

Geometric Pattern Formation on Triangular and Rectangular Grids

Swarm robotics focuses on utilizing a group of simple, cost-effective, and autonomous robots that work collaboratively to accomplish a specific task. This approach offers several advantages over a single, highly capable robot, particularly in terms of scalability, fault tolerance, and development costs. However, a key challenge in designing such systems lies in enabling effective coordination among the robots, which is difficult due to their simplicity. Despite this challenge, researchers have shown growing interest in swarm robotics, as it holds vast potential for applications across various domains, including the military, agriculture, space exploration, and more.

In this work, we address the problem of Geometric Pattern Formation, where a group of arbitrarily deployed robots must arrange themselves into a specified geometric shape (e.g., points, line, circle) based on a given input. Specifically, this thesis focuses on forming points, lines, and circles within grid-like discrete domains. More precisely, we consider two types of grids: triangular and rectangular, both of which have significant applications in various fields, such as programmable matter. We examine this problem under different models, aiming to optimize the capabilities of the robots.

In Chapter 2 and Chapter 3, we focus on the problem of point formation, also known as *Gathering*. In Chapter 2, we investigate the minimum vision range required for myopic robots to gather on an infinite triangular grid. The robots are assumed to be oblivious (i.e., they have no memory of past actions) and silent (i.e., they lack explicit communication). We demonstrate that with a minimal 1-hop vision range, gathering is only possible if all robots share a common axis in their local coordinate systems. Based on this assumption, we propose an algorithm that gathers n robots in $O(n)$ epochs, where an *epoch* is a time span in which each robot is activated at least once, under a semi-synchronous scheduler. Additionally, we prove that this bound is optimal.

In Chapter 3, we explore the Gathering problem on a finite grid with n robots having full visibility, one of which is faulty and can move arbitrarily in any of the four directions, as long as it does not overlap with a functioning robot. We first establish that solving this problem for three robots, where one is faulty, is equivalent to solving it for n robots with a single faulty robot, given that the functional robots can identify the faulty one. We then demonstrate that even under this assumption, the problem remains unsolvable with three robots in a semi-synchronous scheduling model. However, by assuming a fully synchronous scheduler, we propose an algorithm capable of gathering three robots, and consequently, n robots on a finite grid, despite the presence of one faulty robot.

In Chapters 4 and 5, we examine the effects of physical dimensions and obstructed visibility on the Line and Circle formation problems within a discrete domain. In Chapter 4, we address the Line formation problem involving opaque, fat robots on an infinite rectangular grid. Building on this, Chapter 5 focuses on solving the Circle formation problem within the same domain. Both problems are analyzed under the luminous robot model and an asynchronous scheduler.

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