Duality for Many-valued Modal Logic and a Basic Study of Coalgebraic Fuzzy Geometric Logic

Abstract

The thesis focuses on developing a duality for Fitting's many-valued modal logic in a bitopological framework and exploring modal fuzzy geometric logic using coalgebraic logic approaches. The current thesis explores and articulates the structure of duality for many-valued logic and many-valued modal logic by drawing on category theory and universal algebra. Since categorical relationships between systems and algebras, also referred to as frames, already exist in the literature, it is expected that these relationships can be extended to many-valued contexts. This is the goal that this thesis pursues in the first step. However, the investigation of duality for many-valued logic and many-valued modal logic using the methods of bitopological spaces, has drawn greater attention from researchers recently due to the fact that it can offer a more comprehensive viewpoint in this context. In this thesis, natural duality theory and modal natural duality theory are generalized in a bitopological framework by studying bitopological duality theory for Fitting's many-valued logic and many-valued modal logic. Thus, a coalgebraic duality theory is explored for multi-valued modal logics to shed light on more subtle aspects of bitopological duality.

Coalgebraic logic is a proven framework that facilitates the development of an extended version of modal logic. In light of this, the thesis investigates the connections between fuzzy geometric logic and coalgebraic logic.

The thesis is divided into seven main chapters, excluding the introduction and conclusion. Chapter 2 presents the idea of lattice-valued Boolean systems and examines the adjoint and co-adjoint properties of functors that are defined on them. Consequently, a duality for algebras of lattice-valued logic is obtained. Chapter 3 introduces the concept of lattice-valued relational systems, intending to demonstrate a duality between systems and algebras of Fitting's lattice-valued modal logic. Chapter 4 establishes a duality for algebras of Fitting's many-valued logic within the scope of bitopological techniques. In actuality, it extends the natural duality theory in a bi-topological context. Chapter 5 focuses on the extension of the natural duality theory for $\mathbb{ISP}_{\mathbb{I}}(\mathcal{L})$, the class of all isomorphic copies of sub-algebras of intuitionistic power of a finite algebra $\mathcal{L}$. Thus, an intuitionistic version of the natural duality theory is developed. Chapter 6 aims to develop a bitopological duality for algebras of Fitting's many-valued modal logic. This has led to an extension of the natural duality theory for modal algebras. Chapter 7 sheds light on a coalgebraic description of the bitopological duality for Fitting's many-valued modal logic. This yields a coalgebraic duality for Fitting's many-valued modal logic. Chapter 8 applies coalgebra theory to investigate modal fuzzy geometric logic. In other words, this chapter introduces modal operators to the language of fuzzy geometric logic using the methods of coalgebraic logic, to examine how these logics are interpreted in specific fuzzy topological coalgebras.

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