# Comprehensive State Space Model of Hydraulic Servo System Comprising of Directional Control Valve

*A Thesis Submitted in partial fulfilment of the requirement for the Degree of Master in Electrical Engineering (Electrical Engineering Department)*


By

**Ayan Dey**

Registration No.: **163487** of **2022-2023**

Examination Roll No.: **M4ELE24010B**

Under the Guidance of

**Prof. Ranjit Kumar Barui**

Department of Electrical Engineering Jadavpur University, Kolkata-700032, India.


**November, 2024**

FACULTY OF ENGINEERING AND TECHNOLOGY JADAVPUR UNIVERSITY

## CERTIFICATE from Supervisor

This is to certify that the thesis entitled "**Comprehensive State Space Model of Hydraulic Servo System Comprising of Directional Control Valve**" has been carried out by AYAN DEY (University Registration Number **163487** of **2022-2023**, Examination Roll No **M4ELE24010B**) under the guidance of professors from the Electrical Engineering branch and be accepted in partial fulfillment of the requirement for the degree of Master of Engineering in Electrical Engineering with the specialization Control System. The research results presented in the thesis have not been included in any other paper submitted for the award of any degree to any other university or institute.

Prof. Ranjit Kumar Barai

Thesis Supervisor

Dept. of Electrical Engineering

Jadavpur University

FACULTY OF ENGINEERING AND TECHNOLOGY JADAVPUR UNIVERSITY

## CERTIFICATE from HEAD OF THE DEPARTMENT

This is to certify that the thesis entitled "**Comprehensive State Space Model of Hydraulic Servo System Comprising of Directional Control Valve**" has been carried out by AYAN DEY (University Registration Number **163487** of **2022-2023**, Examination Roll No **M4ELE24010B**) under the guidance of professors from the Electrical Engineering branch and be accepted in partial fulfillment of the requirement for the degree of Master of Engineering in Electrical Engineering with the specialization Control System. The research results presented in the thesis have not been included in any other paper submitted for the award of any degree to any other university or institute.

Prof. Biswanath Roy

Head of the Department

Electrical Engineering Department

Jadavpur University

Kolkata-700032

FACULTY OF ENGINEERING AND TECHNOLOGY JADAVPUR UNIVERSITY

## CERTIFICATE from DEAN FET

This is to certify that the thesis titled "**Comprehensive State Space Model of Hydraulic Servo System Comprising of Directional Control Valve**" has been carried out by **Ayan Dey** (University Registration Number **163487** of **2022-2023**, Roll Number **M4ELE24010B**) under the Faculty of Engineering, Electrical Engineering of Jadavpur University, and is accepted in partial fulfillment of the requirement for the degree of Master of Engineering in Electrical Engineering with the specialization Control System. The research results presented in the thesis have not been included in any other paper submitted for the award of any degree to any other university or institute.

Prof. Dipak Laha

Dean, Faculty of Engineering and Technology

Jadavpur University

Kolkata- 700032

FACULTY OF ENGINEERING AND TECHNOLOGY JADAVPUR UNIVERSITY

## CERTIFICATE OF APPROVAL(EXAMINER)

The foregoing thesis is hereby approved as a creditable study of an engineering subject and presented in a manner satisfactory to warrant its acceptance as prerequisite to obtain the degree for which it has been submitted. It is understood that by this approval the undersigned do not necessarily endorse or approve any statement made, opinion expressed or conclusion drawn there in but approve the thesis only for which it is submitted.

**Committee on Final Examination**

**for the Evaluation of the Thesis**

Dated:

______________________

Signature of the Examiner

FACULTY OF ENGINEERING AND TECHNOLOGY JADAVPUR UNIVERSITY

## CERTIFICATE OF APPROVAL(EXAMINER)

The foregoing thesis is hereby approved as a creditable study of an engineering subject and presented in a manner satisfactory to warrant its acceptance as prerequisite to obtain the degree for which it has been submitted. It is understood that by this approval the undersigned do not necessarily endorse or approve any statement made, opinion expressed or conclusion drawn there in but approve the thesis only for which it is submitted.

**Committee on Final Examination**

**for the Evaluation of the Thesis**

________________________

Signature of the Examiner

Prof. Ranjit Kumar Barai

Department of Electrical Engineering

Jadavpur University

Kolkata-700032

FACULTY OF ENGINEERING AND TECHNOLOGY JADAVPUR UNIVERSITY

# DECLARATION OF ORIGINALITY AND COMPLIANCE OF ACADEMIC THESIS

I hereby declare that this thesis entitled "**Comprehensive State Space Model of Hydraulic Servo System Comprising of Directional Control Valve**" contains literature survey and original research work by the undersigned candidate, as part of his degree of Master of Engineering in Electrical Engineering with the specialization Control System.

All information has been obtained and presented in accordance with academic rules and ethical conduct.

I also declare that, as required by these rules and conduct, I have fully cited and referenced all materials and results that are not original to this work.

Name: **Ayan Dey**

Registration No: **163487** of **2022-2023**

Examination Roll No: **M4ELE24010B**

Signature of the candidate

## Table of Contents

## List of Figures

# CHAPTER 1

# INTRODUCTION

## 1.1 Introduction

A hydraulic servo system is a sophisticated assembly that integrates electronic, hydraulic, and mechanical components. It utilizes pressurized fluid to transmit energy and control motion effectively. For a comprehensive and efficient hydraulic servo system design, it is essential to select components optimally, based on precise information about system loads, the capabilities of various subsystems and components, and, more specifically, the dynamic characteristics of the entire system. Developing an accurate state-space model is crucial to understanding these dynamic behaviors and ensuring precise control in hydraulic applications.

Hydraulic servo systems are widely used in applications such as aircraft control, automotive systems, stabilization of military equipment, and various industrial machinery. Despite the increasing adoption of electrical servo systems—driven by advancements in AC motors (especially synchronous motors with permanent magnets) and modern power electronics—hydraulic servo systems remain indispensable in fields that demand high force density and precise control. Electro-hydraulic servo systems continue to play a crucial role in applications like testing equipment, active suspension systems, mining machinery, material testing rigs, flight simulators, paper production machines, marine hydraulics, robotics, and metal rolling mills. The development of a comprehensive state-space model of these systems is essential for improving their dynamic performance and control accuracy across these diverse fields.

Hydraulic servo drives are extensively utilized in industrial applications due to their ability to generate high torques, deliver rapid response times, and perform fast motion and speed reversals. These systems convert low-power electrical control signals into the movement of valve actuators, which, in turn, manage high-power hydraulic actuators for precise control. Compared to electromagnetic drives, modeling hydraulic servo systems is significantly more complex due to the non-linear dynamics and interactions between hydraulic, mechanical, and electronic components.

Hydraulic actuators are particularly valued in industrial settings for their ability to exert large reaction forces on heavy loads, high power-to-volume ratios, robust dynamic performance, inherent rigidity, effective heat dissipation, simple overload protection, and resilience against external impacts. Additionally, they can start under load while delivering maximum power output. Due to their high power-to-weight ratios and precise controllability, hydraulic systems are also extensively used in aerospace applications. The development of a comprehensive state-space model is essential for capturing these dynamic characteristics, enabling improved performance and control in complex hydraulic servo systems.

The theoretical foundations and components of hydraulic servo systems have been explored extensively in various studies [1]-[3], [13]-[18]. In general, any regulated hydrostatic drive can be modeled as a hydraulic servo system, functioning as a closed-loop control system where specific parameters of the drive are regulated through the interaction of hydraulic components. In hydraulic servo systems, the servo mechanism typically controls high-power actuators using low-energy input signals. Within this closed-loop system, the hydraulic servo actuator adjusts the output signal through negative feedback, based on the discrepancy between the reference and actual output signals, while also amplifying the applied power. Developing a precise state-space model for such systems is essential for accurately capturing these dynamics and achieving optimal control performance.

The control performance of hydraulic servo systems relies heavily on key components, including servo controllers, servo valves, hydraulic actuators, position feedback transducers, and the power supply. The dynamic response of such systems is primarily influenced by the frequency characteristics of the servo valve and load. However, control quality can be compromised by various faults within the system, such as internal leakage, friction, or other defects that affect system response.

In this thesis, a hydraulic positioning system serves as the experimental setup. This system comprises a hydraulic power supply with a relief valve and accumulator, a flow control servo valve, a linear actuator (hydraulic cylinder), and a position sensor. The output signal of the system is the electrical signal from the position transducer, which corresponds directly to the actuator's position. Developing an accurate state-space model of this setup allows for a detailed analysis of its dynamic behavior and control accuracy.

## 1.2 Literature Survey

Hydraulic servo-systems are used in a wide variety of industrial fields. They provide many advantages over electrical motors, including high durability and the ability to produce large forces at high speeds. Unfortunately, the dynamic characteristics of these systems are highly non-linear and relatively difficult to control. The non-linearities arise from the compressibility of the hydraulic fluid, the complex flow properties of the servo-valve and the friction in the hydraulic cylinder. Owing to increased computer power and ongoing developments in control theory, expectations regarding modeling of the non-linear dynamic behavior of hydraulic servo-system have increased, the nonlinearities depend on factors which are difficult to measure or estimate online, such as oil bulk modulus, viscosity and temperature. change above paradigm for my thesis work.

In their work, Mohieddine [1] provides an outline of both classical and advanced algorithms for the automatic control of hydraulic drives, highlighting that advanced strategies can yield significant improvements over traditional methods. However, these improvements often come at the cost of more complex design and computational demands. His study also offers valuable structural insights into the behavior of hydraulic servo-systems, addressing the key dynamics and non-linearities that are crucial to understanding their performance. By emphasizing a theoretical, physically-based model of the entire servo-system, Mohieddine's approach presents a comprehensive view, underpinned by the necessary theoretical background to ensure practical applicability. The methodologies explored have been validated through simulations using realistic physical parameters, enabling a deeper understanding of the system's dynamics and non-linear behavior. As control engineering spans multiple disciplines, the monograph reflects an engineer's perspective on the subject. This work by Mohieddine Jelali and Andreas Kroll serves as a strong example of an industrial approach to control engineering, specifically within the context of hydraulic servo-systems, aligning well with the goals of this thesis to improve system modeling and control strategies.

Xingjian [4] emphasizes that achieving high-precision control in electro-hydraulic servo systems requires a comprehensive consideration of practical non-linearities, such as dynamic friction, when modeling and simulating hydraulic systems. Friction plays a critical role in affecting the control precision of these systems in real-world applications, often leading to issues like limit cycles or undesirable stick-slip motion. As one of the dominant non-linear factors in the mathematical modeling of hydraulic servo systems, friction is quantified through various identification and constant velocity experiments. Xingjian's research provides essential friction coefficients found in hydraulic cylinders, which are crucial for enhancing the accuracy of mathematical models and simulations, and for improving control strategies in practical hydraulic servo systems. This understanding of friction and its impact on system performance aligns closely with the focus of this thesis, which aims to address the non-linear behaviors that challenge the modeling and control of hydraulic servo-systems.

Kovari's work [5] investigates the impact of internal leakage on the dynamic behavior of an electro-hydraulic servo position system. Internal leakage is a critical fault in hydraulic systems that can degrade the overall quality of servo control. Kovari focuses on how this leakage affects the performance of electro-hydraulic systems, noting that such defects can lead to the need for actuator replacement or refurbishment, resulting in extended downtime. For manufacturing equipment that operates continuously, such as a hot rolling mill, this downtime can cause significant production and revenue losses. His research provides valuable relationships between internal leakage and the dynamic behavior of the electro-hydraulic servo positioning system. While many defects can influence system response, Kovari highlights internal leakage in the hydraulic cylinder, as it is often difficult to detect. To quantify the effect of internal leakage, Kovari develops a complex non-linear mathematical model that accounts for leakage in the hydraulic cylinder, offering insight into how this issue can affect system performance. This aspect of internal leakage and its impact on system dynamics will be an important consideration in the modeling and control strategies explored in this thesis.

Wonohadidjojo, D.M. Kothapalli, and G. Hassan, M.Y. [6] explore a position control system for electro-hydraulic actuator systems, emphasizing the need for robust control schemes in servo-driven applications. These applications require precise position tracking and smooth response from the actuation system. To achieve robustness and accuracy in position tracking for electro-hydraulic servo systems, the authors consider the key non-linearities of the system, including friction, internal leakage, and the effects of a variable load—specifically simulating a realistic load in a robotic excavator, which is used as a trajectory reference. The authors describe the mathematical modeling of the hydraulic servo system, employing a complete SIMULINK model. To address the non-linearities and uncertainties inherent in the system, they apply a Fuzzy Logic control strategy, which enhances robust tracking performance. This approach to modeling and control, focusing on non-linearities and real-world operating conditions, aligns with the goals of this thesis to improve control strategies for hydraulic servo systems.

Jayan and Joshi [7] conducted a simulation study for a fighter aircraft's flight control system using MATLAB. The simulation, which used inputs such as altitude, Mach number, angle of attack, and control commands, provided key outputs including pressures, flow rates, actuator positions, and accumulator gas volume. While the simulation software offers basic component models, it allows for easy inclusion of additional models. They also highlighted the impact of servo valve nonlinearity, actuator compliance, and friction on the dynamics of flight control surfaces, particularly during deployment through an electro-hydraulic actuation system. This approach to modeling non-linearities is relevant to improving hydraulic servo-system control strategies, which is a focus of this thesis.

Charles M. Close, Dean K. Frederick, and Jonathan C. Newel [8] provide valuable insights into modeling and analysis, particularly for those focused on computer techniques and feedback control systems. Emphasizing hydraulic systems, the authors highlight the importance of understanding the dynamic response of physical systems. After identifying the system dynamics, they develop mathematical models that account for disturbances, noise, and both linear and nonlinear states. This modeling approach is crucial for understanding cause-and-effect relationships within a system, such as determining desired excitations across varying parameter values. This methodology is applicable to enhancing the modeling and control of hydraulic servo-systems in this thesis.

T. Knohl and H. Unbehauen [9] explored adaptive position control using artificial neural networks for a hydraulic system comprising a 4/3 way proportional valve, a differential cylinder, and a load force. They addressed the challenge of large dead zones in the valve by modeling the cylinder and load force as a second-order system and an integrator, respectively. The dynamic model of the hydraulic system is then represented as a series connection of static input non-linearity and a linear system. This approach contributes to enhancing control strategies for hydraulic servo-systems, which is a key focus of this thesis.

Alleyne A [13], and D. McCloy and H. R. Martin [11] provide a detailed analysis of the dynamic behavior of fluids and hydraulic systems. They propose a method for identifying key fluid parameters, which are crucial for eliminating non-linearities and ensuring accurate identification of the hydraulic system's mathematical model. This approach is vital for improving the modeling and control of hydraulic servo-systems, a central focus of this thesis.

T. G. Ling, M. F. Rahmat, and A. R. Husian [12] introduced an Adaptive Neuro-Fuzzy Inference System (ANFIS) modeling technique, demonstrating its ability to accurately model nonlinear systems for precise control of electro-hydraulic actuators. They focused on position tracking accuracy, using ANFIS to develop a simpler model with fewer rules and parameters, which is preferred over more complex models. The authors also evaluated the estimation capabilities of the ANFIS model under varying input amplitudes, operating regions, and limited training data. This method offers an effective approach to system identification, ensuring high accuracy in modeling hydraulic systems, a key component of this thesis.

S. R. Lee and K. Srinivasan [10], and Bona B, Giacomello L, Greco C, and Malandra A [14] focus on fluid power and control in hydraulic systems, specifically applying nonlinear feedback linearization for position control of movable hydraulic actuators. Both studies delve deeply into the dynamic behavior of the hydraulic cylinder and the nonlinearities within the hydraulic system, particularly the role of the proportional flow control servo valve, which is controlled by the current in the command coil. They identify actuator position and velocity as key variables to be controlled. This work contributes valuable insights into controlling non-linear behaviors in hydraulic systems, aligning with the goals of this thesis.

# CHAPTER 2

# INTRODUCTION TO HYDRAULICS SERVO SYSTEM

## 2.1 Servo system

In its simplest form, a servo or servomechanism in a control system measures its own output and forces it to follow a command signal quickly and accurately (see Figure 2.1). This minimizes the impact of anomalies in the control device, load variations, and external disturbances. A servomechanism can be designed to control various physical quantities, including motion, force, pressure, temperature, voltage, or current. This foundational concept is essential to improving control accuracy in hydraulic servo-systems, a key focus of this thesis.

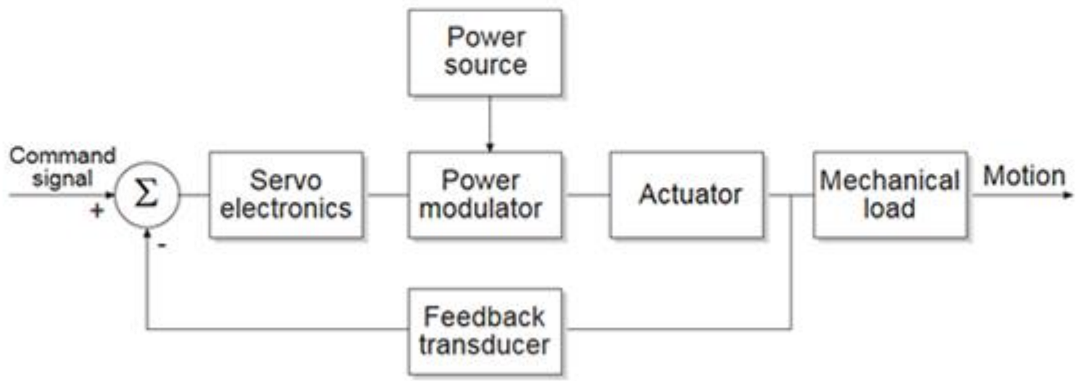



**Figure2.1** Basic servomechanism

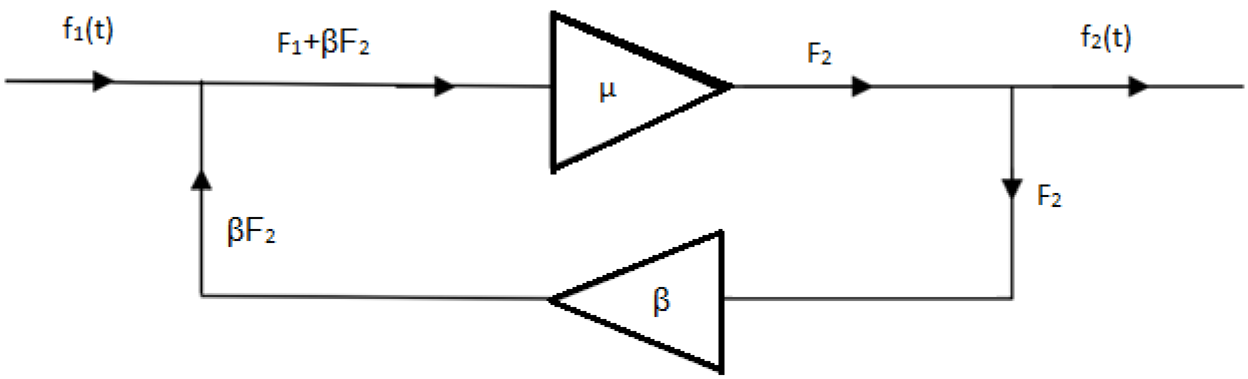
A servo system is a specialized feedback control mechanism, typically incorporating a combination of electrical, mechanical, thermal, and hydraulic elements. Through appropriate design, the behavior of these elements can be characterized using principles from both linear and non-linear system analysis. When sinusoidal signal responses are analyzed, frequency-domain techniques from feedback control theory provide valuable insights. However, given the non-linear characteristics intrinsic to hydraulic servo-systems—such as oil bulk modulus, viscosity, and temperature dependencies—additional considerations beyond linear analysis are required to accurately model and predict system performance under varying operating conditions.

A hydraulic servo system is designed to control an output variable according to a specific function of an input signal. This control mechanism operates in a feedback loop, where a property of the output—such as position or velocity—is continuously monitored and compared to the desired input, generating an error signal. This error signal is then amplified and used to adjust the output, ensuring that it follows the reference input accurately. In hydraulic servo systems, the servo mechanism functions both as a transmission system and as a closed-loop feedback control, where the challenge lies in achieving the desired transmission characteristics while ensuring stability of the feedback loop. Addressing this design problem requires careful modeling of the system dynamics, which can be effectively accomplished through state-space analysis to meet both performance and stability requirements.

The electromechanical servo systems are required to demonstrate high performance indicators in various modes.

The inclusion of elastic elements within hydraulic servo-systems, the inherent uncertainties in system parameters, variations in mass-inertia properties, complex hydraulic circuit designs, and unpredictable external and internal disturbances challenge the achievement of high dynamic performance using conventional control strategies. Implementing state control in complex hydraulic systems, particularly those with significant non-linear and elastic characteristics, enables improved dynamic response and performance. To mitigate the system's sensitivity to parameter fluctuations, structural changes in the control system, and external disturbances, robust control techniques are applied within the state control framework. This approach enhances the reliability and adaptability of hydraulic servo-systems under variable operating conditions.

The simple servo system may be divided into two basic parts, an amplifying circuit and a monitoring or comparison circuit. Such a division is




LOOP TRANSMISSION = $\mu\beta$

TRANSFER CHARACTERISTICS: $F_2 = \frac{\mu}{1-\mu\beta} F_1$

**Figure2.2** Simple servo system

A basic hydraulic servo system can be conceptually divided into two core components: an amplification circuit and a feedback or comparison circuit. This division is illustrated in Figure 2.2, where $\mu$ and $\beta$ denote the transfer characteristics of the amplification and feedback stages, respectively. $F_1$ and $F_2$ represent the sinusoidal components of the input and output signals, $f_1(t)$ and $f_2(t)$, respectively. Here, $\mu$ and $\beta$ are complex functions of the frequency variable $j\omega$, allowing frequency-dependent behavior to be analyzed as detailed in the preceding section. This separation of system functions supports a deeper understanding of the dynamic interactions within the hydraulic servo system, particularly under varying operational conditions.

The return signal $\beta F_2$ from the monitoring circuit is added to the servo input $F_1$ to form a net μ circuit input $F_1+\beta F_2$ . The servo transfer characteristics is found by setting

$$F_2= \mu(F_1+\beta F_2)$$

From which,

$$F_2= \frac{\mu}{1-\mu\beta} F_1$$

The closed system formed by the two basic circuits in tandem is of course a feedback loop, the loop transmission characteristics being given by $\mu\beta$.

## 2.2 Electro-Pneumatic and Electro-Mechanical Servos:

The potential for alternative technologies should be assessed in the light of the well-known capabilities of electro-pneumatic and electro-mechanical servos. High-performance actuation systems are characterized by wide bandwidth frequency response, low resolution, and high stiffness. Additional requirements may include demanding duty cycles and minimization of size and weight. The last mentioned requirements are of special interest in aerospace applications. The most important selection criteria can be summarized as follows:

- Customer performance
- Cost
- Size and weight
- Duty cycle
- Environment: vibration, shock, temperature, etc.

The performance available with electro-hydraulic servos encompasses every industrial and aerospace application. As indicated in Figure-2.3, electro-hydraulic servos will cover applications with higher performance than electro-mechanical and electro-pneumatic servos. This is easily explained because electro-hydraulic servo systems have been designed and developed to accomplish essentially every task that has appeared.

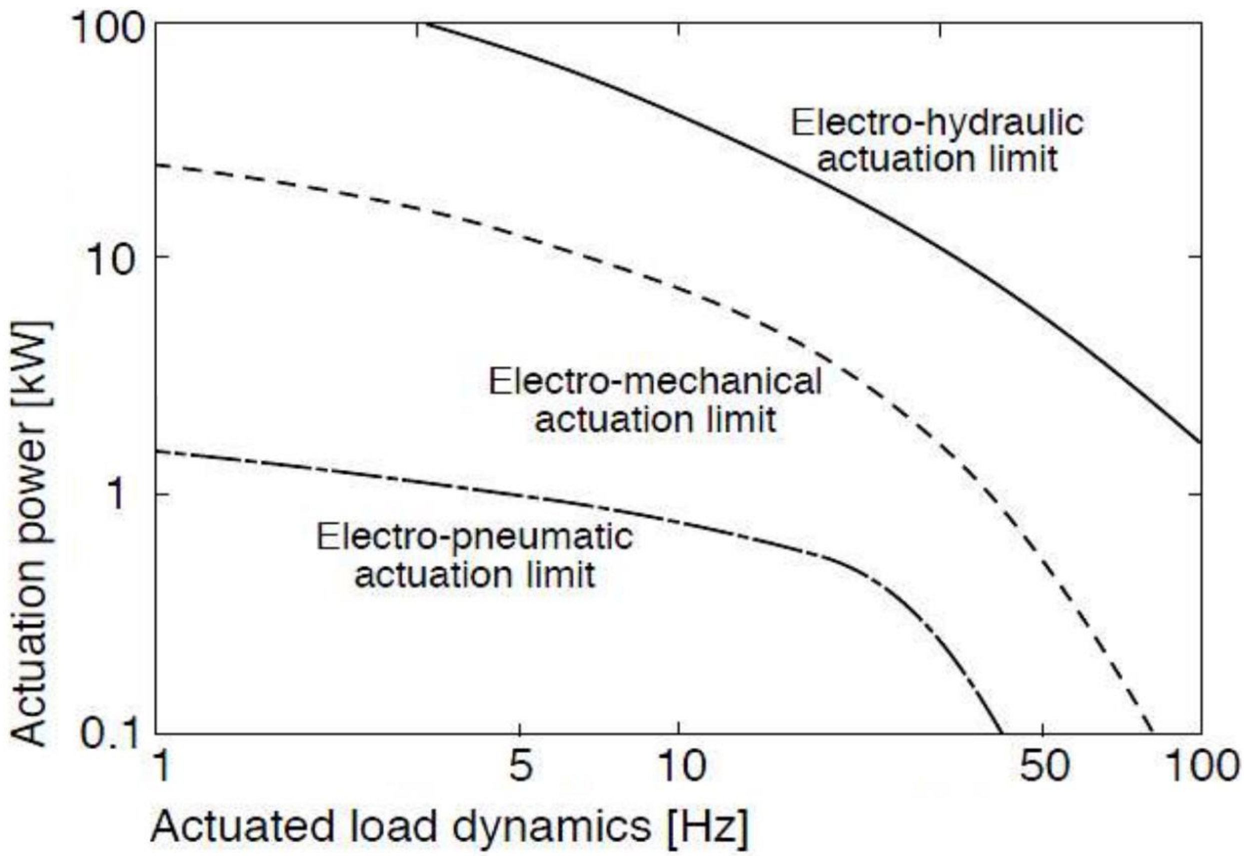



**Figure2.3** Typical performance characteristics for different types of servo actuators

As illustrated in Figure 2.3, applications with lower power and dynamic response requirements may be effectively addressed using electro-pneumatic servos. However, the optimal selection depends on a range of factors, including the selection criteria discussed previously. In most practical applications, cost considerations often play a significant role in determining the most suitable servo technology.

Practical experience suggests that electro-mechanical and electro-pneumatic actuators are generally more cost-effective than electro-hydraulic actuators for applications with lower performance requirements. However, this cost advantage diminishes rapidly as the demands for high power and high dynamic response increase, where electro-hydraulic actuators typically become more suitable.

### 2.2.1 Advantages of Electro-Hydraulic Servo Systems

For applications requiring rapid and precise control of substantial loads, electro-hydraulic servos are often the most effective solution. Hydraulic servo actuators are characterized by their fast response, high force output, and short-stroke capability. Key advantages of hydraulic components include:

- Precise and efficient control of position and velocity
- Excellent stiffness characteristics
- Zero backlash
- Quick response to changes in speed or direction
- Low wear rate over time

These attributes make electro-hydraulic servos particularly suitable for demanding applications where high performance and reliability are essential.

Hydraulic servo drives offer several distinct advantages over electric motor drives:

- **Higher Power-to-Weight Ratio**: Hydraulic drives exhibit significantly higher power-to-weight ratios, leading to increased resonant frequencies in machine frames for a given power level.
- **Greater Stiffness and Accuracy**: Hydraulic actuators are inherently stiffer than electric drives, allowing for higher loop gain, improved accuracy, and enhanced frequency response.
- **Superior Low-Speed Performance and Speed Range**: Hydraulic servo systems deliver smoother performance at low speeds and operate effectively over a wide speed range without requiring specialized control circuits.
- **Self-Cooling Capability and Stall Resistance**: Hydraulic systems are largely self-cooling and can operate in a stall condition indefinitely without risk of damage.
- **Reliability with Proper Maintenance**: Both hydraulic and electric drives offer high reliability when maintained appropriately.
- **Cost-Effectiveness at Higher Power Levels**: Hydraulic servo systems tend to be more cost-effective for applications requiring several horsepower, particularly when a single hydraulic power supply is used to support multiple actuators.

These benefits make hydraulic servo drives highly advantageous for applications demanding high performance, reliability, and efficiency.

### 2.2.2 Fundamental Components and Operation of Electro-Hydraulic Servo Systems

The fundamental components of an electro-hydraulic servo system are illustrated in Figure 2.8. In this setup, a transducer measures the output and converts it into an electrical signal, forming the feedback signal. This feedback signal is then compared with the command signal, generating an error signal. The error signal is amplified by the regulator and electric power amplifier, serving as the input control signal for the servo valve. The servo valve modulates fluid flow to the actuator in proportion to the drive current from the amplifier, thereby driving the actuator to move the load. Any change in the command signal produces an error signal, prompting the load to move in an attempt to reduce this error to zero. With a high amplifier gain, the output closely follows the command signal with high accuracy and responsiveness.

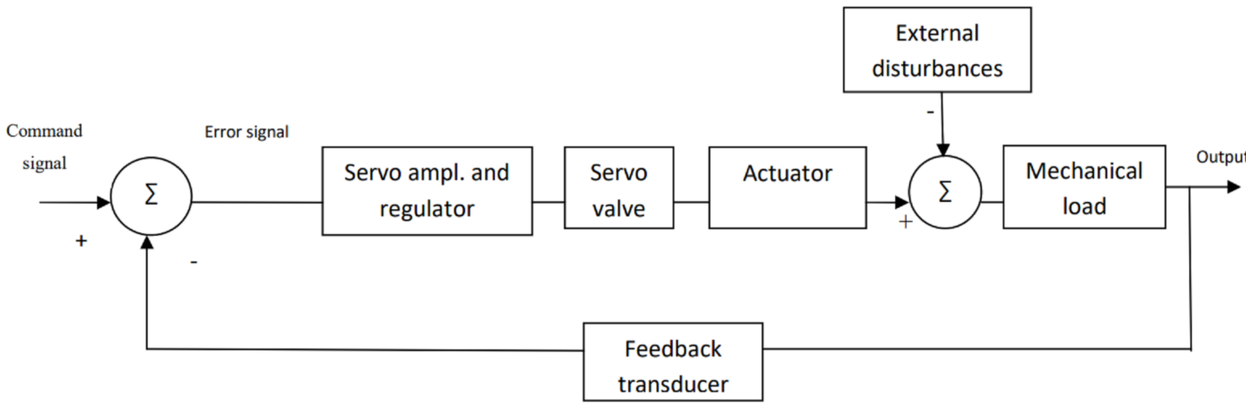



**Figure2.4** Components in an electro-hydraulic servomechanism

External disturbances, such as forces or torques, may cause the load to move independently of the command signal. To counteract these disturbances, the actuator generates an opposing output (as depicted in Figure 2.8). This requires a finite error signal, the magnitude of which decreases as amplifier gain increases. Ideally, a sufficiently high amplifier gain would make the servo system's accuracy dependent solely on the transducer's precision. However, because control loop gain is directly related to amplifier gain, stability constraints limit the achievable gain. In certain applications, stability requirements may restrict the performance level achievable by the servo system.

Electro-hydraulic servos are commonly categorized into three types:

- **Position Servo**: Controls linear or angular position
- **Velocity or Speed Servo**: Controls linear or angular velocity
- **Force or Torque Servo**: Controls force or torque output

### 2.2.2.1 Position Servo

The position servo is one of the most fundamental types of closed-loop control systems. A schematic diagram of a complete position servo system is provided in Figure 2.5. In Figure 2.5, the position of the actuator or load is measured by a position transducer, which outputs an electrical feedback signal $u_f$ in voltage. The servo amplifier compares this feedback signal $u_f$ with the command signal $u_c$ (also in voltage), generating an error signal. This error signal is then amplified by a factor $K_{sa}$ . The resulting output current iii from the amplifier controls the servo valve, regulating fluid flow to the actuator.

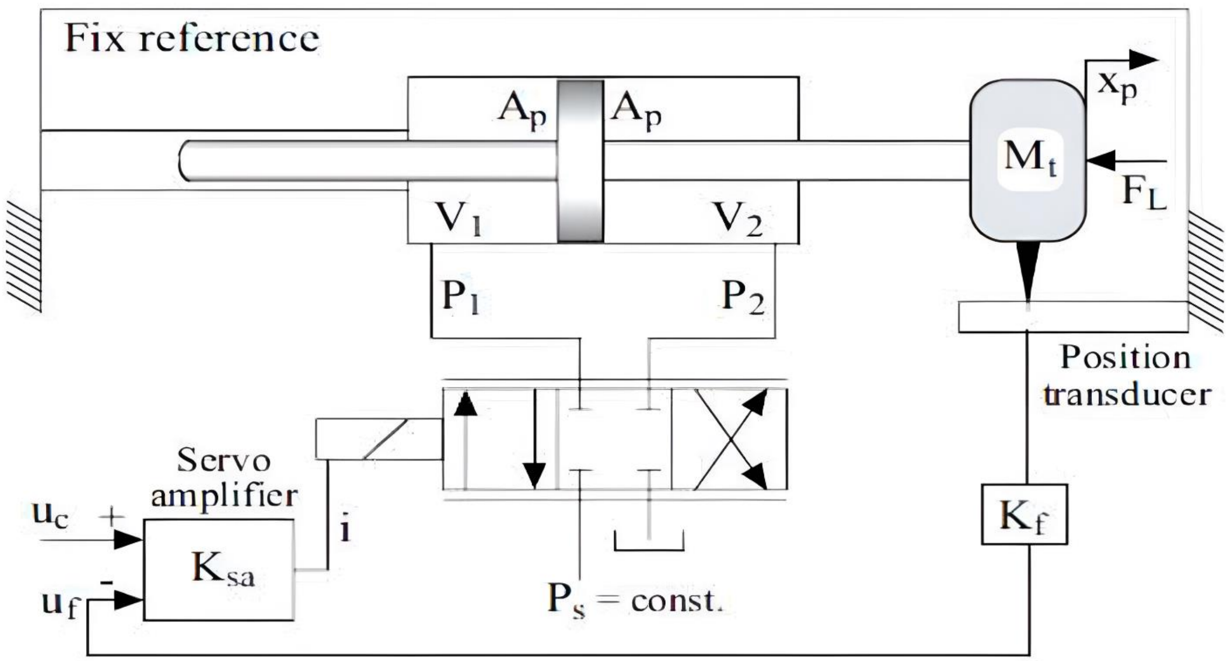



**Figure 2.5** Basic Diagram of Position Servo

### 2.2.2.2 Velocity and force servos:

Other common types of closed-loop control systems include velocity (speed) and force (torque) servos. The configuration of these systems is similar to that of the position servo illustrated in Figure 2.5, with the primary difference being that the transducer measures velocity or force instead of position. Additionally, the controller characteristics may vary. As shown in Figure 2.6, velocity servos are more commonly utilized to control the rotational speed of a hydraulic motor rather than linear velocity.

In a velocity servo, the servo amplifier typically employs an integrating type, as depicted in Figure 2.6. Unlike a position servo, there is no inherent integration between the servo valve displacement and the output velocity in a velocity servo. Consequently, integration is generally provided electronically within the amplifier to reduce static errors and ensure system stability.

In a force servo, the transducer measures the output force, feeding this signal back to the amplifier. A simpler implementation of a force servo can utilize the load pressure within the actuator as a feedback signal. While this method closely approximates a true force servo, it is affected by the frictional force present in the actuator.

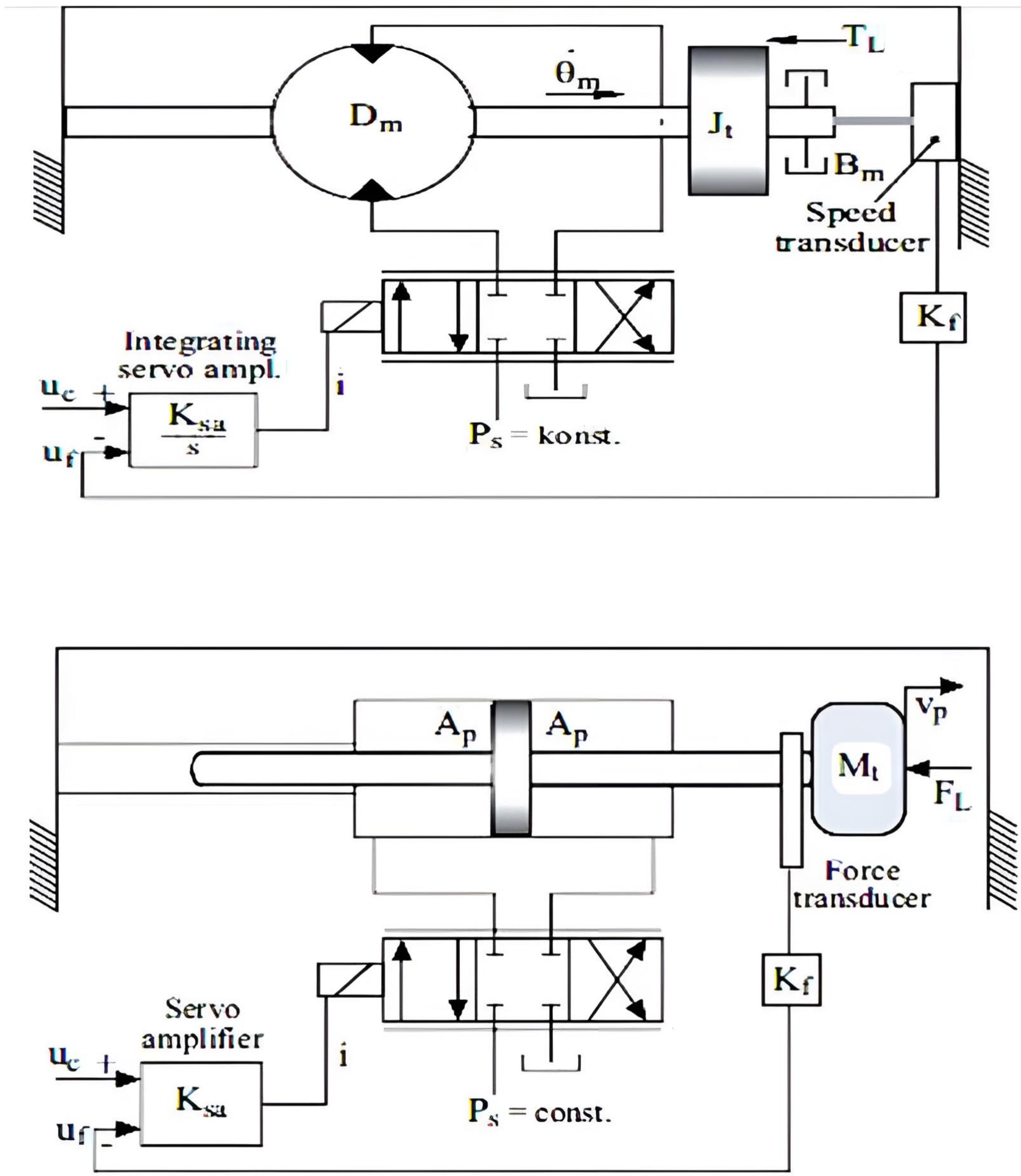



**Figure2.6** Symbol circuit of a speed and force servo respectively

## 2.3 Fundamental Structure of Hydraulic Servo-systems:

A hydraulic servo-system is an assembly of individual components designed to achieve a specific form of hydraulic energy transfer. The fundamental structure of a hydraulic system, illustrated in Figure 2.1, typically comprises the following elements:

- **Hydraulic Power Supply**: Provides the required hydraulic energy for system operation.
- **Control Elements**: Includes valves, sensors, and other components responsible for regulating fluid flow, pressure, and direction.
- **Actuating Elements**: Consists of cylinders and/or motors that convert hydraulic energy into mechanical output.
- **Additional Components**: Includes pipelines, measuring devices, and other elements necessary for system functionality.

The core operation of a hydraulic system (using a standard valve-controlled hydraulic system as an example) can be summarized as follows:

- The pump transforms mechanical energy from the prime mover (e.g., an electric or diesel motor) into hydraulic energy, directing it toward the actuator.
- Valves control the direction and magnitude of pump flow, manage the power output, and regulate the fluid pressure reaching the actuator. The actuator, whether a linear cylinder or rotary motor, then converts hydraulic energy into the desired mechanical output.
- The hydraulic fluid acts as a medium for power transmission and control, while also providing lubrication, sealing, and system cooling.
- Connectors, such as pipelines, link the system's components, channeling pressurized fluid and returning flow to the reservoir.
- Fluid storage and conditioning equipment maintain fluid quality and quantity, ensuring proper system cooling and efficient operation.

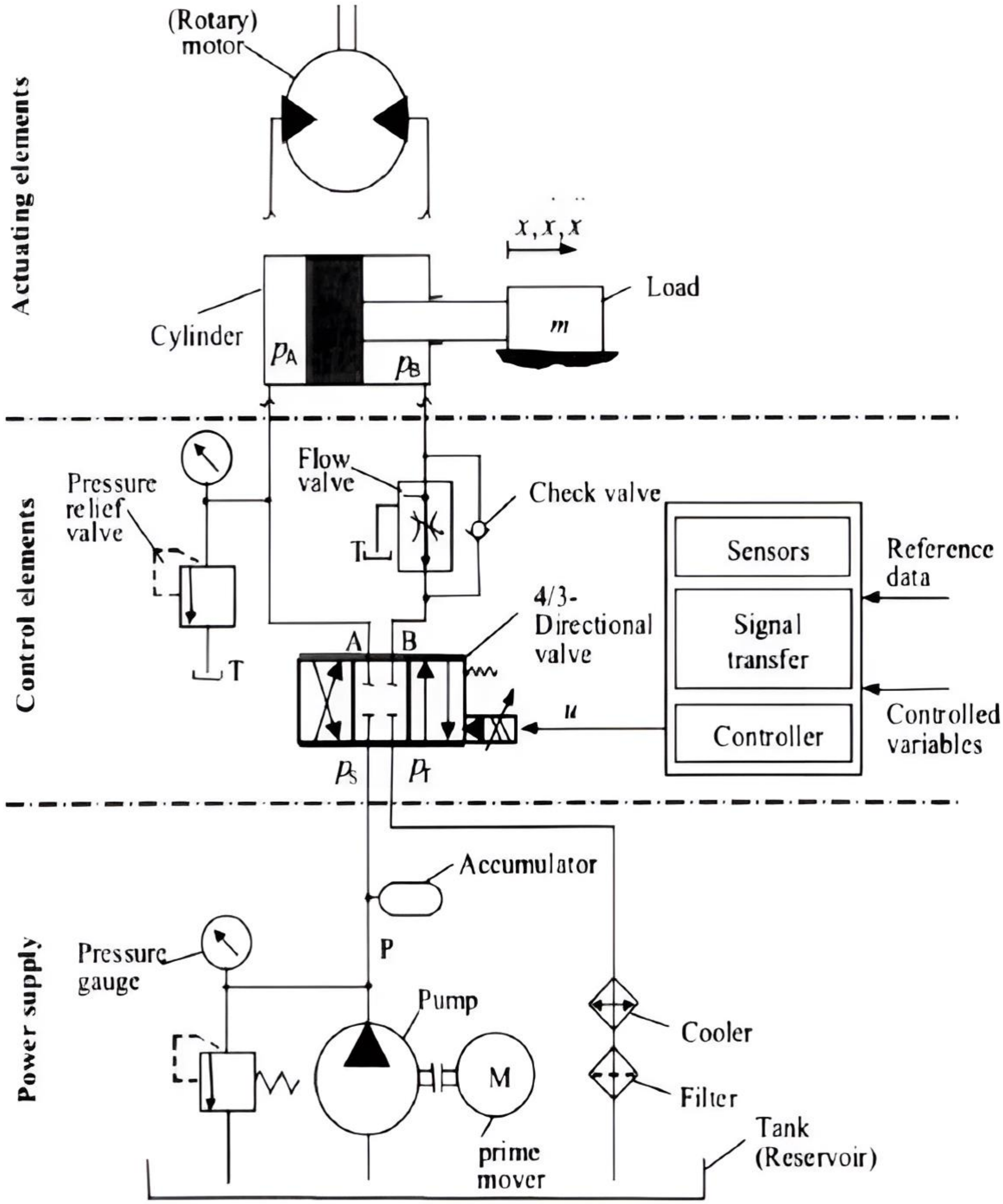



**Figure2.7** Basic structure of hydraulic systems

## 2.4 Description of the Components:

This section offers a comprehensive overview of key hydraulic components (such as pumps, valves, and actuators), detailing their functions and operation. The analysis is presented objectively, independent of the design and development philosophies of individual manufacturers.

### 2.4.1 Valves:

Valves are the most important mechanical (or electrical) link to the fluid interface in hydraulic systems.

#### 2.4.1.1 Valve Types:

There are four primary categories of valves in hydraulic systems:

a) **Pressure-relief valves** are employed to control actuator force and set or regulate pressure levels required for specific machine operations.

- These valves limit the maximum permissible system pressure and divert some or all of the pump's flow to the tank once the pressure setting is reached. Pressure-relief valves are typically "normally closed."
- **Pressure-reducing valves** serve to limit and maintain a constant downstream pressure (subcircuit pressure) lower than the system pressure, regardless of fluctuations in the upstream main circuit. These valves are usually "normally open."

b) **Check valves** are a specialized type of directional control valve that permits fluid flow in only one direction, while blocking flow in the opposite direction. They are further classified into spring-loaded or unloaded check valves and those designed for logic operations, such as OR and AND functions.

c) **Flow control valves** are used to regulate the flow rate between components in a hydraulic system, effectively controlling the maximum speed of cylinders and motors. These valves also limit the available power to sub-circuits by managing the flow in various branches of the circuit.

d) **Directional valves** act as multi-position switches. Prior to the development of servo and proportional valves, they were used to control actuator motion direction, select alternative control circuits, and perform basic logic control functions. Today, however, proportional control valves enable infinitely adjustable settings, allowing rapid and precise control of actuators in terms of force, speed, and stroke position.

### 2.4.1.2 Solenoid-valves to Servo-valves:

The simplest type of valve used for fluid power control is the solenoid valve, which operates as an on-off switch—either fully actuated or unactuated, fully open or fully closed, without any intermediate positions. For nearly a century, solenoid valves have been employed in fluid power applications with minimal changes to the original design. However, recognizing the performance limitations of hydraulic systems using solenoid valves, more advanced control concepts incorporating servo valves were developed in the 1950s, leading to the emergence of servo-hydraulic systems or servo mechanisms. Servo valves are faster-responding directional, pressure, and flow control valves, commonly utilized in closed-loop control systems to achieve the high-frequency response required for modern machinery. Among the earliest and most prominent applications of servo valves are in aerospace vehicles, particularly in primary flight control systems. Today, servo valves are extensively used across various industrial sectors, including mining, steel rolling, agriculture, transportation, and shipping.

### 2.4.1.3 Classification of Spool Valves:

Typical spool valve configurations can be classified based on several criteria:

a) **The number of "ways" (or ports)** through which flow enters and exits the valve. All valves require a supply (P: pressure), a return (T: tank), and at least one line (A, B) to the load. Common valve configurations include three-way and four-way valves, although two-way valves also exist, typically used to open or close the flow path in a single line (on/off valves).

b) **The number of switching (or discrete) positions** available. The number of discrete positions in a spool valve ranges from a single position in simple designs to two or three positions in more common configurations. Some specialized valves may have five positions or more. For example, 5/3 valves are frequently used in mobile hydraulic applications.

c) **The method of valve actuation** that moves the valve mechanism to an alternative position. Actuation can be triggered by external signal commands (such as electrical, manual, or pilot pressure) or internal signal commands (such as pilot pressure or spring force).

d) **The type of center** (open, closed, or critical) when the valve spool is in the neutral position.

### 2.4.1.4 Centre Types:

Spool 'lap' refers to the width of the lands in relation to the width of the ports in the valve bore. There are three possible lap configurations (see Figure 2.8a):

1. **Underlap (Open Centre):** If the width of the land is smaller than the port width in the valve sleeve, the valve is considered to have an open centre or underlap. During a brief period, all valve ports are interconnected, resulting in smooth, pressure-peak-free switching during crossover. However, undesirable actuator movements may occur under certain load conditions. Underlapped valves are typically preferred in closed-loop applications that require continuous flow (such as constant-flow systems or systems that need to maintain a reasonable temperature). A disadvantage of underlapped valves is the relatively high leakage flow in the neutral position, often referred to as centre flow (see Figure 2.8c).

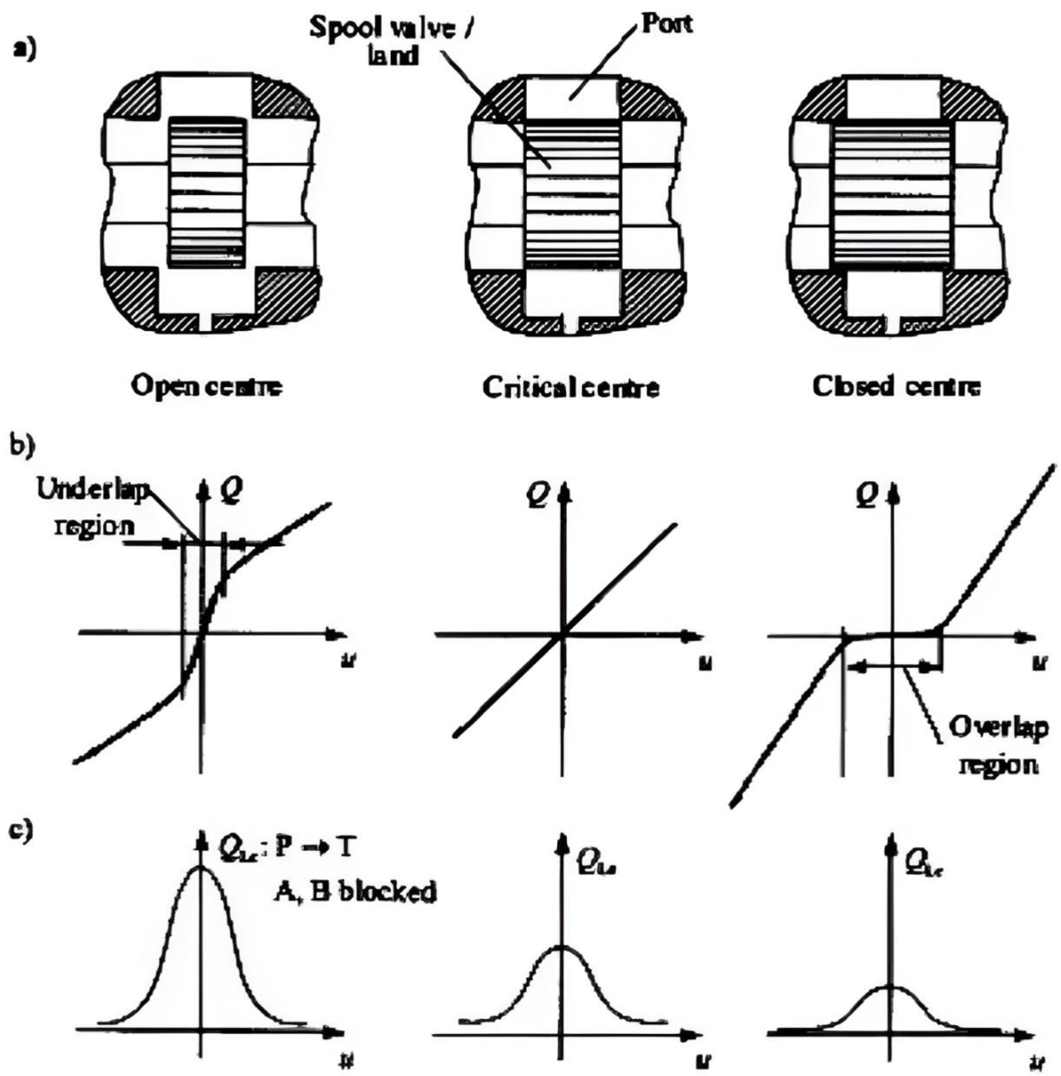



**Figure-2.8** (a) Definition of centre types and their corresponding (b) flow- signal graphs and (c) leakage flow curves

2. **Overlap (Closed Centre):** Closed-centre overlapped valves have a land width greater than the port width when the spool is at the neutral position. During a short period, all valve ports are sealed against each other, preventing system pressure on the actuator from collapsing during crossover. However, the fully closed-centre crossover (dead band) can cause backlash, leading to undesirable pressure peaks in the system, which vary with fluid flow and switching time.

3. **Critical Centre (Zero Lap):** Critical-centre or zero-lap valves have a land width identical to the port width. This configuration is often achieved through practical machining. The majority of commercially available servo-valves are zero-lap (critical-centre) valves, as this design emphasizes a linear flow-signal curve and eliminates oil consumption in the neutral state.

**2.4.1.5 Characteristics of Directional Valves**

Valves are typically named based on their position types and the number of ports. The characteristic names are as follows:

- **Two-port, two-position valves**
- **Three-port, two-position valves**
- **Four-port, two-position valves**
- **Four-port, three-position valves** (the most commonly used variety)

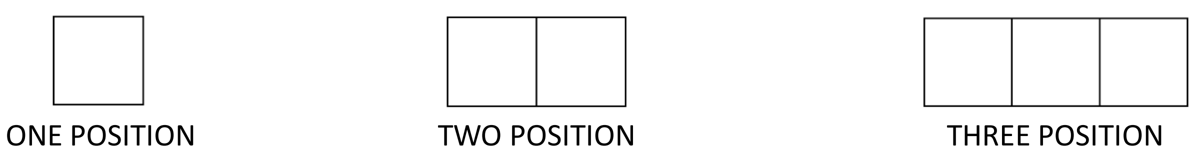



**Figure 2.9** Different types of position

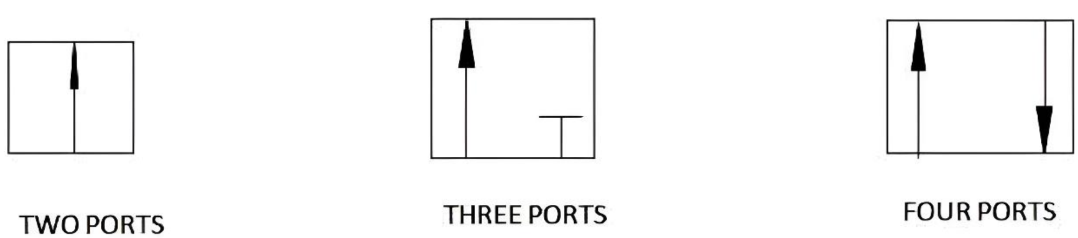



**Figure 2.10** Different typers of ports

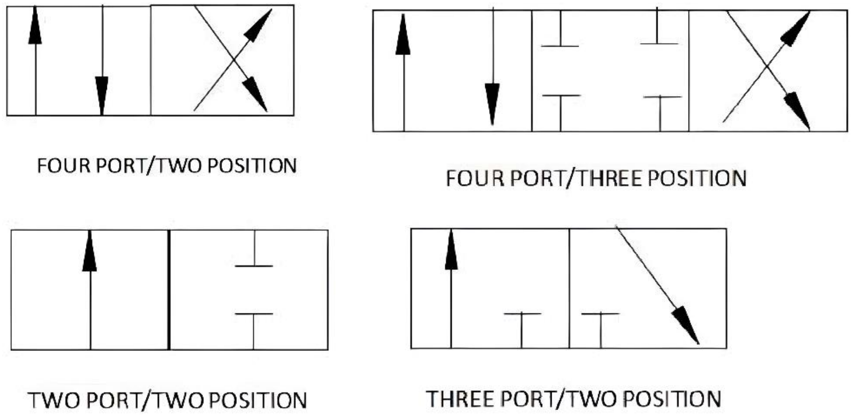



**Figure 2.11** Different typers of valves

The Figure 2.12 illustrate the functional states of a spool valve, a critical component in hydraulic systems. The spool, a cylindrical element with ports (A, B, P, and T), controls fluid flow by aligning or misaligning these ports. The four configurations depicted are:

- **Closed Center:** The spool blocks all ports, halting fluid flow.
- **Straight Through:** The spool aligns the P (pressure) and T (tank) ports, allowing direct fluid flow from pump to tank.
- **Cross-Connect:** The spool connects P to T and A to B, enabling fluid flow between these pairs of ports.
- **Closed Center:** The spool blocks all ports, halting fluid flow.

These valve configurations are indispensable for managing the direction and rate of hydraulic fluid in various applications, such as machinery and robotics.

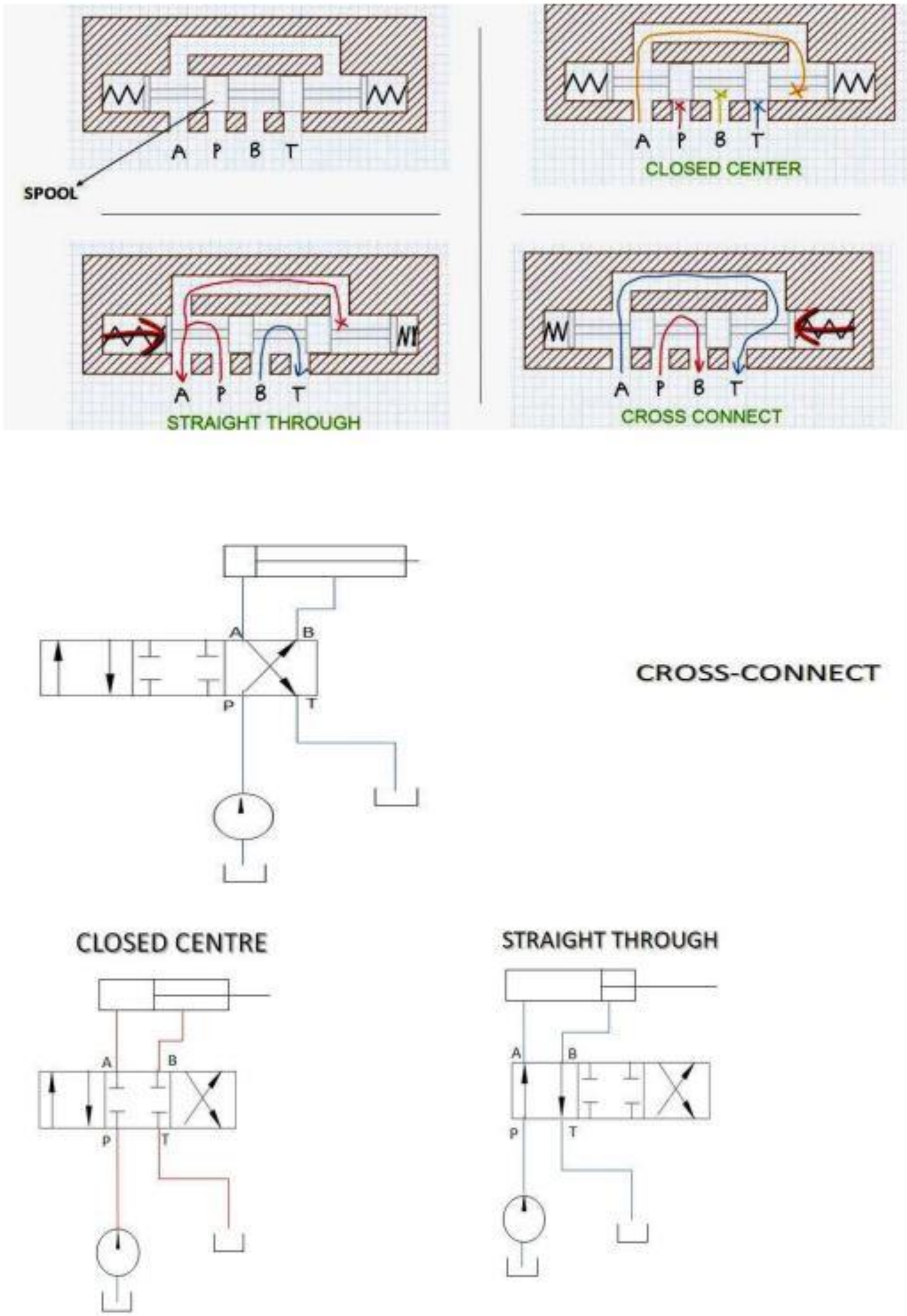



**Figure 2.12** Various Valve positions

### 2.4.2 Pumps

Hydraulic pumps are indispensable devices that efficiently convert mechanical energy into hydraulic energy. Typically driven by rotational motion, these pumps are available in a myriad of shapes, sizes, and pumping mechanisms. Industrial hydraulic pumps, predominantly of the "positive displacement" type, are characterized by their ability to deliver a fixed volume of hydraulic fluid per pump shaft rotation.

Positive displacement pumps are further categorized into gear pumps, vane pumps, and piston pumps based on their pumping mechanism. Piston pumps, available in both axial and radial configurations, are widely recognized as high-performance pumps and are extensively utilized in industrial and mobile hydraulic applications.

Unlike fixed displacement pumps, where the flow rate is predetermined by the pump's design, variable displacement pumps offer greater flexibility by enabling on-demand flow delivery. This adaptability is particularly advantageous in applications requiring precise control over hydraulic fluid flow.

### 2.4.3 Actuators (Cylinders, Motors)

Hydraulic actuators are essential components that harness the power of hydraulic fluid to generate mechanical motion. These devices, driven by the coordinated interplay of pumps and valves, are categorized into two primary types: linear actuators and rotary actuators.
Linear actuators, commonly known as hydraulic cylinders, convert hydraulic energy into linear mechanical force or motion. Single-acting cylinders apply force in a single direction, while double-acting cylinders provide force in both directions. Double-ended, or symmetric cylinders, are designed for applications requiring equal force in both extension and retraction strokes.
Asymmetric, or differential cylinders, are more widely used due to their compact design and ability to generate unequal forces in each direction. These cylinders offer a balance between performance and space efficiency.
Rotary actuators, closely resembling hydraulic pumps in construction, operate on a similar principle but in reverse. Instead of pumping fluid, they are driven by fluid pressure to produce rotational motion and torque.

### 2.4.4 Power Supplies

A key component of hydraulic servo-systems is the pressure controller, which, in conjunction with variable-displacement pumps, regulates the system's pressure levels. Other control strategies may involve flow control, combined pressure and flow control, or input power control, particularly in mobile applications.

The fundamental configurations of hydraulic power supplies designed to maintain constant supply pressure include:

- **Variable-displacement pump with stroke regulator:** This configuration utilizes a variable-displacement pump to adjust the flow rate based on system demands, while a stroke regulator ensures consistent pressure.
- **Switch-off pump:** In this arrangement, the pump is periodically switched on and off to maintain a constant pressure level.
- **Fixed-displacement pump with speed-controller mover:** Here, a fixed-displacement pump is paired with a speed-controller mover to regulate the pump's rotational speed and, consequently, the flow rate.
- **Fixed-displacement pump with pressure-relief valve:** This configuration employs a fixed-displacement pump and a pressure-relief valve to limit the maximum system pressure, preventing excessive pressure buildup.

These configurations provide the foundation for precise and efficient hydraulic servo-control systems, enabling a wide range of applications in industries such as manufacturing, aerospace, and automation.

### 2.4.5 Control Loops

In hydraulic servo-systems, the primary objective is to control specific actuator output variables such as direction, velocity, acceleration, deceleration, position, or force, often in opposition to a resisting load. These systems function as closed-loop hydraulic control systems, with feedback loops that regulate output based on real-time sensor inputs. As depicted in Figure 2.8, each component block within the system can exhibit non-linear behavior, further influencing the system's dynamic response.

| Controlled Variable(s) | Application in Mobile Hydraulics | Application in Industrial Hydraulics |
|---|---|---|
| Position | Medium to Seldom | Frequent |
| Pressure | Frequent | Medium |
| Pressure + Position + Power | Medium | Seldom |

**Table 2.1** Frequency of application of some controlled variables

The system output $y(t)$, typically sensed through electronic measurement devices, is controlled to follow or track the reference signal $y_{ref}(t)$ despite the presence of disturbance inputs $z(t)$, such as load variations, and measurement errors represented by $v(t)$, as illustrated in Figure 2.13. Any deviation between the output and the reference signal (error at the summing point) is fed back to the controller—whether analog or digital, electrical or hydro-mechanical—to prompt corrective action at the valve spool. Consequently, the control system continuously monitors and adjusts for any minor deviations in load behavior to maintain alignment with the desired reference signal. Accurate tracking is essential, even if the dynamics of the plant shift slightly during operation, ensuring robust performance under variable conditions.

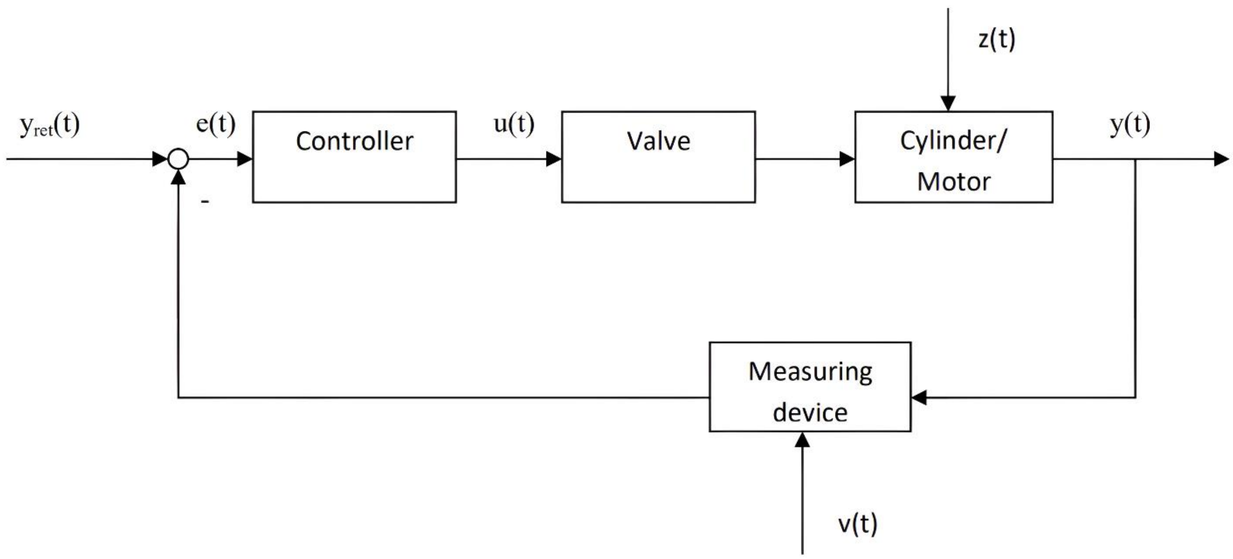



**Figure 2.13** Block diagram of a servo-hydraulic system in a closed control loop

The process of maintaining y(t) close to $y_{net}(t)$, including scenarios where $y_{ref}=0$, is generally referred to as regulation. A system exhibiting effective regulation in the presence of disturbance signals is considered to have good disturbance rejection. Additionally, a system that maintains regulation despite variations in plant parameters is characterized by low sensitivity to these parameters. A system demonstrating both strong disturbance rejection and low sensitivity to parameter changes is described as robust.

# CHAPTER 3

# Fundamental Principles of Hydraulics

The purpose of this section is to summarize the fundamental equations utilized in this thesis, as well as to introduce key concepts in fluid motion and fluid mechanics.

Fluids—whether liquids or gases—are substances that lack a fixed shape and can flow. They undergo shape changes when subjected to external forces, with the rate of deformation depending on the magnitude of the applied force; smaller forces result in slower deformation.

$$\rho = \frac{Force}{Area} = \frac{F}{A} \quad (\mathrm{N/m^2}) \tag{3.1.1}$$

### 3.1.1 Viscosity

The dynamic viscosity coefficient η quantifies the internal resistance of a fluid to shear deformation, reflecting the presence of tangential forces between fluid layers in relative motion. Consider two parallel fluid layers separated by a distance dy, with a relative velocity $dv_x$ between them (as shown in Figure 3.1). The shear stress, τ, that develops due to this velocity gradient is proportional to the dynamic viscosity and the rate of shear deformation.

$$\tau = \frac{Shear\ Force}{Area} = \eta \frac{dvx}{dy} \tag{3.1.2}$$

Thereby, η is a proportionality factor and is called dynamic viscosity.

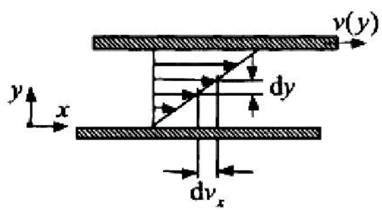



**Figure 3.1** Definition of shear stress

The coefficient of kinematic viscosity, μ, is the ratio of the coefficient of dynamic viscosity to the fluid density, i.e.

$$\mu = \frac{Dynamic\ Viscosity}{Density} = \frac{\eta}{\rho} \tag{3.1.3}$$

The dynamic viscosity of liquids varies considerably with the temperature:

$$\eta_v = \eta 0 e^{-\lambda(\theta - \theta_0)} \tag{3.1.4}$$

where $\eta_0$ is the dynamic viscosity at reference temperature $\theta_0$.

The influence of pressure is given by

$$\eta_v = \eta 0 e^{\alpha p} \quad (3.1.5)$$

where α is the viscosity-pressure coefficient that depends on the temperature.

### 3.1.2 Mass Density and Bulk Modulus

$$\rho = \frac{Mass}{Volume} = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV} \quad (3.16)$$

The density of hydraulic fluids is a function of both pressure and temperature, i.e. $\rho = \rho(p,\theta)$. The approximate equation can be written as

$$\rho = \rho_0 + \left(\frac{\delta\rho}{\delta p}\right)(p-p_0) + \left(\frac{\delta\rho}{\delta\theta}\right)(\theta-\theta_0)$$
$$= \rho_0\left[1 + \frac{1}{E}(p-p_0) - \alpha(\theta-\theta_0)\right] \quad (3.1.7)$$

Equation 3.1.7 of fluids to the simple form can be expressed as the quantity

$$\rho = \rho_i + \frac{\rho i}{E}\rho \quad (3.1.8)$$

The quantity

$$E = \rho_0\left(\frac{\delta p}{\delta\rho}\right)_\theta = -V_0\left(\frac{\delta p}{\delta\rho V}\right)_\theta \quad (3.1.9)$$

is the change in pressure by the fractional change in volume at a constant temperature. It is called the modulus of elasticity, and is also termed as the isothermal bulk modulus or simply bulk modulus of the liquid.

#### 3.1.2.1 Bulk Modulus

While hydraulic fluid is often approximated as incompressible in modeling hydraulic servo-systems, all fluids exhibit some degree of compressibility, which impacts system dynamics under high-pressure conditions. Neglecting fluid compressibility may be valid in applications that do not require high-precision control or involve only moderate pressures and fluid volumes. However, in systems where substantial volumes of fluid are subjected to high pressures, compressibility effects become significant, with notable implications for energy efficiency and system responsiveness.

When high pressures are applied, the energy expended in compressing the fluid effectively stores potential energy within the fluid, leading to two key issues: (1) a delay in actuator response, as energy is initially absorbed in compressing the upstream fluid, and (2) overshoot or continued motion of the actuator after the control valve closes, due to the release of stored compressive energy. The bulk modulus, a parameter quantifying fluid compressibility, becomes critical in such scenarios, as it directly influences the dynamic response of the hydraulic system. In systems operating at pressures of 5000 psi or higher, for instance, ignoring the bulk modulus may lead to compromised response times and control precision.

To enhance the responsiveness and precision of hydraulic servo-systems, minimizing the volume of fluid between the control valve and the actuator is essential. This design strategy reduces the energy loss associated with fluid compression and ensures that applied pressures directly actuate system components rather than being absorbed in fluid compression.

The bulk modulus (K) is a material property that quantifies a substance's resistance to uniform compression. Most substances exhibit a reduction in volume when subjected to an external pressure, indicating a measurable compressibility. This compressive behavior is particularly relevant in hydraulic fluids, as even minimal volumetric changes under pressure can impact system performance.

A typical pressure-volume relationship is depicted in Figure 3.2, illustrating that the volume of a fluid (V) varies as a function of the applied pressure (P), the fluid's compressibility (k), and its initial volume ($V_0$). This relationship highlights the importance of the bulk modulus in hydraulic systems, as it governs the extent to which the fluid will compress under operational pressures. For hydraulic systems, understanding and appropriately incorporating the bulk modulus is essential for predicting system behavior and ensuring optimal response characteristics.

$V = f(P, V_0, k)$

$V_0$ = initial volume, in, l, or $m^3$

P = pressure, psig, Pa, or bar.

k = compressibility, usually negative, $in.^2/lb$

$(V - V_0) \div V$ = specific volume, commonly used for x-axis

The bulk modulus (K) is defined as the inverse of compressibility, representing the slope of the pressure versus specific volume curve, as illustrated in Figure 3.2. Given that specific volume is dimensionless, the units of bulk modulus correspond directly to those of pressure (e.g., psi, bar, Pa, $N/m^2$). Thus, bulk modulus serves as a quantitative measure of a fluid's

resistance to compressibility. A lower bulk modulus, indicated by a gentle slope, reflects a relatively compressible fluid. Conversely, a higher bulk modulus, shown by a steeper slope, signifies a fluid that is stiffer, or minimally compressible.
In hydraulic servo-systems, where high pressures and precise response times are crucial, the bulk modulus of the fluid plays a critical role in determining the system's dynamic behavior. A fluid with a higher bulk modulus will generally support faster response times by limiting volumetric changes under pressure, thereby enhancing the efficiency and stability of the system.

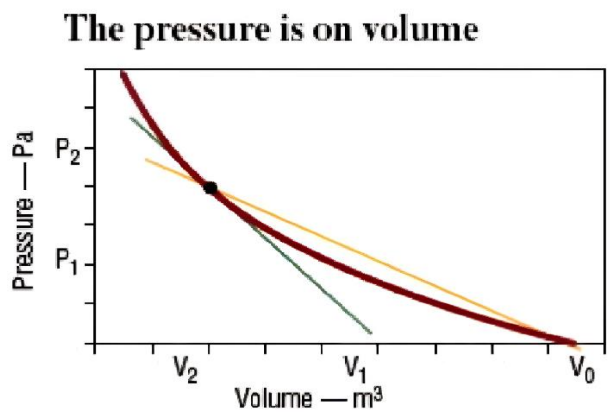



**Figure 3.2** Increasing the pressure applied to a fluid decreases its volume.

The plot shown in Figure 3.2 is nonlinear, with a slope that varies across different points. Consequently, two common approaches are employed to define the slope, or bulk modulus, under such conditions. These approaches allow for a more precise characterization of the fluid's compressibility at specific operating points within the hydraulic system, ensuring accurate modeling of its dynamic response.

The secant bulk modulus is defined as the product of the fluid's initial volume and the slope of the line drawn from the origin to a specific point on the pressure versus specific volume curve, representing the secant line at that point. This measure provides an average bulk modulus over a specified pressure range, offering a practical means to approximate compressibility characteristics when the pressure-volume relationship is nonlinear.

Mathematically, secant bulk modulus, $B_s$, is:
$E_S = (V_0 \times P) \div (V_0 - V)$

The tangent bulk modulus is defined as the product of the fluid volume at a specified pressure and the derivative of pressure with respect to volume at that point, representing the slope of the tangent line on the pressure versus specific volume curve. This method provides an instantaneous measure of bulk modulus at a given pressure, allowing for a more precise characterization of the fluid's compressibility under specific operating conditions within the hydraulic system.

Mathematically, tangent bulk modulus, $E_T$, is:

$$E_T = V_0 \frac{\delta p}{\delta V} \quad (3.1.11)$$

## Temperature and bulk modulus

Temperature plays a significant role in fluid compressibility, as a fluid's volume decreases with rising temperature. As temperature increases, the fluid expands, which in turn generates additional pressure. The rate of compression—whether rapid or gradual—also influences how heat within the fluid is managed. When compression occurs slowly, allowing the generated heat to dissipate, the bulk modulus measured is known as the isothermal (constant temperature) bulk modulus. Conversely, when the fluid is compressed rapidly, capturing both compression effects and thermal expansion, the bulk modulus measured is referred to as the adiabatic or isentropic bulk modulus.

In high-speed, precision-controlled hydraulic systems, conditions are generally assumed to be isentropic due to the rapid movement and limited heat dissipation time. Consequently, the bulk modulus values discussed here primarily refer to the isentropic bulk modulus, as it more accurately represents the compressibility characteristics in these dynamic applications.

## Effect of air on bulk modulus

Caution is advised when using published bulk modulus values, as these are often obtained under controlled laboratory conditions where the fluid is thoroughly degassed prior to compression testing. In practical applications, however, hydraulic fluids typically become aerated, which substantially affects bulk modulus values since air is considerably more compressible than oil. The presence of air can significantly increase fluid compressibility, thereby lowering the effective bulk modulus. Moreover, air solubility in hydraulic fluids rises with pressure, meaning that dissolved air can form bubbles when the pressure decreases, leading to potential cavitation issues. Designers must account for these factors to ensure accurate performance predictions in hydraulic systems.

**Volume lost in pumps and actuators**

The bulk modulus of a hydraulic fluid directly influences both pump output and the positional accuracy of master-slave cylinder configurations. For pumps, a lower bulk modulus results in a percentage volume loss in output, which corresponds to a loss in horsepower, impacting overall system efficiency. In master-slave cylinder arrangements, this volume loss manifests as a reduced stroke from the slave cylinder, impairing positional accuracy and response. Understanding and accounting for these effects is essential for optimizing hydraulic system performance, especially in applications demanding high precision.

When stopping a moving load, such as in the case of a cylinder moving a load at a constant velocity with steady flow, the system must absorb the momentum of the moving load when a valve controlling the upstream and downstream flow is suddenly closed. As the valve closes, the downstream fluid pressure will rise from a nominal value to a peak pressure, as the system absorbs the kinetic energy of the load. Assuming the cylinder and hydraulic lines are rigid, and pressure rises linearly, the fluid's bulk modulus plays a critical role in determining the peak pressure. For a given maximum pressure, fluids with a higher bulk modulus—indicating greater stiffness—will absorb less energy, resulting in less pressure overshoot. Consequently, fluids with higher bulk modulus values contribute to reduced energy absorption and minimized piston overshoot, which ultimately improves positional accuracy and system control.

**Fast load reversals** — Because most fluids are compressible, the fluid in an actuator must be compressed before the cylinder or piston will move a load. In other words, an amount of fluid equal to the compressed volume must be added to an actuator before a load will move. Because this process does not do useful work, it is lost work:

$W_L = P \times V_0$

where P = change in pressure

V = change in volume (increment of stroke × piston area) But $V = P \times (V_0 \div B)$, so:

$WL = (P^2 \times V_0) \div B$

To calculate lost power, divide by time:

$W_L = (P^2 \times V_0) \div (B_t \times 6600)$

Because power loss can be significant at higher pressure ranges, let us examine a typical 3000 psig system, that is, P = 3000 psi. $hpl = (1363\ V_0) \div (B \times t)$ It is now possible to plot lost horsepower versus time for 1 in. of cylinder volume for various bulk moduli, Figure 2. Lost power increases as cylinder size increases and response time decreases.

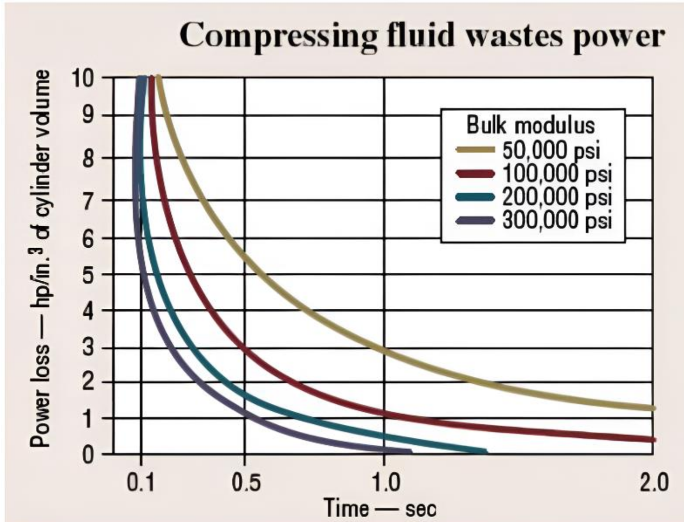



**Figure 3.3** compression of fluid

Figure 3 illustrates the relationship between lost power and response rate for various bulk moduli. While the power loss may initially appear relatively small, it becomes more significant when considering typical cylinder configurations. For instance, assuming a bulk modulus of 200,000 psi, a response rate of 100 Hz, and a stroke length of 10 inches, the power loss is approximately 6.75 hp per square inch of ram area.

Figure 4 correlates power loss to the total available system power. As an example, a 3000-psi, 3.8-gpm system that can supply 6.75 hp would be incapable of moving a load at 100 Hz with a piston, as all the available power would be consumed in compressing the fluid. This highlights the critical impact of fluid compressibility and bulk modulus on system performance, particularly in high-response applications where energy efficiency and power allocation are paramount.

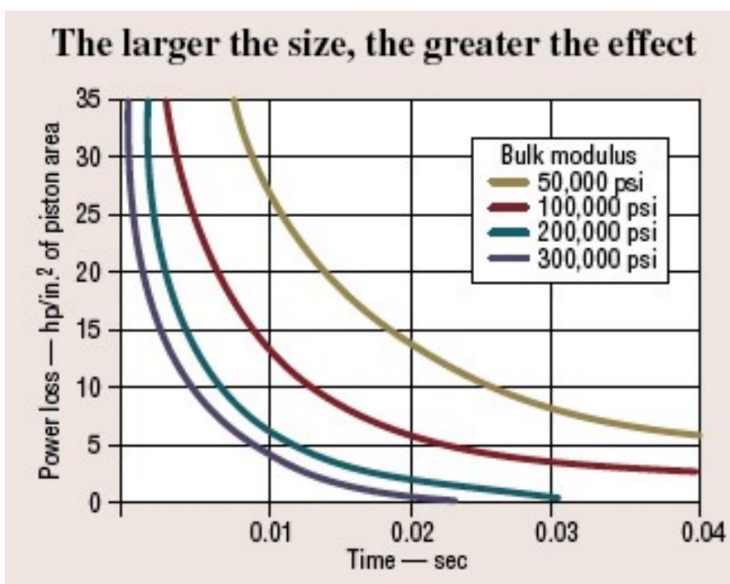



**Figure 3.4**

**Resonance of hydraulic systems**

The natural frequency of a spring mass combination is:

$f = (1 \div 2\pi) \times (kg)^{1/2} \div W$

Where: ƒ = frequency, Hz
W = weight, lb
k = spring rate, lb/in., and
g = acceleration due to gravity, 32.2 ft/sec2

To equate this to a hydraulic system, we only need to substitute bulk modulus for spring rate. Thus, a low modulus also lowers the natural frequency of a system. For example, if 1% air content changes the bulk modulus by 50%, its natural frequency decreases by 30%. This greatly reduces the stability of the system.

## Empirical Effective Bulk Modulus

Other researchers have derived empirical formulae for the calculation of the effective bulk modulus E', including the effects of entrained air and mechanical compliance, based on direct measurements. The commonly used equation for calculation of the bulk modulus E' for hydraulic cylinders in German literature is that of Lee (1977):

$$E'(p)=a_1\, E_{max}\, log(a_2 \frac{P}{P_{max}} + a_3) \qquad (3.1.12)$$

with the parameters $a_1$ = 0.5, a2 = 90, a3 = 3, $E_{max}$ = 18000 bar, and $P_{max}$ = 280 bar. Hoffmann (1981) proposed the formula

$$E'(p)= E_{max}[1\text{-}exp(-0.4\text{-}2x10^{-7}] \qquad (3.1.13)$$

with the pressure p in pascals.
According to Eggerth (1980), the effective bulk modulus can be expressed as

$$E'(p)=\frac{1}{k_1+k_2(p/p_o)^{-\lambda}} \qquad (3.1.14)$$

with the parameters $k_1$ and $k_2$ in Table 3.1; $P_o$ is assumed to be 10 bar.

| Temperature [°C] | $k_1$[$10^{-10}$ m²/N] | $k_2$[$10^{-10}$ m²/N] | λ |
|---|---|---|---|
| 20 | 4.943 | 1.954 | 1.48 |
| 50 | 5.469 | 3.2785 | 1.258 |
| 90 | 5.762 | 4.775 | 1.1 |

**Table 3.3** parameters of Eggerth's formula (Beater, 1999)

The relations for effective bulk modulus E' are plotted in Figure 3.4. Although these formulae are approximate, they are sufficient for design purposes. However, experimental data are always preferable.

## 3.2 General Equations of Fluid Motion

The basic principles of conservation and laws governing fluid flow and associated phenomena are summarized in this section. More detailed derivations are done in McCloy and Martin[11], Blackburn[14].

### 3.2.1 Continuity Equation

Consider a control tube as depicted in Figure 3.5. The integral form of the mass conservation (continuity) equation can be formulated as

$$\int_{(1)}^{(2)} \frac{\partial(\rho A)}{\partial t} dS + \rho_2 v_2 A_2 - \rho_1 v_1 A_1 = 0 \tag{3.2.1}$$

where the density ρ=ρ(t,s) is, in general, not constant.

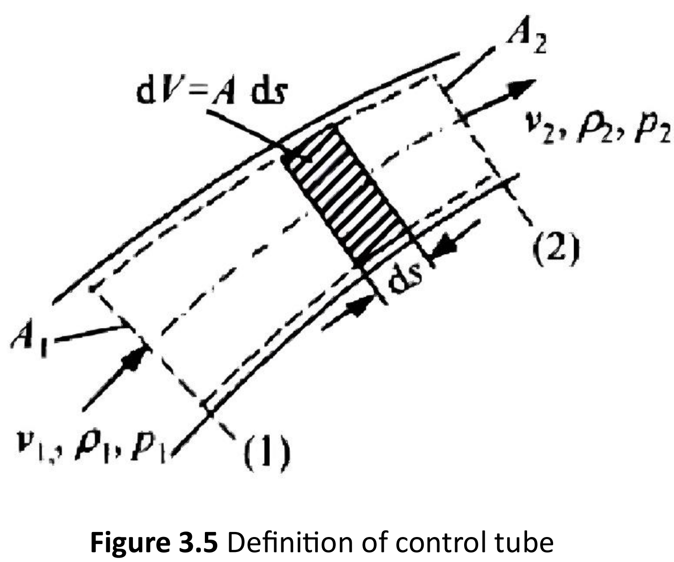



**Figure 3.5** Definition of control tube

For incompressible fluids, i.e., p = const. (which is a standard assumption in hydraulics), Equation 3.2.1 can be reduced to

$$v_1(t)A_1 = v_2(t)A_2 \qquad (\rho = constant) \qquad (3.2.2)$$

or, more generally

$$Q = v(t)A = constant \qquad (volume\ flow) \qquad (3.2.3)$$

For steady flow, the continuity equation becomes

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \qquad (\rho\ is\ not\ constant) \qquad (3.2.4)$$

$$\dot{m} = \rho v A = constant \qquad (3.2.5)$$

Consider the mass conservation equation for a control volume $V$ within a hydraulic servo-system, where the accumulated mass of fluid is denoted by $m$ and the mass density by $\rho$. Given that hydraulic fluid is treated as a continuous medium, the rate at which mass accumulates within the control volume must equal the difference between the incoming and outgoing mass flow rates. This ensures that all fluid movement is accounted for within the system's dynamic behavior, which is crucial in modeling the hydraulic system's non-linear characteristics.

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = \frac{d(\rho V)}{dt} = \rho \dot{V} + V \dot{\rho} \qquad (3.2.6)$$

Taking into account Equation 3.1.8 and dividing Equation 3.2.6 by $\rho$ leads to

$$\sum Q_{in} - \sum Q_{out} = \dot{V} + \frac{V}{\mathrm{E}} \dot{p} \qquad (3.2.7)$$

This equation is fundamental for the description of the pressure dynamics in the hydraulic compartments.

## 3.3 Fluid Flow Regimes in Hydraulic Servo-Systems

In hydraulic servo-systems, two distinct types of fluid flow can occur within passages:

- **Laminar (or viscous) flow**: In this flow regime, each fluid particle follows a well-defined, orderly trajectory, moving only in the primary direction of the flow. This type of flow is stable and predictable, with minimal cross-currents.
- **Turbulent (or hydraulic) flow**: This is the more common flow regime in hydraulic systems. In turbulent flow, each fluid particle moves with fluctuations and crosscurrents in addition to the primary flow direction. This results in chaotic motion, with irregular velocity components perpendicular to the flow direction.

Understanding these flow regimes is crucial, as they impact the system's dynamic behavior, energy losses, and the control accuracy of hydraulic servo-systems.

The Reynolds number, $Re$, is defined as

$$Re = \frac{\rho v d}{\eta} = \frac{vd}{\mu} \tag{3.3.1}$$

In hydraulic servo-systems, the Reynolds number $Re$ serves as a key parameter to characterize flow behavior: at lower $Re$ values, the flow is laminar, while at higher $Re$ values, it becomes turbulent. Here, $v$ represents the average flow velocity, and $d$ is the hydraulic diameter, defined as

$$d = \frac{4A}{S} \tag{3.3.2}$$

where $A$ is the cross-sectional area of the flow, and $S$ is the wetted perimeter of the flow section.

### 3.3.1 Flow Establishment in Pipelines

One basic element of hydraulic systems is cylindrical pipelines, in which flow may be laminar or turbulent. The characteristic length to be used for the Reynolds number is inside pipeline diameter d, i. e. ,

$$Re = \frac{vd}{\mu}$$

Experimental observations in hydraulic systems indicate that the transition from laminar to turbulent flow typically occurs within a Reynolds number range of 2000 < Re < 4000, with the critical Reynolds number for transition generally around Re=2300. For flows with Re<2300, the regime remains laminar, characterized by orderly fluid motion. For Re>4000, flow is usually, though not universally, turbulent, exhibiting chaotic and fluctuating patterns. In specialized conditions, laminar flow may be maintained at Reynolds numbers considerably above 4000 if disturbances—such as vibrations, temperature fluctuations, or roughness in the flow path—are minimized. Understanding and controlling the flow regime is essential in hydraulic servo-systems, as it directly influences the fluid's dynamic response, affecting system stability, energy efficiency, and the predictability of its non-linear behavior.

### 3.3.1.1 Hagen-Poiseuille Law

The Hagen–Poiseuille Law describes laminar flow in a cylindrical pipe with steady conditions. For such flow, the shear stress at the pipe wall is proportional to the pressure gradient along the pipe's length. By integrating this relationship, the velocity profile across the pipe's radius can be determined, with the maximum velocity occurring at the pipe center. Combining this with the continuity equation gives the Hagen–Poiseuille equation for volumetric flow rate $Q$ in terms of the pipe radius $R$, fluid viscosity $\eta$, and pressure gradient $\frac{dP}{dx}$.

$$Q = A\bar{v} = \frac{-\pi R^4}{8\eta}\frac{dP}{dx} \tag{3.3.3}$$

## 3.3.2 Flow Through Orifices

Orifices are sudden restrictions of short length (ideally zero length for a sharp-edged orifice) in the flow passage and may have a fixed or variable area (see Figure 3.8). Orifices are generally used to control flow, or to create a pressure differential (valves). Two types of flow regime exist, depending on whether inertia or viscous forces dominate. The flow velocity through an orifice must increase above that in the upstream region to satisfy the law of continuity. At high Reynolds numbers, the pressure drops across the orifice is caused by the acceleration of the fluid particles from the upstream velocity to the higher jet velocity. At low Reynolds numbers, the pressure drop is caused by the internal shear forces resulting from fluid viscosity.

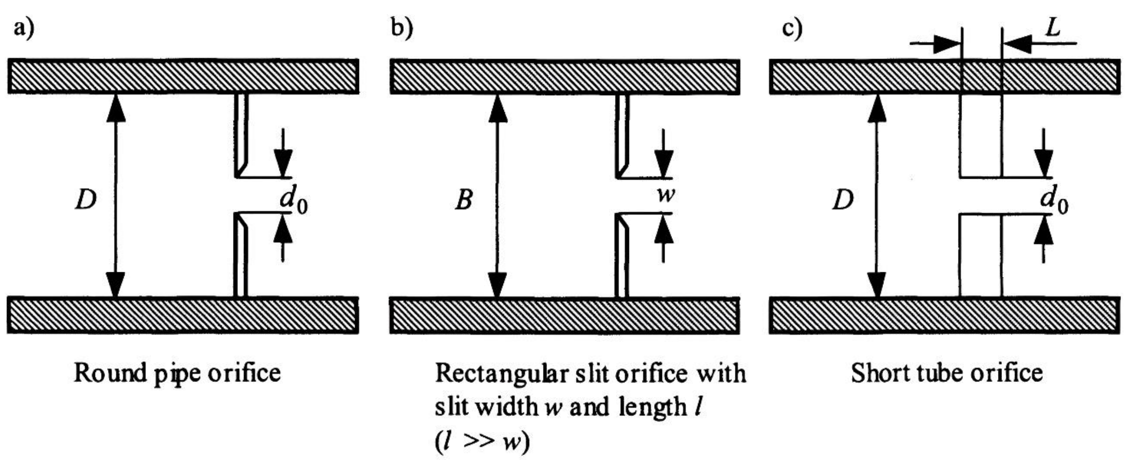



**Figure 3.6** Round, slit-type and short tube orifices

### 3.3.2.1 Orifice Equations for Turbulent Flow

Since most orifice flows occur at high Reynolds numbers, this region is of major importance. Such flows are often referred to as "turbulent" (Figure 3.9b), but the term does not have quite the same meaning as in pipeline flow. Referring to Figure 3.9a, the fluid particles are accelerated up to the jet velocity between sections 1 and 2. The flow between these sections is streamline or potential flow, and experience justifies the use of Bernoulli's theorem in this region.

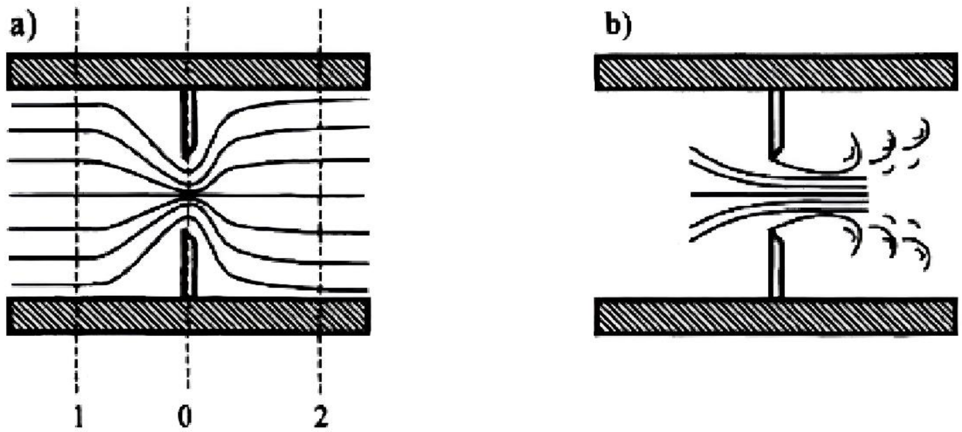



**Figure 3.7** Flow through an orifice: (a) laminar flow; (b) turbulent flow

it is common in the field of hydraulics to use the modified orifice equation

$$Q = a_d A \sqrt{\frac{2}{\rho} \Delta p} \qquad (3.3.4)$$

where $a_d$ is the discharge coefficient. Theoretically, ad = 1t/(1t+2) = 0.611.

Turbulence is ensured only at large enough Reynolds numbers:

$$Re = \frac{2vh}{\mu} \qquad (3.3.4)$$

where h is the smallest dimension of the orifice (rectangular). At low temperatures, low orifice pressure drops, and/or small orifice openings, the Reynolds number may become sufficiently low to permit laminar flow.

Practical experiments carried out proved that very sharp edges in narrow orifices the critical value $Re_{crit}$ is as low as 20, whereas slightly rounded off edges increased $Re_{crit}$ to 80 or higher. Thus, at very sharp edges, $\alpha_d$ may be assumed to be constant at Re > 20.

For ideal laminar region the discharge coefficient as a function of Reynolds number is given as:

$$a_d = \delta\sqrt{Re} \quad (3.3.5)$$

### 3.3.3 Flow through Valves

Flows through valving orifices (Figure 3.10) are usually described by the orifice Equation 3.3.4 with a linear relationship between the valve spool position $X_v$ and the flow area (critical centre), i.e.,

$$Q = Q(x_v, \Delta P) = c_v x_v \sqrt{p_1 - p_2} = c_v x_v \sqrt{\Delta p} \quad (3.3.6)$$

with the flow coefficient

$$c_U = \pi d_v a_d \sqrt{\frac{2}{\rho}} \quad (3.3.7)$$

The Equation 3.3.12 can be written using the valve voltage $u_v$ as :

$$Q = Q(u, \Delta P) = c_v \frac{x_{v,max}}{u_{max}} u \sqrt{p_1 - p_2} = c_{vu} u \sqrt{\Delta p} \quad (3.3.8)$$

This means that the value and the dimension of $v_v$ have to be adapted to the signal used (which can be the valve stroke $x_v$, the valve voltage $u_v$, or the valve current $i_v$).

Finally, a generalised expression for the flow through valve orifices reads:

$$Q = Q(x_v, \Delta P) = a_d A(x_v) \sqrt{\frac{2}{\rho}} \sqrt{\Delta p} \quad (3.3.9)$$

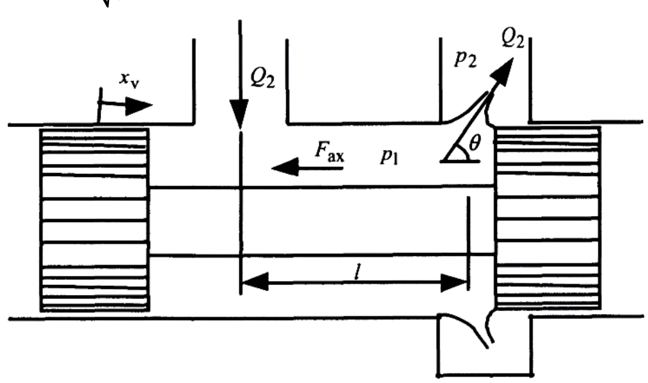



**Figure 3.8** Axial flow force on spool due to unequal jet angles

# CHAPTER 4

# Physically Based Modelling

## 4.1. Modelling of Various Systems

Physical devices are represented by one or more idealized elements that obey laws involving the variables associated with the elements, some degree of approximation is required in selecting the elements to represent a device and the behavior of the combined elements may not correspond exactly to the behavior of the device.

### 4.1.1 Mechanical System

The elements that we include in translational systems are mass, friction, stiffness. The elements law for the three relate the external force to acceleration, velocity, or displacement associated with the element.

**Mass**

Figure (4.1) shows a mass **M**, which has units of kilograms (kg), subjected to a force $f$. Newton's second law states that the sum of the forces acting on a body is equal to the time rate of change of the momentum:

$$\frac{d}{dt}(Mv) = f \tag{4.1.1}$$

which, for a constant mass, can be written as

$$M\frac{dv}{dt} = f \tag{4.1.2}$$

For (4.1.1) and (4.1.2) to hold, the momentum and acceleration must be measured with respect to an inertial reference frame. For ordinary systems at or near the surface of the earth, the earth's surface is a very close approximation to an inertial reference frame, so it is the one we will use. The momentum, acceleration, and force are really vector quantities.

For (4.1.2) to hold, the positive senses of both dv/ dt and the external force f must be same, because the force will cause the velocity to increase in the direction in which the force is acting.

Energy in a mass is stored as kinetic energy if the mass is in motion, and as potential energy if the mass has a vertical displacement relative to its reference position. The kinetic energy is

$$w_k = \frac{1}{2}Mv^2 \tag{4.1.3}$$

and the potential energy, assuming a uniform gravitational fields, is

$$w_p = Mgh \qquad (4.1.4)$$

where g is the gravitational constant and h is the height of the mass above its reference position.

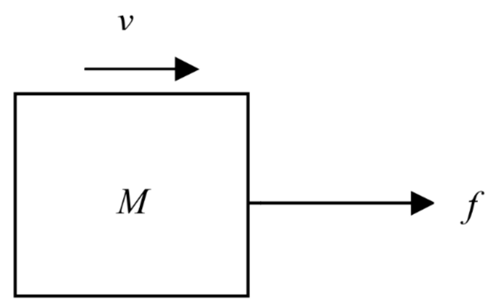



**Figure 4.1**

**Friction**

Forces that are algebraic functions of the relative velocity between two bodies are modeled by friction elements. A mass sliding on an oil film that has laminar flow, as depicted in Figure (4.2), is subject to viscous friction and obeys linear relationship

$$f = B\Delta v \qquad (4.1.5)$$

where $B$ has units of newton-seconds per meter (N.s/m) and where $\Delta v = v_2 - v_1$. The direction of a frictional force will be such as to oppose the motion of the mass. For (4.1.5) to apply to Figure (4.2), the force $f$ exerted on the mass $M$ by the oil film is to the left. By Newton's third law, the mass exerts an equal force f to the right on the oil film.

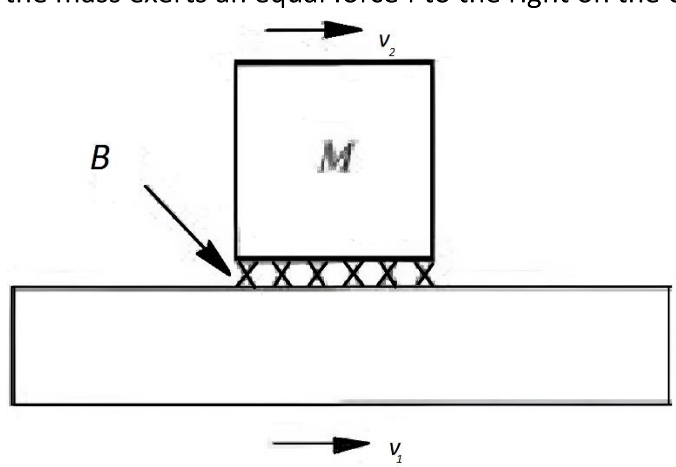



**Figure 4.2** Friction described by (4.1.5) with $\Delta v = v_2 - v_1$

The friction coefficient B is proportional to the contact area and to the viscosity of the oil, and inversely proportional to the thickness of the film. A heavier mass would further compress the oil film, making it thinner and increasing the value of B.

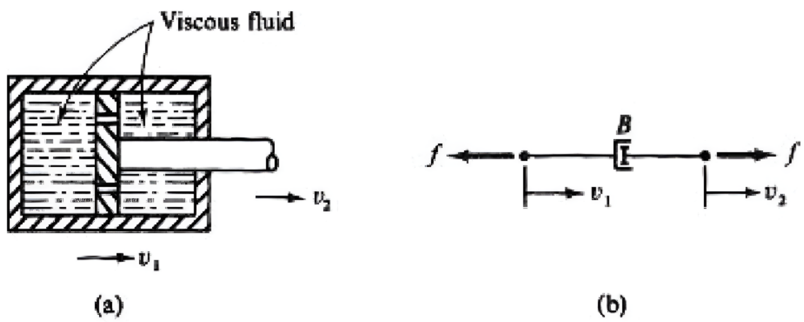



**Figure 4.3** (a) A dashpot, (b) Its representation

As indicated in Figure (4.3) [a], a piston moves through an oil-filled cylinder, and there are small holes in face of the piston though which the oil passes as the parts moves relative to each other. Many dashpot devices involves high rates of fluid flow through the orifices and have nonlinear characteristics.

The viscous friction described by (4.1.5) is a linear element, for which the plot of f versus $\Delta v$ is a straight line passing through the origin, as shown in Figure (4.4, a). Examples of friction that obey nonlinear relationships are dry friction and drag friction. The former is modeled by a force that is independent of the magnitude of the relative velocity as indicated in Figure (4.4, b), and that can be described by the equation

$$f = \begin{cases} -A \; for \; \Delta v < 0 \\ A \; for \; \Delta v > 0 \end{cases}$$

Drag friction is caused by resistance to a body moving through a fluid and can be described by an equation of the form $f = D|\Delta v|\Delta v$, as depicted in Figure (4.4, c). Various other nonlinearities may be encountered in friction elements.

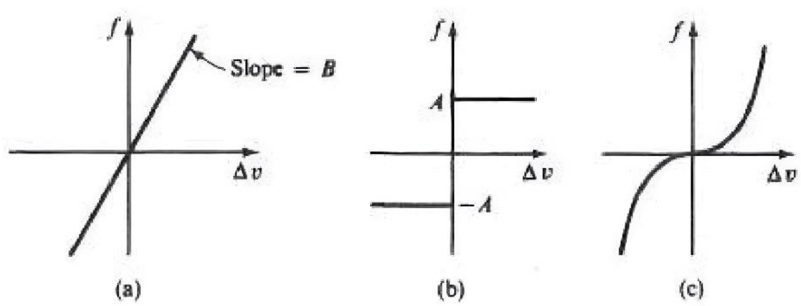



**Figure 4.4**

The power dissipated by friction is the product of the force exerted and the relative velocity of the two ends of the element. This power is immediately converted to heat and thus cannot be returned to the rest of the mechanical system at a later time. Accordingly, we do not usually need to know the initial velocities of the friction elements in order to solve the model of a system.

**Stiffness**

Any mechanical element that undergoes a change in shape when subjected to a force can be characterized by a stiffness element, provided only that an algebraic relationship exists between the elongation and the force. The most common stiffness element is the spring, although most of the mechanical elements undergo some deflection when stressed.

As depicted in the Figure (4.5) the stiffness property refers to the algebraic relationship between $x$ and $f$, because $x$ has been defined as an elongation and the plot shows that $f$ and $x$ always have the same sign. For a linear spring, the curve in Figure(4.5) is a straight line and $f = Kx$, Where $K$ is a constant with units of newtons per meter (N/m).

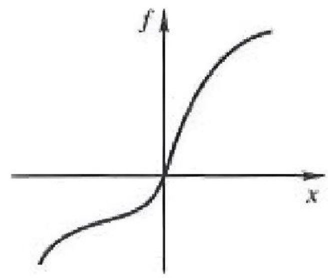



**Figure 4.5**

### 4.1.2 Electrical Systems

The elements in the electrical circuits that are considered are resistors, capacitors, inductors and sources. The first three of these are referred to as passive elements because, although they can store or dissipate energy that is present in the circuit, they cannot introduce additional energy. They are analogous to the dashpot, mass, and spring for mechanical systems. In contrast, sources are active elements that can introduce energy into the circuit and that serve as the inputs. They are analogous to the force or displacement inputs for mechanical systems.

**Resistor**

A resistor is an element for which there is an algebraic relationship between voltage across its terminals and the current through it, that is, an element that can be described by a curve of $e$ versus $i$. A linear resistor is one for which the voltage and current are directly proportional to each other as described in Ohm's law:

$$e = Ri \tag{4.1.6}$$

Or

$$i = \frac{1}{R}e \tag{4.1.7}$$

Where $R$ is the resistance in ohms (Ω). A resistor dissipates any energy supplied to it by converting it into heat( in this it is analogous to the frictional element of mechanical systems).

**Capacitor**

A capacitor is an element that obeys an algebraic relationship between the voltage and the charge, where the charge is the integral of the current. For a linear capacitor, the charge and voltage are related by

$$q = Ce \tag{4.1.8}$$

where $C$ is the capacitance in farads (F). For a fixed linear capacitor, the capacitance is a constant. If (4.1.8) is differentiated and $\dot{q}$ replaced by $i$, the element law for a fixed linear capacitor becomes

$$i = C\frac{de}{dt} \tag{4.1.9}$$

To express the voltage across the terminals of the capacitor in terms of the current, we solve (4.1.9) for $\frac{de}{dt}$ and then integrate, getting

$$e(t) = e(t_0) + \frac{1}{c}\int_{t-0}^{t} i(\lambda)\, d\lambda \tag{4.1.10}$$

where $e(t_0)$ is the voltage corresponding to the initial charge, and where the integral is the charge delivered to the capacitor between the times $t_0$ and $t$.

One form of a capacitor consists of two parallel metallic plates, each of area A, separated by a dielectric material of thickness $d$. Provided that fringing of the electric field is negligible, the capacitance of this element is $C = \epsilon \frac{A}{d}$ , where $\epsilon$ is the permittivity of the dielectric material. The values of practical capacitances are typically expressed in microfarads (μF).

**Inductor**

An inductor is an element for which there is an algebraic relationship between the voltage across its terminal and the derivative of the flux linkage. For a linear inductor,

$$e = L\frac{di}{dt} \qquad (4.1.11)$$

We can find an expression for the current through the inductor by using (4.1.11) to integrate $\frac{di}{dt}$ giving

$$i(t) = i(t_0) + \frac{1}{L}\int_{t-0}^{t} e(\lambda)\, d\lambda \qquad (4.1.12)$$

where i(t0) is the initial current through the inductor.

**Sources**

The inputs for electrical circuits models are provided by ideal voltage and current sources. A voltage source is any device that causes a specified voltage to exist between two points in a circuit, regardless of the current that may flow. A current source causes a specified current to flow through the branch containing the source, regardless of the voltage that may be required. We often represent physical sources by the combination of an ideal source and a resistor.

A voltage source that has a constant value for all time is often represented as shown in Figure (4.6). The symbol E denotes the value of the voltage.

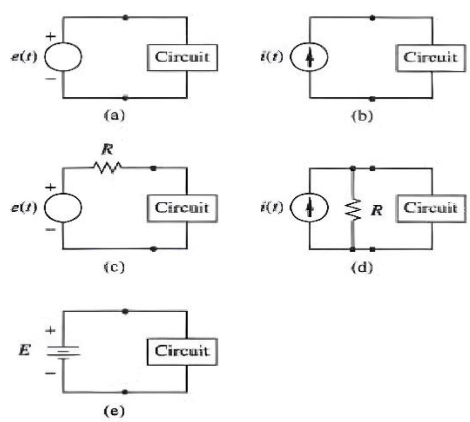



**Figure 4.6** Sources. (a) Voltage (b) Current (c), (d) Non-ideal source (e) Constant voltage source

### 4.1.3 Fluid System

Hydraulic systems exhibit three types of characteristics that can be approximated by lumped elements: capacity, resistance to flow, and inertance. The inertance, which accounts for the the kinetic energy of a moving fluid stream, is usually negligible, and we are not going to consider it.

**Hydraulic Capacitance**

When liquid is stored in an open vessel, there is an algebraic relationship between the volume of the liquid and the pressure at the base of the vessel. If the cross-sectional area of the vessel is given by the function $A(h)$, where $h$ is the height of the liquid level above the bottom of the vessel, then the liquid volume $v$ is the integral of the area from the base of the vessel to the top of the liquid. Hence,

$$v = \int_0^h A(\lambda)\, d\lambda \qquad (4.1.13)$$

where $\lambda$ is a dummy variable of the integration. For a liquid of density ρ expressed in kilograms per cubic meter, the absolute pressure $p$ and the liquid height $h$ are related by

$$p = \rho g h + p_a \qquad (4.1.14)$$

where $g$ is the gravitational constant (9.807 m/s$^2$ ) and where $p_a$ is the atmospheric pressure, which is taken as 1.013×105 N/m$^2$ .

Equations (4.1.13) and (4.1.14) imply that for any vessel geometry, liquid density, and atmospheric pressure, there is a unique algebraic relationship between the pressure $p$ and the liquid volume $v$. A typical characteristics curve describing this relationship is shown in Figure(4.7,a).

If the tangent to the pressure-versus-volume curve is drawn at some point, as shown in Figure(4.7,b), then the reciprocal of the slope is defined to be the hydraulic capacitance, denoted by $C(h)$. As indicated by the $h$ in the parentheses, the capacitance depends on the point on the curve being considered and hence on the liquid height $h$. Now

$$C(h) = \frac{1}{dp/dv} = \frac{dv}{dp}$$

And, from the chain rule of differentiation,

$$C(h) = \frac{dv}{dh}\frac{dh}{dp}$$

We see that $\frac{dv}{dh} = A(h)$ from (4.1.13) and that $\frac{dh}{dp} = \frac{1}{\rho g}$ from (4.1.14). Thus for a vessel of arbitrary shape,

$$C(h) = \frac{A(h)}{\rho g} \qquad (4.1.15)$$

which has units of $m^4 .s^2$ /kg or, equivalent, $m^5$ /N.

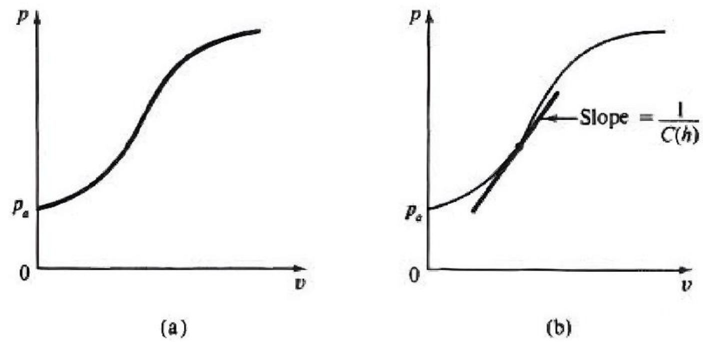



Figure 4.7 Pressure versus liquid volume for a vessel with variable cross-sectional area.

For a vessel with constant cross-sectional area $A$, (4.1.13) reduces to $v = Ah$. We can substitute the height $h = v/A$ into (4.1.14) to obtain the pressure in terms of the volume:

$$p = \frac{\rho g}{A} v + p_a \qquad (4.1.16)$$

Equation (4.1.16) yields a linear plot of pressure versus volume, as shown in Figure 4.7, b). The slope of the line is the reciprocal of the capacitance C, where

$$C = \frac{A}{\rho g} \qquad (4.1.17)$$

The volume of liquid in a vessel at any instant is the integral of the net flow rate into the vessel plus the initial volume. Hence we can write

$$v(t) = V(0) + \int_0^t [w_{in}(\lambda) - w_{out}(\lambda)]\, d\lambda$$

$$\dot{v} = w_{in}(t) - w_{out}(t) \qquad (4.1.18)$$

To obtain expression for the time derivatives of the pressure $p$ and the liquid height $h$ that are valid for vessels with variable cross-sectional areas, we use the chain rule of differentiation to write

$$\frac{dv}{dt} = \frac{dv}{dh}\frac{dh}{dt}$$

Where $\frac{dv}{dt}$ is given by (4.1.18) and where $\frac{dv}{dh} = A(h)$ . Thus the rate of change of the liquid height depends on the net flow rate according to

$$\dot{h} = \frac{1}{A(h)}[w_{in}(t) - w_{out}(t)] \qquad (4.1.19)$$

Alternatively, we can write $\frac{dv}{dt}$ as

$$\frac{dv}{dt} = \frac{dv}{dp}\frac{dp}{dt}$$

where $\frac{dv}{dp} = C(h)$ .Hence the rate of change of the pressure at the base of the vessel is

$$\dot{p} = \frac{1}{C(h)}[w_{in}(t) - w_{out}(t)] \qquad (4.1.20)$$

where $C(h)$ is given by (4.1.15).

Because any of the variables $v$, $h$, and $p$ can be used as a measure of the amount of liquid in a vessel, we generally select one of them as a state variable. Then (4.1.18), (4.1.19), and (4.1.20) will yield the corresponding state-variable equation when $w_{in}$ and $w_{out}$ are expressed in terms of state variables and insputs.

If the cross-sectional area of the vessel is variable, then the coefficient $A(h)$ in (4.1.19) will be a function of $h$, and the system model will be non-linear.

**Hydraulic Resistance**

As liquid flows through a pipe, there is a drop in the pressure of the liquid over the length of pipe. There is likewise a pressure drop if the liquid flows through a valve or through an orifice. The change in pressure associated with a flowing liquid results from the dissipation of energy and usually follows a nonlinear algebraic relationship between the flow rate $w$ and the pressure difference $\Delta p$. A hydraulic valve is used as an energy dissipating element. The expression

$$w = k\sqrt{\Delta p} \qquad (4.1.21)$$

describes an orifice and a valve and is a good approximation for turbulent flow through pipes. In equation (4.1.21), $k$ is a constant that depends on the characteristics of the pipe, valve, or orifice. A typical flow rate versus pressure difference is shown in Figure (4.8,a).
Because (4.1.21) is a nonlinear relationship, we must linearise it about an operating point in order to develop a linear model of a hydraulic system. If we draw the tangent to the curve of $w$ versus $\Delta p$ at the operating point, the reciprocal of the slope is defined as the hydraulic resistance R. Figure(4.8,b) illustrates the geometric interpretation of the resistance, which has units newtons-seconds per meters[5].

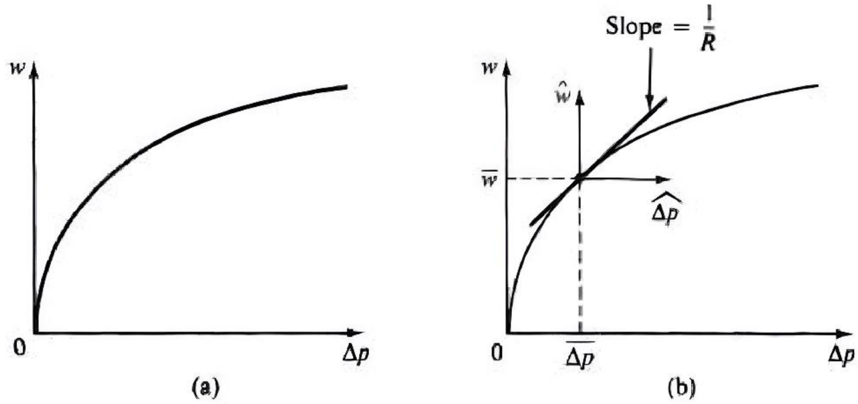



**Figure4.8** (a) Flow rate versus pressure difference given by (21). (b) Geometric interpretation of hydraulic resistance

**Hydraulic Sources**

In most hydraulic systems, the source of energy is a pump that derives its power from an electric motor. Here we shall consider the centrifugal pump driven at a constant speed, which is widely used in chemical processes. Typical input-output relationships for a centrifugal pump being driven at three different constant speeds are shown in Figure (4.9). Pump curves of $\Delta p$ versus $w$ are determined experimentally under steady- state conditions and are quite nonlinear.

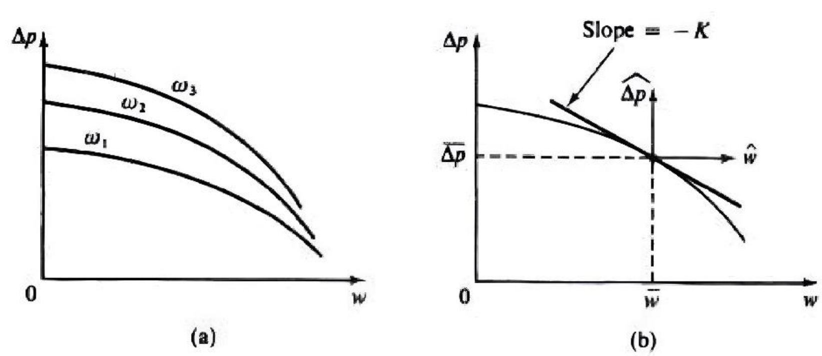



**Figure 4.9** Typical centrifugal pump curves where $\Delta p = p_2 - p_1$. (a) For three different pump speeds (ω1< ω2< ω3). (b) Showing the linear approximation

## 4.2 Characteristics of Subsystems

Although a general characterization of hydraulic servo-systems has been already given in Chapter 2, a more precise description, in view of mathematical modeling of this system is to yet given, including system boundary.

The control input $u$ provided by the valve (actuating element) is generally either a voltage or current, and can be considered as an ideal input signal, i.e., the input impedance for the electrical signal is infinitely large. The control input $u$ of the servo-valve is used to control the oil flow through the valve orifices. Oil is supplied by a power supply unit under constant supply pressure $p_s$, while the return flow is fed to a tank under the return pressure $p_t$.

The actuator (cylinder) consists of two oil chambers, separated by the piston. The resulting oil flows $Q_A$ and $Q_B$ moving into and out of the chambers drive the piston, thereby generating the required pressures $p_A$ and $p_B$, respectively, to move the load of the actuator. In this way, the piston motion depends on the load of the actuator. Actually, for motion systems with free moving bodies, this load can be seen as an inertia $m_p$ plus some external force $F_{ext}$, which might include gravity forces (Figure 4.10).

A constant valve opening and a constant pressure drop over the valve result in a constant oil flow which will generate a linear movement (translation) of constant speed. This explains the basic integrating behavior of a hydraulic actuator between the input $u$ and the output $y$ (piston position). Because the oil in two chambers is compressible, the two oil columns act as two springs. The load is clamped between these springs by the piston. This causes the second-order behavior, which is always found in series with the integrating character of a hydraulic actuator.

In order to reduce the complexity of the modeling of the system inside the defined system boundary, it is useful to distinguish a number of subsystems (Figure 4.11):

1. There is the servo valve which transfers the control input into the oil flows, depending on the actuator pressures. Although the device is designed to be fast and to show linear input-output behavior, its actual behavior is generally not ideal. Because the valve flow drives the actuator, any non-ideal behavior of the valve propagates through the complete servo-system, for which reason the valve is explicitly considered in the modeling as a separate subsystem.
2. There is the hydraulic actuator including load mass, with the driving oil flows QA and $Q_B$ and the external force $F_{ext}$ as inputs, and correspondingly the actuator pressures

$p_A$ and $p_B$, and position $x_p$ or velocity $\dot{x}_P$ as outputs. The hydraulic actuator as subsystem forms the kernel of the complete hydraulic servo-system.

The actuator base should always be designed to be as stiff as possible, with an impedance that is large enough to avoid parasitic energy exchange. In other words, no parasitic motions of the base should occur.

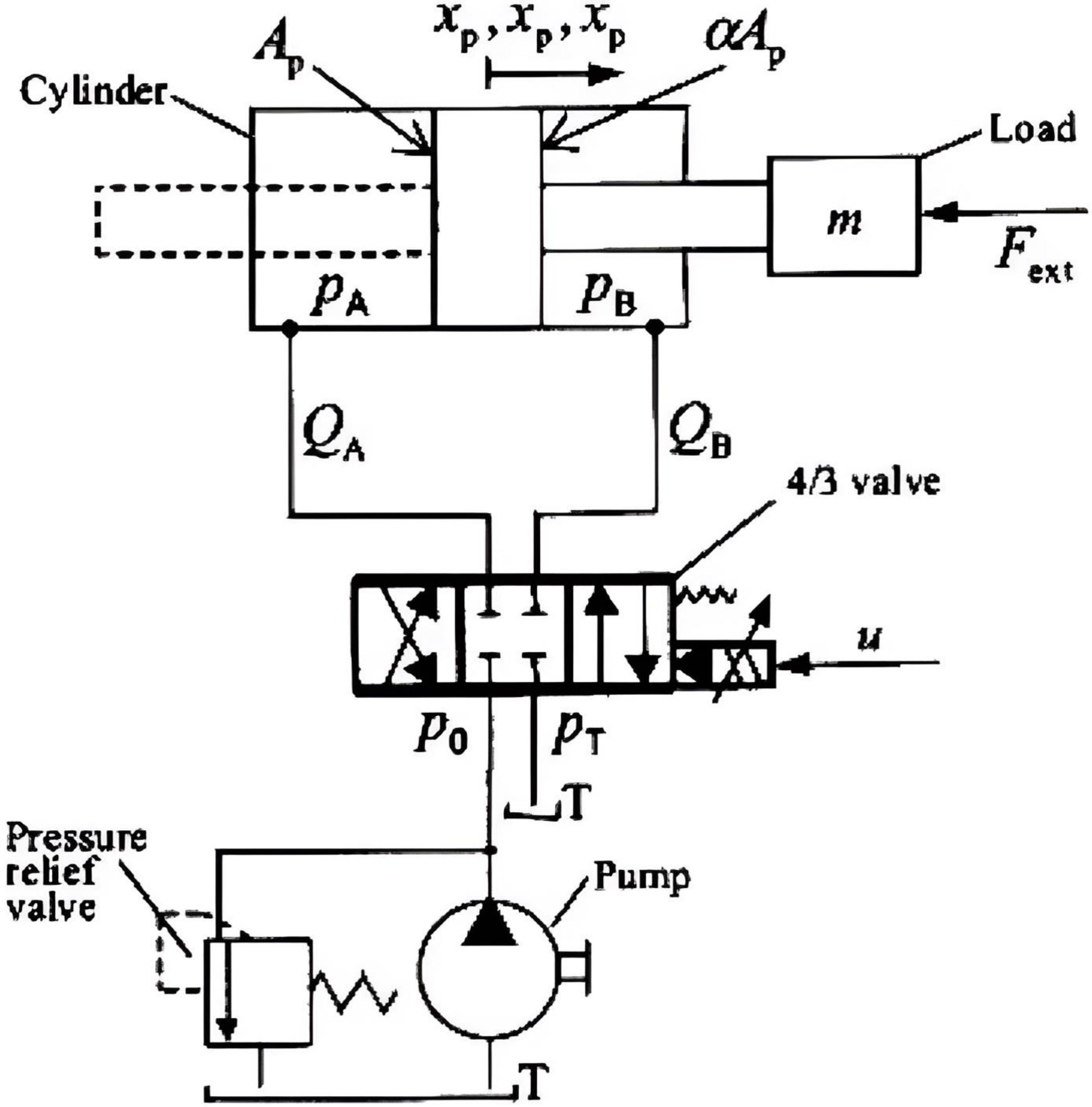



**Figure 4.10** Valve-cylinder combination with power supply

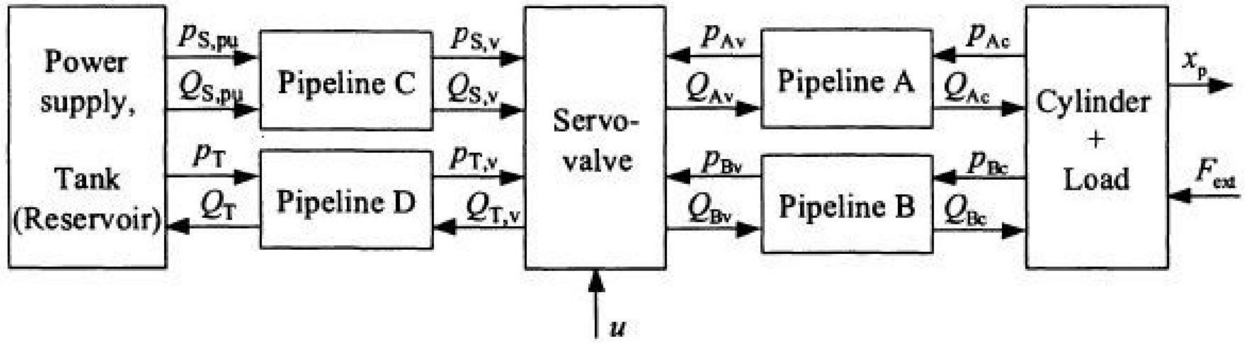



**Figure 4.11** Subsystems of hydraulic servo-systems with interconnections

3. The set of pipelines between the valve and the actuator is to be considered as a third subsystem, which is especially important, for instance, when the actuator has a long stroke. Because of the compressibility and the inertia of the oil, the relatively long pipelines cannot be seen as static devices; pressure waves travel with a finite velocity through the line and are almost ideally reflected at the end of the line. This is indicated in Figure 4.11, where the pressures and the valve-side variables or the cylinder-side variables are concerned; the index Av denotes the valve-side of pipeline A, and the index Ac denotes the cylinder-side; the indices Bv and Bc denotes the valve and cylinder-side of pipeline B respectively.
4. Next, the set of pipelines between the power supply system and the valve, as well as between the tank and the valve, is to considered as a fourth subsystem. This is especially important when the distance between main line and accumulator/valve is large. In many hydraulic servo-control applications, where the valve is placed very close to the cylinder, the dynamics of the return line may be essential for the function and especially for the durability of the hydraulic system.

### 4.2.1 Model Complexity and Applications

In order to obtain insight into the various physical phenomena that play a role in the behavior of a hydraulic servo-system, the modeling approach starts with extensive theoretical modeling of the complete system. Thereby, the models are based on basic physical laws such as mass balances for oil volumes, equations of motion for moving parts, equations for turbulent flow through small restrictions, and so on, as described in Chapter 3. In the theoretical modeling, all effects are included that are expected to influence the dynamic behavior of the system.

A problem with such complex non-linear model is that it is difficult to choose the large number of physical parameters involved in the model such that quantitatively valid simulation results are obtained. Although many of parameter value are known prior , with reasonable accuracy, a large number of parameters are only known within a certain range, and some are even completely unknown. This may be due to manufacturing tolerances, or due to the fact that manufacturers do not provide parameter values.

The consequence of this problem is that the theoretical model is often not useful for quantitative analysis of the hydraulic servo-system behavior. Nevertheless, a lot of qualitative insight can be obtained from simulations, which can be utilized to reduce the hydraulic servo-system model such that only relevant dynamics are taken into account. Furthermore, the simulation results provide the necessary insight to decide which non-linearities of the hydraulic servo-system are relevant. Thus a relatively simple model for the hydraulic servo-system can be derived, which includes relevant nonlinearities and dynamics and which forms a good basis for experimental parameter estimation of the system properties.

Whereas the model of the hydraulic servo system is also to be used for control design, the model should not only be qualitatively correct, but should also have predictive value in a quantitative sense. This means that the model should accurately represent the dynamic and non-linear behavior of a real system. This is possible if there is a clear relation between the input-output behavior of the model and the parameters of the model, so that the parameters can be chosen or adjusted such that the model fits the behavior of some real servo-system. In other words, it should be possible to identify the model parameters from experimental input-output data.

For this purpose, it is at least required that the model is of the same order as the relevant dynamics of the real system, and that only the dominant non-linearities of the real system are included in the model. In other words, the model should not include irrelevant dynamic and/or non-linearities. Actually, the theoretical model, including “all” physical phenomena that possibly play a role in the system behavior, does not fulfil these requirements on the model; it is too complex to be used directly for identification and control design purposes.

## 4.3 Elementary Models

The aim of this section is to collect some of the most fundamental models of hydraulic servo-systems, including valves, cylinders, pipelines and power supplies.

### 4.3.1 Valves

Because a valve is a complicated device, the theoretical modeling is likewise rather complicated, many dynamic and non linearities effects are included, resulting in a complex non-linear model.

Most of the non-linearities present in valves are :

- *Dead band* (for valves with overlap).
- *Square-root function* of the flow equations.
- *Saturations* (of stroke, velocity flow).
- Valve hysteresis, which results from frictional forces between spool and valve body not being evenly distributed over the entire spool stroke.
- *Response sensitivity*, i.e., the amount of electrical input signal change needed to obtain a measurable valve flow change in the same direction from which stopping occurred.
- *Reversal error*, i.e., the amount opf electrical input signal change needed to obtain a measurable valve flow change in the opposite direction from which stopping occurred.
- *Repeatability*, i.e., the valve's ability to produce an identical flow rate when the same input signal is repeated.
- Complicated flow-induced forces and *friction forces.*
- Different non-linearities in the torque motor model.

#### 4.3.1.1 Pressure-Flow Equations for Spool Valves

Flows through valve orifices are described by the orifice Equation 3.3.6, which takes the direction of the pressure drop (flow direction) into account

$$Q = Q(x_v, \Delta P) = c_v x_v \sqrt{|\Delta p|} sign(\Delta p) \tag{4.3.1}$$

Consider the four-way spool valve with an ideal critical centre as shown in Figure 4.12. From this figure, the flow equations can then be written as

$$Q_A = Q_1 - Q_2 = c_{v1} sg(x_v) sign(p_s - p_A)\sqrt{|p_s - p_A|} - c_{v2} sg(-x_v) sign(p_A - p_T)\sqrt{|p_A - p_T|} \tag{4.3.2}$$

$$Q_B = Q_3 - Q_4 = c_{v3}sg(x_v)sign(p_s - p_B)\sqrt{|p_s - p_B|} - c_{v4}sg(-x_v)sign(p_B - p_T)\sqrt{|p_B - p_T|} \quad (4.3.3)$$

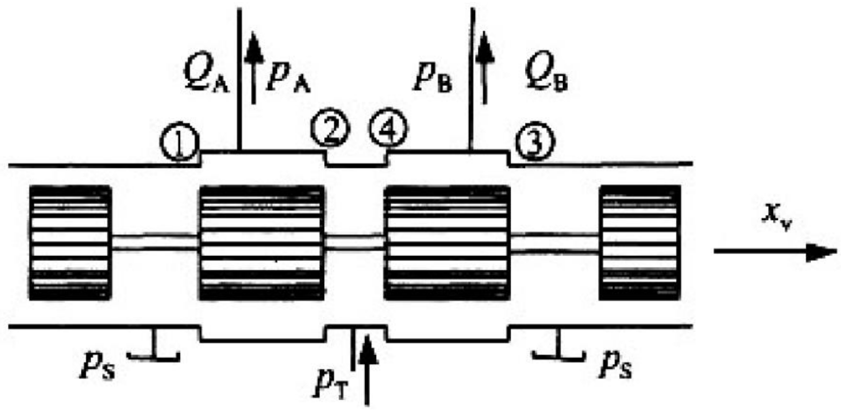



**Figure 4.12** Zero lapped 4\3 (four ports/three switching positions) spool valve

The function sg(x) is defined by

$$sg = \begin{cases} 0, x < 9 \\ x, x \geq 0 \end{cases} \quad 4.3.4$$

The flow direction is determined by the sign of $x_v$. $c_{vi}$, $i$= 1,2,3,4 are discharge coefficients of the valve orifices (also termed as valve constants), which will be equal if all orifices are identical.

So far we have been concerned with the pressure-flow behavior of critical centre valves. However, because the production of an exact critical centre valve is hardly possible, the "practical critical centre valve" has radial clearance and just a little underlap. There always exist some extra leakage flows in the valve-actuator system which influence the performance and behavior of such valves and the associated pressure-flow curves for small openings. Yet, it is better from a practical viewpoint to identify and compensate for these effects using measured data, rather than trying to model these phenomena theoretically and thus introduce more uncertain parameters into the model.

#### 4.3.1.2 Dynamic Characteristics of Servo valves

The dynamic behavior of valves involves a large number of parameters. Thereby, many parameters may only be known within some range, or even completely unknown.

Manufacturers' catalog provides useful information for various sizes and types of valve.

Servo-valves can be approximated by a second-order model of the form [1], [2], [18]:

$$\frac{1}{\omega_v^2}\ddot{x}_v + \frac{2D_v}{\omega_v}\dot{x}_v + x_v + f_{hs}sign(\dot{x}_v) = K_v u_v \qquad (4.3.5)$$

with the normalised valve position ($x_v$), velocity($\dot{x}_v$), acceleration($\ddot{x}_v$) and valve input signal ($u_v$).

The parameters valve gain $K_v$, natural frequency $\omega_v$ and damping coefficient $D_p$ in equation 4.3.5 can be gained from identification or normally extracting data from the manufacturers catalogue information.

Equation 4.3.2, 4.3.3 and 4.3.5 can be combined to form the block diagram of servo-valve in Figure 4.13.

It is usually possible to use a simple dynamic model according to Equation 4.3.5 with its five parameters (in conjugation with the flow Equations 4.3.2 and 4.3.3) regardless of the complexity of the servo-valve.

It should be noted that even for the substantially more complicated servo-valve model described above, the prior knowledge of physical parameters is insufficient to determine the physical parameters in the theoretical model uniquely from identified parameters as in [18]. This means that a kind of gap between servo-valve model and the identified model may remain.

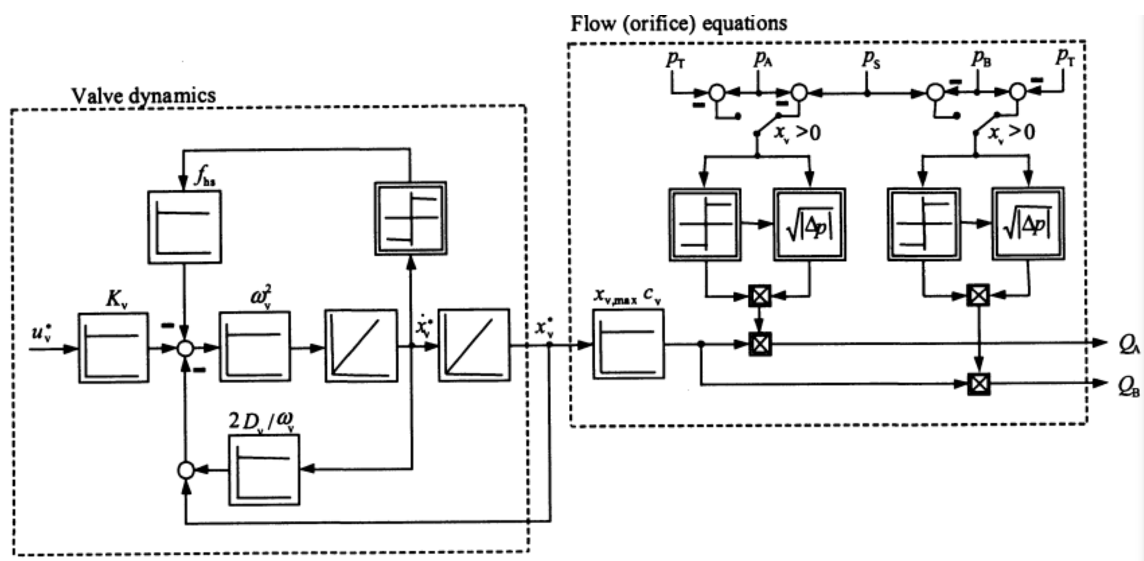



**Figure 4.13** A simplified block diagram of servo-valves

### 4.3.2 Hydraulic Cylinders

Theoretical modelling of the hydraulic actuator is less involved than that of the servo valve [3]. The principal model relations have been given by [11] and [13]. Although most parameters of the theoretical actuator model accurately known prior to use, the quantitative validity of the model can generally be improved by experimental estimation of the parameters. For this purpose, the theoretical model is slightly simplified, neglecting irrelevant dynamics and non-linearities, resulting in a compact model for the actuator.

The most important (non-linear) effects, which contribute to the cylinder model, are[2], [4] and [5]:

- Geometrical asymmetry due to difference in piston and ring side area.
- Pressure dependent *effective bulk modulus* in conjunction with the fluid elasticity as well as the elasticity of mechanical compliance.
- *Position dependent actuator stiffness, i.e.,* the natural frequency, and therefore the damping ratio of the transient dynamics, varies with cylinder position.
- *Friction forces* opposing the piston velocity that encompasses viscous friction and highly non-linear Coulomb friction or dry friction.

#### 4.3.2.1 Pressure Dynamics in Cylinder Chambers

Applying the continuity Equation 3.2.7 to each of the cylinder chambers (see Figure 4.10) yields

$$Q_A - Q_{Li} = \frac{\dot{V}_A}{E(p_A)} \dot{p}_A \tag{4.3.6}$$

$$Q_B + Q_{Li} - Q_{Le} = \dot{V}_B + \frac{V_B}{E(p_B)} \dot{p}_A \tag{4.3.7}$$

where $V_A$ is the volume of piston chamber and $V_B$ is the volume of ring chamber, both including the valve connecting line and chamber volume. $Q_{Li}$and $Q_{Le}$ denote the internal leakage flow and the external leakage flow respectively [1] and [7].

The volumes of the chambers may be written as:

$$V_A = V_{pl,A} + \left(\frac{S}{2} + x_p\right) A_p = V_{A0} + x_p A_p \tag{4.3.8}$$

$$V_B = V_{pl,B} + \left(\frac{S}{2} - x_p\right)\alpha A_p = V_{B0} - x_p \alpha A_p \tag{4.3.9}$$

where, $V_{pl,A}$ and $V_{pl,B}$ are the pipeline volumes at the A-side and B-side respectively. The initial chamber volumes $V_{A0}$ and $V_{B0}$ consist of an efficient part (the volume required to fill only the chambers) and an inefficient part (mainly the volume of pipelines between the valve and the actuator). Since it is assumed that the piston is centred such that initial volume chambers are equal, hence:

$$V_{A0} = V_{B0} = V_0 \tag{4.3.10}$$

Equations 4.3.6 and 4.3.7 can be rearranged to yield the pressure dynamics equations

$$\dot{p}_A = \frac{1}{C_{hA}}(Q_A - \mathrm{A}_p \dot{x}_p + Q_{Li} - Q_{LeA}) \tag{4.3.11}$$

$$\dot{p}_B = \frac{1}{C_{hB}}(Q_B - \alpha \mathrm{A}_p \dot{x}_p + Q_{Li} - Q_{LeB}) \tag{4.3.12}$$

The hydraulic capacitance of each chamber is given by

$$C_{hA} = C_h(p_A, x_q) = \frac{V_A(x_p)}{E_A(p_A)} = \frac{V_{pl,A}(x_{p0} + x_p)A_p}{E_A(p_A)} \tag{4.3.13}$$

$$C_{hB} = C_h(p_B, x_q) = \frac{V_B(x_p)}{E_B(p_B)} = \frac{V_{pl,B}(x_{p0} + x_p)A_p}{E_B(p_B)} \tag{4.3.14}$$

The internal leakage flow (leakage from one chamber to the other) can be calculated as [5],[18]:

$$Q_{Li} = K_{Li}(p_B - p_A) \tag{4.3.15}$$

where $K_{Li}$ is the internal leakage flow coefficient. External leakage (i.e., leakage from each cylinder chamber to case drain or to tank) is neglected [2], [5]:

$$Q_{LeA} = Q_{LeB} = 0$$

#### 4.3.2.2 Equation of Piston Motion

The equation of piston motion governing the load motion arises by applying Newton's second law to the forces on the piston. The resulting force equation is[2],[9]:

$$m_t \ddot{x}_P + F_f(\dot{x}_p) = (p_A - \alpha p_{\boldsymbol{B}})A_p - F_{ext} \tag{4.3.16}$$

The total mass $m_t$ consists of the piston mass $m_p$ and the mass of hydraulic fluid in the cylinder chambers and in the pipelines, $m_{A,fl}$ and $m_{B,fl}$ respectively:

$$m_t = m_p + m_{A,fl} + m_{B,fl} \tag{4.3.17}$$

However, the mass of fluid can usually be neglected compared with the piston mass.

One standard method is to model friction as a function of velocity, e.g., Equation 4.3.18 as referred in [4], [17] and [18], Figure 4.14:

$$F_f(\dot{x}_p) = F_v(\dot{x}_p) + F_c(\dot{x}_p) + F_s(\dot{x}_p)$$

$$F_f(\dot{x}_p) = \sigma\dot{x}_p + sign(\dot{x}_p)[F_{co} + F_{so}\exp\left(-\frac{|\dot{x}_p|}{c_s}\right)] \tag{4.3.18}$$

The three characteristic parts of this curve are: viscous friction $F_v$, static friction $F_s$ and Coulomb friction $F_c$. σ is the parameter for viscous friction, $F_{c0}$ is the parameter for Coulomb friction, $F_{s0}$ and $c_S$ (known as Stribeck velocity) are the parameters for static friction. A typical Stribeck friction curve is shown in Figure 4.14 [1], [2]:

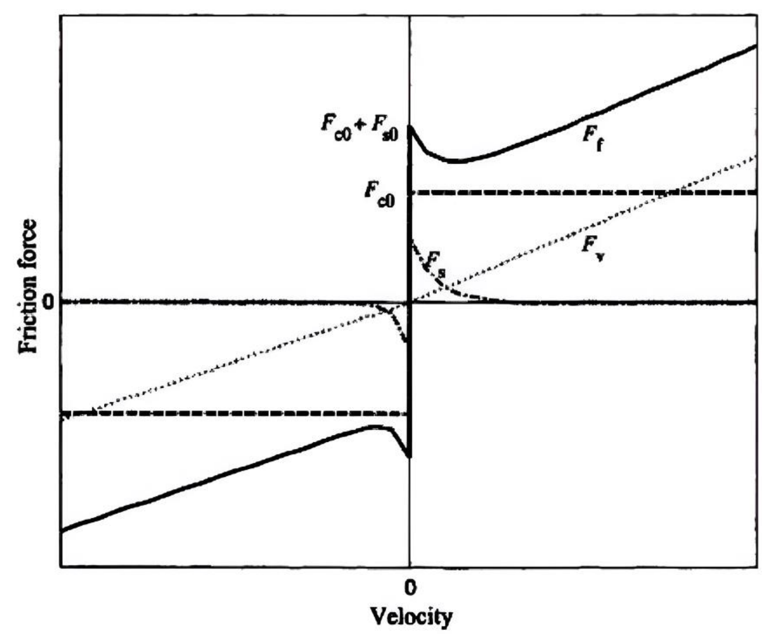



**Figure 4.14** Velocity-dependent friction force (Stribeck curve)

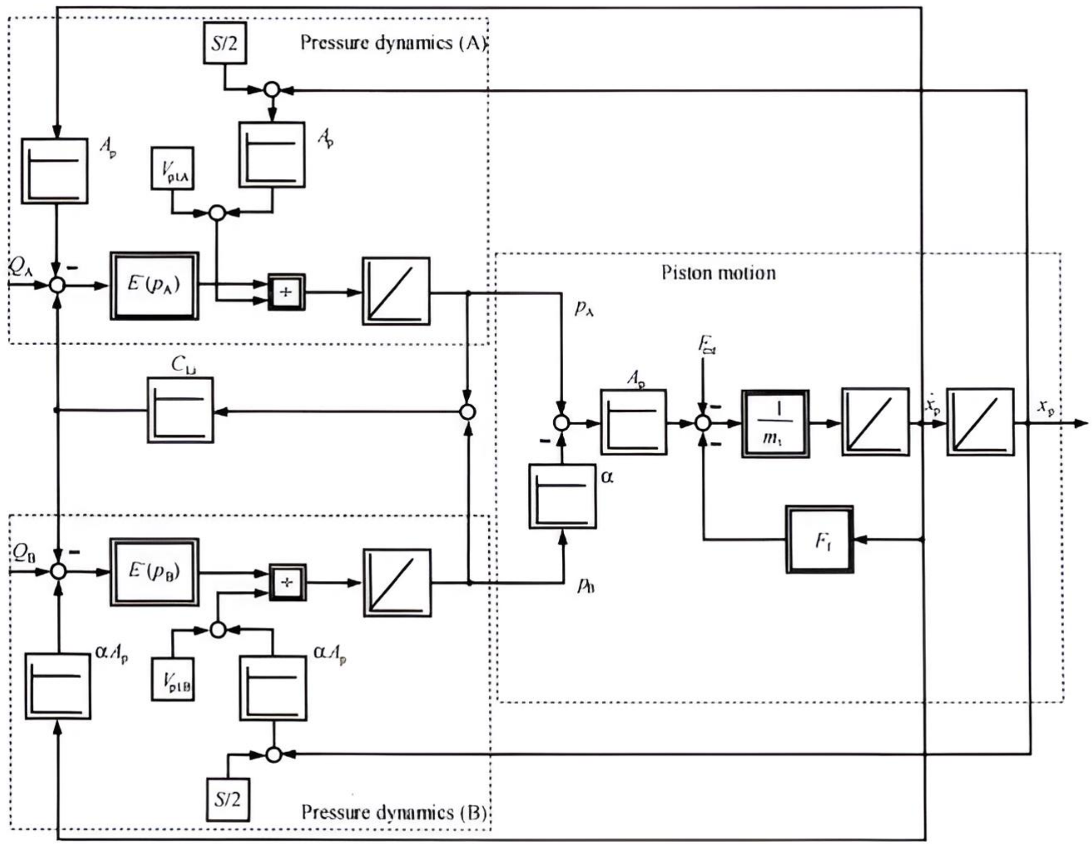



**Figure 4.15** A block diagram of differential cylinders

Equation 4.3.11, 4.3.12 and 4.3.16 can be combined to form the block diagram of the hydraulic cylinder, Figure 4.15, [1] [12].

### 4.3.3 Pipelines

The hydraulic system's components are usually connected by pipelines. As long as the length of pipeline does not exceed a certain limit[3] [5]:

$$l < \frac{c}{10 f_{max}} \qquad (4.3.19)$$

the whole pipeline volume can be added to the corresponding chamber volume of the cylinder model and the pipeline dynamics can be neglected. Thereby, c is the sonic velocity (or wave speed) in oil.

$f_{max}$ denotes the maximum value of the interesting frequency. Otherwise, the dynamic behavior of hydraulic pipelines, having distributed parameters, must be considered, and pipeline modeling has to be performed.

In many industrial applications, the inefficient volumes appear to be significantly large. This is due to installation of several hydraulic safety features between the valve and the actuator.

More specifically, there are many situations in which pipelines effects may play an important role in the hydraulic control systems behavior [1]:

- There are many applications in which oil supply unit is separated from the hydraulic servosystem, often requiring long supply lines. The dynamics due to these long lines may, in practice, lead to undesired oscillations, possibly excited by the pump flow pulsations [18].
- In some other applications, pipeline dynamics have to be considered because the servo-valve cannot directly be placed on the actuator chambers, so that relatively long pipelines between the valve and the actuator chambers are required. This typically applies for long-stroke cylinders and some heavy duty rotary drive applications, where it is practically impossible to place the servo-valve directly on the actuator [18].
- Last but not least, there are some applications, such as hydraulic power assisted car steering and aircraft brake hydraulic systems, where the modeling of supply and return lines is one of the key problems in the modeling of the whole system. On the one hand, these hydraulic lines consist of quite different parts, such as pipes and rubber hoses. On the other hand, they may have great effect on the generation of considerable pressure peaking and oscillations and thus hydraulic noise [7] [17].

#### 4.3.3.1 Basic Model Equations

Simplified (yet sufficient) models for pipeline dynamics have been derived by many reasearchers under following assumptions [2]:

- rigid pipe (finite wall stiffness can be considered by corrected oil bulk modulus B).
- laminar flow
- constant temperature and negligible heat transfer effects
- negligible tangential flow
- negligible differences of p and ρ in the radial direction
- negligible radial speed (Hagen-Poiseuille flow)

The simplified mathematical model consideration for pipelines:

$$\frac{\partial p}{\partial t} = -B\frac{\partial v_x}{\partial x} \qquad (4.3.20)$$

### 4.3.4 Typical Non-linear State-space Model

If the state variables and the input variables are defined as[1] [8] [12]:

$x_1 \equiv x_p$ $\quad x_2 \equiv \dot{x}_p$ $\quad x_3 \equiv p_B$ $\quad x_4 \equiv p_A$

$x_5 \equiv x_v$ $\quad x_6 \equiv \dot{x}_v$

$u_1 \equiv u_v$ (valve input) $\quad u_2 \equiv F_{ext}$ (external disturbance)

then the completely non-linear model of hydraulic servo-systems, which consists of a (two-stage) servo-valve and a (differential) cylinder can be written as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m_t(x_1)}[(x_3 - \alpha x_4)A_p - F_f(x_2) - u_2]$$

$$\dot{x}_3 = \frac{E_A(x_3)}{V_A(x_1)}[Q_A(x_3, x_5) - A_p x_2 + Q_{Li}(x_3, x_4)]$$

$$\dot{x}_4 = \frac{E_B(x_3)}{V_B(x_1)}[Q_B(x_4, x_5) + \alpha A_p x_2 - Q_{Li}(x_3, x_4)]$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = \omega_v^2[u_1 - \frac{2D_v}{\omega_v}x_6 - x_5 - f_{hs}sign(x_5) \qquad (4.3.21)$$

The total mass is given by

$$m_t(x_1) = m_p + \rho[V_{LA} + (x_{10} + x_1)A_A] + \rho[V_{LB} + (x_{10} - x_1)\,\alpha A_p \qquad (4.3.22)$$

The friction forces are given by the equation

$$F_f(x_2) = \sigma x_2 + sign(x_2)[F_{co} + F_{so}\exp\left(-\frac{|x_2|}{c_s}\right)] \qquad (4.3.23)$$

The effective bulk modulus is expressed as

$$E_A(x_3) = a_1 E_{max}\log[100\left(a_2\frac{x_3}{p_{max}} + a_3\right)] \qquad (4.3.24)$$

$$E_B(x_4) = a_1 E_{max}\log[100\left(a_2\frac{x_4}{p_{max}} + a_3\right)] \qquad (4.3.25)$$

The cylinder chamber volumes are given by

$$V_A(x_1) = V_{pl,A} + (x_{10} + x_1)A_p \tag{4.3.26}$$

$$V_B(x_1) = V_{pl,A} + (x_{10} - x_1)\alpha A_p \tag{4.3.27}$$

The flow equations are characterized by

$$Q_A = c_{v1} sg(u_1) sign(p_s - x_3)\sqrt{|p_s - x_3|} - c_{v2} sg(-u_1) sign(x_3 - p_T)\sqrt{|x_3 - p_T|} \tag{4.3.28}$$

$$Q_B = c_{v3} sg(-u_1) sign(p_s - x_4)\sqrt{|p_s - x_4|} - c_{v4} sg(u_1) sign(x_4 - p_T)\sqrt{|x_4 - p_T|} \tag{4.3.29}$$

The (internal) leakage flow can be calculated by

$$Q_{Li} = K(x_4 - x_3) \tag{4.3.30}$$

Note, that a number of standard assumptions that are typically satisfied in practice have been made for the derivation of this non-linear model:

- The hydraulic pump delivers a constant supply pressure, irrespective of the oil flow
- The pressure in the tank in constant.
- A symmetrical critical-centre 4-way valve is modeled.
- The flow through the valve is turbulent
- The inefficient volumes of e.g. the pipelines between valve and actuator can be modeled as addictive inefficient strokes.
- Coulomb, static and viscous friction are acting upon the actuator.
- The surroundings of the actuator and the load are rigid.
- Possible dynamic behavior of the pressure in the pipelines between valve and actuator is assumed to be negligible (i.e., the valve is placed directly upon the actuator). These dynamics can, however, simply be added if they appear to be essential, as explained below.

## 4.4 Linearised Models

A linearisation of the algebraic non-linear equations which describe the pressureflow characteristics of valves is often necessary before (linear) identification or control design techniques can be applied.

### 4.4.1 Valve Sensitivity Coefficients

The expression of Equations 4.3.2 and 4.3.3 as a Taylor series about a particular operating point $P_0$ = $(x_{v0}, p_{ao}, p_{bo})$

$$\Delta Q = \left.\frac{\partial Q}{\partial x_v}\right|_{p_0} \Delta x_v + \left.\frac{\partial Q}{\partial \mathrm{p}}\right|_{p_0} \Delta p + \cdots \tag{4.4.1}$$

leads to

$$\Delta Q_A = K_{Qx,A}\, \Delta x_v + K_{Qp,A}\, \Delta p_A \tag{4.4.2}$$

$$\Delta Q_B = K_{Qx,B}\, \Delta x_v + K_{Qp,B}\, \Delta p_B \tag{4.4.3}$$

Thereby, the flow gains are defined by

$$K_{Qx,A} = \left.\frac{\partial Q_A}{\partial x_v}\right|_{p_0} = \begin{cases} c_v\sqrt{(p_s - p_{A0})}, for\ x_v > 0 \\ c_v\sqrt{(p_{A0} - p_T)}, for\ x_v < 0 \end{cases} \tag{4.4.4}$$

$$K_{Qx,B} = \left.\frac{\partial Q_B}{\partial x_v}\right|_{p_0} = \begin{cases} -c_v\sqrt{(p_{B0} - p_T)}, for\ x_v > 0 \\ -c_v\sqrt{(p_s - p_{B0})}, for\ x_v < 0 \end{cases} \tag{4.4.5}$$

The flow-pressure coefficients are expressed as

$$K_{Qp,A} = \left.\frac{\partial Q_A}{\partial p_A}\right|_{p_0} = \begin{cases} \frac{-c_v x_{v0}}{2\sqrt{(p_S - p_{A0})}}, for\ x_v < 0 \\ \frac{c_v x_{v0}}{2\sqrt{(p_{A0} - p_T)}}, for\ x_v > 0 \end{cases} \tag{4.4.6}$$

$$K_{Qp,B} = \left.\frac{\partial Q_B}{\partial p_B}\right|_{p_0} = \begin{cases} \frac{-c_v x_{v0}}{2\sqrt{(p_{B0} - p_T)}}, for\ x_v < 0 \\ \frac{c_v x_{v0}}{2\sqrt{(p_S - p_{B0})}}, for\ x_v > 0 \end{cases} \tag{4.4.7}$$

Other useful quantities are the pressure sensitivities, defined by

$$K_{px,A} = \left.\frac{\partial p_A}{\partial x_v}\right|_{p_0} \quad (4.4.8)$$

$$K_{px,B} = \left.\frac{\partial p_B}{\partial x_v}\right|_{p_0} \quad (4.4.9)$$

which are related to the other quantities by the well-known relation from calculus.

$$\frac{\partial p}{\partial x_v} = \frac{\partial Q/\partial x_v}{\partial Q/\partial p} \text{ or } K_{px} = \frac{K_{Qx}}{K_{Qp}} \quad (4.4.10)$$

The coefficients $K_{Qx}$. $K_{Qp}$ and $K_{px}$ are referred to as valve sensitivity coefficients and are extremely important in determining stability, frequency response, and other dynamic characteristics.

### 4.4.2 Linear State-space Model

The linearisation of the non-linear Equations 4.3.21-4.3.30 yields the state-space description

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0 & 0 & 0\\ 0 & -\zeta_1 & \zeta_2 & -\alpha\zeta_2 & 0 & 0\\ 0 & -\zeta_3\zeta_5 & \zeta_5(\zeta_7-\zeta_4) & \zeta_4\zeta_5 & \zeta_8\zeta_5 & 0\\ 0 & \alpha\zeta_3\zeta_6 & \zeta_4\zeta_6 & -\zeta_6(\zeta_9+\zeta_4) & \zeta_{10}\zeta_6 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & -\zeta_{11} & -\zeta_{12}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ \zeta_{14}\end{bmatrix}u_1 + \begin{bmatrix}0\\ -\zeta_{13}\\ 0\\ 0\\ 0\\ 0\end{bmatrix}u_2 \quad (4.4.11)$$

with the parameters

$$\zeta_1 = \frac{\sigma}{m_p}, \qquad \zeta_2 = \frac{A_p}{m_p}, \qquad \zeta_3 = A_p, \qquad \zeta_4 = K_{Li}, \qquad \zeta_5 = \frac{E'(x_{30})}{V_A(x_{10})}$$

$$\zeta_6 = \frac{E'(x_{40})}{V_B(x_{10})}, \quad \zeta_7 = K_{Qp,A}, \quad \zeta_8 = K_{Qx,A}, \quad \zeta_9 = K_{Qp,B}, \quad \zeta_{10} = K_{Qx,B}$$

$$\zeta_{11} = \omega_v^2, \qquad \zeta_{12} = 2D_v\omega_v, \ \zeta_{13} = \frac{1}{m_p}, \qquad \zeta_{14} = \frac{\omega_v^2}{u_{max}} \qquad (4.4.12)$$

Thereby, the influence of the piston position change $\Delta x_p$ in $V_A(x_p)$ and $V_B(x_p)$, as well as pressure changes $\Delta p_A$ and $\Delta p_B$ in $E'(p_A)$ and $E'(p_B)$, has been neglected. $\Delta Q_A$ and $\Delta Q_B$ have been replaced by Equations 4.4.2 and 4.4.3 .

# CHAPTER 5

# Conclusion and Future Scope

## 7.1 Conclusion

In this thesis, attempts are made to shed light on Hydraulic Servo systems, which are integral to technology across both industrial and domestic applications. Proper mathematical models have been developed for hydraulic components such as hydraulic actuators, servo valves, and piston chambers, with attention to system requirements, characteristics, and fluid flow properties.

Mathematical models have been used to derive linearized models of these hydraulic system components, emphasizing their structural integrity and practical utility for control design purposes. By using linearized models, time responses can be obtained to help identify key parameters and monitor overall system behavior:

- The linearized models function as virtual representations of hydraulic components, enabling analysis of system behavior with respect to time without physically testing the components.
- Through the use of these models, the performance of hydraulic system components, including subsystems like servo valves, can be evaluated and optimized.
- By adjusting subsystem parameters such as pressure, active annulus area, stroke length, and control current (voltage), designers can identify the optimum parameters for hydraulic servo systems.

This approach of physical modeling, focusing on linearized representations, is ideal for gaining insight into the relevant dynamics of technical systems. Simplified models retain essential nonlinear effects where necessary, including directional valve changes, friction forces, and oil compressibility. However, by using linearization, the models remain manageable and effective for control design, where linear models are advantageous due to their simplicity and compatibility with standard control methods.

The thesis emphasizes reduced, physically structured models that balance complexity with application efficiency, highlighting that simplifications should be application-specific. Comprehensive mathematical derivations of different hydraulic servo system elements are presented, starting from first principles, ensuring the models retain both accuracy and practical applicability.

Further chapters focus on the systematic modeling of the entire system along with its subsystems, with each component modeled individually based on the mathematical principles outlined earlier. This modular approach not only supports thorough validation of each subsystem but also enables a deeper understanding of the integrated Hydraulic Servo system. The final integrated model effectively incorporates nonlinear effects critical to hydraulic systems while providing a streamlined framework suitable for industrial and technical applications.

## 7.2 Future Scope

The future scope of this thesis lies in expanding the application of the developed linearized models to more complex and real-world hydraulic systems. These models can be further refined to account for additional factors like environmental influences, wear and tear of components, and real-time control adjustments. Future research could focus on integrating advanced control strategies, such as adaptive or predictive control, to optimize system performance in dynamic conditions. Moreover, the models can be adapted to various industries, including robotics, aerospace, and automotive, where hydraulic systems play a critical role. Further exploration into hybrid modeling approaches, combining both linear and nonlinear elements, could enhance the accuracy and efficiency of simulations. Additionally, experimental validation of the models using physical prototypes could offer valuable insights, bridging the gap between theoretical models and practical implementations.

# REFERENCES

[1]Mohieddine Jelali, Andreas Kroll, Hydraulic Servo-systems: Modelling, Identification and Control, Berlin: Springer Science & Business Media, 2012.

[2] Mohamed ESME, Magdy ASA, Mohamed AMH, “ Position control of hydraulic servo system using proportional-integral-derivative controller tuned by some evolutionary techniques”, J Vib Control, 2016;22(12):2946-2957.

[3] Mohamed ESME, Magdy ASA, Mohamed AMH, “Position control of hydraulic servo systems using evolutionary techniques (dissertation master thesis)”, Cairo: Faculty of Engineering, Cairo University, Egypt,2014.

[4] Xingjian Wang, Shaoping Wang, “ New approach of friction identification of electrohydraulic servo system based on evolutionary algorithm and statistical logics with experiments.”, J Mech Sci Technol. 2016:30(5):2311-2317.

[5] A. Kovari, “Effect of leakage in Electrohydraulic Servo Systems Based on Complex Nonlinear Mathematical Model and Experimental Results”, Acta Polytechnica Hungaria, vol. 12, no. 3, p. 129-146, 2015

[6] Wonohadidjojo, D.M Kothapalli, G. Hassan,M.Y, “Position control of Electro-hydraulic Actuator System Using Fuzzy Logic Controller Optimized by Particle Swarm optimization.”, International Journal of Automation and Computing, vol. 10, no. 3, p.181-193, 2013.

[7] Joshi A and Jayan P.G, Modelling and Simulation of Aircraft Hydraulic System, AIAA Paper No. AAIA-2002-4611, Proc. of Modelling and Smilutaion Technologies Conference, Monterey, CA, USA, 6-8 August,2002.

[8] Charles M. Close, Dean K. Frederick, Jonathan C. Newel – Modelling and analysis of dynamic systems. John Wiley & Sons, 2002.

[9] T. Knohl, H. Unbehauen, “Adaptive position control of electrohydraulic servo systems using ANN”, Mechatronics, vol. 10, no. 1-2, pp. 127-143, 2000.

[10] S. R. Lee and K. Srinivasan, “On-line identification of process models in closed-loop material testing”, ASME Journal of Dynamic Systems, Measurement and Control, vol. 111, no. 2, pp. 172-179, 1989.

[11] D. McCloy and H. R. Martin, The control of fluid power, Wiley. 1973.

[12] T. G. Ling, M. F. Rahmat and A. R. Husian, “System identification of electro-hydraulic actuator system using ANFIS approach”, Journal Teknologi, vol. 67, no. 5, pp. 41-47, 2014.

[13] Alleyne A, Nonlinear force control of an electro-hydraulic actuator, Proc Japan/USA Symp Flexible Automation, Boston, USA. 1996.

[14] Blackburn JF, Reethof G, Shearer JL, "Fluid Power Control", Technology Press of MIT and Wiley, 1960.

[15] Bona B, Giacomello L, Greco C, Malandra A, Position control of plastic injection moulding machine via feedback linearlisation, 31th Confer Decision Control, Tuscan, USA.1992.

**END**