STUDY OF CONTROLLERS FOR A QUADROTOR UAV

A Thesis

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Master in Control System Engineering

(Electrical Engineering Department)

By

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CERTIFICATE

This is to certify that the dissertation entitled **"Study Of Controllers For A Quadrotor UAV"** has been carried out by SOUMYA KANTI BANERJEE (University Registration No.: 163523 of 2022-2023) under my guidance and supervision and be accepted as partial fulfilment of the requirement for the Degree of Master in Control System Engineering. The research results presented in the thesis have not been included in any other paper submitted for the award of any degree to any other University or Institute.

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ABSTRACT

Quadrotor Unmanned Aerial Vehicles (UAVs) have become vital tools in fields such as surveillance, search-and-rescue, and environmental monitoring due to their flexibility and precise maneuvering capabilities. However, managing the complex and inherently unstable dynamics of quadrotors remains a challenge, requiring effective control systems.

This thesis work presents a detailed mathematical model for a Vertical Take-off and Landing (VTOL) type Unmanned Aerial Vehicle (UAV) known as the quadrotor. The nonlinear dynamic model of the quadrotor is formulated using the Newton-Euler method, the formulated model is detailed including aerodynamic effects and rotor dynamics that are omitted in many literature. The motion of the quadrotor can be divided into two subsystems; a rotational subsystem (attitude) and a translational subsystem (altitude and x and y motion). Although the quadrotor is a 6 DOF underactuated system, the derived rotational subsystem is fully actuated, while the translational subsystem is underactuated.

The derivation of the mathematical model is followed by the development of four control approaches to control the altitude, roll, pitch and yaw of the quadrotor in space. The first approach is based on the linear Proportional-Derivative (PD) controller. The second control approach is based on the nonlinear Sliding Mode Controller (SMC).

The parameters and gains of the forementioned controllers were tuned manually in MATLAB/Simulink in order to track the set point smoothly. Simulation based experiments were conducted to evaluate and compare the performance of the two developed control techniques in terms of dynamic performance and stability.

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Chapter 1

Introduction

This chapter will first discuss the objectives and motivation behind writing this thesis. A brief introduction about Unmanned Aerial Vehicles (UAVs), their history, types and uses will then be presented. We will then move to the quadrotor type UAVs and discuss their concept and architecture. Lastly, the structure of the thesis will be outlined.

1.1 Motivations and Objectives

This thesis work will focus on the modelling and control of a quadrotor type UAV. The reason for choosing the quadrotor is in addition to its advantages that will be addressed later, the research field is still facing some challenges in the control field because the quadrotor is a highly nonlinear, multivariable system and since it has six Degrees of Freedom (DOF) but only four actuators, it is an underactuated system. Underactuated systems are those having a less number of control inputs compared to the system's degrees of freedom. They are very difficult to control due to the nonlinear coupling between the actuators and the degrees of freedom. Although the most common flight control algorithms found in literature are linear flight controllers, these controllers can only perform when the quadrotor is flying around hover, they suffer from a huge performance degradation whenever the quadrotor leaves the nominal conditions or performs aggressive maneuvers. The contributions of this work are: deriving an accurate and detailed mathematical model of the quadrotor UAV, developing linear and nonlinear control algorithms and applying those on the derived mathematical model in computer based simulations [16].

1.2 Unmanned Aerial Vehicles

The definition for UAVs varies from one literature to the other. For our purposes, UAVs are small aircrafts that are own without a pilot. They can either be remotely operated by a human or they can be autonomous; autonomous vehicles are controlled by an onboard computer that can be preprogrammed to perform a specific task or a broad set of tasks. While in other literatures, UAVs may refer to powered or unpowered, tethered or untethered aerial vehicles. The definition used in this thesis is based on that of the American Institute of Aeronautics and Astronautics: An aircraft which is designed or modified not to carry a human pilot and is operated through electronic input initiated by the flight controller or by an on board autonomous flight management control system that does not require flight controller intervention. UAVs were mainly used in military application but recently they are being deployed in civil applications too [16].

1.2.1 Applications of UAVs

In addition to the military use, UAVs can be used in many civil or commercial applications that are too dull, too dirty or too dangerous for manned aircrafts [16]. These uses include but not limited to:

Earth Science observations from UAVs can be used side-to-side with that acquired from satellites. Such missions include:

(a) Measuring deformations in the Earth's crust that may be indications to natural disasters like earthquakes, landslides or volcanos.
(b) Cloud and Aerosol Measurements.
(c) Tropospheric pollution and air quality measurements to determine the pollution sources and how plumes of pollution are transported from one place to
another.
(d) Ice sheet thickness and surface deformation for studying global warming.
(e) Gravitational acceleration measurements, since the gravitational acceleration varies near

Earth, UAVs are used to accurately measure gravitational acceleration at multiple places to define correct references.

(f) River discharge is measured from the volume of water owing in a river at multiple points. This will help in global and regional water balance studies [16].

Search and rescue UAVs equipped with cameras are used to search for survivors after natural disasters like earthquakes and hurricanes or survivors from shipwrecks and aircraft crashes [16].

Wild fire suppression UAVs equipped with infrared sensors are sent to fly over forests prone to fires in order to detect it in time and send a warning back to the ground station with the exact location of the fire before it spreads [16].

Law enforcement UAVs are used as a cost-efficient replacement of the traditional manned police helicopters [16].

Border surveillance UAVs are used to patrol borders for any intruders, illegal immigrants or drug and weapon smuggling [16].

Research UAVs are also used in research conducted in universities to proof certain theories. Also, UAVs equipped with appropriate sensors are used by environmental research institutions to monitor certain environmental phenomena like pollution over large cities [16].

Industrial applications UAVs are used in various industrial applications such as pipeline inspection or surveillance and nuclear factories surveillance [16].

Agriculture development UAVs also have agriculture uses such as crops spraying [16].

1.3 Quadrotors

The quadrotor concept for aerial vehicles was developed a long time ago. It was reported that the Breguet-Richet quadrotor built in 1907 had actually own. A quadrotor is considered to be a rotary-wing UAV due to its configuration that will be discussed later [16].

1.3.1 The Quadrotor Concept

A quadrotor consists of four rotors, each fitted in one end of a cross-like structure as shown in Figure 1.1. Each rotor consists of a propeller fitted to a separately powered DC motor. Propellers 1 and 3 rotate in the same direction while propellers 2 and 4 rotate in an opposite direction leading to balancing the total system torque and cancelling the gyroscopic and aerodynamics torques in stationary flights.

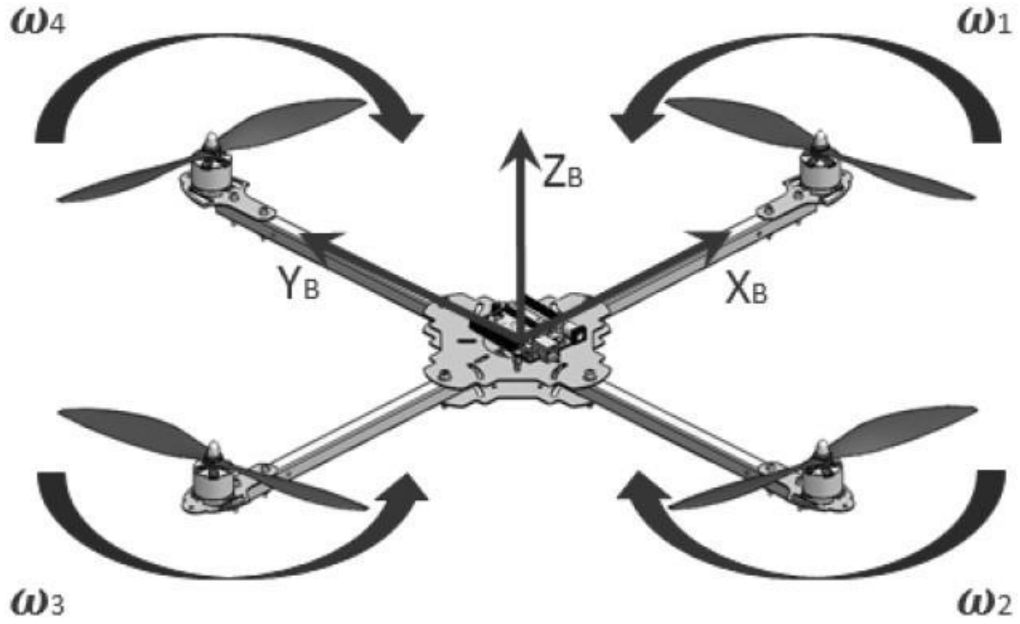

Figure 1.1: Quadrotor Structure [7]

The quadrotor is a 6 DOF object, thus 6 variables are used to express its position in space (x, y, z, ϕ, θ and ψ). x, y and z represent the distances of the quadrotor's centre of mass along the x, y and z axes respectively from an Earth fixed inertial frame. ϕ, θ and ψ are the three Euler angles representing the orientation of the quadrotor. ϕ is called the roll angle which is the angle about the x-axis, θ is the pitch angle about the y-axis, while ψ is the yaw angle about the z-axis. Figure 1.2 clearly explains the Euler Angles. The roll and pitch angles are usually called the attitude of the quadrotor, while the yaw angle is referred to as the heading of the quadrotor. For the linear motion, the distance from the ground is referred to as the altitude and the x and y position in space is often called the position of the quadrotor [16].

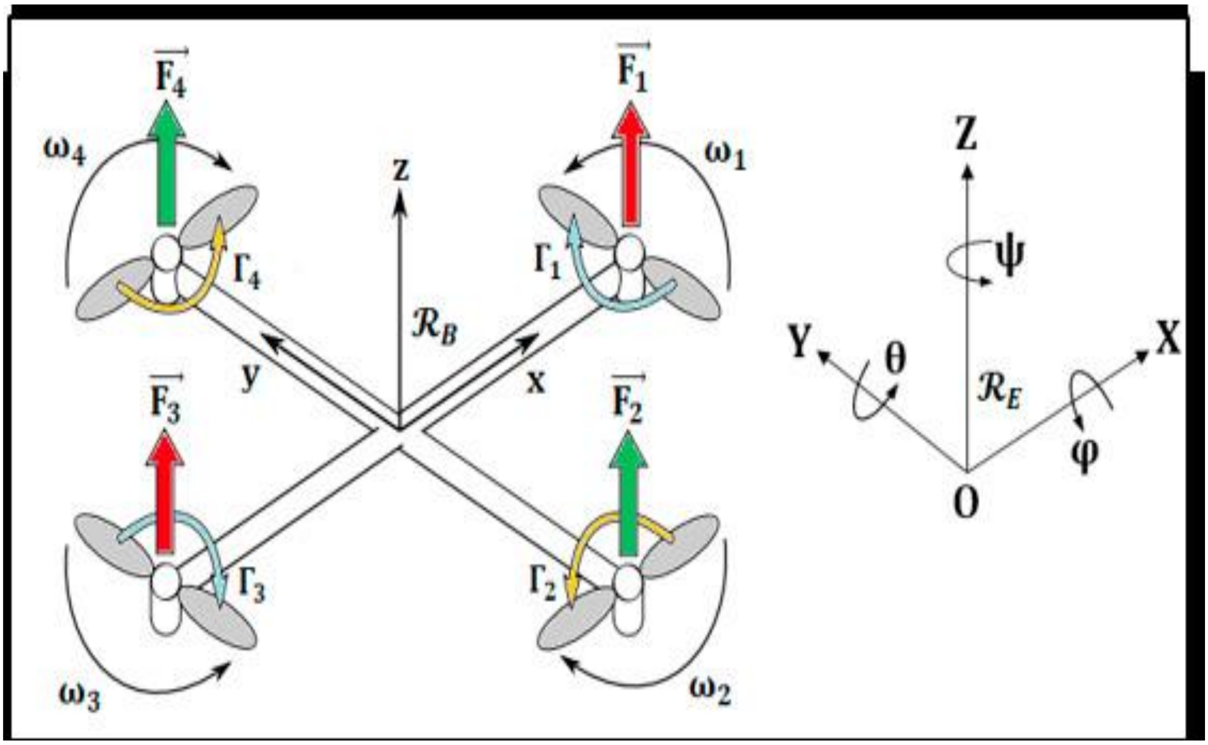

Figure 1.2: Euler Angles for a Quadrotor UAV [8]

To generate vertical upwards motion, the speed of the four propellers is increased together whereas the speed is decreased to generate vertical downwards motion. To produce roll rotation coupled with motion along the y-axis, the second and fourth propellers speeds are changed while for the pitch rotation coupled with motion along the x-axis, it is the first and third propellers speeds that need to be changed. One problem with the quadrotor configuration is that to produce yaw rotation, one need to have a difference in the opposite torque produced by each propeller pair. For instance, for a positive yaw rotation, the speed of the two clockwise turning rotors need to be increased while the speed of the two counterclockwise turning rotors need to be decreased.

Quadrotor Unmanned Aerial Vehicles (UAVs), or drones, have become increasingly popular due to their wide range of applications, from aerial photography to military operations. However, controlling a quadrotor is quite challenging because it is inherently unstable and highly sensitive to external factors like wind or sudden movements. To ensure smooth and stable flight, different control strategies are used [4].
This thesis focuses on three common types of controllers for quadrotors: Proportional-Derivative (PD), and Sliding Mode Control (SMC). Each of these controllers has its own advantages and limitations when it comes to stabilizing and guiding a quadrotor.

- **PD Controller**: This controller adjusts the quadrotor's movement based on its current position and rate of change, offering a simple but effective control strategy.
- **SMC Controller**: The Sliding Mode Controller is a more advanced approach that ensures high performance even in the presence of uncertainties and disturbances, making it robust for challenging conditions.

The goal of this study is to compare these three control methods, examining how well each performs in controlling the flight of a quadrotor UAV. By understanding their strengths and weaknesses, we can determine which controller offers the best solution for stable and reliable flight in various conditions.

Chapter 2

Literature Survey

In recent years, the control of quadrotor Unmanned Aerial Vehicles (UAVs) has attracted a significant amount of research interest due to their applications in both civilian and military fields. As quadrotors are inherently unstable and underactuated systems, selecting an appropriate control strategy is crucial for ensuring stable and efficient flight. A wide range of control methods have been developed and studied, with Proportional-Derivative (PD), Proportional-Integral-Derivative (PID), and Sliding Mode Control (SMC) being some of the most commonly used approaches. This literature survey reviews key research studies related to these control strategies for quadrotor UAVs.

PD Controllers:

The PD controller is one of the simplest forms of control techniques used for quadrotor UAVs. It adjusts the system's response based on the error (the difference between the desired and actual state) and its rate of change. Researchers have widely implemented PD control due to its simplicity and ability to achieve basic stability in quadrotor flight. Bouabdallah et al. (2004) demonstrated the effectiveness of PD controllers for attitude stabilization in quadrotors. Their results showed that PD controllers can provide good performance in maintaining stability during hovering, but may not be sufficient for more complex maneuvers or when exposed to external disturbances.

Gillula et al. (2010) further explored PD control for quadrotors in aggressive flight scenarios. They showed that while PD controllers can handle simple control tasks, their performance significantly degrades in highly dynamic environments or when more precise control is required. As a result, PD controllers are typically used in combination with other control strategies or for less demanding tasks like maintaining a quadrotor's orientation in hover mode.

PID Controllers:

PID controllers extend the basic PD controller by adding an integral component that accounts for accumulated errors over time. This integral term helps reduce steady-state errors, improving long-term accuracy. PID controllers are the most commonly used linear controllers for quadrotors because they offer a good balance between simplicity and performance.

In the work by Hoffman et al. (2007), a PID controller was used to stabilize the altitude and attitude of a quadrotor UAV. Their experiments showed that PID control could effectively handle small disturbances and maintain accurate positioning during flight. However, one of the challenges noted was tuning the gains for the proportional, integral, and derivative terms to optimize the controller's performance under different conditions. Li and Li (2011) applied a self-tuning PID controller for a quadrotor, using adaptive methods to adjust the PID parameters in real-time. Their results indicated that self-tuning PID could improve flight performance in dynamic environments, providing more adaptability to changing conditions such as wind or payload shifts.

While PID control is widely used, it has limitations when dealing with strong external disturbances or highly nonlinear behavior, which are common in quadrotor dynamics. Therefore, researchers have explored more advanced techniques, like nonlinear and robust controllers, to overcome these issues.

SMC Controllers:

Sliding Mode Control (SMC) is a robust nonlinear control strategy that has been increasingly applied to quadrotor UAVs because of its ability to handle uncertainties and disturbances. SMC works by forcing the system's trajectory to slide along a predefined surface, which makes the control robust to model inaccuracies and external disturbances.Madani and Benallegue (2006) successfully applied SMC to stabilize the attitude of a quadrotor. Their results showed that SMC could effectively maintain stability even in the presence of model uncertainties and external forces like wind gusts. However, they also highlighted a common issue with SMC—chattering. Chattering refers to the high-frequency oscillations that can occur around the sliding surface, which can negatively affect control performance and cause mechanical wear on the system.

To address this, González et al. (2009) proposed a modified SMC design that reduced chattering by applying a smoothing function near the sliding surface. Their experiments demonstrated improved performance in reducing chattering while maintaining the robustness of the SMC controller. Other studies, such as those by Lee et al. (2013), have combined SMC with other control strategies, such as adaptive control or backstepping, to further enhance performance and address its limitations.

Comparative Studies:

Several researchers have conducted comparative studies of PD, PID, and SMC controllers to evaluate their relative performance in controlling quadrotor UAVs. For instance, Bouabdallah and Siegwart (2007) compared the performance of PID and SMC controllers in stabilizing a quadrotor in the presence of external disturbances. Their study showed that while PID controllers offer good performance for basic tasks and small disturbances, SMC controllers excel in environments with larger disturbances due to their robustness.

Nagaty et al. (2011) extended this comparison by including a gain-scheduled PID controller and demonstrated that combining gain-scheduling with PID control could improve performance in more dynamic flight conditions. They found that SMC was still the most robust option, particularly in uncertain environments, but required careful design to minimize chattering.

Conclusion:

In summary, the literature shows that PD, PID, and SMC controllers each have strengths and limitations when applied to quadrotor UAVs. PD controllers are simple and effective for basic stability but may not perform well in complex or dynamic environments. PID controllers are widely used due to their balance of simplicity and performance, but they may struggle with strong disturbances or nonlinearities. SMC controllers are more advanced and robust, especially in handling disturbances and model uncertainties, though they require careful design to avoid issues like chattering. This thesis will build on these findings by further exploring and comparing the performance of these controllers in controlling a quadrotor UAV under various conditions.

Chapter 3

Modelling Of Quadrotor

This chapter presents the derivation of kinematic and dynamic models for a quadrotor using the Newton-Euler approach, based on the following assumptions:

- The structure is rigid and symmetrical.
- The quadrotor's center of gravity aligns with the origin of the body-fixed frame.
- The propellers are rigid.
- Thrust and drag are proportional to the square of the propeller speed [1].

Following the development of the kinematic and dynamic models, the chapter will examine the aerodynamic effects on the quadrotor body, along with the rotor dynamics of its actuators. Finally, a state-space model for the quadrotor system will be formulated for use in the next modelling chapter.

From the section of coordinate system, The angle of−

Roll angle =ϕ

Pitch angle = θ

Yaw angle = ψ

And angular velocities are respectively $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$.

These are the 6 states of quadcopter that shows a relationship between the quadcopter and the earth co-ordinate system.

The next 6 states show a physical location within the earth fixed system and denoted as $-\ x\ ,y\ ,z$ and their velocities along axis is $-\ \dot{x}\ ,\dot{y}\ ,\dot{z}$.

So, the 12 states of quadcopter –

$X = \left[\phi \;\; \theta \;\; \psi \;\; \dot{\phi} \;\; \dot{\theta} \;\; \dot{\psi} \;\; x \;\; y \;\; z \;\; \dot{x} \;\; \dot{y} \;\; \dot{z}\right]$

3.1 Kinematic Model Of Quadrotor

To begin modeling the quadrotor, it's essential to define the coordinate frames used. **Figure 3.1** illustrates the Earth reference frame, with axes labeled X_e , Y_e and Z_e , and the body frame, with axes labeled X_b , Y_b , and Z_b. The Earth frame, as an inertial frame, is fixed at a specific ground point and follows the N-W-U convention, with axes oriented toward the North, West, and Upward directions, respectively. In contrast, the body frame is centered on the quadrotor itself, with the x-axis aligned with propeller 1, the y-axis aligned with propeller 2, and the z-axis pointing Upward.

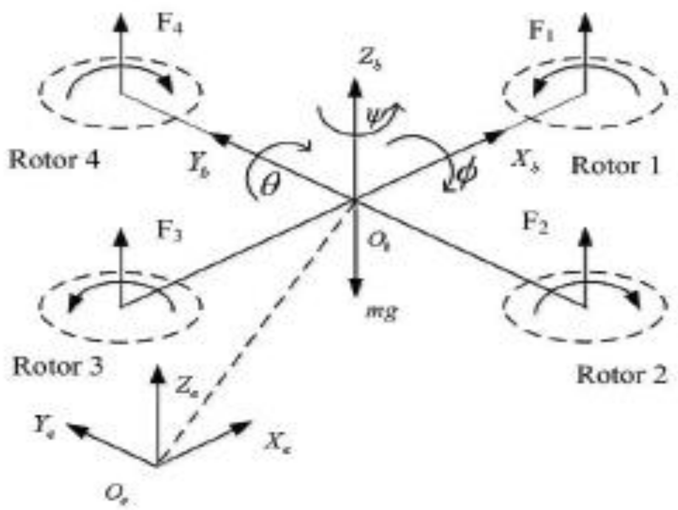

Figure 3.1: Quadrotor Reference Frames [11]

The absolute position of the quadrotor's center of mass, $r = [x\,y\,z]^T$, is defined by the distance between the Earth frame and the body frame. The quadrotor's orientation relative to the inertial frame is represented by a rotation R from the body frame [9]. This orientation is expressed using the roll, pitch, and yaw angles (ϕ, θ, and ψ), which correspond to rotations around the X , Y, and Z-axes, respectively. When these rotations occur in the order of roll, then pitch, and finally yaw, the rotation matrix R, derived from this sequence of principal rotations, is given by:

$$R = \begin{bmatrix} c\theta\, c\psi & s\phi\, s\theta\, c\psi - c\phi\, s\psi & c\phi\, s\theta\, c\psi + s\phi\, s\psi \\ c\theta\, s\psi & s\phi\, s\theta\, s\psi + c\phi\, c\psi & c\phi\, s\theta\, s\psi - s\phi\, c\psi \\ -s\theta & s\phi\, c\theta & c\phi\, c\theta \end{bmatrix} \tag{3.1}$$

where c and s denote cos and sin respectively.

The rotation matrix R is essential for developing the quadrotor's dynamics model, as it allows for conversions between states measured in different frames. Certain states, like thrust forces from the propellers, are recorded in the body frame, while others, such as gravitational forces and the quadrotor's position, are measured in the inertial frame. To connect these different sets of data, a transformation between the two frames is necessary.

Angular velocity information for the quadrotor is typically provided by an onboard Inertial Measurement Unit (IMU), which measures velocity within the body frame [9]. To link the Euler rates $\dot{\eta} = [\dot{\phi}\ \dot{\theta}\ \dot{\psi}]^T$ observed in the inertial frame with the body frame's angular rates ω = $[p\ q\ r\ ]^T$, an appropriate transformation is applied as follows:

$$\omega = R_r\dot{\eta} \tag{3.2}$$

Where,

$$R_r = \begin{bmatrix} 1 & 0 & -sin\theta \\ 0 & cos\phi & sin\phi cos\theta \\ 0 & -sin\phi & cos\phi cos\theta \end{bmatrix}$$

When the quadrotor is near its hover position, the small-angle approximation is applied, where $\cos\phi \approx 1, \cos\theta \approx 1,$ and $\sin\phi = \sin\theta = 0$. With these approximations, R_r simplifies to the identity matrix I.

3.2 Dynamic Model Of Quadrotor

The motion of the quadrotor can be separated into two main subsystems: the rotational subsystem, which includes roll, pitch, and yaw, and the translational subsystem, which includes altitude and the x and y positions. The rotational subsystem is fully actuated, whereas in the translational subsystem x and y position is underactuated.

We know force, $F_i \propto \omega_i^2 \Rightarrow F_i = b\ \omega_i^2$

And Moment of torque, $M_i \propto \omega_i^2 \Rightarrow M_i = d\ \omega_i^2$

where ω_i = angular velocity of i^{th} motor. b and d are thrust coefficient and drag torque coefficient.

- ## Moment Acting On The Quadrotor

$$M_X = (F_2 - F_4)\ l \qquad \text{(Roll moment)} \qquad (3.3)$$

$$M_y = (F_3 - F_1)\ l \qquad \text{(Pitch moment)} \qquad (3.4)$$

$$M_z = (-M_1 + M_2 - M_3 + M_4) \qquad \text{(yaw moment)} \qquad (3.5)$$

combination of each body frame axis gives us a matrix as shown below.

$$M_b = \begin{bmatrix} (F_2 - F_4)\ l \\ (F_3 - F_1)\ l \\ (-M_1 + M_2 - M_3 + M_4) \end{bmatrix} \qquad (3.6)$$

l = Rotor axis to copter center distance(m)

- ## Inertial Matrix:

The inertial matrix for a quadcopter is a diagonal matrix as shown below. The structure of the matrix is because quadcopters are built symmetrically with respect to the coordinate systems.

$$J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{3.7}$$

J_x , J_y , J_z are the moments of inertia about the principal axes on the body frame.

- ## Control Input Vector

The control input is from the controller and the input to a quadcopter is from the force generation by the motor, and for simplicity, Control Input Vector (U) can represent Force (F) or Moment (M) where,

$$U_1 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_3^2) \tag{3.8}$$

$$U_2 = b(\omega_2^2 - \omega_4^2) \tag{3.9}$$

$$U_3 = b(\omega_3^2 - \omega_1^2) \tag{3.10}$$

$$U_4 = d(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2) \tag{3.11}$$

Equations (3.8) to (3.11) can be arranged in a matrix,

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & b & 0 & -b \\ -b & 0 & b & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{3.12}$$

- U_1 represents the combined upward thrust from all four rotors, controlling the quadrotor's altitude and its vertical velocity (z and $\dot{z}$).
- U_2 is the thrust difference between rotors 2 and 4, responsible for roll rotation and its rate of change (ϕ and $\dot{\phi}$).
- U_3 denotes the thrust difference between rotors 1 and 3, which controls pitch rotation and its rate of change (θ and $\dot{\theta}$).
- U_4 captures the torque difference between the clockwise and counterclockwise rotors, producing the yaw rotation and its rate of change (ψ and $\dot{\psi}$).

This control vector U structure separates the quadrotor's rotational dynamics: U_1 directly adjusts altitude, U_2 controls the roll angle, U_3 governs the pitch angle, and U_4 sets the yaw angle. To derive rotor speeds from control inputs, an inverse relationship is necessary. This can be achieved by inverting the matrix in Equation (3.12)

$$\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4b} & 0 & -\frac{1}{2b} & -\frac{1}{4d} \\ \frac{1}{4b} & \frac{1}{2b} & 0 & \frac{1}{4d} \\ \frac{1}{4b} & 0 & \frac{1}{2b} & -\frac{1}{4d} \\ \frac{1}{4b} & -\frac{1}{2b} & 0 & \frac{1}{4d} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \tag{3.13}$$

By taking the square root, the rotor velocities can be determined from the control inputs as follows:

$$\omega_1 = \sqrt{\frac{1}{4b}U_1 - \frac{1}{2b}U_3 - \frac{1}{4d}U_4}$$

$$\omega_2 = \sqrt{\frac{1}{4b}U_1 + \frac{1}{2b}U_2 + \frac{1}{4d}U_4}$$

$$\omega_3 = \sqrt{\frac{1}{4b}U_1 + \frac{1}{2b}U_3 - \frac{1}{4d}U_4}$$

$$\omega_4 = \sqrt{\frac{1}{4b}U_1 - \frac{1}{2b}U_2 + \frac{1}{4d}U_4}$$

(3.14)

From equation 4, the moment acting on the quadcopter in its body frame can be represented as

$$M_b = \begin{bmatrix} U_{tx}\, l \\ U_{ty}\, l \\ U_{dz} \end{bmatrix} \tag{3.15}$$

- ## Drag Moments

Due to Air Friction, there is a drag moment M_a acting on Quadrotor body represented by,

$$M_a = A\dot{\eta} \tag{3.16}$$

Where A is aerodynamic rotation coefficient matrix and $\dot{\eta}$ is the Euler rates.

3.2.1 Rotational Equation Of Motion

The rotational part of the quadcopter is derived from the concept of rotational equation of motion and by using Newton-Euler method derived from the body frame of the quadcopter with a generalized formula as shown below [16].

$$M_b = (j\dot{\omega} + \omega \times j\omega + M_g) \tag{3.17}$$

j : Quadrotor inertia matrix.
ω : represents angular velocity.
M_g : gyroscopic moment generated due to its rotor inertial.
M_b : Moments acting on the quadcopter in its body frame.

The first two terms of Equation 3.13, $j\dot{\omega}$ *and* $\omega \times j\omega$, describe the rate at which angular momentum changes in the body frame. Meanwhile, M_g represents the gyroscopic moments arising from the inertia of the rotors, j_r . These gyroscopic moments are defined as $\omega\times[0\ 0\ j_r\omega_r]^T$. Thus, the rotational dynamics of the quadrotor's motion can be expressed as,

$$M_b = (j\dot{\omega} + \omega \times j\omega + \omega \times [0\ 0\ j_r\omega_r]^T) \tag{3.18}$$

Where,

- j_r : the rotors' inertia
- ω_r is the relative speed of the rotors, defined as $\omega_r = (\omega_1 - \omega_2 + \omega_3 - \omega_4)$

The rotational equations of motion are derived in the body frame rather than the inertial frame to keep the inertia matrix constant over time.

Now According to Rotational equation of motion expressed by equation (3.14) can be rewrite as,

$$(\mathrm{j}\dot{\omega} + \omega \times \mathrm{j}\omega + \omega \times [0\ 0\ \mathrm{j_r}\omega_r]^T) = M_b - M_a \tag{3.19}$$

From Equation (3.15) we get,

$$\begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ j_r\omega_r \end{bmatrix} = \begin{bmatrix} lU_2 \\ lu_3 \\ U_4 \end{bmatrix} - \begin{bmatrix} A_\phi & 0 & 0 \\ 0 & A_\theta & 0 \\ 0 & 0 & A_\psi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{3.20}$$

Where , A_ϕ , A_θ *and* A_ψ are aerodynamic rotation coefficient.

When the matrix is rewritten to have its angular acceleration, we would have –

$$\ddot{\phi} = \left(\frac{j_y - j_z}{j_x}\right)\dot{\theta}\dot{\psi} + \left(\frac{j_r\omega_r}{j_x}\right)\dot{\theta} + \left(\frac{l}{j_x}\right)U_2 - \left(\frac{A_\phi}{j_x}\right)\dot{\phi} \tag{3.21}$$

$$\ddot{\theta} = \left(\frac{j_z - j_x}{j_y}\right)\dot{\phi}\dot{\psi} - \left(\frac{j_r\omega_r}{j_y}\right)\dot{\phi} + \left(\frac{l}{j_y}\right)U_3 - \left(\frac{A_\theta}{j_y}\right)\dot{\theta} \tag{3.22}$$

$$\ddot{\psi} = \left(\frac{j_x - j_y}{j_z}\right)\dot{\phi}\dot{\theta} + \left(\frac{1}{j_z}\right)U_4 - \left(\frac{A_\psi}{j_z}\right)\dot{\phi} \tag{3.23}$$

3.2.2 Translational Equation of Motion

The translation subsystem is based on translational equation of motion and it is based on newton second law which is derived from the earth inertial frame and it is presented in the format below.

$$m\,\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R\,F_{\mathrm{B}} \tag{3.24}$$

Where $r = [x\ y\ z]^T$ is quadrotor distance from inertial frame.

m = mass of quadrotor

g = gravitational acceleration

F_B = Non-gravitational forces acting on the quadrotor in the body frame

R = Rotational matrix (to convert Body frame to Inertial/Earth frame)

Due to air friction drag force, $F_a = A\,\dot{r}$ (A = aerodynamic translational coefficient) equation 16 can be written as ,

$$\mathrm{m}\,\ddot{\mathrm{r}} = \begin{bmatrix} 0 \\ 0 \\ -\mathrm{mg} \end{bmatrix} + \mathrm{R}\,\mathrm{F_b} - \mathrm{F_a} \tag{3.25}$$

By substitution and simplification, we would derive the acceleration along the X, Y, and Z axis.

$$\mathrm{F_b} = \begin{bmatrix} 0 \\ 0 \\ \mathrm{u_1} \end{bmatrix} \tag{3.26}$$

By substituting equation (18) in equation (17) we have,

$$\mathrm{m}\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\mathrm{mg} \end{bmatrix} + \begin{bmatrix} c\theta\, c\psi & s\phi\, s\theta\, c\psi - c\phi\, s\psi & c\phi\, s\theta\, c\psi + s\phi\, s\psi \\ c\theta\, s\psi & s\phi\, s\theta\, s\psi + c\phi\, c\psi & c\phi\, s\theta\, s\psi - s\phi\, c\psi \\ -s\theta & s\phi\, c\theta & c\phi\, c\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \mathrm{u_1} \end{bmatrix} - \begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{3.27}$$

When the matrix is rewritten to have its acceleration we would have,

$$\ddot{x} = \frac{u_1}{m} \left(c\phi \, s\theta \, c\psi + s\phi \, s\psi \right) - A_x \frac{\dot{x}}{m} \tag{3.28}$$

$$\ddot{y} = \frac{u_1}{m} \left(c\phi \, s\theta \, s\psi - s\phi \, c\psi \right) - A_y \frac{\dot{y}}{m} \tag{3.29}$$

$$\ddot{z} = \frac{u_1}{m} \left(c\phi \, c\theta \right) - g - A_Z \frac{\dot{z}}{m} \tag{3.30}$$

Parameter	**Symbol**	**Value**
Quadrotor Mass	m	0.650 kg
Moment Arm Length	l	0.23 m
Gravitational Acceleration	g	9.81 m/s^2
Rotor Inertia	j_r	6×10^{-5} kg. m^2
Quadrotor Mass Moment Of Inertia	j_x, j_y	7.5×10^{-3} kg. m^2
	j_z	1.3×10^{-2} kg. m^2
Thrust Coefficient	b	3.13×10^{-5} N. s^2
Drag Coefficient	d	7.5×10^{-7} N. m. s^2
Aerodynamic Translational coefficient	A_x, A_y	0.1 N/m/s
	A_z	0.15 N/m/s
Aerodynamic Rotation coefficient	A_ϕ, A_θ	0.1 N/rad/s
	A_ψ	0.15 N/rad/s

Table 3.1: Parameters Of Quadrotor [10]

Chapter 4

Open-loop Simulation Of Quadrotor

The output behavior of the mathematical model relies on the input values. Although angular velocities are included as variables in the model, the inputs are set as the RPM of the propeller rotors ($N = \frac{30\omega}{\pi}$), which is a more intuitive method for controlling propeller rotation. The outputs are selected as the position coordinates and the quadrotor's orientation.

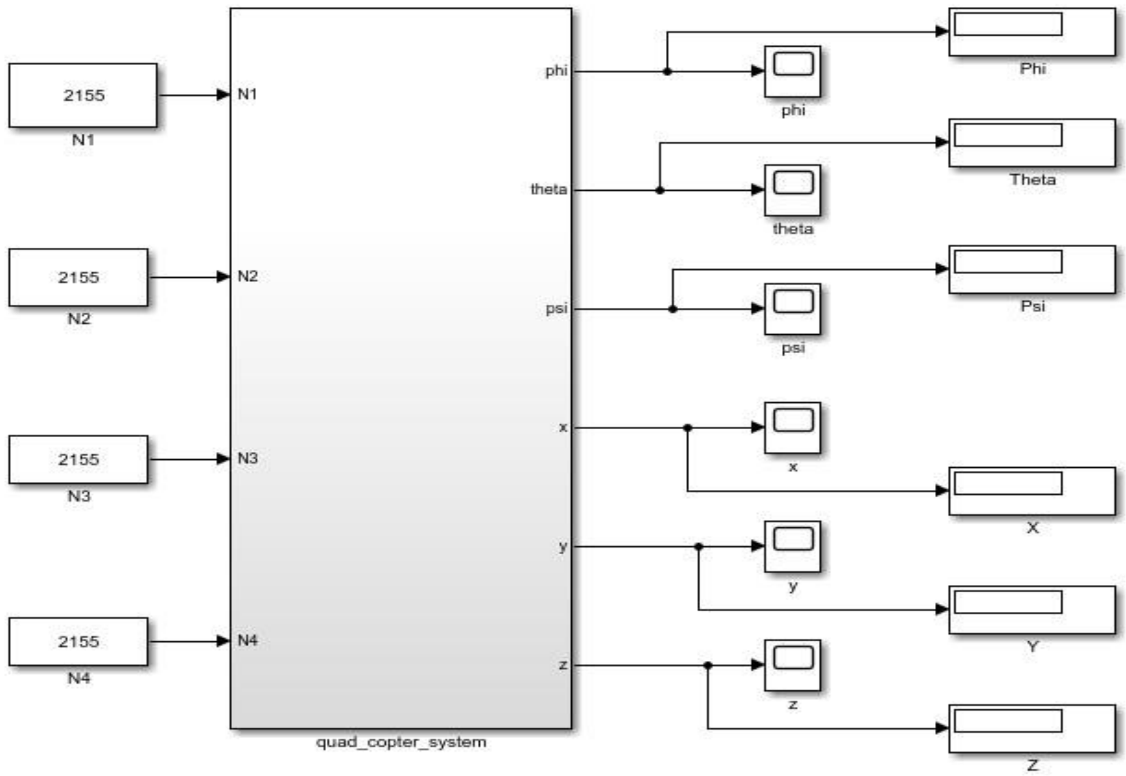

Figure 4.1: Open-loop Simulink Model Of Quadrotor

case	RPM of motors of quadrotor			
	N_1	N_2	N_3	N_4
Hover	2155	2155	2155	2155
Roll	2155	2156	2155	2154
Pitch	2154	2155	2156	2155
Yaw	2154	2156	2154	2156

Table 4.1: Quadrotor speed for different condition

In the first scenario, hovering mode is simulated (Figure 4.2(a)), where all four propellers operate at the same RPM. Here, U_1 matches the gravitational force, while the remaining control variables are set to zero. As a result, the quadcopter's position and orientation remain unchanged, indicating it is in hover mode.

In the second scenario, the quadcopter's response to a change in U_2 is observed . The change in U_2 produces a shift in the roll angle ϕ (Figure 4.2(b))

The third scenario examines the effect of a change in U_3. Like U_2, a small value of U_3 causes a noticeable effect on the quadcopter’s orientation. The change in U_3 results in an alteration of the pitch angle θ (Figure 4.2(d)).

Finally, the fourth scenario evaluates the effect of a change in U_4.

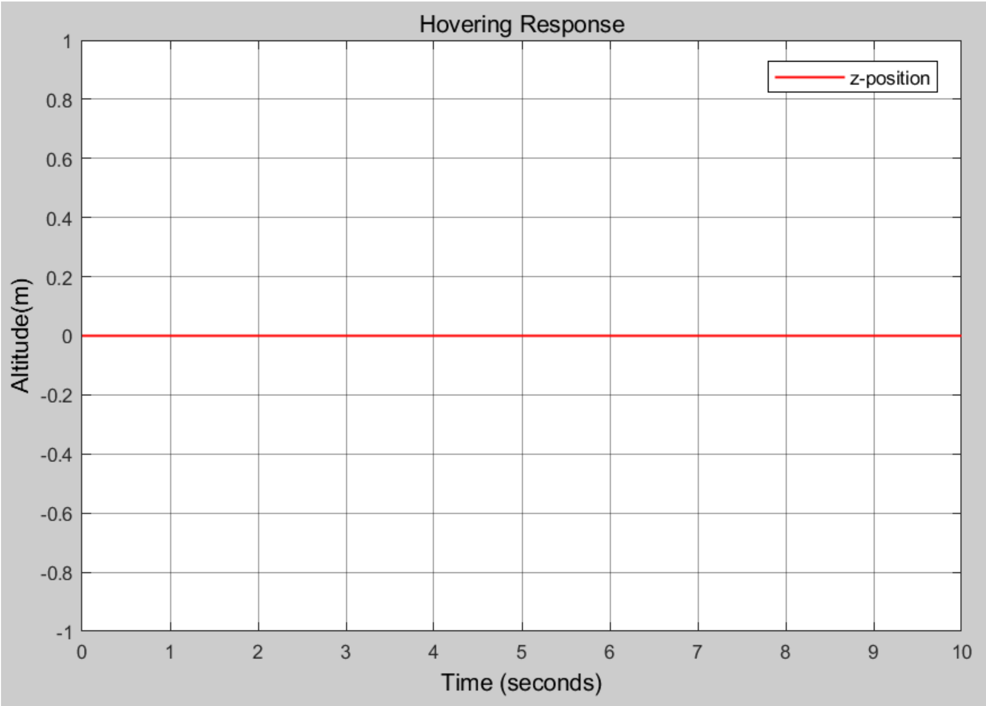

Figure 4.2(a): Hovering Response

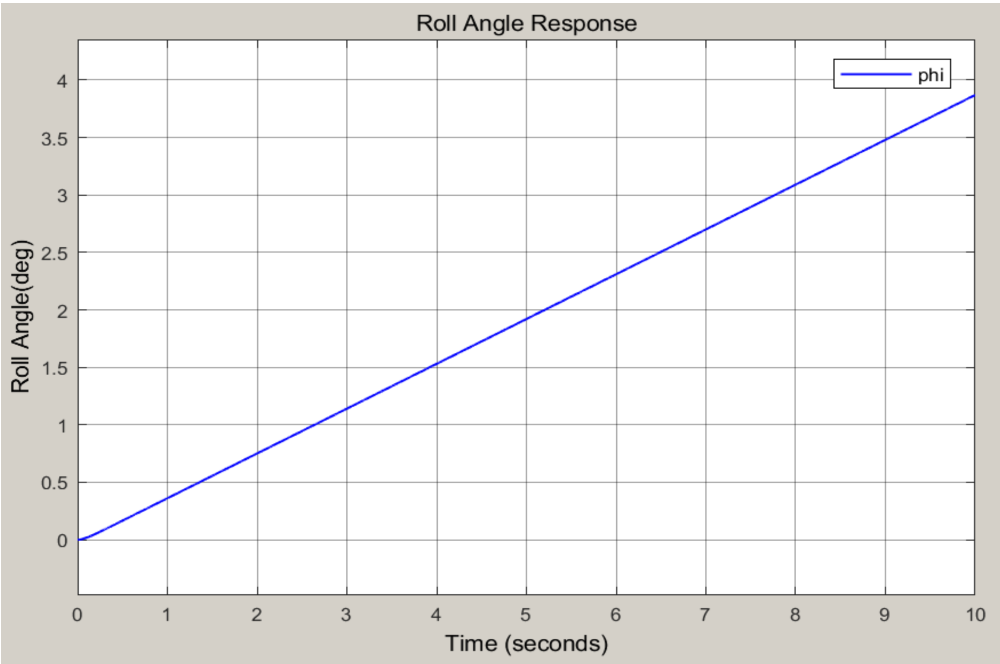

Figure 4.2(b): Roll Angle Response

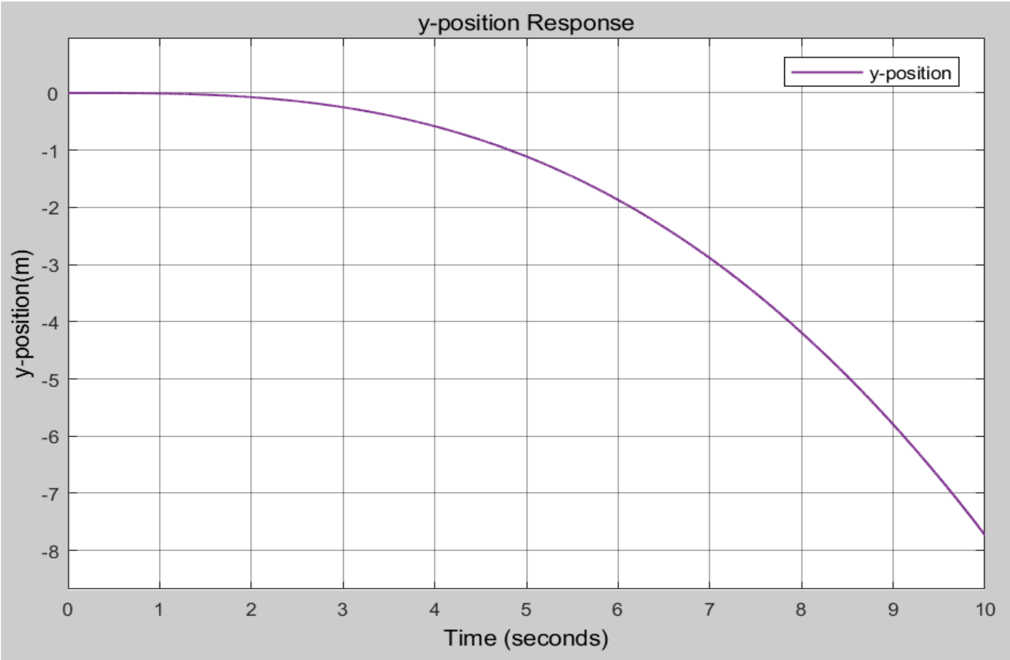

Figure 4.2(c): y-position Response due to Roll Moment

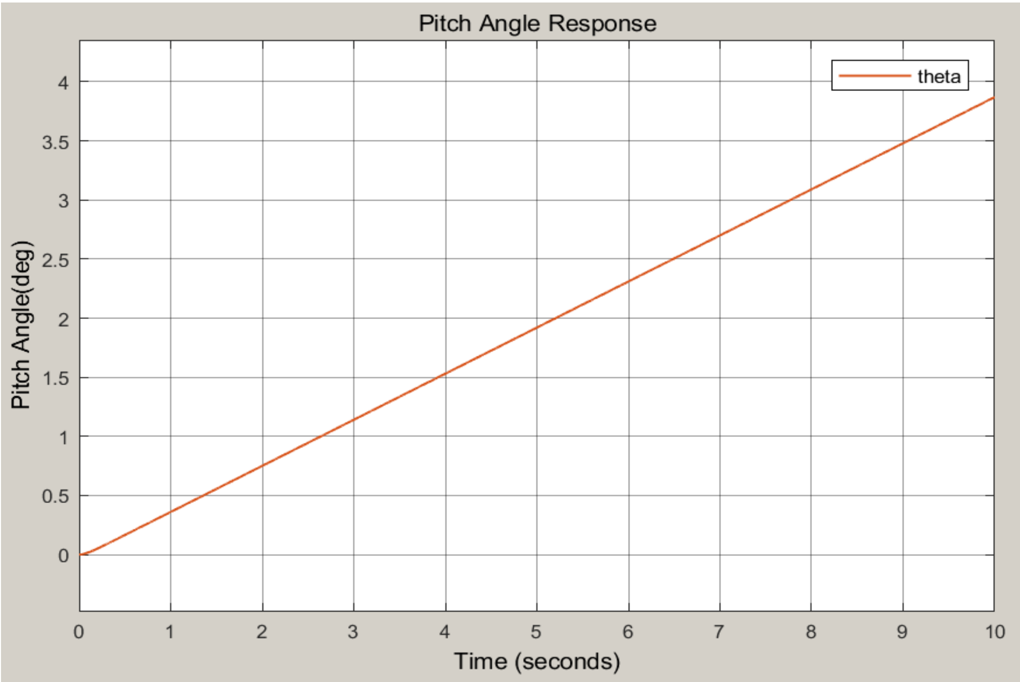

Figure 4.2(d): Pitch Angle Response

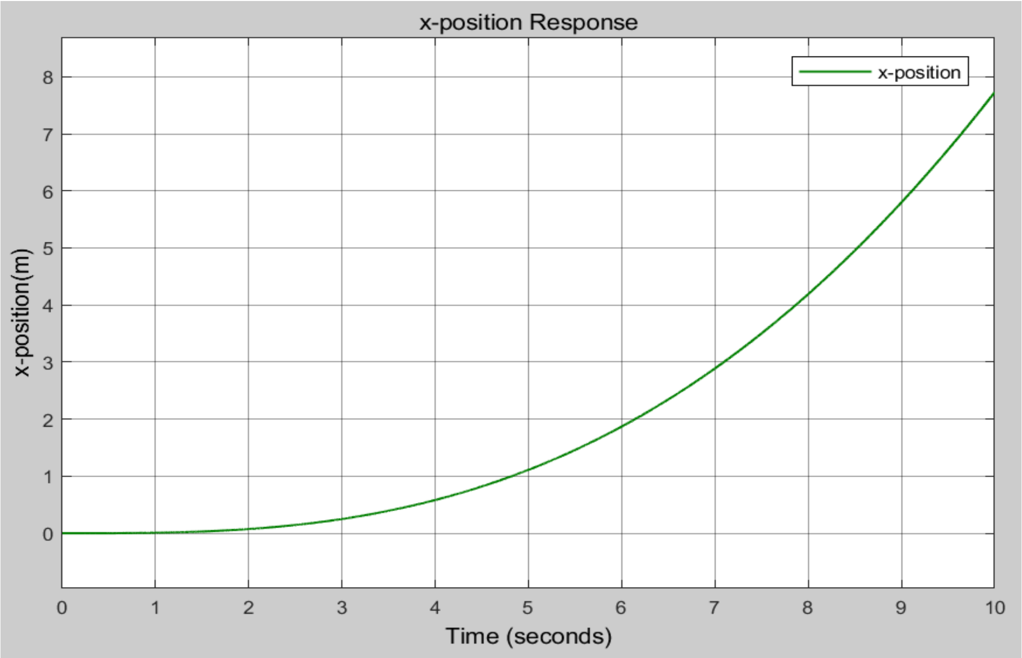

Figure 4.2(e): x-position Response due to Pitch Moment

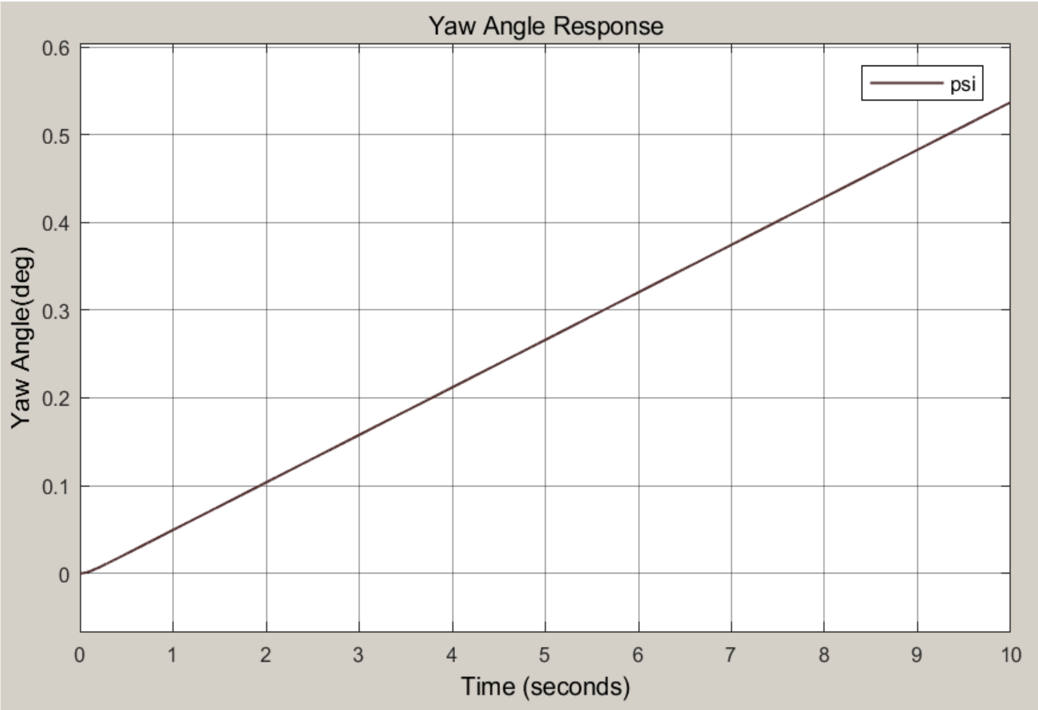

Figure 4.2(f): Yaw Angle Response

Chapter 5

Control Of Quadrotor

5.1 PD Controller

Once the mathematical model of the quadrotor and its open-loop simulation were validated, a PD controller was designed. This controller provides the necessary control inputs for the quadrotor. Figure 5.1 displays the block diagram for the PD controller.

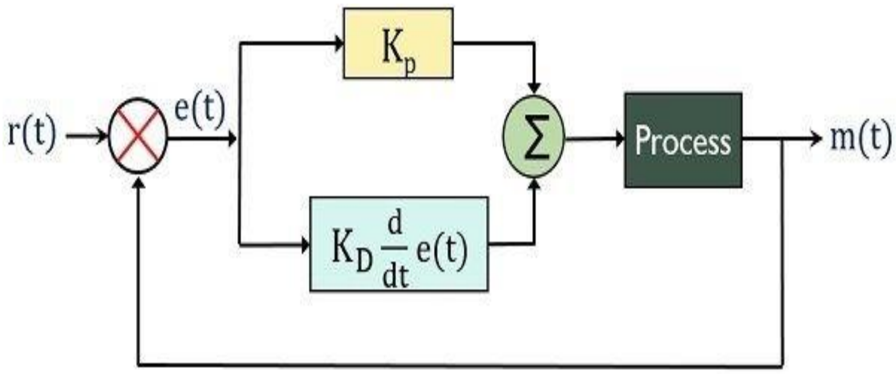

Figure 5.1: PD Controller Block Diagram [15]

5.1.1 Altitude Control

A PD controller is designed to manage the quadrotor's altitude. It produces the acceleration $\ddot{z}_d$.

$$\ddot{z}_d = k_p(z_d - z) + k_d(\dot{z_d} - \dot{z}) \tag{5.1}$$

Now from Equation (3.30) we get the control input U_1, which governs the altitude of the quadrotor according to Equation (3.8). The resulting control law is as follows.

$$U_1 = \left[\frac{m(\ddot{z_d} + g) + A_z\dot{z}}{\cos\phi\cos\theta}\right] \tag{5.2}$$

where

k_p Proportional gain

z_d Desired altitude

k_d Derivative gain

$\dot{z_d}$ Desired altitude rate of change

5.1.2 Attitude Control

5.1.2.1 Roll Controller

An additional PD controller is designed to regulate the roll angle ϕ of the quadrotor. The resulting control law provides the input U_2, which governs the roll angle as follows.

$$U_2 = k_p(\phi_d - \phi) + k_d(\dot{\phi_d} - \dot{\phi}) \quad (5.3)$$

where

k_p Proportional gain

ϕ_d Desired roll angle

k_d Derivative gain

$\dot{\phi_d}$ Desired roll angle rate of change

5.1.2.2 Pitch Controller

A PD controller is designed to manage the pitch angle θ of the quadrotor. The resulting control law produces the input U_3, which regulates the pitch angle as follows.

$$U_3 = k_p(\theta_d - \theta) + k_d(\dot{\theta_d} - \dot{\theta}) \quad (5.4)$$

where

k_p Proportional gain

θ_d Desired pitch angle

k_d Derivative gain

$\dot{\theta_d}$ Desired pitch angle rate of change

5.1.2.3 Yaw Controller

In a manner similar to the pitch and roll controllers, a yaw controller was created to produce the control input U_4 according to the following control law.

$$U_4 = k_p(\psi_d - \psi) + k_d(\dot{\psi}_d - \dot{\psi}) \quad (5.5)$$

where

k_p Proportional gain

ψ_d Desired yaw angle

k_d Derivative gain

$\dot{\psi}_d$ Desired yaw angle rate of change

5.2 PD Controller Simulation

For the altitude controller, optimized PD controller with a desired altitude z_d of 2 m. As no steady state error observed, so PID controller with gain k_i = 0 used. Executing the closed-loop simulation with the obtained gains yielded a settling time of 2.320 seconds.

Likewise, gains for the attitude, heading, and position controllers were optimized. Table 5.1 provides a summary of these optimized control gains along with system performance in terms of settling time.

	Desired Value	k_p	k_d	Settling Time
Altitude(z)	2 m	4.7	3.7	2.320 seconds
Roll(ϕ) and Pitch(θ)	5°	4.5	0.5	1.581 seconds
Yaw(ψ)	5°	6	1	1.703 seconds

Table 5.1: PD Controller Results

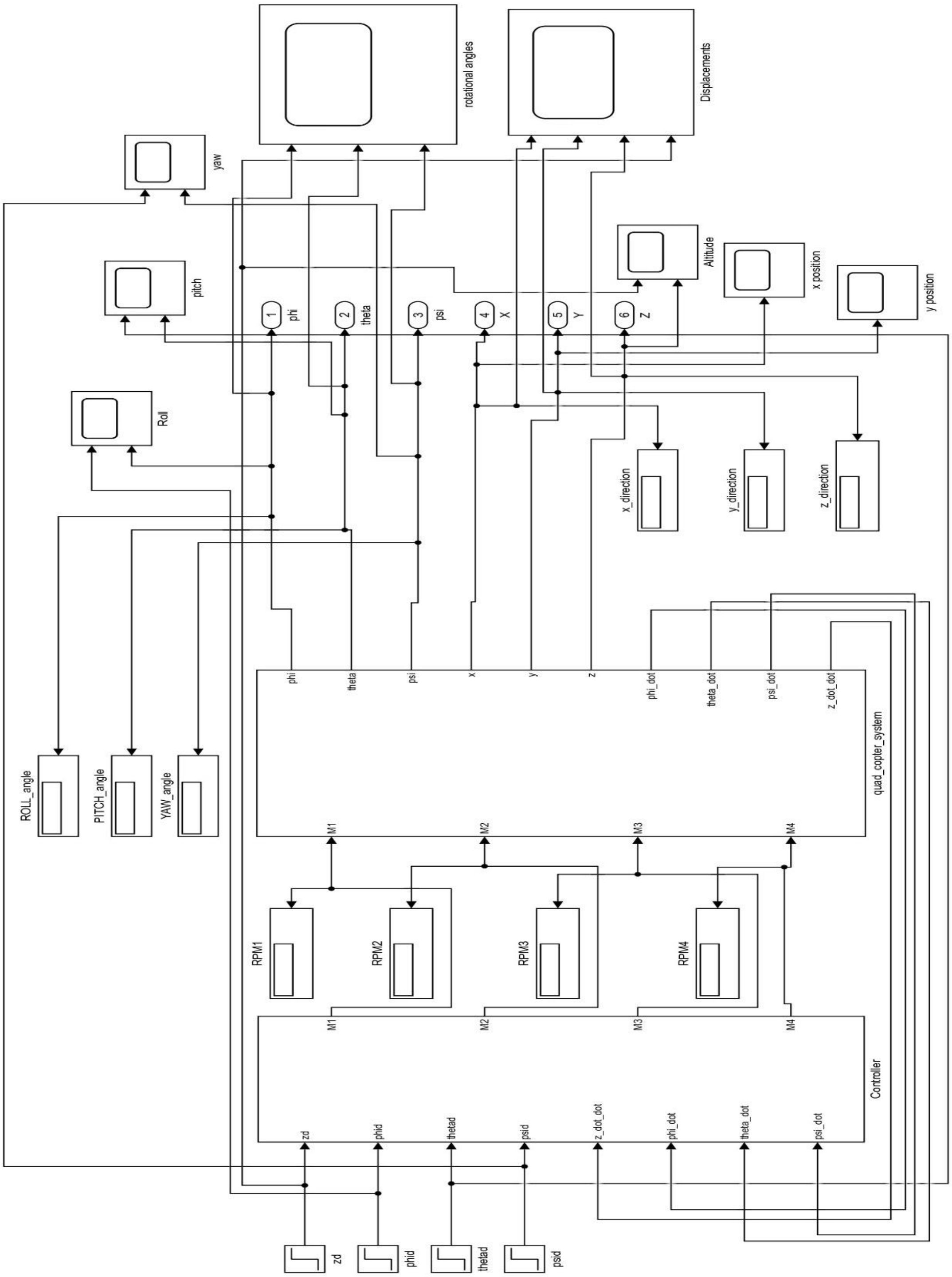

Figure 5.2: Quadrotor Model With PD Controller

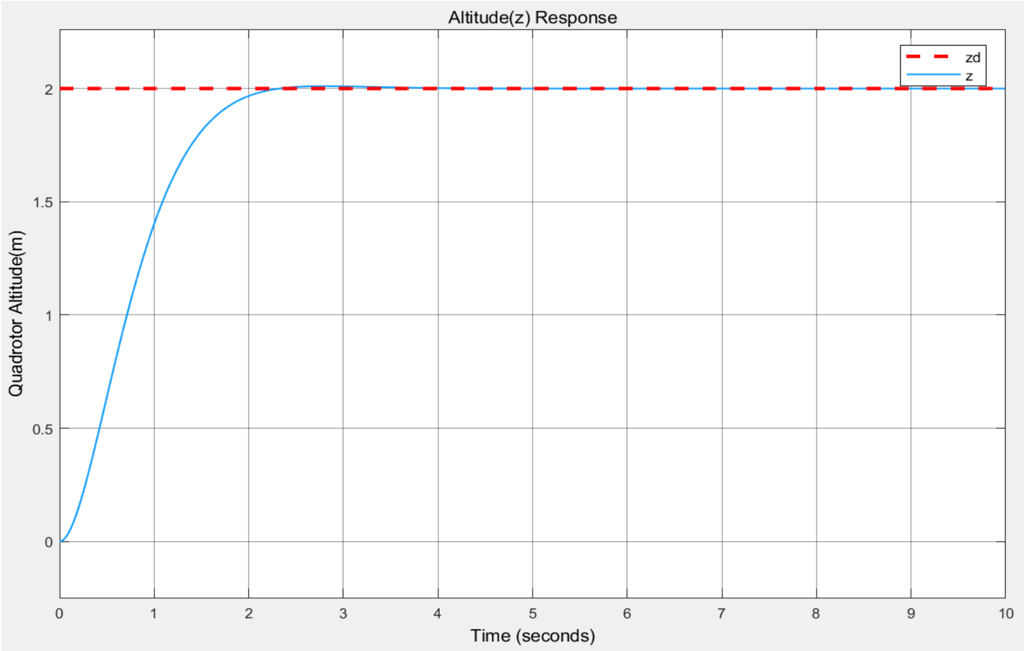

Figure 5.3(a): PD Controller Simulation Result For Altitude Response

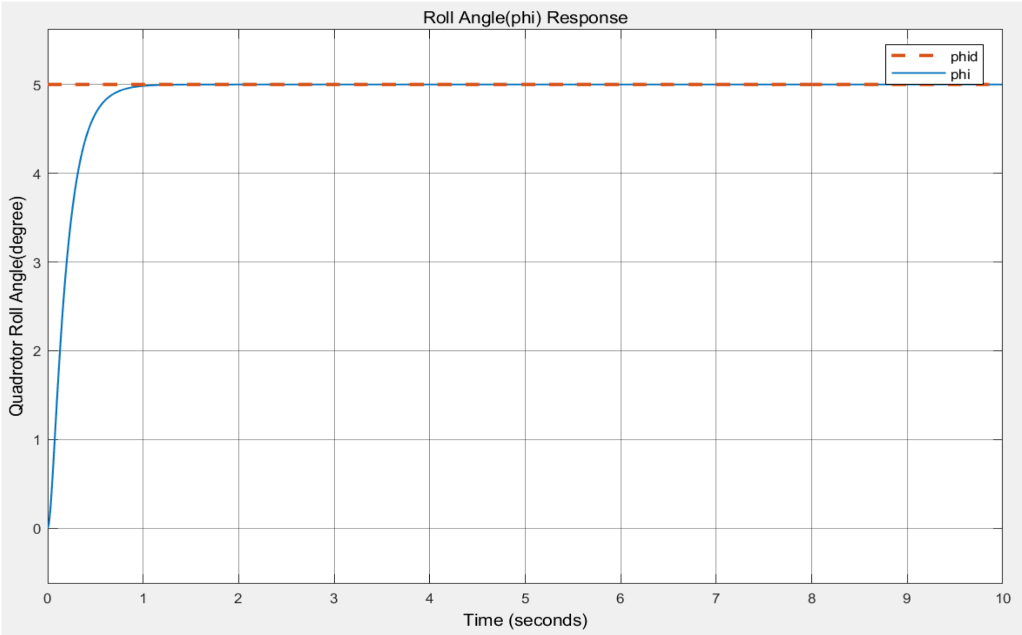

Figure 5.3(b): PD Controller Simulation Result For Roll Angle Response

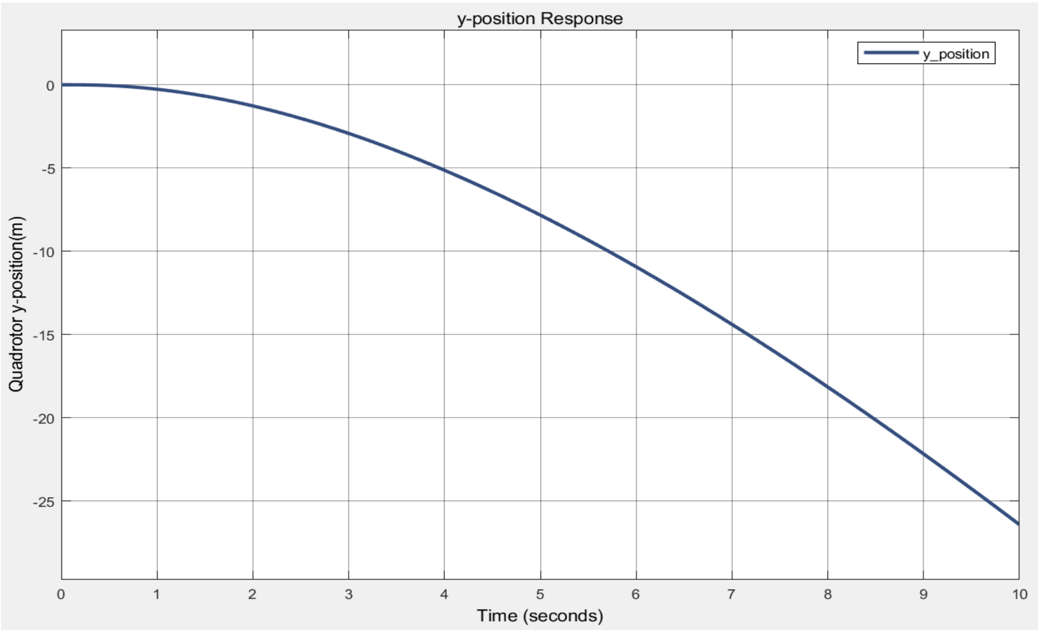

Figure 5.3(c): PD Controller Simulation Result For y-position Control

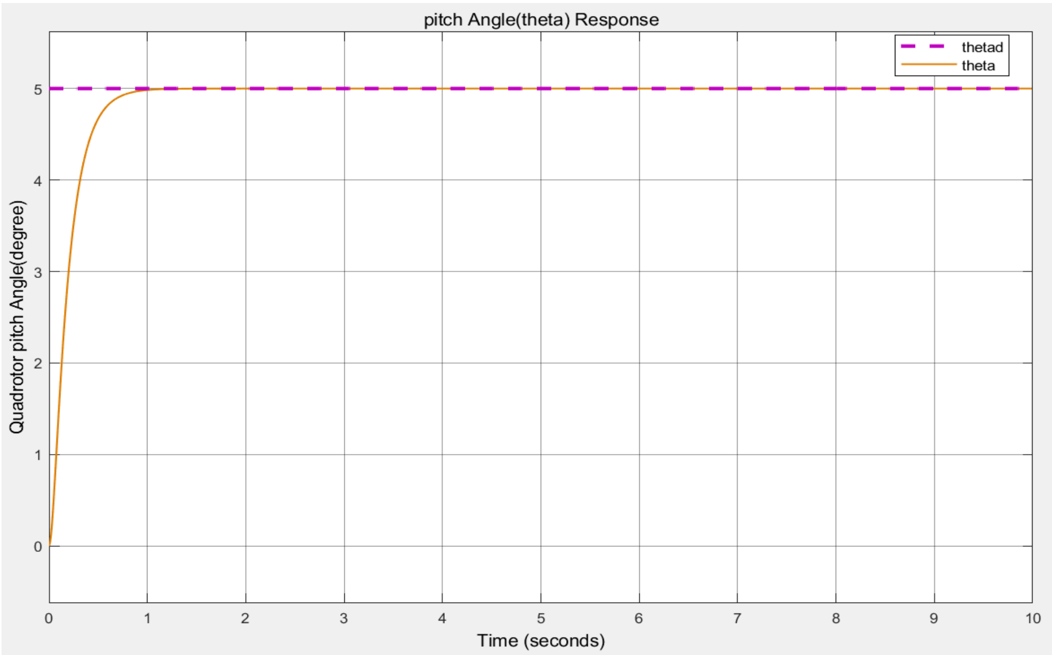

Figure 5.3(d): PD Controller Simulation Result For Pitch Angle Response

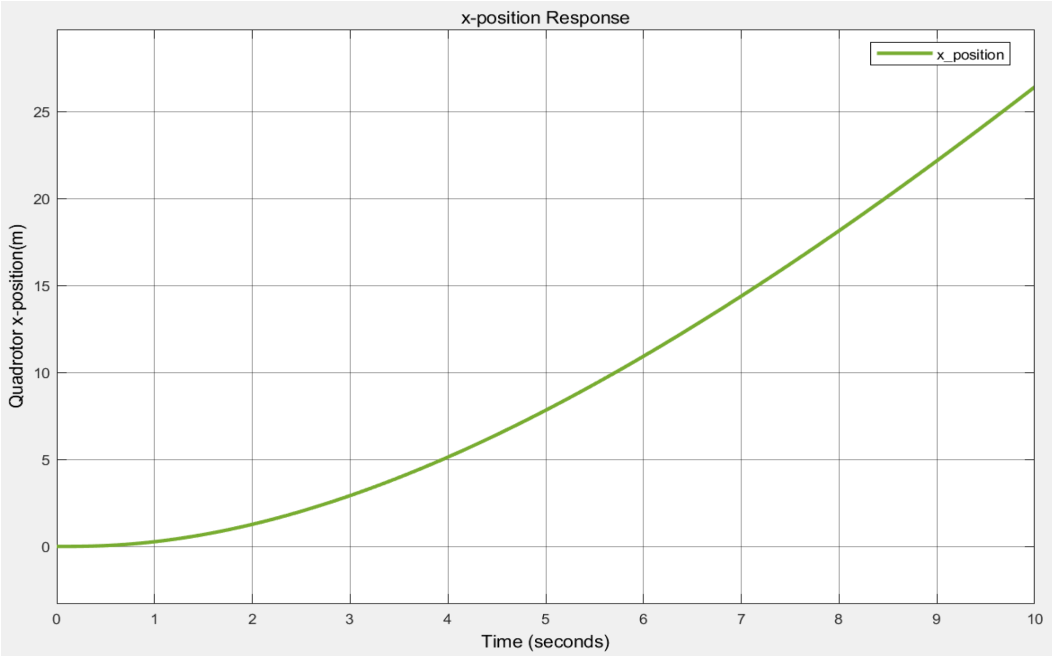

Figure 5.3(e): PD Controller Simulation Result For x-position Control

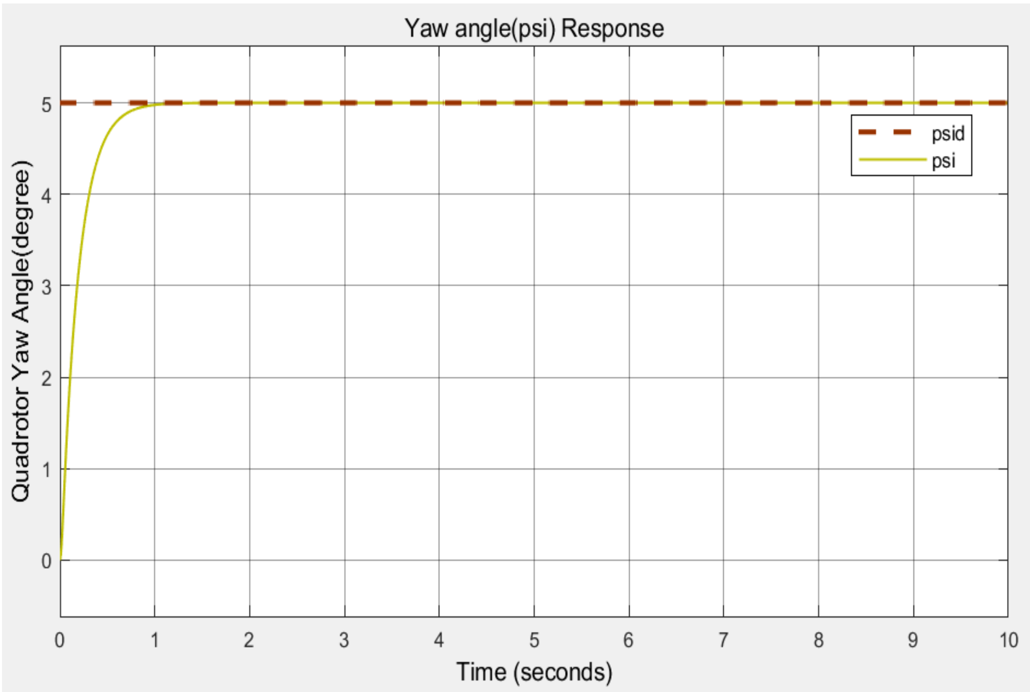

Figure 5.3(f): PD Controller Simulation Result For Yaw Angle Response

5.3 Sliding Mode Controller

Given the nonlinear nature of the quadrotor system, we proposed managing its states using a nonlinear Sliding Mode Controller (SMC).

5.3.1 Introduction to SMC

A Sliding Mode Controller (SMC) is a type of Variable Structure Control (VSC) that uses a high-speed switching control law to keep state trajectories on a predefined surface within the state space and guides them to follow it [14]. As shown in Equation (5.6), the SMC control law consists of two components: an equivalent control term and a corrective control term. To establish the sliding surface, the corrective control compensates for any deviations in state trajectories. Meanwhile, to ensure that trajectories stay on the sliding surface, the control law maintains the time derivative of the surface at zero.

$$\mathrm{U(t)} = \mathrm{U_c}(t) + \mathrm{U_{eq}}(t) \tag{5.6}$$

Where,

$\mathrm{U(t)}$: Control Law

$\mathrm{U_c}(t)$: Corrective Control

$\mathrm{U_{eq}}(t)$: Equivalent Control

A block diagram showing the SMC is shown in Figure 5.3

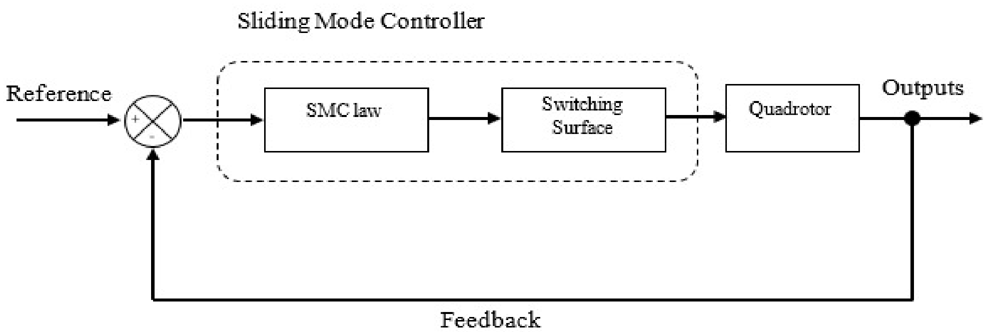

Figure 5.4: SMC Block [13]

5.3.2 Altitude Control

The altitude error is defined as:

$$e = z_d - z \tag{5.7}$$

where, z_d represents the desired altitude of the quadrotor. The sliding surface is defined as:

$$s = c_1 e + \dot{e} \tag{5.8}$$

where c_1 is a constant that must be greater than zero. This is a common form for the sliding surface in tracking problems. Taking the derivative of the sliding surface in Equation (5.8) and substituting Equation (5.7) gives:

$$\dot{s} = c_1 \dot{e} + \ddot{e}$$
$$\dot{s} = c_1(\dot{z_d} - \dot{z}) + (\ddot{z_d} - \ddot{z}) \tag{5.9}$$

A Lyapunov function is then defined to be,

$$V(e, s) = \frac{1}{2}(e^2 + s^2) \tag{5.10}$$

An exponential reaching law is proposed for the sliding mode controller using the Lyapunov function, as shown below.

$$\dot{s} = -k_1 \operatorname{sgn}(s) - k_2 s \tag{5.11}$$

Where,

$$\operatorname{sgn}(s) = \begin{cases} -1 & \text{if } s < 0; \\ 1 & \text{if } s > 0; \end{cases}$$

Here, k_1 and k_2 are design constants. To ensure the sliding mode condition $s\dot{s} < 0$, constraints must be imposed on k_1 and k_2, requiring that $k_1>0$ and $k_2>0$. By setting the proposed reaching law in Equation (5.11) equal to the derivative of the sliding surface from Equation (5.9) and substituting $\ddot{z}$ using its expression from Equation (3.30), the control input U_1 is determined as follows.

$$U_1 = \frac{1}{\cos\phi\cos\theta}[m(k_1 sgn(s) + k_2 s + c_1(\dot{z_d} - \dot{z}) + \ddot{z_d} + g) + A_z\dot{z}] \qquad (5.12)$$

5.3.3 Attitude Control

The sliding mode attitude controller is developed following the method used by Bouabdallah and Siegwart [16].

5.3.3.1 Roll Controller

The SMC is implemented to follow a reference trajectory for the roll angle. The roll error is defined as follows.

$$e = \phi_d - \phi \qquad (5.13)$$

The sliding surface is defined as,

$$s = c_1 e + \dot{e} \qquad (5.14)$$

In this formulation, c_1 represents a constant that must be positive. This structure is commonly used for the sliding surface in tracking control problems. By substituting Equation (5.13) into the derivative of the sliding surface defined in Equation (5.15), we obtain the following expression,

$$\dot{s} = c_1\dot{e} + \ddot{e}$$

$$\dot{s} = c_1(\dot{\phi_d} - \dot{\phi}) + (\ddot{\phi_d} - \ddot{\phi}) \qquad (5.15)$$

A Lyapunov function is then defined to be,

$$V(e, s) = \frac{1}{2}(e^2 + s^2) \qquad (5.16)$$

An exponential reaching law is proposed for the sliding mode controller using the Lyapunov function, as shown below.

$$\dot{s} = -k_1 \mathrm{sgn}(s) - k_2 s \tag{5.17}$$

Where,

$$\mathrm{sgn}(s) = \begin{cases} -1 & \text{if } s < 0; \\ 1 & \text{if } s > 0; \end{cases}$$

Here, k_1 and k_2 are design constants. To ensure the sliding mode condition $s\dot{s} < 0$, constraints must be imposed on k_1 and k_2, requiring that $k_1>0$ and $k_2>0$. By setting the proposed reaching law in Equation (5.17) equal to the derivative of the sliding surface from Equation (5.15) and substituting $\ddot{\phi}$ using its expression from Equation (3.21), the control input U_2 is determined as follows.

$$U_2 = \frac{j_x}{l}\left[k_1 \mathrm{sgn}(s) + k_2 s + c_1(\dot{\phi}_d - \dot{\phi}) + \ddot{\phi}_d - \left(\frac{j_r w_r}{j_x}\right)\dot{\theta} - \left(\frac{j_y - j_z}{j_x}\right)\dot{\theta}\dot{\psi} + \frac{A_\phi \dot{\phi}}{j_x}\right] \tag{5.18}$$

5.3.3.2 Pitch Controller

Using the same approach as for the roll controller, the control input U_3, which generates the pitch rotation θ, is determined as follows:

$$U_3 = \frac{j_y}{l}\left[k_1 \mathrm{sgn}(s) + k_2 s + c_1(\dot{\theta}_d - \dot{\theta}) + \ddot{\theta}_d + \left(\frac{j_r w_r}{j_x}\right)\dot{\phi} - \left(\frac{j_z - j_x}{j_y}\right)\dot{\phi}\dot{\psi} + \frac{A_\theta \dot{\theta}}{j_y}\right] \tag{5.19}$$

5.3.3.3 Yaw Controller

Applying the same method used for the roll and pitch controllers, the control input U_4, which generates the yaw rotation ψ, is derived as follows:

$$U_4 = j_z\left[k_1 \mathrm{sgn}(s) + k_2 s + c_1(\dot{\psi}_d - \dot{\psi}) + \ddot{\psi}_d - \left(\frac{j_x - j_y}{j_z}\right)\dot{\phi}\dot{\theta} + \frac{A_\psi \dot{\psi}}{j_z}\right] \tag{5.20}$$

5.4 SMC Simulation

The design parameter (c, k_1, k_2) optimized for implementation of SMC to achieve minimum settling time for the system. Table 5.2 shows design parameter and resulting settling time for the system.

	Desired Value	c	K_1	K_2	Settling Time
Altitude(z)	2 m	6.98	2.6	6.64	1.308 sec
Attitude (ϕ and θ)	5°	4.68	1.99	1.80	1.567 sec
Heading (ψ)	5°	4.68	1.99	1.80	1.406 sec

Table 5.2 SMC Controller Results

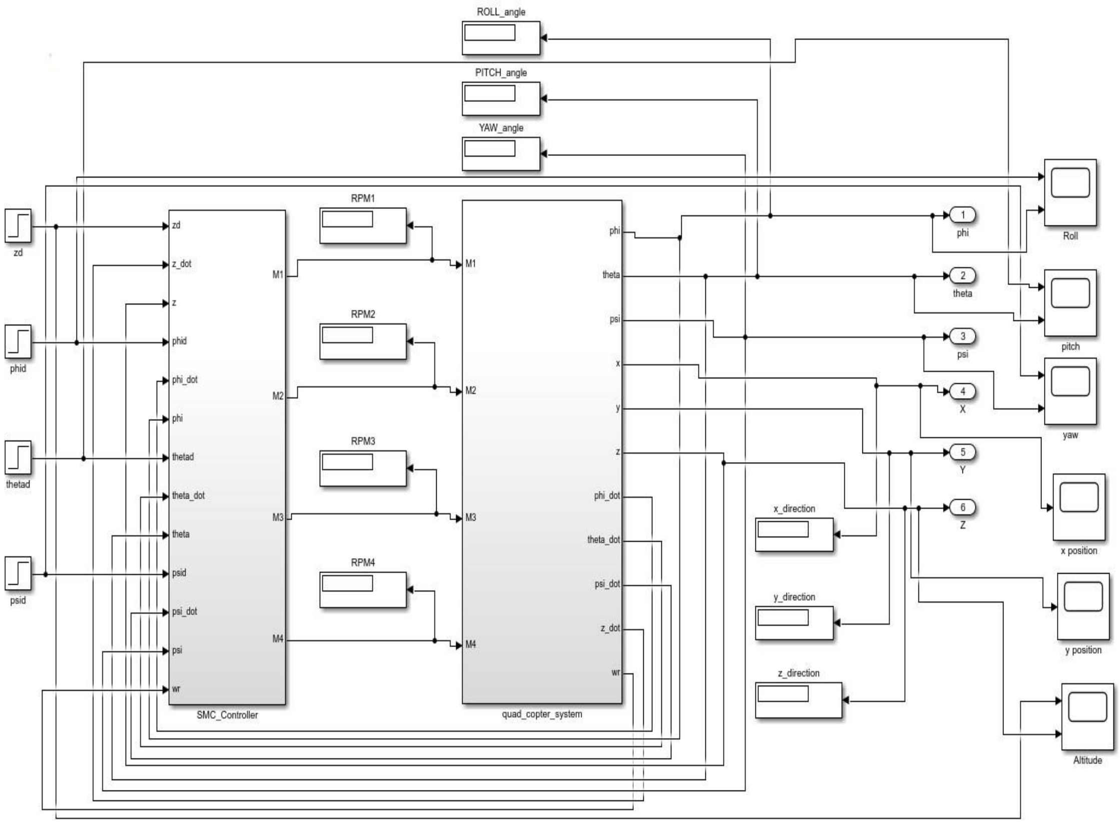

Figure 5.5: Quadrotor Model With SMC Controller

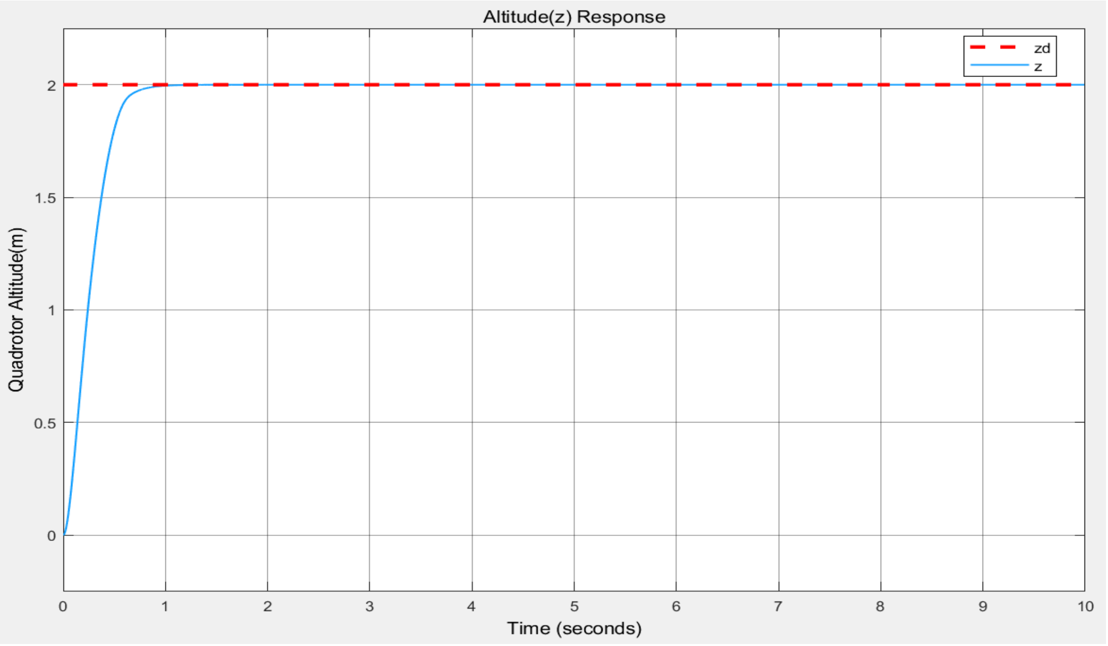

Figure 5.6(a): SMC Controller Simulation Result For Altitude Response

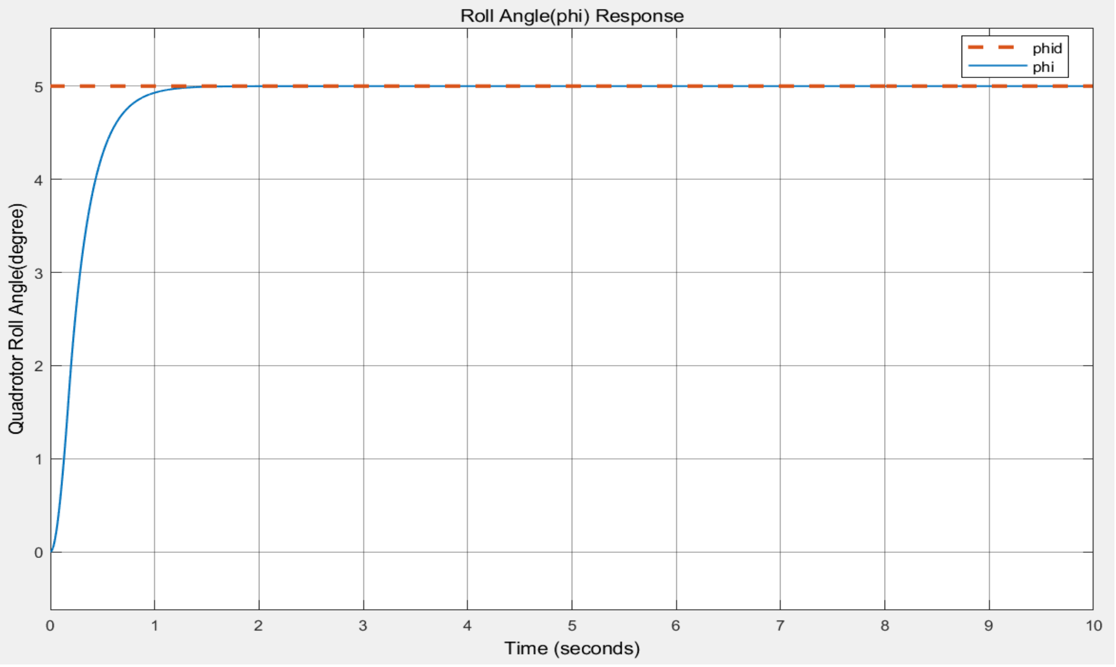

Figure 5.6(b): SMC Controller Simulation Result For Roll Angle Response

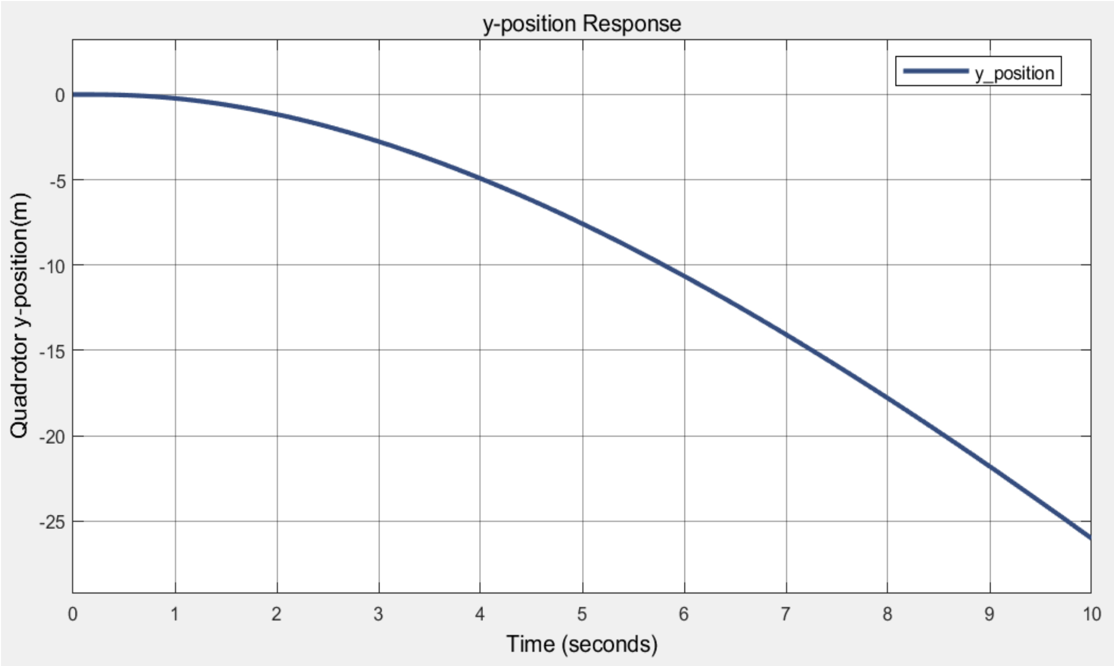

Figure 5.6(c): SMC Controller Simulation Result For y-position Response

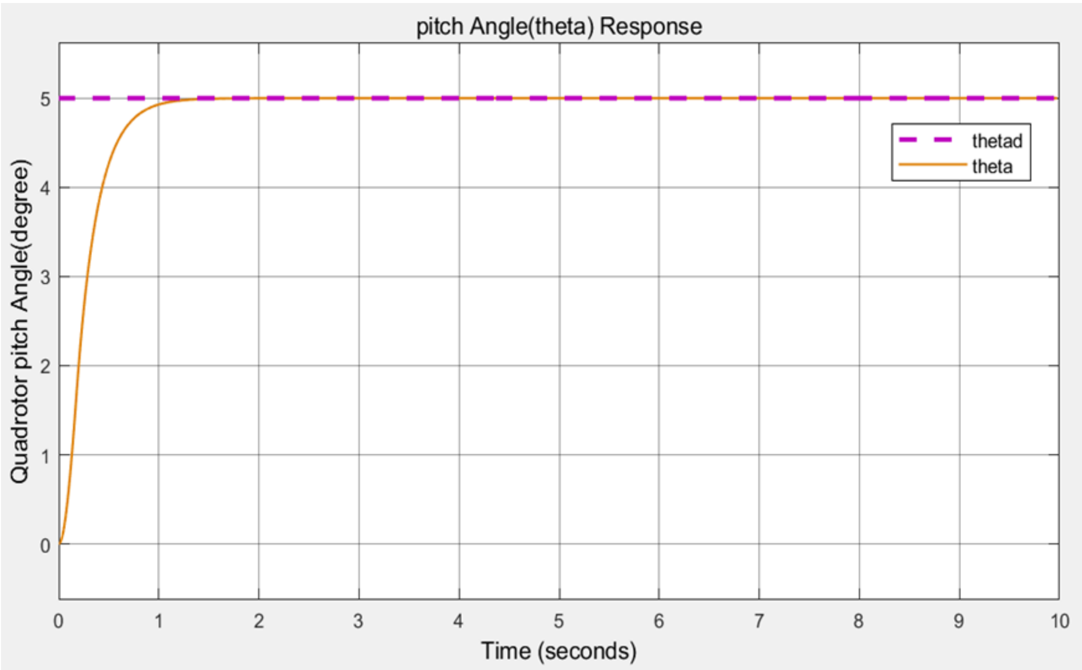

Figure 5.6(d): SMC Controller Simulation Result For Pitch Angle Response

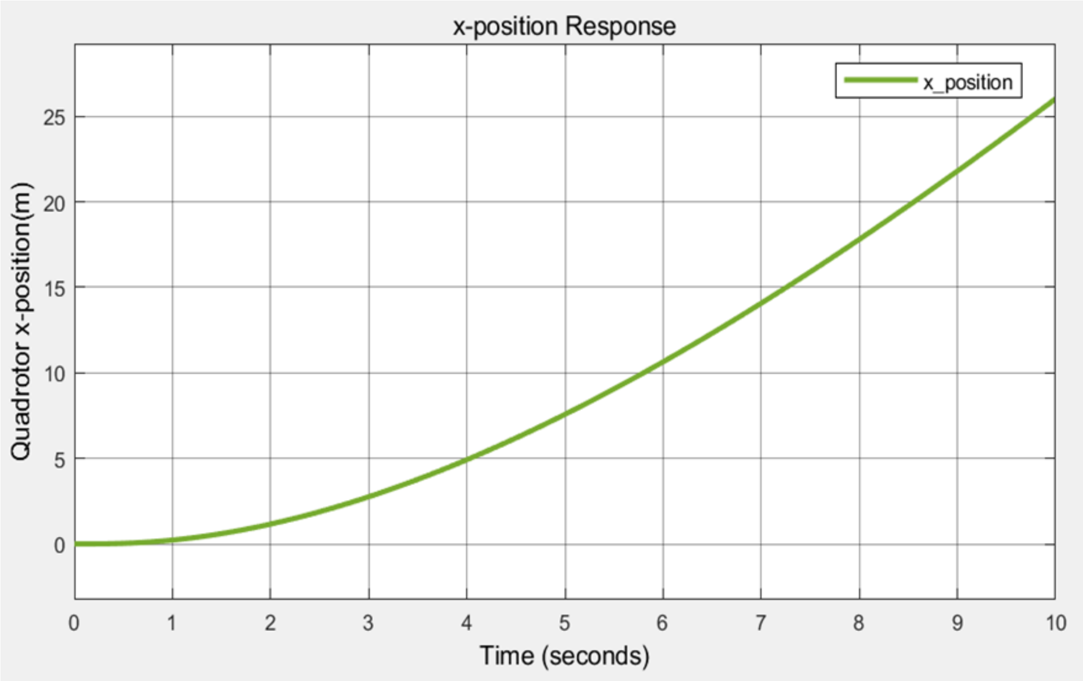

Figure 5.6(e): SMC Controller Simulation Result For x-position Response

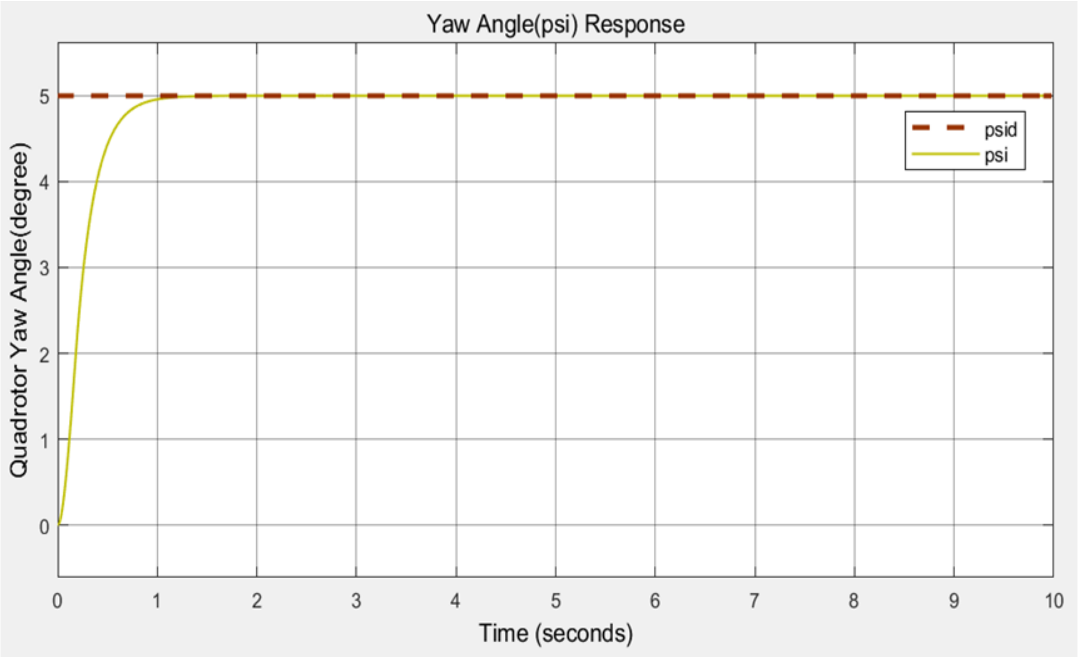

Figure 5.6(f): SMC Controller Simulation Result For Yaw Angle

Chapter 6

Discussion And Conclusion

This work aimed to create a mathematical model for a quadrotor Unmanned Aerial Vehicle (UAV) and to design both linear and nonlinear control algorithms to stabilize the quadrotor's key states altitude and attitude. The controllers' performances were then assessed through simulations for comparative analysis.

The nonlinear dynamic model of quadrotor is derived using Lagrange formalism. The model contains two parts namely translational and rotational dynamics (Euler-angle dynamics). The nonlinear model incudes the gyroscopic moments induced due to rotational motion of quadrotor body & propellers mounted on rotors. Besides, aerodynamic friction moment & force are considered in the modelling, which are often omitted in other studies.

Two distinct control methods were designed: a traditional Proportional-Derivative (PD) controller and a nonlinear Sliding Mode Controller (SMC).

To optimize controller parameters, manual tuning was used in MATLAB/Simulink to track the set point smoothly, with the objective function focusing on dynamic response metrics like settling time and overshoot.

The PD and SMC controller is designed for four output-controlled variables separately.

The controlled variables are altitude, pitch, roll and yaw. From the simulation result it can be concluded that the PD controller has more settling time and overshoot in compare to SMC for controlling of Altitude of the given Quadrotor model. Each controller performed similarly in maintaining hover within a small attitude range (0° to 20°) but here also the SMC allows for quicker attainment of the desired attitude angles compared to the PD controller. The control effort used by the SMC controller to regulate the system is small and within practical limit. Overall, the SMC designed for the quadrotor system is efficient and having very good performance.

Chapter 7

Future Scope Of Work

Here are some possible directions for future work on quadrotor UAVs:

1. **Exploring Advanced Controllers**: Future research could test more advanced control methods, like adaptive sliding mode or backstepping or AI-based controllers, which may improve stability and adaptability in different environments.
2. **Better Parameter Tuning**: Using optimization techniques, like Particle Swarm Optimization (PSO), could help to find the best tuning for controllers to improve overall system stability.
3. **Combining Sensors**: Adding data from multiple sensors, such as GPS and cameras, can make the quadrotor more accurate in controlling its position and attitude, especially in areas without GPS signals.
4. **Testing in Real Conditions**: Testing the controllers in real-world environments, with wind and varying payloads, could improve their performance and reliability outside of simulations.
5. **Energy Efficiency**: Future work could look at reducing the energy each controller uses, which would help extend the UAV’s flight time.
6. **Real-World Testing**: Implementing these controllers on an actual quadrotor would provide insights beyond simulations, revealing more about their effectiveness and potential for improvement in real-life applications.

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