Effect of material and geometric stochasticity in attaining design objective functions for trapezoidal laminated composite plates

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CERTIFICATE OF RECOMMENDATION

This is to certify that the thesis entitled "**Effect of material and geometric stochasticity in attaining design objective functions for trapezoidal laminated composite plates**" is being submitted by **Mr. Rohan Das** in partial fulfillment of the requirements for the award of **'Master of Engineering in Civil Engineering'** from Jadavpur University and it is delightfully declared that it is a record of bonafide research work carried out by him under my supervision in the year of 2023-2024.

This ensures that the outcomes of the present research work have not been submitted to any other university or institution for the award of any degree or diploma.

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The foregoing thesis is hereby approved as a creditable study of an engineering subject carried out and presented in a manner satisfactory to warrant its acceptance as a pre-requisite to the Degree of Master of Engineering in Civil Engineering for which it has been submitted. It is understood that by this approval the undersigned do not necessarily endorse or approve any statement made, opinion expressed, or conclusion drawn therein, but approve the thesis only for the purpose for which it is submitted.

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Abstract

Laminated composite plate structures have numerous applications in construction, aerospace, military, and automotive industries. These materials with high specific strength are being used by the engineers on a very big scale and understanding the limitations of these classes of products is extremely important for confident use. In fact, the aspects of behavior study must include application specific requirements like introducing stiffeners, increasing thickness and cutouts. Hence, an accurate understanding of their structural behavior is required in terms of deflection, stresses, frequency, failure characteristics etc.

This paper highlights the stochastic effects in laminated composite trapezoidal plates to satisfy both dynamic and serviceability based objective function involving the free vibration behavior and maximum deflection under uniformly distributed load respectively. The study considers relevant plate properties as random without changing the plan area which is generally encountered in many industrial and engineering situations. Monte Carlo simulation is carried out in investigating the stochastic effects of material and geometric properties on the objective function. A positive linear trend is found to exist between the coefficients of variation of both the desired frequency and deflection with the input variables, whose slope increases as randomness of input variables increases. The probability of failure in reaching both the desired frequency and deflection is shown to be heavily dependent on the combined variation of material and geometry. The paper establishes that randomness in geometry has a more pronounced effect compared to that of material.

INTRODUCTION

1.1 Application of plates

Plates find widespread applications in various engineering disciplines due to their versatility and ability to withstand loads and distribute forces efficiently. Here are some key applications of plates in different engineering fields:

- Mechanical Engineering:
  - Machine components: Plates are used in the construction of machine components like baseplates, frames, and brackets to provide support and structural integrity.
  - Pressure vessels: Thick plates are used to fabricate pressure vessels for storing and transporting pressurized fluids and gases.
  - Heat exchangers: Plates are employed in heat exchangers to facilitate heat transfer between fluids in industrial processes.
  - Gears and cams: Plates are utilized as components in gear assemblies and cam mechanisms in various machinery.

- Aerospace Engineering:
  - Aircraft fuselage: In aircraft construction, plates are used to form the fuselage, providing the main body structure of the aircraft.
  - Wing and tail surfaces: Plates form the outer surfaces of wings and tail sections, contributing to the aerodynamic performance of the aircraft.

- Electrical Engineering:
  - Circuit boards: Printed circuit boards (PCBs) consist of thin plates with conductive tracks, used to connect electronic components in various devices.
  - Transformers: Laminated steel plates are used in transformer cores to reduce energy losses due to eddy currents.

- Structural Engineering:
  - Buildings: Plates are used as floor slabs, roof slabs, and walls in building construction to provide horizontal and vertical support.
  - Bridges: Plates are used to form the bridge deck, supporting the weight of vehicles and pedestrians.
  - Retaining walls: Plates are utilized in retaining wall structures to hold back soil or other materials.

- Civil Engineering:
  - Roadway surfaces: Steel plates are sometimes used as temporary roadway surfaces during construction or for creating temporary access paths.
  - Culverts and drainage systems: Plates are used in the construction of culverts and drainage channels to manage water flow.

- Automotive Engineering:
  - Vehicle body panels: Plates form the body panels of automobiles, providing the outer structure and shape.
  - Chassis components: Plates are used in the construction of vehicle chassis and frame components.

- Marine Engineering:
  - Shipbuilding: Plates form the hull and deck structures of ships and boats.

- Environmental Engineering:
  - Water treatment: Plates are used in filtration systems and sedimentation tanks for water treatment processes.

- Biomedical Engineering:
  - Prosthetics: Plates are used in the fabrication of certain types of prosthetics to support and replace damaged bones.
  - The selection of plate materials, size, and design depends on the specific requirements and loads that the structure or component will experience in its intended application.

1.2 Advantages of using composite materials

A composite is a material structure that consists of at least two macroscopically identifiable materials that work together to achieve a better result. One constituent is called the reinforcing phase and the other one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibers, particles, or flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibers, etc.

Composites have long been used in the construction industry. Applications range from nonstructural gratings and claddings to full structural systems for industrial supports, buildings, long span roof structures, tanks, bridge components and complete bridge systems. Laminated composite with its high corrosion resistance and low weight have proven attractive in many low stress applications. Construction holds priority for the adaptation of composites in place of conventional materials being used like doors and windows, paneling, furniture, non-structural gratings, long span roof structures, tanks, bridge components and complete bridge systems and other interiors.

Other applications of composites in the civil engineering area are:
- Tunnel supports
- Supports for storage containers
- Airport facilities such as runways and aprons
- Roads and bridge structures
- Marine and offshore structures
- Concrete slabs
- Power plant facilities
- Architectural features and structures such as exterior walls, handrails, etc.

Fiber reinforced polymer (FRP) has been found quite suitable for repair, seismic retrofitting and upgrading of concrete bridges to extend the service life of existing structures. Fiber reinforced composite is also being considered as an economic solution for new bridge structures. The lightweight of composites is especially valuable for the construction of waterway bridges incorporating a lift-up section to permit the passage of boats, and for ease of transportation. Since composite will not spall like concrete during freeze-thaw cycles and will not rust like metal in the moist, corrosive sea environment. The composite bridge decks are quite suitable for replacing conventional/old bridge decks having super structure intact. The replacement can be carried out in a short time with minimal disturbance to the traffic.

Most composite materials are neither isotropic nor homogeneous. Also, the stiffness in the direction parallel to the fibers is higher than in the direction perpendicular to the fibers and thus the properties are not independent of the direction. This makes the composite material anisotropic.

Graphite/epoxy composites are approximately five times stronger than steel on a weight for-weight basis. The most common advanced composites are polymer matrix composites (PMCs) consisting of a polymer (e.g., epoxy, polyester, urethane) reinforced by thin diameter fibers (e.g., graphite, aramids, boron).

The main drawbacks of PMCs (polymer matrix composites) include low operating temperatures, high coefficients of thermal and moisture of expansion, and low elastic properties in certain directions. Specific strength of composites is higher than that of individual material due to which mass of composites reduces and cost also reduces.

A lamina is a thin layer of a composite material that is generally of a thickness on the order of 0.005 in. (0.125 m). A laminate is constructed by stacking several such lamina in the direction of the lamina thickness. Each lamina is represented by the angle of ply and separated from other plies by a slash sign. A schematic diagram of lamina is shown in Figure 1.1, in which the fiber direction is mentioned, represents orientation of fibers with certain direction i.e., the angle made by the fibers with respect to coordinate axes.

Special notations are used for symmetric laminates, laminates with adjacent lamina of the same orientation or of opposite angles, and hybrid laminates. [0°/-45°/90°/60°/0°] denotes the code of the laminate as shown in Figure 1.2. It consists of six plies, each of which has a different angle to the reference *x*-axis. A slash separates each lamina. [0°/–45°/60°]s denotes the symmetric laminate consisting of six plies.

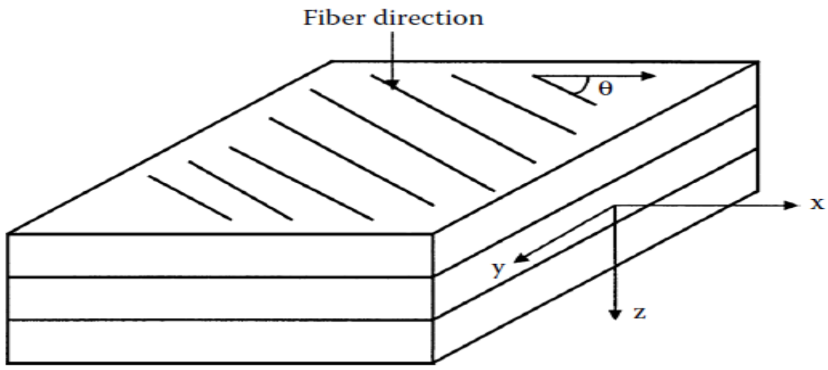


Fig.1: Schematic of laminae

0°
–45°
90°
90°
60°
0°

Fig. 1: Typical laminate

1.3 Design of laminated composite

Design of laminated composites elements in civil engineering includes constraints on optimizing and constraining factors such as:

- Cost.
- Mass as related to the seismic force magnitude.
- Stiffness (to limit deformations) in the structure.
- Fabrication and connection design.

These factors are similar to those used with designing with monolithic materials; thus, the main issue with designing with composites as opposed to monolithic materials involves understanding the orthotropic nature of composite plies. The possibility of different fiber-matrix systems combined with the variables such as fiber volume fraction first dictate the properties of a lamina. Then, laminae can be placed at angles and at distances from the mid plane in the laminate. Then the stresses and strains in the laminate are determined for the stacking sequence. The failure of the composite may be based on the first ply failure (FPF) or the last ply failure (LPF).

Chapter 2

REVIEW OF LITERATURE

The present chapter summarizes some prominent research works that had been conducted on various analysis of laminated composite plates.

2.1 Static bending analysis of laminated composite plates

2.1.1 Rectangular plates

Li et al. [1] used isogeometric analysis (IGA) based on nonuniform rational B-splines (NURBS) and applied it for static and free vibration analysis of laminated composite plates by using the third order shear deformation theory (TSDT). Sahoo et al. [2] presented an investigative study on the deflections and the natural frequencies of the laminated composite flat panels that have been computed numerically using two higher-order mid-plane kinematics and compared with the available published numerical/analytical results. Kumar and Chakrabarti [3] did a failure analysis study on laminated composite plates subjected to transverse loading by a finite element model based on higher order shear deformation theory. Kant and Swaminathan [4] presented analytical formulations and solutions to the static analysis of simply supported composite and sandwich plates which were not previously reported in the literature and the study was based on a higher order refined theory developed by the first author. The displacement field of this theory considered both the transverse shear and normal deformations thus making it more accurate than the first order and other higher order theories considered. Yuksel et al. [5] investigated static analysis of composite laminated plated under uniform loading. The author used the virtual work principle and Kirchhoff-Love plate theory in obtaining the governing equations. In solution of the problem, the Navier method was implemented for a simple supported plate. In obtaining the numerical results and graphs, MATLAB program was used. The displacements of the plate were obtained for different orientation angle of layers. The authors also studied variation of static load on the bending deflections of the composite laminated beam.

2.1.2 Non-rectangular plates

Ghosh and Chakravorty [6] aimed to accurately predict the first ply failure loads of laminated composite hypar shell roofs with different boundary conditions. The first ply failure loads were obtained through different well-established failure criteria including Puck's criterion along with the serviceability criterion of deflection. The close agreement of the published and the results the authors got for different validation problems proved the correctness of the finite element model used in the study. Taqi and Gdade [7] presented an accurate numerical method for computing the time invariant deflection of geometrically thin laminated composite shells subjected to out of plane loading. The transverse and in-plane shear effects were taken into consideration by the higher-order shell kinematics while neglecting the direct transverse stresses. The authors presented results on different geometries including spherical, conical and cylindrical. Moremo et al. [8] demonstrated that a significant characteristic of composite materials was the difference in behaviour under tensile and compressive loads in their principal directions. Upadhyay and Shukla. [9] presented a study on large deformation flexural response of composite laminated skew plates subjected to uniform transverse pressure. The authors used third order shear deformation theory (TSDT) and von-Karman's nonlinearity for the analysis. The effects of geometric nonlinearity, transverse shear, boundary conditions, aspect ratio and modular ratio on the behavior of laminated composite skew plates were discussed in detail. Das and Chakraborty [10] applied a combination of an eight-noded shell element with a three-noded beam element to solve a bending problem of a composite stiffened conoidal shell subjected to a concentrated load at the center. One of the many important conclusion was that the static stiffness of a shell surface is greatly influenced by both its stacking sequence and edge conditions. Chatterjee et al [11] investigated first ply failure behaviour of laminated composite plates under different edge conditions by using some well-established and recent failure criteria.

2.2 Free vibration study of laminated composite plates

2.2.1 Rectangular plates

Belarbi et al. [12] developed a new higher-order layer wise finite element model, to study the free vibration behavior of multilayer sandwich plates. The developed model was based on a proper combination of higher-order and first-order, shear deformation theories and the results obtained by the model were compared with those obtained by the analytical results and other finite element models found in literature. The comparison showed that the element had excellent accuracy and a broad range of applicability. Mehar et al. [13] investigated the nonlinear free vibration behavior of functionally graded carbon nanotube reinforced composite flat panel. The authors checked the validity and the convergence behavior of the nonlinear model. All the nonlinear higher-order terms were included in the mathematical model to achieve the exact flexure of the structure. Most importantly it was observed that the convergence and the comparison studies indicated the accuracy of the present nonlinear HSDT model with and without thermal load. Sinha et al. [14] in their investigation dealt with the numerical and experimental study on the free vibration of woven glass fibre laminated rectangular composite stiffened plates. Thirty-four stiffened plates fabricated from woven glass fibre and binder (epoxy and hardener) were tested in FET analyzer by varying the parameters, such as numbers, types, and orientation of stiffeners, depth of stiffener to thickness of plate ratio, aspect ratio and boundary conditions of plates, to obtain their natural frequencies. A finite element model was used for vibration of laminated composite stiffened plates for validation of the experimental results. The experimental and numerical results were compared, which showed a very good agreement between them. From this study, it was observed that the above-mentioned parameters significantly influence the fundamental frequency. Rajawat et al. [15] presented the free vibration analysis of stiffened laminated plate using Finite element method. Four stiffened laminated plates of different orientation were considered, and their natural frequencies were determined along with the effects of boundary condition on natural frequencies of the plate. The convergence tests and comparison studies were carried out with ANSYS. The obtained results illustrated a good agreement with those available in the literature for different eccentric ratios and different support conditions. Joshi and Duggal [16] investigated natural frequencies of laminates during its progressive failure using finite element method. The finite element formulation was based on seven degrees of freedom displacement field, deduced for an orthotropic material. Free vibration analysis was carried out during ply-by-ply failure for a laminate subjected to uniformly distributed load. The variation of fundamental frequency with material and geometric properties were also presented. The effect of fibre orientation, failure mode and failed ply on natural frequencies was highlighted in this paper. Jacobi iterative method was used to evaluate the natural frequencies of laminate. For the analysis, a computer code was written using FORTRAN based on the finite element formulation. Sharma and Mittal [17] in their study applied FEM for free vibration analysis of moderately thick rectangular laminated composite plates with edges elastically restrained against translation and rotation. The first order shear deformation theory was used to incorporate the effects of transverse shear deformation and rotary inertia. In comparison to the vibration analysis of moderately thick symmetrically laminated plates described by three field variables, five field variables were used. This work, thus, aimed to study the free vibration problem of laminated plates with elastically restrained edges which appeared to have not been studied yet.

2.2.2 Non-rectangular plates

Xie et al. [18] studied free vibration characteristics of parallelogram laminated thin plates under multi-points supported elastic boundary conditions by improved Fourier series method (IFSM). All energy equations of thin plates were established by the classical thin plate elasticity theory after which Rayleigh-Ritz method was used to solve the problem. From the numerical results it was observed that parameters and properties of the plate structure like the stiffness of the supporting spring, the number and position of supporting points all had great influences on free vibration of parallelogram laminated thin plate. Mandal et al. [19] dealt with free vibration analysis of laminated composite skew plates with and without cut-outs. The numerical analysis was carried out by developing a computer code in MATLAB. It was observed that the variation of experimental results with those obtained from present finite element analysis remained within 6%. Chaudhuri et al. [20] in their study dealt with vibration analysis of a hypar shell stiffened along the margin of cut outs with different boundary conditions and antisymmetric angle ply lamination. The formulation was based on first order shear deformation theory. The reduced method of eigen value solution was chosen for the undamped free vibration analysis. Numerical studies are conducted to determine the effects of degree of orthotropy, fibre orientation and width to thickness ratio on the non-dimensional fundamental frequency. The non-dimensional fundamental frequency was found to increase with increase in material anisotropy. Barik and Mukhopadhyay [21] found out that in the analysis of plates with arbitrary geometry, the isoparametric element was widely used because of its capability to model an arbitrary geometry successfully. The authors presented a new approach for the analysis of plates of arbitrary shape. It was concluded from the results that for the analysis of plates with arbitrary shapes, isoparametric element could be considered most suitable because of its capability to model the arbitrary geometry successfully. But some discrepancies in the results were observed when the geometry of the plate became more complex.

2.3 Critical discussions

Aysun Baltaci et al. [22] did buckling analysis of laminated composite plates having circular holes by FEM. Eight node isoparametric shell elements with 24 degrees of freedom are used during the investigation. Variation in perforations and stiffness were not covered. M. Aydin Komura et al. [23] in their study, carried out a buckling analysis out of a woven–glass–polyester laminated composite plate with an circular/elliptical hole, numerically. In the analysis, finite element method (FEM) was applied to perform parametric studies on various plates based on the shape and position of the elliptical hole, however the effect of variation in stiffness was not considered. Sh. Hosseini-Hashemi et al. [24] dealt with the free vibration behavior of laminated transversely isotropic circular plates with axisymmetric grid core attached at the cente, based upon a purely deterministic model. The governing equations of motion are obtained based on Mindlin's first-order shear deformation plate theory. Ajay Kumar et al. [25] calculated failure load using the higher order zig zag theory, with 2D C_0 model, which did not incorporate the variation in shape, geometry and stiffness. Ewa Magnucka-Blandzia [26] performed numerical analysis of the plate with the use of the finite element method (FEM) in the ABAQUS and ANSYS system. The main goal of the numerical investigation was to verify the analytical approach presented in previous sections. The paper was devoted to mathematical modelling of multi-layered structures. The case of a metal seven-layer rectangular plate was considered. The plate was composed of a trapezoidal corrugated main core, two inner flat sheets, two trapezoidal corrugated cores of the faces and two outer flat sheets. However, the uncertainty in response introduced by incorporating so many variations on the base model was not considered. Hong Zhanga et al. [27] presented a unified analysis model for vibration characteristics of rotary composite laminated plate with various elastic boundary conditions by combining SPT with 2D improved Fourier-Ritz method. However, the variation of stiffness was not taken into account. Guojun Nie et al. [28], based on classical laminated plate theory, studied the free vibration of arbitrary straight-sided quadrilateral laminates with variable angle tows (VAT). It was assumed that the fiber orientation angle in each ply of VAT laminate varies linearly along x direction. The influence fiber orientation angles, boundary stiffness and boundary conditions on the natural frequencies and mode shapes is investigated in detail. However, the results were still deterministic. Lekou and Philippidis [29] introduced the effect of variation in material and geometric properties in ply failure theories, thus shifting the mode of study from parametric to stochastic.

SCOPE OF PRESENT STUDY

From the survey of literature, it is found that static bending and free vibration analysis of stiffened and unstiffened non rectangular plates have received very limited attention for few loading cases and boundary conditions. Moreover, due to the uncertainty introduced in determining the strength and stiffness properties of laminates during the manufacturing of the individual composite layers and the laminated structure as a whole, a purely deterministic study can potentially be non-conservative and insufficient. Hence all such material and geometric properties shall be treated as random variables and their uncertainty should be quantified either experimentally or computationally The present research thus intends to perform similar study with more practical parameters using ABAQUS software as the tool.

Experimental data on the mechanical properties of unidirectional glass/polyester showed a coefficient of variation (CV) ranging between 10% to 20% for elastic and shear moduli as well as the material strengths, with variation as high as 24.90% [29]. In unidirectional carbon fibre-reinforced polymers (CFRP), experimental data showed variation as high as 13.1% for tensile strengths [30], with less variation in other material properties. Recent research [31] has shown that, while the elastic properties of carbon fibre/epoxy composites possess a CV of around 5%, the CV of the mechanical strength still ranges from 10% to 20% [32],[33]. Uncertainties in material properties lead to uncertainties in dynamic behavior [34].

Natural frequency enhancement to avoid resonance due to machine installation is often required within a restricted plan area. Again, reduction of maximum central deflection is needed to be executed post installation due to serviceability criteria requirement. The frequency and the deflection are thus two material properties that takes into account the gross response of a structure, which is particularly important in making any decision arising due to practical problems. The geometric and material properties always have elements of uncertainty as a natural occurrence due to the inevitable fabrication inaccuracies in layup and curing. In such cases, even a slight shift in the characteristics of any of the plate properties can have a pronounced effect on the response of the structure.

This study aims to predict the stochastic effects that one faces while dealing with such problems arising due to randomness in material and geometric properties as well as to identify the comparative effect of variation of different combinations at various levels.

TRAPEZOIDAL PLATE
AND
STRUCTURAL MODELLING

The present work mainly aims to perform finite element bending and free vibration analysis of laminated composite trapezoidal plate using ABAQUS software. The determination of natural frequency and central deflection is done with the help using Mindlin-Reissner Plate theory. The type of element chosen in the finite element simulation and analysis and the related reason for choosing such element is discussed.

4.1 Elements used in the software

The commonly used element family available in the ABAQUS library is depicted in the Fig. 1.

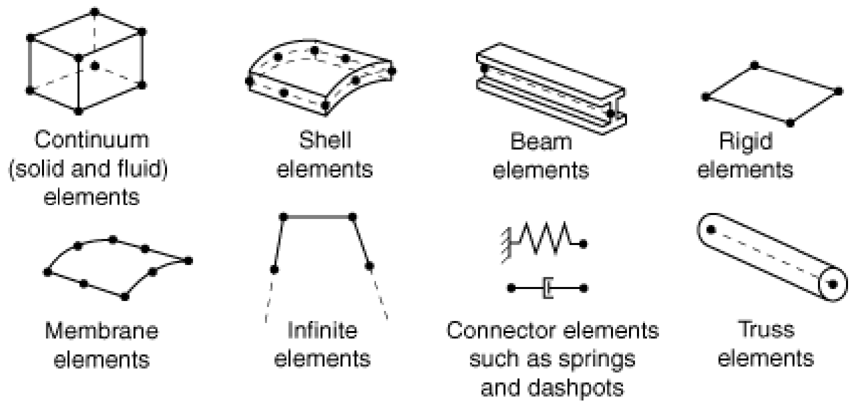


Fig.3: Element family

The ABAQUS shell element library provides elements that allow the modeling of curved, intersecting shells that can exhibit nonlinear material response and undergo large overall motions (translations and rotations). ABAQUS shell elements can also model the bending behavior of composites.

The general-purpose shell elements are axisymmetric elements SAX1, SAX2, and SAX2T and three-dimensional elements S3, S4, S3R, S4R, S4RS, S3RS, and S4RSW, where S4RS, S3RS, and S4RSW are small-strain elements that are available only in ABAQUS/Explicit. The general-purpose elements provide robust and accurate solutions in all loading conditions for thin and thick shell problems. Thus, even in thick shell considerations, solid elements use excessive nodes, which increases the computational time making shell elements more preferable.

The following are the stress/displacement elements which are used by the software (refer Table 1).

- STRI3(S): 3-node triangular facet thin shell, with 1 degree of freedom.
- S3R: 3-node triangular general-purpose shell, finite membrane strains, with 2 degrees of freedom per node
- STRI65(S): 6-node triangular thin shell, using 5 degrees of freedom per node.
- S4: 4-node general-purpose shell, finite membrane strains, with 4/6 degrees of freedom per node

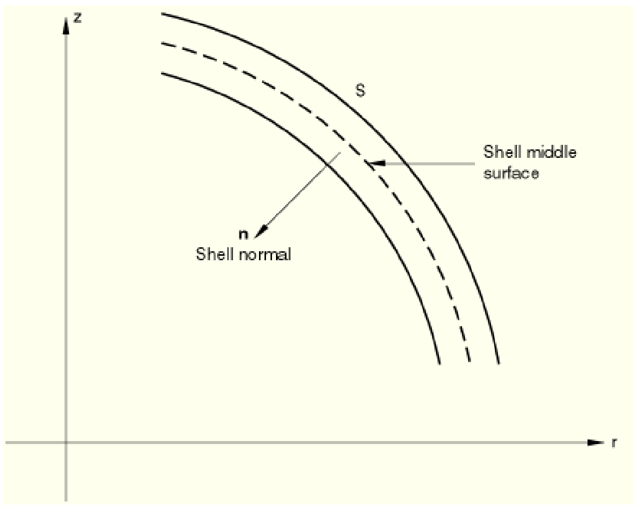


Fig. 4: ABAQUS shell element with middle layer.

Table 1: The list of stress/displacement elements which are used by the software presented

Name of shell element	Type of element	Number of nodes	Degrees of freedom per node
STRI3(S)	Triangular	3	1
S3R	Triangular	3	2
STRI65(S)	Triangular	6	5
S4	Arbitrary	4	4/6

For the element meshing, S4 is being used as a quadrilateral element with 5 degrees of freedom, three in the in-plane direction and two in the out-plane direction.

MODEL VALIDATION
AND
MESH CONVERGANCE

5.1 Model validation

The element and the mesh that is used in the study is first benchmarked with the non dimensional frequency (ω_{nd}) of a clamped graphite-epoxy composite [0º/90º]4 square plate model (235 mm x 235 mm and 2.8 mm thick) presented by Sinha et al. [37]. With a plan dimension of 'b', the fundamental frequency (ω) is made non-dimensional as:

$$\omega_{nd} = \omega b^2 (\rho / E_{22} h^2)^{1/2} \tag{1}$$

ω_{nd} values of two different problems with two and three unidirectional stiffeners (each stiffener 11.2 mm wide and 16.8 mm deep) respectively are calculated by the present approach and furnished in Table 5. The results show a very close agreement for a mesh of 50x50. With this order of finite element mesh, the simulations in this study are performed.

Table 2: Non-dimensional fundamental frequencies (ω_{nd}) of square plate used for benchmarking

Model	Sinha et al. (ref. [37])	Present study	Percent deviation
Two stiffeners along x direction	35	35.019	0.054
Three stiffeners along x direction	37	37.069	0.186

Another analytical validation is performed for the deflection study. Three square plates

(1000mm x 1000mm x 10mm) made of three different materials namely steel (A), composite type-I (B) and composite type-II (C) are analysed under different loadings with different boundary conditions. The determined values are benchmarked against formula derived analytical results showing very low deviation. The material properties and the results are given subsequently in Tables 3 and 4.

Table 3: Mean material properties and parameters

Material	E_{11}(GPa)	E_{22}(GPa)	Poisson's ratio (ν)	G_{12}(GPa)	G_{13}(GPa)	G_{23}(GPa)
Steel	200	-	0.30	76.923	-	-
Composite Type-I	142.5	9.79	0.27	4.72	4.27	1.192
Composite Type-II	175	7	0.25	3.5	3.5	1.4

Table 4: Deflection benchmarking

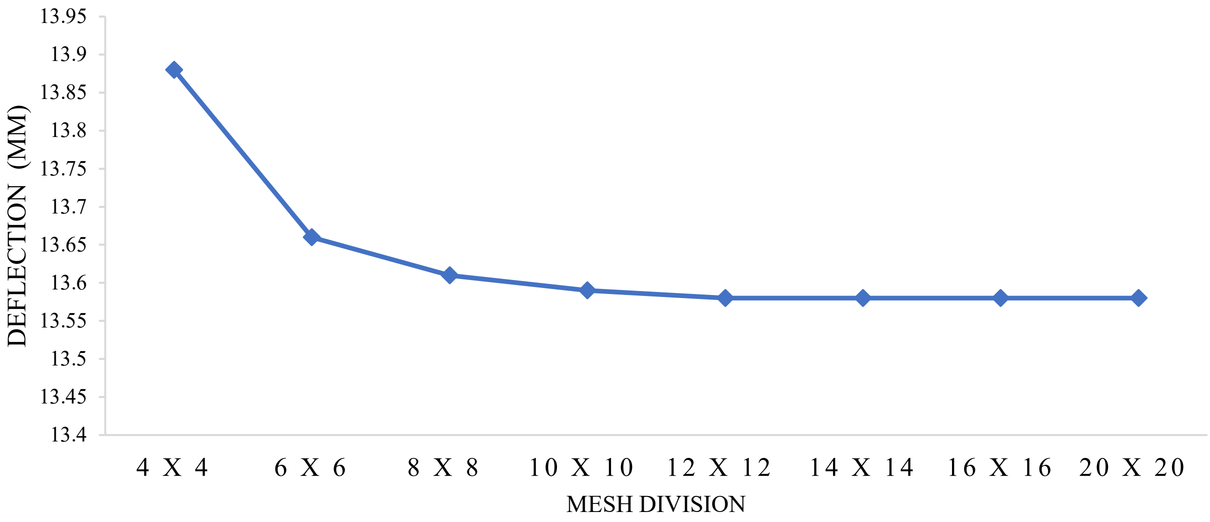
Plate op-tion	Boundary condition	Loading	Deflection (mm)		
			Analytical (Navier)	Present	Percent Devia-tion
A	Clamped (all sides)	Uniformly distributed load (0.2 MPa)	13.81	13.58	1.66
B	Clamped (all sides)	Uniformly distributed load (0.2 MPa)	12.70	12.17	4.17
C	Simply supported (all sides)	Sinusoidal loading	124.06	123.3	0.612

5.2 Study of mesh convergence

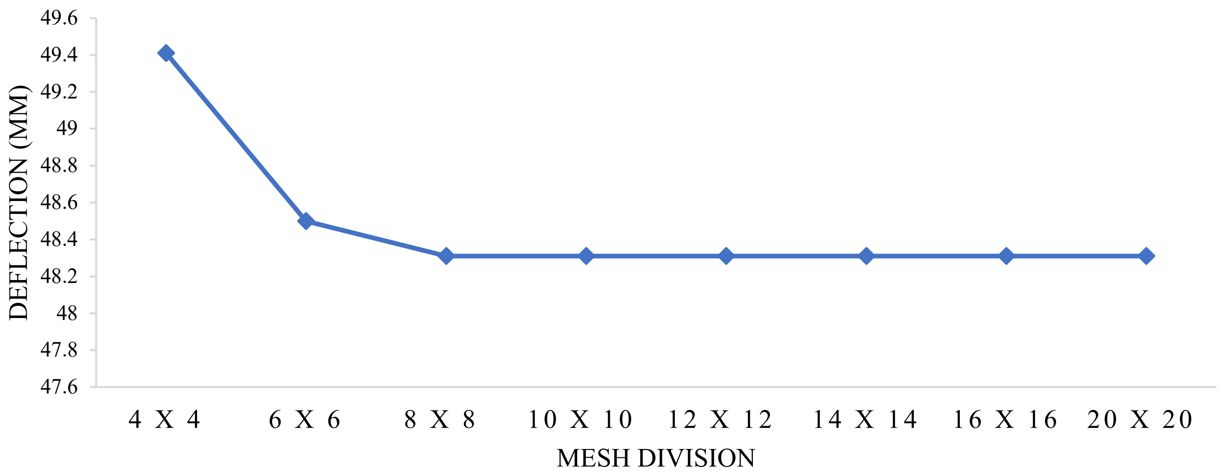
In the present work a mesh convergence study is performed with user defined meshes. Mesh convergence means a particular mesh when the results do not change even with the further increment of meshes. The convergence study is done based on the parameter of deflection check and is presented in Table 5 and Fig. 5.

Table 5: Study for mesh convergence

Plate option	Boundary condition	Deflection (mm) of the plate subjected to 0.2 MPa loading for various mesh size							
		4×4	6×6	8×8	10×10	12×12	14×14	16×16	20×20
A	Clamped (all sides)	13.88	13.66	13.61	13.59	13.58	13.58	13.58	13.58
B	Clamped (all sides)	49.41	48.50	48.31	48.31	48.31	48.31	48.31	48.31



(a) Mesh convergence of plate option A



(b) Mesh convergence of plate option B

Fig. 5: Mesh convergence study (a) for plate option A (b) for Plate option B

The deflection of Plate options A and C are presented below. The meshing follows the above-mentioned mesh convergence study. With increment of meshes, even if the results will remain the same, the visualization of the deflection becomes more pronounced with more contours to add in the figure.

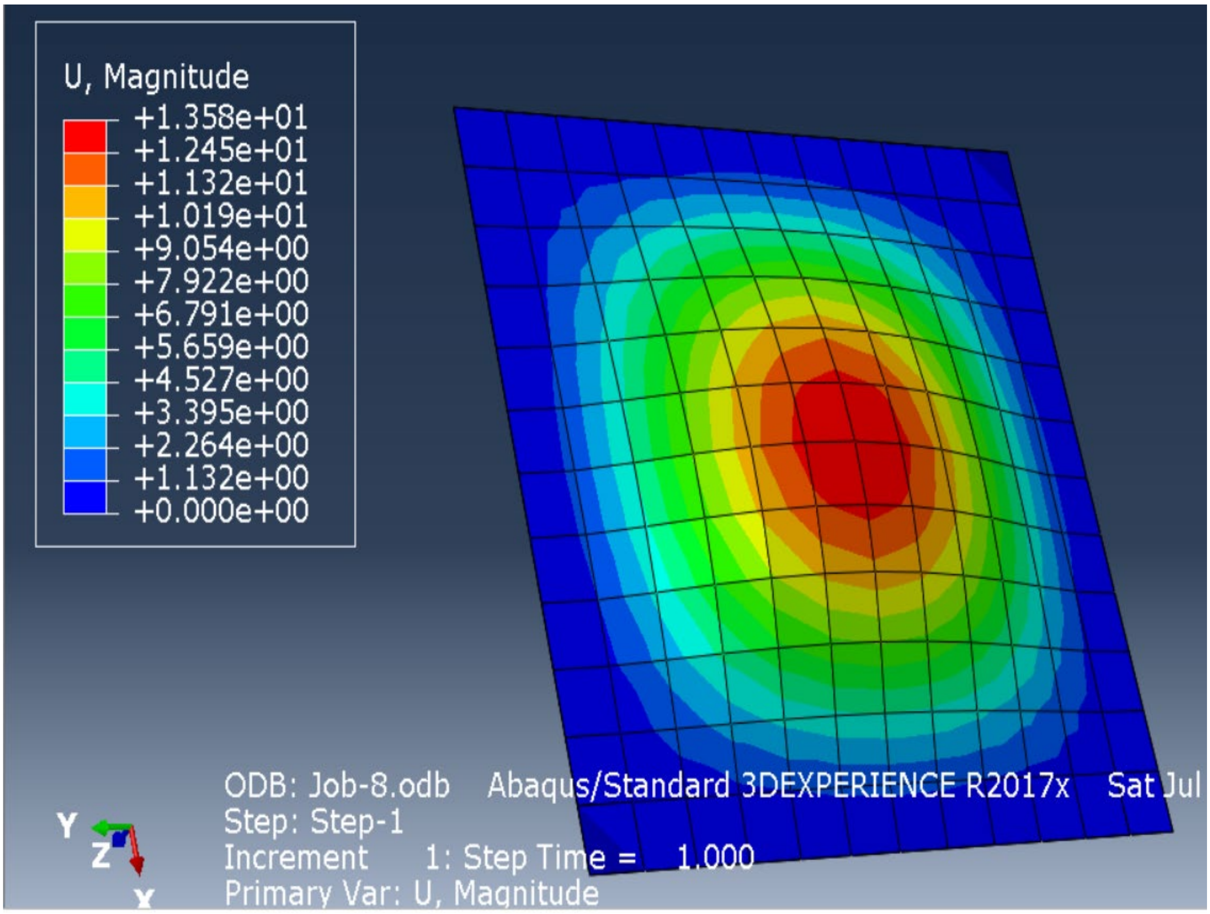


Fig. 6: Deflection results of Plate option A

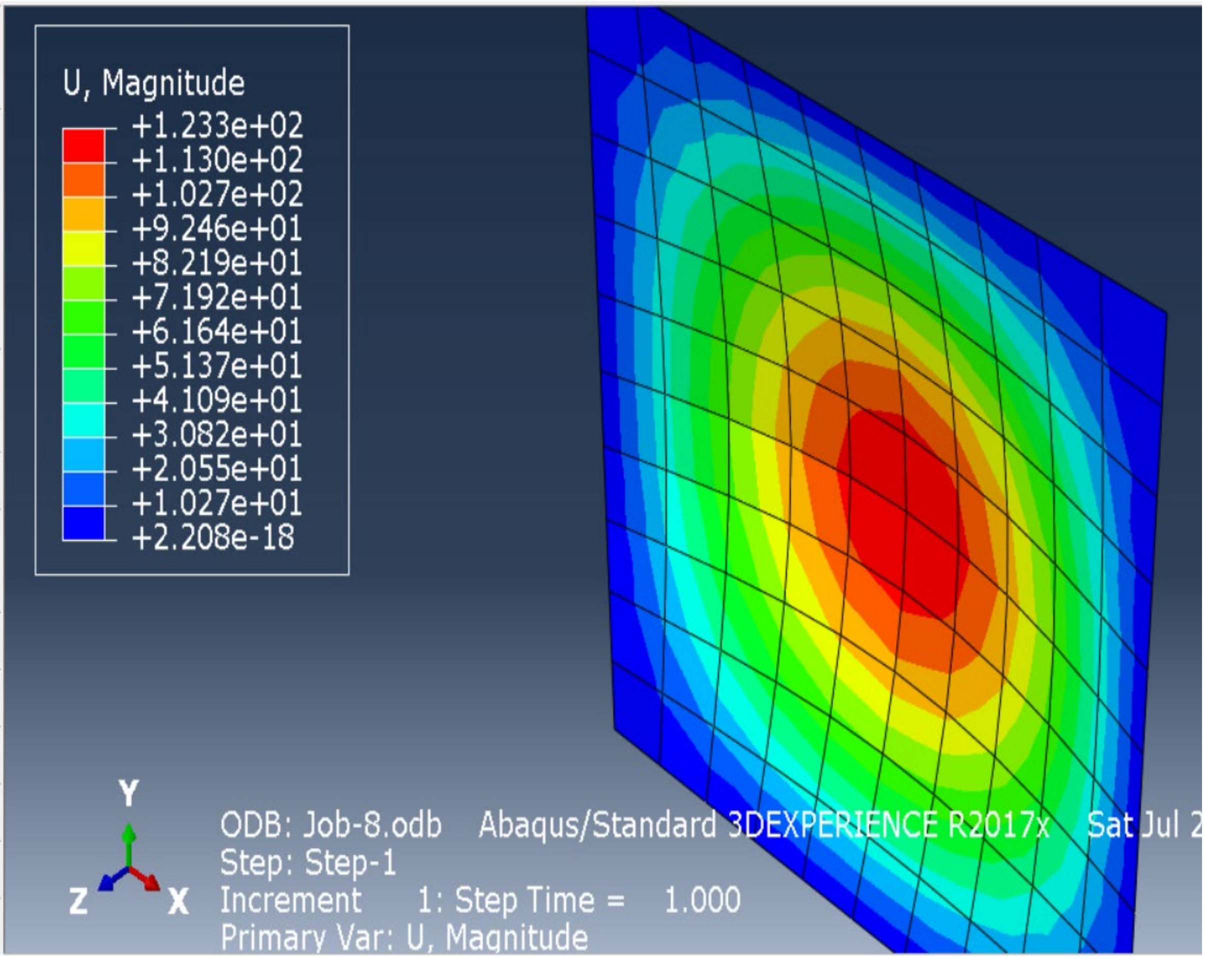


Fig. 7: Deflection results of Plate option D

5.3 Results of free vibration analysis

The following table presents the types of plates taken for free vibration analysis.

Table 6. List of plate options and geometrical properties

Plate type	Material	Stacking order	Nomen-clature	Plate dimension (mm)		
				length	width	thickness
Square	Steel	-	A	1000	1000	10
Skew plate (skew angle 5°)	Steel	-	D	1000	1000	10
Skew plate (skew angle 10°)	Steel	-	E	1000	1000	10

Square	Composite type-1	$(+45°/-45°)$	F	1000	1000	10
Skew plate (skew angle 5°)	Composite type-1	$(+45°/-45°)$	G	1000	1000	10
Square	Composite type-1	$(0°/90°)$	H	1000	1000	10
Skew plate (skew angle 5°)	Composite type-1	$(0°/90°)$	I	1000	1000	10
Skew plate (skew angle 10°)	Composite type-1	$(0°/90°)$	J	1000	1000	10
Square	Composite type-1	$(+45°/-45°/+45°)$	K	1000	1000	10
Skew plate (skew angle 5°)	Composite type-1	$(+45°/-45°)$	L	1000	1000	10
Skew plate (skew angle 10°)	Composite type-1	$(+45°/-45°/+45°)$	M	1000	1000	10
Square	Composite type-1	$(0°/90°/0°)$	N	1000	1000	10
Skew plate (skew angle 5°)	Composite type-1	$(0°/90°/0°)$	O	1000	1000	10
Skew plate (skew angle 15°)	Composite type-1	$(0°/90°/0°)$	P	1000	1000	10

The following table presents the results of the frequencies.

Table 7. Results of natural frequencies of different plate options

Plate Option	Natural frequencies (in rad/sec)		
	1st mode	2nd mode	3rd mode
A	0.005020717	0.010670256	0.010670256
D	0.005054068	0.01052681	0.010954257
E	0.005156082	0.010519333	0.011391756
F	0.001669949	0.003425035	0.003425035
G	0.001682025	0.003375699	0.003523147
H	0.001771906	0.00380853	0.00380853
I	0.001782525	0.003763976	0.003896005
J	0.001815147	0.003759572	0.004032288
K	0.002241532	0.004019665	0.005244889
L	0.002175589	0.003972641	0.005025907
M	0.002145549	0.003949494	0.004929032
N	0.002604116	0.003502978	0.005704341
O	0.002609118	0.003516506	0.005736763
P	0.002624606	0.003565685	0.005835132

With the results obtained from the natural frequency analysis some preliminary graphs are plotted to ascertain the behavior of change of natural frequencies with skewness and with stacking. It is observed that only in the case of cross-ply laminates, the curvature of change in natural frequencies with increasing skewness is convex in nature. This preliminary observation indicates to the future scope of this study where the uncertainty in material and geometric properties can be further extended for frequencies larger than the fundamental frequency.

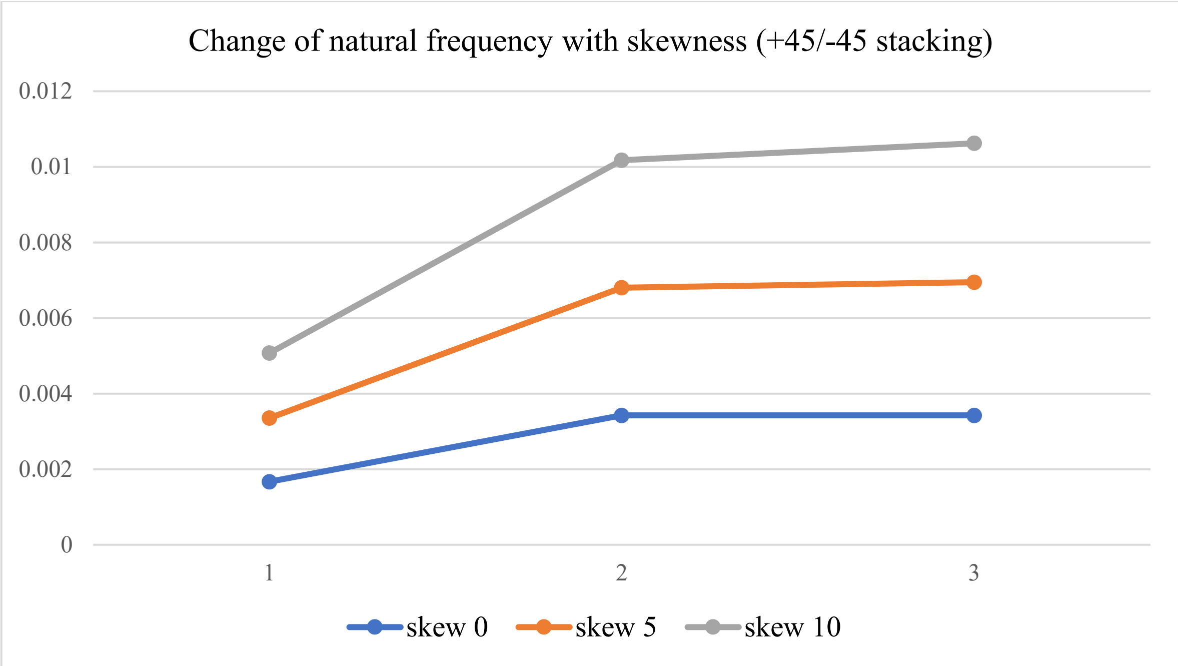


Fig. 8: Change of natural frequency with skewness for composite type 1 of stacking order $(+45° / -45°)$

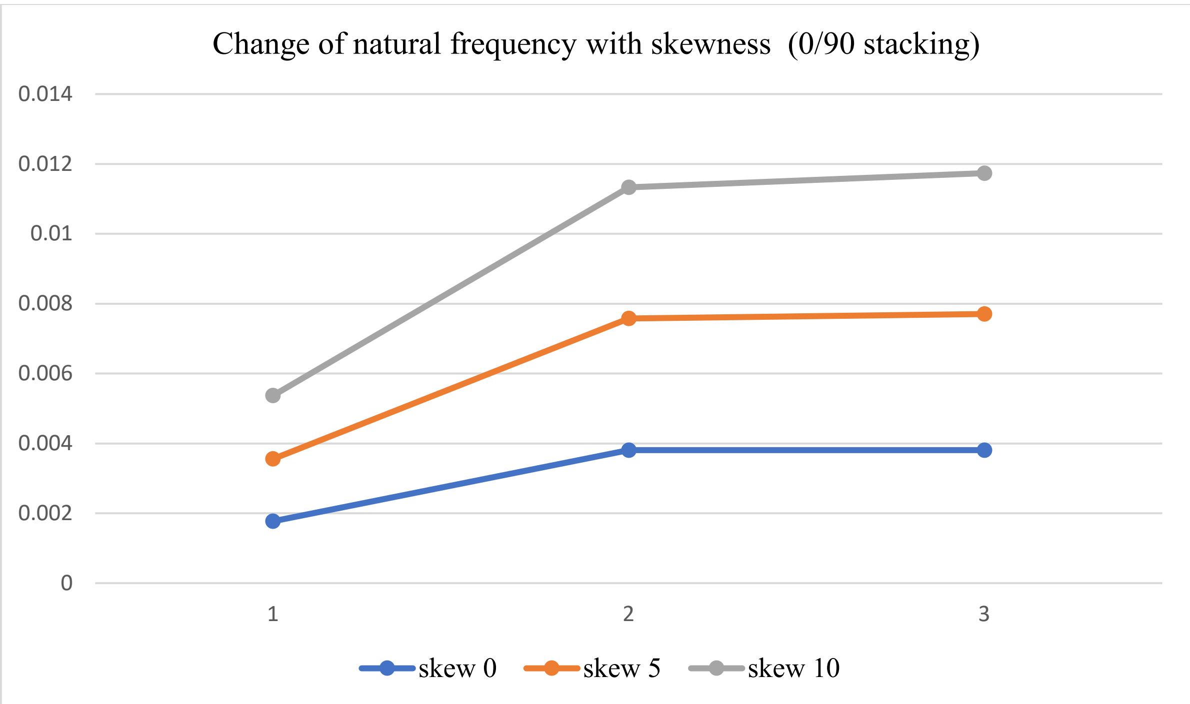


Fig. 9: Change of natural frequency with skewness for composite type 1 stacking order $(0° / 90°)$

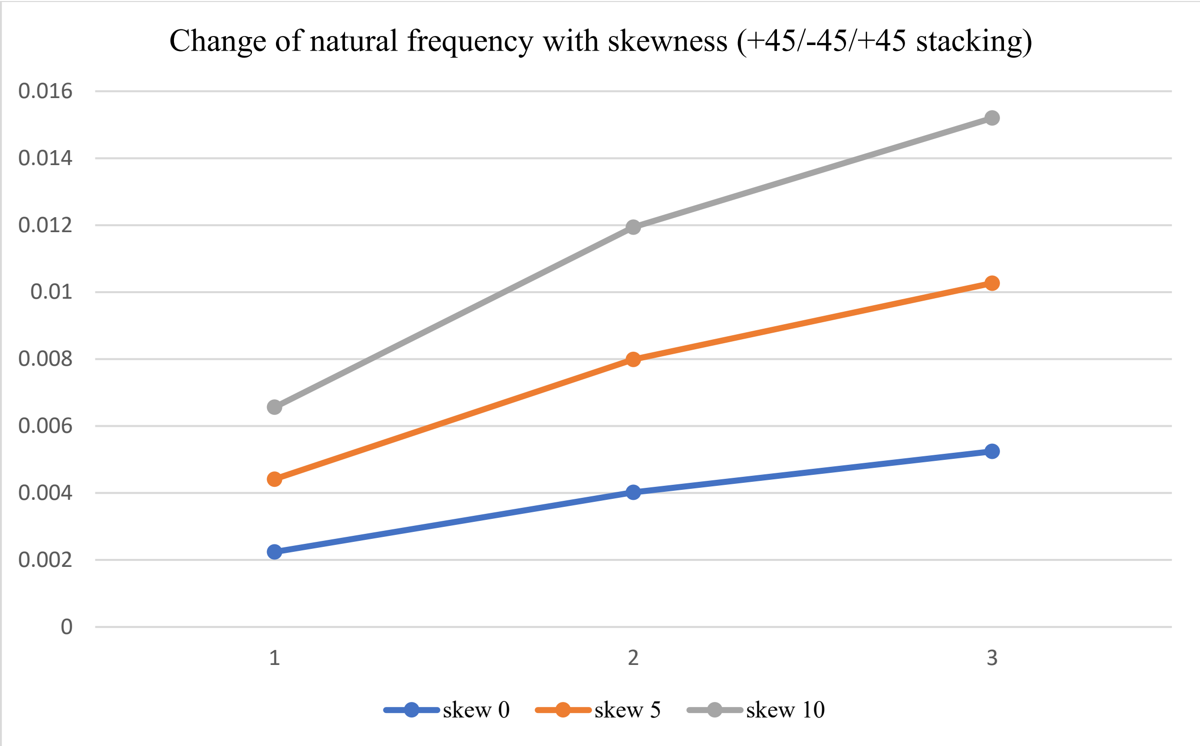


Fig. 10: Change of natural frequency with skewness for composite type 1 of stacking order $(+45°/-45°/+45°)$

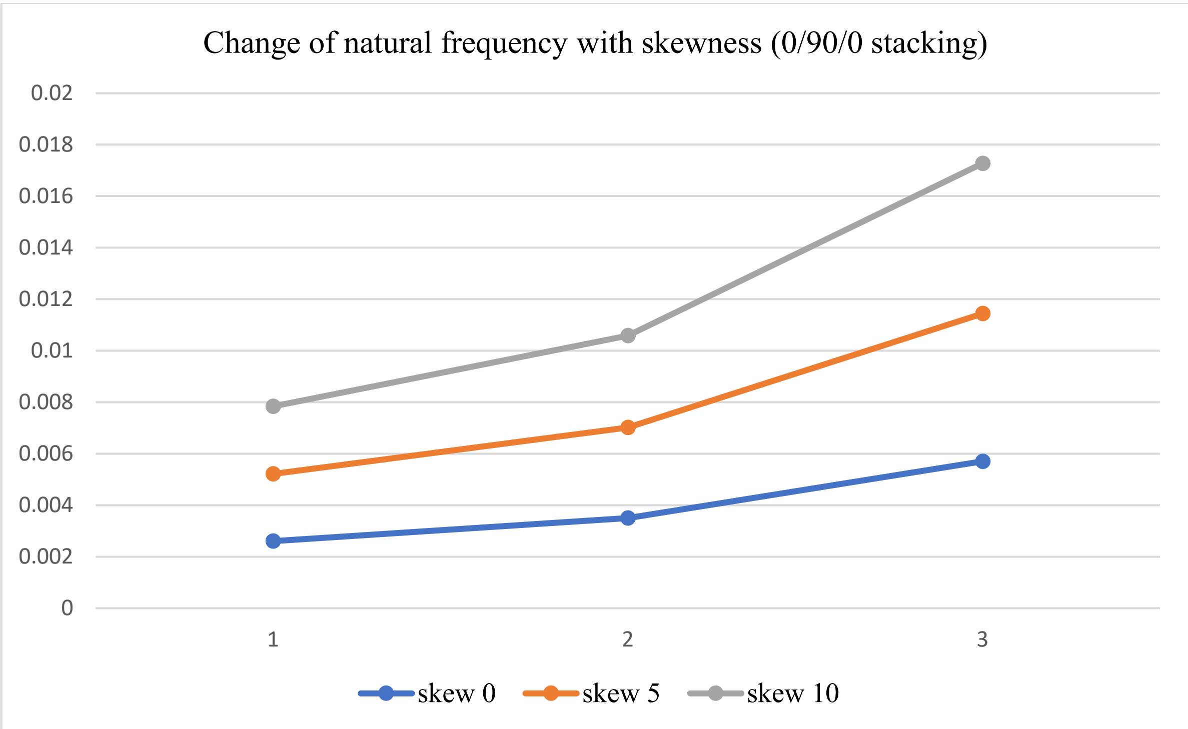


Fig. 11: Change of natural frequency with skewness for composite type 1 of stacking order $(0°/90°/0°)$

PROCEDURE AND SIMULATION SETUP

6.1 Objective function

Two trapezoidal clamped [0º/90º/0º (cross) and 45º/-45º/45º (angle)] plates of the following properties and dimensions as presented in Table 2 and Table 3, is modelled in Abaqus CAE. Fig. 12 demonstrates the dimensions as well as the laminates.

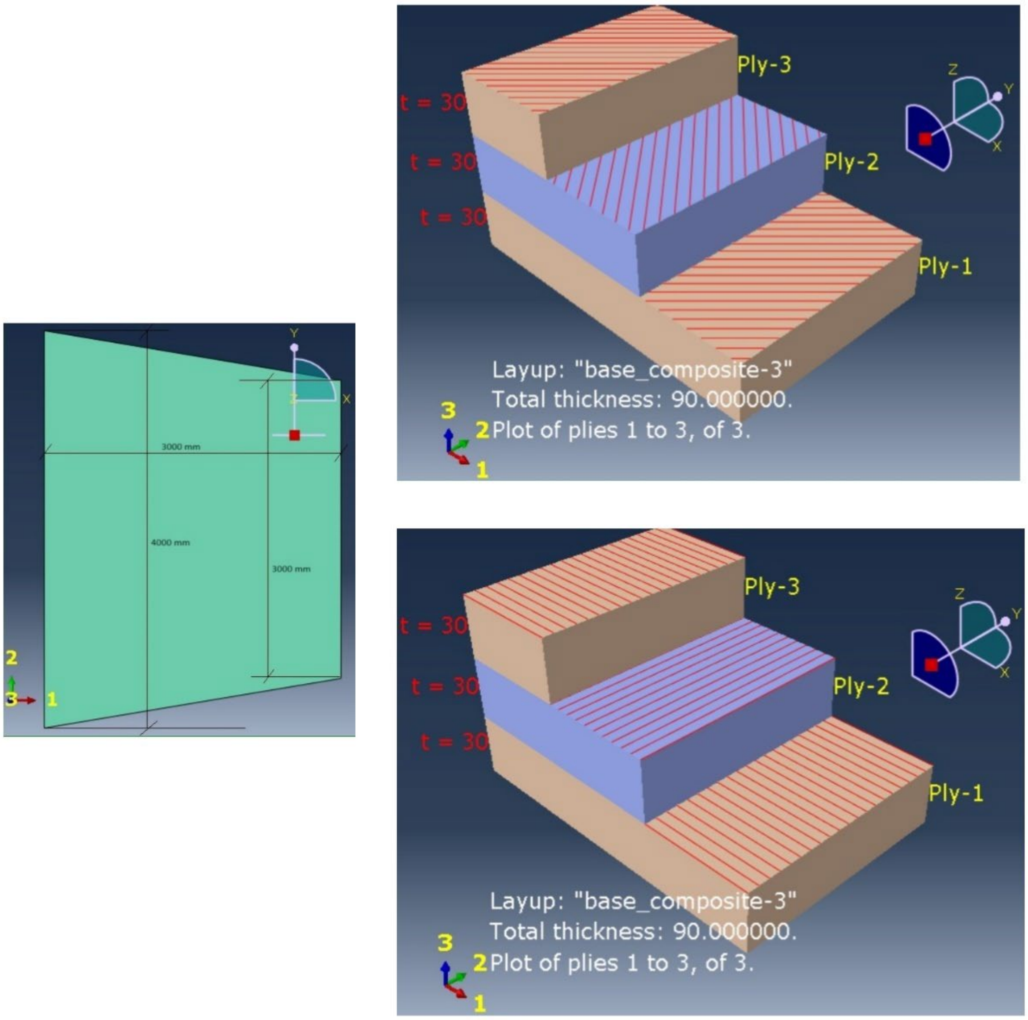


Fig. 12: Clockwise from left: a) Plate dimensions in ABAQUS; b) Angle-ply laminate; c) Cross-ply laminate

A deterministic study is done to note the additional thickness that is needed to have ten percent increase of natural frequency as well as a reduction of ten percent in the maximum deflection. Thus, we define the vibration based objective function as ten percent increment of natural frequency, and ten percent reduction of maximum deflection to be the serviceability based objective function.

Table 8: Mean material properties and parameters

Material	E_{11}(GPa)	E_{22}(GPa)	Poisson's ratio (ν)	G_{12}(GPa)	G_{13}(GPa)	G_{23}(GPa)	t(mm)
Glass/epoxy	38.6	8.27	0.26	4.14	4.14	4.14	33.143

Table 9: Model used in the study

Geometry	Longer side	4000 mm
	Shorter side	3000 mm
	Width	3000 mm
	Thickness	90 mm
	Thickness of individual lamina	30 mm each
0º/90º/0º	Natural frequency	0.542455 Hz
	Desired natural frequency (i.e. ten percent increase)	0.596704 Hz
	Thickness required per lamina to attain the desired natural frequency	33.143 mm
45º/-45º/45º	Natural frequency	0.472956Hz
	Desired natural frequency (i.e. ten percent increase)	0.520252Hz
	Thickness required per lamina to attain the desired natural frequency	33.114 mm
0º/90º/0º	Maximum deflection	27.47 mm
	Desired maximum deflection (i.e. ten percent decrease)	24.723 mm
	Thickness required per lamina to attain the desired maximum deflection	31.105 mm
45º/-45º/45º	Maximum deflection	36.89 mm
	Desired maximum deflection (i.e. ten percent decrease)	33.201 mm
	Thickness required per lamina to attain the desired maximum deflection	31.095 mm

6.2 Frequency and deflection ratio

By the objective function described above, reaching the desired frequency is a Boolean (i.e., 'true' or 'false') value, but they provide very little information about how much the thickness can be decreased or increased if the laminate yields a frequency greater or lesser than the desired frequency respectively. Thus, a frequency ratio (fr) is introduced and is accordingly used to compare all objective function failure scenarios in this study and the ratio is simply a metric defined as:

$$\text{fr} = \frac{\nu_{desired}}{\nu_{model}} \tag{2}$$

Where ν_{model} is the frequency of the model and $\nu_{desired}$ is the ten percent increased frequency. By the same logical argument we can define the deflection ratio (dr) also as:

$$\text{dr} = \frac{d_{desired}}{d_{model}} \tag{3}$$

6.3 Monte Carlo simulation

In this study, Monte Carlo simulation (MCS), a simple computational approach to stochastic physical problems involving many degrees of freedom, is used to generate the data. To obtain numerical results, MCS involves repeated sampling of random variables to simulate an arbitrarily large number of experiments [7]. For each trial in this study, the material properties, material strengths, ply orientations and ply thicknesses form a vector X of basic random variables and is given by:

$$\mathbf{X} = [\,(E_{11}\ E_{12}\ G_{12}\ \theta\ t)_1 \quad (E_{11}\ E_{12}\ G_{12}\ \theta\ t)_2 \quad (E_{11}\ E_{12}\ G_{12}\ \theta\ t)_3\,] \tag{4}$$

All material and geometric properties are allowed to vary at the ply level, but for simplicity G13 and G23 are assumed to be equal for all plies in each laminate. In this study, with 15 stochastic degrees of freedom, the vector X in Eq. (3) highlights the increasing complexity and uncertainty of both the laminate frequency ratio and deflection ratio as random variables.

6.4 Probability of failure and correction factor

The probability of failure is defined in this study as the proportion of laminates in a simulation of N trials whose resulting frequency is less than the predicted deterministic value desired and can be calculated as:

$$P_f = \frac{1}{N}\sum_{i=1}^{N} I(\nu_{model_i} < \nu_{desired}) \tag{5}$$

where $I(\nu_{model} < \nu_{desired})$ is the indicator of the objective function to reach the desired frequency, defined as:

$$I(\nu_{model_i} < \nu_{desired}) = \begin{cases} 1 \; if \; \nu_{model_i} \leq \nu_{desired} \\ 0 \; if \; \nu_{model_i} > \nu_{desired} \end{cases} \tag{6}$$

and the frequency of the ith model ν_{model_i} is the manifestation of the random variable ν_{model} defined as a function of the random variable X in trial i, i.e. $\nu_{model_i} = \nu_{model}(\boldsymbol{X}_i)$.

Similarly, for a simulation of M trials whose resulting deflection is more than the predicted deterministic value $\delta_{desired}$, probability of failure can be determined as:

$$P_f = \frac{1}{M}\sum_{i=1}^{M} I(d_{model_i} > d_{desired}) \tag{7}$$

where $I(d_{model_i} > d_{desired})$, the indicator of the objective function to reach the desired deflection is defined as:

$$I(d_{model_i} > d_{desired}) = \begin{cases} 1 \; if \; d_{model_i} > d_{desired} \\ 0 \; if \; d_{model_i} \leq d_{desired} \end{cases} \tag{8}$$

and the deflection of the i^{th} model d_{model_i} is the manifestation of the random variable d_{model} defined as a function of the random variable X in trial i, i.e. $d_{model_i} = d_{model}(\boldsymbol{X}_i)$.

Defining failure probability this way would neither adequately reflect to what extent the objective is achieved nor would indicate the extent to what the objective remains unattained. To address this, a correction factor (CF) is defined as:

$$\mathrm{CF} = \frac{\nu_{desired}}{\nu_{model_{failed}}} \tag{9}$$

where $\nu_{model_{failed}}$ is the frequency of the model of the failed laminate (i.e. $\nu_{model_{failed}} = \nu_{model_i}$ when $\nu_{model_i} \leq \nu_{desired}$). Correction factor (CF) for deflection can similarly be defined as:

$$\mathrm{CF} = \frac{d_{desired}}{d_{model_{failed}}} \tag{10}$$

where $d_{model_{failed}}$ is the deflection of the model of the failed laminate (i.e. $d_{model_{failed}} = d_{model_i}$ when $d_{model_i} > d_{desired}$). Hence, CF is defined only when the laminate fails. CF can be interpreted as the factor by which the frequency or deflection of a laminate must be multiplied to reach the predicted desired deterministic value, thus acting as a tool to compare trials.

6.5 The empirical distribution function and sample quantiles

The cumulative distribution function (CDF) of a random variable, Y, is used to describe the distribution of that random variable and is defined as:

$$F_Y(y) = \mathrm{P}(Y \leq y), \qquad y \in R \tag{11}$$

More importantly, the CDF effectively contains all information about the random variable and completely determines the shape of its distribution. The CDF can be approximated by:

$$\hat{F}_Y(y) = \frac{1}{N,M} \sum_{i=1}^{N,M} I(Y_i < y) \tag{12}$$

which may be defined here as the empirical distribution function (EDF). The EDF of the frequency can therefore be defined as:

$$\hat{F}_y(x) = \frac{1}{N}\sum_{i=1}^{N} I(y_{model_i} < x), \qquad min\{y_{model_i}\} \leq x < max\{y_{model_i}\} \qquad (13)$$

where min $\{y_{model_i}\}$ and max $\{y_{model_i}\}$ are the lowest and highest realizations of the frequencies or deflections of all trials in each simulation.

6.6 Simulation set up

Abaqus CAE is used to compute the frequencies and maximum deflections of various combinations of laminates. Frequency and deflection data are generated using MCS for two different simulations which consider combinations of material and laminate geometric parameters to explore the interaction effects of randomness in each type of simulation. Five different coefficients of variation are taken up for each simulation.

Each simulation is repeated for 50 trials, resulting in a total of 1000 simulations for each combination of material properties and another 1000 simulations for each combination of material and geometric properties.

Now for frequency analysis, angle ply is considered for an extra simulation to highlight the uncertainty introduced by geometry variation alone, which involves 200 additional simulations. The same is conducted for cross ply in deflection analysis for another 200 simulations. Each time, the model is reset to eliminate the interaction effects, if any, between simulations. Table 4 provides a summary. Materials examined and their corresponding mean values are summarized in Table 8. These materials were used in [36] as well. The standard deviations of all material properties are defined functions of their mean values and CV. An exception is made for the ply orientation, whose standard deviation is assumed to have a maximum of 1.8° at the upper bound of simulated CV values and varies linearly with CV (e.g., the deviation is 0.45° for CV = 0.05) as in [6].

Table 10: List of simulations performed

Simulation	Random parameters
1	Material properties (E_{11}, E_{22}, G_{12})
2	Material and geometric properties ($E_{11}, E_{22}, G_{12}, \theta, t$)
3	Geometric properties (θ, t)

Experimental data from Maekawa [30] regarding material properties of unidirectional carbon fibre-reinforced laminates shows that the distribution of basic material parameters can be closely approximated by a Gaussian or normal distribution. However, [37] suggests to mostly use Normal and Weibull distributions for modelling strength properties of both metallic and composite materials. Gamma distribution is also selected as it is a basic distribution of statistics for variables bounded at one end (such as the strength properties and the stiffness moduli of the composite material). Asymptotic extreme value distributions type I for the smallest and largest elements are chosen, since they have been used to model the distribution of the breaking strength of some materials [37]. Using the Kolmogorov-Smirnov test, Lekou and Philippidis [29] showed that the assumption of normally distributed mechanical properties for unidirectional glass/polyester cannot be rejected at 5% significance level. Thus suggested from K-S Test results in [29], the study takes E11, E22, and G12 to follow Weibull, Extreme type I of Largest element distribution and Gamma distribution respectively.

These assumptions are considered in this study to assign a distribution function during drawing random variables for each trial.

RESULTS FROM THE SIMULATION

7.1 Frequency coefficient of variation

Figs. 13-16 show monotonic increase in frequency coefficient of variation (CV(ν)) and deflection coefficient of variation (CV(δ)) with input coefficient of variation (CV(X)) for the symmetric trapezoidal composite laminates considered in this study. The polynomial fit-curve with R^2 are also shown alongside.

It is noted that either a cubic or a quadradic polynomial fits when both material and geometry is varying, which signifies that the coefficient of variation is not linear. For simulations where only the material properties are varying, a liner fit curve is observed. It is apparent that the linear increase in natural frequency and displacement coefficients of variation as reported in ref. [39] is not mirrored in this study which shows a clear non-linear dependence of the frequency and deflection coefficient of variation with variation of the input variables.

The figures indicates that across all simulations, each unit change in CV(X) results in disproportionate change in the CV(ν). Similarly, disproportionate change in CV(δ) is also concluded. However, Figs. 13 and 14 indicates that the greatest effect on CV(ν) as well as CV(δ) is due to variations of geometric properties particularly thickness as reported in [34] and [40]. The importance of thickness was also reported by Gohari et al. [41] who found that slight changes in shell lay-up thickness caused considerable fluctuations in failure strength. In the present study, it is clearly shown that geometric variations alone effects both the natural frequency and maximum deflection in a much profound manner than material variation and is nearly as high as combined variation.

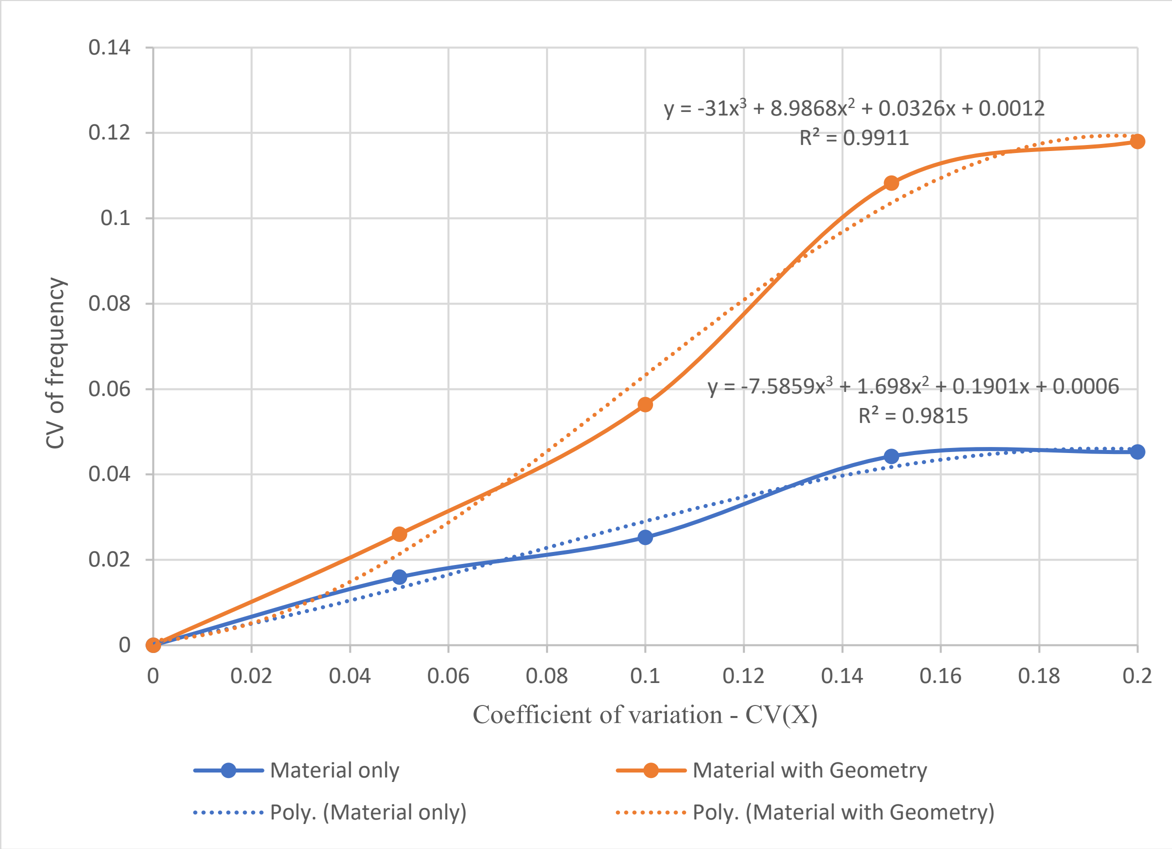


Fig. 13: Coefficients of variation: Frequency vs Input variables for cross-ply.

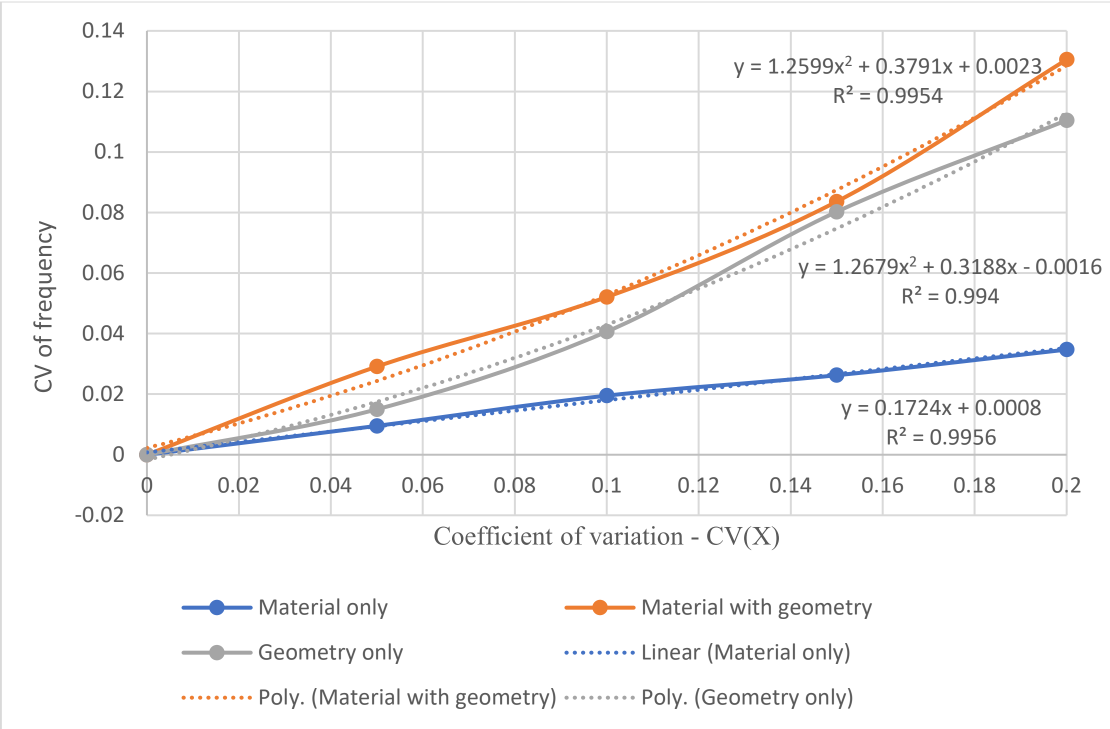


Fig. 14: Coefficients of variation: Frequency vs Input variables for angle-ply.

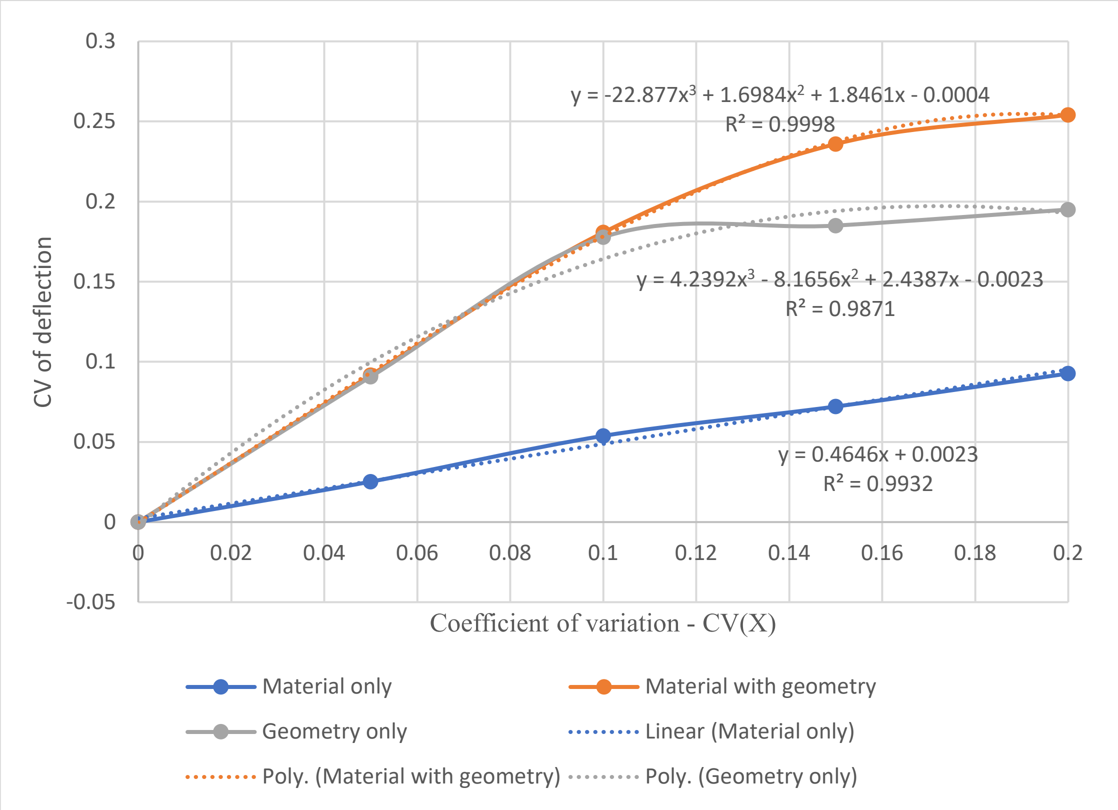


Fig. 15: Coefficients of variation: Deflection vs Input variables for cross ply

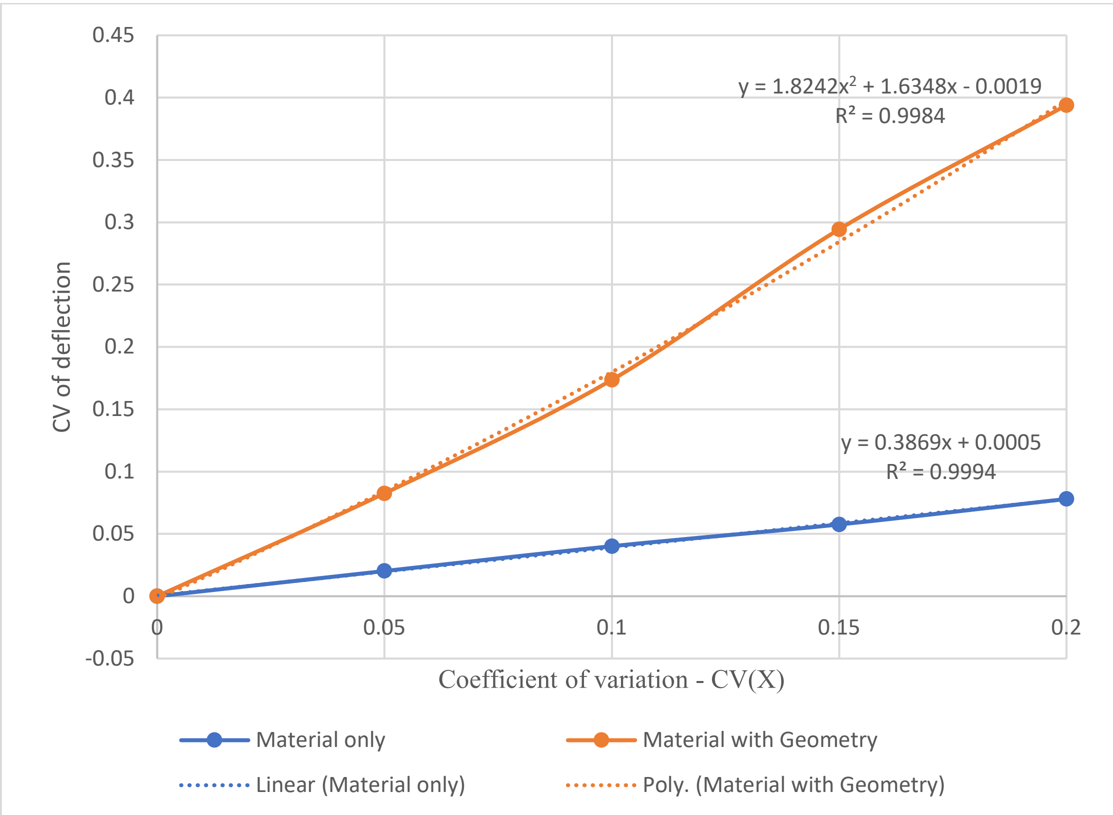


Fig. 16: Coefficients of variation: Deflection vs Input variables for angle ply

7.2. Probability of failure

Figs. 17-20 show the probability of failure for the first two combinations of randomness. It is concluded that this metric is generally non-linear and of an overlapping nature over its range of coefficients of variation in input variables. Whether a combination of material and geometric randomness has more impact than only material or geometry randomness in the failure probability cannot be concluded. However, this study suggests the range in which the variation will occur. At higher values of variation coefficient, the failure probability takes a leap for all the cases.

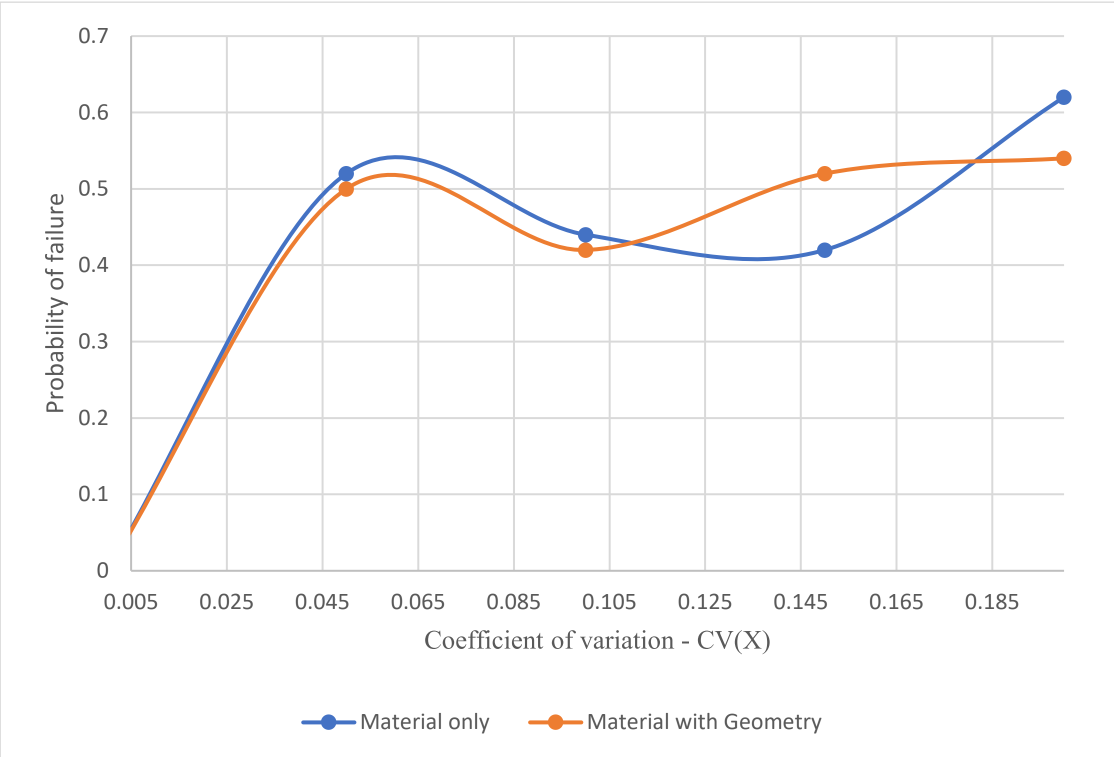


Fig. 17: Probability of failure for cross ply: Frequency simulation comparison

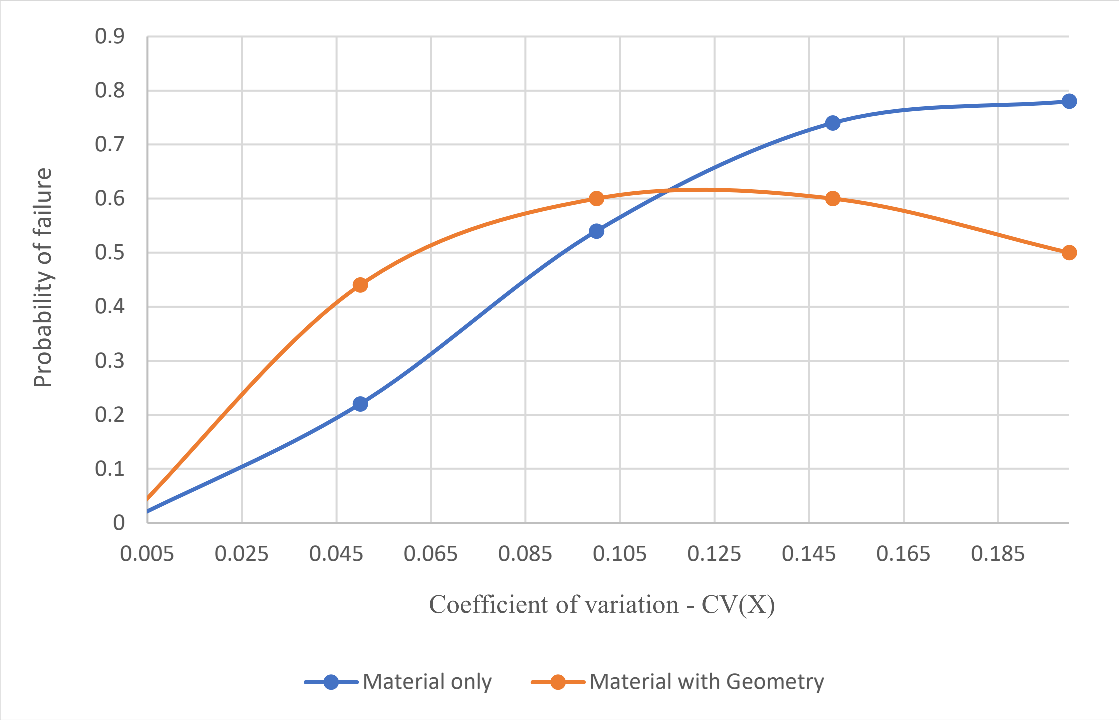


Fig. 18: Probability of failure for angle ply: Frequency simulation comparison.

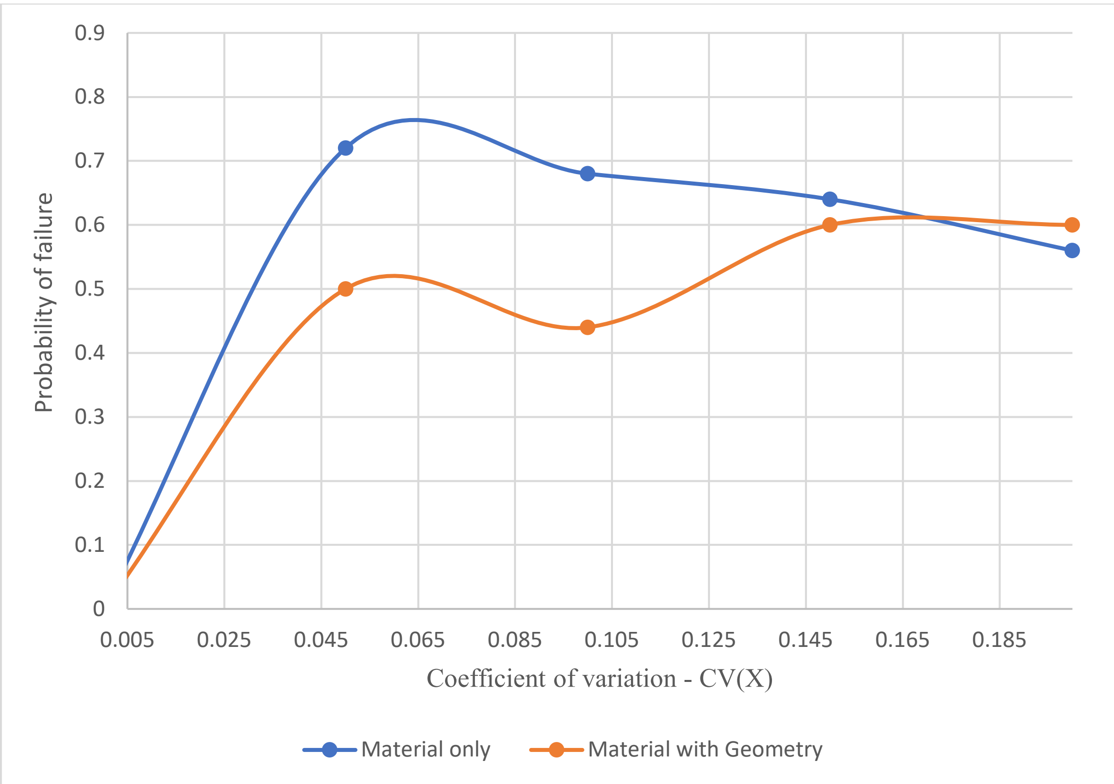


Fig. 19: Probability of failure for cross ply: Deflection simulation comparison.

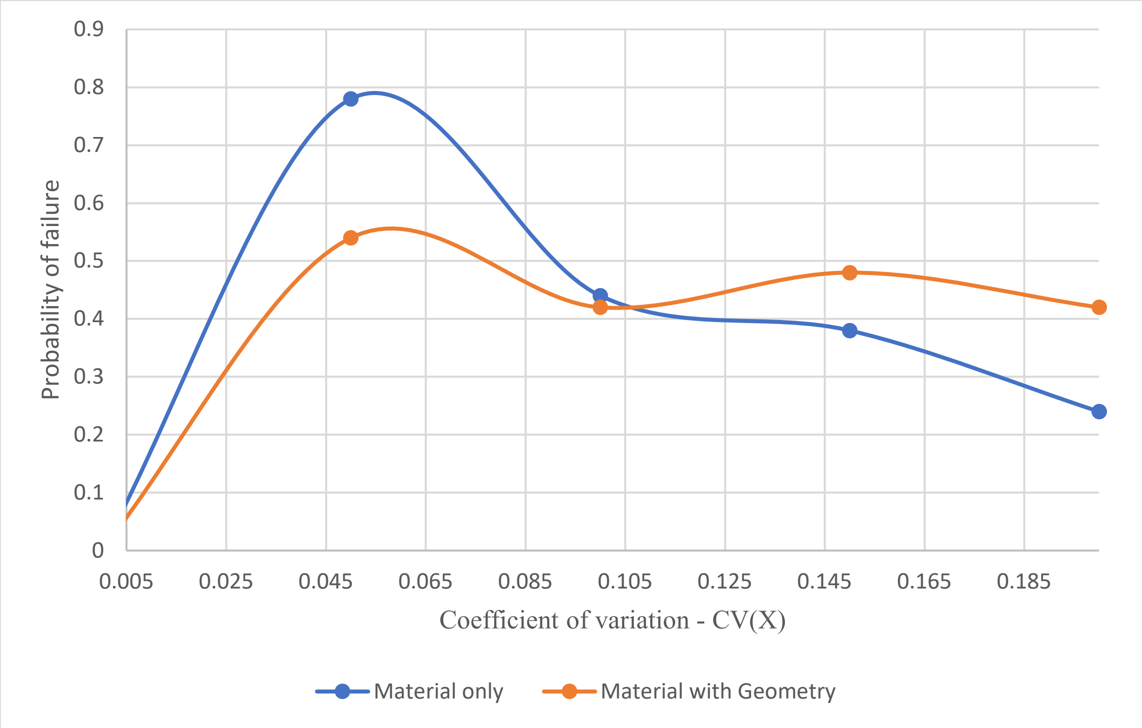


Fig. 20: Probability of failure for angle ply: Deflection simulation comparison.

7.3. Correction factor

Figs. 21-24 show a strong, positive correlation between the mean correction factor and CV(X). The mean is preferred over the median when dealing with data containing outliers because it is more sensitive to extreme values, making it a more appropriate measure of central tendency. It shows, if not exact, an approximate linear behaviour for both types of simulations. However, with higher values of variable X coefficient of variation, it is noticed that the correction factor for the simulation where both material and geometry are varied rises progressively.

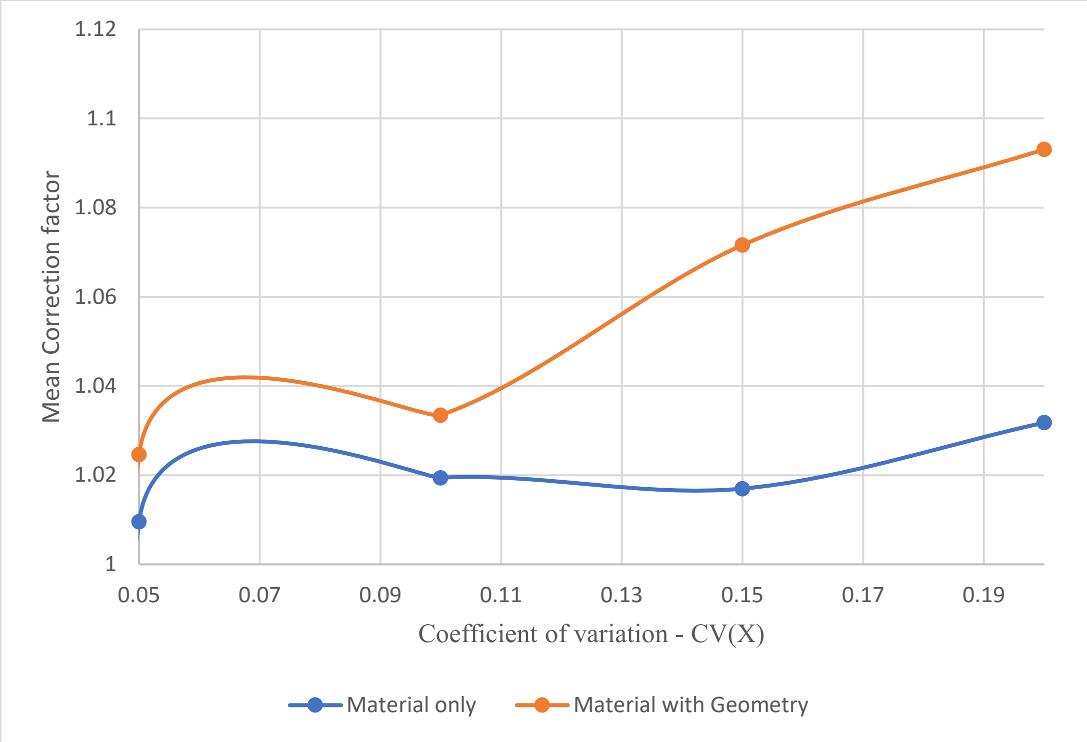


Fig. 21: Correction factor for cross-ply: Frequency simulation comparison

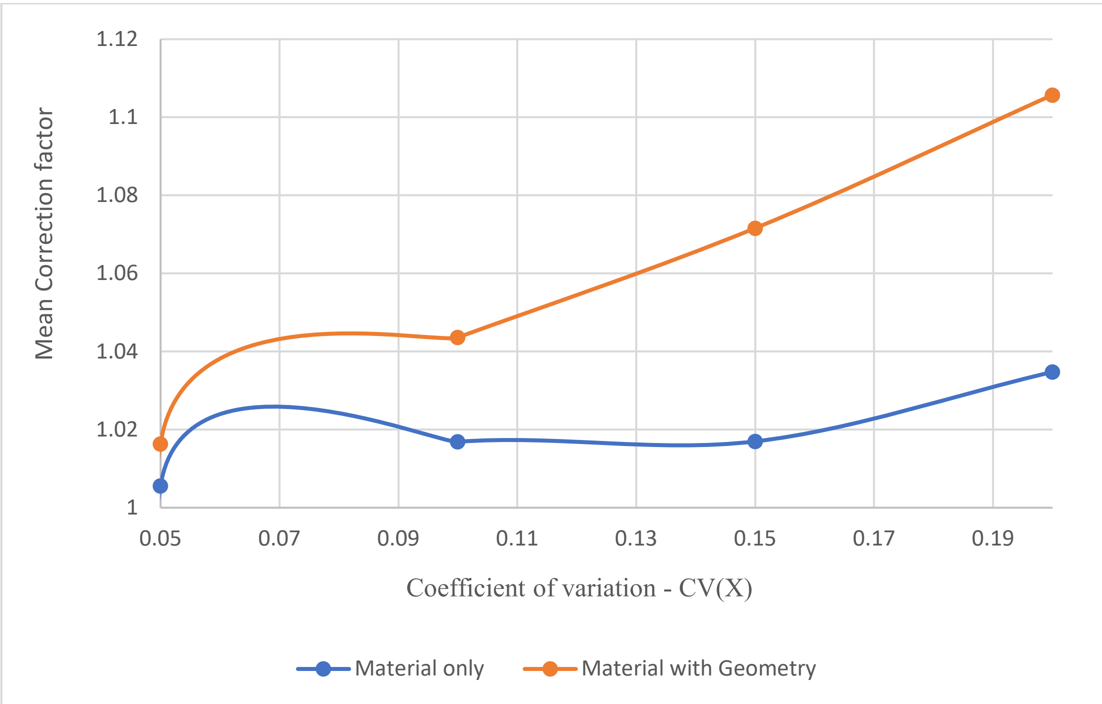


Fig. 22: Correction factor for angle-ply: Frequency simulation comparison

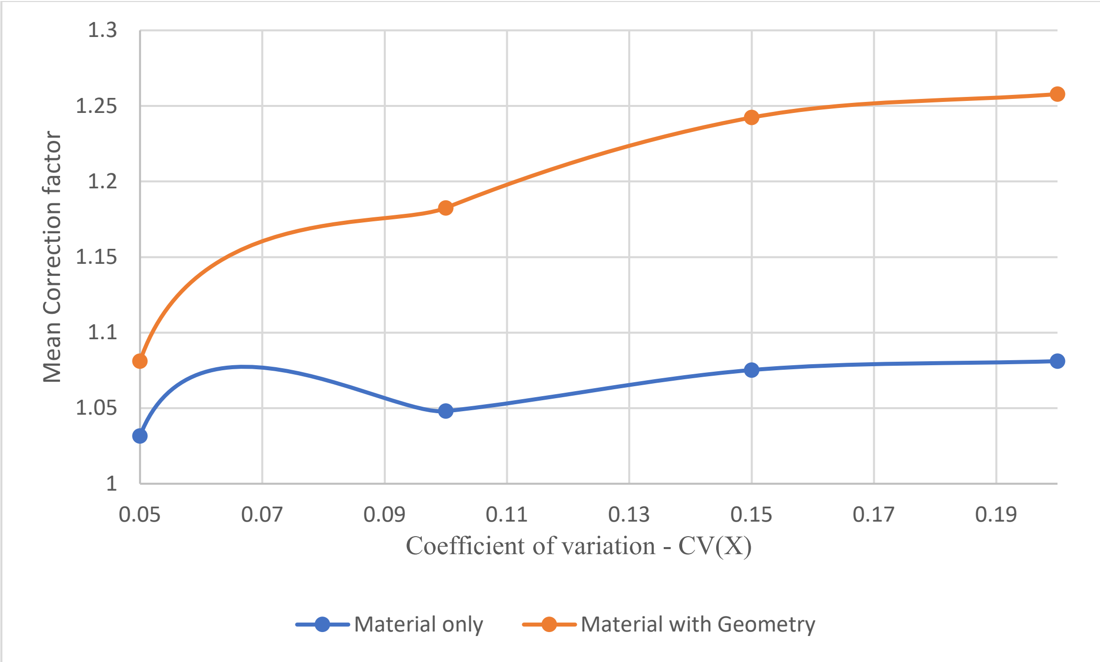


Fig. 23: Correction factor for cross ply: Deflection simulation comparison

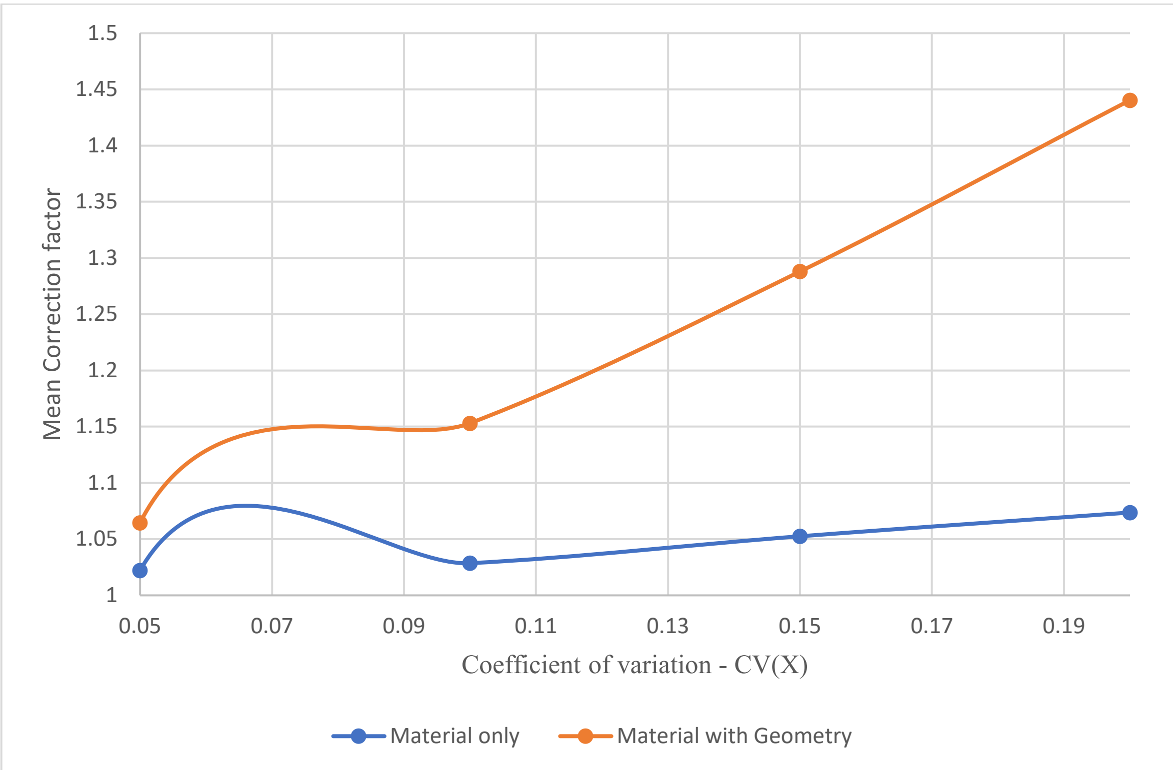


Fig. 24: Correction factor for angle-ply: Deflection simulation comparison

7.4. Empirical distribution function

Figs. 25-32 show the EDF of the correction factor for first and second types of simulations respectively. The EDFs clearly illustrate a notable rise in sample quantiles of safety factors as the coefficient of variation of variable X increases. For second simulation of Glass/Epoxy (Fig. 13), 90% of simulated laminates have a correction factor less than or equal to 1.016 when CV(X) is 0.05 but is 1.06 when CV(X) is 0.15. In other words, with CV = 0.05, 90% of laminates must be manufactured 1.6% thicker to ensure attainment of the expected deterministic frequency, compared to 3.6% when considering CV = 0.15. Similar findings for failure strength studies were noted in ref. [42]. It is also clearly noted from the two figures that the addition of randomness in geometry variation incurs more penalty than the variation in material property alone.

Again for CV = 0.15, when only material properties are varying, Fig. 21 shows that 90% of the laminates need to manufactured 3.6% thicker to ensure that desired frequency is attained. However, from Fig. 22 we can see that when both material and geometry is varying, we need to manufacture 90% of the laminates with 8.9% more thickness. The progressive rise in percentage of extra thickness required to attain the desired objective function for different levels of coefficient of variation of input variables is illustrated in the subsequent tables (Tables 11-14). For clarity in deriving conclusions, the Tables 11-14 have been prepared from the raw data corresponding to a particular cumulative probability of 0.90. The values in the table clearly establishes that for dynamic and serviceability based objective function, the effect of geometric variations is profoundly much higher than material variations.

Table 11: Percentage of extra thickness needed for frequency attainment in cross ply

CV of input variables	Variation in material	Variation in material and geometry
0.05	1.6 %	3.0 %
0.10	2.7 %	2.9 %
0.15	3.6 %	6.9 %
0.20	5.7 %	7.6 %

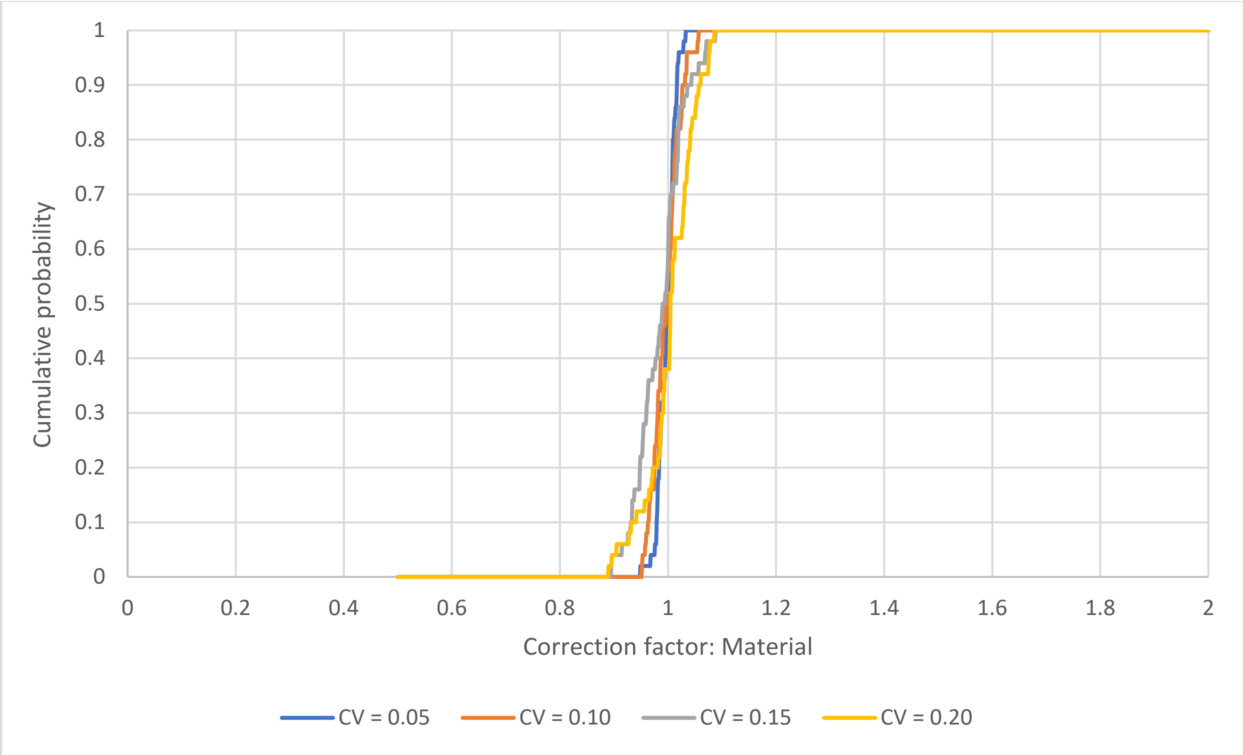


Fig. 25: Frequency correction factor EDF: Material as random variable in cross ply.

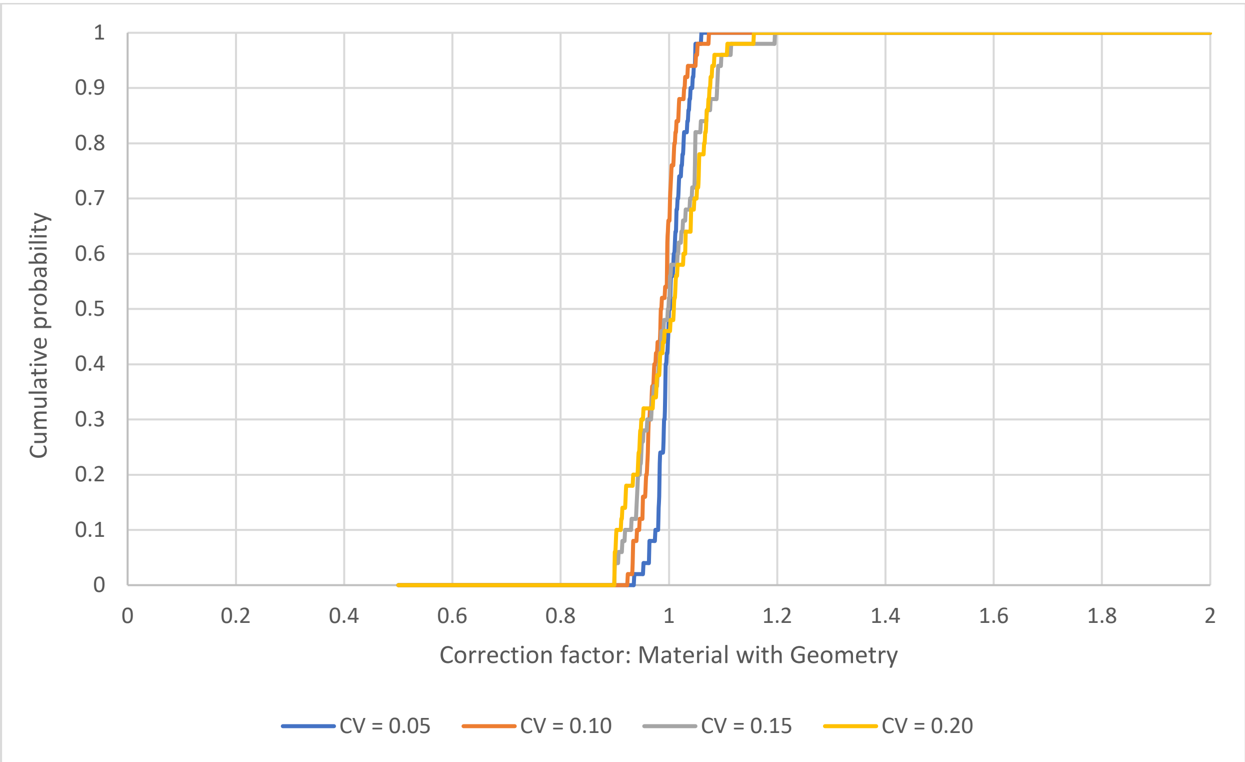


Fig. 26: Frequency correction factor EDF: Material and geometry as random variable in cross ply.

Table 12: Percentage of extra thickness needed for frequency attainment in angle ply

CV of input variables	Variation in material	Variation in material and geometry
0.05	1 %	2.9 %
0.10	3.7 %	8.3 %
0.15	3.7 %	14 %
0.20	6.4 %	15 %

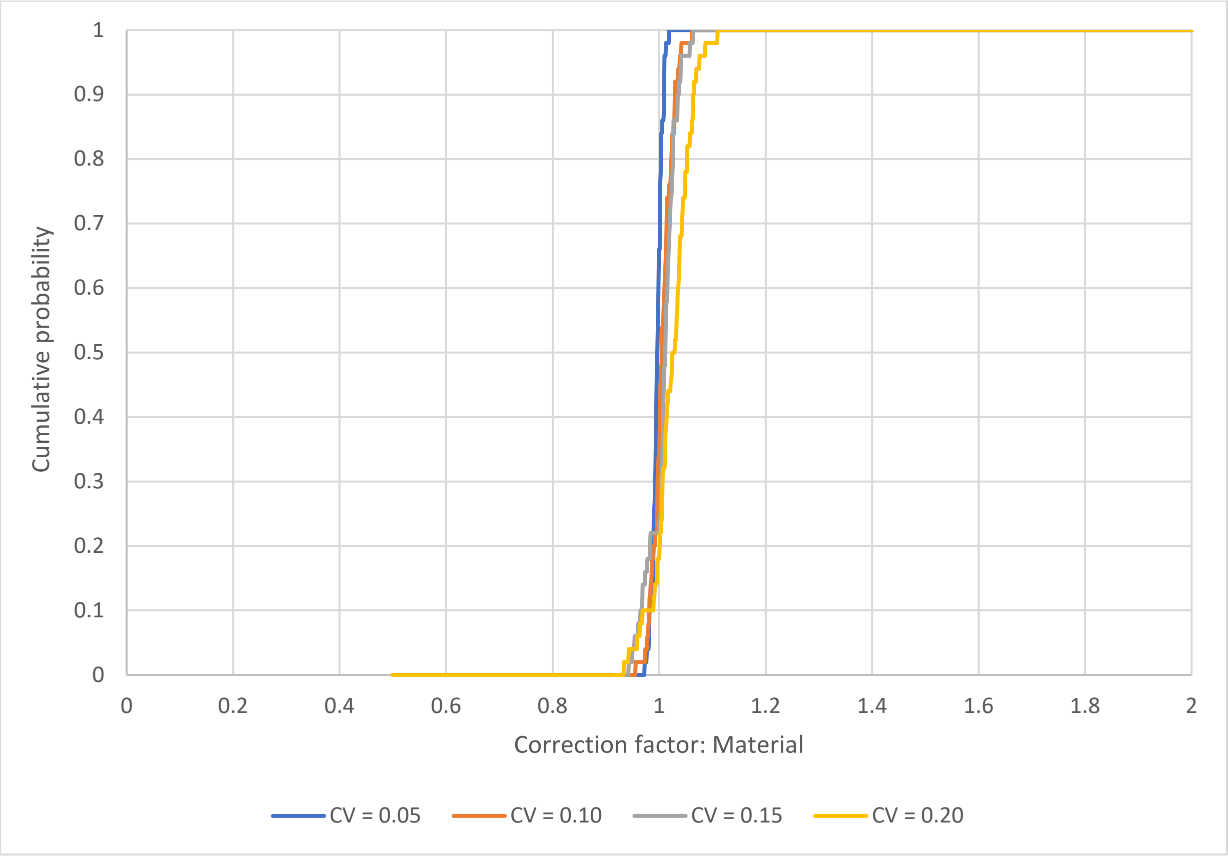


Fig. 27: Frequency correction factor EDF: Material as random variable in angle ply

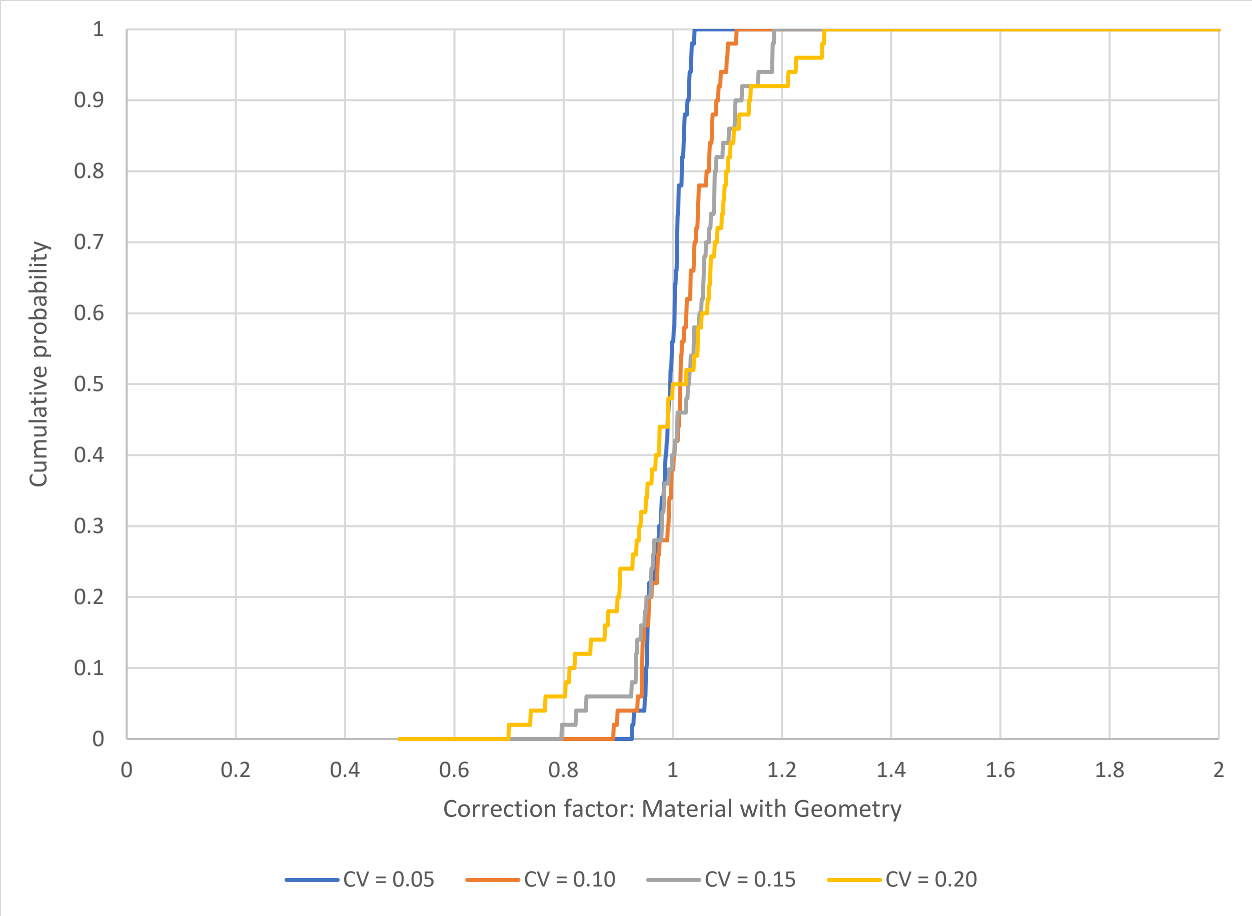


Fig. 28: Frequency correction factor EDF: Material and geometry as random variable in angle ply

Table 13: Percentage of extra thickness needed for deflection reduction in cross ply

CV of input variables	Variation in material	Variation in material and geometry
0.05	4.9 %	10.4 %
0.10	8.1 %	27.8 %
0.15	14.1 %	39.4 %
0.20	14.2 %	39.5 %

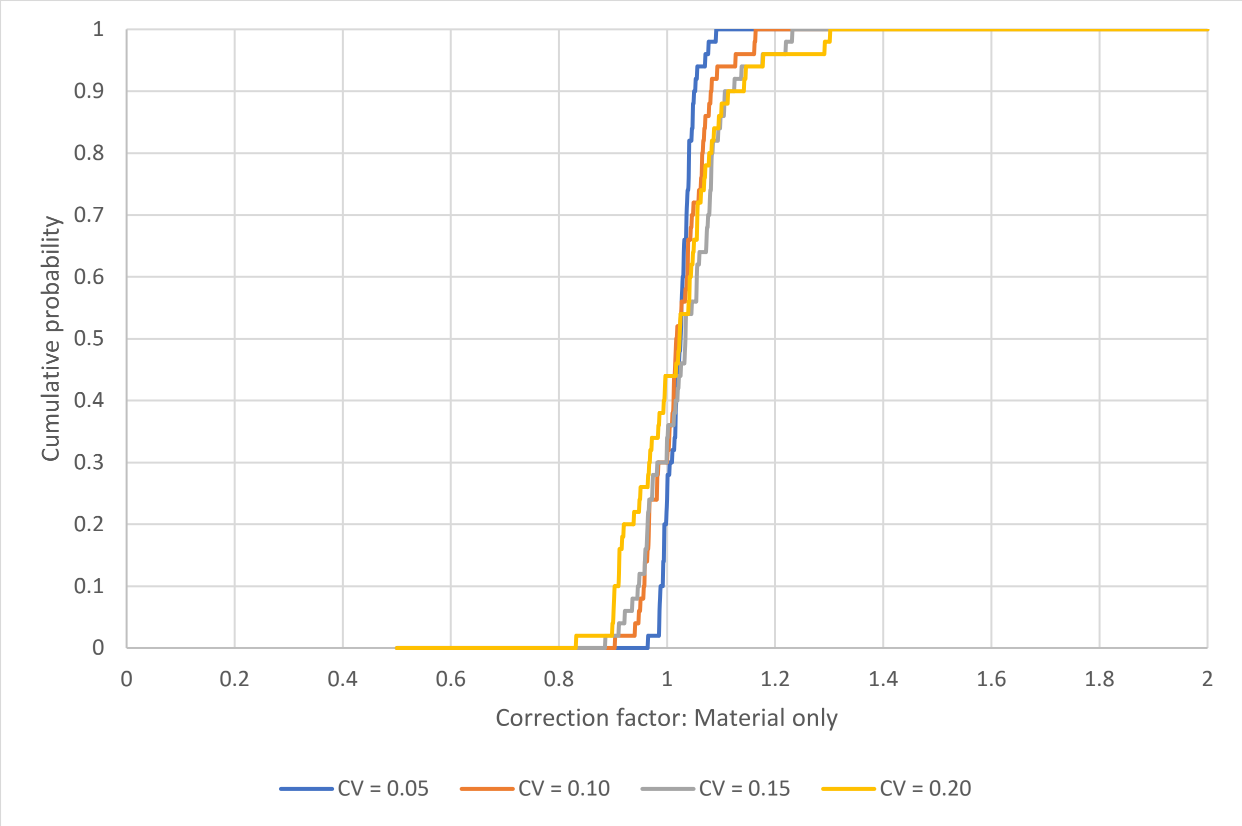


Fig. 29: Deflection correction factor EDF: Material as random variable in cross ply

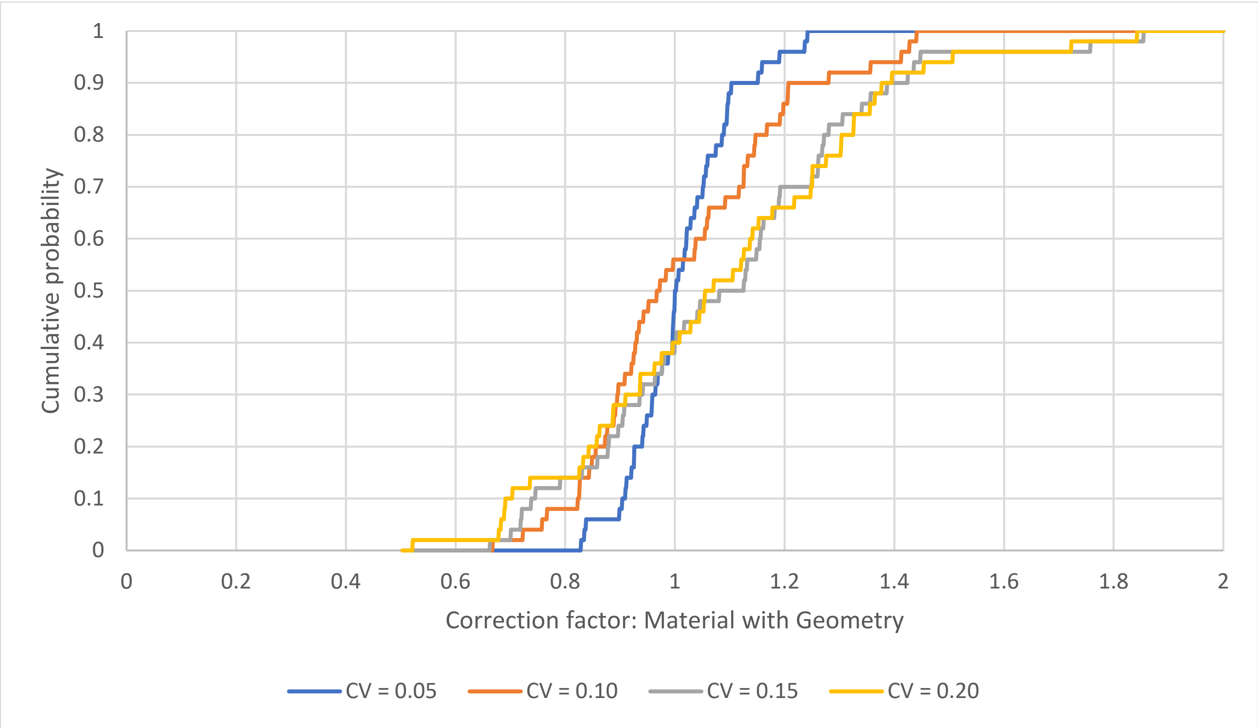


Fig. 30: Deflection correction factor EDF: Material and geometry as random variable in cross ply

Table 14: Percentage of extra thickness needed for deflection reduction in angle ply

CV of input variables	Variation in material	Variation in material and geometry
0.05	4.5 %	10.1 %
0.10	4.5 %	23.9 %
0.15	9.2 %	49 %
0.20	10.6 %	75.2 %

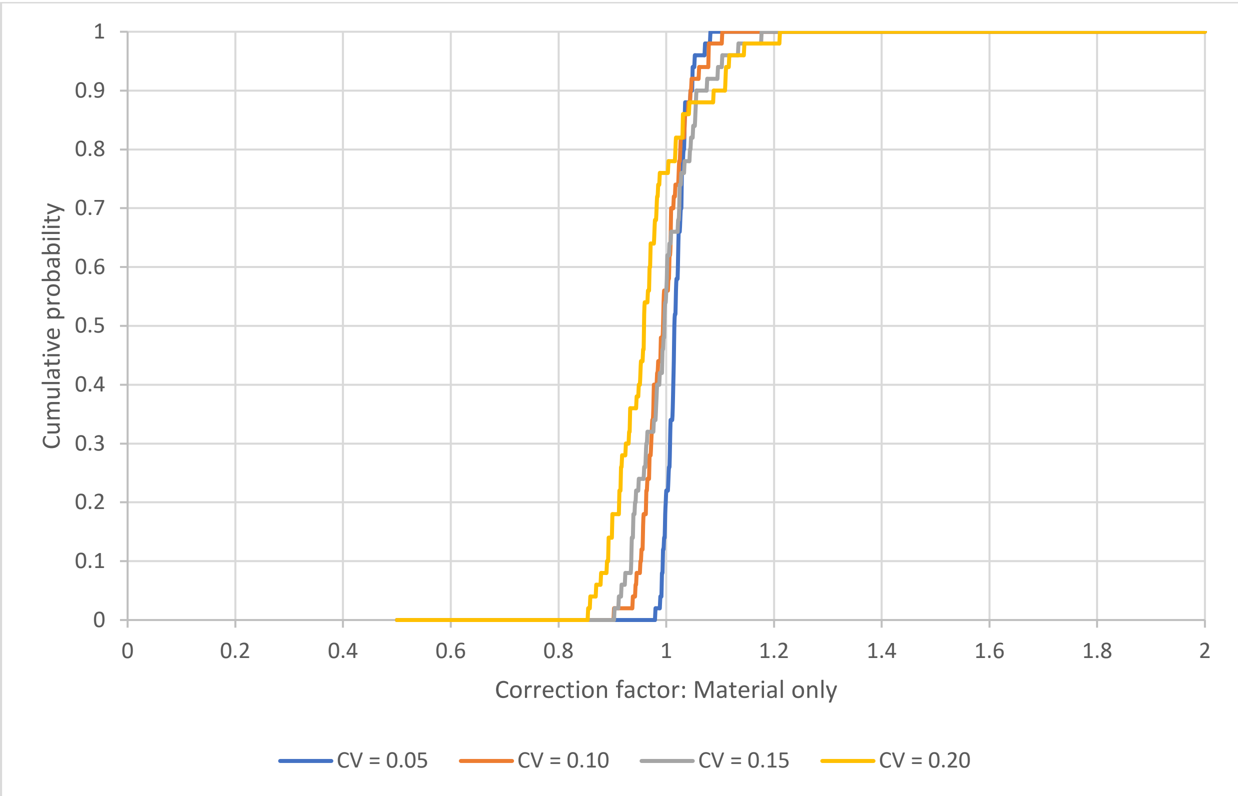


Fig. 31: Deflection correction factor EDF: Material as random variable in angle ply

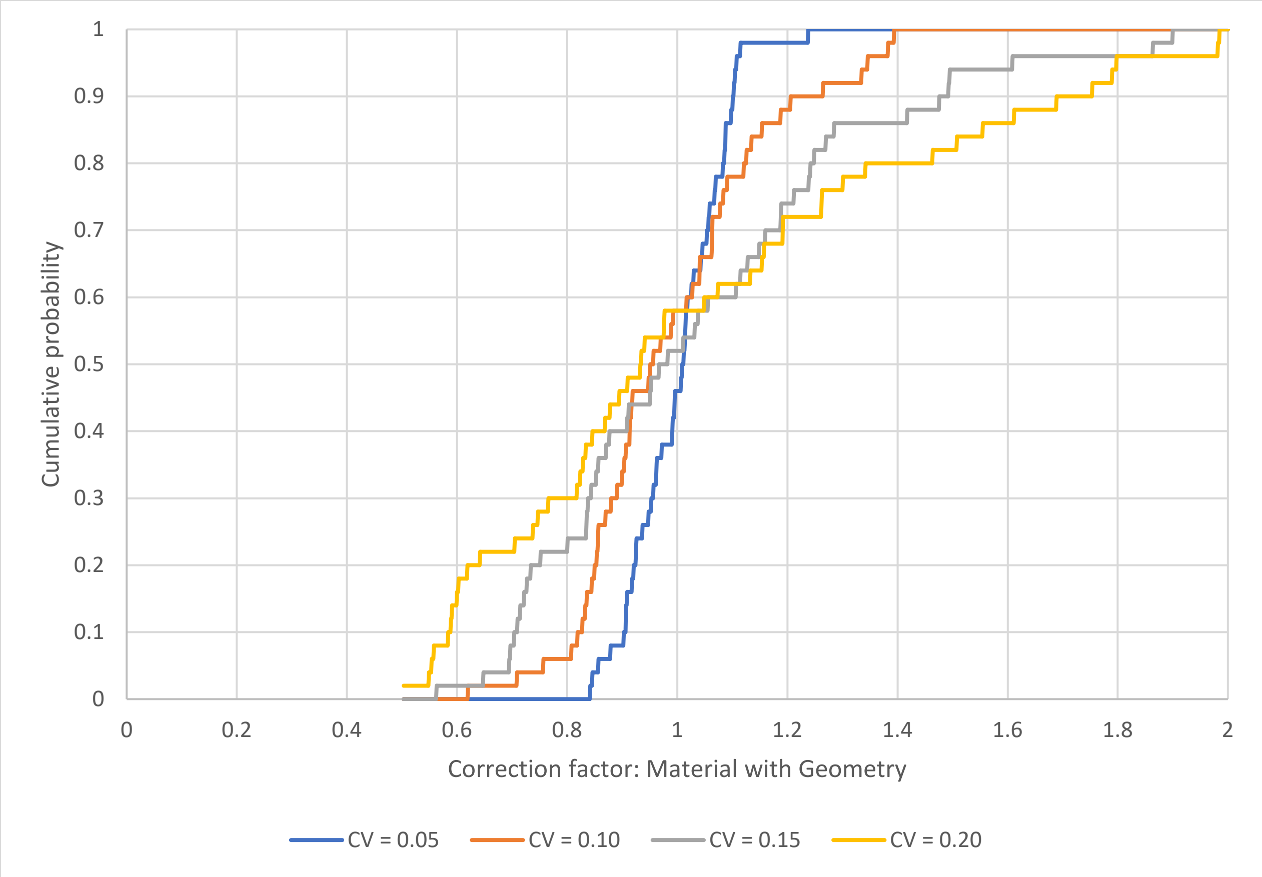


Fig. 32: Deflection correction factor EDF: Material and geometry as random variable in angle ply

7.5. Conclusions from frequency analysis of various options

- Respective modes have greater frequencies when the skewness increases.
- The rate of increase of modal frequencies decreases with increase of skewness for all except the $0^0/90^0/0^0$ stacking

CONCLUSONS FROM THE PRESENT STUDY

Natural frequencies and maximum deflection predicted by deterministic analysis are often inaccurate when considering randomness in material properties and laminate parameters. The study:

- determines the comparative effects of material and geometric randomness when dealing with vibration and serviceability-based problems.
- finds the penalty incurred for different cumulative probability for varied levels of variation coefficient. Thus underscores the significance of accounting randomness in achieving a targeted natural frequency and maximum deflection by augmenting thickness in a symmetric ply when the plan area is kept unaltered which is generally encountered in practical situations.
- states that a non-linear relationship exists between coefficient of variation of all input variables and frequency as well as deflection. A near liner relationship exist when only material properties are varying. The probability of failure and correction factor is generally nonlinear.
- points out disproportionate sensitivity of the coefficient of variation of natural frequency (CV(ν)) and deflection CV(δ) to the coefficient of variation of all material and geometric parameters (CV(X)), particularly focusing on ply thickness. Thus, the study accentuates the importance of incorporating more manufacturing accuracy.
- establishes that control over variation in geometric properties will alone lead to better results in terms of vibration and serviceability.
- points out that the penalty incurred to ensure that a laminate aligns with the anticipated deterministic frequency or maximum deflection escalates much more with the coefficient of variation (CV) of variable X when geometric parameters vary randomly.

These findings highlight the importance of considering randomness in every available factor of design of composite laminates, with a focus on geometric properties control in the manufacturing process.

FUTURE SCOPE

The present study establishes the importance of considering stochasticity in the various design parameters of a laminated composite plate. The study can be extended both in geometric and material variation and in analytical parameter variation.

Geometric variation

- This study deals with clamped boundary condition. It can be extended for other boundary conditions.
- Stiffeners can be introduced in the plate geometry consideration keeping practical applicability in mind.
- The study deals with on trapezoidal plates. This can be extended to other shapes and cut-outs can also be introduced.

Material variation

- The study uses Glass epoxy as the constituent material which can be extended for Graphite epoxy and other constituent materials also.
- Further study on the comparative sensitivity analysis of different constituent materials towards geometric and material variation can be done.

Analytical parameter variation

- The study considers maximum central deflection and first natural frequency as the design parameters. It can be extended to check for higher frequencies with suitable practical needs. Other positions where deflection check is necessary accordingly can also be pursued with the same principle.
- The study can be extended towards using stress limit as an analytical parameter for design considerations along with frequency and deflection.

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LIST OF PUBLICATIONS

International Conference Paper Accepted

Title	Conference	Authors	Time and Venue
Effect of stochastic randomness in natural frequency increment of a trapezoidal laminated composite plate	9th International Conference on Structural Engineering and Concrete Technology, Imperial College, London	Rohan Das Dr. Dipankar Chakravorty Dona Chatterjee	April 14 – 16, 2024 London, United Kingdom