Abstract

Title: Study of Berezin number inequalities of operators on reproducing kernel Hilbert space

Submitted by: Anirban Sen

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In this dissertation, we develop new Berezin number inequalities for operators on reproducing kernel Hilbert spaces. We derive several interesting upper bounds of the Berezin number for bounded linear operators acting on reproducing kernel Hilbert spaces. By using the Cartesian decomposition of bounded linear operators, we obtain some lower bounds for the Berezin number. Our results are more precise than the previously established results. We then obtain inequalities involving the Berezin number and the Berezin norm of operators on reproducing kernel Hilbert spaces. As special cases, we prove several inequalities that refine the existing ones. From these inequalities, we prove an interesting equality between the Berezin number and the Berezin norm. We show that for all positive bounded linear operators defined on reproducing kernel Hilbert spaces, the Berezin number and the Berezin norm are equal. We also provide an example to demonstrate that this equality may not hold for general self-adjoint operators. Next, we define a new norm on the space of all bounded linear operators on reproducing kernel Hilbert spaces, which generalizes both Berezin number and Berezin norm. We mention this norm as the α-Berezin norm and study some basic properties of this new norm norm. Furthermore, we develop various inequalities of the α-Berezin norm and the Berezin number. By applying the relationship between the α-Berezin norm and the Berezin number, we obtain new upper bounds for the Berezin number. Next, we establish upper bounds for the Berezin number and the Berezin norm of $n \times n$ operator matrices. As a special case, for 2×2 operator matrices, our results are sharper than the existing bounds. We also provide computational examples to estimate the bounds of the Berezin number and the Berezin norm for some concrete operators acting on the Hardy-Hilbert space. Finally, we conclude with some open questions and remarks, together with a few examples which may be useful for further study in this direction.

Prof. Kallol Paul 09.12.24

Professor
DEPARTMENT OF MATHEMATICS
Jadavpur University
Kolkata - 700 032, West Bengal

Anirban Sen 04.12.24

Anirban Sen