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Title of thesis: Some problems associated with graphs related to algebraic structures.

Abstract

Algebraic graph theory is an intriguing area of mathematics that investigates the connections between graphs and matrices. Algebraic graph theory affords efficient techniques for analyzing and understanding many different structures, encompassing social and computer networks. In this thesis, we constructed new graphs over groups, rings, and other algebraic structures and explored their features, particularly different topological indices of graph-theoretic and algebraic properties. The thesis consists of 7 chapters.

In **Chapter 1**, we discuss the historical backdrop and objective of this effort, as well as the theoretical preliminaries and some important results from the literature reviewed.

In **Chapter 2**, we define a new algebraic graph by adding new condition on square element graph, $S_q(G)$ over a group G , which is called the order two element graph over a group G . We denote this graph by $S_2(G)$. In this chapter, we characterize those commutative groups G for which the graph $S_2(G)$ is connected. We also characterize the structures of the graph $S_2(\mathbb{Z}_n)$ for any positive integer n . Moreover we look into the structure and other graph-theoretic properties of the symmetric group S_n , the dihedral group D_n for any positive integer n . Finally, we give a list of finite groups of order ≤ 16 with their corresponding order two element graph structures.

In **Chapter 3**, we explore the Merrifield-Simmons index, independence number, and domination number of the order two element graph over a group. We start by establishing some lovely results on the Merrifield-Simmons index of the order two element graph over a group. The product of two order two element graphs over two groups is then described. Additionally, we examine some significant findings on the domination number of order two element graph over a group.

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In **Chapter 4**, we discuss a number of distance-based topological indices over the order two element graph of a group, including the Wiener index, Hyper-Wiener index, Harary index, and Gutman index. We ascertain the precise formula for the Wiener index, Hyper-Wiener index, Harary index, and Gutman index of the order two element graph over the finite groups \mathbb{Z}_n and D_n .

In **Chapter 5**, we explore the co-maximal graph over a commutative ring R with identity which is denoted by $\Gamma'(R)$. In this chapter, we characterize Merrifield-Simmons Index of a co-maximal graph over a commutative ring. Furthermore, we characterised the Merrifield-Simmons index of co-maximal graph over the ring \mathbb{Z}_n .

In **Chapter 6**, we explore the zero-divisor graph over a commutative ring R . This graph is denoted by $\Gamma(R)$. In this chapter, we compute the Merrifield-Simmons index of the zero-divisor graph over the ring \mathbb{Z}_n (the ring of integer modulo n), for $n = p^m, pq, p^2q, p^2q^2, pqr$, where p, q, r are distinct prime numbers. Furthermore, we derive the Merrifield-Simmons index for the zero-divisor graph $\Gamma(\mathbb{Z}_{pq}[x]/\langle x^2 \rangle)$. Additionally, we compute the Merrifield-Simmons index of zero divisor graphs of some small finite commutative rings.

Finally in **Chapter-7**, we focus on two distance based topological indices, the sum-connectivity Gourava index and the product-connectivity Gourava index of a zero-divisor graph over a commutative ring. We compute the sum-connectivity Gourava index and the product-connectivity Gourava index of the zero-divisor graph over the ring \mathbb{Z}_n , for $n = p^m, pq, p^2q, p^2q^2, pqr$, where p, q are distinct prime numbers. Furthermore, we compute the sum and product connectivity Gourava index for the zero-divisor graph $\Gamma(\mathbb{Z}_{pq}[x]/\langle x^2 \rangle)$. In the final section, we compute the sum and product connectivity Gourava index of zero divisor graphs from several small finite commutative rings.

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