1-m_2)\theta(t - t_2)}\right)\Big\}, & (t_2 \leqslant t \leqslant T), \end{cases} \tag{5.21}$$

where $I(t_2) = (Q - S)e^{-(1-m_2)\theta(t_2 - t_l)} - w(a - bp_0 + \alpha g)(t_2 - t_l)e^{-(1-m_2)\theta t_2}$, origin of t_2 is mentioned in the later section, and the specific expression of t_2 has already been provided in section 5.2.2. The boundary condition $I(T) = 0$ further gives the explicit form of the order quantity as

$$\begin{aligned} Q \quad = \quad & S + \frac{w(a + \alpha g + Rb)}{2}\left\{(T - t_2)e^{-(1-m_2)\theta t_l} - \frac{e^{-(1-m_2)\theta t_l}}{(1 - m_2)\theta}\left(1 - e^{-(1-m_2)\theta(T - t_2)}\right)\right\} \\ & + w(a - bp_0 + \alpha g)(t_2 - t_l)e^{-(1-m_2)\theta t_l}. \end{aligned} \tag{5.22}$$

The holding cost is $HC = h\int_{t_l}^{T} I(t)dt$

$$
\begin{aligned}
= & \left((Q-S)e^{-(1-m_2)\theta t_l} + w(a-bp_0+\alpha g)t_l\right)\frac{\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta t_2}\right)}{(1-m_2)\theta} \\
& -\frac{w(a-bp_0+\alpha g)}{(1-m_2)^2\theta^2}\left\{(1+(1-m_2)\theta t_l)e^{-(1-m_2)\theta t_l} - (1+(1-m_2)\theta t_2)e^{-(1-m_2)\theta t_2}\right\} \\
& +I(t_2)\frac{\left(1-e^{-(1-m_2)\theta(T-t_2)}\right)}{(1-m_2)\theta} - \frac{(a+\alpha gRb)w}{2}\times \\
& \left[\left\{\frac{((1-m_2)\theta t_2+1)e^{-(1-m_2)\theta t_2} - ((1-m_2)\theta T+1)e^{-(1-m_2)\theta T}}{(1-m_2)^2\theta^2}\right\}\right. \\
& \left. +\left\{\left(\frac{e^{-(1-m_2)\theta(T-t_2)}}{(1-m_2)\theta} - t_2\right)\frac{e^{-(1-m_2)\theta t_2} - e^{-(1-m_2)\theta T}}{(1-m_2)\theta}\right\} - \frac{e^{-(1-m_2)\theta T}}{(1-m_2)\theta}(T-t_2)\right].
\end{aligned}
$$

The deterioration cost is $DC = c_d\left\{(1-m_2)\theta\left(\int_{t_l}^{T} I(t)dt\right) + (Q_2 - Q)\right\}$.

The total revenue is given by

$$
\begin{aligned}
& p_0S + \int_{t_l}^{T} p(t)\{a-bp(t)+\alpha g\}q(t)dt \\
= & p_0S + wp_0(a-bp_0+\alpha g)\frac{\left(e^{-(1-m_2)\theta t_2} - e^{-(1-m_2)\theta t_l}\right)}{(1-m_2)\theta} + \frac{w}{4b}\left[\frac{((a+\alpha g)^2 - b^2R^2)}{(1-m_2)\theta}\times\right. \\
& \left(e^{-(1-m_2)\theta t_2} - e^{-(1-m_2)\theta T}\right) - \frac{(a+\alpha g+bR)^2}{(1-m_2)\theta}\left(e^{-(1-m_2)\theta T} - e^{-(1-m_2)\theta(2T-t_2)}\right) \\
& \left. -2bR(a+\alpha g+bR)e^{-(1-m_2)\theta T}(T-t_2)\right].
\end{aligned}
$$

Noting that the preservation investment, greening cost, and lead time reduction cost are identical to those of the earlier case, the average profit function which is to be maximized is $AP_2 = \frac{TP}{T}$ subject to $t_d \leqslant t_l \leqslant T$, where $TP_2(p_0, \xi_1, \xi_2, \phi, T, g) = TR(p_0, \xi_i, \xi_2, \phi, T, g) - PC(\xi_1, \xi_2, \phi, g, T) - DC(p_0, \xi_1, \xi_2, \phi, T, g) - HC(p_0, \xi_1, \xi_2, \phi, T, g) - GC(g) - PTC(\xi_1, \xi_2) - LC(\phi) - SC(p_0, \xi_1, \phi, g)$.

5.2.3.3 Case 3: $t_l \leqslant T \leqslant t_d$

Figure 5.5 represents inventory dynamics of this case. This case demonstrates the scenario where the cycle ends before the deterioration takes place. The rate of change of inventory level at any time t can be described by

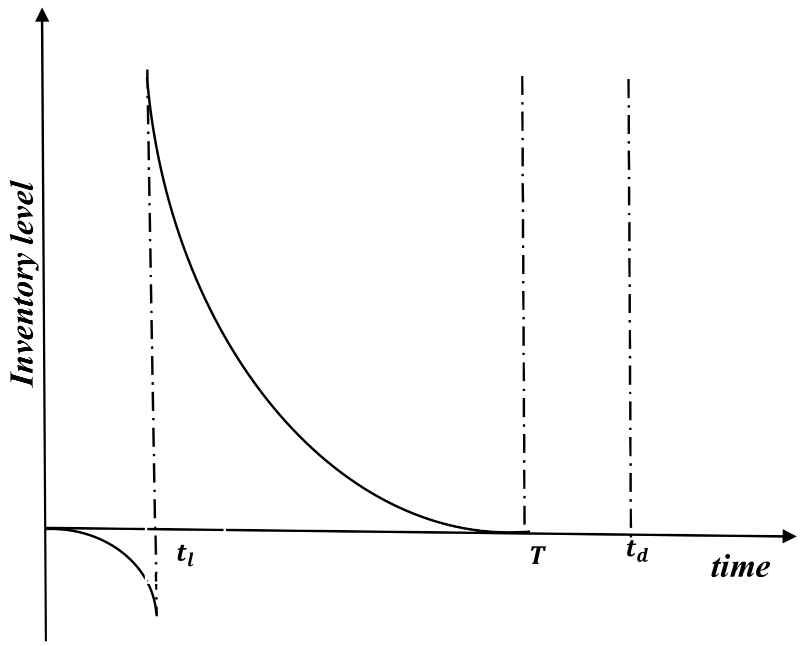


Figure 5.5: Schematic diagram of inventory level for model 3

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\eta(t_l - t)} D(p, g, q_{t_l}) & \text{with } I(0) = 0, \quad 0 \leqslant t \leqslant t_l \\ -D(p, g, q) \quad \text{with } I(t_l) = Q - S, & t_l < t \leqslant T. \end{cases} \tag{5.23}$$

Solving, we get

$$I(t) = \begin{cases} -\frac{(a - bp_0 + \alpha g) y e^{-(1-m_1)\theta t_l}}{\eta} e^{-\eta t_l} \left(e^{\eta t} - 1 \right), & (0 \leqslant t \leqslant t_l) \\ (Q - S) - \frac{w(a - bp_0 + \alpha g)}{(1-m_2)\theta} \left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta t} \right), & (t_l \leqslant t \leqslant t_3) \\ I(t_3) - \frac{hbTw}{2} \left(e^{-(1-m_2)\theta t_3} - e^{-(1-m_2)\theta t} \right) + \frac{hbw}{2} \times & \\ \left\{ \frac{(1 + (1-m_2)\theta t_3) e^{-(1-m_2)\theta t_3} - (1 + (1-m_2)\theta t) e^{-(1-m_2)\theta t}}{(1-m_2)\theta} \right\}, & (t_3 \leqslant t \leqslant T), \end{cases} \tag{5.24}$$

with $I(t_3) = (Q-S) - \frac{w(a-bp_0+\alpha g)}{(1-m_2)\theta}\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta t_3}\right)$, where t_3 is explained later with expression provided in notations section 5.2.2.
In a similar manner, order quantity in this case is derived from $I(T) = 0$ as

$$\begin{aligned} Q &= S + \frac{(a-bp_0+\alpha g)w}{2(1-m_2)\theta}\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta t_3}\right) + \frac{hbTw}{2(1-m_2)\theta} \times \\ &\left(e^{-(1-m_2)\theta t_3} - e^{-(1-m_2)\theta T}\right) - \frac{hbw}{2(1-m_2)\theta}\Big\{(1+(1-m_2)\theta t_3)e^{-(1-m_2)\theta t_3} \\ &-(1+(1-m_2)\theta T)e^{-(1-m_2)\theta T}\Big\}. \end{aligned} \tag{5.25}$$

Total holding cost is $HC = h\left(\int_{t_l}^{T} I(t)dt\right)$

$$\begin{aligned} &= h\Bigg[\left(Q - S - \frac{w(a-bp_0+\alpha g)}{(1-m_2)\theta}e^{-(1-m_2)\theta t_l}\right)(t_3 - t_l) + \frac{w(a-bp_0+\alpha g)}{(1-m_2)^2\theta^2} \times \\ &\left(e^{-(1-m_2)\theta t_l} - e^{-(1-m_2)\theta t_3}\right)\left(I(t_3) - \frac{hbTw}{2(1-m_2)\theta}e^{-(1-m_2)\theta t_3} + \frac{hbw}{2(1-m_2)\theta} \times \right. \\ &\left.(1+(1-m_2)\theta t_3)e^{-(1-m_2)\theta t_3}\right)(T-t_3) + \frac{hbw(T-1)}{2(1-m_2)^2\theta^2}\left(e^{-(1-m_2)\theta t_3} - e^{-(1-m_2)\theta T}\right) \\ &-\frac{hbw}{2(1-m_2)\theta}\Big((1+(1-m_2)\theta t_3)e^{-(1-m_2)\theta t_3} \\ &-(1+(1-m_2)\theta T)e^{-(1-m_2)\theta T}\Big)\Bigg]. \end{aligned}$$

The total revenue is $TR(p_0, \xi_1, \xi_2, \phi, T, g)$

$$\begin{aligned} & p_0 S + \int_{t_l}^{T} wp(t)(a - bp(t) + \alpha g)q(t)dt \\ =\; & p_0 S + wp_0\frac{(a-bp_0+\alpha g)}{(1-m_2)\theta}\left(e^{-(1-m_2)\theta t_3} - e^{-(1-m_2)\theta t_l}\right) \\ & + \frac{w}{4b}\Bigg[\frac{(a+\alpha g)^2 - (a+\alpha g - hbT)^2}{(1-m_2)\theta}\left(e^{-(1-m_2)\theta t_3} - e^{-(1-m_2)\theta T}\right) \\ & - \frac{h^2b^2}{(1-m_2)^3\theta^3}\Big((2+2(1-m_2)\theta t_3 + (1-m_2)^2\theta^2 t_3^2)e^{-(1-m_2)\theta t_3} \\ & -(2+2(1-m_2)\theta T^3 + (1-m_2)^2\theta^2T^2)e^{-(1-m_2)\theta T}\Big) - \frac{2hb(a+\alpha g)}{(1-m_2)\theta} \times \end{aligned}$$

$$\left(e^{-(1-m_2)\theta t_3} - e^{-(1-m_2)\theta T}\right) \frac{2h^2b^2}{(1-m_2)\theta}\Big((1+(1-m_2)\theta t_3)e^{-(1-m_2)\theta t_3}$$
$$-(1+(1-m_2)\theta T)e^{-(1-m_2)\theta T}\Big)\Big].$$

The shortage cost is $SC = c_s y e^{-(1-m_1)\theta t_l}\left\{(a-bp_0+\alpha g)t_l - \frac{(a-bp_0+\alpha g)}{\eta}\left(1-e^{-\eta t_l}\right)\right\}$. The total profit and the average profit in this case thus can likewise be derived respectively as

$$\begin{aligned} TP_3(p_0,\xi_1,\xi_2,\phi,T,g) &= TR(p_0,\xi_i,\xi_2,\phi,T,g) - PC(\xi_1,\xi_2,\phi,g,T) \\ &\quad -HC(p_0,\xi_1,\xi_2,\phi,T,g) - GC(g) \\ &\quad -PTC(\xi_1,\xi_2) - LC(\phi) - SC(p_0,\xi_1,\phi,g) \end{aligned}$$

and $AP_3 = \frac{TP_3}{T}$ subject to $t_l \le T \le t_d$.

5.2.4 Solution Methodology

A two-period pricing strategy is adopted here to solve each of the developed models. For the shortage period, a static pricing strategy is adopted; the derived price is then used as the initial price of the stock-in period. Different pricing policies for all three cases will be derived one by one under required conditions; for a given set of parameter values, the case producing the highest average profit will be adopted.

Let us derive the profit function during the shortage period first. It is observed that the profit dynamics during the shortage period are identical in cases 1 and 3, while it contains an extra component $c_d(Q_2 - Q)$ in case 5.2.4.2. Since both Q_2 and Q are related to the dynamic price, we choose to consider the component while optimizing the stock-in period. This also makes the profit during shortage period identical in all three cases. The price-related components are thus taken into account only to rewrite the profit function as

$$\begin{aligned} \Pi_S &= (p_0 - c_p + c_s)y\frac{(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l}}{\eta}\left(1-e^{-\eta t_l}\right) \\ &\quad -c_s(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l}t_l. \end{aligned}$$

The waiting customers are aware that the quality of the items depends on the time of delivery but is independent of the time of order. Therefore, the quality component of the demand will remain the same during the shortage period as it was at the time $t = t_l$. The following proposition ensures the existence of optimal static pricing during the shortage period.

Proposition 5.7. *The profit function* Π_S *is concave in* p_0*, and the optimal price is given by* $p_0 = \frac{a+\alpha g}{2b} + \frac{(c_p - c_s)}{2} + \frac{\eta c_s t_l}{2(1-e^{-\eta t_l})}$.

Proof: It is straightforward to derive that

$$\begin{aligned}
\frac{\partial \Pi_S}{\partial p_0} &= -(p_0 - c_p + c_s)e^{-(1-m_1)\theta t_l}\frac{by}{\eta}\left(1 - e^{-\eta t_l}\right) \\
&\quad + \frac{(a - bp_0 + \alpha g)y}{\eta}e^{-(1-m_1)\theta t_l}\left(1 - e^{-\eta t_l}\right) + c_s b e^{-(1-m_1)\theta t_l} y t_l, \\
\text{and } \frac{\partial^2 \Pi_S}{\partial p_0^2} &= -\frac{2be^{-(1-m_1)\theta t_l}(1 - e^{-\eta t_l})}{\eta} < 0,
\end{aligned}$$

which ensures the concavity of Π_S. Setting $\frac{\partial \Pi_S}{\partial p_0}$=0, we obtain the optimal price as provided.

We now characterize the optimal price in the following property.

Property 1. *(i)* $\frac{\partial p_0}{\partial \alpha} > 0$; *(ii)* $\frac{\partial p_0}{\partial c_s} < 0$; *(iii)* $\frac{\partial p_0}{\partial \eta} > 0$; *(iv)* $\frac{\partial p_0}{\partial t_0} > 0$; *(v)* $\frac{\partial S}{\partial g} > 0$; *(vi)* $\frac{\partial S}{\partial p_0} < 0$.

Proof: Proofs of (i) and (ii) are straightforward.

For (iii), we have $\frac{\partial p_0}{\partial \eta} = \frac{c_s t_l}{2\left(1-e^{-\eta t_l}\right)}\left(1 - \frac{\eta t_l e^{-\eta t_l}}{\left(1-e^{-\eta t_l}\right)}\right)$. Now, $\left(1 - \frac{\eta t_l e^{-\eta t_l}}{\left(1-e^{-\eta t_l}\right)}\right) < 0$ leads to $(1 + \eta t_l) > e^{\eta t_l}$, which is false. Hence the result (iii) is proved. Result (iv), (v), (vi) can be proven similarly and hence omitted.

Property 1 indicates that the price is likely to increase with customers' sensitivity towards green products. Since the shortage cost has a negative effect on profit, the price is reduced with a higher shortage cost. The price may be raised to offset the profit loss when the backlog rate increases. Finally, the price increases with the initial lead time, the possible reason being the increasing cost related to lead time reduction, shortage, and spoilage.

5.2.4.1 Case 1: $t_l \leqslant t_d \leqslant T$

We now derive the optimal dynamic pricing policy during the stock-in period using Pontryagin's maximum principle. The objective function is rewritten as

$$\begin{aligned}\max_{p(.)} J_L &= \frac{1}{T}\Bigg[\int_0^{t_l} p_0 y(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l} B(t_l-t)dt \\ &+ \int_{t_l}^{T} p(t)w(a-bp(t)+\alpha g)q(t)dt - c_d(1-m_2)\theta\int_{t_d}^{T} I(t)dt \\ &- h\int_{t_l}^{T} I(t)dt - \xi_1 t_l - \xi_2(T-t_l) - \frac{1}{2}kg^2 - \phi - A - c_p Q \\ &- c_s y e^{-(1-m_1)\theta t_l}\left\{(a-bp_0+\alpha g) - \frac{(a-bp_0+\alpha g)}{\eta}\left(1-e^{-\eta t_l}\right)\right\}\Bigg], \quad (5.26)\end{aligned}$$

subject to the stationary conditions

$$\dot{I}(t) = \begin{cases} -yB(t-t_l)(a-bp(t)+\alpha g) & (0 \leqslant t \leqslant t_l), \\ -w(a-bp(t)+\alpha g)q(t) & (t_l \leqslant t \leqslant t_d), \\ -w(a-bp(t)+\alpha g)q(t) - (1-m_2)\theta I(t) & (t_d \leqslant t \leqslant T). \end{cases}$$

The Hamiltonian function is given by

$$H(I,p,\lambda,t) = \begin{cases} p_0 y(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l}B(t_l-t) - \phi & \\ -\lambda y B(t_l-t)(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l} - \xi_1, & (0 \leqslant t \leqslant t_l) \\ p(t)w(a-bp(t)+\alpha g)q(t) & \\ -\lambda w(a-bp(t)+\alpha g)q(t) - hI(t) - \xi_2, & (t_l \leqslant t \leqslant t_d) \\ p(t)w(a-bp(t)+\alpha g)q(t) - \xi_2 - hI(t) & \\ -\lambda\{w(a-bp(t)+\alpha g)q(t) + (1-m_2)\theta I(t)) & \\ -c_d(1-m_2)\theta I(t) & (t_d \leqslant t \leqslant T\}, \end{cases} \quad (5.27)$$

λ being the costate variable. According to Pontryagin's maximum principle, the necessary condition for the existence of optimal pricing policy is the existence of continuous and piecewise continuously differentiable function $\lambda(\cdot)$ satisfying costate equation $\dot{\lambda} = -\frac{\partial H}{\partial I}$, i.e.

$$\dot{\lambda} = \begin{cases} 0 & (0 \leqslant t \leqslant t_l) \\ h & (t_l \leqslant t \leqslant t_d) \\ h + c_d(1-m_2)\theta + \lambda(1-m_2)\theta & (t_d \leqslant t \leqslant T) \end{cases}$$

$$\text{with the transversality condition } \lambda(T) = \text{constant.} \tag{5.28}$$

The Hamiltonian maximizing condition becomes
$H(I^*(t), p^*(t), \lambda(t), t) = \max_{0 \leqslant p(t) \leqslant \frac{a}{b}} H(I^*(t), p(t), \lambda(t), t)$.
Since the state variable is always non-negative, the Hamiltonian function is concave in price. The optimal pricing policy is derived as

$$p^*(t) = \begin{cases} p_0 & \lambda \leqslant -\frac{\alpha g + a - 2bp_0}{b} \\ \frac{a+\lambda b+\alpha g}{2b} & -\frac{a+\alpha g-2bp_0}{b} \leqslant \lambda \leqslant \frac{\alpha g+a}{b} \\ \frac{a+\alpha g}{b} & \lambda \geqslant \frac{a+\alpha g}{b} \end{cases}. \tag{5.29}$$

We now solve the problem as follows (Dorfman, 1969):
$p(T)w(a - bp(T) + \alpha g)q(T) - \xi_2 - hI(T) - c_d(1-m_2)\theta I(T) - \lambda(T)w((a - bp(T) + \alpha g)q(T) + (1-m_2)\theta I(T)) = 0$,
i.e. $p(T)(a - bp(T) + \alpha g)q(T) = \lambda(T)((a - bp(T) + \alpha g)q(T))$ (since $\xi_2 = 0$ at $I(T) = 0$) which yields $\lambda(T) = p(T)$.
If $p(T) = \frac{a+\lambda b+\alpha g}{2b}$, we obtain $\lambda(T) = \frac{a+\alpha g}{b}$. During the period $[t_l, t_d]$, the optimal price as given in 5.29 is substituted in the costate equation 5.28 to obtain following

four regimes:

$$\begin{aligned}
R_{1a} &: \lambda(t) \leqslant -\frac{a+\alpha g-2bp_0}{b},\ p^*(t)=p_0,\ \dot{I}(t)=-(a-bp_0+\alpha g)q(t),\\
&\quad \dot{\lambda}(t)=h;\\
R_{2a} &: -\frac{a+\alpha g-2bp_0}{b} < \lambda(t) \leqslant 0,\ p^*(t)=\frac{a+\lambda b+\alpha g}{2b},\\
&\quad \dot{I}(t)=-\frac{(a-\lambda b+\alpha g)}{2},\ \dot{\lambda}(t)=h;\\
R_{3a} &: 0<\lambda(t)<\frac{a+\alpha g}{b},\ p^*(t)=\frac{a+\lambda b+\alpha g}{2b},\ \dot{I}(t)=-\frac{(a-\lambda b+\alpha g)}{2},\\
&\quad \dot{\lambda}(t)=h;\\
\text{and } R_{4a} &: \lambda(t)\geqslant \frac{a+\alpha g}{b},\ p^*(t)=\frac{a+\alpha g}{b},\ \dot{I}(t)=-\alpha I(t),\ \dot{\lambda}(t)=h,
\end{aligned}$$

on the basis of which it is seen that

$$\begin{aligned}
&\dot{\lambda}_{R_{2a}}(t)\Big|_{\lambda_{R_{2a}}(t)\to-\frac{a+\alpha I(t)-2bp_0}{b}} = \dot{\lambda}_{R_{1a}}(t)\Big|_{\lambda_{R_{1a}}(t)\to-\frac{a+\alpha I(t)-2bp_0}{b}} = h,\\
&\dot{\lambda}_{R_{3a}}(t)\Big|_{\lambda_{R_{3a}}(t)\to 0} = \dot{\lambda}_{R_{2a}}(t)\Big|_{\lambda_{R_{2a}}(t)\to 0} = h,\\
&\dot{\lambda}_{R_{3a}}(t)\Big|_{\lambda_{R_{3a}}(t)\to\frac{a+\alpha g}{b}} = \dot{\lambda}_{R_{4a}}(t)\Big|_{\lambda_{R_{4a}}(t)\to\frac{a+\alpha g}{b}} = h.
\end{aligned}$$

In a similar manner, the following regimes are derived for the period $[t_d, T]$:

$$\begin{aligned}
R_{1b} &: \lambda(t) \leqslant -\frac{a+\alpha g-2bp_0}{b},\ p^*(t)=p_0,\ \dot{I}(t)=-(a-bp_0+\alpha g)q(t),\\
&\quad \dot{\lambda}(t)=h+c_d(1-m_2)\theta+\lambda(1-m_2)\theta;\\
R_{2b} &: -\frac{a+\alpha g-2bp_0}{b} < \lambda(t) \leqslant 0,\ p^*(t)=\frac{a+\lambda b+\alpha g}{2b},\ \dot{I}(t)=-\frac{(a-\lambda b+\alpha g)}{2},\\
&\quad \dot{\lambda}(t)=h+c_d(1-m_2)\theta+\lambda(1-m_2)\theta;\\
R_{3b} &: 0<\lambda(t)<\frac{a+\alpha g}{b},\ p^*(t)=\frac{a+\lambda b+\alpha g}{2b},\ \dot{I}(t)=-\frac{(a-\lambda b+\alpha g)}{2},\\
&\quad \dot{\lambda}(t)=h+c_d(1-m_2)\theta+\lambda(1-m_2)\theta;\ \text{and}\\
R_{4b} &: \lambda(t)\geqslant \frac{a+\alpha g}{b},\ p^*(t)=\frac{a+\alpha g}{b},\ \dot{I}(t)=-\alpha I(t),
\end{aligned}$$

$$\dot{\lambda}(t) = h + c_d(1-m_2)\theta + \lambda(1-m_2)\theta,$$

on the basis of which it is seen that

$$\begin{aligned}
&\dot{\lambda}_{R_{2b}}(t)\Big|_{\lambda_{R_{2b}}(t)\to -\frac{a+\alpha I(t)-2bp_0}{b}} = \dot{\lambda}_{R_{1b}}(t)\Big|_{\lambda_{R_{1b}}(t)\to -\frac{a+\alpha I(t)-2bp_0}{b}} \\
&= h + \left(\frac{a+\alpha g - 2bp_0 + c_d}{b}\right)(1-m_2)\theta \\
&\dot{\lambda}_{R_{3b}}(t)\Big|_{\lambda_{R_{3b}}(t)\to 0} = \dot{\lambda}_{R_{2b}}(t)\Big|_{\lambda_{R_{2b}}(t)\to 0} = h + c_d(1-m_2)\theta \\
&\dot{\lambda}_{R_{3b}}(t)\Big|_{\lambda_{R_{3b}}(t)\to \frac{a+\alpha g}{b}} = \dot{\lambda}_{R_{4b}}(t)\Big|_{\lambda_{R_{4b}}(t)\to \frac{a+\alpha g}{b}} = h - \left(\frac{a-\alpha g - bc_d}{b}\right)(1-m_2)\theta.
\end{aligned}$$

Solving $\dot{\lambda}(t) = h + c_d(1-m_2)\theta + \lambda(1-m_2)\theta$ with $\lambda(T) = \frac{a+\alpha g}{b}$, we get

$$\lambda(t) = -\frac{(h+c_d(1-m_2)\theta)}{(1-m_2)\theta} + \left(\frac{a+\alpha g}{b} + \frac{(h+c_d(1-m_2)\theta)}{(1-m_2)\theta}\right)e^{-(1-m_2)\theta(T-t)}, \tag{5.30}$$

so that $\lambda(t_d) = -\frac{(h+c_d(1-m_2)\theta)}{(1-m_2)\theta} + \left(\frac{a+\alpha g}{b} + \frac{(h+c_d(1-m_2)\theta)}{(1-m_2)\theta}\right)e^{-(1-m_2)\theta(T-t_d)}$.

We use this as boundary condition to solve $\dot{\lambda}(t) = h$ during $[t_l, t_d]$ and obtain

$$\begin{aligned}
\lambda(t) \quad = \quad & -\frac{(h+c_d(1-m_2)\theta)}{(1-m_2)\theta} + \left(\frac{a+\alpha g}{b} + \frac{(h+c_d(1-m_2)\theta)}{(1-m_2)\theta}\right)e^{-(1-m_2)\theta(T-t_d)} \\
& + h(t-t_d).
\end{aligned} \tag{5.31}$$

Putting λ in 5.26, we summarize the result in the next proposition. For the first case, the possible sequences are given by $R_{1a} \to R_{1b} \to R_{2b} \to R_{3b} \to R_{4b}$, $R_{1a} \to R_{2a} \to R_{2b} \to R_{3b} \to R_{4b}$, $R_{1a} \to R_{2a} \to R_{3a} \to R_{3b} \to R_{4b}$, $R_{1a} \to R_{2a} \to R_{3a} \to R_{4a} \to R_{4b}$, $R_{2a} \to R_{3a} \to R_{4a} \to R_{4b}$, $R_{2a} \to R_{3a} \to R_{3b} \to R_{4b}$, $R_{2a} \to R_{2b} \to R_{3b} \to R_{4b}$, $R_{3a} \to R_{4a} \to R_{4b}$, $R_{3a} \to R_{3b} \to R_{4b}$, $R_{4a} \to R_{4b}$.

For the second case, the possible sequences are, $R_{1b} \to R_{2b} \to R_{3b} \to R_{4b}$, $R_{2b} \to R_{3b} \to R_{4b}$ or $R_{3b} \to R_{4b}$. For the third case, the possible sequence is given by, $R_{1a} \to R_{2a} \to R_{3a} \to R_{4a}$ and for the second is $R_{2a} \to R_{3a} \to R_{4a}$ or $R_{3a} \to R_{4a}$.

Proposition 5.8. *Optimal dynamic pricing is stated as follows:*

(i) If $p_0 < \frac{1}{2}\left\{\frac{(a+\alpha g)}{b}\left(1+e^{-(1-m_2)\theta(T-t_l)}\right) - R\left(1-e^{-(1-m_2)\theta(T-t_l)}\right) - h(t_d-t_l)\right\}$*, the*

possible sequence of the $\lambda(t)$ values is given by $R_{2a} \rightarrow R_{2b} \rightarrow R_{3b} \rightarrow R_{4b}$, $R_{2a} \rightarrow R_{3a} \rightarrow R_{3b} \rightarrow R_{4b}$, $R_{2a} \rightarrow R_{3a} \rightarrow R_{4a} \rightarrow R_{4b}$, and the pricing strategy is

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_l) \\ \frac{a+\alpha g+\left\{(a+\alpha g+Rb)e^{-(1-m_2)\theta(T-t_d)}-Rb+hb(t-t_d)\right\}}{2b} & (t_l \leqslant t \leqslant t_d) \\ \frac{a+\alpha g+\left\{(a+\alpha g+Rb)e^{-(1-m_2)\theta(T-t)}-Rb\right\}}{2b} & (t_d \leqslant t \leqslant T) \end{cases} . \quad (5.32)$$

(ii) If $\frac{1}{2}\left\{\frac{(a+\alpha g)}{b}\left(1+e^{-(1-m_2)\theta(T-t_l)}\right) - R\left(1-e^{-(1-m_2)\theta(T-t_l)}\right) - h(t_d - t_l)\right\} < p_0 < \frac{1}{2}\left\{\frac{(a+\alpha g)}{b}\left(1+e^{-(1-m_2)\theta(T-t_l)}\right) - R\left(1-e^{-(1-m_2)\theta(T-t_d)}\right)\right\}$ *, the possible sequences are given by $R_{1a} \rightarrow R_{2a} \rightarrow R_{2b} \rightarrow R_{3b} \rightarrow R_{4b}$, $R_{1a} \rightarrow R_{2a} \rightarrow R_{3a} \rightarrow R_{3b} \rightarrow R_{4b}$, $R_{1a} \rightarrow R_{2a} \rightarrow R_{3a} \rightarrow R_{4a} \rightarrow R_{4b}$, and the corresponding pricing strategy is*

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_1) \\ \frac{a+\alpha g+\left\{(a+\alpha g+Rb)e^{-(1-m_2)\theta(T-t_d)}-Rb+hb(t-t_d)\right\}}{2b} & (t_1 \leqslant t \leqslant t_d) \\ \frac{a+\alpha g+\left\{(a+\alpha g+Rb)e^{-(1-m_2)\theta(T-t)}-Rb\right\}}{2b} & (t_d \leqslant t \leqslant T) \end{cases} , \quad (5.33)$$

where t_1 is the time when the value of $\lambda(t)$ in 5.31 is equal to $-\frac{\alpha g+a-2bp_0}{b}$.
(iii) If $p_0 > \frac{1}{2}\left\{\frac{(a+\alpha g)}{b}\left(1+e^{-(1-m_2)\theta(T-t_l)}\right) - R\left(1-e^{-(1-m_2)\theta(T-t_d)}\right)\right\}$*, the possible sequence of $\lambda(t)$ is given by $R_{1a} \rightarrow R_{1b} \rightarrow R_{2b} \rightarrow R_{3b} \rightarrow R_{4b}$, and the corresponding pricing strategy is*

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_1') \\ \frac{a+\alpha g+\left\{(a+\alpha g+Rb)e^{-(1-m_2)\theta(T-t)}-Rb\right\}}{2b} & (t_1' \leqslant t \leqslant T) \end{cases} , \quad (5.34)$$

where t_1' is the time when the value of $\lambda(t)$ in 5.30 is equal to $-\frac{\alpha g+a-2bp_0}{b}$.
(iv) In particular, if $p_0 = \frac{1}{2}\left\{\frac{(a+\alpha g)}{b}\left(1+e^{-(1-m_2)\theta(T-t_l)}\right) - R\left(1-e^{-(1-m_2)\theta(T-t_d)}\right)\right\}$,

the corresponding pricing strategy is given by

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_d) \\ \frac{a+\alpha g+\left\{(a+\alpha g+Rb)e^{-(1-m_2)\theta(T-t)}-Rb\right\}}{2b} & (t_d \leqslant t \leqslant\leqslant T) \end{cases}. \quad (5.35)$$

5.2.4.2 Case 2: $t_d \leqslant t_l \leqslant T$

The objective function in this case is rewritten as

$$\begin{aligned} \max_{p(.)} J_L \;=\; & \frac{1}{T}\Bigg[\int_0^{t_l} yp_0(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l}B(t-t_l)dt \\ & + \int_{t_l}^{T} wp(t)(a-bp(t)+\alpha g)q(t)dt - \xi_1 t_l - \xi_2(T-t_l) - c_d(1-m_2)\theta \times \\ & \int_{t_l}^{T} I(t)dt - h\int_{t_l}^{T} I(t)dt - \frac{1}{2}kg^2 - \phi - c_pQ - c_d(Q-Q_2) - c_s e^{-(1-m_1)\theta t_l} \times \\ & \left\{(a-bp_0+\alpha g)wt_l - \frac{(a-bp_0+\alpha g)}{\eta}\left(1-e^{-\eta t_l}\right)\right\}\Bigg], \end{aligned} \quad (5.36)$$

subject to the stationery conditions

$$\begin{aligned} \dot{I}(t) &= -yB(t-t_l)(a-bp_0+\alpha g) && (0 \leqslant t \leqslant t_l), \\ \text{and}\quad \dot{I}(t) &= -w(a-bp(t)+\alpha g)q(t) - (1-m_2)\theta I(t) && (t_l \leqslant t \leqslant T). \end{aligned}$$

The Hamiltonian function is given by

$$H(I,p,\lambda,t) = \begin{cases} yp_0((a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l})B(t-t_l) - \\ \lambda(B(t-t_l)y(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l}) - \phi - \xi_1, & (0 \leqslant t \leqslant t_l) \\ wp(t)(a-bp(t)+\alpha g)q(t) - \xi_2 - hI(t) - c_d(1-m_2)\theta I(t) \\ -\lambda(w(a-bp(t)+\alpha g)q(t) + (1-m_2)\theta I(t)) & (t_l \leqslant t \leqslant T). \end{cases} \quad (5.37)$$

According to Pontryagin's maximum principle, the costate equation is given by

$$\dot{\lambda} = \begin{cases} 0 & (0 \leqslant t \leqslant t_l) \\ h + c_d(1-m_2)\theta + \lambda(1-m_2)\theta & (t_l \leqslant t \leqslant T) \end{cases} \tag{5.38}$$

with the transversality condition $\lambda(T)$ =constant.
The Hamiltonian maximizing condition is
$H(I^*(t), p^*(t), \lambda(t), t) = \max_{0 \leqslant p(t) \leqslant \frac{a}{b}} H(I^*(t), p(t), \lambda(t), t).$
The optimal pricing policy therefore is

$$p^*(t) = \begin{cases} p_0 & \lambda \leqslant -\frac{\alpha g + a - 2bp_0}{b} \\ \frac{a + \lambda b + \alpha g}{2b} & -\frac{\alpha g + a - 2bp_0}{b} \leqslant \lambda \leqslant \frac{a - \alpha g}{b} \\ \frac{a}{b} & \lambda \geqslant \frac{a-\alpha g}{b} \end{cases}. \tag{5.39}$$

Since only the second regime of the first model is feasible in this case, taking it into account while solving $\dot{\lambda}(t) = h + c_d(1-m_2)\theta + \lambda(1-m_2)\theta$, $t \in [t_l, T]$ with $\lambda(T) = \frac{a}{b}$, we get

$$\lambda(t) = -\frac{(h + c_d(1-m_2)\theta)}{(1-m_2)\theta} + \left(\frac{a + \alpha g}{b} + \frac{(h + c_d(1-m_2)\theta)}{(1-m_2)\theta}\right) e^{-(1-m_2)\theta(T-t)}. \tag{5.40}$$

Putting in 5.39, the dynamic pricing strategy can be derived as given in following proposition.

Proposition 5.9. *The pricing strategy in this case is*

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_2) \\ \frac{(a + \alpha g + \left\{(a + \alpha g + Rb)e^{-(1-m_2)\theta(T-t)} - Rb\right\}}{2b} & (t_2 \leqslant t \leqslant T), \end{cases} \tag{5.41}$$

where t_2 is explained in section 5.2.2, the value of $\lambda(t)$ in 5.40 is equal to $-\frac{\alpha g+a-2bp_0}{b}$, or

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_l) \\ \frac{(a+\alpha g+\{(a+\alpha g+Rb)e^{-(1-m_2)\theta(T-t)}-Rb\}}{2b} & (t_l \leqslant t \leqslant T), \end{cases} \tag{5.42}$$

according as $p_0 > \frac{1}{2}\left\{\frac{(a+\alpha g)}{b}\left(1+e^{-(1-m_2)\theta(T-t_l)}\right) - R\left(1-e^{-(1-m_2)\theta(T-t_l)}\right)\right\}$ *or* $p_0 < \frac{1}{2}\left\{\frac{(a+\alpha g)}{b}\left(1+e^{-(1-m_2)\theta(T-t_l)}\right) - R\left(1-e^{-(1-m_2)\theta(T-t_l)}\right)\right\}$*, respectively. The possible feasible sequence for the first case is* $R_{1b} \to R_{2b} \to R_{3b} \to R_{4b}$*, and for the second case is* $R_{2b} \to R_{3b} \to R_{4b}$ *or* $R_{3b} \to R_{4b}$*.*

5.2.4.3 Case 3: $t_l \leqslant T \leqslant t_d$

The objective function in this case is

$$\begin{aligned} \max_{p(.)} J_L &= \frac{1}{T}\Bigg[\int_0^{t_l} yp_0(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l}B(t-t_l)dt \\ &+ \int_{t_l}^{T} p(t)w(a-bp(t)+\alpha g)q(t)dt - \xi_1 t_l - \xi_2(T-t_l) \\ &-c_s\left\{(a-bp_0+\alpha g)wt_l - \frac{(a-bp_0+\alpha g)ye^{-(1-m_1)\theta t_l}}{\eta}(1-e^{-\eta t_l})\right\} \\ &-h\int_{t_l}^{T} I(t)dt - \frac{1}{2}kg^2 - \phi - c_pQ\Bigg], \end{aligned} \tag{5.43}$$

subject to the stationery conditions

$$\begin{aligned} \dot{I}(t) &= -yB(t-t_l)(a-bp_0+\alpha g) && (0 \leqslant t \leqslant t_l) \\ \text{and} \quad \dot{I}(t) &= -w(a-bp(t)+\alpha g)q(t) && (t_l \leqslant t \leqslant T). \end{aligned}$$

The Hamiltonian function is given by

$$H(I,p,\lambda,t)=\begin{cases} p_0y((a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l})B(t-t_l)-\phi \\ -\lambda(yB(t-t_l)(a-bp_0+\alpha g)e^{-(1-m_1)\theta t_l})-\xi_1, & (0\leqslant t\leqslant t_l) \\ p(t)w(a-bp(t)+\alpha g)q(t)-\xi_2 \\ -hI(t)\lambda w(a-bp(t)+\alpha g)q(t), & (t_l\leqslant t\leqslant T). \end{cases} \tag{5.44}$$

Similar to the first two models, the costate equation is given by the Pontryagin maximal principle

$$\dot{\lambda}=\begin{cases} 0 & (0\leqslant t\leqslant t_d) \\ h & (t_l\leqslant t\leqslant T) \end{cases} \tag{5.45}$$

with the transversality condition $\lambda(T)$=constant.
The Hamiltonian maximizing condition is

$$H(I^*(t),p^*(t),\lambda(t),t)=\max_{0\leqslant p(t)\leqslant \frac{a}{b}} H(I^*(t),p(t),\lambda(t),t).$$

From the optimal pricing policy

$$p^*(t)=\begin{cases} p_0 & \lambda\leqslant -\frac{\alpha g+a-2bp_0}{b} \\ \frac{a+\lambda b+\alpha g}{2b} & -\frac{\alpha g+a-2bp_0}{b}\leqslant\lambda\leqslant\frac{a-\alpha g}{b} \\ \frac{a}{b} & \lambda\geqslant\frac{a-\alpha g}{b} \end{cases}, \tag{5.46}$$

it is seen that only the first regime of the first model is active here. So we take this regime into account during the interval $[t_l, T]$, and solve $\dot{\lambda}(t)=h$ with $\lambda(T)=\frac{a+\alpha g}{b}$ to get

$$\lambda(t)=\frac{a+\alpha g}{b}-h(T-t) \tag{5.47}$$

Proposition 5.10. *The optimal pricing strategy is given by*

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_3) \\ \frac{2a - hb(T-t) + 2\alpha g}{2b} & (t_3 \leqslant t \leqslant T), \end{cases} \tag{5.48}$$

where t_3 is explained in section 5.2.2, to be obtained by setting $\lambda(t)$ in 5.47 to $-\frac{\alpha g + a - 2bp_0}{b}$; or

$$p^*(t) = \begin{cases} p_0 & (0 \leqslant t \leqslant t_l) \\ \frac{2a - hb(T-t) + 2\alpha g}{2b} & (t_l \leqslant t \leqslant T), \end{cases} \tag{5.49}$$

according as $p_0 > \frac{1}{2}\left\{\frac{2(a+\alpha g)}{b} - h(T - t_l)\right\}$ or $p_0 < \frac{1}{2}\left\{\frac{2(a+\alpha g)}{b} - h(T - t_l)\right\}$, respectively. The possible sequence for the first case is $R_{1a} \to R_{2a} \to R_{3a} \to R_{4a}$ and for the second is $R_{2a} \to R_{3a} \to R_{4a}$ or $R_{3a} \to R_{4a}$.

Due to complexity, we are unable to examine the concavity of the objective function with respect to the decision variables. To ensure that an optimal solution exists, all three cases are numerically illustrated in the next section.

5.2.4.4 Numerical illustration

In this section, we demonstrate the developed model through numerical examples. We use in-built numerical optimization function in the software Mathematica 11.3 for numerical computations. Following Modak et al., 2024a, we set the parameter values for **Example 1** as $a = 500$ units/week, $b = 10$ units/\$/week, $\alpha = 3$ units/week, $c_m = \$10$/unit, $\theta = 0.1$/week, $h = \$1$/unit/week, $t_d = 2$ weeks, $c_d = \$3$/unit, $k = 200$. Keeping consistency with the parameter values, remaining values are further set as $y = 1$, $\gamma_1 = 0.006$, $\gamma_2 = 0.005$, $\eta = 0.9$, $t_0 = 3$ weeks, $c_s = \$20$/unit, $\rho = 0.005$, and $A = \$100$. Considering the model as the base one, **Example 2** illustrates a high deterioration scenario ($\theta = 0.3$/week) with costly disposal of deteriorated items ($c_d = \$5$/unit), less efficient preservation system during storage ($\gamma_2 = 0.0005$) but more efficient preservation system during transportation ($\gamma_1 = 0.06$). The dynamics of the decision variables are shown in Table 5.4, which will help the business managers make better decisions on inventory policies. Example 1 is further selected for sensitivity and robustness analysis, leading to possible managerial insights. The numerical findings are summarized in Table 5.5, and analysis is provided as follows.

Table 5.4: Optimal results for different examples

Model	p_0	ξ_1	ξ_2	c_l	t_l	g	T	AP
1	1	41.9	401	0	81	2	2.16	27
2	41.64	0	316.6	81.1	2	2.55	20	1300
3	32.4	76	114.27	361.5	0.492	0.1	0.4923	1325
2	1	41.65	77.23	0	81.1	2	0.586	10.6
2	41.6	0	0	81.1	2	0.213	5.175	412.8
3	32.5	65.7	28.3	353.8	0.511	0.107	0.5115	1493.3

Sensitivity analysis

- *Effect of price sensitivity*: A higher b reflects greater price sensitivity, causing demand to drop more significantly as prices rise, thereby resulting in lesser profit too. This suggests that businesses should be cautious with price adjustments, as higher sensitivity can lead to sharper reductions in both demand and profit. Conversely, lower values of b yield higher profits, indicating that consumers are less responsive to price changes and will maintain demand despite price increases. Additionally, as b decreases, the optimal choice of case shifts from 2 to 3, with a marked reduction in cycle length. This aligns with Dell's business strategy, affirming that when customers are less sensitive to price, a fully customized approach with minimal inventory holding is optimal. A higher

Table 5.5: Sensitivity analysis

Parameter	Value	Model	p_0	ξ_1	ξ_2	c_l	t_l	g	T	Pricing strategy	Profit
	12	2	37.58	0	258.5	81.1	2	1.45	15.48	$p^*(t) = \begin{cases} 37.58, (0 \leq t \leq 11.26) \\ \frac{977.38+31.32e^{-0.03(15.48-t)}}{24}, \\ (11.26 \leq t \leq 15.48) \end{cases}$	872.9
b	10	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
	8	3	38.39	125.36	132	383.2	0.44	0.126	0.44	$p^*(t) = 38.39\ (0 \leq t \leq 0.44)$	2458
	5	3	56.76	210	112	424.4	0.36	0.2	0.36	$p^*(t) = 56.76\ (0 \leq t \leq 0.36)$	6181
	5	2	42.9	0	364.7	81.09	2	5.26	22.8	$p^*(t) = \begin{cases} 42.9, \ (0 \leq t \leq 11.67) \\ \frac{1175.65-123.05e^{-0.02(22.8-t)}}{20}, \\ (11.67 \leq t \leq 22.8) \end{cases}$	1366
	4	2	42.3	0	334.9	81.09	2	3.7	21.1	$p^*(t) = \begin{cases} 42.3, \ (0 \leq t \leq 11.15) \\ \frac{1078.37-48.85e^{-0.02(21.1-t)}}{20}, \\ (11.15 \leq t \leq 21.1) \end{cases}$	1327
α	3	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
	2	3	32.4	76	154.2	360	0.49	0.07	0.49	$p^*(t) = 32.4\ (0 \leq t \leq 0.49)$	1322
	1	3	32.38	75.6	325.76	361.5	0.49	0.03	0.49	$p^*(t) = 32.38\ (0 \leq t \leq 0.49)$	1320
	0.5	3	32.49	344	242	361.5	0.49	0.1	0.49	$p^*(t) = 32.49\ (0 \leq t \leq 0.49)$	1057

	0.3	3	32.5	259	241	361.4	0.49	0.1	0.49	$p^*(t) = 32.49\ (0 \leq t \leq 0.49)$	1142
θ	0.1	3	32.5	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
	0.05	3	32.43	0	6	360	0.54	0.1	0.54	$p^*(t) = 32.43\ (0 \leq t \leq 0.54)$	1437
	0.01	2	42	0	0	81	2	2.87	25.1	$p^*(t) = \begin{cases} 42, & (0 \leq t \leq 12.86) \\ \frac{1538-521.39e^{-0.01(25.1-t)}}{20}, & (12.86 \leq t \leq 25.1) \end{cases}$	1450
	6	1	49.6	0	447	0	3	4.29	28.34	$p^*(t) = \begin{cases} 49.6, & (0 \leq t \leq 26.18) \\ \frac{1477.52-451.78e^{-0.01(28.34-t)}}{20}, & (26.18 \leq t \leq 28.34) \end{cases}$	1680
	4	1	49.5	0	429.8	0	3	3.71	26.3	$p^*(t) = \begin{cases} 49.5, & (0 \leq t \leq 24.3) \\ \frac{1398.76-376.5e^{-0.012(26.3-t)}}{20}, & (24.33 \leq t \leq 26.3) \end{cases}$	1465.7
t_d	3	2	49.45	0	419	0	3	3.4	25.16	$p^*(t) = \begin{cases} 49.45, & (0 \leq t \leq 23.25) \\ \frac{1352.74-332.32e^{-0.012(25.16-t)}}{20}, & (23.25 \leq t \leq 25.16) \end{cases}$	1347
	2	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
	1	3	32.49	76	138	361.5	049	0.1	0.49	$p^*(t) = 32.49\ (0 \leq t \leq 0.49)$	1322
	10	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
c_d	7	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323

	3	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
	1	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
	0.01	3	32.39	75.9	504.5	361.5	0.49	0.1	0.49	$p^*(t) = 32.39\ (0 \leq t \leq 0.49)$	1325
	5	2	42	0	319	183.2	2	2.57	20.13	$p^*(t) = \begin{cases} 42, & (0 \leq t \leq 10.93) \\ \frac{1030.53-15.13e^{-0.02(20.13-t)}}{20}, & (10.93 \leq t \leq 20.13) \end{cases}$	1295
	4	2	42	0	318	138.6	2	2.56	20.09	$p^*(t) = \begin{cases} 42, & (0 \leq t \leq 10.92) \\ \frac{1028.05-12.69e^{-0.02(20.09-t)}}{20}, & (10.92 \leq t \leq 20.09) \end{cases}$	1297
t_0	3	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323
	2	3	32.2	68.5	245.5	294	0.46	0.094	0.46	$p^*(t) = 32.2\ (0 \leq t \leq 0.46)$	1496
	1	3	30.49	53.6	225	182	0.4	0.09	0.4	$p^*(t) = 30.49\ (0 \leq t \leq 0.4)$	1818
	30	2	47.7	0	322	81.1	2	2.526	20.29	$p^*(t) = \begin{cases} 47.7, & (0 \leq t \leq 17.25) \\ \frac{1037.87-22.69e^{-0.02(20.29-t)}}{20}, & (17.25 \leq t \leq 20.29) \end{cases}$	1285
	25	2	44.84	0	321	81.1	2	2.55	20.24	$p^*(t) = \begin{cases} 44.84, & (0 \leq t \leq 14.19) \\ \frac{1035.44-20.14e^{-0.02(20.24-t)}}{20}, & (14.19 \leq t \leq 20.24) \end{cases}$	1289
c_s	20	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \leq t \leq 0.51)$	1323

	10	3	31.36	100.7	164	339.2	0.55	0.114	0.55	$p^*(t) = 31.36\ (0 \le t \le 0.55)$	1678
	5	3	30.75	116	209	324	0.59	0.13	0.59	$p^*(t) = 30.75\ (0 \le t \le 0.59)$	1878
γ_1	0.015	3	32.46	100.4	489	358	0.51	0.1	0.51	$p^*(t) = 32.46\ (0 \le t \le 0.51)$	1413
	0.01	3	32.44	95	234.5	356	0.51	0.1	0.51	$p^*(t) = 32.44\ (0 \le t \le 0.51)$	1372
	0.006	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \le t \le 0.51)$	1323
	0.003	3	32.37	0.19	31.1	351	0.49	0.1	0.49	$p^*(t) = 32.37\ (0 \le t \le 0.49)$	1307
	0.001	3	32.34	0	30.1	351	0.48	0.1	0.48	$p^*(t) = 32.34\ (0 \le t \le 0.48)$	1307
γ_2	0.015	2	42.1	0	181.4	81.1	2	3.8	27.7	$p^*(t) = \begin{cases} 42.1, & (0 \le t \le 13.75) \\ \frac{2060.95-1038.15e^{-0.01(27.7-t)}}{20}, & (13.75 \le t \le 27.7) \end{cases}$	1551
	0.01	2	42.1	0	231.1	81.1	2	3.4	25.28	$p^*(t) = \begin{cases} 42.1, & (0 \le t \le 12.94) \\ \frac{1548.65-528.25e^{-0.01(25.28-t)}}{20}, & (12.94 \le t \le 25.28) \end{cases}$	1474
	0.005	3	32.49	83	149.2	353.8	0.51	0.1	0.51	$p^*(t) = 32.49\ (0 \le t \le 0.51)$	1323
	0.002	3	32.39	76	435.8	361.5	0.49	0.1	0.49	$p^*(t) = 32.39\ (0 \le t \le 0.49)$	1325
	0.001	3	32.39	76	435.8	361.5	0.4922	0.1	0.49	$p^*(t) = 32.39\ (0 \le t \le 0.49)$	1325

price sensitivity compels the manager to invest less in reducing lead time and delaying deterioration during transit, and investing more on preservation during stock-in period.

- *Effect of green sensitivity of customers*: Profit levels positively correlate with green sensitivity. Higher values of α indicate greater responsiveness to greening efforts, thereby boosting demand for environmentally friendly products. This trend underscores businesses' growing focus on appealing to eco-conscious consumers. However, the profit increase shows diminishing returns beyond a certain threshold, implying that while investments in greening are advantageous, their scalability is limited. With high green sensitivity, the model emphasizes increased green investment, prioritizing sustainability over freshness considerations. Consequently, investments in lead time reduction and preservation during shortages tend to decline, as the focus shifts toward meeting eco-conscious demand rather than maximizing product freshness. The model transitions from case 2 to case 3 with lesser green sensitivity, with a corresponding decrease in cycle length. This shift highlights the complementary nature of greening and freshness: as green sensitivity diminishes, prioritizing the freshest possible products becomes optimal to meet customer expectations.

- *Effect of deterioration rate*: A negative relationship between deterioration rate and profit is observed. Higher deterioration rates accelerate product spoilage, diminishing inventory quality and thus lowering demand. Reducing θ through preservation investments is directly linked to higher profits, as it lessens spoilage losses. Therefore, minimizing deterioration is crucial for managing perishable product models. In cases of high perishability, a short-term business model proves optimal, as shown in the sensitivity analysis. The data reveal that with higher deterioration rates, lead time reduction investments increase while total cycle length decreases. A decreasing deterioration rate transitions the model from case 3 to case 2, reflecting reduced spoilage, minimal freshness loss, and lower lead time reduction costs, underscoring the importance of maintaining low deterioration rates in perishable inventory systems.

- *Effect of non-deterioration period*: The non-deterioration period is crucial for managing perishable items. In the current model, an extended non-deterioration period results in decreased lead time reduction investments and accommodates a longer lead time, both of which positively impact profit. A longer non-deterioration period leads to fewer items being discarded, thereby reducing losses due to spoilage, which is essential for perishable goods. For dynamic preservation investment, the model suggests that for items with a longer non-deterioration period, starting with a lower preservation investment and gradually increasing it over time is more effective. This approach minimizes costs while maintaining product quality as the risk of spoilage rises later in the cycle. Preservation technology investments that extend the non-deterioration period are essential to sustain high profit levels. With an increase in t_d, as product loss decreases, the model configuration shifts from case 3 to case 2, and eventually to case 1, once the non-deterioration period surpasses the initial lead time. This reflects an optimized cycle where minimal deterioration losses allow for higher profitability and efficient inventory management.

- *Effect of initial lead time*: Lengthy initial lead time extends the waiting period before the first replenishment. This leads to a higher risk of lost sales and shortages during this interval. Preservation investments may also need to increase to maintain product quality over the prolonged holding time. A longer lead time influences the dynamic pricing strategy $p(t)$, potentially requiring price reductions to boost demand once stock becomes available following an extended shortage. This extended lead time ultimately reduces profits, as reflected in the decreasing profit values observed for larger t_0. The demand may decline due to longer wait times, prompting firms to adjust prices dynamically. During this period, preservation investments ξ_1 and ξ_2 are essential for managing product quality, preventing deteriorated inventory from exacerbating losses due to extended lead times. Given the impact of lead time on demand and pricing, the optimal profit oscillates between the cases 2 and 3, indicating the delicate balance required in such scenarios.

- *Effect of shortage cost*: Higher shortage costs increase the financial penalties for unmet customer demand. With a higher shortage cost, the manager should shift from case 3 to case 2. While in case 3 which is a no-holding model, a rise in shortage cost compels the manager to shift the investment from preservation to lead time reduction, resulting in reduce stock-out period as well as business cycle; however, while in case 2, the manager should try to offset the loss by charging higher price and focusing on eco-friendliness of the product. These adjustments emphasizes a balance between reduced shortages and sustainable practices. A higher shortage cost prompts managers to adopt competitive pricing to avoid backorders, while a lower cost allows for more flexible pricing options. Although preservation investments may become less prioritized as focus shifts toward reducing shortages, they remain crucial for maintaining inventory quality during high-cost shortage periods. As c_s increases, total profits decline unless inventory and pricing strategies are optimized to manage shortages.

- *Effect of* γ_1: As γ_1 increases, indicating greater investment in preservation during the shortage period, deterioration rates during this time are reduced, improving product quality and reducing disposal costs. Dynamic pricing $p(t)$ can remain more stable since products maintain their quality. Preservation investment ξ_1 during transportation becomes more active and hence profitable as γ_1 increases, shifting the focus to managing shortages rather than just maintaining stock freshness. A higher γ_1 improves profits by reducing spoilage and maintaining product quality during shortage periods.

- *Effect of* γ_2: As γ_2 increases, enhanced preservation during the stock-in period effectively extends product shelf life, minimizing spoilage risk. This allows businesses to hold inventory for longer durations, potentially increasing the cycle length. With reduced impact of product deterioration, dynamic pricing can adopt more aggressive strategies without the constraint of rapid quality loss. A higher γ_2 reduces the need for preservation investments in other areas, as products maintain freshness over extended periods. This preservation effectiveness provides greater flexibility in inventory and pricing strategies, enabling longer holding periods and cycle lengths, aligning with traditional inventory models. As preservation investment effectiveness rises, the model configuration tends to shift toward a longer cycle length, transitioning from case 3 to case 2, highlighting the advantages of extended shelf life and inventory duration.

Robustness analysis

Price sensitivity: For the dataset $\{5, 8, 10, 12\}$, we have the following observations: p_0 has a mean of 41.3, standard deviation of 8.24, and a CV of 20.1%. ξ_1 has a mean of 83.67, standard deviation of 79.62, and a CV of 95.16%. ξ_2 has a mean of 165, standard deviation of 50.9, and a CV of 30%. c_l has a mean of 248.5, standard deviation of 173, and a CV of 69.69%. t_l has a mean of 1.26, standard deviation of 1.05, and a CV of 83.96%. g has a mean of 0.45, standard deviation of 0.5, and a CV of 111.15%. T has a mean of 5.51, standard deviation of 6.39, and a CV of 116%. Profit has a mean of 2270.78, standard deviation of 2061.36, and a CV of 90.78%. As we may see from the dataset, the high variability of all the decision variables indicates that the system is highly sensible to price. Dynamic pricing strategies must be optimized based on market responsiveness to price changes.

Green sensitivity: For the dataset $\{1, 2, 3, 4, 5\}$, we have the following observations: p_0 has a mean of 36.5, standard deviation of 5, and a CV of 13.67%. ξ_1 has a mean of 46.92, standard deviation of 38.4, and a CV of 81.84%. ξ_2 has a mean of 265.75, standard deviation of 94, and a CV of 35.37%. c_l has a mean of 247.49, standard deviation of 135.89, and a CV of 54.9%. t_l has a mean of 1.09, standard deviation of 0.73, and a CV of 66.86%. g has a mean of 1.83, standard deviation of 2.21, and a CV of 121.12%. T has a mean of 9.09, standard deviation of 10.54, and a CV of 115.91%. Profit has a mean of 1331.6, standard deviation of 17.35, and a CV of 1.3%. As we may see from the analysis, low variability in p_0 (CV 13.67%), and profit (CV 1.3%), shows that the system is highly robust to changes in green sensitivity. This means environmental factors have little impact on pricing and profit. Preservation investments, particularly ξ_1, cycle length T, green investment g have higher variability, suggesting that greening efforts should be complemented with more effective preservation strategies.

Deterioration rate: For the dataset $\{0.01, 0.05, 0.1, 0.3, 0.5\}$, we observe that p_0 has a mean of 34.38, standard deviation of 3.81, and a CV of 11.08%. ξ_1 has a mean of 137.2, standard deviation of 140.12, and a CV of 102.12%. ξ_2 has a mean of , standard deviation of , and a CV of %. c_l has a mean of 301.34, standard deviation of 110.27, and a CV of 36.59%. t_l has a mean of 0.8, standard deviation of 0.6, and a CV of 74%. g has a mean of 0.65, standard deviation of 1.1, and a CV of 169.42%. T has a mean of 5.43, standard deviation of 9.83, and a CV of 181.26%. Profit has a mean of

1281.8, standard deviation of 157.57, and a CV of 12.29%. From the analysis, a good robustness is observed for p_0 (CV 34.38%) and profit (CV 12.29%). Preservation investments (ξ_1 and ξ_2) and green investments g are highly sensitive to changes in deterioration rate. Moderate robustness was observed for the lead time reduction investment c_l. Overall, the system manages deterioration well, but preservation techniques need careful adjustments to maintain profitability as deterioration rates change.

Non-deterioration period: For the dataset $\{1, 2, 3, 4, 6\}$, we find that p_0 has a mean of 42.69, standard deviation of 8.37, and a CV of 19.6%. ξ_1 has a mean of 31.8, standard deviation of 39, and a CV of 122.67%. ξ_2 has a mean of 316.6, standard deviation of 141.58, and a CV of 44.72%. c_l has a mean of 143.06, standard deviation of 175.23, and a CV of 122.49%. t_l has a mean of 2, standard deviation of 1.22, and a CV of 61%. g has a mean of 2.32 , standard deviation of 1.83, and a CV of 79.1%. T has a mean of 16.16, standard deviation of 12.83, and a CV of 79.36%. Profit has a mean of 1427.54, standard deviation of 146.93, and a CV of 9.59%. The analysis shows that, the preservation investment during the shortage period (xi_1) and the lead time reduction cost (c_l), total cycle length T are highly sensitive to the non deterioration period. On the other hand, the preservation investment during stock in (ξ_2), the shortage price p_0 are moderately sensitive to the changes in t_d. the deterioration period helps reduce spoilage and improves profits, but preservation and cycle length must be adjusted accordingly. Preservation investments need to be dynamically managed to ensure the freshness of goods throughout extended periods.

Deterioration cost: For the dataset $\{0.01, 1, 3, 7, 10\}$, we observe that p_0 has a mean of 32.47, standard deviation of 0.04, and a CV of 0.12%. ξ_1 has a mean of 81.58, standard deviation of 2.84, and a CV of 3.48%. ξ_2 has a mean of 220.26, standard deviation of 142.12, and a CV of 64.52%. c_l has a mean of 355.34, standard deviation of 3.08, and a CV of 0.87%. t_l has a mean of 0.51, standard deviation of 0.01, and a CV of 1.53%. g has a mean of 0.1, standard deviation of 0.001, and a CV of 0.8%. T has a mean of 0.51, standard deviation of 0.01, and a CV of 1.51%. Profit has a mean of 1323.4, standard deviation of 0.8, and a CV of 0.1%. The dataset reveals very high robustness is observed for all variables except for preservation investment ξ_2. Changes in deterioration cost have a negligible impact on the system if the preservation investment are chosen wisely. This means that the supply chain is well-optimized for deterioration management, with minimal fluctuation in pricing, preservation investments, and profit.

Initial lead time: For the dataset $\{1, 2, 3, 4, 5\}$, p_0 has a mean of 35.84, standard deviation of 5.08, and a CV of 14.17%. ξ_1 has a mean of 41.02, standard deviation of 34.75, and a CV of 84.74%. ξ_2 has a mean of 251.34, standard deviation of 63.53, and a CV of 25.27%. c_l has a mean of 230.32, standard deviation of 80.33, and a CV of 34.88%. t_l has a mean of 1.07, standard deviation of 0.75, and a CV of 70.47%. g has a mean of 1.08, standard deviation of 1.21, and a CV of 111.76%. T has a mean of 8.32, standard deviation of 9.62, and a CV of 115.75%. Profit has a mean of 1445.8, standard deviation of 200.5, and a CV of 13.86%. From the robustness data, low robustness is observed for profit, preservation investment during stock in and shortage price, with high variability (CV $>$ 70%) in ξ_1 and g and T. The system is sensitive to changes in lead time, which directly affects inventory availability and lost sales. Reducing t_0 is critical for maintaining profit and optimizing the shortage period.

Shortage cost: For the dataset $\{5, 10, 20, 25, 30\}$, p_0 has a mean of 37.43, standard deviation of 7.3, and a CV of 19.5%. ξ_1 has a mean of 59.94, standard deviation of 50.04, and a CV of 83.49%. ξ_2 has a mean of 233.04, standard deviation of 74.86, and a CV of 32.13%. c_l has a mean of 235.84, standard deviation of 126.7, and a CV of 53.72%. t_l has a mean of 1.13, standard deviation of 0.7, and a CV of 62.77%. g has a mean of 1.08, standard deviation of 1.19, and a CV of 109.69%. T has a mean of 8.44, standard deviation of 9.66, and a CV of 114.47%. Profit has a mean of 1490.6, standard deviation of 243.39, and a CV of 16.33%. The lead time reduction investment, stock out preservation and greening investment shows high sensitivity. Other than these, moderate robustness is shown across other variables. Preservation costs have a moderate CV, suggesting the need to optimize inventory management and preservation investments when shortage costs increase. Profit remains relatively stable, showing that adjusting for shortages with adequate stock levels is manageable.

Preservation coefficient for shortage period (γ_1): For the dataset $\{0.001, 0.003, 0.006, 0.01, 0.015\}$, p_0 has a mean of 32.42, standard deviation of 0.06, and a CV of 0.17%. ξ_1 has a mean of 55.71, standard deviation of 45.76, and a CV of 82.14%. ξ_2 has a mean of 186.78, standard deviation of 169.59, and a CV of 90.8%. c_l has a mean of 353.96, standard deviation of 2.76, and a CV of 0.78%. t_l has a mean of 0.5, standard deviation of 0.01, and a CV of 2.64%. g has a mean of 0.1, standard deviation of 0, and a CV of 0%. T has a mean of 0.5, standard deviation of 0.013, and a CV of 2.64%. Profit has a mean of 1344.4, standard deviation of 41.77, and a CV of 3.11%. For γ_1, high robustness is observed in most of the decision variables along with profit and p_0, with low CV values. Preservation costs are highly variable, indicating that preservation investment during shortages needs careful optimization. Dynamic preservation strategies will help maintain profit stability in response to changing market conditions.

Preservation coefficient for stock-in period (γ_2): For the dataset $\{0.001, 0.003, 0.006, 0.01, 0.015\}$, p_0 has a mean of 36.29, standard deviation of 4.74, and a CV of 13%. ξ_1 has a mean of 47, standard deviation of 38.46, and a CV of 81.83%. ξ_2 has a mean of 286.66, standard deviation of 124.54, and a CV of 43.44%. c_l has a mean of 247.8, standard deviation of 136.13, and a CV of 54.94%. t_l has a mean of 1.1, standard deviation of 0.74, and a CV of 66.92%. g has a mean of 1.5, standard deviation of 1.72, and a CV of 114.7%. T has a mean of 10.9, standard deviation of 12.76, and a CV of 117.08%. Profit has a mean of 1399.6, standard deviation of 95.35, and a CV of 6.81%. Very high robustness across all decision variables other than the preservation investments, indicating that the system efficiently handles preservation for the stock-in period. Changes in preservation coefficient have low effects on profit and other decision variables, suggesting that the system is well-optimized for stock preservation management.

Managerial insights

The following managerial insights may be drawn from the above analysis. Managers should focus on strategies like *brand building* or *product differentiation* to make customers less sensitive to price changes. Lower price sensitivity helps maintain demand even with moderate price increases, allowing firms to optimize pricing without losing customers. Managers may further earn benefit from *marketing the eco-friendliness* of their products, but the return on investment in greening initiatives diminishes after a certain threshold. There should be initiatives from the Government too such as rebate in tax or subsidized electric bill to encourage production of eco-friendly products. Extending the non-deterioration period (t_d) through preservation investments is a key factor for maintaining higher profits. Products that stay fresh for longer attract more customers, especially for perishable goods. *As long* *as deterioration rate remains significant, investments in advanced preservation technology* will pay off by keeping the product quality high and reducing the cost of spoilage. Increasing lead time (t_0) negatively impacts profitability due to increased lost sales during the shortage period. Therefore, *shortening lead time* should be a priority as long as products start deteriorating soon. Managers should focus on *lead time reduction strategies* like streamlining logistics, improving inventory management, and reducing transit time to prevent stock shortages. Higher deterioration costs (c_d) significantly reduce profits, especially for models more sensitive to deterioration. Managers should focus on *minimizing deterioration costs* by improving storage conditions and transportation methods. Manager may further plan to re-use the spoiled products in some other industries too. A shorter inventory cycle T and more competitive pricing strategy should be implemented to minimize the financial burden of shortages. Preservation investments must be carefully balanced to ensure sufficient stock levels without excessive spoilage. Managers should increase preservation spending when shortages are frequent or unavoidable, as this helps maintain product quality and profitability even during supply chain disruptions.

5.2.4.5 Conclusion

This part of the chapter explores the optimal pricing strategy for managing perishable inventory systems, emphasizing the dual role of freshness and greening as primary demand drivers. Recognizing that perishable food items rapidly lose freshness, the study underscores lead time as a pivotal factor, which the authors incorporate through planned shortages to enhance the overall model efficiency. Key aspects examined include optimizing cycle length, lead time reduction investments, and investments in preservation technologies to maintain product quality over time. The findings reveal a complementary relationship between greening and freshness, as well as the effectiveness of a no-holding business model, particularly relevant to e-commerce settings. Through a range of numerical cases, the proposed model demonstrates practical applications and uncovers managerial insights that can support corporate managers in tailoring strategies to align with specific business models and product quality standards. By strategically managing shortages and implementing an exact pricing approach, the study maximizes the system's total profit potential. Additionally, the insights drawn from numerical analysis offer valuable guidance for business owners to make informed, quality-driven decisions. The key aspect of this modeling is its ability to detail various scenarios, guiding business owners in choosing the optimal strategy based on product quality. It helps determine whether an inventory-holding model, an e-commerce dropshipping model, or a customized business approach will best maximize outcomes for the business depending on the quality of the particular item.

While this research significantly advances the literature on perishable inventory systems, it does present some limitations. There are a few avenues for further improvement, such as expanding the model to include a multi-level supply chain perspective or implementing discounting strategies during shortage periods to enhance demand responsiveness. Currently, this study primarily examines dynamic pricing; future extensions could integrate other control parameters, allowing for dynamic investment decisions or introducing stochastic price behavior as demonstrated in Das Roy and Sana (2017). Moreover, incorporating inventory level-dependent demand within a multi-period framework would offer a richer understanding of demand dynamics in response to stock levels, especially when perishability is a central concern. The model could also benefit from examining deterioration effects during transportation more closely, where a disruption model would align well with the potential for unforeseen interruptions, thereby adding robustness to real-world applications. These suggested extensions would broaden the applicability and strategic insights of the model, supporting decision-makers in managing complex, perishable inventory systems more effectively.

CHAPTER 6

Production and Preservation Strategies in a Two-Echelon Supply Chain under Revenue Sharing Contract

6.1 Introduction

Majority of supply chain models primarily focuses on short-term strategies, often overlooking the long-term effect of time on the chain's performance. When planning a long-term business strategy, especially for perishable items, relying solely on short-term strategies may lead to sub-optimal outcomes. It is wise to adjust the price to align with changes in the parameters of demand. It is noteworthy that an increase in revenue does not necessarily equate to higher profitability since costs are an integral part of profit, making inventory control more complex when considering multiple factors. Moreover, the presence of deterioration reduces the effective inventory level. A more prudent approach for a long-term business manager is to avoid accumulating excessive inventory all at once, opting for smaller batch sizes and multiple shipments instead. These considerations underline the justification for developing a multi-period inventory model over a longer time horizon. In order to study such long-term business scenarios, the need arises for the development of a multi-period perishable supply chain that deals with time and price-dependent demand. Furthermore, the manager should properly invest in preservation strategy too to generate profit. No supply chain model has considered dynamic pricing strategy as yet, let alone considering production too. Since a centralized model is ideal but impossible to implement, profit enhancement through some contract is desirable too. In presence of dynamic retail pricing, it is quite natural to allow the manufacturer to set dynamic wholesale prices as well, a pertinent issue further neglected in supply chain literature. The present chapter thus develops a production-supply chain model under revenue sharing contract and desires to address following research questions:

RQ 1: How would the model work in the long run for an item with the specified demand pattern?

RQ 2: For perishable items, which strategies should be used in order to get maximum profit?

RQ 3: How would the non-instantaneous production work for supply chain of such items under revenue sharing contract?

To answer these questions, the current study considers a two-echelon supply chain with price and quadratic time dependent demand. Shah and Naik, 2018 developed their model with similar demand pattern but focused only on the retailer's inventory. This work considers time as an integral component of demand and introduces a discrete dynamic pricing policy for the retailer. The wholesale price set by the manufacturer is also allowed to vary in accordance with changing order quantities over time intervals. Building on the idea presented by Giri and Bardhan, 2012, this work adopts the concept of non-instantaneous production, recognizing that deterioration is typically a gradual process and not an instantaneous occurrence, motivating the inclusion of deterioration only at the retailer's end and investing in preservation to combat it. To enhance the overall performance of the chain, a revenue sharing contract is adopted, accounting for the dynamic nature in subsequent intervals, thereby implementing dynamic revenue sharing where the shared fraction is allowed to vary. The model is applicable for branded electronic products such as mobile phones, television, *etc* sold in e-commerce platforms, and also for agro-business industries such as packaged food products. These items, though differing in perishability levels, require careful inventory management and preservation strategies to maintain their value and usability. For electronic products, 'perishability' might refer to technological obsolescence, while for packaged foods, it involves actual spoilage. The model accounts for these factors by incorporating time- and price-sensitive demand functions, making it highly relevant for such industries. The multi-period business model assumes varying pricing and revenue-sharing strategies over time, which can be applied to real-world scenarios. For example, in retail or e-commerce platforms, shipments for a product might arrive at different intervals, but the overall business cycle for selling the product stretches across several periods. The dynamic pricing and preservation strategies allow businesses to adapt to changing demand conditions, optimize profits, and manage inventory effectively. Thus, the model serves as a valuable tool for supply chain managers seeking to implement flexible pricing and inventory strategies over the product's life cycle, especially in industries where perishability is a critical factor.

6.2 Notations and Assumptions

The following notations are used throughout the chapter.

Table 6.1: Notations

w_i	:	unit wholesale price of the manufacturer in ith cycle (decision variables of the manufacturer)
p_i	:	unit selling price in ith cycle (decision variables of the retailer)
ξ	:	preservation technology investment per unit time (decision variable of the retailer)
T	:	length of one cycle (decision variable of the retailer)
H	:	entire business horizon
n	:	number of cycles, $H = nT$
P	:	production rate of the manufacturer
t_{m_i}	:	set up time/ machine maintenance time before the ith cycle for the manufacturer
c_m	:	unit production cost of the manufacturer
c_d	:	unit deterioration cost
h_r	:	per unit holding cost of the retailer per unit time
h_m	:	per unit holding cost of the manufacturer per unit time

S_r	:	ordering cost of the retailer per order
S_m	:	set up cost of the manufacturer per set up
θ	:	constant deterioration rate per unit time
t_d	:	time point after which the deterioration starts taking place
$m(\xi)$	:	per unit reduction in deterioration rate for preservation investment ξ
$D(p_i, t)$	:	demand rate
Q_i	:	lot size for the i^{th} period
B_i	:	total items sold in the i^{th} period
ϕ_i	:	shared fraction of revenue by the retailer in i^{th} period, $0 < \phi_i < 1$

The following assumptions are made to formulate the model:

- A multi-period two level supply chain model with one supplier and one retailer is examined. The customer demand is assumed to depend on both price and time, the dependence being linear in price and quadratic in time. Since the demand is time sensitive, as a compelling response, the retailer is allowed to set different selling prices during different intervals. The specific demand function for the i^{th} period is given by $D(p_i, t) = a + bt - ct^2 - \beta p_i$, where a denotes base demand, b and c are time sensitivity parameters, and β denotes price sensitivity. It is imposed that $a + bH - cH^2 - \beta p_i > 0$ for all possible choices of retail prices in order to guarantee non-negative demand within the business horizon.

- After the manufacturer finishes the production for a particular cycle, he transport them to the retailer. The retailer then sells it to the consumers. To reduce holding cost as much as possible, the manufacturer delays the production and starts it at the time point t_{m_i} such that production reaches desired amount just at the starting point of the next cycle (Giri and Bardhan, 2012). Clearly, the items produced during the time interval $[t_{m_i}, iT]$ are stored at the manufacturer's warehouse during the said period and are delivered to the retailer at time iT. They are sold during the period $[iT, (i+1)T]$. The manufacturer is allowed to charge different wholesale prices in different intervals.

- Following Maihami et al., 2017, lead time is assumed to be deterministic, and the non-deterioration period is larger than lead time, thereby removing the possibility of spoilage during transportation.

- Although deterioration is a continuous process that gradually makes the product obsolete, it is often visible only after a certain time point. Food products such as vegetables remains fresh during initial periods but starts loosing freshness after some time; the period is referred to as non-deterioration period (Li et al., 2019). Accordingly, non-instantaneous deterioration at constant rate has been considered here which takes place in retailer's warehouse only. Consequently, the retailer is the only one to invest in preservation to prevent spoilage. With ξ amount of investment, $m(\xi)$ fraction of deterioration is reduced, where $m(\xi)$ is a continuous, concave, growing function of preservation investment with $m(0) = 0$ and $\lim_{\xi \to \infty} m(\xi) = 1$. The condition $m''(\xi) < 0$ ensures diminishing return on investment, thereby restricting the retailer from investing huge amount of money in it.

- As the retail as well as wholesale prices are dynamic in nature, a dynamic revenue sharing contract is also adopted to enhance profits of both the entities in the decentralized setting to optimize different cycle more accurately and efficiently, where the retailer is allowed to offer different revenue sharing fractions in different cycles.

6.3 The Model

In this section, we first derive the profit functions of the channel members. Then the centralized model and the decentralized model under wholesale price only contract will be discussed. Finally a dynamic revenue sharing mechanism will be implemented to enhance profit level under decentralized setting.

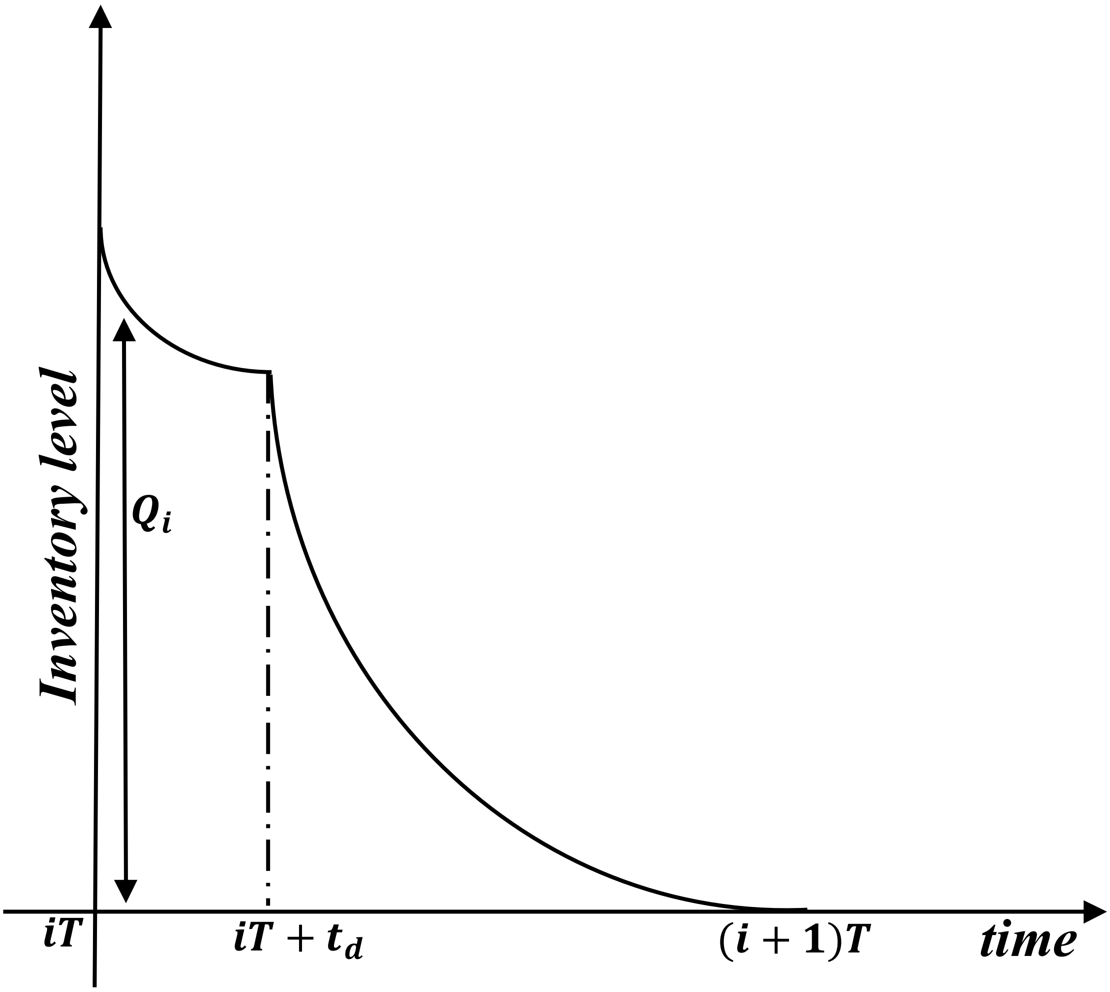


Figure 6.1: Schematic diagram of the retailer's inventory

6.3.1 Retailer's profit

The schematic diagram of retailer's inventory is provided in Figure 6.1. The rate of change of inventory level for the retailer at any time $t \in [iT, (i+1)T]$ is given by

$$\frac{dI}{dt} = \begin{cases} -(a + bt - ct^2 - \beta p_i) & iT \leqslant t \leqslant iT + t_d \\ -(a + bt - ct^2 - \beta p_i) - (1-m)\theta I(t) & iT + t_d \leqslant t \leqslant (i+1)T \end{cases}. \quad (6.1)$$

Solving the above equations with the conditions $I(iT) = Q_i$ and using the continuity of inventory level at t_d, we get the explicit form of inventory level at any time $t \in$

$[iT,(i+1)T]$ as

$$I(t)=\begin{cases} Q_i+iT\left(a+\frac{biT}{2}-\frac{ci^2T^2}{3}-\beta p_i\right)-t\left(a+\frac{bt}{2}-\frac{ct^2}{3}-\beta p_i\right), \\ \qquad\qquad iT\leqslant t\leqslant iT+t_d \\ e^{(1-m)\theta(iT+t_d-t)}\Big\{Q_i-t_d\left(a+\frac{b(iT+t_d)}{2}-\frac{c(iT+t_d)^2}{3}-\beta p_i\right) \\ -iT\left(\frac{bt_d}{2}-\frac{c(2t_diT+t_d^2)}{3}\right)+\frac{(a+b(iT+t_d)-c(iT+t_d)^2-\beta p_i)}{(1-m)\theta} \\ -\frac{(b-2c(iT+t_d))}{(1-m)^2\theta^2}-\frac{2c}{(1-m)^3\theta^3}\Big\}-\Big\{\frac{(a+bt-ct^2-\beta p_i)}{(1-m)\theta}-\frac{(b-2ct)}{(1-m)^2\theta^2} \\ -\frac{2c}{(1-m)^3\theta^3}\Big\}, \qquad iT+t_d\leqslant t\leqslant (i+1)T \end{cases}. \tag{6.2}$$

From $I((i+1)T)=0$, we get the expression for ordering quantity Q_i as

$$\begin{aligned} Q_i &= e^{(1-m)\theta(T-t_d)}\left\{\frac{(a+b(i+1)T-c(i+1)^2T^2-\beta p_i)}{(1-m)\theta}-\frac{b-2c(i+1)T}{(1-m)^2\theta^2}\right. \\ &\left.-\frac{2c}{(1-m)^3\theta^3}\right\}+t_d\left(a+\frac{b(iT+t_d)}{2}-\frac{c(iT+t_d)^2}{2}-\beta p_i\right) \\ &+iT\left(\frac{bt_d}{2}-\frac{c(t_d^2+2iTt_d)}{3}\right)-\frac{a+b(iT+t_d)-c(iT+t_d)^2-\beta p_i}{(1-m)\theta} \\ &+\frac{b-2c(iT+t_d)}{(1-m)^2\theta^2}+\frac{2c}{(1-m)^3\theta^3}. \end{aligned} \tag{6.3}$$

The total inventory held by the retailer in $[iT,(i+1)T]$ is

$$\begin{aligned} &\int_{iT}^{(iT+t_d)} I(t)dt+\int_{(iT+t_d)}^{(i+1)T} I(t)dt \\ =\;& t_d\left(Q_i+aiT+\frac{bi^2T^2}{2}-\frac{ci^3T^3}{3}-\beta p_i iT\right)-t_d\Big[\frac{a}{2}\left(t_d+2iT\right)+\frac{b}{6}(3i^2T^2+3t_diT \\ &+t_d^2)-\frac{c}{12}\left\{(2iT+t_d)((iT+t_d)^2+(iT)^2)\right\}-\frac{\beta p_i}{2}\left(t_d+2iT\right)\Big] \\ &+\Bigg[\frac{K\left(1-e^{-(1-m)\theta(T-t_d)}\right)}{(1-m)\theta}-(T-t_d)\Bigg\{\left(\frac{(a-\beta p_i)}{(1-m)\theta}-\frac{b}{(1-m)^2\theta^2}\right. \end{aligned}$$

$$-\frac{2c}{(1-m)^3\theta^3}\bigg) + \frac{(T+2iT+t_d)}{2}\left(\frac{b}{(1-m)\theta} + \frac{2c}{(1-m)^2\theta^2}\right)$$
$$-\frac{c}{3(1-m)\theta}\left(T^2+Tt_d+t_d^2+3i^2T^2+3iT^2+3iTt_d\right)\Bigg\}\Bigg],$$

$$\text{where } K = e^{(1-m)\theta(T-t_d)}\Bigg\{\frac{(a+b(i+1)T-c(i+1)^2T^2-\beta p_i}{(1-m)\theta} - \frac{(b-2c(i+1)T)}{(1-m)^2\theta^2}$$
$$-\frac{2c}{(1-m)^3\theta^3}\Bigg\},$$

so that the holding cost $HC_i = h_r\left(\int_{iT}^{iT+t_d} I(t)dt + \int_{(iT+t_d)}^{(i+1)T} I(t)dt\right)$.

Total items sold during the time period $[iT,(i+1)T]$ is

$$B_i = \int_{iT}^{(i+1)T}(a+bt-ct^2-\beta p_i)dt = T\left\{(a-\beta p_i) + \frac{bT}{2}(2i+1) - \frac{cT^2}{3}\left(3i^2+3i+1\right)\right\},$$

from which the total revenue in the ith period may be obtained as

$$TR_i = p_iB_i = p_iT\left\{(a-\beta p_i) + \frac{bT}{2}(2i+1) - \frac{cT^2}{3}\left(3i^2+3i+1\right)\right\}.$$

The deterioration cost for the ith cycle is derived as

$$\begin{aligned} DC_i &= c_d(1-m)\theta\int_{(iT+t_d)}^{(i+1)T} I(t)dt \\ &= c_d(1-m)\theta\Bigg[\frac{K\left(1-e^{-(1-m)\theta(T-t_d)}\right)}{(1-m)\theta} - (T-t_d)\Bigg\{\bigg(\frac{a-\beta p_i}{(1-m)\theta} - \frac{b}{(1-m)^2\theta^2} \\ &\quad -\frac{2c}{(1-m)^3\theta^3}\bigg) + \frac{(T+2iT+t_d)}{2}\left(\frac{b}{(1-m)\theta}+\frac{2c}{(1-m)^2\theta^2}\right) - \frac{c}{3(1-m)\theta}\times \\ &\quad \left(T^2+Tt_d+t_d^2+3i^2T^2+3iT^2+3iTt_d\right)\Bigg\}\Bigg]. \end{aligned}$$

Purchasing cost and preservation technology investment cost are given by w_iQ_i and ξT, respectively, from which the profit of the retailer in the i^{th} interval is given by

$\Pi_i^r = TR_i - HC_i - DC_i - \xi T - w_i Q_i$, and the total profit over the entire business horizon is given by $\Pi^r(p_i, \xi, T) = \sum_{i=1}^{n} \Pi_i^r$.

6.3.2 Manufacturer's profit

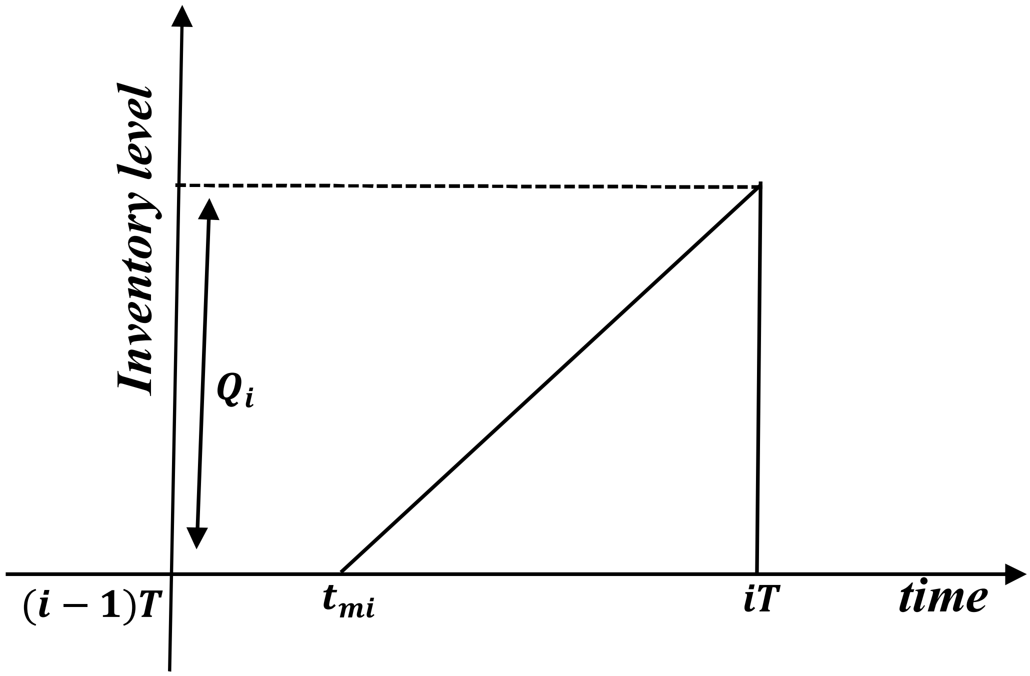


Figure 6.2: Schematic diagram of the manufacturer's inventory

Figure 6.2 provides a schematic diagram of the manufacturer's inventory. Total quantity produced by the manufacturer during $[t_{m_i}, iT]$ equates Q_i, so that $\int_{t_{m_i}}^{iT} P dt = Q_i$, from which we derive

$$t_{m_i} = iT - \frac{Q_i}{P}. \tag{6.4}$$

Purchasing cost for the ith interval is given by $c_1 Q_i$, and the holding cost is $\frac{h_m(iT - t_{m_i})}{2} Q_i$, from which it is easy to derive the total profit of the manufacturer in the ith cycle as

$$\begin{aligned} \Pi_i^m(w_i) &= w_i Q_i - c_m Q_i - S_m - \frac{h_m(iT - t_{m_i})}{2} Q_i \\ &= \left\{ w_i - c_m - \frac{h_m(iT - t_{m_i})}{2} \right\} Q_i - S_m. \end{aligned}$$

Total profit of the manufacturer for the entire business horizon is $\Pi^m(w_i) = \sum_{i=1}^{n} \Pi_i^m$.

6.3.3 Centralized model

In the centralized setting, both the channel members act together and jointly maximize their total profit, so that internal transfer of payment has no effect on the total profit, thereby eliminating double marginalization. Total profit of the centralized model is summed up as $\Pi^c(p_i, \xi, T) = \Pi^m + \Pi^r$. The following proposition ensures existence of dynamic pricing policy in each interval.

Proposition 6.1. *Total profit function is concave in retail price in each interval.*

Proof:
We have

$$\begin{aligned}
\frac{\partial \Pi^c}{\partial p_i} &= T\left\{a + \frac{bT}{2}(2i+1) - \frac{cT^2}{3}(3i^2+3i+1)\right\} - 2\beta T p_i + c_m\left[\frac{\beta}{(1-m)\theta} \times \right.\\
&\left.\left\{e^{1-m)\theta(T-t_d)} - 1\right\} + \beta t_d\right] - \beta h_r t_d \left\{\frac{\left(e^{(1-m)\theta(T-t_d)} - 1\right)}{(1-m)\theta} + \frac{t_d}{2} - iT\right\}\\
&- \frac{(h_r + c_d(1-m)\theta)\beta}{(1-m)\theta}\left\{\frac{(e^{(1-m)\theta(T-t_d)} - 1)}{(1-m)\theta} + (T - t_d)\right\}\\
&- \frac{h_m(iT - t_{mi})\beta}{2}\left\{\frac{\left(e^{((1-m)\theta(T-t_d)} - 1\right)}{(1-m)\theta} + t_d\right\},
\end{aligned}$$

and $\dfrac{\partial^2 \Pi}{\partial p_i^2} = -2\beta T < 0$. This completes the proof.

Further, the following lemma straightforwardly specifies the price explicitly for each interval $i = 1, 2, \cdots, n$, which may be derived from the first order optimality condition provided above.

Lemma 1: *The optimal retail price in the ith period is given by*

$$p_i^c = \frac{1}{2\beta T}\left[aT + \frac{bT^2}{2}(2i+1) - \frac{cT^3}{3}(3i^2+3i+1) + c_m\beta\left\{\frac{e^{(1-m)\theta(T-t_d)} - 1}{(1-m)\theta} + t_d\right\}\right.$$

$$+\beta h_r t_d \left\{ \frac{e^{(1-m)\theta(T-t_d)} - 1}{(1-m)\theta} + \frac{t_d}{2} \right\} + \frac{\{h_r + c_d(1-m)\theta\}\beta}{(1-m)\theta} \times$$

$$\left\{ \frac{e^{(1-m)\theta(T-t_d)} - 1}{(1-m)\theta} + T - t_d \right\} + \frac{h_m(iT - t_{m_i})\beta}{2} \left\{ \frac{e^{(1-m)\theta(T-t_d)} - 1}{(1-m)\theta} + t_d \right\} \Bigg].$$

The following characteristics of the optimal price are derived and summarized in Property 1.

Property 1: *(i)* $\frac{\partial p_i^c}{\partial c_m} > 0$ *(ii)* $\frac{\partial p_i^c}{\partial a} > 0$ *(iii)* $\frac{\partial p_i^c}{\partial h_m} > 0$ *(iv)* $\frac{\partial p_i^c}{\partial h_r} > 0$ *(v)* $\frac{\partial p_i^c}{\partial P} < 0$ (vi) $\frac{\partial Q_i^c}{\partial p_i} < 0$.

Proof:

(i) $\frac{\partial p_i}{\partial c_m} = \frac{1}{2T} \left\{ \frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + t_d \right\} > 0;$

(ii) $\frac{\partial p_i}{\partial a} = \frac{1}{2\beta} > 0;$ (iii) $\frac{\partial p_i}{\partial h_m} = \frac{(iT-t_{m_i})}{4T} \left\{ \frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + t_d \right\} > 0;$

(iv) $\frac{\partial p_i}{\partial h_r} = \frac{t_d}{2T} \left\{ \frac{\left(e^{(1-m)\theta(T-t_d)}-1\right)}{(1-m)\theta} + \frac{t_d}{2} \right\} + \frac{1}{2T(1-m)\theta} \left\{ \frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + T - t_d \right\} > 0;$

(v) $\frac{\partial p_i}{\partial P} = -\frac{h_m Q_i}{4TP^2} \left\{ \frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + t_d \right\} < 0$ (vi) $\frac{\partial Q_i}{\partial p_i} = -\frac{\beta}{(1-m)\theta} \left\{ e^{(1-m)\theta(T-t_d)} - 1 \right\} - \beta t_d < 0.$

The properties 1-3 provide valuable managerial insights which are summarized in Sections 6.4.1 and 6.4.2.

6.3.4 Decentralized model: wholesale price only contract

Although the centralized model always provides the best result in terms of generating profit, in real world the scenario is different as both the retailer and manufacturer works for maximizing their individual profits in general, resulting in double marginalization. To depict the scenario, we adopt the most primitive contract between them - the wholesale price only contract where the manufacturer declares per unit wholesale price, depending on which the retailer decides retail price. In the dynamic scenario adopted here, the manufacturer has the option to declare different wholesale prices in different periods based on which the retailer is allowed to set different retail prices in different periods as well. In addition, the retailer decides optimal preservation investment, and number of shipments in which he wishes to receive the product. Given a fixed time horizon, the number of shipments directly fixes one business cycle length. Stackelberg game structure requires solving the model using backward substitution method, starting with optimizing the profit of the retailer. The following proposition ensures the existence of optimal retail price in each interval.

Proposition 6.2. *Retailer's total profit in the ith period is concave in price, and the optimal price is*

$$\begin{aligned} p_i^w &= \frac{1}{2\beta T}\Bigg[aT + \frac{bT^2}{2}(2i+1) - \frac{cT^3}{3}(3i^2+3i+1) + w_i\beta\left\{\frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + t_d\right\} \\ &+\beta h_r t_d\left\{\frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + \frac{t_d}{2}\right\} + \frac{\{h+c_d(1-m)\theta\}\beta}{(1-m)\theta} \times \\ &\left\{\frac{\left(e^{(1-m)\theta(T-t_d)}-1\right)}{(1-m)\theta} + (T-t_d)\right\}\Bigg]. \end{aligned}$$

Proof:
We have

$$\begin{aligned} \frac{\partial \Pi^r}{\partial p_i} &= \left\{a + \frac{bT}{2}(2i+1) - \frac{cT^2}{3}(3i^2+3i+1)\right\} - 2\beta T p_i + w_i\Bigg(\frac{\beta}{(1-m)\theta} \\ &\left\{e^{1-m)\theta(T-t_d)} - 1\right\} + \beta t_d\Bigg) + \beta h_r t_d\left\{\frac{\left(e^{(1-m)\theta(T-t_d)}-1\right)}{(1-m)\theta} + \frac{t_d}{2} - iT\right\} \\ &+\frac{(h_r + c_d(1-m)\theta)\beta}{(1-m)\theta}\left\{\frac{(e^{(1-m)\theta(T-t_d)}-1)}{(1-m)\theta} - (T-t_d)\right\}, \end{aligned}$$

and $\frac{\partial^2 \Pi^r}{\partial p_i^2} = -2\beta T < 0$. This completes the proof.

Property 2: *(i)* $\frac{\partial p_i^w}{\partial w_i} > 0$ *(ii)* $p_i^c < p_i^w$.

Proof:
(i) $\frac{\partial p_i^w}{\partial w_i} = \beta\left(\frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + t_d\right) > 0$; (ii) Assuming the parameters ξ and T to remain unchanged, we use the expression of w_i given in Proposition 6.3 to derive $p_i^w - p_i^c = \frac{(1-m)\theta Q_i}{e^{(1-m)\theta(T-t_d)}-1+(1-m)\theta t_d}$. The proof thus follows since $e^{(1-m)\theta(T-t_d)} > 1$.
We now optimize the manufacturer's profit. Given the optimal decisions of the retailer, the manufacturer maximizes his profit by suitably choosing the wholesale price for each interval.

Proposition 6.3. *Manufacturer's total profit in the ith period is concave in wholesale price, and the optimal wholesale price is given by*

$$w_i = \frac{2TQ_i}{\beta\left(t_d + \frac{e^{(1-m)\theta(T-t_d)}}{(1-m)\theta} - \frac{1}{(1-m)\theta}\right)\left(\frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + t_d\right)} + c_m + \frac{h_m(iT - t_{m_i})}{2},\ i = 1, 2, \cdots, n.$$

Proof:

Noting that $\frac{\partial p_i}{\partial w_i} = \frac{1}{2T}\left\{\frac{e^{(1-m)\theta(T-t_d)}-1}{(1-m)\theta} + t_d\right\}$, we have

$$\begin{aligned}\frac{\partial \Pi_i^m}{\partial w_i} &= \beta\left(w_i - c_m - \frac{h_m(iT - t_{m_i})}{2}\right) \times \\ &\quad \left\{\left(\frac{1}{(1-m)\theta} - t_d - \frac{e^{(1-m)\theta(T-t_d)}}{(1-m)\theta}\right)\frac{\partial p_i}{\partial w_i}\right\} + Q_i,\\ \text{so that } \frac{\partial^2 \Pi_i^m}{\partial w_i^2} &= -2\beta\left\{\left(t_d + \frac{e^{(1-m)\theta(T-t_d)}}{(1-m)\theta} - \frac{1}{(1-m)\theta}\right)\frac{\partial p_i}{\partial w_i}\right\} < 0.\end{aligned}$$

The expression of w_i may be found out by solving the first order optimality condition mentioned above.

The effects of the associated parameters on wholesale price may be summarized as follows:

Property 3: *(i)* $\frac{\partial w_i}{\partial c_m} > 0$ *(ii)* $\frac{\partial w_i}{\partial h_m} > 0$.

6.3.5 Revenue Sharing Contract

Property 2(ii) indicates that the product is more costly under decentralized setting, so less revenue is expected to get generated, thereby resulting in less total profit for the channel. Since in the real-world scenario, the centralized model is not applicable and the decentralized model is significantly less profitable, we apply revenue sharing contract in order to increase the profit level for both the manufacturer and retailer under decentralized setting. We assume that the retailer shares ϕ_i portion of his revenue with the manufacturer in the i^{th} interval and consequently the manufacturer provides the products at lower wholesale price to the retailer. The expressions for holding and deterioration costs for the retailer will remain the same as obtained in Section 6.3.1, while the total revenue for the retailer in the i^{th} period will be

$$TR^{RS} = (1-\phi_i)p_i^{RS}T\left\{\left(a-\beta p_i^{RS}\right)+\frac{bT}{2}(2i+1)-\frac{cT^2}{3}\left(3i^2+3i+1\right)\right\}.$$

Since the profit of the retailer in each interval should be higher than its counterpart under price only contract, we may derive from the inequality $\Pi_i^r < \Pi_i^{r^{RS}}$ that

$$\begin{aligned}
\phi_i < \ & \frac{1}{p_i^{RS}B_i^{RS}}\Bigg[\left(p_i^{RS}B_i^{RS}-p_iB_i\right)+\left(w_iQ_i-w_i^{RS}Q_i^{RS}\right)+\left(\xi-\xi^{RS}\right)T \\
& +h_r\Bigg(t_d\left(Q_i-Q_i^{RS}\right)+\frac{\beta t_d}{2}\left(p_i^{RS}-p_i\right)+\Bigg[\frac{\left(e^{(1-m)\theta(T-t_d)}-1\right)}{(1-m)\theta}\times \\
& \left\{\frac{(a+b(i+1)T-c(i+1)^2T^2-\beta p_i)}{(1-m)\theta}-\frac{(b-2c(i+1)T)}{(1-m)^2\theta^2}-\frac{2c}{(1-m)^3\theta^3}\right\} \\
& -\frac{\left(e^{(1-m^{RS})\theta(T-t_d)}-1\right)}{(1-m^{RS})\theta}\Bigg\{\frac{(a+b(i+1)T-c(i+1)^2T^2-\beta p_i^{RS})}{(1-m^{RS})\theta} \\
& -\frac{(b-2c(i+1)T)}{(1-m^{RS})^2\theta^2}-\frac{2c}{(1-m^{RS})^3\theta^3}\Bigg\}\Bigg]-(T-t_d)\Bigg\{\Bigg(\Bigg(\Bigg(\frac{(a-\beta p_i)}{(1-m)\theta} \\
& -\frac{(a-\beta p_i^{RS})}{(1-m^{RS})\theta}\Bigg)-b\left(\frac{1}{(1-m)^2\theta^2}-\frac{1}{(1-m^{RS})^2\theta^2}\right)-2c\Bigg(\frac{1}{(1-m)^3\theta^3} \\
& -\frac{1}{(1-m^{RS})^3\theta^3}\Bigg)-\frac{c}{3}\left(T^2+Tt_d+t_d^2+3i^2T^2+3iT^2+3iTt_d\right)\Bigg(\frac{1}{(1-m)\theta} \\
& -\frac{1}{(1-m^{RS})\theta}\Bigg)+\frac{(T+2iT+t_d)}{2}\Bigg(b\left(\frac{1}{(1-m)\theta}-\frac{1}{(1-m^{RS})\theta}\right) \\
& +2c\left(\frac{1}{(1-m)^2\theta^2}-\frac{1}{(1-m^{RS})^2\theta^2}\right)\Bigg)\Bigg\}\Bigg)\Bigg)+c_d(1-m)\theta\times \\
& \Bigg(\frac{\left(e^{(1-m)\theta(T-t_d)}-1\right)}{(1-m)\theta}\Bigg\{\frac{(a+b(i+1)T-c(i+1)^2T^2-\beta p_i)}{(1-m)\theta} \\
& -\frac{(b-2c(i+1)T)}{(1-m)^2\theta^2}-\frac{2c}{(1-m)^3\theta^3}\Bigg\}-(T-t_d)\Bigg\{\Bigg(\frac{(a-\beta p_i)}{(1-m)\theta}-\frac{b}{(1-m)^2\theta^2}
\end{aligned}$$

$$-\frac{2c}{(1-m)^3\theta^3}\bigg) + \frac{(T+2iT+t_d)}{2}\left(\frac{b}{(1-m)\theta} + \frac{2c}{(1-m)^2\theta^2}\right)$$

$$-\frac{c}{3(1-m)\theta}\left((T^2+Tt_d+t_d^2+3i^2T^2+3iT^2+3iTt_d)\right)\bigg\}\bigg) - c_d\left(1-m^{RS}\right)\times$$

$$\theta\Bigg(\frac{\left(e^{(1-m^{RS})\theta(T-t_d)}-1\right)}{(1-m^{RS})\theta}\Bigg\{\frac{(a+b(i+1)T-c(i+1)^2T^2-\beta p_i^{RS})}{(1-m^{RS})\theta}$$

$$-\frac{(b-2c(i+1)T)}{(1-m^{RS})^2\theta^2} - \frac{2c}{(1-m^{RS})^3\theta^3}\bigg\} - (T-t_d)\bigg\{\bigg(\frac{(a-\beta p_i^{RS})}{(1-m^{RS})\theta} - \frac{b}{(1-m^{RS})^2\theta^2}$$

$$-\frac{2c}{(1-m^{RS})^3\theta^3}\bigg) + \frac{(T+2iT+t_d)}{2}\left(\frac{b}{(1-m^{RS})\theta} + \frac{2c}{(1-m^{RS})^2\theta^2}\right) - \frac{c}{3(1-m^{RS})\theta}\times$$

$$\left(T^2+Tt_d+t_d^2+3i^2T^2+3iT^2+3iTt_d\right)\bigg\}\bigg)\bigg]. \tag{6.5}$$

In a similar manner, the profit for the manufacturer in the i^{th} cycle under this contract is given by

$$\begin{aligned}\Pi_i^{m^{RS}}(w_i) &= w_i^{RS}Q_i^{RS} + \phi_i p_i B_i^{RS} - c_m Q_i^{RS} - S_m - \frac{h_m(iT-t_{m_i})}{2}Q_i^{RS} \\ &= \left\{w_i^{RS} - c_m - \frac{h_m(iT-t_{m_i})}{2}\right\}Q_i^{RS} + \phi_i p_i^{RS}B_i^{RS} - S_m\end{aligned}$$

which should be higher than its counterpart under price only contract Π_i^m for each i, so that the manufacturer is indeed inclined to join the contract. The above condition provides us the inequality

$$\begin{aligned}\phi_i &> \frac{1}{p_i^{RS}B_i^{RS}}\bigg\{\left(w_iQ_i - w_i^{RS}Q_i^{RS}\right) - c_m\left(Q_i - Q_i^{RS}\right) - \frac{h_m iT}{2}\left(Q_i - Q_i^{RS}\right) \\ &\quad + \frac{h_m}{2}\left(t_{mi}Q_i - t_{mi}^{RS}Q_i^{RS}\right)\bigg\}.\end{aligned} \tag{6.6}$$

It is thus seen that a feasible range of values exists within which any value may be adopted as the revenue sharing fraction. It is to be noted that the actual value of the fraction depends upon the bargaining capacity of the channel members. Since different sharing fractions at different intervals may be adopted, the changed demand

and other dynamics may play a vital role in negotiating the values in different intervals. It should also be noted that the conditions provided by equations 6.5 and 6.6 are necessary, but not sufficient; there may be some other conditions which are required to be satisfied (see Figures 6.4a, 6.6a, 6.7a and 6.8a for instance). It is now left to establish the benefit generated by the contract in terms of profit. Solving the first order optimality condition for the profit of the respective channel member, the optimal retail and wholesale pricing policies may be derived, which are summarized in the next proposition.

Proposition 6.4. *The optimal retail and wholesale prices in i^{th} interval under revenue sharing contract are given by*

$$\begin{aligned}
p_i^{RS} &= \frac{1}{2\beta}\left(a+\frac{bT}{2}(2i+1)-\frac{cT^2}{3}(3i^2+3i+1)\right)\\
&+\frac{w_i^{RS}}{2T(1-\phi_i)}\left(\frac{\left(e^{(1-m^{RS})\theta(T-t_d)}-1\right)}{(1-m^{RS})\theta}+t_d\right)\\
&+\frac{h_r t_d}{2T(1-\phi_i)}\left(\frac{\left(e^{(1-m^{RS})\theta(T-t_d)}-1\right)}{(1-m^{RS})\theta}+\frac{t_d}{2}\right)\\
&+\frac{\left(h_r+c_d\left(1-m^{RS}\right)\theta\right)}{2T(1-\phi_i)}\left(\frac{\left(e^{(1-m^{RS})\theta(T-t_d)}-1\right)}{(1-m^{RS})^2\theta^2}-\frac{(T-t_d)}{(1-m^{RS})\theta}\right),\\
\textit{and}\ \ w_i^{RS} &= c_m+\frac{h_m(iT-t_{m_i})}{2}+\frac{\left(1-m^{RS}\right)\theta}{\beta\left(e^{(1-m^{RS})\theta(T-t_d)}-1\right)+\beta\left(1-m^{RS}\right)\theta t_d}\times\\
&\left\{\phi_i B_i^{RS}-\phi_i p_i^{RS}\beta T+\frac{2T\left(1-m^{RS}\right)\theta(1-\phi_i)Q_i^{RS}}{\left(\left(e^{(1-m^{RS})\theta(T-t_d)}-1\right)+t_d\left(1-m^{RS}\right)\theta\right)}\right\}.
\end{aligned}$$

The complex forms of the decision variables restrict us from establishing analytically that both retail and wholesale prices under this contract are lesser than those obtained under wholesale price only contract. Nevertheless, we shall establish it numerically in the next section.

6.4 Numerical illustration

Table 6.2: Total profit of the centralized model for different values of n

n	Example 1	Example 2	Example 3	Example 4
1	313	440255	-12396	-44090
2	17293	473461	10769	-11688
3	23170	478632	20448	-338
4	**23966**	**479120**	23530	3967
5	23799	478445	**23922**	5813
6	22886	474823	23757	6019
7	21638	473845	22937	**6145**
8	20170	472480	21837	5647

In this section, we illustrate the developed models numerically. For this, we consider four examples for four different cases. For **Example 1**, we assume the parameter values as $a = 500$ units/month, $b = 5$, $c = 0.05$, $h_m = \$3$/unit/month, $h_r = \$1$/unit/month, $\beta = 10$ units/\$/month, $\theta = 0.1$, $t_d = 0.2$ months, $H = 10$ months, $S_r = \$100$/order, $S_m = \$2000$/setup, $c_1 = \$10$/unit, $c_d = \$2$/unit, $P = 2000$ units/month. The reduction in deterioration rate is expressed by a function in exponential form specified as $m = 1 - e^{-\gamma\xi}$ with $\gamma = 0.001$. Considering this one as the basic one, **Example 2** demonstrates the case of a high demand scenario with $a = 1000$ units/month, $b = 10$, and $\beta = 5$ units/\$/month; **Example 3** depicts a high spoilage and low preservation efficiency scenario with $\theta = 0.25$/month and $\gamma = 0.0001$; and finally **Example 4** illustrates the scenario of an inverted holding costs with $h_m = \$1$/unit/month and $h_r = \$3$/unit/month, keeping all other parameter values unchanged. The in-built multi-objective optimization application using genetic algorithm in Matlab 2018b is used to derive the optimal results. The program is run for different integer values of n, and the outputs for the centralized model are presented in Table 6.2. The concavity with respect to n (and consequently with respect to T too) is established in all the examples. The optimal results are provided in Table 6.3. It is noteworthy to see that investment in preservation indeed

Table 6.3: Results of centralized model

	n	Price ($)	ξ ($\$/month$)	Profit ($)
Example 1	4	33.72; 33.4; 33.96; 34.43	22	23966
Example 2	4	111.3; 113.7; 116; 118.3	12	479120
Example 3	5	33.17; 33.67; 34.14; 34.5; 35	22	23922
Example 4	7	32.3; 32.6; 32.9; 33.3; 33.6; 33.88; 34.1	0	6373.7

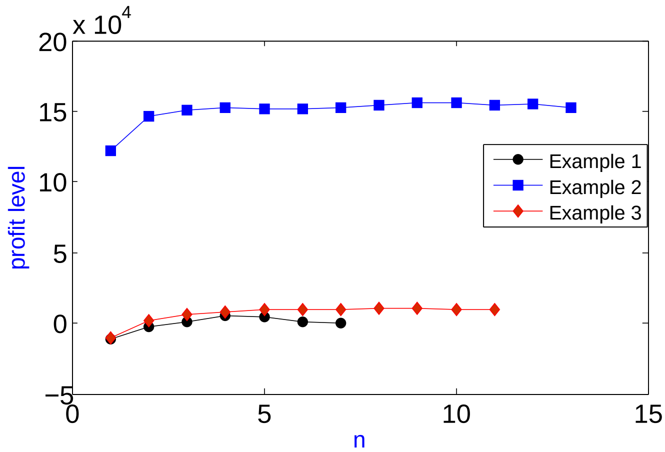


Figure 6.3: Example with different n values

boosts the profit up to some extent under a wide range of business scenarios. In a similar manner, the maximum profit of the retailer under wholesale price only contract for different shipment sizes are plotted in Figure 6.3, establishing the concavity in all examples. The optimal results under price only contract are provided in Table 6.4. Based on the observation that the retailer's profit declines in subsequent intervals, we choose the revenue sharing fractions as $\phi_1 = 0.45$, $\phi_2 = 0.4$, $\phi_3 = 0.35$, and $\phi_4 = 0.3$ under example 1, so that the shared portion of revenue is reduced as well. The profits of the channel members in each interval is provided in Table 6.5, establishing that win-win situation is achieved for both the channel members.

Table 6.4: Results of decentralized model (price-only contract)

	n	w_i	p_i	ξ	Π_m	Π_r	$\Pi = \Pi_m + \Pi_r$
Example 1	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
Example 2	9	71.37; 72.17; 72.78; 73.6; 74.23; 74.9; 75.77; 76.24; 76.9	171.54; 173.3; 175.1; 176.8; 178.6; 180.3; 181.9; 183.7; 185.3	10	62880	155752	218632
Example 3	9	21.75; 21.96; 22; 22.28; 22.43; 22.5; 22.77; 22.73; 22.9	44.4; 44.8; 45.3; 45.7; 46.1; 46.5; 46.8; 47.3; 47.6	25	-10738	9929	-809

Table 6.5: Effect of revenue sharing contract (Example 1)

	1st cycle	2nd cycle	3rd cycle	4th cycle	Total
Π_i^r	2094	1589	1101	638	5422
Π_i^m	256	214	180	145	795
Π_i^{rRS}	2387.3	1854.8	1378	847.9	6468
Π_i^{mRS}	376.9	496.3	609.8	572.6	2055.6

Optimal prices and preservation investments are obtained as $p_1 = \$44.24$, $p_2 = \$45.3$, $p_3 = \$46.2$, $p_4 = \$47.3$; $w_1 = \$2.78$, $w_2 = \$4.84$, $w_3 = \$7.03$, $w_4 = \$9.19$; and $\xi = \$52.45$. It is to be observed that, with higher share of revenue, the manufacturer is inclined to reduce wholesale price, even lower than the production cost itself, a result that corroborates with the findings of Cachon and Lariviere, 2005. It is further observed that the retail prices are also lower than those in wholesale price only contract, establishing that the contract is capable of generating more demand and extracting more profit out of it.

6.4.1 Sensitivity analysis

Table 6.6: Sensitivity analysis of the centralized model

Parameter	values	n	p_i	ξ	Total profit
c_m	5	4	30; 30.59; 31.1; 31.6	174	32966
	8	4	31.6; 32.25; 32.75; 33.29	111.5	26924
	10	4	33.72; 33.4; 33.96; 34.43	22	23966
	12	4	33.88; 34.45; 34.95; 35.5	22	19602
	15	5	34.8; 35.3; 35.74; 36.18; 36.64	21.94	12976
c_d	1	4	32.74; 33.29; 33.95; 34.3	17.28	24210
	1.5	4	32.75; 33.3; 33.88; 34.39	18.7	24075
	2	4	33.72; 33.4; 33.96; 34.43	22	23966
	2.5	4	32.83; 33.37; 33.94; 34.43	35.5	23779
	3	5	32.85; 33.39; 34; 34.47; 19.43	183	23726
h_r	0.5	4	32.39; 33; 33.52; 34	50	28953
	0.7	4	32.61; 33.1; 33.65; 34.19	44.7	26821
	1	4	33.72; 33.4; 33.96; 34.43	22	23966
	1.5	5	32.5; 32.94; 33.38; 33.85; 34.26	36.66	18487
	2	5	32.64; 33.22; 33.63; 34.1; 34.5	26.37	13954
h_m	1.5	4	32.53; 33.16; 33.65; 34.14	16.24	24143
	2	4	32.6; 33.13; 33.7; 34.1	19	23982
	3	4	33.72; 33.4; 33.96; 34.43	22	23966
	4.5	4	32.97; 33.55; 34; 34.51	56.59	22889
	6	4	33.09; 33.74; 34.38; 34.84	61.98	22356
	0.1	4	33.72; 33.4; 33.96; 34.43	22	23966
	0.2	5	32.65; 33.12; 33.52; 34; 34.32	155.37	23771

θ	0.3	5	33.15; 33.6; 34.1; 34.6; 35	176	22666
	0.4	5	34.05; 34.52; 35; 35.4; 35.8	207	20919
P	1000	5	32.54; 33.01; 33.52; 33.9; 34.23	37.4	22572
	1500	5	32.25; 32.78; 33.23; 33.67; 34	33.8	23101
	2000	4	33.72; 33.4; 33.96; 34.43	22	23966
	2500	4	32.7; 33.3; 33.8; 34.3	10	24282
	3000	4	32.6; 33.18; 33.7; 34.23	0	24570
c	0.01	4	32.8; 33.46; 34.13; 34.63	18.53	25145
	0.03	4	32.75; 33.33; 34; 34.47	23.63	24478
	0.05	4	33.72; 33.4; 33.96; 34.43	22	23966
	0.075	4	32.77; 33.3; 33.65; 34.1	28.4	22528
	0.1	4	32.7; 33.25; 33.67; 34.06	30	22280

Robustness of the proposed model is established in Tables 6.6 and 6.7. With slight changes in parameter values, optimal values of the decision variables along with profits also deviate in a pattern, establishing a trend from which following sensitivity analysis may be drawn.

- Table 6.3 reveals that in a high demand and less price sensitive market, prices increase, but strong demand offsets the impact of price hikes. High demand also accelerates product turnover, reducing spoilage risks and lowering the need for preservation investment. Since total profit is directly linked to demand, businesses benefit from a high-demand scenario. For highly perishable items with low preservation efficiency, business strategies shift. With preservation being less effective, investment in preservation remains unchanged. To counteract deterioration, the manager shortens replenishment cycles and increases replenishment frequency, mitigating spoilage. Additionally, higher holding costs at the retailer further justify shorter cycles, leading to reduced preservation investment, though total profit declines. Under a wholesale price-only contract, high demand or high spoilage significantly influences retailer

Table 6.7: Sensitivity analysis of the decentralized model

Parameter	values	n	w_i	p_i	ξ	Π^m	Π^r	Π
c_m	5	4	18.83; 19; 19.5; 19.78	44.38; 45.4; 46.2; 47.05	251	4022	6365	10387
	8	4	20.53; 20.94; 21.28; 21.55	45; 45.93; 46.8; 47.65	152	1931	6030	7961
	10	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
	12	4	23.25; 23.4; 23.78; 24.22	45.67; 46.7; 47.56; 48.32	68	129	2707	2836
	15	4	25.79; 24.7; 24.4; 26.2	45.9; 47.63; 48.86; 48.87	55	-996	858	-138
c_d	1	4	21.7; 21.87; 22; 22.19	45.24; 45.9; 46.85; 47.81	80	490	7621	8111
	1.5	4	21.75; 21.98; 22.2; 22.45	45.32; 46.1; 47; 47.88	89	653.36	6690	7343
	2	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
	2.5	4	21.87; 22.2; 22.48; 22.83	45.36; 46.3; 47.2; 48	117	1037	3177	4214
	3	4	21.83; 22.14; 22.48; 22.79	45.38; 46.33; 47.2; 48	117	1009	3153	4162
h_r	0.5	4	22.1; 22.39; 22.72; 23.05	45.24; 46.2; 47.08; 47.89	115	1197	8193	9350
	0.7	4	21.92; 22.34; 22.66; 23.01	45.33; 46.22; 47.11; 47.9	110	1089.3	6190	7279.3
	1	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
	1.2	6	22.04; 22.49; 22.85; 23.05; 23.31; 23.5	44.59; 45.1; 45.67; 46.2; 46.67; 47.16	92	-3174	4214	1040

	1.5	6	22.06; 22.5; 22.85; 23.16; 23.37; 23.6	44.87; 45.4; 46.08; 46.54; 47.12; 47.77	90	-3227	1297	-1930
h_m	1.5	4	21.73; 22.03; 22.36; 22.64	45.34; 46.3; 47.18; 48	26.5	288.34	5994	6282.34
	2	4	21.72; 22.08; 22.4; 22.67	45.36; 46.29; 47.17; 48.02	32	330.74	5875	6205.74
	3	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
	4.5	4	21.9; 22.12; 22.43; 22.74	45.37; 46.34; 47.22; 48.05	138	1118.9	4671	5789.9
	6	4	21.92; 22.07; 22.47; 22.74	45.38; 46.4; 47.27; 48.1	143	1101.5	4705	5806.5
θ	0.1	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
	0.2	4	20.64; 20.97; 21.27; 21.57	45.58; 46.5; 47.39; 48.21	221	-2797	5726	2929
	0.3	7	21.64; 21.98; 22.13; 22.18; 22.32; 22.5; 22.84	44.63; 45.12; 45.69; 46.29; 46.83; 47.3; 47.69	430	-9291	5857	-3434
	0.4	7	19.96; 20.13; 20.3; 20.46; 21.5; 21.1; 21.08	44.99; 45.56; 46.1; 46.64; 46.61; 47.45; 48.03	669	-10005	6113	-3892
P	1000	4	21.73; 22; 22.3; 22.6	45.4; 46.35; 47.18; 48.05	34	181.8	6323	6141.2
	1500	4	21.77; 22.12; 22.49; 22.77	45.4; 46.33; 47.2; 48.04	58.4	506	5925	6431
	2000	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
	2500	4	21.75; 22.03; 22.35; 22.67	45.36; 46.29; 47.2; 48	120	974.8	4896	5870.8
	3000	4	21.85; 22.33; 22.63; 22.88	45.36; 46.22; 47.12; 47.98	138	1308	2835	4143

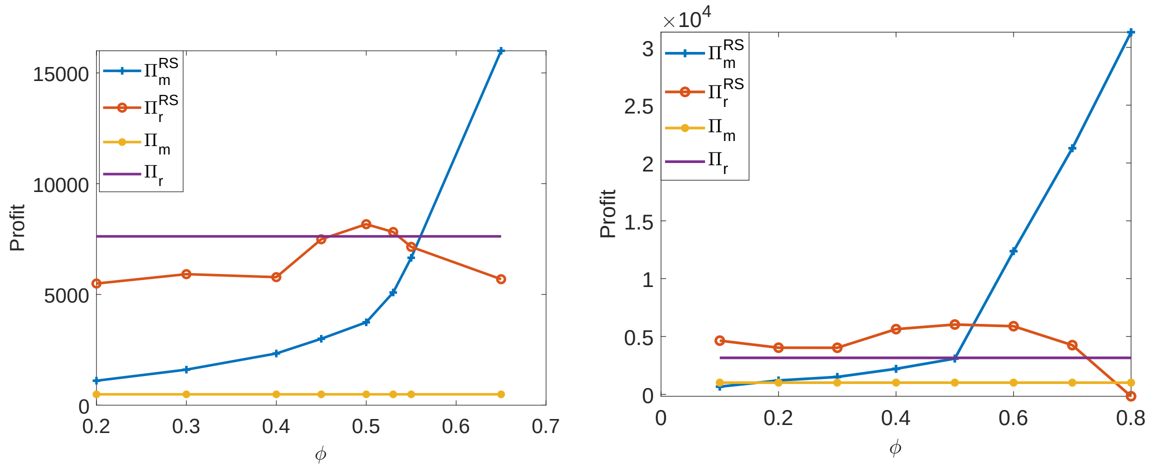
	0.01	4	21.87; 22.01; 22.65; 23	45.38; 46.46; 47.51; 48.46	131	715.6	6120	6835.6
	0.03	4	21.83; 22.16; 22.47; 22.84	45.37; 46.37; 47.33; 48.23	121	833.4	5769	6602.4
c	0.05	4	21.8; 22.03; 22.32; 22.64	45.36; 46.3; 47.18; 48	100	795	5422	6217
	0.075	4	21.79; 22.04; 22.37; 22.67	45.34; 46.26; 47.05; 47.76	74	831.6	3474	4305.6
	0.1	4	21.73; 22.03; 22.29; 22.47	45.33; 46.15; 46.88; 47.5	67	929.45	3039	3968.45

decisions compared to a centralized model. In high-demand scenarios, retailers exploit non-instantaneous deterioration by shortening business cycles, prioritizing pricing over preservation, and investing less in preservation than in moderate-demand conditions. Interestingly, under moderate demand, retailers invest more in spoilage reduction than in a centralized model, whereas in high demand, spoilage concerns are deprioritized. However, a high spoilage rate combined with low preservation efficiency creates an unfavorable business environment under a price-only contract. While the retailer adapts by shortening cycles and sustaining operations, the manufacturer faces losses, ultimately leading to the breakdown of the price-only contract.

- An escalation in production costs corresponds to a concurrent increase in price level. This, in turn, signifies a decline in demand, while the model's framework assumes uniform cycle lengths for all cycles. Collectively, these factors result in a noticeable decrease in revenue. The surge in expenses, coupled with the drop in earnings, ultimately translates into reduced profit. The pricing-only contract illustrates that both wholesale and retail prices rise in response to higher production costs. Elevated production costs impact the order quantity, subsequently reduces the overall inventory level. This, in turn, leads to a decrease in the investment made in preservation technology, a trend observable in both centralized and decentralized scenarios. The price-sensitivity of customers restricts the channel members from setting arbitrarily high price, resulting in potential loss for the manufacturer under very high production cost.

- An increase in the deterioration or disposal costs leads to a decrease in profit level. Higher disposal expenses result in elevated overall costs, ultimately reduce profit. Notably, the pricing structure is not adjusted to reward a more efficient inventory flow. Sensitivity analysis reveals a positive correlation between rising disposal costs and increased investment in preservation technology under both the scenarios. However, the manufacturer's inventory management presents a distinct narrative. This study suggests that the manufacturer initiates the transfer of inventory to the retailer before the potential onset of deterioration, effectively insulating the manufacturer's profit from any adverse impacts resulting from an increase in disposal costs. In the event of heightened disposal costs, it is plausible to observe a corresponding increase in manufacturer's profit margin, implying a corresponding surge in the quantity of orders placed at the manufacturer. The high disposal cost affects the retailer mostly, so that he has to invest in reducing spoilage even higher than centralized model.

- An increase in the retailer's unit holding cost contributes to an escalation in the total cost of the system. This cost factor exerts an influence on the overall inventory level, necessitating a reduction to minimize costs. The business manager should opt to increase prices in response to lower inventory level and fixed cycle length. This decision will result in an increase in revenue, albeit with a corresponding decrease in product quantity. Additionally, the manager should scale back investment in preservation due to the rising cost of holding inventory. In the context of decentralized system, it can be observed that an increase in holding cost prompts the retailer to transfer a greater proportion of the holding time to the manufacturer. Reduced length of a business cycle and higher number of shipments in the decentralised model have a significant impact on the business dynamics of the manufacturer, resulting in a decline in profit and even potential losses under certain adverse conditions. As evidenced by the sensitivity table, the aggregate profit of the supply chain experiences a decline.



(a) range of ϕ for $c_d = 1$

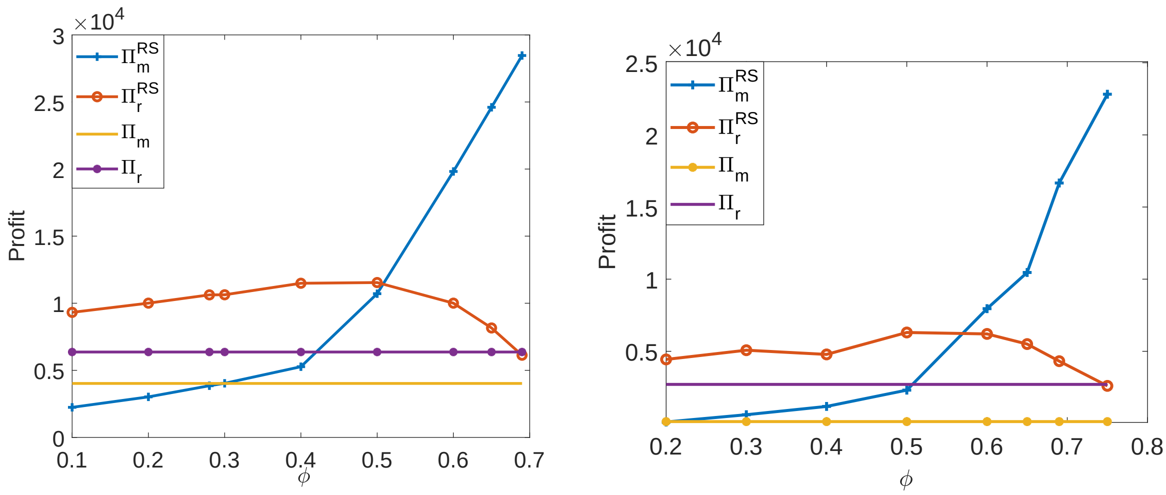
(b) range of ϕ for $c_d = 3$

Figure 6.4: Ranges of ϕ for different deterioration cost

- The escalation in the manufacturer's holding cost is found to have a discernible impact on the profitability of the centralized supply chain. However, the magnitude of the alterations is not as pronounced as that of other sensitivity parameters, given that the goods in question tend to have a shorter shelf life in the manufacturer's inventory. It can be inferred that an increase in the holding cost for the manufacturer results in a corresponding increase in the total cost incurred. Consequently, the manufacturer may be disinclined to take action and instead opt to raise the wholesale price. An augmented wholesale price for the retailer connotes an elevated cost, the retail price thus also experiences an increase. Consequently, the rate of demand experiences a decline. The relationship between wholesale and retail prices and marginal value of products is noteworthy. Table 6.7 indicates that a higher price corresponds to a greater marginal value, thereby increasing the cost of deterioration and emphasising the importance of investment in preservation technology. The confluence of these factors collectively exerts a downward pressure on the aggregate profit level, thereby rendering the prevailing situation unfavourable.

- The process of deterioration, which is evidently an undesirable circumstance for the majority of inventories, has exhibited an adverse effect on the inventory model under consideration. With an increase in the rate of deterioration, business cycle length is reduced. The non-instantaneous nature of the deterioration implies that an increase in the number of cycles results in a reduction of the vulnerable time for the products, thereby leading to a decrease in the overall deterioration. A higher value of θ indicates a lower optimal ordering quantity, thereby reducing inventory levels in warehouses and providing retailers with the opportunity to increase prices, coupled with the fact that there should be more investment in preservation too to safeguard the products. All of these factors ultimately have a negative impact on the overall profitability. In the context of decentralized system, it appears that a higher spoilage rate affects the retailer more adversely since the cycle length even more short here. The retailer is transferring the incurred loss to the manufacturer. The potential for loss incurred by the manufacturer in this scenario may result in a reduction of overall supply chain profitability. It is worth noting that, for a range of values of θ, the manufacturer faces a loss while the total profit is positive.

So the retailer has to share some of his revenue in those cases so that the manufacturer may sustain in business, thereby justifying the necessity of revenue sharing contract.



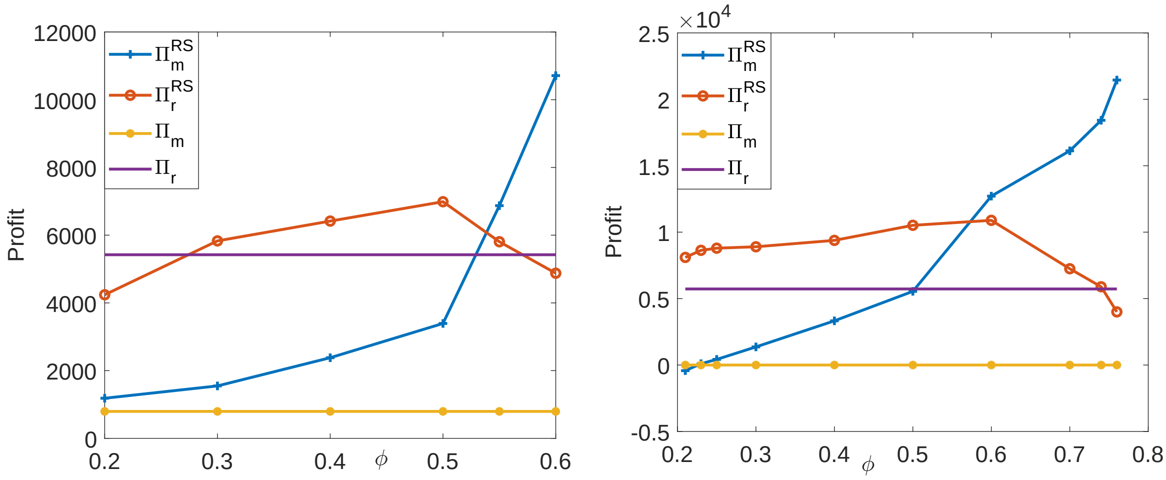
(a) range of ϕ for $c_m = 5$ **(b)** range of ϕ for $c_m = 12$

Figure 6.5: Range of ϕ for different production cost

- An increased production rate has the potential to generate higher profits under centralized model. However, the retailer does not derive substantial benefits from the heightened production rate under price-only contract, as indicated by the sensitivity table. The key contributing factor is the positive correlation between the production rate and the preservation investment, as reflected in Table 6.7. This correlation implies a corresponding increase in the effective level of deterioration due to larger storage time.

- An elevation in the value of c results in a reduction of the profit level in both centralized and price-only contract models. With the increased quadratic time dependence c which indicates faster reduction in demand, the optimal wholesale price falls faster as well in subsequent intervals. The necessity for investment in preservation technology is also reduced. In the decentralised scenario, an increase in c indicates reduced price.

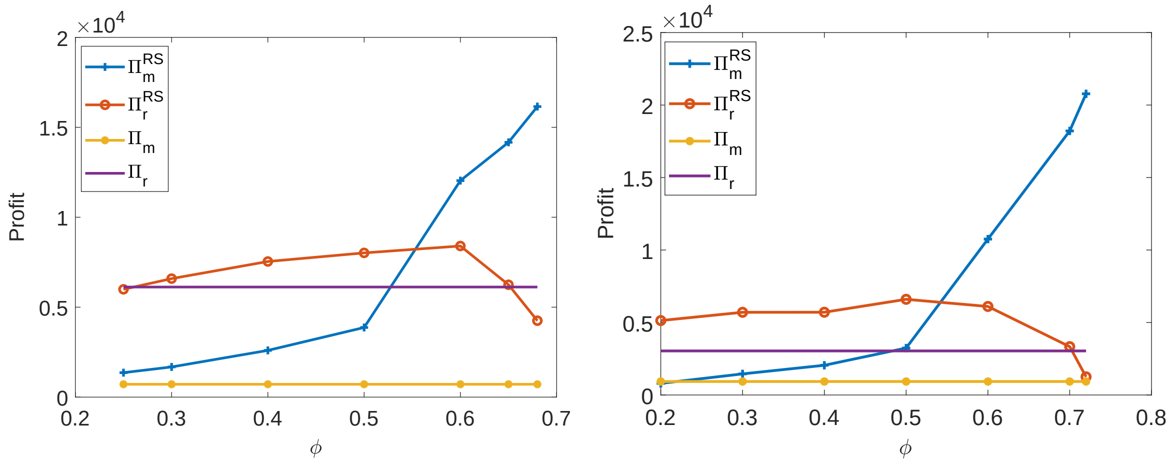
- The effect of double marginalization on both the channel members is evident from Property 2(ii) and Tables 6.6 and 6.7. While optimal retail price is found to be around 30 to 32 in the centralized model, the price jumps to around 42 to 45 under price only contract. The effect of double marginalization is severe



(a) range of ϕ for $\theta = 0.1$ **(b)** range of ϕ for $\theta = 0.2$

Figure 6.6: Range of ϕ for different deterioration rate

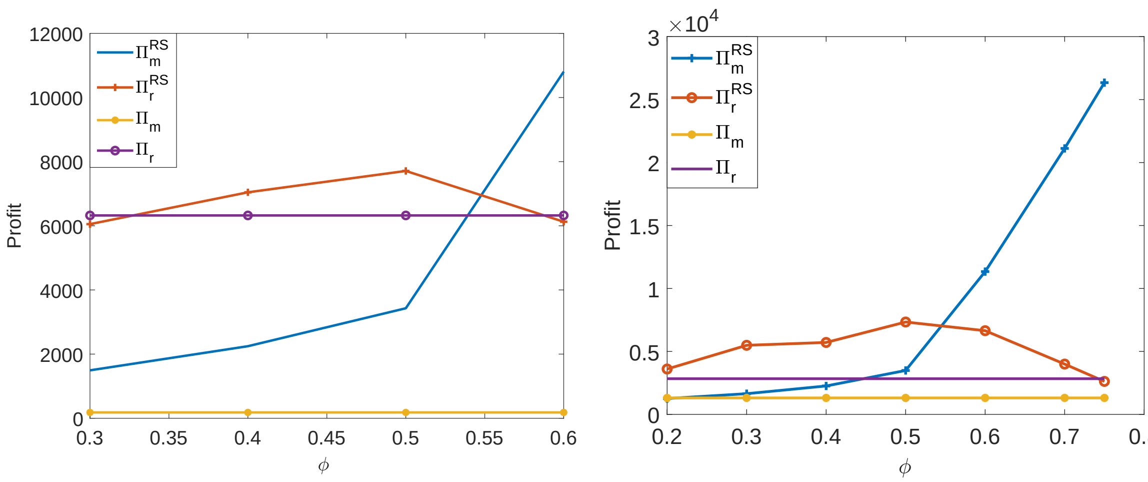
when raw material cost is high. Marginal value of any product increases because of double marginalization, making preservation investment even more important. The incorporation of a contract ensuring win-win is thus necessary to reduce the double marginalization. Regarding applicability of revenue sharing contract, since profits of the channel members drop down in subsequent cycles, shared revenue should also be reduced.



(a) Range of ϕ for $c = 0.01$ **(b)** Range of ϕ for $c = 0.1$

Figure 6.7: Range of ϕ for different values of c

- Figures 6.4-6.8 depict the changes in the feasible range of ϕ with varying parameter values. The study is conducted while considering the same value of ϕ for all intervals. It explores an important characteristic of the relationship between retailer's profit and shared revenue fraction ϕ. As the retailer shares higher fraction of revenue with the manufacturer, the manufacturer in turn reduces wholesale price, thereby reducing double marginalization effect as well. This results in surge in profit for both the channel members. However, reduction in wholesale price is stopped when it reaches zero, representing a scenario with no double marginalization at all. The members may be considered to act as one unit, then the supply chain is equivalent to a centralized model, and revenue sharing may be seen as profit sharing in this case. However, when ϕ is kept on increasing, the manufacturer continues to increase his profit level, but the retailer is unable to extract any more benefit, so his profit starts declining, resulting in a concave pattern altogether. The feasibility range is to be determined based on all the critical points when profits of both the channel members are beyond the predetermined level (those obtained under price-only contract). Hike in parameter values which have negative effect on business dynamics (such as deterioration rate, deterioration and production costs) necessitates the implementation of profit enhancing contract more. To sustain in those adverse situations, both the channel members have to be more flexible, leaving more room for bargaining, resulting in wider range of feasible values of ϕ. Since a higher production rate results in more holding cost and spoilage while a higher quadratic demand sensitivity c has a negative effect on demand, the above observation is equally true for a faster production scenario or faster reducing demand scenario as well.



(a) range of ϕ for $P = 1000$ **(b)** range of ϕ for $P = 3000$

Figure 6.8: Range of ϕ for different production rates

6.4.2 Managerial insights

From the analytical results and sensitivity analysis, a few important managerial insights can be drawn.

- The retailer should reduce the length of business cycle under higher spoilage rate or higher disposal cost so as to get benefit from non-instantaneous spoilage. For higher disposal cost or higher rate of spoilage, retailer should invest more in preservation- even more than the investment under centralized scenario. He needs to share some of his revenue too with the manufacturer so as to protect him from making losses.

- When production rate is much higher, the centralized model is adjusted to focus on pricing and not on preservation, while the price-only contract compels the retailer to invest more in preservation. Price should be reduced with higher production rate.

- For higher holding costs at retailer's inventory, the manager should reduce the length of the business cycle and plan to deliver products in higher number of shipments.

- For faster reducing demand, the manager should set lower retail prices in subsequent intervals to attract demand, and should reduce investment in preservation.

- The manager should hike retail price with the increase of raw material cost, holding costs, and base demand. If the raw material cost is too high, manager should not indulge in business unless the base demand or production rate is high.

- The retailer should hike wholesale price with higher raw material cost or higher holding cost of the manufacturer. The manufacturer should adjust production time period to mitigate the negative effect of higher holding cost. The manufacturer should reduce wholesale price when demand falls down fast.

- For higher unit deterioration cost, production rate and deterioration rate, the feasible range of share of revenue is wide; the channel members should properly bargain to ensure their individual share of revenue. However, lower values of the parameters leave little scope for bargaining. Fast reducing demand also compels the retailer to offer higher revenue share.
- With higher production cost, the manufacturer may accept the offer of revenue sharing contract even for a lesser share of revenue.

6.5 Concluding remarks

In the current world of fluctuating demand dynamics, where demand varies over time, dynamic pricing emerges as a more practical strategy compared to static pricing. This chapter delves into a multi-period supply chain model characterized by demand sensitivity to both price and time. With the inclusion of non-instantaneous deterioration for the retailer and non-instantaneous production for the manufacturer, this study derives optimal pricing strategies for different business intervals, along with preservation investment. The research establishes the existence of optimal pricing in various business scenarios through rigorous analytical proofs. Upon demonstrating that the wholesale price-only contract yields sub-optimal results in comparison to the centralized model, the study introduces a revenue sharing contract. This contract is observed to significantly boost profits, particularly under adverse conditions where one of the channel members might otherwise opt out of the business. Interestingly, it is observed that profits for both channel members decrease in subsequent intervals under the price-only contract, despite the upward adjustments in both wholesale and retail prices. In light of this finding, it is proposed that the shared revenue portion should also decrease in subsequent intervals to preserve the retailer's share of profit. Moreover, a higher shared fraction of revenue prompts the manufacturer to reduce wholesale prices to a greater extent. The study underscores the importance of preservation investment in generating higher profits, a finding substantiated numerically. The sensitivity analysis further illuminates how changes in various parameters impact the total profit. The managerial insights gleaned from this analysis have the potential to greatly benefit businesses if implemented wisely and effectively.

The current study offers opportunities for several extensions, and a few potential avenues are outlined below. One potential enhancement involves replacing the discrete dynamic pricing policy with a continuous dynamic pricing strategy. This change would allow for more flexible price adjustments, leading to more precise optimization of the situation. Another interesting extension could involve making the cycle length dynamic, adapting it to changing conditions. The assumption of a constant production rate for the manufacturer is a simplification, and it could be refined by considering factors such as random yield or the possibility of the production process shifting to an out-of-control state, resulting in the production of substandard materials. Further consideration of green investment to reduce carbon emission could also be worth examination in the context of global environment. Consideration of other demand controlling parameters such as reference price or stock level would also be interesting to study. These extensions could lead to a more comprehensive and realistic modeling of the supply chain dynamics.

CHAPTER 7

Exploring Random Pricing Strategies in a Closed-Loop Supply Chain with Greening Investment

The pricing strategy plays a crucial role in supply chain management, directly influencing demand, profitability, and overall market competitiveness. In a closed-loop supply chain (CLSC), where products circulate between manufacturers, retailers, and consumers with the inclusion of reverse logistics for recycling or remanufacturing, pricing decisions become even more complex. Unlike traditional supply chains, CLSCs must account for not only forward logistics and production costs but also the recovery and resale value of returned products. In such a system, a well-structured pricing strategy is essential to balance profitability, sustainability, and market demand. While it is true that there is always a predetermined price, it is quite unrealistic to commit to that value in real life. To illustrate through an example, let us consider the maximum retail price (MRP) of a product. Majority of the supermarkets set actual retail price at the MRP; however, during the sale, the product price is impacted by various events such as discounts or changes in demand, thereby resulting in a stochastic behavior of price. Given the price's randomization, demand will now likewise act randomly, leaving the retailer with either a lack of stock or product spoilage.

While many studies have focused on static or deterministic pricing models, real-world markets exhibit inherent uncertainties that necessitate more flexible approaches.

Random pricing strategies have gained attention as a means to navigate demand fluctuations, customer behavior variations, and competitive market dynamics. By incorporating stochastic elements into pricing decisions, businesses can enhance their adaptability and optimize revenue in uncertain environments. This approach is particularly relevant in CLSCs, where the unpredictability of product returns, varying remanufacturing costs, and fluctuating consumer willingness to pay make the fixed pricing strategies less effective. Random pricing strategies in CLSC operations help the businesses to achieve better demand-supply equilibrium, optimize resource utilization, and improve long-term financial performance. This chapter explores the role of random pricing strategies in a closed-loop supply chain, analyzing how price fluctuations can influence inventory management, profitability, and customer retention. It examines key factors such as demand uncertainty, return rates, and remanufacturing costs, offering insights into how firms can leverage stochastic pricing to enhance their competitive edge. Additionally, the chapter also elaborates the remanufacturing strategy in detailed manner and how it is helpful to manage the uncertainties in market. The findings of this chapter provide valuable guidance for supply chain managers and policymakers in designing more resilient and adaptive pricing strategies in an increasingly uncertain market landscape. The chapter is divided into two parts.

7.1 Optimal Pricing, Greening and Warranty Investments under Random Yield

7.1.1 Introduction

In real-world business scenarios, an effective pricing strategy is crucial for capturing market demand. Properly setting retail prices requires understanding their impact on demand and overall supply chain performance. Additionally, growing consumer awareness of environmental sustainability and product warranties must be considered in pricing decisions. Random pricing is particularly relevant for electronic products and online retail, where price fluctuations are common.
This study aims to address the following research questions:
RQ1: How can a pricing strategy be developed for products where random pricing is applicable?
RQ2: How does a warranty offer influence demand and other business decisions?
RQ3: Given rising environmental awareness, what measures should managers take to adopt eco-friendly business practices?
RQ4: How does a closed-loop supply chain perform under random pricing, considering warranty-driven remanufacturing?
To explore these questions, this study develops a closed-loop supply chain model where demand depends on price, warranty period, and product sustainability. Unlike existing works, the model integrates both deterministic and stochastic components in pricing, making it more realistic. It also incorporates remanufacturing as a strategy to reduce environmental impact. This work extends existing studies by considering random pricing alongside green investment and warranty policies, which have been overlooked in prior research. Furthermore, the study examines different contractual mechanisms, including wholesale price and buyback contracts, to enhance supply chain performance. By providing a structured framework for implementing random pricing in a closed-loop system, this research contributes valuable insights for businesses aiming to optimize pricing while balancing profitability, sustainability, and consumer expectations.

7.1.2 Notations and Assumptions

The following notations are used throughout the chapter.

Table 7.1: Notations

w_m	:	unit wholesale price (decision variable of the manufacturer)
g	:	green level of the product (decision variable of the manufacturer)
ω	:	warranty period (decision variable of the manufacturer)
p_0	:	unit deterministic part of the selling price (decision variable of the retailer)
Q	:	ordering quantity (decision variable of the retailer)
ϵ	:	a variable denoting randomness in the price function
p	:	realized retail price of the product
$f(\epsilon)$	:	probability density function of the variable ϵ
μ	:	investment coefficient associated with green investment
λ	:	investment coefficient associated with warranty period
$D(p, g, \omega)$	:	demand rate
D'	:	returned quantity of the product
c_m	:	per unit production cost
$c_{r_1} (< c_m)$	:	per unit production cost of re-manufactured item
c_{r_2}	:	per unit production cost of refurbished item
w'_m	:	unit wholesale price of the remanufactured items at secondary market
c_s	:	per unit shortage cost
c_d	:	per unit salvage value of the unsold item
β	:	sensitivity of warranty period on demand
γ	:	sensitivity of green level on demand
δ	:	fraction of sold amount which gets returned
η	:	fraction of returned product which can be re-manufactured
y	:	a random variable denoting randomness in return of the products

with support $[0, 1]$ and pdf $h(y)$

The following assumptions are made for developing the proposed model:

- A closed-loop supply chain system is considered where the manufacturer produces a single product and sells it to the retailer who then sells those products to the potential customers. By offering a warranty, the manufacturer provides the retailer a chance to return any defective items.
- Following Ghosh and Shah, 2015 and Giri et al., 2018, the market demand is assumed to depend simultaneously on price, green level and warranty period. The specific pattern is $D = a - bp + \beta\omega + \gamma g$. The retail price is random in nature, but it has a deterministic component which may be optimized by the retailer. It is assumed that $p = p_0 + \epsilon$, where p_0 is the deterministic part and ϵ is the random part that follows probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$ over the range $[l, u]$.
- During the warranty period, the customers can return the defective items which will be replaced with new ones. The defective items are collected and remanufactured by the manufacturer. The amount of returned items during warranty period depends on the length of the warranty period ω as well as the amount of sold items, multiplied by a random variable y to ensure random return. The actual return rate is thus assumed to be $\delta y D\omega$. Note that the warranty period may extend beyond a single business period, which could cause the actual return amount in a particular period to exceed the total sales amount for that period. To prevent this, the return function ensures that return-related costs are accounted for in the business period during which the item was originally sold. To exclude the impossible situation when returned amount is more than sold amount, it is to be assumed that $\delta < \frac{1}{\omega}$. Out of the returned items, $(1 - \eta)$ fraction can be refurbished and made operative as it was earlier, and sent back to the customers; the rest η portion has to be remanufactured, but those cannot be made same as earlier and therefore are sold in a secondary market at a cheaper price, and the same amount is reproduced to meet customers' demand.

- Similar to Giri et al., 2018, the greening cost and warranty investment cost are assumed to be μg^2 and $\lambda\omega^2$ for g unit of greening level and ω warranty period, respectively. The convex patterns ensures diminishing return, thereby preventing the manufacturer from spending infinitely large amounts in these categories.

7.1.3 Model Formulation

The expected demand is given by $E(D) = \int_l^u (a - bp_0 + \beta\omega + \gamma g - b\epsilon) f(\epsilon) d\epsilon$, and the expected sales is given by $E(S) = E[\min\{Q, (a - bp_0 + \beta\omega + \gamma g - b\epsilon)\}]$. The expected returned amount is given by $E(D') = \int_0^1 \int_l^u \delta\omega y \min\{Q, D\} f(\epsilon) h(y) d\epsilon dy$.

The expected profits of the manufacturer, retailer and the whole system are given by

$$\begin{aligned}
\Pi_m(w_m, \omega, g) &= (w_m - c_m)Q + (w'_m - c_{r_1})\eta E(D') - c_m\eta E(D') - (1-\eta)E(D')c_{r_2} \\
&\quad -\lambda\omega^2 - \mu g^2, \qquad (7.1)\\
\Pi_r(p_0, Q) &= E[(p_0 + \epsilon)\min\{Q, D\}] - w_m Q + c_d E[(Q - D)^+] \\
&\quad -c_s E[(D - Q)^+], \text{ and} \qquad (7.2)\\
\Pi(p_0, \omega, Q, g) &= \Pi_m(w_m, \omega, g) + \Pi_r(p_0, Q). \qquad (7.3)
\end{aligned}$$

With the profit functions at hand, we shall now proceed to develop different business scenarios. The centralized model will first be studied as the benchmark one; the wholesale price only contract will then be studied to represent the primitive market scenario; a suitable profit enhancing contract will then be developed so as to ensure win-win situation for both the channel members.

7.1.3.1 Centralized case

In the centralized scenario, both the channel members act together to optimize the joint profit. The expected profit function is given by

$$\begin{aligned}
\Pi_c(p_0,\omega,g,Q) &= \int_l^A (p_0+\epsilon)Qf(\epsilon)d\epsilon + \int_A^u (p_0+\epsilon)\{\alpha(p_0,\omega,g)-b\epsilon\}f(\epsilon)d\epsilon \\
&+c_d\int_A^u \{Q-\alpha(p_0,\omega,g)+b\epsilon\}f(\epsilon)d\epsilon - c_m\eta \\
&-c_s\int_l^A \{\alpha(p_0,\omega,g)-b\epsilon-Q\}f(\epsilon)d\epsilon - c_mQ + \{(w'_m-c_{r_1})\eta \\
&-(1-\eta)c_{r_2}\}\omega\int_0^1 \delta y\Big[\int_l^A Qf(\epsilon)d\epsilon \\
&+\int_A^u \{\alpha(p_0,\omega,g)-b\epsilon\}f(\epsilon)d\epsilon\Big]h(y)dy - \lambda\omega^2 - \mu g^2, \qquad (7.4)
\end{aligned}$$

where $\alpha(p_0,\omega,g) = a - bp_0 + \beta\omega + \gamma g$ and $A = \frac{\alpha(p_0,\omega,g)-Q}{b}$. It is not been possible to establish the joint concavity of $\Pi_c(p_0,\omega,g,Q)$ analytically with respect to all four decision variables. Nevertheless, the profit function is characterized in the following propositions.

Proposition 7.1. *(a) Given ω and Q, $\Pi_c(p_0,\omega,g,Q)$ is jointly concave in p_0 and g.*
(b) $\Pi_c(p_0,\omega,g,Q)$ is jointly concave in Q and ω when $f(A) > \frac{bP^2}{2\{(\lambda-\beta P)(B+(c_s-c_d)+P\omega)\}}$ $F^2(A)$.

Proof: Denoting $\{(w'_m-c_{r_1})\eta - c_m\eta - (1-\eta)c_{r_2}\}\delta\overline{y}$ by P, it is straightforward to derive from equation 7.4 that

$$\begin{aligned}
\frac{\partial^2\Pi_c}{\partial p_0^2} &= -\Big[(a+\beta\omega+\gamma g+bc_s-bc_d-Q)f(A)+bP\omega f(A)+2b\overline{F(A)}\Big], \\
\frac{\partial^2\Pi_c}{\partial Q^2} &= -\frac{1}{b}\Big[Bf(A)+(c_s-c_d)f(A)+P\omega f(A)\Big], \\
\frac{\partial^2\Pi_c}{\partial \omega^2} &= -\frac{\beta^2}{b}(B+c_s-c_d)f(A)-\beta P\left(\frac{\omega\beta}{b}f(A)-2\overline{F(A)}\right)-2\lambda, \\
\frac{\partial^2\Pi_c}{\partial g^2} &= -\frac{\gamma^2}{b}f(A)\Big[c_s-c_d+P\omega+B\Big]-2\mu, \\
\frac{\partial^2\Pi_c}{\partial p_0\partial g} &= \gamma f(A)\Big[c_d-c_s+B+P\omega\Big]+\gamma\overline{F(A)}, \text{ and} \\
\frac{\partial^2\Pi_c}{\partial Q\partial \omega} &= \frac{\beta Bf(A)}{b}+(c_s-c_d)\frac{\beta}{b}f(A)+PF(A)+P\omega\frac{\beta}{b}f(A),
\end{aligned}$$

so that for the Hessian matrix $H = \begin{bmatrix} \frac{\partial^2 \Pi_r^c}{\partial Q^2} & \frac{\partial^2 \Pi_r^c}{\partial \omega \partial Q} \\ \frac{\partial^2 \Pi_r^c}{\partial \omega \partial Q} & \frac{\partial^2 \Pi_r^c}{\partial \omega^2} \end{bmatrix}$, $|H_1| < 0$ and $|H_2| = \frac{1}{b^2}\Big(2\lambda bB$ $f(A) - 2Bb\beta P f(A) + 2\lambda(c_s - c_d)bf(A) - 2(c_s - c_d)b\beta P f(A) - 2\beta\omega b P^2 f(A) - b^2 P^2$ $F^2(A) + 2\lambda\omega P b f(A)\Big) > 0$. Further, under assumed condition for $J = \begin{vmatrix} \frac{\partial^2 \Pi_r^c}{\partial p_0^2} & \frac{\partial^2 \Pi_r^c}{\partial g \partial p_0} \\ \frac{\partial^2 \Pi_r^c}{\partial g \partial p_0} & \frac{\partial^2 \Pi_r^c}{\partial g^2} \end{vmatrix}$, $|J_1| < 0$ and $|J_2| = Bf^2(A)\gamma^2(c_s - c_d) + 2Bbf(A)\mu + \omega P\gamma^2(c_s - c_d)f^2(A) + 2\mu(c_s - c_d)bf(A) + (c_s - c_d)\gamma^2 Bf^2(A) + 2(c_s - c_d)\gamma^2 f(A)\overline{F(A)} + 4\mu b\overline{F(A)} + \omega P\gamma^2(c_s - c_d)$ $f^2(A) + 2\mu b\omega P f(A) - \gamma^2(\overline{F(A)})^2 + 2\gamma^2(c_s - c_d)f(A)\overline{F(A)} + 2B\gamma^2(c_s - c_d)f^2(A) + 2\gamma^2\omega(c_s - c_d)Pf^2(A) > 0$. Hence the proof.

However, as is seen, the investments in greenness and warranty is relevant only in presence of *sufficient* awareness or inclination of the customers towards these attributes. Following Yu et al., 2020, we derive propositions which specify the *sufficiency levels* in terms of providing a threshold level of the awareness or sensitivity.

Proposition 7.2. *(a) For given $p_0, Q,$ and g, if $\Delta_\omega < 0$, then it is always beneficial to offer warranty period; otherwise, the warranty offer is profitable only if $\beta > \Delta_\omega$.*
(b) For given $p_0, Q,$ and ω, if $\Delta_g > 0$ then it is always beneficial to invest in greenness.

Proof:
(a) Denoting $A^g = \frac{a - bp_0 + \beta\omega - Q}{b}$, $A^\omega = \frac{a - bp_0 + \gamma g - Q}{b}$, $A^Q = \frac{a - bp_0 + \beta\omega + \gamma g}{b}$, $A^{c_m} = \frac{a - bc_m + \beta\omega + \gamma g - Q}{b}$, we set $\frac{\partial \Pi_c}{\partial \omega}\Big|_{\omega=0} > 0$ to deduce
$\beta > \frac{\Big((w'_m - c_{r1})\eta - c_m\eta - (1-\eta)c_{r2}\Big)\Big(\int_l^{A^\omega} Qf(\epsilon)d\epsilon + \int_{A^\omega}^u (a - bp_0 + \gamma g - b\epsilon)f(\epsilon)d\epsilon\Big)}{\Big(\int_{A^\omega}^u (p_0 + \epsilon)f(\epsilon)d\epsilon - c_d\overline{F(A^\omega)} - c_s F(A^\omega)\Big)}$. Setting
$\Delta_\omega = \frac{\Big((w'_m - c_{r1})\eta - c_m\eta - (1-\eta)c_{r2}\Big)\Big(\int_l^{A^\omega} Qf(\epsilon)d\epsilon + \int_{A^\omega}^u (a - bp_0 + \gamma g - b\epsilon)f(\epsilon)d\epsilon\Big)}{\Big(\int_{A^\omega}^u (p_0 + \epsilon)f(\epsilon)d\epsilon - c_d\overline{F(A^\omega)} - c_s F(A^\omega)\Big)}$, the proof follows.
(b) $\frac{\partial \Pi_c}{\partial g}\Big|_{g=0} > 0$ yields $\int_{A^g}^u (p_0 + \epsilon)f(\epsilon)d\epsilon + \{(w'_m - c_{r1})\eta - c_m\eta - (1-\eta)c_{r2}\} \times$ $\omega\delta\bar{y}\overline{F(A^g)} - c_d\overline{F(A^g)} - c_s F(A^g) > 0$. Setting $\Delta_g = \int_{A^g}^u (p_0 + \epsilon)f(\epsilon)d\epsilon + \Big\{(w'_m - c_{r1})\eta$ $- c_m\eta - (1-\eta)c_{r2}\Big\}\omega\delta\bar{y}\overline{F(A^g)} - c_d\overline{F(A^g)} - c_s F(A^g)$, the proof follows.

It is now straightforward to derive the following first order conditions which are to be solved simultaneously to obtain optimal decision variables p_0, Q, ω and g when both Δ_ω and Δ_g are positive:

$$\begin{aligned}
\frac{\partial \Pi_c}{\partial Q} &= \int_l^A \epsilon f(\epsilon)d\epsilon + c_d\overline{F(A)} - c_m + [\{(w'_m - c_{r_1})\eta - c_m\eta - (1-\eta)c_{r_2}\}\omega\delta\overline{y} \\
&\quad + p_0 + c_s]F(A) = 0, \\
\frac{\partial \Pi_c}{\partial \omega} &= \int_A^u (p_0+\epsilon)\beta f(\epsilon)d\epsilon - c_d\int_A^u \beta f(\epsilon)d\epsilon - c_s\int_l^A \beta f(\epsilon)d\epsilon + \{(w'_m - c_{r_1})\eta \\
&\quad - c_m\eta - (1-\eta)c_{r_2}\} \times \Bigg[\omega\int_0^1\int_A^u \beta\delta y f(\epsilon)h(y)d\epsilon dy \\
&\quad + \int_0^1 \delta y\left\{\int_l^A Qf(\epsilon)d\epsilon + \int_A^u (\alpha(p_0,\omega,g) - b\epsilon)f(\epsilon)d\epsilon\right\}h(y)dy\Bigg] \\
&\quad - 2\lambda\omega = 0, \\
\frac{\partial \Pi_c}{\partial g} &= -c_d\int_A^u \gamma f(\epsilon)d\epsilon - c_s\int_l^A \gamma f(\epsilon)d\epsilon + \int_A^u (p_0+\epsilon)\gamma f(\epsilon)d\epsilon \\
&\quad + \{(w'_m - c_{r_1})\eta - c_m\eta - (1-\eta)c_{r_2}\}\omega\int_0^1 \delta y\int_A^u \gamma f(\epsilon)h(y)d\epsilon dy \\
&\quad - 2\mu g = 0, \\
\text{and} \quad \frac{\partial \Pi_c}{\partial p_0} &= (Q + bc_s)F(A) + \int_A^u ((\alpha(p_0,\omega,g) - b\epsilon) - b(p_0+\epsilon))f(\epsilon)d\epsilon \\
&\quad + bc_d\overline{F(A)} - b\{(w'_m - c_{r_1})\eta - c_m\eta - (1-\eta)c_{r_2}\}\omega\delta E[y]\overline{F(A)} = 0.
\end{aligned}$$

The joint concavity property of the profit function and the existence of unique optimal solution are established numerically in Section 7.1.4. An iterative method is provided below in the form of an algorithm to obtain the optimal solution.

Algorithm

Step 1: Set $g_0 = 0$, $p_{0_0} = c_m$, $\Pi^* = 0$ and $i = 1$.
Step 2: Obtain unique Q_i and ω_i using g_{i-1} and $p_{0_{i-1}}$ (by virtue of Proposition 7.1).
step 3: Obtain unique g_i and p_{0_i} using Q_i and ω_i derived in step 2 (by virtue of Proposition 7.1).
Step 4: Calculate Π_{c_i} using Q_i, ω_i, g_i and p_{0_i}.
Step 5: If $|\Pi_{c_i} - \Pi^*| < 0.01$, stop. Else, set $\Pi^* = \Pi_{c_i}$, $i = i+1$ and go to step 2.

The values of the decision variables obtained at the ith level are the desired optimal values.

7.1.3.2 Wholesale price only contract

Under wholesale price only contract, the manufacturer first declares his optimal decisions for wholesale price, greening investment, and warranty period, based on which retailer decides his choice of the deterministic part of the retail price and order quantity to maximize his own profit. The model is solved through backward substitution method. To start with, the expected profit of the retailer is considered. The following proposition ensures existence of optimal solution for the retailer.

Proposition 7.3. *The expected profit of the retailer is jointly concave in Q and p_0.*

Proof:
From equation 7.2, it is straightforward to derive

$$\begin{aligned}
\frac{\partial \Pi_r}{\partial Q} &= \int_l^A \epsilon f(\epsilon) d\epsilon + (p_0 + c_s - c_d)F(A) - w_m + c_d, \\
\frac{\partial^2 \Pi_r}{\partial Q^2} &= -\frac{Af(A)}{b} - \frac{(p_0 + c_s - c_d)f(A)}{b} \\
\frac{\partial^2 \Pi_r}{\partial p_0 \partial Q} &= F(A) - (A + p_0 + c_s - c_d)f(A), \\
\frac{\partial \Pi_r}{\partial p_0} &= \int_A^u (\alpha(p_0, \omega, g) - 2b\epsilon - bp_0 + bc_d)f(\epsilon)d\epsilon + (Q + bc_s)F(A), \\
\frac{\partial^2 \Pi_r}{\partial p_0^2} &= -2b \int_A^u f(\epsilon)d\epsilon - (A + p_0 - c_d + c_s)bf(A).
\end{aligned}$$

From the Hessian matrix $H = \begin{bmatrix} \frac{\partial^2 \Pi_r}{\partial Q^2} & \frac{\partial^2 \Pi_r}{\partial p_0 \partial Q} \\ \frac{\partial^2 \Pi_r}{\partial p_0 \partial Q} & \frac{\partial^2 \Pi_r}{\partial p_0^2} \end{bmatrix}$, it is now easy to show that $|H_1| < 0$ and $|H_2| = F(A)\{4f(A)(A + p_0 + c_s - c_d) - F(A)\} > 0$, establishing the concavity property. The joint concavity ensures the existence of unique optimal solution, which is provided below.

Proposition 7.4. *For given set of manufacturer's decisions, the retailer's optimal decisions are obtained by solving the following equations*

$$\int_l^A \epsilon f(\epsilon)d\epsilon + (p_0 + c_s - c_d)F(A) \;=\; w_m - c_d, \tag{7.5}$$

$$\text{and} \int_A^u (2b\epsilon + bp_0 - bc_d - \alpha(p_0, \omega, g))f(\epsilon)d\epsilon \;=\; (Q + bc_s)F(A) \tag{7.6}$$

for Q and p_0.

Based on the retailer's decisions, the optimal decisions of the manufacturer are obtained. Differentiating equations 7.5 and 7.6 with respect to ω partially, we have

$$\begin{aligned}
\left[\frac{Af(A)}{b}\left\{b\frac{\partial p_0}{\partial\omega} - \beta + \frac{\partial Q}{\partial\omega}\right\}\right] &= \frac{\partial p_0}{\partial\omega}F(A) + (p_0 + c_s - c_d)f(A) \times \\
&\quad \left[\frac{1}{b}\left\{-b\frac{\partial p_0}{\partial\omega} + \beta - \frac{\partial Q}{\partial\omega}\right\}\right], \\
2Af(A)\left\{b\frac{\partial p_0}{\partial\omega} - \beta + \frac{\partial Q}{\partial\omega}\right\} &= F(A)\frac{\partial Q}{\partial\omega} + (Q + bc_s)f(A) \times \\
&\quad \left[\frac{1}{b}\left\{-b\frac{\partial p_0}{\partial\omega} + \beta - \frac{\partial Q}{\partial\omega}\right\}\right] \\
&\quad + \int_A^u \left(\beta - 2b\frac{\partial p_0}{\partial\omega}\right) f(\epsilon)d\epsilon \\
&\quad - [\alpha(p_0, \omega, g) - bp_0 - bc_d)]\, f(A) \times \\
&\quad \left[\frac{1}{b}\left\{-b\frac{\partial p_0}{\partial\omega} + \beta - \frac{\partial Q}{\partial\omega}\right\}\right],
\end{aligned}$$

solving which we have $\frac{\partial p_0}{\partial\omega} = \frac{(R_2S_3 + R_3S_2)\beta}{(S_1R_2 + S_2R_1)}$ and $\frac{\partial Q}{\partial\omega} = \frac{(S_3R_1 - S_1R_3)\beta}{(S_1R_2 + S_2R_1)}$, where $R_1 = 2Af(A)b + (Q + bc_s)f(A) + 2b\overline{F(A)} - \{\alpha(p_0, \omega, g) - bp_0 - bc_d\}f(A)$, $R_2 = F(A) - \frac{(Q+bc_s)f(A)}{b} + \frac{\alpha(p_0,\omega,g) - bp_0 - bc_d}{b} - 2Af(A)$, $R_3 = \left(\frac{(Q+bc_s)f(A)}{b} + \overline{F(A)} - \{\alpha(p_0, \omega, g) - bp_0 - bc_d\}\frac{f(A)}{b} + 2Af(A)\right)$, $S_1 = Ab$, $S_2 = p_0 + c_s - c_d + A$, and $S_3 = \frac{p_0 + c_s - c_d + A}{f(A)}$. In a similar manner, the equations are differentiated partially with respect to g to produce the equations

$$\begin{aligned}
\left[\frac{Af(A)}{b}\left\{b\frac{\partial p_0}{\partial g}-\gamma+\frac{\partial Q}{\partial g}\right\}\right] &= \frac{\partial p_0}{\partial g}F(A)+(p_0+c_s-c_d)f(A) \\
&\quad \left[\frac{1}{b}\left\{-b\frac{\partial p_0}{\partial g}+\gamma-\frac{\partial Q}{\partial g}\right\}\right], \\
2Af(A)\left\{b\frac{\partial p_0}{\partial g}-\gamma+\frac{\partial Q}{\partial g}\right\} &= F(A)\frac{\partial Q}{\partial g}+(Q+bc_s)f(A) \\
&\quad \left[\frac{1}{b}\left\{-b\frac{\partial p_0}{\partial g}+\gamma-\frac{\partial Q}{\partial g}\right\}\right] \\
&\quad +\int_A^u\left(\gamma-2b\frac{\partial p_0}{\partial g}\right)f(\epsilon)d\epsilon-\Big[\alpha(p_0,\omega,g)-bp_0 \\
&\quad -bc_d)\Big]f(A)\left[\frac{1}{b}\left\{-b\frac{\partial p_0}{\partial g}+\gamma-\frac{\partial Q}{\partial g}\right\}\right],
\end{aligned}$$

solving which we get, $\frac{\partial p_0}{\partial g}=\frac{(R_2S_3+R_3S_2)\gamma}{(S_1R_2+S_2R_1)}$, $\frac{\partial Q}{\partial g}=\frac{(S_3R_1-S_1R_3)\gamma}{(S_1R_2+S_2R_1)}$, $\frac{\partial p_0}{\partial w_m}=\frac{T_4}{T_1T_4+T_2T_3}$, and partial derivatives with respect to w_m produce

$$\begin{aligned}
F(A)\frac{\partial p_0}{\partial w_m} &= 1+\left[\frac{Af(A)}{b}\left\{b\frac{\partial p_0}{\partial w_m}+\frac{\partial Q}{\partial w_m}\right\}\right]+(p_0+c_s-c_d)\frac{f(A)}{b}\times \\
&\quad \left\{b\frac{\partial p_0}{\partial w_m}+\frac{\partial Q}{\partial w_m}\right\} \\
2Af(A)\left\{b\frac{\partial p_0}{\partial w_m}+\frac{\partial Q}{\partial w_m}\right\} &= F(A)\frac{\partial Q}{\partial w_m}-(Q+bc_s)\frac{f(A)}{b}\left[b\frac{\partial p_0}{\partial w_m}+\frac{\partial Q}{\partial w_m}\right]-2b\frac{\partial p_0}{\partial w_m}\times \\
&\quad \int_A^u f(\epsilon)d\epsilon+\Big[\alpha(p_0,\omega,g)-bp_0-bc_d)\Big]\frac{f(A)}{b}\times \\
&\quad \left[b\frac{\partial p_0}{\partial w_m}+\frac{\partial Q}{\partial w_m}\right],
\end{aligned}$$

solving which we get $\frac{\partial Q}{\partial w_m}=-\frac{T_3}{T_1T_4+T_2T_3}$, where $T_1=F(A)-(A+p_0+c_s-c_d)f(A)$, $T_2=\frac{A+p_0+c_s-c_d}{b}f(A)$; $T_3=2Af(A)b+(Q+bc_s)f(A)+2b\overline{F(A)}-\{\alpha(p_0,\omega,g)-bp_0-bc_d\}f(A)$, and $T_4=2Af(A)-F(A)+\frac{(Q+bc_s)}{b}f(A)-\frac{\{\alpha(p_0,\omega,g)-bp_0-bc_d\}f(A)}{b}$. Differentiating Equation 7.1 partially with respect to the decision variables, we get

$$\frac{\partial \Pi_m}{\partial w_m} = Q-\frac{1}{T_1T_4+T_2T_3}\left\{T_3((w_m-c_m)+\omega PF(A))+T_4bP\overline{F(A)}\omega\right\},$$

$$\begin{aligned}\frac{\partial \Pi_m}{\partial \omega} &= \frac{1}{S_1R_2+S_2R_1}\Big\{((w_m-c_m)+P\omega F(A))\beta(S_3R_1-S_1R_3)\\ &-P\omega b\overline{F(A)}\beta(R_2S_3+S_2R_3)\Big\}+X_1,\\ \frac{\partial \Pi_m}{\partial g} &= \frac{1}{S_1R_2+S_2R_1}\Big\{((w_m-c_m)+P\omega F(A))\gamma(S_3R_1-S_1R_3)\\ &-P\omega b\overline{F(A)}\gamma(R_2S_3+S_2R_3)\Big\}+(P\omega\gamma\overline{F(A)}-2\mu g),\end{aligned}$$

where $X_1 = PQF(A) + P\int_A^u(\alpha(p_0,\omega,g) - b\epsilon)f(\epsilon)d\epsilon + P\omega\beta\overline{F(A)} - 2\lambda\omega$. The optimal decisions can be obtained from the first order conditions. The complicated form of the manufacturer's profit function restricts us from establishing analytically the second order conditions for optimality. Nevertheless, the existence of the optimal solution will be numerically verified in Section 7.1.4. An algorithm is provided below to obtain the optimal values of the decision variables.

Algorithm

Step 1: Set $\epsilon = 0.01$, $\omega = 0$, $w_m = c_m$, $g = 0$, $\Pi_m^* = 0$, $\Pi_r^* = 0$, and go to Step 2.
Step 2: Set predetermined values of Ω, G and w_m, and go to step 3.
Step 3: Set $w_m = w_m + \epsilon$, and go to step 4.
Step 4: Set $g = g + \epsilon$, and go to step 5.
Step 5: Set $\omega = \omega + \epsilon$, and go to step 6.
Step 6: Obtain unique Q and p_0 (by virtue of propositions 7.3 and 7.4), and calculate Π_m and Π_r. If $\Pi_m^* < \Pi_m$, set $\Pi_m^* = \Pi_m$, $\Pi_r^* = \Pi_r$, $w_m^* = w_m$, $\omega^* = \omega$, $g^* = g$, $Q^* = Q$ and $p^* = p_0$. Go to step 7.
Step 7: If $w_m < W_m$ and $g < G$ and $\omega < \Omega$, go to step 5; if $w_m < W_m$ and $g < G$ and $\omega \geq \Omega$, set $\omega = 0$ and go to step 4; if $w_m < W_m$ and $g \geq G$, set $g = 0$, $\omega = 0$ and go to step 3; else print w_m^*, g^*, ω^*, p_0^*, and Q^* as optimal values of the decision variables, and Π_r^* and Π_m^* as optimal profits of the manufacturer and the retailer, respectively.

The following proposition exhibits the relationship of the optimal retail price with the salvage value.

Proposition 7.5. *Optimal retail price decreases with salvage price, i.e.* $\frac{\partial p_0}{\partial c_d} < 0$.

Proof:

$$\frac{\partial p_0}{\partial c_d} = \frac{U_2U_6 - U_5U_3}{U_1U_5 - U_4U_2} = \frac{-F(A)\overline{F(A)}}{F^2(A) - 2bU_2} < 0,$$

where $U_1 = (A + p_0 + c_s - c_d)f(A) - F(A)$; $U_2 = \frac{Af(A)}{b} + \frac{(p_0+c_s-c_d)}{b}f(A)$; $U_3 = F(A) - 1$; $U_4 = (a + \beta\omega + \gamma g + bc_s - bc_d - Q)f(A) + 2b\overline{F(A)}$; $U_5 = \frac{(a+\beta\omega+\gamma g+bc_s-bc_d-Q)f(A)}{b} - F(A)$; $U_6 = -b\overline{F(A)}$.

As is seen from the above proposition, the price only contract compels the retailer to solely bear the risk of over- and under-stocking- occurring due to randomness in price- thereby generating double marginalization effect. If the risk is partly shared by the manufacturer, the retailer will naturally be seeking to reserve more amount. In addition, products can be sold at a lesser retail price which may be achieved through higher salvage value. The finding paves the way to implement the buyback contract.

7.1.3.3 Buyback contract

In a buyback contract, the manufacturer takes back the unsold amount from the retailer by paying a price c_b per unit, and thus partially bears the risk due to uncertainty in price. The expected profit of the retailer under buyback contract is thus

$$\begin{aligned}\Pi_r^b(p_0, Q) &= E[(p_0 + \epsilon)\min\{Q, D\}] - w_m Q + c_b E[(Q - D)^+] \\ &\quad - c_s E[(D - Q)^+]. \end{aligned} \tag{7.7}$$

Obviously $c_b > c_d$ so as to make the retailer interested in the contract rather than selling the product at salvage value. It is also to be noted that $c_b = w_m$ would imply that the manufacturer himself bears the entire risk of uncertainty, leading to worse result for him. A feasible range is thus obtained as $c_d < c_b < w_m$; the exact rate depends on the bargaining power of the entities. The following proposition (analogous to Proposition 7.3) ensures existence of optimal decisions for the retailer under this contract.

Proposition 7.6. *The retailer's expected profit function Π_r^b is jointly concave in Q and p_0, and optimal decision variables Q^b and p_0^b satisfy*

$$\int_l^{A^b} \epsilon f(\epsilon) d\epsilon + (p_0^b + c_s - c_b) F(A^b) = w_m^b - c_d, \quad (7.8)$$

$$\text{and} \quad \int_{A^b}^u (2b\epsilon + bp_0^b - bc_b - \alpha(p_0^b, \omega^b, g^b)) f(\epsilon) d\epsilon = (Q^b + bc_s) F(A^b). \quad (7.9)$$

The manufacturer sells the unsold amount received from the retailer in the secondary market, so that his expected profit becomes

$$\begin{aligned} \Pi_m^b(w_m, \omega, g) &= w_m E[\min Q, D] - (c_b - c_d) E[(Q - D)^+] - c_m Q + (w_m' - c_{r_1}) \eta E(D') \\ &\quad - c_m \eta E(D') - (1 - \eta) E(D') c_{r_2} - \lambda \omega^2 - \mu g^2. \end{aligned} \quad (7.10)$$

Again, the complicated form of the profit function 7.10 restricts us from deriving any analytical result for the manufacturer. The applicability of the contract is demonstrated numerically in the next section.

7.1.4 Numerical illustration

The following parameter-values are taken from Giri et al., 2018: $a = 65$ units/-month, $b = 0.11$ units/\$/month, $\beta = 0.94$, $\gamma = 0.32$ units/month, $\delta = 0.001$, $c_m = \$60$/unit, $c_{r_1} = \$45$/unit, $c_{r_2} = \$25$/unit, $w_m' = \$70$/unit, $\mu = 20$, $\lambda = 10$, $v = 3$, $z = 5$, and $\eta = 0.45$. Maintaining feasibility and compatibility of the business scenario, we additionally assume $c_d = \$40$/unit and $c_s = \$20$/unit. The randomness in price is assumed to be normally distributed with mean 0 and variance 1. Although the price is theoretically allowed to assume all values (even negative), the probability that the variable will assume values within the range $(-3, 3)$ is 0.997, so it seems justified to consider the distribution. With these values, optimal results are provided in Table 7.2. The table further illustrates how the total profit is enhanced through a buyback contract ensuring a win-win situation for both the channel members. The sensitivity analysis by allowing the value of one parameter to deviate while all other parameter values are unchanged is performed to examine the stability of the solution. The result is plotted in Figures 7.1-7.4. A robustness analysis is

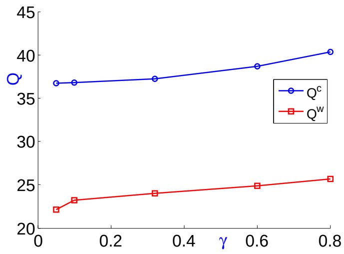
Table 7.2: Optimal results

Scenario	c_b ($/units)	w_m ($/unit)	p_0 ($/units)	Q (units)	ω (months)	g	Π_m ($)	Π_r ($)	$\Pi_m + \Pi_r$ ($)
centralized	-	-	396.78	37.17	15.73	2.683	-	-	9787.37
price-only contract	-	310	459.55	24	8.8	1.13	6393	1195	7588
buy back contract	60	316.6	452.3	24.68	8.9	1.18	6799	1265	8064
	80	321.2	447.6	26.21	9.35	1.26	6712	1324	8036
	100	329.8	438	28.9	10.1	1.41	6589	1399	7988
	120	342.4	431	30.2	10.88	1.63	6478	1447	7925
	140	364.7	426.1	30.8	11.45	1.76	6415	1495	7910

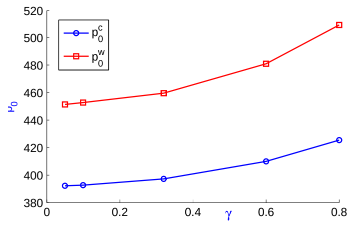
also described to extract managerial insights which are further summarized.

Sensitivity analysis

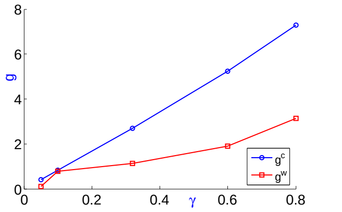
- *Effect of green investment:* In the absence of green investment, the optimal profit for the centralized model is derived as 9646.23$, and the decision variables as $p_0 = \$391.9$, $Q = 36.6$ units, $\omega = 15.5$ months. Investing for environmental reasons is thus proven to be a wise decision, both financially and environmentally. The best course of action is to boost green investment as consumer awareness towards eco-friendly products grows (Fig 7.1c). This will guarantee a profit gain (Fig 7.1e) that will permit a price increase without even jeopardizing demand. Increased green investment also gives the management freedom to extend the product's warranty (Fig 7.1d) which attracts more customers and positively influences profit margin. But, if the cost of becoming green exceeds a certain threshold, it would be wiser to make a compromise and settle for a product with a lower green level because the excess cost would make the product more expensive overall, something that most buyers would attempt to avoid. The similar trend could also be observed in the wholesale price only contract.



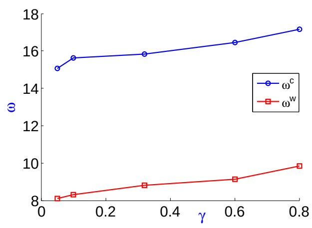
(a) γ vs Q



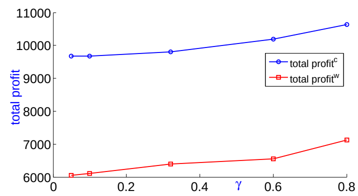
(b) γ vs p_0



(c) γ vs g



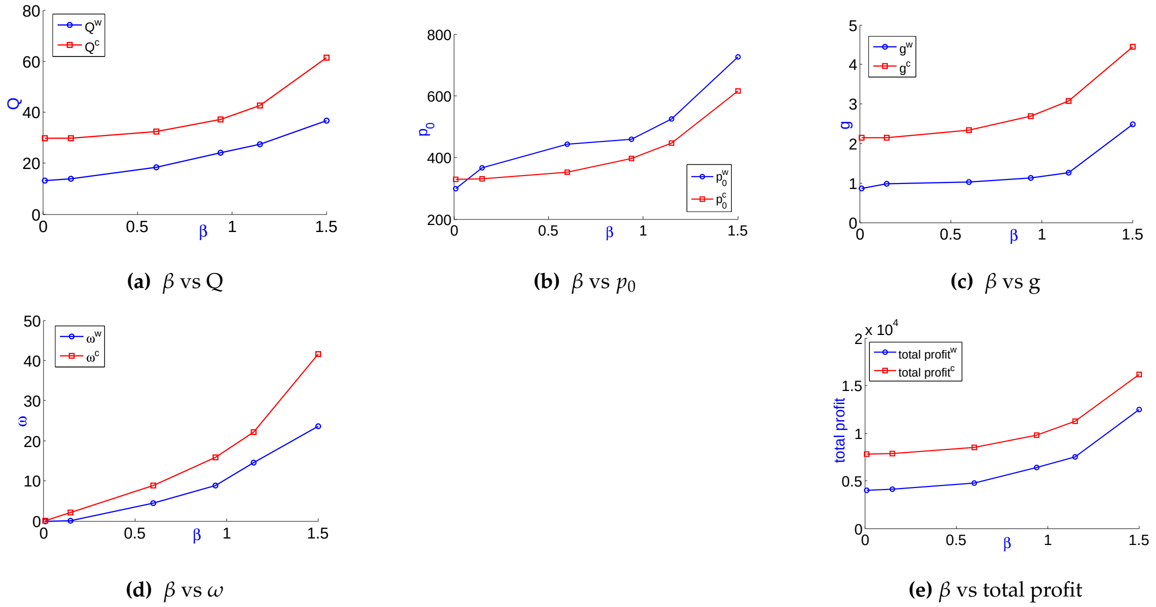
(d) γ vs ω



(e) γ vs total profit

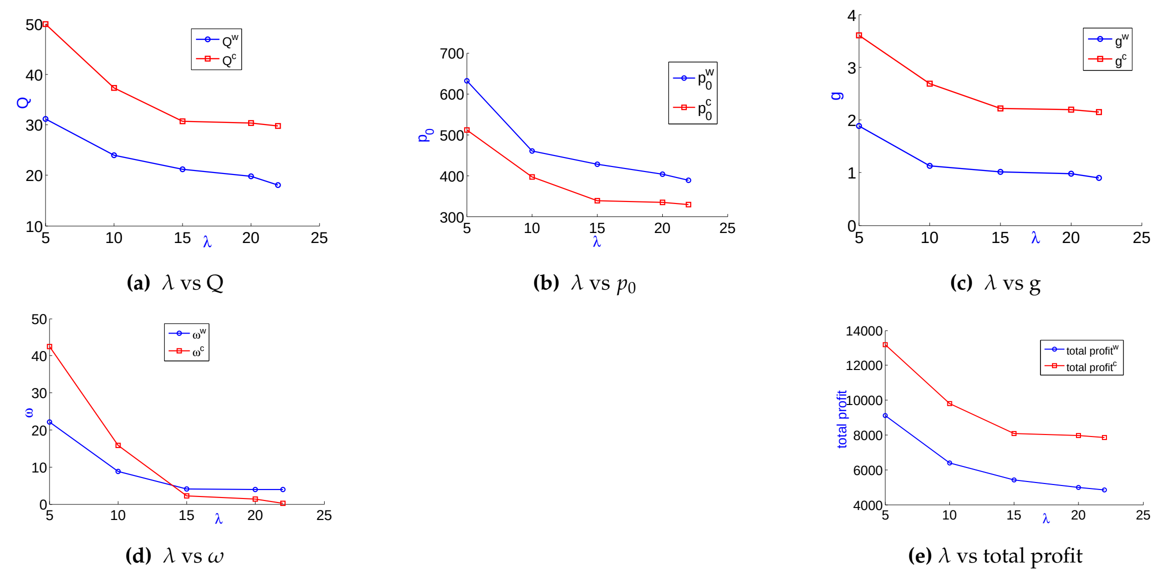
Figure 7.1: Sensitivity with respect to γ

- *Effect of warranty period:* In absence of warranty period, the optimal profit for the centralized model is derived as \$7812, and the decision variables as $p_0 = \$328.63$, $Q = 29.7$ units and $g = 2.14$. It may be argued that the warranty duration has a bigger influence on overall inventory choices and profit margins. The business manages should make investments for items that are more sensitive to the warranty duration (Fig 7.2d). On the other hand, it is acceptable to be cautious when it comes to items with short warranty impulses, since offering a longer warranty term would result in more costs and a higher product price, which buyers will reject in that case. For the wholesale price only contract, the retailer may be seen relatively to get more advantage from it than the manufacturer. An increased warranty period allows the price to increase faster (Fig 7.2b) as lengthier warranty periods attract the customers, letting the retailer free space for price allocation. Similar growth may also be observed for the greening level also (Fig 7.2c). With the same warranty sensitivity, however, it is imprudent to maintain the same warranty length as the warranty related cost goes up otherwise. When both the greening level and warranty are low, the retailer is forced to reduce the price to compel the customers to choose the product. All these factors ultimately affect the profit level of the supply chain (Fig 7.2e).



(a) β vs Q **(b)** β vs p_0 **(c)** β vs g **(d)** β vs ω **(e)** β vs total profit

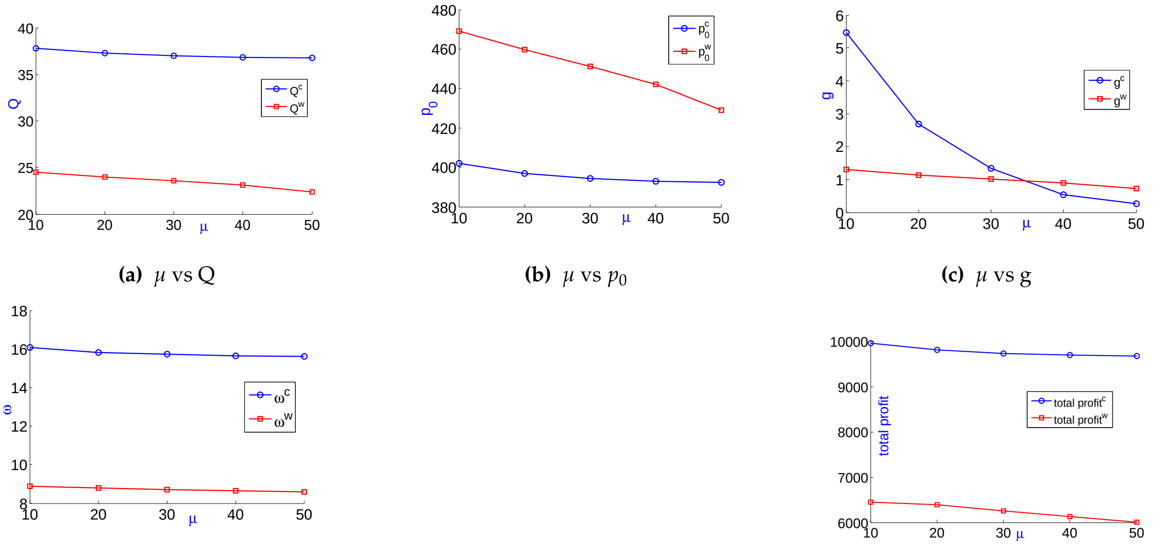
Figure 7.2: Sensitivity with respect to β



(a) λ vs Q **(b)** λ vs p_0 **(c)** λ vs g **(d)** λ vs ω **(e)** λ vs total profit

Figure 7.3: Sensitivity with respect to λ

- *Effect of warranty and greening cost:* Figures 7.3 and 7.4 provide a guideline about how the decision variables should be realigned with varying greening and warranty costs in both the centralized and decentralized cases. For higher values of λ, the price may be seen to have a steeper curve under wholesale price contract than the centralized one (Fig 7.3b), indicating that warranty cost, although solely bore by the manufacturer, has negative impact on retailer's revenue too. On the other hand, the warranty period may be witnessed to face more steep decline in the centralized one (Fig 7.3d). Overall business is negatively impacted (Fig 7.3a) along with other variables (Fig 7.3c) and the total profit (Fig 7.3e) is impacted in a similar manner that undergoes a downturn with the increment in this cost. For greening cost parameter μ, one may notice that the warranty period remains almost unaffected in both the centralized and decentralized cases (Fig 7.4d), although other decision variables including the total profit in the decentralized one are impacted more than the centralized ones. The greening investment undergoes sharper decline in the centralized case indicating the correlation between the two costs.



(a) μ vs Q **(b)** μ vs p_0 **(c)** μ vs g

(d) μ vs ω **(e)** μ vs total profit

Figure 7.4: Sensitivity with respect to μ

- *Effect of budget constraint:* For the centralized model, the total cost in greening and warranty is calculated as \$2651.74. Budget constraints on the sum of these two investments result in much lower profit level despite lower price. However, warranty investment may be seen as taking the lead due to the benefit of remanufacturing (Table 7.3). However, when the effect is further studied under same values of cost and demand controlling parameters (Table 7.4), it

Table 7.3: Effect of budget constraint (centralized results)

Budget ($)	Q	p_0	g	ω	Total Profit
500	-	-	-	-	-1480
1000	13.9	450	0.6	10	4170
1500	29	381.443	0.714	10.06	8928.81
2000	36.2	387.55	2.33	13.75	9772.7
2500	37	394.87	2.61	15.37	9707
2651.74	37.17	396.78	2.683	15.73	9787.37

is identified that there is indeed no discrimination on choosing one field of investment over others. It is the cost to return ratio that plays vital role in deciding where to invest more when there is fund crunch. Shrinking the budget for too much results in business loss which indicates a no business sign without enough capital.

Table 7.4: Effect of budget constraint under $\lambda = \mu = 20; \beta = \gamma = 0.9$ (centralized results)

Budget ($)	Q	p_0	g	ω	Total Profit
500	32.55	354.4	3.532	3.5395	9000
1000	33.87	366.4	4.99	5	9290
1500	34.88	375.5	6.117	6.13	9417
2000	35.74	383.3	7.06	7.079	9461.48
2134.82	35.95	385.2	7.297	7.313	9463

- *Effect of uncertainty in price:* Uncertainty in the retail price is seen to affect the supply chain profit in a negative way which is evidenced in Table 7.5. For the decentralized settings, the retailer is the one to deal with the customers directly. Thus the increasing randomness of price is seen to have more severe impact on the retailer's profit level in the decentralized case. Increasing the randomness further produces situations where the centralized case shows some robustness with the increased standard deviation of ϵ. So there are situations where the price only contract ceases to perform whereas the centralized one still works fine. This induces an incentive for the business members to come under some risk sharing contract when the uncertainty is fierce.

Table 7.5: Effect of randomness in price

s.d		Q	p_0	g	ω	w_m	Π_m	Π_r	Π
1	Centralized	37.17	396.78	2.69	15.73	x	x	x	9787
	Wholesale price only contract	24	459.55	1.13	8.8	310	5198	1195	6393
2	Centralized	28.01	406	0.914	5.35	x	x	x	2869
	Wholesale price only contract	20	569	0.35	1.34	376	812	324	1136
3	Centralized	19.53	431.36	0.333	1.95	x	x	x	764
	Wholesale price only contract								-ve
4	Centralized	12	487.6	0.138	0.81	x	x	x	198.6
	Wholesale price only contract								-ve
5	Centralized	-							-108
	Wholesale price only contract								-ve

Robustness analysis

Robustness with γ: For the centralized scenario, to conduct the robustness analysis of γ, the outcome metrics under consideration are Q, p_0, g, and ω along with the profit function Π_c. The mean values for these metrics are given by 37.75 for Q, 401.44 for p_0, 2.75 for g and 15.9 for ω, 9940.61 for Π_c. The standard deviations are calculated as 1.36 for Q, 12.33 for p_0, 2.67 for g and 0.67 for ω and 358.83 for Π_c. The coefficient of variations are 3.6% for Q, 3.07% for p_0, 97.14% for g, 4.23% for ω and 3.61% for Π_c. On the other hand, for the decentralized case, the retailer's and manufacturer's separate profit should also be taken into consideration. Here, the mean values for these metrics are given by 23.98 for Q, 470.78 for p_0, 315.2 for w, 1.41 for g and 8.84 for ω, 5259.2 for Π_m, 1186.6 for Π_r and 6445.8 for Π. The standard deviations are calculated as 1.26 for Q, 21.92 for p_0, 21.47 for w, 1.04 for g, 0.63 for ω, 322.02 for Π_m, 67.14 for Π_r and 384.27 for Π. The coefficient of variations are 5.26% for Q, 4.66% for p_0, 6.81% for w, 73.83% for g and 7.08% for ω, 6.12% for Π_m, 5.66% for Π_r and 5.96% for Π. As can be seen from the robustness result, the green investment is highly sensitive towards the green sensitivity of the customers whereas all the other parameters are remaining robust along with the profit function for both the centralized and decentralized case.

Robustness with μ: The mean values of the metrics under consideration are derived as 37.05 for Q, 395.17 for p_0, 1.71 for g, 15.75 for ω, and 9757.17 for Π_c. The standard deviations are calculated as 0.38 for Q, 3.44 for p_0, 1.89 for g, 0.16 for ω and 100.41 for Π_c. The coefficient of variations are 1.01% for Q, 0.88% for p_0, 110.19% for g and 1.02% for ω and 1.03% for Π_c. For the decentralized case, the mean values for the metrics are derived as 23.52 for Q, 450.13 for p_0, 307.46 for w, 1.01 for g, 8.73 for ω, 5084.8 for Π_m, 1161.2 for Π_r and 6246 for Π. The standard deviations are calculated as 0.71 for Q, 13.84 for p_0, 14.27 for w, 0.2 for g and 0.1 for ω, 126.6 for Π_m, 38.7 for Π_r and 165.26 for Π. The coefficient of variations are 3.06% for Q, 3.08% for p_0, 4.64% for w, 19.9% for g, 1.18% for ω, 2.49% for Π_m, 3.33% for Π_r and 2.64% for Π. The model is seen to be robust with respect to green cost coefficient; a volatile cost coefficient affect the greenness level of the product.

Robustness with β: The mean values for the metrics are given by 38.87 for Q, 411.61 for p_0, 2.81 for g and 15.1 for ω, 10235.99 for Π_c. The standard deviations are calculated as 11.04 for Q, 100.02 for p_0, 0.8 for g, 14.08 for ω and 2918.82 for Π. The coefficient of variations are 28.4% for Q, 24.30% for p_0, 28.55% for g, 93.31% for ω and 28.52% for Π_c. On the other hand, the mean values for these metrics in decentralized case are given by 22.26 for Q, 469.54 for p_0, 327.5 for w, 1.29 for g, 8.57 for ω, 5332 for Π_m, 1214.5 for Π_r and 6546.5 for Π. The standard deviations are calculated as 8.21 for Q, 135.23 for p_0, 104.31 for w, 0.55 for g and 8.41 for ω, 2277.05 for Π_m, 665.33 for Π_r and 2933.68 for Π. The coefficient of variations are 36.87% for Q, 28.8% for p_0, 31.85% for w, 42.54% for g, 98.04% for ω, 42.71% for Π_m, 54.78% for Π_r and 44.81% for Π. The robustness results indicate that all business factors are highly sensitive to the warranty period. The warranty is thus proven to be a critical parameter in determining the order quantity. The retailer's profit is also significantly affected by the warranty cost beyond the manufacturer's profit.

Robustness with λ: The mean values for the metrics under consideration are 35.62 for Q, 382.18 for p_0, 2.57 for g, 12.42 for ω, and 9377.8 for Π_c. The standard deviations are calculated as 7.68 for Q, 69.43 for p_0, 0.56 for g, 16.1 for ω, and 2030.18 for Π_c. The coefficient of variations are 21.58% for Q, 18.17% for p_0, 21.59% for g, 129.64% for ω and 21.65% for Π_c. For the decentralized case, the mean values under consideration are given by 22.86 for Q, 462.39 for p_0, 360.08 for w, 1.18 for g, 8.57 for ω, 5080.8 for Π_m, 1070.8 for Π_r and 6151.6 for Π. The standard deviation are calculated as 4.6 for Q, 19.05 for p_0, 41.77 for w, 0.36 for g and 7.03 for ω, 1136.02 for Π_m, 440.16 for Π_r and 1573.48 for Π. The coefficient of variations are 20.12% for Q, 19.06% for p_0, 11.60% for w, 30.52% for g, 82.05% for ω, 22.36% for Π_m, 41.11% for Π_r and 25.58% for Π. The robustness analysis of warranty cost reveals that adjustments to greening efforts and order quantities are highly sensitive to this parameter, as is the profit of supply chain members. These aspects will be thoroughly addressed in the managerial insights section.

Managerial insights

This subsection delivers critical managerial insights drawn from the analysis, offering practical implications for real-world applications. These findings can be effectively utilized in green supply chains and provide valuable guidance for non-green supply chain managers. They help pinpoint scenarios where transitioning to a green supply chain or investing in remanufacturing would be advantageous. The insights assist managers in fine-tuning decision variables to maximize profitability, streamline production, and bolster sustainability.

- Sensitivity analyses of parameters λ (warranty cost) and μ (greening cost) highlight the importance of adaptive strategies. Managers should continuously monitor cost changes and adjust investment levels in greening and warranty policies to align with shifting consumer demand and operational capabilities.
- Since green investment is highly influenced by customers' green sensitivity, business managers should prioritize green marketing or, more specifically, green awareness campaigns to strike a balance between environmental and economic factors.
- When constrained by budgets, managers must evaluate the cost-to-return ratio of greening and warranty investments. A strategic allocation based on marginal returns ensures optimal use of limited resources while avoiding excessive cost burdens that could hinder profitability.
- Price randomness negatively impacts profits, especially for retailers in decentralized scenarios. Managers should adopt centralized decision-making in volatile markets or implement contracts that include risk-sharing mechanisms to protect stakeholders from excessive losses.

7.1.5 Concluding remarks

Addressing random pricing problems is a pertinent issue in today's business scenario. Coupled with customers' inclination towards sustainability and dependence on warranty, the problem needs to be analyzed in order to sustain in competitive business. This chapter paves a way towards providing optimal results and analyzing a closed-loop supply chain with price, greening and warranty period dependent demand where the price of the product is considered to have a random component in it. A random return rate during warranty period is considered as well. Both the centralized and decentralized models are discussed, where it is established that the coordination generates more profit. Based on the relationship between price and salvage value, a risk sharing buyback contract is proposed to ensure win-win situation. From the numerical section it is clear that investing in environmental cause and providing warranty service help to enhance supply chain profit. It is also identified how the manager should act when there is a budget constraint. The robustness analysis highlights the pressing necessity for a comprehensive and well-structured greenness awareness campaign targeted at customers. This underscores the importance of fostering a deeper understanding and commitment among consumers toward sustainable and eco-friendly practices. Such a campaign would aim to not only inform but also inspire action, ensuring that the principles of environmental responsibility resonate strongly with the customer base. An increased uncertainty in price affects the profit in a negative way, and after a certain limit, the price only contract ceases to perform. This establishes the necessity of having a risk-sharing contract implemented to sustain in adverse business situations. The analysis provides valuable managerial insights to follow.

The current work can be extended in many ways. Production yield has been assumed to be deterministic here, which can be generalized to incorporate random yield. Considering different qualities for returned items and making the remanufacturing cost a decision variable would also be worth studying. Competition among retailers may also be studied under the extended scenario. The issue of supply disruption coupled with random pricing is also an interesting but challenging task. Finally, the model may be extended to a multi-echelon supply chain.

7.2 Optimal Pricing and Remanufacturing Strategies under Carbon Emissions and Government Regulations

7.2.1 Introduction

Concerns about the environment are growing at the same rate now-a-days as business prosperity. The impact of business on the environment have become apparent more and more; the manufacturing, marketing, and sale of everything have an adverse effect on the environment. As mentioned by Jauhari et al., 2023, the manufacturing industry ranks among the highest in emissions. To help meet emission reduction target, governments have implemented various carbon policies, including carbon taxes, carbon caps, and cap-and-trade systems. For instance, in 2020, the Norwegian government raised its carbon tax by 8.6%, while the Portuguese government increased its carbon tax by 84.6% compared to 2019. In addition to other commercial factors, the business manager's concerns eventually expanded to include environmental problems as well. The concern initiated the process of remanufacturing items. Remanufacturing is not only motivated by just environmental considerations, though; as identified by Van and Van, 2018, economic factor also plays vital role while considering investment in it. In case of a bottle of shampoo, customer use it and throw it away when it becomes empty. However, with one's television, the customer goes to the store and often exchanges the older one while buying a new one. The marginal value of the used product in the first example is so low that most individuals are not even concerned about it in general. However, when environmental issues are considered, a planned remanufacturing of items with even lower marginal values adds some contribution to the environment as well. The process of remanufacturing the used goods always results in preserving some units of raw material which may be utilized for another project. In addition, the process of remanufacturing is also less expensive than new production (Mondal and Giri, 2020). Thus, it is worth examining how remanufacturing impacts the overall business picture. The process of remanufacturing is initiated with the collection of the used products. Not every unit of collected product can make it to the production facility; each item must first pass screening before the chosen things may proceed to

*This part of the chapter is based on the work published in *Journal of Cleaner Production* (2025), volume 486, article id 144523.

the next stage of the production process (Wei et al., 2018). While it is true that factors such as the quality of the raw materials and the conditions of the production facility affect how much a product can be manufactured (or remanufactured), it is also true that an inherent randomness known as 'random yield' exists in the production process which results in variability in the actual volume of output or production (Jones et al., 2001). Depending on the business environment, the manufacturer may deliver simply the items which are produced, or in certain business situations, the manufacturer is required by business contract to supply the pre-specified quantity. Since the primary event that determines all other product-related regulations is production, the complete system is somewhat impacted by this production instability. Following manufacture, the goods are on display in the marketplace before being purchased by clients. Unsold items stored in the inventories may further be remanufactured in the subsequent period. The salvage goods will be sold as scrap at the end of the second phase. Given that the circumstances in the two periods differ, pricing is likewise allowed to change in two periods, allowing the business owner greater flexibility to adjust to the altered circumstances.

Motivation of the current work

The literature review inspires to consider a closed-loop stochastic supply chain with remanufacturing and environment consideration. The model should be made as close to real life as possible: regarding supply, it should consider random yield in production; the pricing should be considered random as well; remanufacturing should be adopted to save cost; and environment issues should be considered to reduce carbon footprint and make sustainable supply chain. Although there are number of studies on closed-loop supply chains and remanufacturing, no closed-loop supply chain has yet taken the unpredictability of pricing strategy into consideration. Articles such as Mondal and Giri, 2020 and Das Roy and Sana, 2017 considering random pricing assumed that the randomness follows a uniform distribution which appears to be an unrealistic assumption. In reality, the actual price fluctuates around a predetermined value set by the retailer. To represent the scenario, the current work chooses the pricing technique to be of hybrid type in nature: the stochastic component following any reasonable distribution pattern while the deterministic part may be optimized by the business managers. Further, most of the closed-loop models exclude the remanufacturing cost to be a decision variable. Remanufacturing is less expensive than new manufacturing, but investment

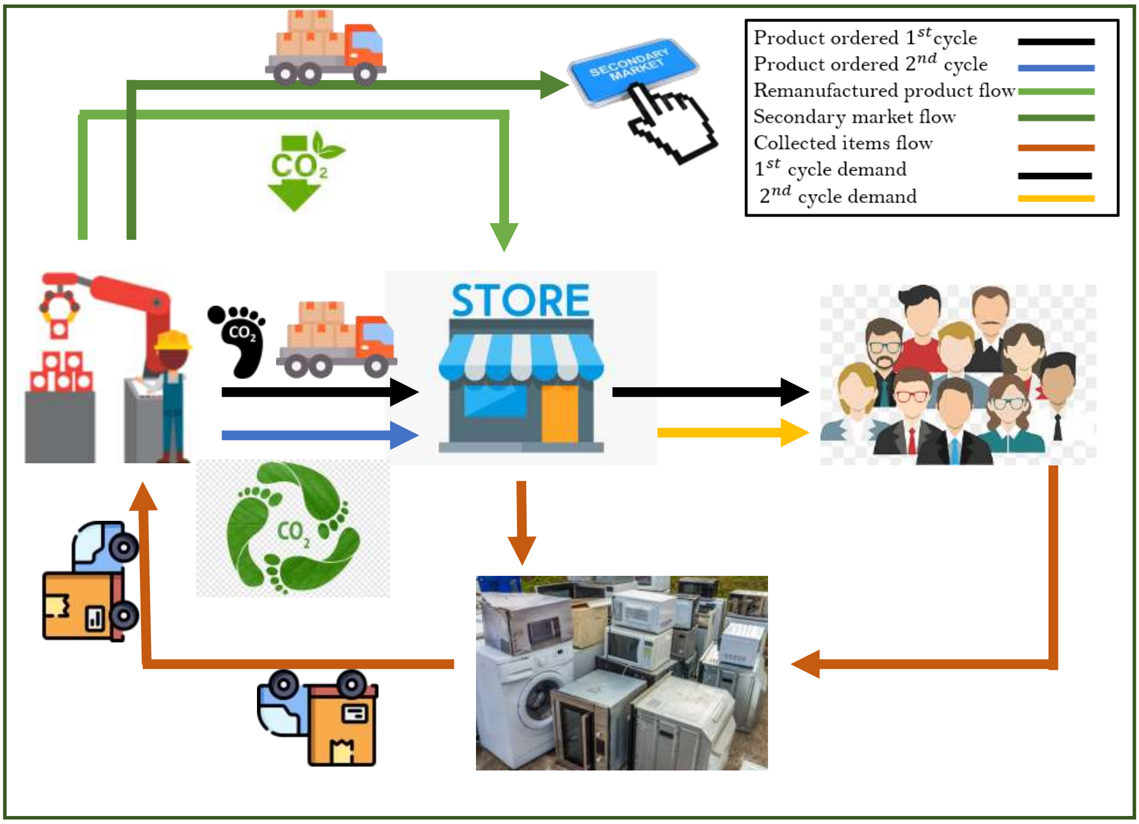


Figure 7.5: Schematic diagram of the model

in remanufacturing and associated activities such as product collection, screening, packaging, and delivering back to factories, acceptance by the customers, *etc* is an important factor to consider by the companies while maximizing total profit. It is reasonable to assume that the manufacturer has the power to decide how much he should invest in remanufacturing, based on the above-mentioned aspects. A sold item cannot be picked up immediately after it is sold; it is thus justified to consider two periods contemplated to precisely investigate the scenario. The products sold in the initial phase are made using fresh raw materials. Throughout this period, the sold goods are also collected. Consequently, in the second period, both newly made and remanufactured items are included in the sold items. The work thus develops a two-period supply chain where after selling all the fresh items in the first period, a portion of the used items are collected, scrutinized, remanufactured, and finally sold in the second period along with some freshly produced items. The process of remanufacturing not only reduces the production cost but also has an impact on the environment as well. To study the effect on environment and associated government policies on production process, the carbon emission associated with the production is considered. To move towards an environment-friendly business scenario, the greening investment to reduce carbon emissions is also considered. A carbon cap-and-trade policy is considered to restrict environmental pollution. To examine the impacts of environmental consideration on business dynamics, two separate models- with and without environmental consideration have been developed and the results are compared. The effect of carbon emission in the absence of green investment is also discussed, thus providing a complete view of the associated scenario. In light of the carbon emissions resulting from both newly produced and remanufactured items, the chapter concludes by discussing the environmental impact of remanufacturing which generally is the primary concern of the consumers no less than other attributes. The work aims to address following research questions:

RQ1: What will be the business dynamics with the randomness in pricing? How does the introduction of randomness in pricing influence demand and revenue in a closed-loop supply chain? What strategies can businesses employ to manage the impact of price variability on customer behavior and market stability?

RQ2: What ordering and pricing strategies should be adopted in presence of random yield in production? How the remanufacturing strategy is affected by yield variations?

RQ3: What changes in business policies should be adopted if the carbon emission factor along with government regulations is considered? How does the inclusion of carbon emission reduction in the cost-benefit analysis of remanufacturing affect decision-making processes? How do carbon emission considerations influence the overall sustainability and economic viability of remanufacturing operations? What are the potential trade-offs between economic gains and environmental impact while prioritizing green technology in remanufacturing?

RQ4: What are the key constraints affecting the remanufacturing process, and how do they impact operational efficiency and output quality?

The contribution of the current work in the field of supply chain research is manifold. Firstly, this is the first work in the closed-loop scenario that takes into account the randomness in the pricing strategy. Secondly, it considers investment in remanufacturing to be a decision variable instead of just investing a fixed amount for remanufacturing. Thirdly, investment is made to reduce carbon emission in the closed-loop supply chain. Finally, consideration of random yield makes the work worth finding a position in the existing literature.

7.2.2 Notations and assumptions

The following notations have been used throughout this chapter.

Table 7.6: Notations

τ	:	rate of old goods collected, $0 \leq \tau \leq 1$ (decision variable)
R_i	:	planned production quantity in period i, $i = 1, 2$ (decision variables)
Q_i	:	order quantity in period i, $i = 1, 2$ (decision variables)
c_r	:	per unit remanufacturing cost (decision variable)
G_i	:	green investment in the ith cycle, $i = 1, 2$ (decision variables)
p_{ri}	:	deterministic part of retail price in period i, $i = 1, 2$ (decision variables)
p_i	:	random selling price in period i, $i = 1, 2$
$D(p_i)$	:	price dependent demand rate
c_m	:	per unit fresh production cost of the manufacturer
w_R	:	unit salvage value for the items which could not be remanufactured
c_{sr}	:	unit shortage cost for the retailer
c_{ms}	:	price of the finished product at secondary market
ϵ	:	randomness related to the price with pdf $f(\epsilon)$
y	:	randomness related to the production with pdf $h(y)$
e_n	:	per unit carbon emission due to manufacturing
e_r	:	per unit carbon emission due to remanufacturing
c_e	:	per unit carbon emission cost
V	:	carbon cap declared by the Government
$R(G_i)$	:	reduction in carbon emission with G_i investment in cycle i

The following assumptions are made to formulate the model for this section.

- A two-echelon closed-loop supply chain is developed in the work. Production process is subject to random yield; for planned production R, the actual produced amount is yR, y being the associated randomness with pdf $h(y)$. When the yield is lower than the ordering quantity, the shortfall is managed by collecting it from outside at a higher rate c_{ms}. When the yield is higher than the ordering quantity, the remaining items are stored and undergone remanufacturing before selling it in the next period.

- Following Mondal and Giri, 2020, the demand is assumed to depend linearly on selling price and is specified by $D_i = a - bp_i, i = 1, 2$, where a and b denotes base price and price sensitivity respectively. The retail price is assumed to be random in nature with a deterministic decisive component, added a random part to it. The pricing is of the form $p_i = p_{ri} + \epsilon$, where ϵ follows pdf $f(\epsilon)$.

- Due to the closed loop supply chain, here, the possibility of product returns is taken into account. Returned amount is a fraction τ of the sold items in 1st period. $\tau = 0$ or 1 indicate no or full return, respectively. The associated expenses to ensure τ fraction of return is $H\tau^2$, where $H > 0$ is a parameter, may be considered as a measure of customers' unawareness in storing used products and retailer's inefficiency in collecting used products.

- The manufacturer re-manufactures the collected products. However, only a fraction of the returned goods can be successfully reproduced by the manufacturer; the said fraction depends on the investment in remanufacturing cost of the product. To be precise, the fraction is $\sqrt{\frac{c_r}{c_m}}$; the more the investment, the more amount can be successfully remanufactured. Rest of the products are then sold at the salvage value w_R per unit.

- A fraction η of the remanufactured products are of the same quality as new ones and are sold as new products in the next period, while the remaining $1 - \eta$ fraction is of lower quality and is sold at secondary market at a reduced price. The products purchased in one cycle are collected in the next period.

- Fresh production emits more carbon than remanufacturing.

- Following Sepehri et al., 2021, we consider investment in green technology to reduce the carbon emission cost. For a G amount of green investment, the carbon reduction is $R(G) = \lambda\left(1 - e^{-mG}\right)$, where the technology's efficiency is denoted by m. In consistent with general intuition, $R'(G) > 0$ with $R(0) = 0$ and $R(G) \to \lambda$ as $G \to \infty$.

- Shortage cost is greater than the salvage value.

7.2.3 Model Formulation

Two models shall be developed and analyzed in this section in two different cases. The first case does not consider carbon emission issues while the second one considers it.

Model 1

The entire business cycle is divided in two periods. During the first period, the retailer sets the deterministic component of the retail price, predicts the demand and places an order of amount Q_1, based on which manufacturer plans his production quantity R_1, and is left with actually produced amount yR_1. Since he has to arrange the shortfall of production from outside sources, the expected sale is thus $E[S] = \min\{Q_1, a - b(p_{r1} + \epsilon)\}$, and expected revenue is $E[(p_{r1} + \epsilon)\min\{Q_1, a - b(p_{r1} + \epsilon)\}]$. Combining with shortage, production and purchase cost from secondary market, the expected profit in the first cycle is derived as

$$\begin{aligned}\Pi_1(p_{r1}, Q_1, R_1) \quad = \quad & \int_l^{A_1} (p_{r1} + \epsilon)Q_1 f(\epsilon)d\epsilon + \int_{A_1}^{u} (p_{r1} + \epsilon)\big(\alpha(p_{r1}) - b\epsilon\big) f(\epsilon)d\epsilon \\ & -c_m R_1 - c_{sr}\int_l^{A_1} \big(\alpha(p_{r1}) - b\epsilon - Q_1\big) f(\epsilon)d\epsilon \\ & -c_{ms}\int_s^{\frac{Q_1}{R_1}} (Q_1 - yR_1)h(y)dy, \qquad (7.11)\end{aligned}$$

where $\alpha(z) = a - bz$, and $A_1 = \frac{a - bp_{r1} - Q_1}{b}$. Following proposition ensures the existence of unique optimal solution that maximizes the expected profit.

Proposition 7.7. *Π_1 is jointly concave in p_{r1}, Q_1, and R_1.*

Proof:

We have, from 7.11, $\frac{\partial \Pi_1}{\partial p_{r1}} = Q_1 F(A_1) + \int_{A_1}^{u} (a - 2bp_{r1} - 2b\epsilon) f(\epsilon)d\epsilon + bc_{sr}F(A_1)$, $\frac{\partial^2 \Pi_1}{\partial p_{r1}^2} =$

$-\left[(a+bc_{sr}-Q_1)f(A_1)+2b\overline{F(A_1)}\right]<0$, $\frac{\partial \Pi_1}{\partial Q_1}=\int_l^{A_1}(p_{r1}+\epsilon)f(\epsilon)d\epsilon+c_{sr}F(A_1)-c_{ms}H\left(\frac{Q_1}{R_1}\right)$, $\frac{\partial^2 \Pi_1}{\partial Q_1^2}=-\frac{1}{b^2}(a+bc_{sr}-Q_1)f(A_1)-\frac{c_{ms}}{R_1}h\left(\frac{Q_1}{R_1}\right)<0$, $\frac{\partial \Pi_1}{\partial R_1}=-c_m+c_{ms}\int_s^{\frac{Q_1}{R_1}}yh(y)dy$, $\frac{\partial^2 \Pi_1}{\partial R_1^2}=-c_{ms}\frac{Q_1^2}{R_1^3}h\left(\frac{Q_1}{R_1}\right)<0$, $\frac{\partial^2 \Pi_1}{\partial R_1\partial Q_1}=\frac{\partial^2 \Pi_1}{\partial Q_1\partial R_1}=c_{ms}\left(\frac{Q_1}{R_1^2}\right)h\left(\frac{Q_1}{R_1}\right)$, $\frac{\partial^2 \Pi_1}{\partial R_1\partial p_{r1}}=\frac{\partial^2 \Pi_1}{\partial p_{r1}\partial R_1}=0$, and $\frac{\partial^2 \Pi_1}{\partial p_{r1}\partial Q_1}=\frac{\partial^2 \Pi_1}{\partial Q_1\partial p_{r1}}=-\left(\frac{a-Q_1}{b}+c_{sr}\right)f(A_1)+F(A_1)$, so that from the Hessian matrix $\begin{bmatrix}\frac{\partial^2 \Pi_1}{\partial Q_1^2} & \frac{\partial^2 \Pi_1}{\partial Q_1\partial p_{r1}} & \frac{\partial^2 \Pi_1}{\partial Q_1\partial R_1}\\ \frac{\partial^2 \Pi_1}{\partial p_{r1}\partial Q_1} & \frac{\partial^2 \Pi_1}{\partial p_{r1}^2} & \frac{\partial^2 \Pi_1}{\partial p_{r1}\partial R_1}\\ \frac{\partial^2 \Pi_1}{\partial R_1\partial Q_1} & \frac{\partial^2 \Pi_1}{\partial R_1\partial p_{r1}} & \frac{\partial^2 \Pi_1}{\partial R_1^2}\end{bmatrix}$, it is derived that $\frac{\partial^2 \Pi_1}{\partial Q_1^2}<0$, $\begin{vmatrix}\frac{\partial^2 \Pi_1}{\partial Q_1^2} & \frac{\partial^2 \Pi_1}{\partial Q_1\partial p_{r1}}\\ \frac{\partial^2 \Pi_1}{\partial p_{r1}\partial Q_1} & \frac{\partial^2 \Pi_1}{\partial p_{r1}^2}\end{vmatrix}=\frac{1}{R_1}c_{ms}(a-Q_1+bc_{sr})f(A_1)h\left(\frac{Q_1}{R_1}\right)+\frac{2}{b}(a-Q_1+bc_{sr})f(A_1)+\frac{2b}{R_2}c_{ms}h\left(\frac{Q_1}{R_1}\right)\overline{F(A_1)}-F^2(A_1)>0$, and finally $\begin{vmatrix}\frac{\partial^2 \Pi_1}{\partial Q_1^2} & \frac{\partial^2 \Pi_1}{\partial Q_1\partial p_{r1}} & \frac{\partial^2 \Pi_1}{\partial Q_1\partial R_1}\\ \frac{\partial^2 \Pi_1}{\partial p_{r1}\partial Q_1} & \frac{\partial^2 \Pi_1}{\partial p_{r1}^2} & \frac{\partial^2 \Pi_1}{\partial p_{r1}\partial R_1}\\ \frac{\partial^2 \Pi_1}{\partial R_1\partial Q_1} & \frac{\partial^2 \Pi_1}{\partial R_1\partial p_{r1}} & \frac{\partial^2 \Pi_1}{\partial R_1^2}\end{vmatrix}=-c_{ms}\left\{\frac{2f(A_1)}{b}(a+bc_{sr}-Q_1)+F^2(A_1)\right\}\frac{Q_1^2}{R_1^3}h\left(\frac{Q_1}{R_1}\right)>0$, completing the proof.

The following proposition is straightforward from the above proposition, specifying the optimal values as follows.

Proposition 7.8. *The optimal values of p_{r1}, Q_1 and R_1 simultaneously satisfy*

$$(Q_1+bc_{sr})F(A_1)+(a-2bp_{r1})\overline{F(A_1)} = 2b\int_{A_1}^{u}\epsilon f(\epsilon)d\epsilon, \tag{7.12}$$

$$\int_l^{A_1}(p_{r1}+\epsilon)f(\epsilon)d\epsilon+c_{sr}F(A_1) = c_{ms}H\left(\frac{Q_1}{R_1}\right), \tag{7.13}$$

$$\textit{and} \int_s^{\frac{Q_1}{R_1}}yh(y)dy = \frac{c_m}{c_{ms}}. \tag{7.14}$$

Let us now look at the constituent components of the expected profit for second period. Note that the supply contract is the same as previous one, *i.e.* the manufacturer has to balance the shortfall of production from secondary market to meet the placed demand Q_2. Denoting $A_2 = \frac{a-bp_{r2}-Q_2}{b}$, the expected revenue by selling the items directly is derived as $E[\min\{Q_2, a-b(p_{r2}+\epsilon)\}] = \int_l^{A_2}(p_{r2}+\epsilon)Q_2 f(\epsilon)d\epsilon + \int_{A_2}^u (p_{r2}+\epsilon)(\alpha(p_{r2})-b\epsilon)f(\epsilon)d\epsilon$. Revenue earned by salvage consists of items supplied to retailer in second period but remained unsold, overproduced items in second period due to random yield, the portion of collected used items which was found inappropriate for remanufacturing after scrutiny, a fraction of unsold and overproduced products from first period, and the portion of collected used products which couldn't be remanufactured, *i.e.*

$$w_R \int_{A_2}^u (Q_2-\alpha(p_{r2})+b\epsilon)f(\epsilon)d\epsilon + w_R\int_{\frac{Q_2}{R_2}}^t (yR_2-Q_2)h(y)dy + w_R(1-\eta)\times \left[\tau\left\{\int_l^{A_1} Q_1 f(\epsilon) + \int_{A_1}^u(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon\right\} + \int_{\frac{Q_1}{R_1}}^t (yR_1-Q_1)h(y)dy + \int_{A_1}^u (Q_1-\alpha(p_{r1}) + b\epsilon)f(\epsilon)d\epsilon\right] + w_R\left(1-\sqrt{\frac{c_r}{c_m}}\right)\eta\left\{\tau\int_l^{A_1} Q_1 f(\epsilon)d\epsilon + \tau\int_{A_1}^u(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon + \int_{\frac{Q_1}{R_1}}^t (yR_1-Q_1)h(y)dy + \int_{A_1}^u(Q_1-\alpha(p_{r1})+b\epsilon)f(\epsilon)d\epsilon\right\}.$$

The manufacturer plans to produce total R_2 items out of which an amount is already available to him due to remanufacturing of both a fraction of unused items as well as collected used products of previous interval, so that the actual cost of production in second period is

$$c_m\left[R_2 - \sqrt{\frac{c_r}{c_m}}\tau\eta\left\{\int_l^{A_1} Q_1 f(\epsilon)d\epsilon + \int_{A_1}^u(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon\right\} - \eta\sqrt{\frac{c_r}{c_m}}\int_{\frac{Q_1}{R_1}}^t (yR_1-Q_1)h(y)dy - \eta\sqrt{\frac{c_r}{c_m}}\int_{A_1}^u (Q_1-\alpha(p_{r1})+b\epsilon)f(\epsilon)d\epsilon\right].$$

Cost to mitigate the shortfall between demand and production is $c_{ms}\int_s^{\frac{Q_2}{R_2}}(Q_2 - yR_2)h(y)dy$.

Investment in collection of used product is $H\tau^2$ while shortage cost is $c_{sr}\int_l^{A_2}(\alpha(p_{r2}) - b\epsilon - Q_2)f(\epsilon)d\epsilon$.

Finally, cost of remanufacturing the unused items and τ fraction of collected used items is

$$c_r\eta\left[\int_{A_1}^u(Q_1-\alpha(p_{r1})+b\epsilon)f(\epsilon)d\epsilon + \int_{\frac{Q_1}{R_1}}^t (yR_1-Q_1)h(y)dy + \tau\left\{\int_l^{A_1} Q_1 f(\epsilon)d\epsilon + \int_{A_1}^u(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon\right\}\right].$$

Summing up, the total expected profit in the second period is given by

$$\begin{aligned}
\Pi_2 \;=\; & \int_l^{A_2}(p_{r2}+\epsilon)Q_2 f(\epsilon)d\epsilon+\int_{A_2}^{u}(p_{r2}+\epsilon)(\alpha(p_{r2})-b\epsilon)f(\epsilon)d\epsilon \\
& +w_R\int_{A_2}^{u}(Q_2-\alpha(p_{r2})+b\epsilon)f(\epsilon)d\epsilon-H\tau^2-c_m\Bigg[R_2-\sqrt{\frac{c_r}{c_m}}\tau\eta\times \\
& \left\{\int_l^{A_1}Q_1 f(\epsilon)d\epsilon+\int_{A_1}^{u}(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon\right\}-\eta\sqrt{\frac{c_r}{c_m}}\times \\
& \int_{\frac{Q_1}{R_1}}^{t}(yR_1-Q_1)h(y)dy-\eta\sqrt{\frac{c_r}{c_m}}\int_{A_1}^{u}(Q_1-\alpha(p_{r1})+b\epsilon)f(\epsilon)d\epsilon\Bigg] \\
& +w_R\left(1-\sqrt{\frac{c_r}{c_m}}\right)\eta\Bigg\{\tau\int_l^{A_1}Q_1 f(\epsilon)d\epsilon+\tau\int_{A_1}^{u}(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon \\
& +\int_{\frac{Q_1}{R_1}}^{t}(yR_1-Q_1)h(y)dy+\int_{A_1}^{u}(Q_1-\alpha(p_{r1})+b\epsilon)f(\epsilon)d\epsilon\Bigg\} \\
& +w_R\int_{\frac{Q_2}{R_2}}^{t}(yR_2-Q_2)h(y)dy+w_R(1-\eta)\times\Bigg[\tau\Bigg\{\int_l^{A_1}Q_1 f(\epsilon) \\
& +\int_{A_1}^{u}(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon\Bigg\}+\int_{\frac{Q_1}{R_1}}^{t}(yR_1-Q_1)h(y)dy \\
& +\int_{A_1}^{u}(Q_1-\alpha(p_{r1})+b\epsilon)f(\epsilon)d\epsilon\Bigg]-c_{ms}\int_s^{\frac{Q_2}{R_2}}(Q_2-yR_2)h(y)dy \\
& -c_{sr}\int_l^{A_2}(\alpha(p_{r2})-b\epsilon-Q_2)f(\epsilon)d\epsilon-c_r\eta\int_{A_1}^{u}(Q_1-\alpha(p_{r1})+b\epsilon)f(\epsilon)d\epsilon \\
& -c_r\eta\int_{\frac{Q_1}{R_1}}^{t}(yR_1-Q_1)h(y)dy-c_r\eta\tau\Bigg\{\int_l^{A_1}Q_1 f(\epsilon)d\epsilon \\
& +\int_{A_1}^{u}(\alpha(p_{r1})-b\epsilon)f(\epsilon)d\epsilon\Bigg\}. \qquad (7.15)
\end{aligned}$$

Our next task is to establish the joint concavity of $\Pi_2(Q_2, R_2, p_{r2}, \tau, c_r)$ with respect to the decision variables. However, the expression is too complicated to derive the desired result directly, so it has been subdivided in two parts to be solved separately which is provided in the next proposition.

Proposition 7.9. *The expected profit $\Pi_2(Q_2, R_2, p_{r2}, \tau, c_r)$ satisfies the following:*

(a) *For given τ and c_r, Π_2 is jointly concave in Q_2, p_{r2} and R_2;*

(b) *For given Q_2, p_{r2} and R_2, Π_2 is jointly concave in τ and c_r.*

Proof:

(a) From equation 7.15, it is straightforward to derive that $\frac{\partial \Pi_2}{\partial p_{r2}} = Q_2 F(A_2) + \int_{A_2}^{u} (a - 2bp_{r2} - 2b\epsilon) f(\epsilon) d\epsilon + w_R b\overline{F(A_2)} + bc_{sr}F(A_2)$, so that $\frac{\partial^2 \Pi_2}{\partial p_{r2}^2} = -(a - Q_2 + bc_{sr} - bw_R) f(A_2) - 2b\overline{F(A_2)} < 0$; $\frac{\partial \Pi_2}{\partial Q_2} = \int_l^{A_2} (p_{r2} + \epsilon) f(\epsilon) d\epsilon + w_R \left(\overline{F(A_2)} - \overline{H\left(\frac{Q_2}{R_2}\right)} \right) - c_{ms} H\left(\frac{Q_2}{R_2}\right) + c_{sr}F(A_2)$, so that $\frac{\partial^2 \Pi_2}{\partial Q_2^2} = -\frac{1}{b^2} (a - Q_2 + bc_{sr} - bw_R) f(A_2) - \frac{1}{R_2}(c_{ms} - w_R) h\left(\frac{Q_2}{R_2}\right) < 0$; and $\frac{\partial \Pi_2}{\partial R_2} = -c_m + w_R \int_{\frac{Q_2}{R_2}}^{t} yh(y)dy + c_{ms} \int_s^{\frac{Q_2}{R_2}} yh(y)dy$, so that $\frac{\partial^2 \Pi_2}{\partial R_2^2} = -(c_{ms} - w_R) \frac{Q_2^2}{R_2^3} h\left(\frac{Q_2}{R_2}\right) < 0$. The matrix $K = \begin{bmatrix} \frac{\partial^2 \Pi_2}{\partial Q_2^2} & \frac{\partial^2 \Pi_2}{\partial Q_2 \partial p_{r2}} & \frac{\partial^2 \Pi_2}{\partial Q_2 \partial R_2} \\ \frac{\partial^2 \Pi_2}{\partial p_{r2} \partial Q_2} & \frac{\partial^2 \Pi_2}{\partial p_{r2}^2} & \frac{\partial^2 \Pi_2}{\partial p_{r2} \partial R_2} \\ \frac{\partial^2 \Pi_2}{\partial R_2 \partial Q_2} & \frac{\partial^2 \Pi_2}{\partial R_2 \partial p_{r2}} & \frac{\partial^2 \Pi_2}{\partial R_2^2} \end{bmatrix}$ now gives $|K_1| < 0$, $|K_2| = \begin{vmatrix} \frac{\partial^2 \Pi_2}{\partial Q_2^2} & \frac{\partial^2 \Pi_2}{\partial Q_2 \partial p_{r2}} \\ \frac{\partial^2 \Pi_2}{\partial p_{r2} \partial Q_2} & \frac{\partial^2 \Pi_2}{\partial p_{r2}^2} \end{vmatrix} = \frac{1}{R_2}(c_{ms} - w_R)(a - Q_2 + b(c_{sr} - w_R)) f(A_2) h\left(\frac{Q_2}{R_2}\right) + \frac{2}{b}(a - Q_2 + b(c_{sr} - w_R)) f(A_2) + \frac{2b}{R_2}(c_{ms} - w_R) h\left(\frac{Q_2}{R_2}\right) \overline{F(A_2)} - F^2(A_2) > 0$,

and $|K_3| = \begin{vmatrix} \frac{\partial^2 \Pi_2}{\partial Q_2^2} & \frac{\partial^2 \Pi_2}{\partial Q_2 \partial p_{r2}} & \frac{\partial^2 \Pi_2}{\partial Q_2 \partial R_2} \\ \frac{\partial^2 \Pi_2}{\partial p_{r2} \partial Q_2} & \frac{\partial^2 \Pi_2}{\partial p_{r2}^2} & \frac{\partial^2 \Pi_2}{\partial p_{r2} \partial R_2} \\ \frac{\partial^2 \Pi_2}{\partial R_2 \partial Q_2} & \frac{\partial^2 \Pi_2}{\partial R_2 \partial p_{r2}} & \frac{\partial^2 \Pi_2}{\partial R_2^2} \end{vmatrix}$
$= -(c_{ms} - w_R) \left\{ \frac{2f(A_2)}{b}(a + b(c_{sr} - w_R) - Q_2) + F^2(A_2) \right\} \frac{Q_2^2}{R_2^3} h\left(\frac{Q_2}{R_2}\right) > 0$.

(b) For given values of Q_2, R_2, and p_{r2}, we have
$\frac{\partial \Pi_2}{\partial \tau} = -2H\tau - c_m + \sqrt{c_r c_m} \eta E(S_1) + w_R \left(1 - \sqrt{\frac{c_r}{c_m}}\right) \eta E(S_1) + w_R(1 - \eta) E(S_1) - c_r \eta E(S_1)$, so that $\frac{\partial^2 \Pi_2}{\partial \tau^2} = -2H < 0$, and $\frac{\partial \Pi_2}{\partial c_r} = \frac{1}{2\sqrt{c_r c_m}} \Big[(c_m - w_R) \tau \eta E(S_1) + (c_m - w_R) \eta \int_{\frac{Q_1}{R_1}}^{t} (yR_1 - Q_1) h(y) dy + (c_m - w_R) \eta \times \int_{A_1}^{u} (Q_1 - a + bp_{r1} + b\epsilon) f(\epsilon) d\epsilon + c_e \Big\{ (e_n - e_r) \tau \eta \bar{y} E(S_1) - \eta \tau E(S_1)$, so that $\frac{\partial^2 \Pi_2}{\partial c_r^2} = -\frac{1}{4\sqrt{c_m}(c_r)^{\frac{3}{2}}} \Big[(c_m - w_R) \tau \eta E(S_1) + (c_m - w_R) \eta \int_{\frac{Q_1}{R_1}}^{t} (yR_1 - Q_1) h(y) dy + (c_m - w_R) \eta \int_{A_1}^{u} (Q_1 - a + bp_{r1} + b\epsilon) f(\epsilon) d\epsilon \Big] - \eta \tau E(S_1)$, and finally $\frac{\partial^2 \Pi_2}{\partial \tau \partial c_r} =$

$\frac{1}{2\sqrt{c_r c_m}}\Big\{(c_m - w_R) - 1\Big\}\eta E(S_1)$. It is now easy to derive that $\begin{vmatrix} \frac{\partial^2 \Pi_2}{\partial \tau^2} & \frac{\partial^2 \Pi_2}{\partial c_r \partial \tau} \\ \frac{\partial^2 \Pi_2}{\partial \tau \partial G_1} & \frac{\partial^2 \Pi_2}{\partial c_r^2} \end{vmatrix} > 0$, completing the proof.

The following algorithm is useful to obtain the optimal decision variables by virtue of the above proposition.

Algorithm

Step 1: Set $\delta = 0.001$, $p_{r2}^* = c_m$, $Q_2^* = 0$, $R_2^* = 0$, $\tau^* = 0$, $c_r^* = 0$ and go to Step 2.

Step 2: Set $p_{r2} = p_{r2}^*$, $Q_2 = Q_2^*$, $R_2 = R_2^*$, optimize Π_2 using any optimization technique to obtain values of τ and c_r, and go to step 3.

Step 3: If both $|\tau - \tau^*| < \delta$ and $|c_r - c_r^*| < \delta$, go to step 6, else $\tau^* = \tau$ and $c_r^* = c_r$ and go to step 4.

Step 4: Using τ^* and c_r^*, optimize Π_2 again to get new values of Q_2, R_2, p_{r2}, and go to step 5.

Step 5: If $|Q_2 - Q_2^*| < \delta$, $|R_2 - R_2^*| < \delta$ and $|p_{r2} - p_{r2}^*| < \delta$ simultaneously, go to step 6, else go to step 2.

Step 6: The values $p_{r2}^*, Q_2^*, R_2^*, c_r^*, \tau^*$ are optimal values of the decision variables.

Model 2: Consideration of carbon emission

The aim of this subsection is to extend the previous model by considering the effect of carbon emission from both manufacturing as well as remanufacturing. As mentioned in the notation table, e_n and e_r denote per unit carbon emission from manufacturing and remanufacturing respectively. Since remanufacturing harms the environment lesser, it is justified to assume that $e_n > e_r$. The company further invests G_i amount in ith period to reduce carbon emission during production which has a reduction fraction $R(G_i)$. There is a carbon cap V imposed on every company by the government such that if the total carbon emission in any business period crosses that threshold, the company has to pay a per-unit penalty for the extra emission; if the company emits lesser carbon, it does not need to pay anything, it rather may earn some revenue by selling the remaining carbon limit to other companies at the same rate. Figure 7.5 provides a schematic diagram of the model. Similar to case 1, the expected profit in the first cycle may be given by

$$\begin{aligned}\Pi_1^c(p_{r1}^c, Q_1^c, R_1^c, G_1) &= \int_l^{A_1^c}(p_{r1}^c+\epsilon)Q_1^c f(\epsilon)d\epsilon + \int_{A_1^c}^{u}(p_{r1}^c+\epsilon)(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon \\ &-c_{sr}\int_l^{A_1^c}(\alpha(p_{r1}^c)-b\epsilon-Q_1^c)f(\epsilon)d\epsilon - c_{ms}\int_s^{\frac{Q_1^c}{R_1^c}}(Q_1^c-yR_1^c)h(y)dy \\ &-c_m R_1^c - c_e e_n \bar{y} R_1^c\left[1-\lambda\left(1-e^{-mG_1}\right)\right] + c_e V - G_1. \qquad (7.16)\end{aligned}$$

The expected profit here consists of the same revenue and cost functions provided in model 1, along with costs and revenue related to reduced carbon emission, green investment, and utilization of carbon cap. The following proposition ensures the existence of unique optimal results which maximizes the expected profit function.

Proposition 7.10. *The expected profit $\Pi_1^c(Q_1^c, p_{r1}^c, R_1^c, G_1)$ satisfies the following:*

(a) *For given R_1^c and G_1, Π_1^c is jointly concave in Q_1^c and p_{r1}^c;*

(b) *For given Q_1^c and p_{r1}^c, Π_1^c is jointly concave in R_1^c and G_1 when* $G_1 > \frac{1}{m}\ln\left[\frac{R_1^{c2} c_e e_n \bar{y}\lambda}{c_{ms} Q_1^{c2} m h\left(\frac{Q_1^c}{R_1^c}\right)}\right]$.

Proof:

Part (a) follows from proposition 7.7.

For (b), $\frac{\partial \Pi_1^c}{\partial R_1^c} = -c_m + c_{ms}\int_s^{\frac{Q_1^c}{R_1^c}} yh(y) - c_e e_n \bar{y}\left\{1-\lambda\left(1-e^{-mG_1}\right)\right\}$, so that $\frac{\partial^2 \Pi_1^c}{\partial R_1^{c2}} = -c_{ms}\frac{Q_1^{c2}}{R_1^{c3}}h\left(\frac{Q_1^c}{R_1^c}\right) < 0$; and $\frac{\partial \Pi_1^c}{\partial G_1} = c_e e_n m \bar{y} R_1^c \lambda e^{-mG_1} - 1$, so that $\frac{\partial^2 \Pi_1^c}{\partial G_1^2} = -c_e e_n \bar{y} R_1^c \lambda m^2 \times e^{-mG_1} < 0$. Further, $\frac{\partial^2 \Pi_1^c}{\partial R_1^c \partial G_1} = c_e e_n m \bar{y}\lambda e^{-mG_1}$, so that for $K = \begin{bmatrix} \frac{\partial^2 \Pi_1^c}{\partial G_1{}^2} & \frac{\partial^2 \Pi_1^c}{\partial G_1 \partial R_1^c} \\ \frac{\partial^2 \Pi_1^c}{\partial R_1^c \partial G_1} & \frac{\partial^2 \Pi_1^c}{\partial R_1^{c2}} \end{bmatrix}$,

$|K_1| < 0$ and $|K_2| = c_e e_n \bar{y}\lambda e^{-mG_1}\left\{ m c_{ms}\frac{Q_1^{c2}}{R_1^{c2}}h\left(\frac{Q_1^c}{R_1^c}\right) - c_e e_n \bar{y}\lambda e^{-mG_1}\right\}$. In order to $|K_2| > 0$, the given condition has to be satisfied, completing the proof.

A similar algorithm may be provided to derive optimal values of the decision variables. Looking at the second period now, it is observed that the expected profit

in that period consists of same cost and revenue components provided in equation 7.15, along with components generated from consideration of reduced carbon emission, green investment, and carbon cap. Noting that the carbon emission related to newly produced items is
$e_n\bar{y}\Bigg\{R_2^c - \sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\Bigg\{\int_l^{A_1^c} Q_1^c f(\epsilon)d\epsilon + \int_{A_1^c}^u(\alpha(p_{r1}^c) - b\epsilon)f(\epsilon)d\epsilon\Bigg\}$
$-\eta\sqrt{\frac{c_r^c}{c_m}}\int_{\frac{Q_1^c}{R_1^c}}^t(yR_1^c - Q_1^c)h(y)dy - \eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^u(Q_1^c - \alpha(p_{r1}^c) + b\epsilon)f(\epsilon)d\epsilon\Bigg\}$, and related to remanufacturing is
$e_r\bar{y}\Bigg\{\sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\left\{\int_l^{A_1^c} Q_1^c f(\epsilon)d\epsilon + \int_{A_1^c}^u(\alpha(p_{r1}^c) - b\epsilon)f(\epsilon)d\epsilon\right\}$
$+\eta\sqrt{\frac{c_r^c}{c_m}}\int_{\frac{Q_1^c}{R_1^c}}^t(yR_1^c - Q_1^c)h(y)dy + \eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^u(Q_1^c - \alpha(p_{r1}^c) + b\epsilon)f(\epsilon)d\epsilon\Bigg\}$ which is further reduced to a fraction $[1 - R(G_2)]$ vide green investment G_2, the expected profit of the second cycle will be

$$\begin{aligned}
\Pi_2^c = & \int_l^{A_2^c}(p_{r2}^c+\epsilon)Q_2^c f(\epsilon)d\epsilon + \int_{A_2^c}^u(p_{r2}^c+\epsilon)(\alpha(p_{r2}^c) - b\epsilon)f(\epsilon)d\epsilon \\
& + w_R\int_{A_2^c}^u(Q_2^c - \alpha(p_{r2}^c) + b\epsilon)f(\epsilon)d\epsilon - H\tau_c^2 - c_m\left[R_2^c - \sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\,\times\right. \\
& \left\{\int_l^{A_1^c} Q_1^c f(\epsilon)d\epsilon + \int_{A_1^c}^u(\alpha(p_{r1}^c) - b\epsilon)f(\epsilon)d\epsilon\right\} - \eta\sqrt{\frac{c_r^c}{c_m}}\,\times \\
& \left.\int_{\frac{Q_1^c}{R_1^c}}^t(yR_1^c - Q_1^c)h(y)dy - \eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^u(Q_1^c - \alpha(p_{r1}^c) + b\epsilon)f(\epsilon)d\epsilon\right] \\
& + w_R\left(1 - \sqrt{\frac{c_r^c}{c_m}}\right)\eta\left\{\tau_c\int_l^{A_1^c} Q_1^c f(\epsilon)d\epsilon + \tau_c\int_{A_1^c}^u(\alpha(p_{r1}^c) - b\epsilon)f(\epsilon)d\epsilon\right. \\
& \left. + \int_{\frac{Q_1^c}{R_1^c}}^t(yR_1^c - Q_1^c)h(y)dy + \int_{A_1^c}^u(Q_1^c - \alpha(p_{r1}^c) + b\epsilon)f(\epsilon)d\epsilon\right\} + w_R\,\times \\
& \int_{\frac{Q_2^c}{R_2^c}}^t(yR_2^c - Q_2^c)h(y)dy + w_R(1-\eta)\left[\tau_c\left\{\int_l^{A_1^c} Q_1^c f(\epsilon) + \int_{A_1^c}^u(\alpha(p_{r1}^c) - b\epsilon)f(\epsilon)d\epsilon\right\}\right. \\
& \left. + \int_{\frac{Q_1^c}{R_1^c}}^t(yR_1^c - Q_1^c)h(y)dy + \int_{A_1^c}^u(Q_1^c - \alpha(p_{r1}^c) + b\epsilon)f(\epsilon)d\epsilon\right] - c_{ms}\times
\end{aligned}$$

$$\int_{s}^{\frac{Q_2^c}{R_2^c}}(Q_2^c-yR_2^c)h(y)dy-c_{sr}\int_{l}^{A_2^c}(\alpha(p_{r2}^c)-b\epsilon-Q_2^c)f(\epsilon)d\epsilon-c_r^c\times$$
$$\eta\int_{A_1^c}^{u}(Q_1^c-\alpha(p_{r1}^c)+b\epsilon)f(\epsilon)d\epsilon-c_r^c\eta\int_{\frac{Q_1^c}{R_1^c}}^{t}(yR_1^c-Q_1^c)h(y)dy-c_e\bar{y}\Bigg[e_n\Bigg\{R_2^c$$
$$-\sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\left\{\int_{l}^{A_1^c}Q_1^cf(\epsilon)d\epsilon+\int_{A_1^c}^{u}(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon\right\}$$
$$-\eta\sqrt{\frac{c_r^c}{c_m}}\int_{\frac{Q_1^c}{R_1^c}}^{t}(yR_1^c-Q_1^c)h(y)dy-\eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^{u}(Q_1^c-\alpha(p_{r1}^c)+b\epsilon)f(\epsilon)d\epsilon\Bigg\}$$
$$+e_r\Bigg\{\sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\left\{\int_{l}^{A_1^c}Q_1^cf(\epsilon)d\epsilon+\int_{A_1^c}^{u}(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon\right\}+\eta\sqrt{\frac{c_r^c}{c_m}}\int_{\frac{Q_1^c}{R_1^c}}^{t}(yR_1^c$$
$$-Q_1^c)h(y)dy+\eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^{u}(Q_1^c-\alpha(p_{r1}^c)+b\epsilon)f(\epsilon)d\epsilon\Bigg\}\Bigg]\left[1-\lambda\left(1-e^{-mG_2}\right)\right]$$
$$+c_eV-G_2-c_r^c\eta\tau_c\left\{\int_{l}^{A_1^c}Q_1^cf(\epsilon)d\epsilon+\int_{A_1^c}^{u}(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon\right\}. \tag{7.17}$$

Again, the complicated form restricted us to provide an analytical proof of joint concavity of the profit function. The following proposition however helps to some extend in that direction.

Proposition 7.11. *The expected profit* $\Pi_2(Q_2, R_2, p_{r2}, \tau, c_r, G_2)$ *satisfies the following:*

(a) *For given* τ*,* c_r*, and* G_2*,* Π_2 *is jointly concave in* Q_2*,* p_{r2} *and* R_2*;*

(b) *For given* Q_2*,* p_{r2}*,* R_2*, and* G_2*,* Π_2 *is jointly concave in* τ *and* c_r*.*

(c) *For given* Q_2*,* p_{r2}*,* R_2*,* τ *and* c_r*,* Π_2 *is concave in* G_2*.*

Proof:
Parts (a) and (b) are similar to those provided in proposition 7.9, hence skipped. For (c), note that

$$\frac{\partial\Pi_2^c}{\partial G_2} = mc_e\bar{y}\Bigg[e_n\Bigg\{R_2^c-\sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\left\{\int_{l}^{A_1^c}Q_1^cf(\epsilon)d\epsilon+\int_{A_1^c}^{u}(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon\right\}$$
$$-\eta\sqrt{\frac{c_r^c}{c_m}}\int_{\frac{Q_1^c}{R_1^c}}^{t}(yR_1^c-Q_1^c)h(y)dy-\eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^{u}(Q_1^c-\alpha(p_{r1}^c)+b\epsilon)f(\epsilon)d\epsilon\Bigg\}$$
$$+e_r\Bigg\{\sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\left\{\int_{l}^{A_1^c}Q_1^cf(\epsilon)d\epsilon+\int_{A_1^c}^{u}(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon\right\}+\eta\sqrt{\frac{c_r^c}{c_m}}\times$$

$$\int_{\frac{Q_1^c}{R_1^c}}^{t}(yR_1^c-Q_1^c)h(y)dy+\eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^{u}(Q_1^c-\alpha(p_{r1}^c)+b\epsilon)f(\epsilon)d\epsilon\Bigg\}\Bigg]\lambda e^{-mG_2}$$

-1, so that

$$\frac{\partial^2\Pi_2^c}{\partial G_2^2}=-m^2c_e\bar{y}\lambda e^{-mG_2}\Bigg[e_n\Bigg\{R_2^c-\sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\left\{\int_l^{A_1^c}Q_1^cf(\epsilon)d\epsilon+\int_{A_1^c}^{u}(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon\right\}$$

$$-\eta\sqrt{\frac{c_r^c}{c_m}}\int_{\frac{Q_1^c}{R_1^c}}^{t}(yR_1^c-Q_1^c)h(y)dy-\eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^{u}(Q_1^c-\alpha(p_{r1}^c)+b\epsilon)f(\epsilon)d\epsilon\Bigg\}$$

$$-e_r\Bigg\{\sqrt{\frac{c_r^c}{c_m}}\tau_c\eta\left\{\int_l^{A_1^c}Q_1^cf(\epsilon)d\epsilon+\int_{A_1^c}^{u}(\alpha(p_{r1}^c)-b\epsilon)f(\epsilon)d\epsilon\right\}+\eta\sqrt{\frac{c_r^c}{c_m}}\times$$

$$\int_{\frac{Q_1^c}{R_1^c}}^{t}(yR_1^c-Q_1^c)h(y)dy+\eta\sqrt{\frac{c_r^c}{c_m}}\int_{A_1^c}^{u}(Q_1^c-\alpha(p_{r1}^c)+b\epsilon)f(\epsilon)d\epsilon\Bigg\}\Bigg]<0,$$

since the terms inside brackets are production quantity in second period and remanufactured items, hence positive. In a similar manner, another algorithm is provided below to extract the optimal values of the decision variables by virtue of the above theorem.

Algorithm

Step 1: Set $\delta=0.001$, $p_{r2}^*=c_m$, $Q_2^*=0$, $R_2^*=0$, $\tau^*=0$, $c_r^*=0$, $G_2=0$ and go to step 2.

Step 2: Set $p_{r2}=p_{r2}^*$, $Q_2=Q_2^*$, $R_2=R_2^*$, $\tau=\tau^*$, $c_r=c_r^*$ optimize Π_2 using any optimization technique to obtain values of G_2 and go to step 3.

Step 3: If $|G_2^*-G_2|<\delta$ go to step 8, else set $G_2^*=G_2$ and go to step 4.

Step 4: Using G_2^*, p_{r2}^*, Q_2^*, R_2^* optimize Π_2 again to get new values of τ, c_r and go to step 5.

Step 5: If $|\tau-\tau^*|<\delta$ and $|c_r-c_r^*|<\delta$ simultaneously, go to step 8, else set $\tau^*=\tau$ and $c_r^*=c_r$, and go to step 6.

Step 6: Using G_2^*, τ^* and c_r^*, optimize Π_2 again to get new values of Q_2, R_2, p_{r2} and go to step 7.

Step 7: If $|Q_2-Q_2^*|<\delta$, $|R_2-R_2^*|<\delta$, and $|p_{r2}-p_{r2}^*|<\delta$ simultaneously, go to step 8, else go to step 2.

Step 8: The values $p_{r2}^*, Q_2^*, R_2^*, c_r^*, \tau^*, G_2^*$ are optimal values of the decision variables.

7.2.4 Numerical illustration

This section aims to illustrate the developed models numerically through a few examples. As identified by Xin et al., 2022, true data cannot be accessed unconditionally due to business confidentiality. Hence, data sets which resemble the proposed supply chain are chosen. The following parameter-values are chosen: $\eta = 0.8$, $w_R =$ \$1/unit, $a = 400$ units/month, $b = 3$ units/\$/month, $H = 100$, $c_m =$ \$10/unit, $c_{sr} =$ \$4/unit, $c_{ms} =$ \$30/unit, $c_e =$ \$4/unit, $e_n = 5$/unit, $e_r = 2$/unit, $V = 300$ units, $m = 0.1$, and $\lambda = 0.6$. The randomness inherent in retail price is assumed to follow normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 5$, and the random yield is assumed to follow Beta distribution with parameters $(2, 3)$. It is to be noted that although a normal distribution has a spectrum ranging over the entire real line, $P(|X - \mu| < 5\sigma) > 0.999$, it is reasonable to consider such a distribution here. Based on the values, scenario-based analysis is performed to represent different business scenarios. Example 2 considers $e_n = 10$/unit, $e_r = 5$/unit, $c_e =$ \$10/unit, and $V = 100$ units to represent an inefficient production process emitting high carbon, a stringent government imposing a higher penalty on carbon emission as well as setting lower carbon cap; Example 3 represents a cheaper business scenario with $c_m =$ \$5/unit, $w_R =$ \$2/unit, and $c_{ms} =$ \$40/unit which is somewhat a favorable business environment; and Example 4 represents a costlier scenario with $c_m =$ \$15/unit, $w_R =$ \$0/unit, and $c_{ms} =$ \$10/unit which is unfavorable for business. Optimal results derived from both models are provided in Table 7.7. The findings are summarized in Section 7.2.4.3 to gain managerial insights.

7.2.4.1 Sensitivity analysis

The results of sensitivity analysis- performed by varying one parameter value at a time - are provided in Figures 7.9-7.14 and in Table 7.8. Following business strategies may be derived out of it.

Ordering strategy: Under carbon cap, the retailer should reduce the order quantity a bit. Higher production costs or high price sensitivity of the customers compels the retailer to reduce order quantity, while he places higher

Table 7.7: Optimal results for both the models

Example	Model	p_{r1} (\$)	Q_1 (units)	R_1 (units)	G_1 (\$)	p_{r2} (\$)	Q_2 *units*	R_2 (units)	G_2 (\$)	c_r (\$/unit)	τ	Π_1 (\$)	Π_2 (\$)	$\Pi = \Pi_1 + \Pi_2$ (\$)
Example 1	I	79.79	166.4	238.7	-	79.7	165.47	238.7	-	0.54	1	7907	8157	16064
	II	81.6	159.4	159.4	43.37	81.5	158.4	158.4	41.85	0.79	1	8439	8738	17177
Example 2	I	79.79	166.4	238.7	-	79.7	165.47	238.7	-	0.541	1	7907	8157	16064
	II	87.7	136.8	136.8	57.9	87.6	135.6	135.6	55.3	1.93	1	6328	8973	15301
Example 3	I	76.63	178.9	450.5	-	76.39	179.4	477.4	-	0.143	1	9053	9557	18610
	II	80.0	165.45	325.9	50.5	79.9	165.0	334.0	50.1	0.337	1	8969	9394	18355
Example 4	I	76.73	178.5	178.5	-	76.66	177.3	177.3	-	0.9995	1	9016	9215	18231
	II	78.3	172.3	172.3	44	78.2	171.0	171.0	42.6	1.2733	1	8680	9867	18547

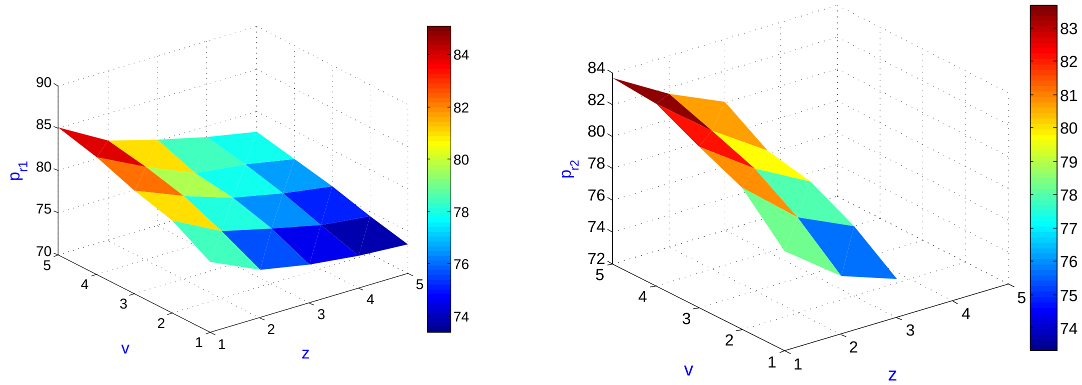
Table 7.8: Robustness analysis

Parameter	Value	% Change	p_1 (\$)	Q_1 (units)	R_1 (units)	p_2 (\$)	Q_2 (units)	R_2 (units)	c_r (\$/ units)	τ	Π_1 (\$)	Π_2 (\$)	$\Pi = \Pi_1 + \Pi_2$ (\$)
	40	-60	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8177	16084

H	80	-20	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8177	16084
	100	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064
	120	20	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8136.97	160445.97
	160	60	79.79	166.4	238.7	79.7	165.47	238.76	0.545	0.888	7907	8098	16005
c_m	4	-60	74.45	187.95	459.4	74.32	187.68	474.1	0.177	1	9892.3	10140.5	20032.8
	8	-20	78.3	172.23	291.48	78.23	171.39	293.3	0.421	1	8436	8671.71	17107.71
	10	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064
	12	20	81.02	161.72	161.74	80.93	160.7	161.36	0.665	1	7485.61	7756	15241.61
	16	60	82.9	154.52	154.52	82.82	153.44	153.45	0.925	1	6853.17	7176.11	14029.28
$s.d.$	3	-40	80	163.6	234.6	80	163.06	235.28	0.53	1	8121	8347.44	16468.44
	4	-20	79.9	165	236.1	79.9	164	236.2	0.536	1	8054	8258.9	16312.9
	5	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064
	6	20	79.6	167.2	240	79.55	166.8	239.6	0.546	1	7801	8061	15862
	7	40	79.5	168.9	242.3	79.41	167.57	241.8	0.551	1	7679.45	7951.8	15631.25
w_R	0.5	-50	79.79	166.4	238.7	79.71	165.32	237.6	0.59	1	7907	8087	15994
	0.8	-20	79.79	166.4	238.7	79.71	165.4	238.38	0.565	1	7907	8127.6	16034.6
	1	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064

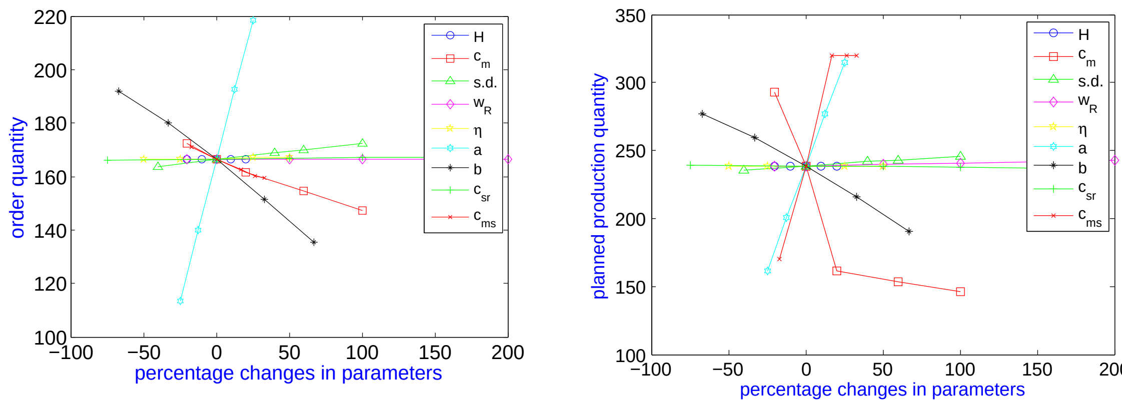
	1.2	20	79.79	166.4	238.7	79.71	165.52	239.1	0.513	1	7907	8197	16104
	1.5	50	79.79	166.4	238.7	79.72	165.6	239.7	0.482	1	7907	8231	16138
	300	-25	62.9	113.6	162.9	62.7	112	161.7	0.547	0.962	3514.8	3676.7	7191.5
	350	-12.5	71.36	140.1	201	71.25	138.9	200.5	0.543	1	5500	5704.76	11204.76
a	400	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064
$month)$	450	12.5	88.2	192.5	276.1	88.1	191.77	276.7	0.539	1	10736.2	11030.8	21767
	500	25	96.6	218.5	313.4	96.55	217.9	314.4	0.538	1	13984.8	14325.3	28310.1
	1	-66.7	213.5	192	275.4	213.5	192	277	0.527	1	34455	34720.2	69175.2
	2	-33.3	113.3	180	258.3	113.3	179.8	259.5	0.535	1	14492.9	14747.3	29240.2
b	3	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064
$/month)$	4	33.3	62.9	151.5	217.2	62.74	149.4	215.65	0.545	1	4686.5	4935.4	9640.9
	5	66.7	52.6	135.5	194.3	52.37	131.9	190.3	0.548	1	2822	3079	5901
	2	-50	79.77	166.1	238.2	79.7	165.9	239.36	0.543	0.998	7914	8148	16062
	3	-25	79.78	166.3	238.5	79.7	165.6	239	0.542	1	7910	8151	16061
c_{sr}	4	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064
	5	25	79.79	166.5	238.8	79.69	165.2	238.5	0.541	1	7904	8161	16065
	6	50	79.8	166.7	239	79.69	165.1	238.3	0.54	1	7900.76	8165.6	16066.36

c_{ms}	20	-33.3	77.2	176.6	176.6	77.16	175.73	175.73	0.535	1	8838.36	9096.4	17934.76
	25	-16.7	78.6	170.9	171.1	78.57	170	170	0.537	1	8317.1	8567	16884.1
	30	0	79.79	166.4	238.7	79.7	165.47	238.7	0.541	1	7907	8157	16064
	35	16.7	80.76	162.7	263	78.9	168.65	320	2.02	0.715	7573.7	7650.15	15223.85
	40	33.3	81.6	159.4	281	80.21	163.5	320	2	0.712	7284.3	7405.3	14689.6



(a) period 1 **(b)** period 2

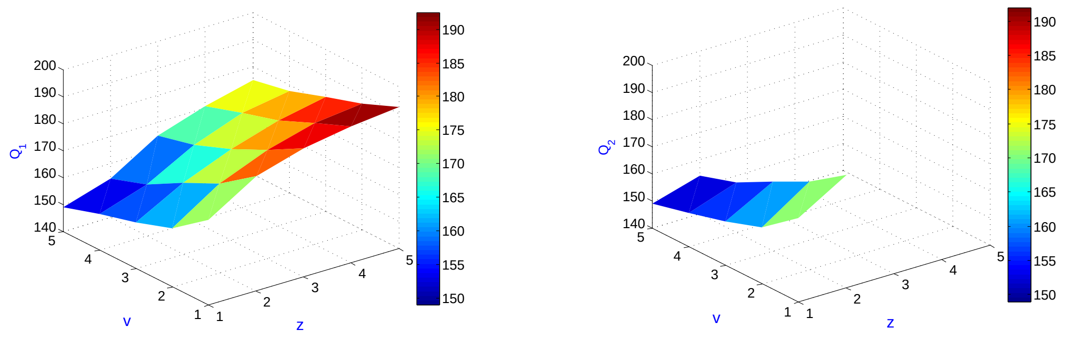
Figure 7.6: Effect of random yield on pricing



(a) period 1 **(b)** period 2

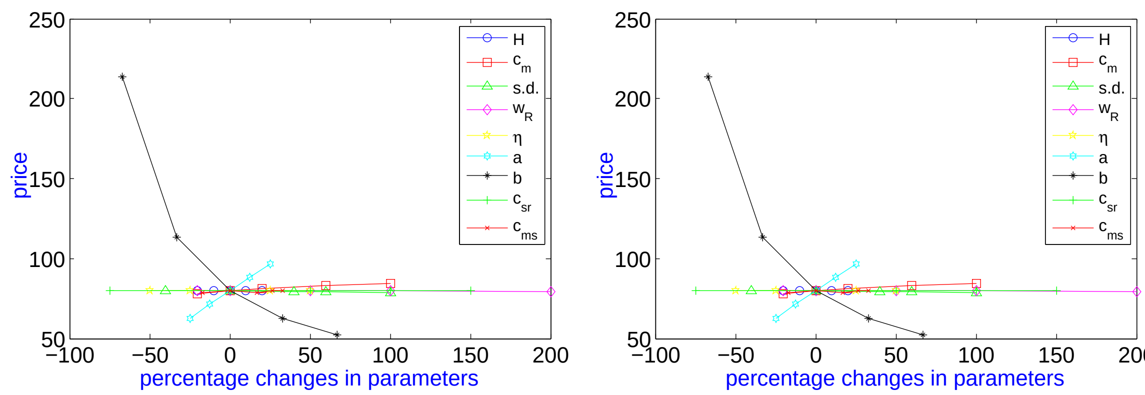
Figure 7.7: Effect of parameters on order quantities

orders to deal with the higher variation in price that may result in higher demand. With reduced expected yield in production, the business manager is to reduce order quantity. The retailer should order more to mitigate high demand or high shortage costs as extra security to avoid lost sales, while he should marginally reduce order quantity when the price of the ready-made goods at the secondary market is high. With higher per unit carbon emission, the retailer tends to place fewer orders, while he may place more orders with higher efficiency of the green technology. However, the ordering is seen to be unaffected by changes in cost parameters related to product collection or carbon emission during remanufacturing.



(a) period 1 **(b)** period 2

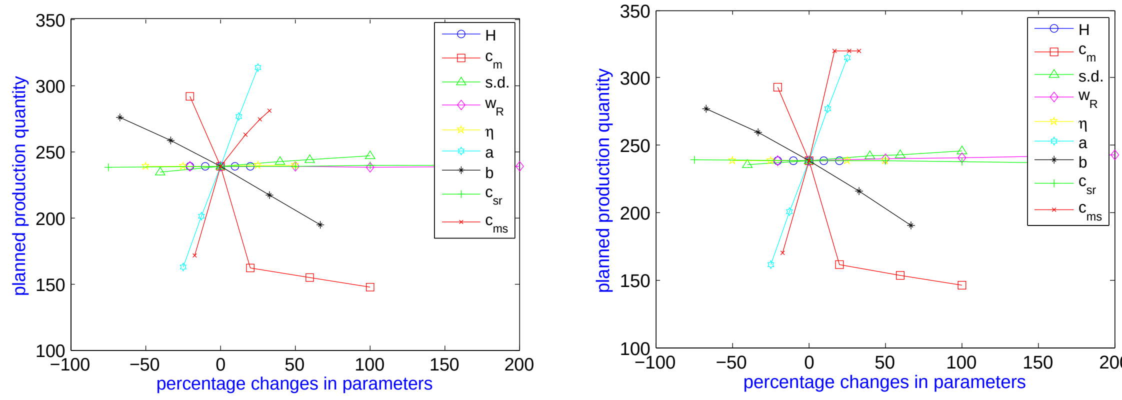
Figure 7.8: Effect of random yield on order quantities



(a) **(b)**

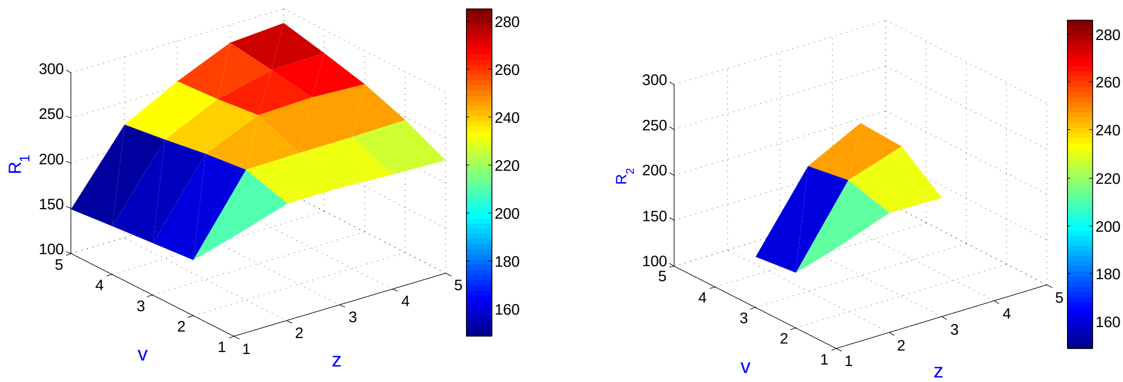
Figure 7.9: Effect of parameters on pricing

Production strategy: In absence of carbon emission consideration, the primary aim of the manufacturer would be to produce more to mitigate the effect of random yield. However, when carbon emission cost on production is imposed and retailer is capable of reducing it with green investment, the manufacturer should reduce production quantity and focus more on remanufacturing. Lower expected yield compels the manufacturer to reduce production quantity and rely more on the secondary market for the finished product. Higher production cost reduces production level while higher deviation in price or higher demand attracts the manufacturer to produce more. The gap between the ordering quantity and planned production quantity gets shrunk with an increased raw material cost, aiming at reducing wastage due to randomness in yield. Since increasing price sensitivity hurts demand, the production shall thereby be reduced too. When the price of the finished good in the secondary



(a) period 1 **(b)** period 2

Figure 7.10: Effect of parameters on planned production quantity



(a) period 1 **(b)** period 2

Figure 7.11: Effect of random yield on planned produciton quantities

market is high, the production level should also be raised to produce a sufficient amount even after the inherent yield. The ratio between the ordering quantity and planned production quantity gets lower with higher values of c_{ms}, which is an effort to avoid the shortage to the highest extent. Production levels should be reduced when associated carbon emission costs are high. A higher efficiency of green technology allures the manufacturer to produce more.

Investment in remanufacturing: Consideration of the carbon emission compels the manufacturer to focus more on remanufacturing. With higher costs related to product collection, the collected items are marginally more costly and are to be utilized more wisely, necessitating a rise in remanufacturing invest-

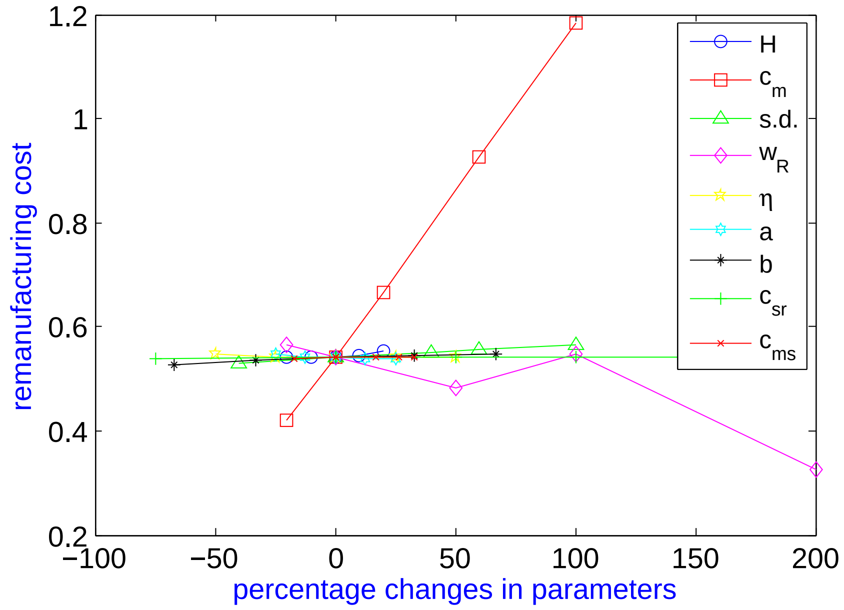
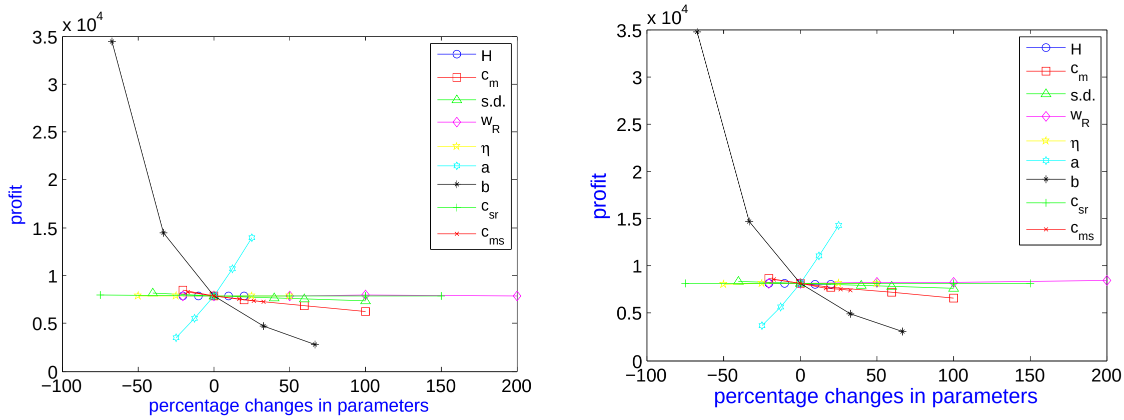


Figure 7.12: Effect of parameters on remanufacturing investment

ment. An increased raw material cost indirectly enhances the marginal value of the remanufacturing cost, so the business managers are advised to invest more in remanufacturing. With higher variation in price, the manager should invest more in remanufacturing since it costs lesser than manufacturing. With the increase in the salvage value which in turn generates more profit, the remanufacturing investment should be reduced. Higher demand indicates more product collection, thereby reduction in remanufacturing investment may be observed. With higher price sensitivity, the manufacturer should focus more on the remanufacturing policy and invest more in the remanufacturing cost. The remanufacturing investment should be increased with higher shortage cost. High price of final product at secondary market leaves more surplus quantity, necessitating more investment in remanufacturing and pointing to the potential for producing repurposed and recycled materials to support the growth of the other farm. Remanufacturing investment should rise in the second cycle aiming at reducing new production volume when corresponding carbon emission is high. On the contrary, financial benefit out of remanufacturing is much diminished with higher carbon emission cost related to remanufacturing, indicating less investment in it. With higher linear efficiency of technology λ, remanufacturing is efficient since it produces less carbon emissions. As a result, when greening investments are already reducing emissions overall, remanufacturing costs may also be slightly reduced. Recycling the entire item is also beneficial to the environment since it reduces the trash and pollution while disposing them of, and it also improves the performance of other farms because of their interdependence. Higher expected yield in production allures the manufacturer to invest more on remanufacturing.

Product collection: Used product collection is inversely related to the associated cost coefficient; the more per unit cost, the less investment in it. With lesser demand, the retailer should invest less in product collection due to comparatively lesser marginal return, and with high price of final product at secondary market, higher production quantity results in more surplus quantity as well, making the manufacturer inclined to reduce investing for the product collection.

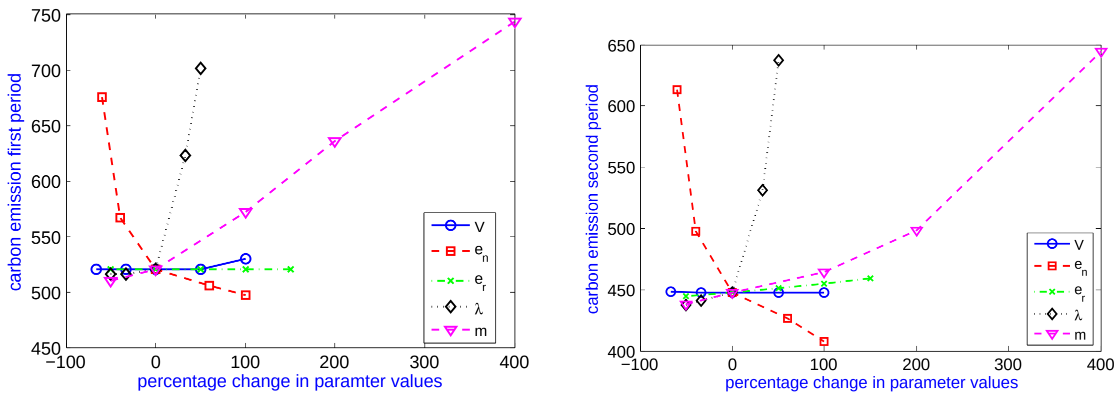
Effect on profit: Profit in the second period is always higher than the first one, the reason being remanufacturing at least some of the items at a reduced rate, and at a lower carbon emission as well in case of later model. Higher cost related to product collection affects the profit in second period only. The profit level decreases with a larger variation in price, since the shortage or salvage both increase in this case. With a higher per unit production cost or price at secondary market, the total profit is bound to reduce. With a higher salvage value or high demand, the business witnesses rise in profit. However, when customers are more prone to price sensitivity, the business environment becomes adverse. Higher carbon cap helps to generate more profit in two ways: by not getting charged penalty for carbon emission as well as generating revenue by selling unused carbon limit to other companies. On the contrary, higher per unit carbon emission costs reduce the profit significantly. The profit in the second period is higher than the first even when the remanufacturing carbon emission is higher than new production, which is due to the lower production cost of remanufacturing. Finally, the manufacturer should opt for introducing efficient green technology to generate more profit.



(a) period 1 **(b)** period 2

Figure 7.13: Effect of parameters on profit

Green investment: It is indeed beneficial to invest in reducing carbon footprints in the environment. As is seen in Table 7.7, the positive effect is evident in both the periods. The cost of greening investments must rise in tandem with



(a) period 1 **(b)** period 2

Figure 7.14: Effect of parameters on carbon emission

the total growth in carbon emissions which is due to per unit carbon emission costs. Higher carbon emission cost during manufacturing compels the manufacturer to enhance green investment in both the periods, such a higher investment results in reduced carbon emission. However, with higher emission in remanufacturing, only the investment in second period is to be increased. With higher exponential efficiency m, even a lower green investment is expected to produce same output, thereby allowing the manufacturer to reduce investment. However, gross carbon emission is seen to increase a bit in this case. Carbon cap seems to have no effect on green investments.

7.2.4.2 Robustness analysis

This subsection aims to exhibit the robustness of the model numerically. The parameter data set from example 1 has been chosen to perform the checking. Keeping other parameter values fixed, one parameter value is deviated up to 66% in both the directions at a time to demonstrate its effect on variations in optimal decisions.

Effect of H

To conduct the robustness analysis for the parameter H given the data points provided, we first summarize the values for each corresponding variable. The outcome metrics under consideration are p_{r1}, Q_1, R_1, p_{r2}, Q_2, R_2, c_r, τ, along with the profit functions Π_1, Π_2, and Π. The mean values for these metrics are 79.79 for p_{r1}, 166.4 for Q_1, 238.7 for R_1, 79.7 for p_{r2}, 165.47 for Q_2, 238.71 for R_2, 0.542 for c_r, 0.9776 for τ, 7907 for Π_1, 8129.394 for Π_2, and 16056.402 for Π. The standard deviations are calculated as 0 for p_{r1}, 0 for Q_1, 0.024 for R_1, 0 for p_{r2}, 0 for Q_2, 0.024 for R_2, 0.002 for c_r, 0.0494 for τ, 0 for Π_1, 32.473 for Π_2, and 175.314 for the Π. The coefficients of variation (CV) are 0% for p_{r1}, 0% for Q_1, 0.01% for R_1, 0% for p_{r2}, 0% for Q_2, 0.01% for R_2, 0.37% for c_r, 5.05% for τ, 0% for Π_1, 0.4% for Π_2, and 1.09% for the Π. This analysis indicates that while most metrics show minimal variation (suggesting robustness), τ exhibits the highest sensitivity.

Effect of production cost

The data points for c_m are {4, 8, 10, 12, 16}. The mean values of the decision variables under these variations are calculated as follows: p_{r_1} has a mean of

79.292, Q_1 has a mean of 168.964, and R_1 has a mean of 261.968. Similarly, p_{r2} has a mean of 79.196, Q_2 has a mean of 167.326, and R_2 has a mean of 264.102. The mean value for c_r is 0.546, τ remains constant at 1, Π_1 has a mean of 8114.216, Π_2 has a mean of 8440.564, and Π has a mean of 16870.781. The standard deviations for these parameters are: p_{r1} at 3.174, Q_1 at 10.692, R_1 at 112.822, p_{r2} at 3.159, Q_2 at 10.610, R_2 at 113.110, c_r at 0.252, τ at 0.0, Π_1 at 1070.192, Π_2 at 1031.382, and Π at 2064.451. The coefficients of variation (CV) are: p_{r1} at 4.00%, Q_1 at 6.33%, R_1 at 43.08%, p_{r2} at 3.99%, Q_2 at 6.34%, R_2 at 42.84%, c_r at 46.15%, τ at 0.0%, Π_1 at 13.19%, Π_2 at 12.22%, and the Π at 12.24%, indicating that production cost mostly affects investment in remanufacturing and planned production quantity.

Effect of standard deviation of demand

The data points for $s.d.$ are {3, 4, 5, 6, 7}. The resulting values for various parameters are: p_{r1} with a mean of 79.76, Q_1 with a mean of 166.22, R_1 with a mean of 238.34, p_{r2} with a mean of 79.71, Q_2 with a mean of 165.58, and R_2 with a mean of 238.12. Additionally, c_r has a mean of 0.541, τ is constant at 1, Π_1 has a mean of 7912.89, Π_2 has a mean of 8155.03, and Π has a mean of 16067.92. The standard deviations for these variables are: p_{r1} at 0.193, Q_1 at 2.169, R_1 at 3.031, p_{r2} at 0.196, Q_2 at 2.196, R_2 at 2.739, c_r at 0.007, τ at 0, Π_1 at 159.1, Π_2 at 158.63, and Π at 317.56. The coefficients of variation (CV) are: p_{r1} at 0.24%, Q_1 at 1.31%, R_1 at 1.27%, p_{r2} at 0.25%, Q_2 at 1.33%, R_2 at 1.15%, c_r at 1.32%, τ at 0.0%, Π_1 at 2.01%, Π_2 at 1.95%, and the Π at 1.98%.

Effect of salvage value

The robustness analysis for w_R is conducted with the following values: {0.5, 0.8, 1, 1.2, 1.5}. The results show that all decision variables for 1st cycle remain constant. For p_{r2}, the values are slightly varied around 79.71. Q_2 varies slightly around 165.47, and R_2 varied around 238.7, with a slight increase as w_R is increased. The variable c_r decreases from 0.59 to 0.482 as w_R is increased from 0.5 to 1.5. The goods collection rate τ remains constant at 1 across all values. The profit Π_1 is also constant at 7907, while Π_2 increases from 8087 to 8231 as w_R is increased. Π thus shows a slight increase from 15994 to 16138. This analysis demonstrates that p_{r2}, Q_2, and R_2 are moderately sensitive with salvage value.

Effect of base demand

The data points chosen for base demand a are $\{300, 350, 400, 450, 500\}$. The resulting values for various parameters are as follows: p_{r1} with a mean of 79.97, Q_1 with a mean of 166.42, R_1 with a mean of 238.62, p_{r2} with a mean of 79.88, Q_2 with a mean of 165.61, and R_2 with a mean of 238.88. Additionally, c_r has a mean of 0.542, τ has a mean of 0.992, Π_1 has a mean of 7907, Π_2 has a mean of 8115.85, and Π has a mean of 16022.85. The standard deviations for these variables are: p_{r1} at 12.52, Q_1 at 40.58, R_1 at 60.18, p_{r2} at 12.53, Q_2 at 39.88, R_2 at 61.08, c_r at 0.003, τ at 0.015, Π_1 at 4482.27, Π_2 at 4527.26, and Π at 9007.94. The coefficients of variation (CV) are: p_{r1} at 15.66%, Q_1 at 24.38%, R_1 at 25.22%, p_{r2} at 15.68%, Q_2 at 24.08%, R_2 at 25.57%, c_r at 0.55%, τ at 1.51%, Π_1 at 56.68%, Π_2 at 55.81%, and Π at 56.19%. This detailed analysis highlights the impact of a on the stability and variability of the different parameters.

Effect of price sensitivity

For the data set $\{1, 2, 3, 4, 5\}$, the resulting values for various variables are: p_{r1} has a mean of 104.22, Q_1 has a mean of 165.08, R_1 has a mean of 236.78, p_{r2} has a mean of 104.05, Q_2 has a mean of 163.73, and R_2 has a mean of 236.03. Additionally, c_r has a mean of 0.539, τ has a mean of 1, Π_1 has a mean of 12872.88, Π_2 has a mean of 13127.98, and Π with the mean of 26000.85. The standard deviations for these variables are: p_{r1} at 63.13, Q_1 at 21.32, R_1 at 31.77, p_{r2} at 63.24, Q_2 at 21.09, R_2 at 32.57, c_r at 0.008, τ at 0.0, Π_1 at 11634.18, Π_2 at 11654.77, and Π at 23253.07. The coefficients of variation (CV) are: p_{r1} at 60.56%, Q_1 at 12.92%, R_1 at 13.42%, p_{r2} at 60.76%, Q_2 at 12.88%, R_2 at 13.8%, c_r at 1.48%, τ at 0%, Π_1 at 90.42%, Π_2 at 88.82%, and the Π at 89.42%. This detailed analysis highlights that price sensitivity affects pricing and profit mostly.

Effect of shortage cost

The data points for c_{sr} are $\{2, 3, 4, 5, 6\}$. The resulting values for various parameters are as follows: p_{r1} with a mean of 79.79, Q_1 with a mean of 166.4, R_1 with a mean of 238.64, p_{r2} with a mean of 79.68, Q_2 with a mean of 165.45, and R_2 with a mean of 238.97. Additionally, c_r has a mean of 0.541, τ has a mean of 1, Π_1 has a mean of 7907, Π_2 has a mean of 8156.6, and Π has a mean of 16063.6. The standard deviations for these variables are: p_{r1} at 0.012, Q_1 at 0.206, R_1 at 0.3, p_{r2} at 0.037, Q_2 at 0.308, R_2 at 0.538, c_r at 0.001, τ at 0, Π_1 at 4.47,

Π_2 at 5.72, and Π at 10.18. The coefficients of variation (CV) are: p_{r1} at 0.01%, Q_1 at 0.12%, R_1 at 0.13%, p_{r2} at 0.05%, Q_2 at 0.19%, R_2 at 0.22%, c_r at 0.18%, τ at 0.0%, Π_1 at 0.06%, Π_2 at 0.07%, and the Π at 0.06%. This detailed analysis highlights the impact of c_{sr} on the stability and variability of the different parameters.

Effect of price at secondary market

The data points for c_{ms} are $\{20, 25, 30, 35, 40\}$. The resulting values for various parameters are: p_{r1} with a mean of 79.79, Q_1 with a mean of 166.4, R_1 with a mean of 238.7, p_{r2} with a mean of 78.56, Q_2 with a mean of 168.87, and R_2 with a mean of 224.69. Additionally, c_r has a mean of 1.127, τ has a mean of 0.885, Π_1 has a mean of 7984.49, Π_2 has a mean of 8175.57, and the Π has a mean of 16160.06. The standard deviations for these variables are: p_{r1} at 1.714, Q_1 at 6.892, R_1 at 41.95, p_{r2} at 1.454, Q_2 at 2.037, R_2 at 84.32, c_r at 0.612, τ at 0.133, Π_1 at 543.6, Π_2 at 502.2, and the Π at 1033.41. The coefficients of variation (CV) are: p_{r1} at 2.15%, Q_1 at 4.14%, R_1 at 17.57%, p_{r2} at 1.85%, Q_2 at 1.21%, R_2 at 37.53%, c_r at 54.3%, τ at 15.03%, Π_1 at 6.81%, Π_2 at 6.14%, and the Π at 6.39%.

7.2.4.3 Managerial insights

This subsection provides valuable managerial insights derived from the analysis for real world implications. The observations may be adopted in any green supply chain, and it will also help non-green supply chain managers to identify situations when shifting to green supply chain and/or investment in remanufacturing will be beneficial to them. These insights guide managers on how to adjust decision variables to optimize profitability, production, and sustainability. By carefully managing production costs, demand stability, salvage values, green investments, and market pricing strategies, companies can enhance their financial performance and achieve their operational and environmental goals.

- For baseline scenarios (Example 1), maintaining balanced production and pricing strategies with moderate green investments ensures stable profitability.

Retail prices should be adjusted based on production costs and market conditions to maintain profitability while managing demand. In a balanced moderate business scenario, Model II outperforms Model I due to the additional focus on sustainability and efficiency. Focus should be on stringent cost control and strategic investments to maintain profitability in unfavorable business conditions. Maintaining a consistent rate of old goods collection and remanufacturing strategy is crucial for efficient inventory and waste management.

- The optimal retail price should be set significantly higher to compensate for increased production and environmental compliance costs. Order and production quantities should also be reduced, reflecting a strategy to minimize waste and manage higher costs. High levels of green investment are necessary to mitigate the higher environmental penalties, emphasizing the importance of sustainability. The high costs and penalties result in lower profits, especially in Model II, highlighting the need for more efficient production and emission reduction strategies. Managers should therefore strive to minimize fresh production costs through process improvements or cost-effective sourcing of materials. Government may also provide business incentives in terms of tax rebate or other to encourage green investment.

- Lower retail prices should be set to attract more customers in a low-cost business scenario (example 3), supported by lower production costs. Higher order and production quantities reflect confidence in the favorable market conditions and capacity to meet increased demand. Adequate green investments are to be made as well to maintain compliance without significantly impacting profitability. Both models achieve the highest profits due to favorable production and market conditions, making this scenario highly lucrative.

- In a high-cost business scenario, the focus should be on cost control, moderate production, and strategic green investments to comply with regulations and minimize losses. Investments in green technology are necessary but should be kept moderate due to higher overall costs. Despite unfavorable conditions, profitability may remain reasonable due to effective cost management and strategic pricing.

- Higher deviation in the stochastic part of the demand implies higher coefficients of variation, indicating greater instability. Managers should focus on demand forecasting and stabilization strategies to reduce uncertainty and maintain stable operations.

- Higher salvage values may lead to marginal improvements in profitability, though the effect is not as pronounced. This suggests that while optimizing salvage value is beneficial, its impact on overall profitability might be limited. Managers should balance salvage value optimization with other cost-saving measures.

- Higher values of η (related to green investment) lead to better environmental outcomes and can enhance profitability due to potential cost savings from reduced emissions. Managers should invest in green technologies and practices to achieve sustainability goals while also benefiting from cost reductions.

- Since higher base demand has a positive effect on overall profit, it underscores the importance of market expansion and demand stimulation strategies for enhancing financial performance. Managers should focus on marketing and customer engagement to boost base demand, and carefully set prices to balance demand and revenue, avoiding overly high prices that could significantly deter customers. He should further optimize inventory levels and ensure robust supply chains to minimize shortages and associated costs. Finally, managers should balance secondary market pricing to avoid cannibalizing primary market sales while maximizing overall revenue.

7.2.5 Conclusion

Summary

The subchapter considers random pricing strategy in a closed-loop supply chain facing price-dependent demand under random yield in both production as well as remanufacturing. Consideration of optimized investment for both the product collection as well as remanufacturing are two real-life business practices which have been incorporated here. Further consideration of carbon emission during production, carbon cap and tax policy, and possibly reducing the carbon footprint through green investment has made the work applicable in the field of sustainable supply chain. The joint and pairwise concavities are established analytically to ensure the applicability of the model. Two algorithms are also provided to obtain desired result. Various numerical examples are considered to represent various business situations and examine whether consideration of green investment will be fruitful in those situations or not. Given that inherent randomness is therein demand, price, and production, the numerical section offers guidance on what to do when uncertainty fluctuates. Important managerial insights on the remanufacturing, product collection, green investment, and pricing are listed. The work extends the models of Mondal and Giri, 2020 and Das Roy and Sana, 2017; the first one is based on a closed-loop supply chain model while the second one is a random pricing model with defective production. The current work proposes a hybrid pricing system that incorporates both random and deterministic components. As is seen, green investment affects production planning decisions in general, except when the production is already costlier and manufacturer has already reduced production quantities a lot. Green investment boosts remanufacturing too, resulting in overall more profit. However, the investment comes at a cost which is to be leveraged by the end customers; the environment awareness of customers and inclination of paying more for it plays role in setting higher retail price. The manager should boost the awareness more through campaigns and offers. The proposed model is significant for the industries where both green and non-green type products may be produced, and the manager wants to decide the most profitable one. The model also helps the manager to identify which parameters are more vulnerable and sensitive, and take appropriate measure to ensure least deviation of such parameters.

Limitations and future research directions

Although the work fills research gaps and contributes to existing literature, it still has its limitations. The demand has been considered to depend only on price, while in reality there are other factors too which stimulate demand. While considering remanufacturing, the screening process is always considered to be perfect. Further, the model aims at examining when investing in green initiatives will be beneficial under various cost and business scenarios, and neglected the inter-dynamics of adjacent supply chain members. Such barriers are to be crossed to extend the research. The current work may be extended in various ways, some of which we propose here. One of the main focus of the chapter was to discuss the greening investment policy. It showed that once carbon emissions and penalty on carbon emissions are imposed, green investments becomes necessary. The business manager can regulate the emission level with the aid of the remanufacturing process, and the greening investment additionally proves out to be profitable. Consideration of a single carbon cap and trade policy which may regulate the carbon emission to these extent will surely be a fine extension of the current version. The consideration of a perfect scrutiny process regarding identifying remanufacturable products may also be relaxed as the effect of such imperfection is also a good point to analyze. Recycling and upcycling are further options in addition to remanufacturing. The present study may be extended to consider those issues as well. Consideration of an imperfect production process would be more realistic. An advanced customer base may also be considered where customers are environment aware, and degree of greenness has a positive effect on demand. Demand stimulating efforts such as providing free servicing may also be considered. Finally, the centralized model may be extended in a dominant manufacturer business scenario where Stackleberg game scenario would be interesting to apply.

CHAPTER 8

Conclusion and Future Research Prospects

Effective pricing strategies are fundamental to managing perishable inventory efficiently. Given the complexities of perishability, price sensitivity, time-dependency of demand, and preservation efforts, businesses must carefully design pricing models to balance profitability, minimize waste, and enhance customer satisfaction. This thesis has explored various pricing strategies, including static and dynamic pricing, discounting policies, preservation investments, and sustainability-focused approaches. By integrating factors such as learning effects, capital and space constraints, supply chain coordination, and green-conscious demand, our research provides a comprehensive framework for optimizing pricing in perishable inventory management.

A key contribution of this work is the development of multi-period inventory models that incorporate dynamic pricing, service-level improvements, and preservation investment decisions. These models enable businesses to adapt their pricing strategies over time, responding to shifting market conditions while ensuring operational efficiency. Furthermore, the inclusion of closed-loop supply chains highlights the growing importance of sustainable practices, demonstrating how green investments and remanufacturing can enhance both environmental responsibility and long-term profitability.

Moreover, this research extends traditional pricing models by integrating revenue-sharing contracts and closed-loop supply chains, which highlights the importance of collaboration between retailers and manufacturers. These mechanisms ensure that pricing decisions are aligned across different supply chain partners, leading to greater efficiency, risk-sharing, and mutually beneficial pricing strategies. By bridging the gap between pricing theory and practical implementation, this study offers valuable insights into how businesses can navigate the complexities of perishable inventory management while maintaining financial sustainability.

8.1 Managerial Implications

The findings of this study offer several practical insights for managers handling perishable inventory. First, pricing must be approached dynamically, with adjustments based on demand patterns, stock levels, and customer preferences. Rigid pricing models can lead to overstocking, lost sales, and higher disposal costs. Second, preservation investments are crucial not only for reducing spoilage but also for maintaining product quality and extending shelf life, both of which directly impact consumer trust and willingness to pay. Third, the learning effect plays a vital role in improving operational efficiency—managers should leverage it to optimize order quantities, shorten shortage periods, and enhance service quality over time.

Moreover, sustainability initiatives- such as investing in greener products and remanufacturing efforts- can create competitive advantages. Consumers are becoming increasingly environmentally conscious, and businesses those align their pricing and inventory strategies with green practices can strengthen brand loyalty while meeting regulatory expectations. Additionally, revenue-sharing contracts in supply chains can improve coordination between retailers and manufacturers, leading to mutually beneficial pricing policies.

8.2 Limitations & Future Research Directions

While this research has provided a solid foundation for pricing strategies in perishable inventory management, several avenues remain open for further exploration.

One promising direction is the integration of real-time pricing algorithms driven by machine learning and artificial intelligence. These techniques could help businesses dynamically adjust prices based on real-time market conditions, competitor pricing, and demand forecasting.

Another key area for future study is the role of consumer behavior and psychological pricing strategies. Understanding how consumers perceive value, respond to discounts, and make purchase decisions in the presence of perishability constraints could refine existing pricing models. Additionally, the impact of emerging sustainability regulations on pricing and inventory decisions warrants further investigation, particularly as governments and organizations worldwide push for greener supply chain practices.

Finally, risk and uncertainty in supply chains—such as disruptions due to climate change, pandemics, or geopolitical issues—present significant challenges for perishable inventory management. Future research could explore how businesses can develop more resilient pricing strategies that account for unpredictable demand fluctuations and supply chain uncertainties.

8.3 Conclusions

This thesis contributes to the field of perishable inventory and supply chain management by offering practical, data-driven pricing strategies that align with both profitability and sustainability goals. By integrating preservation investments, learning effects, green-conscious demand, and dynamic pricing models, our research provides a valuable guide for businesses navigating the complexities of perishable supply chains. Moving forward, continued advancements in technology, sustainability, and behavioral economics will shape the future of pricing strategies, ensuring that businesses remain competitive and responsive in an evolving global market.

Bibliography

Abbasi, S., & Erdebilli, B. (2023). Green closed-loop supply chain networks' response to various carbon policies during covid-19. *Sustainability*, *15*(4), 3677. https://doi.org/https://doi.org/10.3390/su15043677

Adak, S., & Mahapatra, G. (2022). Effect of reliability on multi-item inventory system with shortages and partial backlog incorporating time dependent demand and deterioration. *Annals of Operations Research*, *315*(2), 1551–1571.

Adenso-Díaz, B, Lozano, S, & Palacio, A. (2017). Effects of dynamic pricing of perishable products on revenue and waste. *Applied Mathematical Modelling*, *45*, 148–164.

Adnan, Z. H., & Özelkan, E. (2023). Supply chain price variability under the buyback contract. *Journal of Industrial and Production Engineering*, *40*(4), 301–322.

Afshari, H., Jaber, M. Y., & Searcy, C. (2019). Investigating the effects of learning and forgetting on the feasibility of adopting additive manufacturing in supply chains. *Computers & Industrial Engineering*, *128*, 576–590.

Aggarwal, A, Sangal, I., & Singh, S. (2017). Optimal policy for non-instantaneous decaying inventory model with learning effect with partial shortages. *International Journal of Computer Applications*, *161*(10), 13–18.

Ahmad, B., & Benkherouf, L. (2020). On an optimal replenishment policy for inventory models for non-instantaneous deteriorating items with stock dependent demand and partial backlogging. *RAIRO-Operations Research*, *54*(1), 69–79.

Alamri, O. A., Jayaswal, M. K., Khan, F. A., & Mittal, M. (2022). An EOQ model with carbon emissions and inflation for deteriorating imperfect quality items under learning effect. *Sustainability*, *14*(3), 1365. https://doi.org/https://doi.org/10.3390/su14031365

Alqahtani, A. Y., & Gupta, S. M. (2017). Warranty as a marketing strategy for remanufactured products. *Journal of Cleaner Production*, *161*, 1294–1307.

Asadkhani, J., Fallahi, A., & Mokhtari, H. (2022). A sustainable supply chain under vmi-cs agreement with withdrawal policies for imperfect items. *Journal of Cleaner Production*, *376*, 134098. https://doi.org/https://doi.org/10.1016/j.jclepro.2022.134098

Ayres, R., Ferrer, G., & Van Leynseele, T. (1997). Eco-efficiency, asset recovery and remanufacturing. *European Management Journal*, *15*(5), 557–574.

Bakker, M., Riezebos, J., & Teunter, R. H. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, *221*(2), 275–284.

Bandaly, D., Satir, A., & Shanker, L. (2016). Impact of lead time variability in supply chain risk management. *International Journal of Production Economics*, *180*, 88–100.

Bardhan, S., Modak, I., & Giri, B. C. (2025). A multi-period inventory model with price, time and service level dependent demand under preservation technology investment. *European Journal of Industrial Engineering*, *19*(1), 18–44.

Bardhan, S., Pal, H., & Giri, B. C. (2019). Optimal replenishment policy and preservation technology investment for a non-instantaneous deteriorating item with stock-dependent demand. *Operational Research*, *19*(2), 347–368.

Barman, D., & Mahata, G. C. (2022). Two-echelon production inventory model with imperfect quality items with ordering cost reduction depending on controllable lead time. *International Journal of System Assurance Engineering and Management*, *13*(5), 2656–2671.

Barron, Y. (2023). Integrating replenishment policy and maintenance services in a stochastic inventory system with bilateral movements. *Mathematics*, *11*(4), 864. https://doi.org/10.3390/math11040864

Bart, N., Chernonog, T., & Avinadav, T. (2021). Revenue-sharing contracts in supply chains: A comprehensive literature review. *International Journal of Production Research*, *59*(21), 6633–6658.

Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, *51*(1), 30–44.

Chang, H.-J., & Dye, C.-Y. (1999). An EOQ model for deteriorating items with time varying demand and partial backlogging. *Journal of the Operational Research Society*, *50*(11), 1176–1182.

Chang, W.-S., & Lin, Y.-T. (2019). The effect of lead-time on supply chain resilience performance. *Asia Pacific Management Review*, *24*(4), 298–309.

Chen, J., Dong, M., Rong, Y., & Yang, L. (2018). Dynamic pricing for deteriorating products with menu cost. *Omega*, *75*, 13–26.

Chen, X., Li, L., & Zhou, M. (2012). Manufacturer's pricing strategy for supply chain with warranty period-dependent demand. *Omega*, *40*(6), 807–816.

Chen, Z.-S., Wu, S., Govindan, K., Wang, X.-J., Chin, K.-S., & Martíınez, L. (2022). Optimal pricing decision in a multi-channel supply chain with a revenue-sharing contract. *Annals of Operations Research*, *318*(1), 67–102.

Christopher, M. (1998). Logistics and supply chain management. london: Financial times. *Prentice Hall.(2000)" The Agile Supply Chain." Industrial Marketing Management*, *29*(1), 37–44.

Cui, L., Guo, S., & Zhang, H. (2020). Coordinating a green agri-food supply chain with revenue-sharing contracts considering retailers' green marketing efforts. *Sustainability*, *12*(4), 1289. https://doi.org/10.3390/su12041289

Das, S. C., Zidan, A., Manna, A. K., Shaikh, A. A., & Bhunia, A. K. (2020). An application of preservation technology in inventory control system with price dependent demand and partial backlogging. *Alexandria Engineering Journal*, *59*(3), 1359–1369.

Das Roy, M., & Sana, S. S. (2017). Random sales price-sensitive stochastic demand: An imperfect production model with free repair warranty. *Journal of Advances in Management Research*, *14*(4), 408–424.

Dey, B. K., Bhuniya, S., & Sarkar, B. (2021). Involvement of controllable lead time and variable demand for a smart manufacturing system under a supply chain management. *Expert Systems with Applications*, *184*, 115464. https://doi.org/https://doi.org/10.1016/j.eswa.2021.115464

Dorfman, R. (1969). An economic interpretation of optimal control theory. *The American Economic Review*, *59*(5), 817–831.

Dye, C.-Y. (2020). Optimal joint dynamic pricing, advertising and inventory control model for perishable items with psychic stock effect. *European Journal of Operational Research*, *283*(2), 576–587.

Dye, C.-Y., Chang, H.-J., & Teng, J.-T. (2006). A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. *European Journal of Operational Research*, *172*(2), 417–429.

Dye, C.-Y., & Hsieh, T.-P. (2012). An optimal replenishment policy for deteriorating items with effective investment in preservation technology. *European Journal of Operational Research*, *218*(1), 106–112.

Dye, C.-Y., & Yang, C.-T. (2016). Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects. *Omega*, *62*, 52–67.

Emmos, H, & Gilbert, S. (1998). The role of returns policies in pricing and inventory decisions for catalogue goods. *Management Science*, *44*(2), 276–283.

Evans, G. C. (1924). The dynamics of monopoly. *The American Mathematical Monthly*, *31*(2), 77–83.

Fan, T., Xu, C., & Tao, F. (2020). Dynamic pricing and replenishment policy for fresh produce. *Computers & Industrial Engineering*, *139*, 106127. https://doi.org/https://doi.org/10.1016/j.cie.2019.106127

Feng, L. (2019). Dynamic pricing, quality investment, and replenishment model for perishable items. *International Transactions in Operational Research*, *26*(4), 1558–1575.

Gaula, A. K., & Jha, J. (2024). Pricing strategy with quality improvement in a dual collection channel closed-loop supply chain under return uncertainty. *Operational Research*, *24*, 27. https://doi.org/https://doi.org/10.1007/s12351-024-00836-7

Ghosh, D., & Shah, J. (2015). Supply chain analysis under green sensitive consumer demand and cost sharing contract. *International Journal of Production Economics*, *164*, 319–329.

Ghosh, S. K., & Chaudhuri, K. (2006). An EOQ model with a quadratic demand, time-proportional deterioration and shortages in all cycles. *International Journal of Systems Science*, *37*(10), 663–672.

Giri, B., & Bardhan, S. (2014). Coordinating a supply chain with backup supplier through buyback contract under supply disruption and uncertain demand. *International Journal of Systems Science: Operations & Logistics*, *1*(4), 193–204.

Giri, B., Mondal, C, & Maiti, T. (2018). Analysing a closed-loop supply chain with selling price, warranty period and green sensitive consumer demand under revenue sharing contract. *Journal of Cleaner Production*, *190*, 822–837.

Giri, B. C., & Bardhan, S. (2012). Supply chain coordination for a deteriorating item with stock and price-dependent demand under revenue sharing contract. *International Transactions in Operational Research*, *19*(5), 753–768.

Glickman, T. S., & Berger, P. D. (1976). Optimal price and protection period decisions for a product under warranty. *Management Science*, *22*(12), 1381–1390.

Goyal, S. K., & Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research*, *134*(1), 1–16.

Gross, D., & Soriano, A. (1969). The effect of reducing leadtime on inventory levels-simulation analysis. *Management Science*, *16*(2), B61–B76.

Halat, K., Hafezalkotob, A., & Sayadi, M. K. (2021). Cooperative inventory games in multi-echelon supply chains under carbon tax policy: Vertical or horizontal? *Applied Mathematical Modelling*, *99*, 166–203.

Hollier, R., & Mak, K. (1983). Inventory replenishment policies for deteriorating items in a declining market. *International Journal of Production Research*, *21*(6), 813–836.

Hsieh, T.-P., & Dye, C.-Y. (2013). A production–inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. *Journal of Computational and Applied Mathematics*, *239*, 25–36.

Hsieh, T.-P., & Dye, C.-Y. (2017). Optimal dynamic pricing for deteriorating items with reference price effects when inventories stimulate demand. *European Journal of Operational Research*, *262*(1), 136–150.

Hsu, P., Wee, H., & Teng, H. (2010). Preservation technology investment for deteriorating inventory. *International Journal of Production Economics*, *124*(2), 388–394.

Huang, Y.-S., Hsu, C.-S., & Ho, J.-W. (2014). Dynamic pricing for fashion goods with partial backlogging. *International Journal of Production Research*, *52*(14), 4299–4314.

Jaber, M. Y. (2006). Learning and forgetting models and their applications. *Handbook of Industrial and Systems Engineering*, *30*(1), 30–127.

Jaber, M., Goyal, S., & Imran, M. (2008). Economic production quantity model for items with imperfect quality subject to learning effects. *International Journal of Production Economics*, *115*(1), 143–150.

Jauhari, W. A., Ramadhany, S. C. N., Rosyidi, C. N., Mishra, U., & Hishamuddin, H. (2023). Pricing and green inventory decisions for a supply chain system with green investment and carbon tax regulation. *Journal of Cleaner Production*, *425*, 138897. https://doi.org/https://doi.org/10.1016/j.jclepro.2023.138897

Jayaswal, M. K., Mittal, M, Sangal, I, & Tripathi, J. (2021a). Fuzzy-based EOQ model with credit financing and backorders under human learning. *International Journal of Fuzzy System Applications (IJFSA)*, *10*(4), 14–36.

Jayaswal, M., Mittal, M, & Sangal, I. (2021b). Ordering policies for deteriorating imperfect quality items with trade-credit financing under learning effect. *International Journal of System Assurance Engineering and Management*, *12*, 112–125.

Jayaswal, M., Sangal, I, & Mittal, M. Learning effect on stock-policies with imperfect quality and deteriorating items under trade credit. In: In *2019 Amity International Conference on Artificial Intelligence (AICAI)*. IEEE. 2019, 499–504.

Jha, J., & Shanker, K. (2013). Single-vendor multi-buyer integrated production-inventory model with controllable lead time and service level constraints. *Applied Mathematical Modelling*, *37*(4), 1753–1767.

Jian, J., Li, B., Zhang, N., & Su, J. (2021). Decision-making and coordination of green closed-loop supply chain with fairness concern. *Journal of Cleaner Production*, *298*, 126779. https://doi.org/https://doi.org/10.1016/j.jclepro.2021.126779

John, E. P., & Mishra, U. (2023a). A sustainable three-layer circular economic model with controllable waste, emission, and wastewater from the textile and fashion industry. *Journal of Cleaner Production*, *388*, 135642. https://doi.org/https://doi.org/10.1016/j.jclepro.2022.135642

John, E. P., & Mishra, U. (2023b). Sustainable circular economy production system with emission control in led bulb companies. *Environmental Science and Pollution Research*, *30*(21), 59963–59990.

Jones, P. C., Lowe, T. J., Traub, R. D., & Kegler, G. (2001). Matching supply and demand: The value of a second chance in producing hybrid seed corn. *Manufacturing & Service Operations Management*, *3*(2), 122–137.

Karim, R., & Nakade, K. (2021). Modelling a one retailer–one manufacturer supply chain system considering environmental sustainability and disruption. *International Journal of Systems Science: Operations & Logistics*, *8*(4), 297–320.

Keller, A., Vogelsang, M., & Totzek, D. (2022). How displaying price discounts can mitigate negative customer reactions to dynamic pricing. *Journal of Business Research*, *148*, 277–291.

Keshavarz-Ghorbani, F, & Arshadi Khamseh, A. (2022). Modeling and optimizing a multi-period closed-loop supply chain for pricing, warranty period, and quality management. *Journal of Ambient Intelligence and Humanized Computing*, *13*(4), 2061–2089.

Khan, M. A.-A., Shaikh, A. A., & Cárdenas-Barrón, L. E. (2021). An inventory model under linked-to-order hybrid partial advance payment, partial credit policy, all-units discount and partial backlogging with capacity constraint. *Omega*, *103*, 102418. https://doi.org/https://doi.org/10.1016/j.omega.2021.102418

Khan, M., Jaber, M., & Wahab, M. (2010). Economic order quantity model for items with imperfect quality with learning in inspection. *International Journal of Production Economics*, *124*(1), 87–96.

Khanna, A., Pritam, P., & Jaggi, C. K. (2020). Optimizing preservation strategies for deteriorating items with time-varying holding cost and stock-dependent demand. *Yugoslav Journal of Operations Research*, *30*(2), 237–250.

Khanra, S., Ghosh, S. K., & Chaudhuri, K. (2011). An EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. *Applied Mathematics and Computation*, *218*(1), 1–9.

Konstantaras, I, Skouri, K, & Jaber, M. (2012). Inventory models for imperfect quality items with shortages and learning in inspection. *Applied Mathematical Modelling*, *36*(11), 5334–5343.

Kotler, P., & Keller, K. (2009). *Marketing management*. Pearson Education India.

Kumar, S., Handa, N., Singh, S., & Yadav, D. (2015). Production inventory model for two-level trade credit financing under the effect of preservation technology and learning in supply chain. *Cogent Engineering*, *2*(1), 1045221. https://doi.org/https://doi.org/10.1080/23311916.2015.1045221

Lai, X., Chen, Z., & Bidanda, B. (2019). Optimal decision of an economic production quantity model for imperfect manufacturing under hybrid maintenance policy with shortages and partial backlogging. *International Journal of Production Research*, *57*(19), 6061–6085.

Li, G., He, X., Zhou, J., & Wu, H. (2019). Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items. *Omega*, *84*, 114–126.

Li, J., Lai, K. K., & Li, Y. (2024). Remanufacturing and low-carbon investment strategies in a closed-loop supply chain under multiple carbon policies. *International Journal of Logistics Research and Applications*, *27*(1), 170–192.

Li, M., & Mizuno, S. (2022). Comparison of dynamic and static pricing strategies in a dual-channel supply chain with inventory control. *Transportation Research Part E: Logistics and Transportation Review*, *165*, 102843. https://doi.org/https://doi.org/10.1016/j.tre.2022.102843

Li, R., Xiao, L., & Yao, D. (2023). Dynamic pricing and production control in a two-item make-to-stock system with flexible dual sourcing and lost sales. *Production and Operations Management*, *32*(10), 3119–3137.

Liao, C.-J., & Shyu, C.-H. (1991). An analytical determination of lead time with normal demand. *International Journal of Operations & Production Management*, *11*(9), 72–78.

Lin, H.-J. (2017). Two-critical-number control policy for a stochastic production inventory system with partial backlogging. *International Journal of Production Research*, *55*(14), 4123–4135.

Lin, K. Y. (2006). Dynamic pricing with real-time demand learning. *European Journal of Operational Research*, *174*(1), 522–538.

Liu, A., Li, D., Jie, M., & Li, Z. (2022). Coordination through revenue sharing contract in an e-commerce supply chain with consumer preference. *Managerial and Decision Economics*, *43*(6), 2467–2479.

Liu, G., Zhang, J., & Tang, W. (2015). Joint dynamic pricing and investment strategy for perishable foods with price-quality dependent demand. *Annals of Operations Research*, *226*(1), 397–416.

Liu, H., & Yan, Q. (2024). Revenue-sharing contract in a three-tier online supply chain under eb platform financing. *Finance Research Letters*, *59*, 104773. https://doi.org/https://doi.org/10.1016/j.frl.2023.104773

Lu, C.-J., Gu, M., Lee, T.-S., & Yang, C.-T. (2022). Impact of carbon emission policy combinations on the optimal production-inventory decisions for deteriorating items. *Expert Systems with Applications*, *201*, 117234. https://doi.org/https://doi.org/10.1016/j.eswa.2022.117234

Lu, J., Zhang, J., Jia, X., & Zhu, G. (2019). Optimal dynamic pricing, preservation technology investment and periodic ordering policies for agricultural products. *RAIRO-Operations Research*, *53*(3), 731–747.

Luo, R., Zhou, L., Song, Y., & Fan, T. (2022). Evaluating the impact of carbon tax policy on manufacturing and remanufacturing decisions in a closed-loop supply chain. *International Journal of Production Economics*, *245*, 108408. https://doi.org/https://doi.org/10.1016/j.cie.2019.106127

Macías-López, A., Cárdenas-Barrón, L. E., Peimbert-García, R. E., & Mandal, B. (2021). An inventory model for perishable items with price-, stock-, and time-dependent demand rate considering shelf-life and nonlinear holding costs. *Mathematical Problems in Engineering*, *2021*, 6630938. https://doi.org/https://doi.org/10.1155/2021/6630938

Mahata, S., & Debnath, B. K. (2022). A profit maximization single item inventory problem considering deterioration during carrying for price dependent demand and preservation technology investment. *RAIRO-Operations Research*, *56*(3), 1841–1856.

MahmoumGonbadi, A., Genovese, A., & Sgalambro, A. (2021). Closed-loop supply chain design for the transition towards a circular economy: A systematic literature review of methods, applications and current gaps. *Journal of Cleaner*

Production, *323*, 129101. https://doi.org/https://doi.org/10.1016/j.jclepro.2021.129101

Maihami, R., Karimi, B., & Fatemi Ghomi, S. M. T. (2017). Effect of two-echelon trade credit on pricing-inventory policy of non-instantaneous deteriorating products with probabilistic demand and deterioration functions. *Annals of Operations Research*, *257*, 237–273.

Malleeswaran, B, & Uthayakumar, R. (2022). A vendor-managed inventory model for imperfect production process using sustainability investment and energy consumption under different carbon policies. *Process Integration and Optimization for Sustainability*, *6*(3), 707–722.

Mishra, U., Cárdenas-Barrón, L. E., Tiwari, S., Shaikh, A. A., & Treviño-Garza, G. (2017). An inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. *Annals of Operations Research*, *254*(1), 165–190.

Mishra, U., Tijerina-Aguilera, J., Tiwari, S., & Cárdenas-Barrón, L. E. (2021). Production inventory model for controllable deterioration rate with shortages. *RAIRO-Operations Research*, *55*, S3–S19.

Modak, I., Bardhan, S., & Giri, B. C. (2024a). Dynamic pricing and preservation investment strategy for perishable products under quality, price and greenness dependent demand. *Journal of Industrial and Management Optimization*, *20*(7), 2260–2281.

Modak, I., Bardhan, S., & Giri, B. C. (2025). Random pricing, product collection, and green investment strategies in a closed-loop supply chain with price dependent demand and remanufacturing. *Journal of Cleaner Production*, *486*, 144523. https://doi.org/https://doi.org/10.1016/j.jclepro.2024.144523

Modak, I., Bardhan, S., & Giri, B. C. (2024b). Dynamic pricing and replenishment policy under price, time, and service level-dependent demand and preservation investment. *Journal of Management Analytics*, *11*(2), 276–301.

Momeni, M. A., Jain, V., Govindan, K., Mostofi, A., & Fazel, S. J. (2022). A novel buy-back contract coordination mechanism for a manufacturer-retailer circular supply chain regenerating expired products. *Journal of Cleaner Production*, *375*, 133319. https://doi.org/https://doi.org/10.1016/j.jclepro.2022.133319

Monahan, G. E., Petruzzi, N. C., & Zhao, W. (2004). The dynamic pricing problem from a newsvendor's perspective. *Manufacturing & Service Operations Management*, *6*(1), 73–91.

Mondal, C., & Giri, B. C. (2020). Pricing and used product collection strategies in a two-period closed-loop supply chain under greening level and effort dependent demand. *Journal of Cleaner Production*, *265*, 121335. https://doi.org/https://doi.org/10.1016/j.jclepro.2020.121335

Mondal, R., Das, S., Das, S. C., Shaikh, A. A., & Bhunia, A. K. (2023). Pricing strategies and advance payment-based inventory model with partially backlogged shortages under interval uncertainty. *International Journal of Systems Science: Operations & Logistics*, *10*(1), 2070296. https://doi.org/https://doi.org/10.1080/23302674.2022.2070296

Murphy, P. R., & Poist, R. F. (2003). Green perspectives and practices: A "comparative logistics" study. *Supply Chain Management: An International Journal*, *8*(2), 122–131.

Padmanabhan, G, & Vrat, P. (1995). EOQ models for perishable items under stock dependent selling rate. *European Journal of Operational Research*, *86*(2), 281–292.

Padmanabhan, V., & Png, I. P. (1995). Returns policies: Make money by making good. *Sloan Management Review*, *37*, 65–65.

Pal, B., & Adhikari, S. (2019). Price-sensitive imperfect production inventory model with exponential partial backlogging. *International Journal of Systems Science: Operations & Logistics*, *6*(1), 27–41.

Pal, B., Sana, S. S., & Chaudhuri, K. (2017). A stochastic production inventory model for deteriorating items with products' finite life-cycle. *RAIRO-Operations Research*, *51*(3), 669–684.

Palsule-Desai, O. D. (2013). Supply chain coordination using revenue-dependent revenue sharing contracts. *Omega*, *41*(4), 780–796.

Pan, J. C.-H., & Yang, J.-S. (2002). A study of an integrated inventory with controllable lead time. *International Journal of Production Research*, *40*(5), 1263–1273.

Pang, Z. (2011). Optimal dynamic pricing and inventory control with stock deterioration and partial backordering. *Operations Research Letters*, *39*(5), 375–379.

Park, K. S. (1982). Inventory model with partial backorders. *International Journal of Systems Science*, *13*(12), 1313–1317.

Pervin, M., Roy, S. K., & Weber, G.-W. (2018). Analysis of inventory control model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration. *Annals of Operations Research*, *260*, 437–460.

Petruzzi, N. C., & Dada, M. (2002). Dynamic pricing and inventory control with learning. *Naval Research Logistics (NRL)*, *49*(3), 303–325.

Prakash, D., & Spann, M. (2022). Dynamic pricing and reference price effects. *Journal of Business Research*, *152*, 300–314.

Rabbani, M., Zia, N. P., & Rafiei, H. (2016). Joint optimal dynamic pricing and replenishment policies for items with simultaneous quality and physical quantity deterioration. *Applied Mathematics and Computation*, *287*, 149–160.

Rajan, A., Rakesh, & Steinberg, R. (1992). Dynamic pricing and ordering decisions by a monopolist. *Management Science*, *38*(2), 240–262.

Rajput, R., & Venkataraman, S. V. (2024). Revenue-sharing contract with government subsidy: A case of the indian sugar supply chain. *Computers & Industrial Engineering*, *191*, 110159. https://doi.org/https://doi.org/10.1016/j.cie.2024.110159

Ramandi, M. D., & Bafruei, M. K. (2020). Effects of government's policy on supply chain coordination with a periodic review inventory system to reduce greenhouse gas emissions. *Computers & Industrial Engineering*, *148*, 106756. https://doi.org/https://doi.org/10.1016/j.cie.2020.106756

Rout, C., Paul, A., Kumar, R. S., Chakraborty, D., & Goswami, A. (2020). Cooperative sustainable supply chain for deteriorating item and imperfect production under different carbon emission regulations. *Journal of Cleaner Production*, *272*, 122170. https://doi.org/https://doi.org/10.1016/j.jclepro.2020.122170

Saga, R. S., Jauhari, W. A., Laksono, P. W., & Dwicahyani, A. R. (2019). Investigating carbon emissions in a production-inventory model under imperfect production, inspection errors and service-level constraint. *International Journal of Logistics Systems and Management*, *34*(1), 29–55.

Salami, M. S., Eslamipirharati, M., Bakhshi, A., Aghsami, A., Jolai, F., & Yazdani, M. (2022). Does a buyback contract coordinate a reverse supply chain facing remanufacturing capacity disruption and returned product quality uncertainty? *Sustainability*, *14*(23), 15939. https://doi.org/https://doi.org/10.3390/su142315939

Samanta, B., Giri, B. C., & Chaudhuri, K. (2023). A supply chain model with two competitive buyers under a hybrid greening cost and revenue-sharing contract. *Journal of Management Analytics*, *10*(2), 270–307.

Sana, S. S. (2011). The stochastic EOQ model with random sales price. *Applied Mathematics and Computation*, *218*(2), 239–248.

Sana, S. S. (2012). Price sensitive demand with random sales price–a newsboy problem. *International Journal of Systems Science*, *43*(3), 491–498.

Sana, S. S. (2020). Price competition between green and non green products under corporate social responsible firm. *Journal of Retailing and Consumer Services*, *55*, 102118. https://doi.org/https://doi.org/10.1016/j.jretconser.2020.102118

Sarada, Y, & Sangeetha, S. (2022). Coordinating a reverse supply chain with price and warranty dependent random demand under collection uncertainties. *Operational Research*, *22*(4), 4119–4158.

Sarkar, A., & Pal, B. (2022). Pricing and service strategies in a dual-channel supply chain under return–refund policy. *International Journal of Systems Science: Operations & Logistics*, *9*(3), 281–301.

Sarkar, B., Dey, B. K., Sarkar, M., & AlArjani, A. (2021). A sustainable online-to-offline (o2o) retailing strategy for a supply chain management under controllable lead time and variable demand. *Sustainability*, *13*(4), 1756. https://doi.org/https://doi.org/10.1016/j.eswa.2021.115464

Sarkar, S., Tiwari, S., & Giri, B. (2022). Impact of uncertain demand and lead-time reduction on two-echelon supply chain. *Annals of Operations Research*, *315*(2), 2027–2055.

Sarkar, T, Ghosh, S. K., & Chaudhuri, K. (2012). An optimal inventory replenishment policy for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles. *Applied Mathematics and Computation*, *218*(18), 9147–9155.

Sepehri, A., Mishra, U., & Sarkar, B. (2021). A sustainable production-inventory model with imperfect quality under preservation technology and quality improvement investment. *Journal of Cleaner Production*, *310*, 127332. https://doi.org/https://doi.org/10.1016/j.jclepro.2021.127332

Sethi, S., & Thompson, G. (2000, January). Applications to production and inventory. https://doi.org/10.1007/0-387-29903-3_6

Shah, N. H., Chaudhari, U., & Jani, M. Y. (2016). An integrated production-inventory model with preservation technology investment for time-varying deteriorating item under time and price sensitive demand. *International Journal of Inventory Research*, *3*(1), 81–98.

Shah, N. H., & Naik, M. K. (2018). Inventory model for non-instantaneous deterioration and price-sensitive trended demand with learning effects. *International Journal of Inventory Research*, *5*(1), 60–77.

Shekarian, E. (2020). A review of factors affecting closed-loop supply chain models. *Journal of Cleaner Production*, *253*, 119823. https://doi.org/https://doi.org/10.1016/j.jclepro.2019.119823

Shi, J., Du, Q., Lin, F., Li, Y., Bai, L., Fung, R. Y., & Lai, K. K. (2020). Coordinating the supply chain finance system with buyback contract: A capital-constrained newsvendor problem. *Computers & Industrial Engineering*, *146*, 106587. https://doi.org/https://doi.org/10.1016/j.cie.2020.106587

Singh, S., & Saxena, N. (2013). A closed loop supply chain system with flexible manufacturing and reverse logistics operation under shortages for deteriorating items. *Procedia Technology*, *10*, 330–339.

Sodhi, M. S., Sodhi, N. S., & Tang, C. S. (2014). An EOQ model for mro customers under stochastic price to quantify bullwhip effect for the manufacturer. *International Journal of Production Economics*, *155*, 132–142.

Srivastava, S. K. (2007). Green supply-chain management: A state-of-the-art literature review. *International Journal of Management Reviews*, *9*(1), 53–80.

Taleizadeh, A. A., Alizadeh-Basban, N., & Niaki, S. T. A. (2019). A closed-loop supply chain considering carbon reduction, quality improvement effort, and return policy under two remanufacturing scenarios. *Journal of Cleaner Production*, *232*, 1230–1250.

Tang, O., Grubbström, R. W., & Zanoni, S. (2007). Planned lead time determination in a make-to-order remanufacturing system. *International Journal of Production Economics*, *108*(1-2), 426–435.

Tatsiopoulos, I., & Kingsman, B. (1983). Lead time management. *European Journal of Operational Research*, *14*(4), 351–358.

Tharani, S, & Uthayakumar, R. (2020). A novel approach to safety stock management in an integrated supply chain with controllable lead time and ordering cost reduction using present value. *RAIRO-Operations Research*, *54*(5), 1327–1346.

Tian, B., Yu, L., & Chang, X. (2024). Examining supply chain financing: A comparative and coordination analysis under a buyback contract. *RAIRO-Operations Research*, *58*(2), 1899–1925.

Tiwari, S., Sana, S. S., & Sarkar, S. (2018). Joint economic lot sizing model with stochastic demand and controllable lead-time by reducing ordering cost and setup cost. *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas*, *112*(4), 1075–1099.

Van, P., Loon, & Van, L. N., Wassenhove. (2018). Assessing the economic and environmental impact of remanufacturing: A decision support tool for oem suppliers. *International Journal of Production Research*, *56*(4), 1662–1674.

Wangsa, I. (2017). Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions. *International Journal of Industrial Engineering Computations*, *8*(4), 453–480.

Wei, J., Wang, Y., & Zhao, J. (2018). Interaction between greening and remanufacturing strategies in a manufacturer-retailer supply chain. *Journal of Cleaner Production*, *189*, 585–601.

Whitin, T. M. (1957). *Theory of inventory management*. Princeton University Press.

Wright, T. P. (1936). Factors affecting the cost of airplanes. *Journal of the Aeronautical Sciences*, *3*(4), 122–128.

Wu, K.-S., Ouyang, L.-Y., & Yang, C.-T. (2006). An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. *International Journal of Production Economics*, *101*(2), 369–384.

Xin, C., Zhou, Y., Sun, M., & Chen, X. (2022). Strategic inventory and dynamic pricing for a two-echelon green product supply chain. *Journal of Cleaner Production*, *363*, 132422. https://doi.org/https://doi.org/10.1016/j.jclepro.2022.132422

Xu, Z., Pokharel, S., & Elomri, A. (2023). An eco-friendly closed-loop supply chain facing demand and carbon price uncertainty. *Annals of Operations Research*, *320*(2), 1041–1067.

Yadav, D., Singh, S., Kumar, S, & Cárdenas-Barrón, L. (2022). Manufacturer-retailer integrated inventory model with controllable lead time and service level constraint under the effect of learning-forgetting in setup cost. *Scientia Iranica*, *29*(2), 800–815.

Yang, N., & Zhang, R. (2014). Dynamic pricing and inventory management under inventory-dependent demand. *Operations Research*, *62*(5), 1077–1094.

Yazdian, S. A., Shahanaghi, K., & Makui, A. (2016). Joint optimisation of price, warranty and recovery planning in remanufacturing of used products under linear and non-linear demand, return and cost functions. *International Journal of Systems Science*, *47*(5), 1155–1175.

Ye, P., Wang, S., Xu, X., & Jiang, A. (2022). An integrated decision making model for dynamic pricing and inventory control of substitutable products based on demand learning. *Journal of Industrial Engineering and Management*, *15*(3), 484–497.

Yu, C., Qu, Z., Archibald, T. W., & Luan, Z. (2020). An inventory model of a deteriorating product considering carbon emissions. *Computers & Industrial Engineering*, *148*, 106694. https://doi.org/https://doi.org/10.1016/j.cie.2020.106694

Yu, H., Mi, J., & Xu, N. (2024). Recycling, pricing decisions, and coordination in the dual-channel closed-loop supply chain: A dynamic perspective. *Journal of Cleaner Production*, *456*, 142297. https://doi.org/https://doi.org/10.1016/j.jclepro.2024.142297

Zhang, J., Bai, Z., & Tang, W. (2014). Optimal pricing policy for deteriorating items with preservation technology investment. *Journal of Industrial & Management Optimization*, *10*(4), 1261–1277.

Zhang, J., Lu, J., & Zhu, G. (2023). Optimal shipment consolidation and dynamic pricing policies for perishable items. *Journal of the Operational Research Society*, *74*(3), 719–735.

Zhang, J., Wang, Y., Lu, L., & Tang, W. (2015). Optimal dynamic pricing and replenishment cycle for non-instantaneous deterioration items with inventory-level-dependent demand. *International Journal of Production Economics*, *170*, 136–145.

Zhu, Q., Sarkis, J., & Geng, Y. (2005). Green supply chain management in china: Pressures, practices and performance. *International Journal of Operations & Production Management*, *25*(5), 449–468.

List of publications

1. Nabin Sen, **Indrani Modak**, Sudarshan Bardhan, and Bibhas C. Giri, 2025. VMI-CS policy in a price-and-stock-dependent demand scenario under buyer's space limitation and preservation investment. Accepted for publication in *IMA Journal of Management Mathematics*, DoI: 10.1093/imaman/dpaf009.

2. **Indrani Modak**, Sudarshan Bardhan, and Bibhas C. Giri, 2025. Optimal dynamic pricing and preservation policies under quality, stock and price sensitive demand with partial backlogging and controllable lead time. Accepted for publication in *Journal of System Science: Operations & Logistics*, vol 12, issue 1, article id 2462098.

3. **Indrani Modak**, Sudarshan Bardhan, and Bibhas C. Giri, 2025. Random pricing, product collection, and green investment strategies in a closed-loop supply chain with price dependent demand and remanufacturing. *Journal of Cleaner Production*, vol 486, article id 144523.

4. Sudarshan Bardhan, **Indrani Modak** and Bibhas C. Giri, 2025. A multi-period inventory model with price, time and service level dependent demand under preservation technology investment. *European Journal of Industrial Engineering*, vol 19, issue 1, pages 18-44.

5. **Indrani Modak**, Sudarshan Bardhan and Bibhas C. Giri, 2024. Dynamic pricing and preservation investment strategy for perishable products under quality, price and greenness dependent demand. *Journal of Industrial and Management Optimization*, vol 20, issue 7, pages 2260-2281.

6. **Indrani Modak** and Bibhas C. Giri, 2024. Note on "Joint dynamic pricing and freshness-keeping effort strategy for perishable products with price-, freshness-, and stock-dependent demand". *OPSEARCH*, DoI: 10.1007/s12597-024-00837-2.

7. **Indrani Modak**, Sudarshan Bardhan, and Bibhas C. Giri, 2024. Dynamic pricing and replenishment policy under price, time, and service level-dependent demand and preservation investment. *Journal of Management Analytics*, vol 11, issue 2, pages 276-301.

INTERNATIONAL JOURNAL OF SYSTEMS SCIENCE: OPERATIONS & LOGISTICS
2025, VOL. 12, NO. 1, 2462098
https://doi.org/10.1080/23302674.2025.2462098

Optimal dynamic pricing and preservation policies under quality-, stock- and price-sensitive demand with partial backlogging and controllable lead time

Indrani Modak [a], Sudarshan Bardhan [b] and Bibhas C. Giri [a]

[a]Department of Mathematics, Jadavpur University, Kolkata, India; [b]Department of Mathematics, Kanchrapara College, Kanchrapara, West Bengal, India

ABSTRACT

In any inventory system, price is one of the key factors to stimulate demand and generate profit. Apart from price, market demand usually depends on product's stock level and its freshness. However, freshness declines with time and customers tend to avoid such items. In such a dynamic environment, price is expected to vary with time as well. Reducing the transport time is crucial for perishable products particularly when products start deteriorating soon after production. This paper addresses the dynamic pricing problem under price, stock, and freshness-dependent demand. Investments in both preservation and lead time reduction are considered. The business period under consideration is divided into two parts: the first one is the stock-out period where investments for preserving freshness and reducing lead time are considered, and the second one is the stock-in period. The optimal dynamic pricing strategy is determined using Pontryagin's maximum principle. Numerical results reveal that, even for products with inventory level-dependent demand (or similar characteristics), the stock-in price can be lower than the stock-out price and the slope of the price function depends on stock sensitivity.

ARTICLE HISTORY
Received 24 June 2024
Accepted 25 January 2025

KEYWORDS
Dynamic pricing; inventory level-dependent demand; freshness; shortages; controllable lead time; preservation investment

1. Introduction

Price can be looked upon as the most dictating factor for the demand. As Katharine Paine quoted, 'The moment you make a mistake in pricing, you're eating into your reputation or your profits'[1]. The management thereby needs to be very careful while deciding the pricing strategy, especially when it comes to maximising the profit. Pricing can be the game-changing strategy in this age of technology since the customers are completely aware of what they are buying or they can get the information with only a click. Along with this, the price factor could also be used as an incentive for the customers when the other factors of demand are not in favour of business owners. To make the price most effective, the best idea is to avoid keeping it static and go with the flow.

Deterioration of inventory is defined as the loss of a product's value over time. When it comes to perishable products, fruits and vegetables top the list. These are also the types of products for which freshness is the most important consideration for consumers. Fresh foods and veggies are considered to be healthier choices. Customers won't hesitate to pay relatively more for a product that they feel is better for them. However, deterioration is a completely natural process and every product will begin to deteriorate over time, no matter how carefully one treats them. The type of degradation may vary from product to product. It may be freshness loss for food or evaporation for gasoline, but whatever be the process, it ultimately results in less sales and increased costs for management. That is why preservation technology is so important for business managers nowadays. Preservation can reduce the deterioration rate significantly; a smart preservation method keeps the product fresh for a longer period of time, resulting in business revenue. The product gradually begins to lose its freshness over time. The expiry date can be considered as the time limit after which the product is considered to be unusable. This paper typically considers that the business cycle is shorter than the product expiration date and preservation technology investment slows the process down.

It is often seen as profitable for companies to maintain shortages for a smaller business period. With a shortage, the business managers make a profit without sparing the costs related to holding the items. Further, if anybody wants to study a cycle completely, transportation should also be considered. Perishable products often undergo changes right from production. The preservation investment during transportation is thus also very

CONTACT Sudarshan Bardhan sudarshan.bardhan@gmail.com Department of Mathematics, Kanchrapara College, West Bengal 743145, India

IMA Journal of Management Mathematics (2025) **00**, 1–27
https://doi.org/10.1093/imaman/dpaf009

Vendor managed inventory under price-and-stock-dependent demand, buyer's space limitations and preservation investment

Nabin Sen
School of Business, UPES, Dehradun, Uttarakhand 248007, India

Indrani Modak and Bibhas C. Giri
Department of Mathematics, Jadavpur University, Kolkata, West Bengal 700032, India

and

Sudarshan Bardhan†
Department of Mathematics, Kanchrapara College, Kanchrapara, West Bengal 743145, India
†Corresponding author: Email: sudarshan@kpcoll.ac.in

[Received on 10 June 2023; accepted on 2 February 2025]

Accepted by: M. Zied Babai

This study explores a two-level vendor managed inventory model for deteriorating items facing demand dependent on stock and price, while operating under a consignment stock policy. It presents a joint vendor-buyer model that considers varying deterioration rate and preservation technology investment for both parties. The model is developed under the constraint of limited warehouse space at the buyer's end. The study offers an analytical discourse on the model that aims to determine the optimal values for the selling price, preservation technology investment and shipment time. An assessment is conducted to determine the optimal model for a specific business through a comparative analysis of the capacitated and uncapacitated models. The applicability and robustness of the proposed model are established through the inclusion of numerical illustration and sensitivity analysis. The proposed model is deemed suitable for industries that specializes in clothing, food and pharmaceutical products, provided that all underlying assumptions of the model are adequately met.

Keywords: consignment stock; vendor managed inventory; space limitation; price and stock dependent demand; preservation investment.

1. Introduction

The coordinated implementation of vendor managed inventory (VMI) and consignment stock (CS) policy exerts a significant influence on the performance of inventory models. Within the context of VMI policy, the vendor assumes responsibility for both production and delivery of goods to the buyer's location. Additionally, the vendor is accountable for monitoring inventory levels, establishing a replenishment schedule and ensuring accurate information is collected from the buyer. According to the CS policy, the goods are shipped in limited quantities to the buyer's warehouse. At that point, the buyer has the authority to distribute the product and he/she is responsible for paying the product's market value when it is sold. The vendor retains legal ownership of the product until it is sold. Numerous researchers have

Journal of Industrial and Management Optimization
Vol. 20, No. 7, July 2024, pp. 2260–2281
doi:10.3934/jimo.2024001

DYNAMIC PRICING AND PRESERVATION INVESTMENT STRATEGY FOR PERISHABLE PRODUCTS UNDER QUALITY, PRICE AND GREENNESS DEPENDENT DEMAND

Indrani Modak*[1], Sudarshan Bardhan[2] and Bibhas C. Giri[1]

[1]Department of Mathematics, Jadavpur University
Kolkata - 700032, India

[2]Department of Mathematics, Kanchrapara College
West Bengal - 743145, India

(Communicated by Abolfazl Mirzazadeh)

Abstract. The most significant factor to consider for proper operation of a firm is demand, emphasizing the importance of selecting efficient strategies to regulate it. Price has always been the main factor influencing demand, but as people's awareness about environmental issues grows, customers are now keeping an eye on product's level of greenness too in addition to other qualities. The current paper takes into account all these factors for modelling inventory of perishable products which makes preservation technology investment an intrinsic factor as well. The article's major contribution is to propose optimal pricing strategy that can be used in various situations where demand follows the presumed specific pattern. In addition to pricing, other policies like preservation and greening investments are also taken into consideration, and several important managerial insights are derived analytically and with the help of numerical examples. The model's applicability in numerous contexts is also demonstrated.

1. **Introduction.** Business plays a crucial role in the expansion of the economy. Not only money but also a solid business plan ensures that consumers receive high-quality products, which promotes the general welfare of consumers. In order to diligently design a scenario, one must carefully analyze every aspect associated with it. In order to establish and operate a business, it is imperative to consider the market demand for the product or service being offered. Whitin [47] was one of the pioneers to address non-constant demand pattern by choosing the demand to depend on price. Numerous research works considering various types of price dependency have been carried out since then which have been extensively reviewed by Petruzzi and Dada [33], Yano and Gilbert [50], Den Boer [9], and Chen & Chen [6]. The dynamic nature of consumer preferences necessitates a corresponding responsiveness in product development and pricing as well. Thomas [45] was the first to jointly optimize the inventory policy along with pricing policy considering fixed ordering cost and stochastic demand. Liu et al. [24] worked on an inventory model with price and quality dependent demand to find out the optimal dynamic pricing and

2020 *Mathematics Subject Classification.* 90B05, 93-10.
Key words and phrases. Dynamic pricing, preservation investment, freshness, greening effect.
* Corresponding author: Indrani Modak .

Journal of Cleaner Production 486 (2025) 144523

Contents lists available at ScienceDirect

Journal of Cleaner Production

journal homepage: www.elsevier.com/locate/jclepro

Random pricing, product collection, and green investment strategies in a closed-loop supply chain with price dependent demand and remanufacturing

Indrani Modak [a], Sudarshan Bardhan [b],*, Bibhas C. Giri [a]

[a] Department of Mathematics, Jadavpur University, India
[b] Department of Mathematics, Kanchrapara College, India

ARTICLE INFO

Handling Editor: Xin Tong

Keywords:
Random pricing
Green investment
Product collection
Remanufacturing
Carbon emission
Closed loop supply chain

ABSTRACT

The most crucial parameter in all commercial ventures is a product's price. However, there always lies a gap between what the retailer wants to charge and what the market is prepared to pay. This propensity coupled with a few other variables lead to an unpredictability in the real price, which in turn causes a randomness in demand. In the modern world, environmental issues are also important. Investments in greening initiatives and product recycling are thought to have a good impact on environment in addition to raising corporate profits. Keeping these factors into consideration, the current study explores a two-period closed-loop integrated supply chain model while taking the product collection into account. Considering both the manufacturing and remanufacturing to be uncertain in nature, the authors first illustrate the traditional model without the environmental consideration, and extend the model considering the carbon emission & carbon cap-and-trade policy under greening investment. The model is further elaborated through numerical examples to establish the business opportunity associated with the said investments. Green investment is seen to boost profit when carbon emission is considered, and remanufacturing is also seen to be a profit-enhancing option. The model suggests that the business manager should focus on market expansion, demand forecasting, and boosting customers' awareness towards using green products in general, and on efficient production process and cost control in a high-cost business scenario. Sensitivity analysis provides elaborate managerial insights on how the manager should take action with change in model parameters.

1. Introduction

Concerns about the environment are growing at the same rate now-a-days as business prosperity. The impact of business on the environment have become apparent more and more; the manufacturing, marketing, and sale of everything have an adverse effect on the environment. As mentioned by Jauhari et al. (2023), the manufacturing industry ranks among the highest in emissions. To help meet emission reduction target, governments have implemented various carbon policies, including carbon taxes, carbon caps, and cap-and-trade systems. For instance, in 2020, the Norwegian government raised its carbon tax by 8.6%, while the Portuguese government increased its carbon tax by 84.6% compared to 2019. In addition to other commercial factors, the business manager's concerns eventually expanded to include environmental problems as well. The concern initiated the process of remanufacturing items. Remanufacturing is not only motivated by just environmental considerations, though; as identified by Van and Van (2018), economic factor also plays vital role while considering investment in it. In case of a bottle of shampoo, customer use it and throw it away when it becomes empty. However, with one's television, the customer goes to the store and often exchanges the older one while buying a new one. The marginal value of the used product in the first example is so low that most individuals are not even concerned about it in general. However, when environmental issues are considered, a planned remanufacturing of items with even lower marginal values adds some contribution to the environment as well. The process of remanufacturing the used goods always results in preserving some units of raw material which may be utilized for another project. In addition, the process of remanufacturing is also less expensive than new production (Mondal and Giri, 2020). Thus, it is worth examining how remanufacturing impacts the overall business picture. The process of remanufacturing is initiated with the collection of the used products. Not every unit of collected product can make it to the production facility; each item must first pass screening before the chosen things may proceed to the next stage of the production process (Wei et al., 2018). While it is true that factors such as the quality of the raw materials and the conditions of the production facility affect how much a product can

* Corresponding author.
E-mail addresses: indranim.math.rs@jadavpuruniversity.in (I. Modak), sudarshan@kpcoll.ac.in (S. Bardhan), bibhasc.giri@jadavpuruniversity.in (B.C. Giri).

https://doi.org/10.1016/j.jclepro.2024.144523
Received 23 March 2024; Received in revised form 14 November 2024; Accepted 17 December 2024
Available online 26 December 2024

A multi-period inventory model with price, time and service level dependent demand under preservation technology investment

Sudarshan Bardhan

Department of Mathematics,
Kanchrapara College, India
Email: sudarshan.bardhan@gmail.com

Indrani Modak* and Bibhas Chandra Giri

Department of Mathematics,
Jadavpur University, India
Email: indranimodak33@gmail.com
Email: bcgiri.jumath@gmail.com
*Corresponding author

Abstract: Price and time are two important parameters having significant impact on market demand, especially for fashion items, newly launched electronic products, etc. After-sale service facility offered by the retailers is seen to boost demand while investing in preservation technology reduces product spoilage. All these issues are taken into consideration while developing a multi-period inventory model where market demand depends on all three of the above-mentioned factors. The replenishment cycles are all of equal length, but due to the time-dependent nature of demand, the stock-in (and consequently stock-out) periods in the cycles are allowed to vary. The policy of planned shortages followed by replenishment in each cycle is adopted and seen to be fruitful indeed. Learning effect in holding and ordering costs are taken into account. The effects of limited capital and warehousing space are investigated. Numerical examples are employed to demonstrate the developed model and gain managerial insights from it. [Received: 23 December 2022; Accepted: 3 August 2023]

Keywords: price dependent demand; learning effect; preservation technology; service level; multi-period inventory model.

Reference to this paper should be made as follows: Bardhan, S., Modak, I. and Giri, B.C. (2025) 'A multi-period inventory model with price, time and service level dependent demand under preservation technology investment', *European J. Industrial Engineering*, Vol. 19, No. 1, pp.18–44.

Biographical notes: Sudarshan Bardhan is an Assistant Professor of Mathematics in the Kanchrapara College, India. He received his MS and PhD both from the Jadavpur University, India. His research interest lies in supply chain management and inventory modelling under various deterministic and stochastic scenarios. He has got research works published in various journals

OPSEARCH
https://doi.org/10.1007/s12597-024-00837-2

THEORETICAL ARTICLE

Note on "Joint dynamic pricing and freshness-keeping effort strategy for perishable products with price-, freshness-, and stock-dependent demand"

Indrani Modak[1] · Bibhas C. Giri[1]

Accepted: 5 August 2024

Abstract
Recently Shi and You (J. Ind. Manage. Optim. 19(9):6572–6592, 2023) developed an inventory model with price and freshness dependent demand. However, after studying their model carefully, it is observed that some of their calculations were erroneous, thereby nullifying subsequent optimal pricing policy and numerical results. The current work aims at pointing the error out and rectifying it.

Keywords Dynamic pricing · Freshness-keeping effort · Perishable products · Optimal control · Maximum principle

1 Introduction

[2] Conducted an investigation on a dynamic optimal control problem considering the market demand to depend on price, freshness, and inventory level simultaneously. The distinguishing assumption of their model lies in its demand pattern. In contrast to prevailing inventory models that employ continuous dynamic pricing and exhibit a demand that is multiplicatively dependent on inventory levels, the above-mentioned study employs an additive demand pattern. The authors posit that the process of deterioration under consideration is of qualitative nature, characterised by an instantaneous occurrence. Their objective is to determine the optimal dynamic pricing and preservation investment strategy which maximize the total profit of the system. The investment in preservation technology is assumed to follow a linear control model. [2] Determined the optimal inventory level and associated factors through analysis of optimal pricing and preservation technology investment strategies.

Indrani Modak
indranimodak33@gmail.com

Bibhas C. Giri
bcgiri.jumath@gmail.com

[1] Department of Mathematics, Jadavpur University, Kolkata 700032, India

Published online: 17 September 2024

Journal of Management Analytics, 2024
Vol. 11, No. 2, 276–301, https://doi.org/10.1080/23270012.2024.2306616

Dynamic pricing and replenishment policy under price, time, and service level-dependent demand and preservation investment

Indrani Modak [a], Sudarshan Bardhan [b*] and Bibhas Chandra Giri [a]

[a]*Department of Mathematics, Jadavpur University, Kolkata, India;* [b]*Department of Mathematics, Kanchrapara College, Kanchrapara, India*

(*Received 9 March 2023; revised 11 July 2023; accepted 14 January 2024*)

Time, price, and servicing are three important factors which affect the demand for any product. In a multi-period inventory model, different pricing in different periods may have a positive impact on demand. Offering discounts to customers during a shortage period is an efficient way to reduce lost sales. To address all these issues, a multi-period inventory model is developed and analyzed in this paper to obtain optimal pricing and replenishment periods where the market demand depends on price, time and service level provided by the retailer. The time horizon is fixed; the replenishment cycles are of equal length, but the shortage period may vary in different cycles. The retailer is allowed to charge different prices in different periods and provide discounts if someone places demand during a shortage period. Learning effects in holding and ordering costs are considered. Under a dynamic pricing framework, this study examines service improvement through learning and price discounts during shortages as an incentive for customers. It demonstrates that discounting indeed helps to produce more revenue when the back-order rate is low, and the service level is critical in generating demand when the stock-out period is lengthy, revealing a critical relationship between shortage periods and service level. A numerical experiment is performed to investigate the optimal policy in different situations and obtain some managerial insights.

Keywords: price-dependent demand; learning effect; preservation technology; dynamic pricing; price discount

1. Introduction

In today's world, business decisions have a significant impact on a nation's economic development, and so making intelligent business decisions is very important. To make the business flourish ever than before, business managers are trying every technique at their disposal and it is the researchers who analyze more or less every aspect of it to make their implementation rational rather than driven by some mere instinct. Products' demand usually varies over time due to changes of demand parameters which cannot be controlled by the managers. In this scenario, to maintain the demand level and keep the business running smoothly, it is important to be flexible enough to change the pricing rather than keeping it static throughout the business period. Dynamic pricing is a policy where the price of a product may fluctuate over time per changes in demand, freshness or a combination of these factors.

*Corresponding author. Email: sudarshan.bardhan@gmail.com