ENHANCED COMMON SPATIAL PATTERN FOR MOTOR IMAGERY CLASSIFICATION

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By

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CERTIFICATE

This is to certify that the dissertation entitled **"Enhanced Common Spatial Pattern for Motor Imagery Classification"** has been carried out by SUDIP BANDYOPADHYAY (University Registration No.: 160201 of 2021-2022) under my guidance and supervision and be accepted in partial fulfillment of the requirement for the degree of Master of Electronics and Telecommunication Engineering. The research results presented in the thesis have not been included in any other paper submitted for the award of any degree to any other University or Institute.

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DECLARATION OF ORIGINALITY AND COMPLIANCE OF ACADEMIC THESIS

I hereby declare that the thesis entitled d **“Enhanced Common Spatial Pattern for Motor Imagery Classification”** contains literature and original research work by the undersigned candidate, as part of his Degree of Master of Electronics and Telecommunication Engineering. All information have been obtained and presented in accordance with academic rules and ethical conduct. It is hereby declared that, as required by these rules and conduct, all materials and results that are not original to this work have been properly cited and referenced.

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PREFACE

The idea behind the development of next generation Brain Computer Interfaces (BCI) is to provide healthy individuals an easier and better life while providing diseased individuals assistance and rehabilitation. Such devices are based on some form of input bio-modality that is processed efficiently to provide the desired results while being interfaced with the computer. BCI based control of rehabilitative aids, communicative devices and robotic equipment are gaining a tremendous significance as observed from recent literature. The direction of research is the focus of this thesis. Different aspects of BCI applications related to cognitive and haptic tasks using the bio-modality of brain signals through (EEG) have been used in this research.

The common spatial pattern which is gaining popularity in the recent years due to its tremendous abilities to segregate various classes of BCI data has been modified in this work. The previous methods made the traditional CSP very robust and more accurate and in this work we three new methods has been discussed. The first method is based on the penalization of the objective function with the signal to noise ratio parameter in the objective function itself to provide some prior to the function and make the solution more generalized. The second method also uses the same penalty matrix parameter but rather on the covariance matrix and the matrix formation is also being different. The third method is completely new and can be used flexibly to great extent where the covariance matrix is penalized in a parameterized manner. This method can be useful to combine various types of penalties together so that the results can be made even better and more accurate. The parameter tuning is required in this approach and can be easily done using hyperparameter tuning techniques. This work provides the betterment of the CSP algorithm and can be implemented in numerous sectors due to its improved accuracy and robustness. All necessary data are acquired from the datasets and each section of work is supported by sufficient experimental results to validate the proposed methodologies and algorithms.

LIST OF PUBLICATIONS

1. Sudip Bandyopadhyay and Amit Konar, "Enhanced Common Spatial Pattern for Motor Imagery Classification", *Med Biol Eng Comput* (Submitted)

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Chapter 1:

INTRODUCTION

This chapter discusses about the basics of Brain Computer Interfacing (BCI) and its applications. It also deals with the various types of BCI along with advantages and disadvantages. It also provides a basic framework explanation for the general EEG processing.

1.1 Brain Computer Interfaces

1.1.1 Introduction:

Brain-Computer Interfacing (BCI) is an innovative field that focuses on establishing a direct communication pathway between the human brain and external devices. By leveraging the electrical or hemodynamic signals generated by the brain, BCI systems enable bidirectional interaction, opening up possibilities for enhanced human-machine interfaces, medical applications, and advances in neuroscience. The term Brain Computer Interfacing was coined to the research field in 1970s by Jacques Vidal at the University of California, Los Angeles (UCLA). The first work to mention the BCI was published by Vidal in 1973 [1].

1.1.2 Definition of Brain-Computer Interfacing:

Brain-Computer Interfacing refers to the technology and methodology used to facilitate communication between the human brain and external devices through the acquisition, processing, and interpretation of neurophysiological signals. These signals can be electrical, such as those obtained through electroencephalography (EEG) or electrocorticography (ECoG), or hemodynamic, as measured by functional near-infrared spectroscopy (fNIRS) or functional magnetic resonance imaging (fMRI).

The acquired brain signals are processed using various signal processing techniques, such as filtering, feature extraction, and classification algorithms, to enhance the quality of the signals and extract relevant information. Advanced computational methods, including machine learning and pattern recognition algorithms, are often employed to decode the user's intentions, mental states, or commands from the processed brain signals. The decoded information is then translated into meaningful outputs or commands that can be used to control external devices, such as prosthetic limbs, computer interfaces, robotic systems, or to provide feedback to the user.

Overall, BCI aims to bridge the gap between the human brain and technology, opening up new possibilities for human-machine interaction, medical applications, and advancing our understanding of the brain's complexities. Ongoing research and technological advancements in signal processing techniques, decoding algorithms, and device development are constantly pushing the boundaries of BCI, with the goal of improving system performance, usability, and accessibility.

1.1.3 Components of Brain-Computer Interfacing:

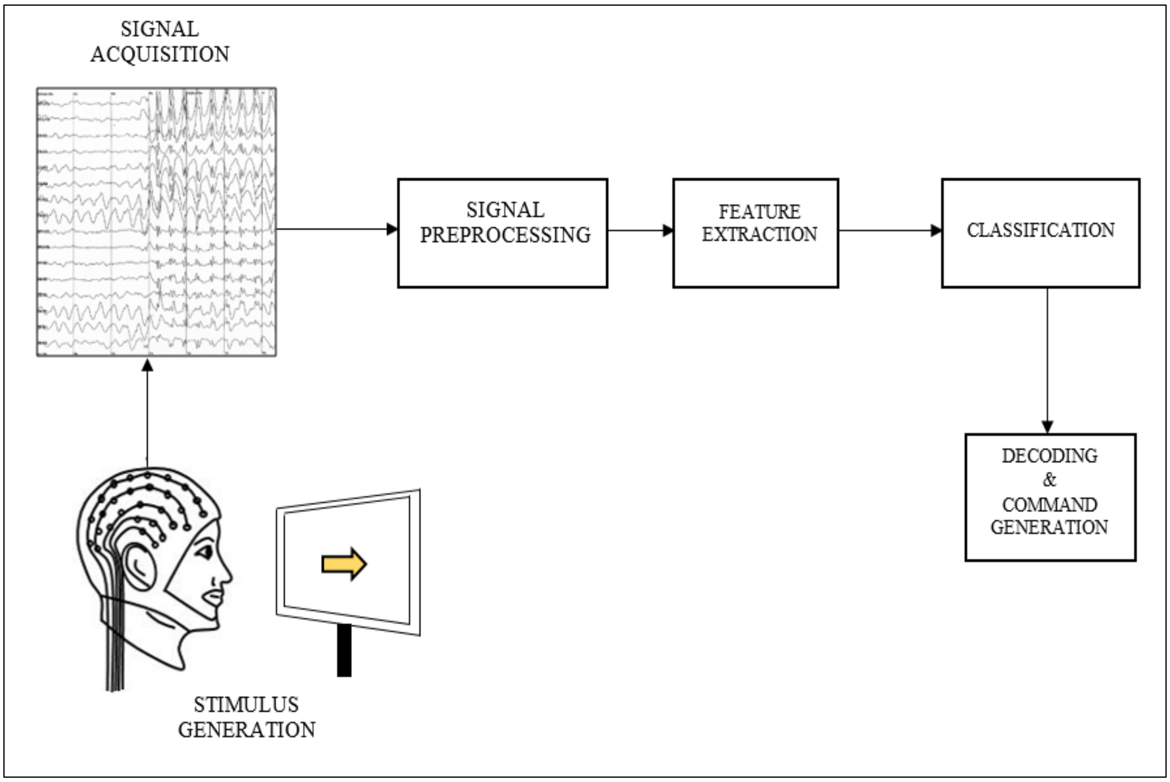

Fig. 1.1. Main components of BCI

The field of Brain-Computer Interfacing (BCI) involves several key components that work together to establish a communication link between the human brain and external devices. These components play crucial roles in acquiring, processing, and translating brain signals into actionable commands. These main components of BCI are shown in the **picture 1** and they are as follows:

- **Signal Acquisition:** The first step in BCI is the acquisition of neurophysiological signals from the brain. Various techniques are employed to capture these signals, including non-invasive methods such as electroencephalography (EEG) [2], functional near-infrared spectroscopy (fNIRS) [3], and magnetoencephalography (MEG) [4]. Invasive methods, such as electrocorticography (ECoG) [5] or implanted microelectrodes [6], provide higher spatial and temporal resolution. These methods capture electrical or hemodynamic signals associated with brain activity, which serve as the input for the BCI system.

- **Signal Preprocessing:** Once the brain signals are acquired, they undergo preprocessing to improve their quality and extract relevant information. Preprocessing techniques may involve filtering to remove noise, artifacts, and unwanted frequency components, as well as amplification and calibration to enhance the signal-to-noise ratio. Preprocessing is crucial for ensuring the accuracy and reliability of subsequent signal processing steps.

- **Feature Extraction:** Feature extraction is the process of identifying and extracting meaningful characteristics or patterns from the preprocessed brain signals. This step aims to capture relevant information that can be used to decode the user's intentions or mental states. Commonly used features include spectral power, event-related potentials (ERPs) [7], frequency bands, or statistical measures derived from the signal.

- **Signal Processing and Classification:** Signal processing and classification algorithms play a central role in BCI systems. These algorithms analyze the extracted features and map them to specific commands or mental states. Machine learning techniques, such as support vector machines (SVM) [8], hidden Markov models (HMM) [9], or deep neural networks (DNN) [10], are often employed to build models that can accurately classify brain signals and predict user intentions based on the extracted features.

- **Decoding and Command Generation:** The decoded information from the signal processing and classification stage is used to generate commands or outputs that control external devices. These commands can be in the form of controlling a robotic arm, moving a cursor on a computer screen, or providing feedback to the user. The specific mapping between decoded brain signals and desired actions depends on the application and user requirements.

1.1.4 Applications of Brain-Computer Interfacing:

Brain-Computer Interfacing (BCI) has a wide range of applications across various fields, ranging from healthcare to gaming and assistive technology. These applications harness the power of neural communication to improve human-machine interaction, enhance medical interventions, and provide assistance to individuals with disabilities. Here are some notable applications of BCI:

- **Restoring Motor Function:** BCI holds great potential in restoring motor function for individuals with paralysis or limb loss [11, 12]. By decoding neural signals associated with movement intentions, BCI systems enable users to control prosthetic limbs, exoskeletons, or robotic devices. This technology allows individuals to regain independence and perform daily tasks more effectively.

- **Assisting Communication:** BCI enables effective communication for individuals with severe communication impairments, such as locked-in syndrome or amyotrophic lateral sclerosis (ALS) [13, 14]. By translating brain activity related to language or movement into text or speech output, BCI systems allow these individuals to express their thoughts, needs, and emotions. This application greatly enhances their quality of life and promotes social interaction.

- **Cognitive Enhancement:** BCI research also explores ways to augment human cognition [15]. Neurofeedback-based BCI systems provide real-time feedback to users about their brain activity patterns [16]. Through this feedback, users can learn to regulate their brainwaves, enhancing attention, memory, and learning capabilities. BCI-based cognitive training holds potential in educational settings and rehabilitation programs.

- **Gaming and Virtual Reality:** BCI is increasingly integrated into gaming and virtual reality (VR) applications [17]-[19]. By using brain signals as input, users can control game characters, navigate virtual environments, and interact with virtual objects, creating a more immersive and engaging gaming experience. BCI in gaming has potential for entertainment, therapeutic interventions, and neurorehabilitation.

- **Brain-Computer Interfaces for Healthcare:** BCI has promising applications in healthcare beyond motor function and communication. It can be used for monitoring and diagnosing brain disorders, such as epilepsy and sleep disorders, by analyzing brain activity patterns. BCI technology can also assist in neurofeedback therapy, helping individuals with conditions like attention-deficit/hyperactivity disorder (ADHD) and anxiety disorders [20].

- **Brain-Machine Interfaces in Research:** BCI is a valuable tool for neuroscience research, enabling scientists to study brain activity and understand neural mechanisms. BCI systems provide insights into brain functioning during various tasks, perception, and decision-making processes. These interfaces contribute to the advancement of neuroscientific knowledge and promote a deeper understanding of the human brain.

- **Military Applications:** BCI has military applications, particularly in the field of unmanned aerial vehicles (UAVs) or drones [21]. BCI can be used to control UAVs using the pilot's thoughts, allowing for faster and more accurate responses to changing situations. BCI can also be used to enhance the performance of soldiers in the field, by providing real-time feedback on their cognitive and physiological state [22].

1.2 Types of Brain Computer Interfacing

There are many types of BCI in current use. These are mainly categorized into the non-invasive, Partially Invasive and invasive BCI. These techniques are very crucial for obtaining various kinds of signals and data from the subjects in order to get numerous results and findings.

1.2.1 Non-Invasive BCI

Non-invasive Brain-Computer Interfacing (BCI) techniques refer to methods of acquiring neural signals and establishing a communication link between the brain and external devices without the need for surgical implantation. These techniques have gained significant attention due to their non-invasive nature, ease of use, and potential for widespread applications. Here are some common non-invasive BCI modalities:

1.2.1.1 Electroencephalography (EEG)

Fig. 1.2. EEG setup

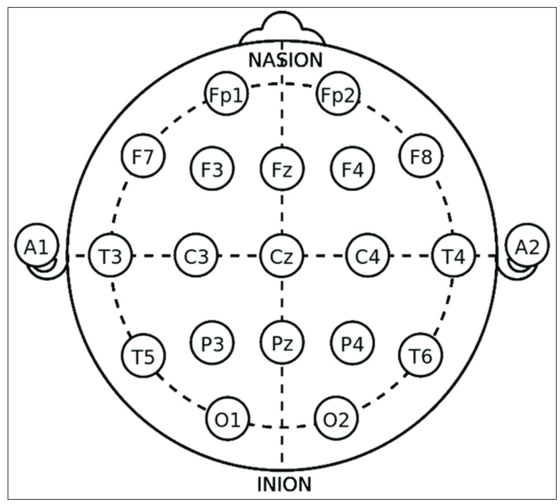

Fig. 1.3. International 10-20 system for electrode placement

EEG (electroencephalography) is a non-invasive method of measuring the electrical activity of the brain using electrodes placed on the scalp. EEG measures the summation of the electrical activity of millions of neurons firing together, providing insight into the brain's functional connectivity and activity patterns. German physiologist and psychiatrist Hans Berger recorded the first human EEG in 1924 [2]. EEG is widely used in clinical and research settings to diagnose and study various neurological and psychiatric conditions such as epilepsy, sleep disorders, and cognitive impairments. It can also be used to investigate the effects of drugs, brain stimulation, and other interventions on brain activity.

During an EEG recording, electrodes are placed on the scalp in specific locations to measure the electrical activity of the brain. The typical setup of EEG on a human subject is shown in **Fig 1.2**. The electrode locations are specified by various electrode placing systems like international 10-20 system, 10-10 system, 10-5 system, Modified Combinations [23]. The International 10-20 System which is shown in **Fig 1.3** is the most widely used standard for EEG electrode placement [24]. It provides a systematic method for positioning electrodes based on the distances between anatomical landmarks on the scalp. The system divides the scalp into regions and assigns electrode positions based on a percentage of the total distance between the landmarks. Electrode positions are denoted by letters and numbers, such as Fz, Cz, and Pz, representing frontal, central, and parietal regions, respectively. EEG works on the principle of measuring the electrical activity of the brain using electrodes placed on the scalp. The brain's electrical activity is generated by the activity of millions of neurons firing together, creating electrical fields that can be measured on the scalp. When the neurons in the brain are activated, they produce small electrical currents that travel through the brain tissue and the skull to the scalp. The electrodes placed on the scalp detect these electrical signals, which are then amplified, filtered, and digitized. The EEG signal is typically analyzed by dividing it into different frequency bands, such as delta (0.5-4 Hz), theta (4-8 Hz), alpha (8-13 Hz), beta (13-30 Hz), and gamma (>30 Hz) [26]. Each frequency band is associated with different brain states, such as sleep, wakefulness, and attention. The range of frequencies observed in EEG are typically between 1 and 30 Hz and amplitudes vary from 20 to 100 μV. These signals are feeble and filled with various types of noise thus the signals are first amplified and then the noise along with unwanted artifacts are removed and suppressed to make the signals clean to work with and also providing accurate results. Like any technology, EEG has advantages and disadvantages that need to be considered when deciding whether to use it for a particular application.

Advantages of EEG:

- **High temporal resolution:** EEG has very high temporal resolution, meaning it can capture changes in brain activity with millisecond precision. This makes it ideal for studying dynamic changes in brain activity, such as those associated with cognitive processes and sensory perception.
- **Non-invasive:** EEG is a non-invasive technique that does not require surgery or the injection of contrast agents. This makes it safer and less uncomfortable for patients compared to other neuroimaging techniques such as fMRI or PET.

- **Cost-effective:** Compared to other neuroimaging techniques, EEG is relatively inexpensive and can be used in a variety of settings, including hospitals, clinics, and research laboratories.

Disadvantages of EEG:

- **Limited spatial resolution:** The spatial resolution of EEG is relatively poor compared to other neuroimaging techniques such as fMRI or PET. EEG can only provide information about the overall pattern of brain activity in different regions of the brain, but cannot provide detailed information about the location of specific neural structures.
- **Susceptible to noise:** EEG signals are easily contaminated by noise, including muscle artifacts, environmental interference, and other electrical signals. This can make it difficult to obtain clean and reliable data, especially in noisy environments.
- **Limited depth of measurement:** EEG can only measure the activity of neurons located near the surface of the brain, which limits its ability to investigate deeper brain structures. This can be a disadvantage for studying certain brain disorders or processes that involve deep brain structures.

Electroencephalography (EEG) has a wide range of applications in various fields due to its ability to measure and record electrical activity in the brain. The uses of EEG are as follows:

- **Clinical Diagnosis and Monitoring:** EEG is commonly used in clinical settings to diagnose and monitor various neurological disorders. It helps in the diagnosis of epilepsy by detecting abnormal brainwave patterns during seizures. EEG is also used to assess brain function in conditions such as sleep disorders, coma, and brain injuries. It can aid in monitoring the effects of anesthesia during surgery and evaluating brain activity in intensive care units.

- **Neurofeedback and Brain Training:** EEG-based neurofeedback allows individuals to self-regulate their brain activity by providing real-time feedback about their brainwave patterns. It is used in therapeutic interventions for conditions like attention-deficit/hyperactivity disorder (ADHD), anxiety, and cognitive impairments. Neurofeedback training aims to enhance attention, reduce symptoms, and improve cognitive performance.

- **Cognitive and Neuroscience Research:** EEG is widely used in cognitive neuroscience research to investigate brain processes associated with attention, perception, memory, and decision-making. It helps researchers understand the neural mechanisms underlying cognitive functions and the effects of various stimuli or interventions. EEG-based studies provide insights into brain dynamics during cognitive tasks and can help elucidate brain disorders and their neural correlates.

- **Brain-Computer Interfaces (BCIs):** EEG is a fundamental component of Brain-Computer Interfaces (BCIs), which establish a direct communication link between the brain and external devices. EEG-based BCIs enable users to control devices, such as prosthetic limbs or computer interfaces, using their brain activity. These systems are especially valuable for individuals with motor disabilities, providing them with the ability to interact with their environment and regain independence.

- **Sleep and Dream Research:** EEG is crucial for studying sleep and dream patterns. It helps classify different stages of sleep, such as rapid eye movement (REM) and non-REM sleep, by analyzing characteristic brainwave patterns [26]. EEG recordings during sleep allow researchers to investigate sleep disorders, study dream activity, and understand the relationship between brain activity and sleep-related phenomena.

- **Human-Computer Interaction and Gaming:** EEG has been integrated into human-computer interaction and gaming applications. By using brainwave patterns as input, EEG-based systems allow users to control computer interfaces or interact with virtual environments in gaming. This technology creates immersive experiences and novel ways of interaction, enhancing the gaming experience and expanding possibilities in virtual reality (VR) applications.

These applications represent just a few examples of how EEG is utilized across different domains. Ongoing advancements in technology, signal processing, and analysis techniques continue to expand the potential applications of EEG, making it a valuable tool for understanding brain function, diagnosing neurological disorders, and developing innovative interventions.

1.2.1.2 Functional Near-Infrared Spectroscopy fNIRS

Functional Near-Infrared Spectroscopy (fNIRS) is a non-invasive neuroimaging technique that measures changes in blood oxygenation levels in the brain. When neurons in the brain become active, they consume oxygen and glucose to meet the increased metabolic demands. This leads to changes in blood flow and oxygenation levels, resulting in a hemodynamic response. fNIRS detects these hemodynamic changes as an indicator of neural activity [27]. It utilizes near-infrared light in the range of 650 to 950 nm to penetrate the scalp and skull and interact with chromophores in the brain tissue, primarily oxygenated and deoxygenated hemoglobin. The near-infrared light, which with longer wavelengths penetrate biological tissues more deeply than visible light. Near-infrared light is less absorbed and scattered by the scalp, skull, and brain tissue, allowing it to reach the cerebral cortex and provide measurements from superficial cortical regions. Near-infrared light is sensitive to changes in the concentrations of oxygenated and deoxygenated hemoglobin. Oxygenated hemoglobin absorbs more near-infrared light, while deoxygenated hemoglobin absorbs less. By comparing the absorption of two or more wavelengths of light, fNIRS can estimate the relative concentrations of these hemoglobin types. By measuring the absorption and scattering of light, fNIRS provides an indirect measure of neural activity in specific brain regions. A typical setup of fNIRS on human subject is shown in **Fig 1.4**. fNIRS employs optodes [29], which consist of a light source and a detector, to emit and detect near-infrared light. Optodes are placed on the scalp, and the light is transmitted through the tissue and detected on the other side. The distance between the source and detector determines the depth of the measured brain region. A typical arrangement of optodes is shown in **Fig. 1. 5** with 10 Transmitters and 8 Receivers.

Fig. 1.4. fNIRS setup

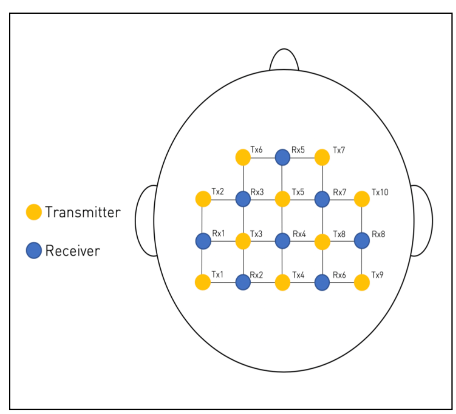

Fig. 1.5. fNIRS optode placement

Functional Near-Infrared Spectroscopy (fNIRS) has several advantages and disadvantages that should be considered when using this neuroimaging technique.

Advantages of fNIRS:

- **Non-invasiveness:** fNIRS is a non-invasive technique that does not require the use of ionizing radiation or contrast agents. It is considered safe and well-tolerated, making it suitable for use in various populations, including infants, children, and individuals with medical conditions.

- **Portability:** fNIRS systems are typically portable and relatively easy to set up. This allows for flexibility in data collection, enabling studies to be conducted in various environments, such as clinical settings, research labs, and even during naturalistic tasks in the field.

- **Real-time Measurements:** fNIRS provides real-time measurements of changes in brain activity. It has a relatively high temporal resolution, allowing researchers to track rapid changes in hemodynamic responses and neural activity during tasks or interventions.

- **Spatial Resolution:** Although fNIRS has lower spatial resolution compared to techniques like fMRI, it can provide reasonable localization of brain activity, especially in the superficial layers of the cortex. Multiple optodes placed over specific regions of interest can improve the spatial accuracy of fNIRS measurements.

- **Versatility:** fNIRS can be used to study a wide range of cognitive functions, including language processing, attention, memory, and motor control. It can also be integrated with other neuroimaging techniques, such as EEG or fMRI, to provide complementary information and enhance the understanding of brain function.

Disadvantages of fNIRS:

- **Limited Penetration Depth:** Near-infrared light used in fNIRS has limited penetration depth in biological tissues, which restricts the measurement to the cortical surface. This makes it challenging to study deeper brain regions and limits the application of fNIRS in certain research areas.

- **Vulnerability to Motion Artifacts:** fNIRS measurements can be affected by motion artifacts caused by head movements. Even slight movements can introduce noise and distort the recorded signals, potentially compromising data quality. Measures such as head stabilization or artifact rejection algorithms are employed to mitigate this issue.

- **Sensitivity to Scalp and Skull:** The scalp and skull can affect the propagation of near-infrared light, leading to variations in the recorded signals. Variability in scalp and skull thickness, composition, and optical properties among individuals can introduce confounds and influence the accuracy of fNIRS measurements.

- **Limited Coverage of Brain Areas:** Due to the limited number of optodes used in fNIRS setups, the coverage of brain areas can be constrained. This restricts the ability to simultaneously measure activity from a large number of brain regions and limits the spatial resolution for detailed mapping.

- **Difficulty in Differentiating Hemodynamic Sources:** fNIRS measures changes in blood oxygenation levels, but it does not directly measure neural activity. Disentangling the contributions of different hemodynamic sources, such as changes in cerebral blood flow and oxygen consumption, can be challenging and require complex modeling approaches.

1.2.1.3 Magnetoencephalography (MEG)

Magnetoencephalography (MEG) is a non-invasive neuroimaging technique that measures the magnetic fields generated by the electrical activity of neurons in the brain [29]. It provides high temporal resolution and excellent spatial resolution, allowing researchers and clinicians to study the timing and location of neural activity with great precision. MEG is based on the principle of magnetoencephalic effect, which refers to the generation of magnetic fields by electric currents. MEG measures the weak magnetic fields generated by electrical currents in the brain. Neuronal activity generates small electrical currents that create associated magnetic fields perpendicular to the electrical current flow. These magnetic fields can be detected using highly sensitive magnetometers. MEG uses Superconducting Quantum Interference Device (SQUID) Sensors to measure the tiny magnetic fields generated by the brain [30]. SQUID sensors are extremely sensitive to magnetic fields and are typically cooled to cryogenic temperatures to enhance their performance. MEG systems typically consist of an array of sensors arranged in a helmet-like structure, providing whole-head coverage. This allows for comprehensive measurement of brain activity from various regions simultaneously. A human subject having the MEG system setup is shown in **Fig 1.6**.

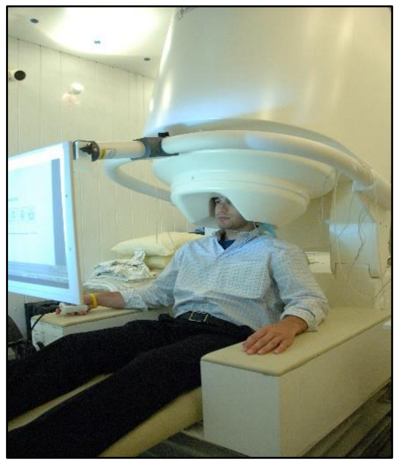

Fig. 1.6. MEG setup

Magnetoencephalography (MEG) has several advantages and disadvantages that should be considered when utilizing this neuroimaging technique.

Advantages of MEG:

- **Excellent Temporal Resolution:** MEG offers exceptional temporal resolution, providing precise timing information of neural activity. It can detect neural events at the millisecond level, making it suitable for studying rapid cognitive processes and capturing the dynamics of brain activity.

- **High Spatial Resolution:** MEG provides relatively good spatial resolution, allowing for the localization of neural activity with accuracy. It can identify the sources of brain signals and provide detailed information about the specific brain regions involved in a particular task or cognitive process.

- **Whole-Head Coverage:** MEG systems typically have an array of sensors that cover the entire head, enabling comprehensive recording of brain activity from various regions simultaneously. This allows for a comprehensive understanding of neural dynamics and interactions across the brain.

- **Direct Measurement of Neural Activity:** MEG directly measures the magnetic fields generated by neural electrical currents, providing a direct reflection of underlying neural activity. This makes MEG a valuable tool for investigating the functional organization and connectivity of the brain.

- **Non-Invasive and Safe:** MEG is a non-invasive technique that does not involve the use of ionizing radiation or contrast agents. It is considered safe and can be used in a wide range of populations, including children and individuals with medical conditions.

Disadvantages of MEG:

- **Sensitivity to Magnetic Interference:** MEG is highly sensitive to magnetic interference from the environment, such as metallic objects and electrical devices. Special magnetically shielded rooms are required to minimize the impact of external magnetic fields and reduce noise levels.

- **Limited Spatial Resolution for Deep Brain Structures:** While MEG provides good spatial resolution for the cortical surface, it has limitations in accurately localizing activity from deep brain structures. The magnetic fields attenuate as they pass through the skull, which can make it challenging to precisely identify sources in subcortical regions.

- **Cost and Availability:** MEG systems are relatively expensive to acquire and maintain, which can limit their availability to certain research institutions or clinical centers. The high cost of MEG equipment and the need for specialized infrastructure and expertise may pose barriers to its widespread adoption.

- **Limited Compatibility with Metallic Objects:** MEG is not compatible with metallic objects, such as dental fillings, braces, or certain medical implants. These objects can cause artifacts and distortions in the measured magnetic fields, potentially compromising data quality.

- **Limited Sensitivity to Neuronal Activity Orientation:** MEG is more sensitive to neural activity that is perpendicular to the scalp surface compared to activity parallel to the scalp. This sensitivity limitation can affect the accuracy of source localization, particularly for deep sources and sources with complex orientations.

- **Integration with Structural Imaging:** While MEG provides functional information, it does not provide detailed structural information about the brain. Integration with other imaging techniques, such as MRI, is necessary to combine functional and anatomical data for comprehensive analysis.

1.2.1.4 Magnetic Resonance Imaging (MRI)

Magnetic Resonance Imaging (MRI) is a non-invasive medical imaging technique that uses a strong magnetic field and radio waves to generate detailed images of the body's internal structures [31]. MRI provides high-resolution images of soft tissues, organs, and bones, making it a valuable tool in diagnosing and monitoring a wide range of medical conditions. MRI utilizes a powerful magnetic field and radio waves to manipulate the hydrogen atoms present in the body's tissues. The magnetic field aligns the hydrogen atoms, while the radio waves cause them to emit signals that are captured by the MRI machine. MRI offers excellent tissue contrast, making it particularly useful for distinguishing between different types of soft tissues. This enables detailed visualization of organs, blood vessels, muscles, ligaments, and nerves, aiding in the detection and characterization of abnormalities. MRI allows imaging in multiple planes, including axial (horizontal), sagittal (vertical from side to side), and coronal (vertical from front to back). This multiplanar capability provides a comprehensive view of the anatomy and facilitates accurate assessment of the location and extent of abnormalities. Contrast agents can be used in MRI to enhance the visualization of specific structures or to highlight areas of abnormality. These agents are injected intravenously and can help in the detection of tumors, blood vessel abnormalities, or areas of inflammation. Functional MRI measures changes in blood flow and oxygenation levels in the brain to map neural activity. It enables the study of brain function and connectivity during specific tasks or at rest, providing insights into cognitive processes and brain disorders. A MRI machine along with a typical MRI scan result is shown in **Fig 1.7 and Fig 1.8**. The advantages and disadvantages of MRI are as follows:

Fig. 1.7. MRI setup

Fig. 1.8. MRI scan result

Advantages of Magnetic Resonance Imaging (MRI):

- **Excellent Soft Tissue Contrast:** MRI provides excellent soft tissue contrast, allowing for detailed visualization and differentiation of various anatomical structures. This makes it particularly useful for detecting abnormalities in organs, muscles, ligaments, and nerves.

- **Multiplanar Imaging:** MRI can acquire images in multiple planes (axial, sagittal, and coronal), providing a comprehensive view of the anatomy and facilitating accurate diagnosis and surgical planning.

- **Non-Invasive:** MRI is a non-invasive imaging technique that does not use ionizing radiation. It uses a powerful magnetic field and radio waves to generate images, making it a safer option for patients, especially those who require multiple scans or children.

- **No Known Health Risks:** MRI has no known health risks associated with the magnetic field or radio waves used in the procedure. It does not involve exposure to ionizing radiation, making it a preferred choice for long-term monitoring or repeated imaging studies.

- **Versatile and Wide Range of Applications:** MRI can be used to image various parts of the body, including the brain, spine, joints, abdomen, pelvis, and blood vessels. It is useful in diagnosing and monitoring a wide range of medical conditions, such as tumors, strokes, neurological disorders, musculoskeletal injuries, and cardiovascular diseases.

- **Functional and Dynamic Imaging:** In addition to structural imaging, MRI offers functional and dynamic imaging capabilities. Functional MRI (fMRI) can map brain activity and connectivity, while dynamic contrast-enhanced MRI can assess blood flow and tissue perfusion.

Disadvantages of Magnetic Resonance Imaging (MRI):

- **Cost:** MRI equipment is expensive to purchase, operate, and maintain. The high cost of MRI scans may limit accessibility, especially in healthcare systems with limited resources. This can result in longer waiting times for patients.

- **Time-Consuming:** MRI scans typically take longer to acquire compared to other imaging modalities. The duration can range from several minutes to over an hour, depending on the type of scan and the area being imaged. Patients need to remain still during the scan, which can be challenging for individuals, especially children or those with movement disorders.

- **Claustrophobia and Patient Discomfort:** The MRI machine requires the patient to lie still inside a narrow, enclosed space. This can induce feelings of claustrophobia and anxiety in some individuals, potentially leading to discomfort and the need for sedation in certain cases. Open-bore or wide-bore MRI machines are available to accommodate patients who may feel more comfortable in a larger space.

- **Metallic Implants and Contraindications**: MRI is contraindicated for individuals with certain metallic implants, such as pacemakers, cochlear implants, or aneurysm clips. The strong magnetic field of the MRI machine can interact with these implants and pose potential risks to the patient. Additionally, patients with certain conditions or medical devices may need to be screened for compatibility with MRI.

- **Noise and Sensory Experience:** MRI scans produce loud knocking or buzzing noises during the imaging process, which can be bothersome for patients. Ear protection is provided, but some individuals may still find the noise unsettling. Additionally, the confined space and the need to lie still for an extended period may cause discomfort or anxiety for some patients.

- **Limited Availability:** MRI machines may not be readily available in all healthcare facilities, especially in remote or underserved areas. This limited availability can result in longer travel distances or waiting times for patients in need of MRI scans.

1.2.1.5 Transcranial Magnetic Stimulation (TMS)

Fig. 1.8. TMS setup

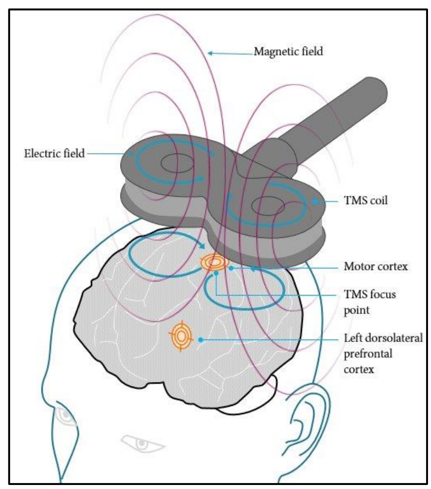

Fig. 1.9. Stimulation using TMS coil

Transcranial Magnetic Stimulation (TMS) is a non-invasive neurostimulation technique that uses magnetic fields to induce electrical currents in specific regions of the brain [32] [33]. It involves the use of a device called a TMS coil, which is placed on the scalp and delivers focused magnetic pulses to targeted brain areas. A typical TMS setup is shown in **Fig 1.8**. TMS can modulate neural activity and has therapeutic applications in various neurological and psychiatric disorders. TMS works on the principle of electromagnetic induction. When a rapidly changing magnetic field is applied to the scalp, it induces small electrical currents in the underlying brain tissue. These currents can either enhance or inhibit neuronal activity, depending on the stimulation parameters and target region. There are different types of TMS techniques like Single-Pulse TMS In which a single magnetic pulse is delivered at a specific intensity and timing to elicit a neural response or assess motor cortex excitability, Repetitive TMS (rTMS) which involves the repeated delivery of magnetic pulses over time. It can be used to modulate brain activity and induce longer-lasting effects, Theta Burst Stimulation (TBS) which is a specific pattern of rTMS that delivers bursts of pulses at high frequency (theta range) and is used to either enhance or suppress cortical excitability. TMS can be applied to specific brain regions by positioning the TMS coil over the desired target area. The stimulation of a particular area of brain using the TMS coil is shown in the **Fig 1.9**. Different brain regions can be targeted to study their function or modulate their activity for therapeutic purposes. TMS can modulate neural activity by either exciting or inhibiting the targeted brain regions. Excitatory TMS increases cortical excitability and can enhance neuronal firing, while inhibitory TMS decreases cortical excitability and can suppress neuronal activity. Advantages and Disadvantages of TMS are:

Advantages of Transcranial Magnetic Stimulation (TMS):

- **Non-Invasive:** TMS is a non-invasive technique that does not require surgery or implantation of electrodes. It can be applied externally to the scalp, making it a safe and well-tolerated procedure.

- **Targeted Stimulation**: TMS allows for precise targeting of specific brain regions, enabling researchers and clinicians to study and modulate the function of those areas.

- **Reversible and Adjustable Effects:** The effects of TMS are reversible and adjustable, meaning that the stimulation can be discontinued or modified as needed. This flexibility allows for tailored treatment or research protocols.

- **Minimal Systemic Side Effects:** TMS primarily affects the targeted brain regions and does not have widespread systemic side effects, making it a favorable option compared to certain pharmacological treatments.

Disadvantages of Transcranial Magnetic Stimulation (TMS):

- **Limited Penetration Depth:** TMS is limited in its ability to reach deep brain structures due to the physical properties of the magnetic field. This restricts its application to cortical and near-surface brain regions.

- **Variable Response:** The response to TMS can vary among individuals, and not everyone may benefit from the treatment. Factors such as individual variability in brain anatomy and functioning may contribute to the variability in treatment response.

- **Treatment Duration and Frequency:** TMS typically requires multiple treatment sessions over several weeks to achieve therapeutic effects. This can be time-consuming and may pose challenges for patient compliance.

- **Cost:** The cost of TMS treatment can be a limiting factor for some individuals, as it may not be covered by insurance or available in all healthcare settings.

1.2.1.6 Positron Emission Tomography (PET):

Positron Emission Tomography (PET) is a medical imaging technique that allows for the visualization and quantification of metabolic and functional processes in the body [34] [35]. It involves the use of radioactive tracers, known as radiotracers, which emit positrons. By detecting these emitted positrons, PET scanners can generate detailed three-dimensional images of the distribution of the radiotracer in the body. PET imaging is widely used in clinical practice and research for various applications. PET imaging relies on the detection of gamma rays emitted by a radiotracer. The radiotracer is a biologically active molecule labeled with a positron-emitting radioactive isotope, such as fluorine-18 (18F), carbon-11 (11C), or oxygen-15 (15O). When the radiotracer is administered to the patient, it undergoes specific biochemical interactions and emits positrons. When a positron encounters an electron in the body, it undergoes annihilation, resulting in the emission of two gamma rays in opposite directions. These gamma rays are detected by a ring of detectors surrounding the patient, and a computer reconstructs the data to produce an image. PET provides information about functional and metabolic processes in the body. It allows for the measurement of physiological functions, such as blood flow, glucose metabolism, oxygen utilization, and receptor binding. This makes PET particularly useful for studying diseases that involve alterations in these processes, such as cancer, neurodegenerative disorders, and cardiovascular diseases. PET imaging uses a variety of radiotracers specific to different physiological processes. For example, the radiotracer [18F]fluorodeoxyglucose (FDG) is commonly used to assess glucose metabolism, which is helpful in diagnosing and staging various types of cancer. Other radiotracers target specific receptors, enzymes, or molecules involved in disease processes, allowing for targeted imaging of specific organs or systems. The PET setup is shown in **Fig 1.10** and the imaging obtained for the human brain is shown in **Fig 1.11**.

Fig. 1.10. PET setup

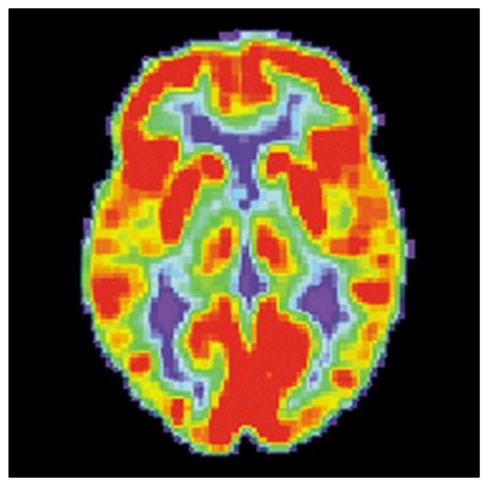

Fig. 1.11. PET image of human brain

Advantages of Positron Emission Tomography (PET):

- **Functional and Metabolic Information:** PET provides functional and metabolic information that complements the anatomical information obtained from other imaging modalities, such as CT or MRI. This enables a more comprehensive understanding of disease processes and can guide treatment decisions.

- **Molecular Targeting:** PET imaging allows for the visualization and quantification of specific molecular targets, such as receptors or enzymes involved in disease processes. This molecular targeting capability has significant implications for personalized medicine and targeted therapies.

Disadvantages of Positron Emission Tomography (PET):

- **Cost:** PET imaging is a relatively expensive modality compared to other imaging techniques. The cost is primarily associated with the production and short half-life of the radiotracers, the specialized equipment required, and the need for dedicated facilities and personnel.

- **Limited Availability:** PET scanners may not be widely available, especially in smaller healthcare facilities or certain regions. This limited availability can lead to longer waiting times for patients requiring PET imaging.

- **Radiation Exposure:** PET involves exposure to ionizing radiation due to the use of radioactive tracers. While the radiation doses are considered safe, precautions are taken to minimize unnecessary exposure, especially in pregnant women and pediatric patients.

- **Image Resolution:** PET imaging has lower spatial resolution compared to other imaging modalities like CT or MRI. The resolution limitations can affect the detection and characterization of small or subtle abnormalities.

1.2.2 Partially Invasive BCI

Partially invasive BCI (Brain-Computer Interface) refers to a type of BCI system that combines both invasive and non-invasive approaches for recording and decoding neural activity. This approach aims to leverage the advantages of invasive techniques, such as high-quality neural signals, while minimizing some of the associated risks and limitations. In a partially invasive BCI system, some components are implanted or inserted into the body, while others remain external. The invasive components may include implantable electrodes or arrays placed directly on or into the brain tissue to record neural signals, while non-invasive components can involve sensors or devices placed outside the body, such as EEG electrodes or wearable sensors.

1.2.2.1 Electrocorticography (ECoG)

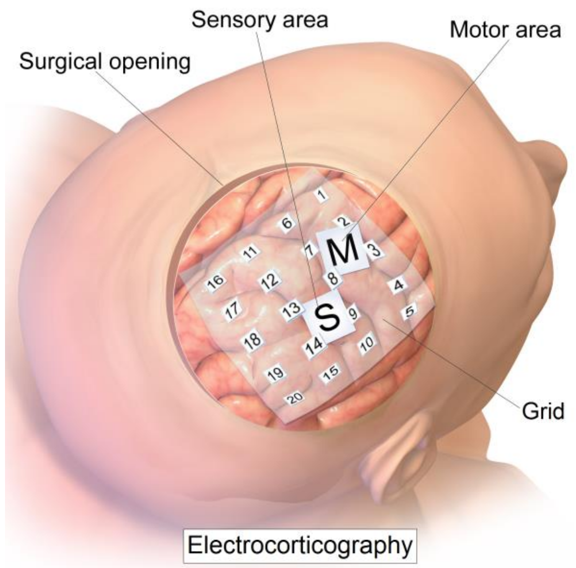

Fig. 1.12. ECoG setup

Electrocorticography (ECoG) is a neurophysiological technique that involves the placement of an electrode grid directly on the surface of the brain to record electrical activity [36] [37]. ECoG is an invasive method of brain recording that provides high-resolution neural data with relatively less invasiveness compared to penetrating microelectrode arrays used in intracortical BCIs. In an ECoG procedure, a neurosurgeon places a thin grid of electrodes, typically made of platinum or stainless steel, over the cortex. The electrode grid is positioned on the surface of the brain, beneath the dura mater (the protective membrane covering the brain). This allows the electrodes to pick up electrical signals directly from the cortical tissue. ECoG electrodes capture electrical potentials generated by the collective activity of large populations of neurons near the surface of the brain. These electrical signals, known as electrocorticograms, provide information about neural activity in specific brain regions. ECoG has been widely used in clinical and research settings to study brain activity associated with various functions, such as motor control, language processing, sensory perception, and epileptic seizures. It has also been utilized in the development of brain-computer interfaces (BCIs) for applications such as communication and control of external devices. One advantage of ECoG is its ability to capture high-resolution neural signals with good signal-to-noise ratio. Compared to non-invasive techniques like electroencephalography (EEG), which measures electrical activity from the scalp, ECoG provides more spatially precise information due to its closer proximity to the brain surface. This makes it suitable for decoding specific neural patterns and developing more precise BCIs. ECoG has a relatively lower risk profile compared to penetrating intracortical electrodes. It requires a surgical procedure for electrode placement, but the surgery is less invasive than implanting electrodes directly into the cortical tissue. This reduces the risk of tissue damage and allows for longer-term recordings. Electrocorticography (ECoG) offers several advantages and disadvantages as a neurophysiological technique. The key advantages and disadvantages of ECoG are as follows:

Advantages:

- **High spatial and temporal resolution:** ECoG provides relatively high spatial and temporal resolution compared to non-invasive techniques like electroencephalography (EEG). The electrodes are placed directly on the surface of the brain, allowing for more precise localization of neural activity.

- **Good signal quality:** ECoG signals have a higher signal-to-noise ratio compared to EEG. The electrodes in ECoG are in closer proximity to the cortical tissue, resulting in cleaner and more reliable neural signals.

- **Localization of cortical activity:** ECoG allows for the identification and localization of specific cortical areas involved in various functions, such as motor control or language processing. This information can be valuable in both clinical and research contexts.

- **Long-term recordings:** ECoG can provide stable and long-term recordings, as the electrodes are placed on the surface of the brain rather than being implanted deep into the cortical tissue. This can be beneficial for studying brain dynamics over extended periods.

- **Potential for clinical applications:** ECoG has been used in clinical settings for mapping epileptic foci, preoperative planning for epilepsy surgery, and monitoring brain activity during surgical procedures.

Disadvantages:

- **Invasiveness:** ECoG requires a surgical procedure to place the electrode grid on the surface of the brain. Although it is considered less invasive than intracortical techniques, it still carries some surgical risks, such as infection or bleeding.

- **Limited coverage:** The electrode grid used in ECoG covers a limited area of the brain's surface, typically a few square centimeters. This restricts the spatial coverage and may not capture activity from deeper cortical regions or subcortical structures.

- **Limited portability:** ECoG systems typically require external recording and amplification equipment, which limits the portability of the technique. It is not as easily deployable for ambulatory or real-world applications compared to non-invasive techniques like EEG.

- **Ethical considerations:** As with any invasive technique, ethical considerations must be taken into account, including obtaining informed consent, ensuring participant safety, and addressing privacy concerns related to the use of ECoG data.

1.2.3 Invasive BCI

Invasive Brain-Computer Interfacing (BCI) refers to the use of surgical techniques to implant electrodes or other devices directly into the brain in order to establish a direct communication pathway between the brain and an external device or computer system. This approach allows for the recording and interpretation of neural activity and enables individuals to control external devices or receive sensory feedback directly from the brain.

1.2.3.1 Penetrating Intracortical Electrodes

Penetrating intracortical electrodes are a type of neuroprosthetic technology that involves the implantation of fine electrodes directly into the cortical tissue of the brain [39] [40]. Unlike surface-based techniques such as electrocorticography (ECoG), which place electrodes on the brain's surface, penetrating intracortical electrodes are inserted into the brain to record or stimulate neural activity at a more localized level. The advantages and disadvantages associated with penetrating intracortical electrodes are as follows:

Advantages:

- **Enhanced spatial resolution:** Penetrating intracortical electrodes can provide high spatial resolution due to their close proximity to individual neurons. This allows for more precise recording and stimulation of specific brain regions or even individual neurons.

- **Long-term stability:** Once implanted, penetrating intracortical electrodes can provide stable and reliable recordings over extended periods of time. This stability is particularly important for long-term neuroprosthetic applications, where consistent and accurate neural signals are crucial.

- **Improved selectivity:** The fine, penetrating nature of these electrodes enables them to selectively target specific populations of neurons or neural circuits. This selectivity can enhance the precision of decoding neural signals and enable more accurate control of prosthetic devices or other applications.

- **Fine temporal resolution:** In addition to spatial resolution, intracortical electrodes can capture neural activity with high temporal resolution. This is important for accurately tracking the timing and dynamics of neural processes, such as motor control or sensory perception.

Disadvantages:

- **Invasive surgical procedure:** Implanting penetrating intracortical electrodes requires a more invasive surgical procedure compared to surface-based techniques like ECoG. This increases the risk of complications, such as infection, bleeding, or damage to brain tissue during the implantation process.

- **Tissue damage and immune response:** The insertion of electrodes into the brain tissue can cause localized damage and trigger an immune response. This response can lead to tissue scarring or inflammation, potentially affecting the stability and longevity of the electrode recordings.

- **Limited coverage:** Penetrating intracortical electrodes typically provide recordings from a limited number of sites or channels within the brain. This restricts the spatial coverage compared to techniques like ECoG, which can cover larger surface areas.

- **Technical challenges:** Penetrating intracortical electrodes pose technical challenges in terms of maintaining stable and reliable recordings over time. Factors such as electrode drift, signal attenuation, and tissue-electrode interface stability need to be carefully managed to ensure optimal performance.

1.3 EEG Processing

EEG is a very powerful tool when it comes to the research and analysis of the brain. This technique is not only very popular but also very powerful in the sense that it can provide a pretty deep understanding of the functioning of the brain. The signals obtained by this technique is not however directly used for analysis it is required to pass these signals through a process to get the clean signals and also the most out of it. A general EEG processing framework is shown in the **Fig 1.13**.

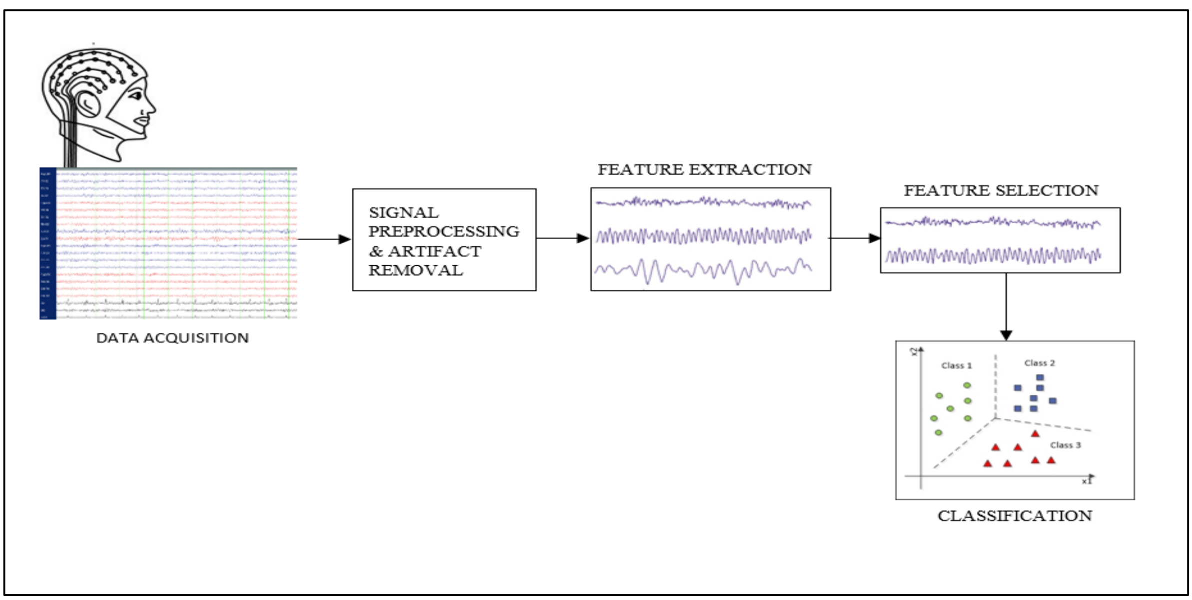

Fig. 1.13. EEG Processing framework

1.3.1 Data Acquisition

EEG (electroencephalography) data acquisition involves the process of recording electrical brain activity using electrodes placed on the scalp. The acquisition system typically consists of electrodes, amplifiers, analog-to-digital converters (ADCs), and a recording device or software. The key steps involved in EEG data acquisition are:

- **Electrode Placement:** EEG electrodes are positioned on the scalp according to standardized electrode placement systems such as the International 10-20 system or 10-10 system. The specific electrode positions depend on the research or clinical protocol and the desired brain regions to be monitored.

- **Electrode Preparation:** Before applying the electrodes, the scalp is usually cleaned to remove any oil, dirt, or dead skin cells that may interfere with the electrode-skin contact. Conductive gel or paste is then applied to improve the electrical conductivity between the electrode and the scalp.

- **Amplification:** The EEG signals picked up by the electrodes are typically very small (microvolts) and require amplification to increase their strength. EEG amplifiers are designed to amplify the weak electrical signals while minimizing noise and interference. The amplifiers may also include filters to remove unwanted frequencies outside the range of interest.

- **Analog-to-Digital Conversion:** The amplified analog EEG signals are then converted into digital form using analog-to-digital converters (ADCs). The ADC samples the continuous analog signal at a specific sampling rate (typically ranging from 100 to 1000 Hz) and assigns numerical values to represent the signal amplitude at each sample point. The choice of sampling rate depends on the desired temporal resolution and the bandwidth of the EEG signals being recorded.

- **Signal Recording:** The digitized EEG data is stored on a recording device or computer for further processing and analysis. The data is typically saved in a standardized file format, such as EDF (European Data Format) or BDF (Biosemi Data Format), which allows for compatibility with various EEG analysis software.

It's worth noting that during the data acquisition process, it's important to maintain a controlled environment to minimize artifacts and noise. Factors such as patient or participant movement, environmental electrical noise, and equipment-related artifacts can affect the quality of the recorded EEG signals. Taking precautions, such as reducing electromagnetic interference, properly grounding the equipment, and ensuring comfortable and stationary positioning of the participant, can help improve data quality. Also the number and type of electrodes used in EEG data acquisition can vary depending on the specific research or clinical requirements. One may choose from different electrode configurations, including standard configurations with a limited number of channels or high-density arrays that provide more spatial coverage of the scalp.

1.3.2 Signal Processing and Artifact Removal

Preprocessing of EEG signals is a crucial step in analyzing and interpreting the data. It involves various techniques to clean and enhance the raw EEG signals, reducing noise, artifacts, and other unwanted components that can interfere with the accurate analysis of brain activity. The common preprocessing steps for EEG signals:

- **Filtering:** Filtering is used to remove noise and unwanted frequency components from the EEG signals. Two types of filters are commonly applied:

i. **Low-pass filter:** It attenuates high-frequency noise and artifacts while allowing low-frequency brain activity to pass through. The cutoff frequency is typically set below 30 Hz.

ii. **High-pass filter:** It removes low-frequency drifts and baseline fluctuations, enhancing the higher frequency components that contain the desired brain activity. The cutoff frequency is typically set above 0.5 Hz.

iii. Additional filters, such as band-pass filters and notch filters, can be used to further target specific frequency ranges or remove power line interference (50 or 60 Hz) if present.

- **Artifact Removal:** EEG signals can be contaminated by various artifacts, such as eye blinks, eye movements, muscle activity, and electrode pops. These artifacts can distort the underlying brain activity. Techniques for artifact removal include:

i. **Independent Component Analysis (ICA):** ICA decomposes the EEG signals into independent components, allowing the separation of brain-related activity from artifacts. Components corresponding to artifacts can be identified and removed [40].

ii. **Template-based or regression methods:** Specific artifacts like eye blinks can be detected using predefined templates or regression techniques and subsequently subtracted from the EEG signals [41].

- **Epoching:** EEG data is often divided into epochs or segments around specific events or stimuli. Epoching is important for event-related analysis, as it allows the extraction of event-related potentials (ERPs) and other transient brain responses. Segments can be extracted based on stimulus onset, response onset, or other relevant triggers.

- **Baseline Correction:** A baseline correction is applied to remove the average or median signal amplitude of a specific period before the event or stimulus onset. This step helps in normalizing the data and reducing the influence of baseline fluctuations.

- **Re-referencing:** EEG signals are typically recorded with respect to a reference electrode. Re-referencing involves transforming the recorded data to a new reference point, which can help in reducing common noise sources or spatial biases. Common re-referencing options include average referencing (referencing to the average of all electrodes) or using a reference electrode located on a neutral site (e.g., linked mastoids or a nose reference).

- **Interpolation:** In cases where electrodes are faulty or missing, interpolation methods can be employed to estimate the signal at these locations based on the surrounding electrodes' activity. This ensures a complete set of electrode data for subsequent analysis.

These preprocessing steps may vary depending on the specific research question, experimental design, or analysis approach. There are several software packages and programming libraries available (e.g., EEGLAB, MNE-Python, FieldTrip) that provide tools for EEG preprocessing and analysis.

1.3.3 Feature Extraction

The feature extraction is done the preprocessed signals and various features are mined from the signals for training the classification models and also to get various insights about the data. The features extracted from the signals are either time domain features, frequency domain features, wavelet domain features (time-frequency), empirical mode decomposition features or the spatial features. These features provide various information about various domains and are very useful in the upcoming stages.

1.3.3.1 Time Domain Features

Time domain features in EEG refer to characteristics of the EEG signals that are analyzed directly in the time domain. These features provide information about the temporal properties of the EEG waveform and can reveal important patterns or characteristics of brain activity. These are some common time domain features used in EEG analysis:

- **Amplitude:** The amplitude of the EEG signal represents the magnitude or strength of the electrical activity. It can be calculated as the peak-to-peak amplitude, which measures the difference between the highest positive peak and the lowest negative peak within a specific time window. Amplitude features can provide insights into the overall signal strength and intensity.

- **Mean:** The mean of the EEG signal represents the average value of the waveform within a given time window. It provides information about the baseline or average level of the signal. Mean features can be useful for comparing the overall signal levels between different conditions or time periods.

- **Variance:** Variance quantifies the variability or spread of the EEG signal values around the mean. It measures the average squared deviation from the mean. Variance features provide information about the signal's stability or irregularity. Higher variance may indicate more complex or diverse brain activity.

- **Skewness:** Skewness measures the asymmetry of the EEG signal's distribution. It quantifies the degree to which the signal is skewed to the left or right of the mean. Positive skewness indicates a longer tail on the right side, while negative skewness indicates a longer tail on the left side. Skewness features can provide insights into the shape or asymmetry of the EEG waveform.

- **Kurtosis:** Kurtosis characterizes the shape of the EEG signal's distribution. It measures the peakedness or flatness of the distribution relative to a normal distribution. High kurtosis indicates a more peaked distribution, while low kurtosis indicates a flatter distribution. Kurtosis features can capture the sharpness or broadness of specific EEG waveform patterns.

- **Zero-crossing rate:** Zero-crossing rate calculates the number of times the EEG signal crosses the zero-axis within a specific time window. It provides information about the frequency or rate of signal fluctuations and can be useful for detecting rapid changes or transitions in the EEG waveform.

- **Hjorth Parameters:** Hjorth parameters are descriptors of the EEG signal that capture its complexity, mobility, and activity [42]. They include the activity parameter (related to signal power), the mobility parameter (related to signal frequency), and the complexity parameter (related to signal waveform irregularity). Hjorth parameters provide insights into the dynamic properties of the EEG waveform.

These time domain features are often computed over specific segments or epochs of the EEG signal, such as event-related potentials (ERPs) or other time-locked responses. They can be used for various purposes, including comparing different conditions, detecting abnormalities, or characterizing specific temporal patterns in EEG data. Additionally, combining time domain features with other feature extraction methods, such as frequency or connectivity measures, can provide a more comprehensive analysis of EEG signals.

1.3.3.2 Frequency Domain Features

Frequency domain features in EEG analysis involve analyzing the EEG signals in the frequency domain to extract relevant information about the spectral characteristics of brain activity. These features provide insights into the distribution of power or energy across different frequency bands, which can be indicative of specific brain states or processes. Some common frequency domain features used in EEG analysis are as follows:

- **Power Spectral Density (PSD):** PSD represents the distribution of power or energy in the EEG signal across different frequency bands. It provides information about the relative strength or activity in different frequency ranges. Common frequency bands of interest include delta (0.5-4 Hz), theta (4-8 Hz), alpha (8-13 Hz), beta (13-30 Hz), and gamma (above 30 Hz). PSD features can be derived by calculating the power within each frequency band or by computing the power spectral density using techniques such as the Fast Fourier Transform (FFT) or the Welch method.

- **Band Power:** Band power refers to the total power or energy within a specific frequency band. It represents the sum of power across the frequencies within the band of interest. Band power features can be calculated for different frequency bands, such as alpha power, beta power, or theta power. They provide information about the relative contribution or dominance of specific frequency ranges.

- **Peak Frequency:** Peak frequency is the frequency with the highest power or activity in the EEG spectrum. It represents the frequency at which the EEG signal exhibits the most pronounced oscillatory activity. Identifying the peak frequency can be useful for understanding dominant brain rhythms and their relationship to specific cognitive or perceptual processes.

- **Relative Power:** Relative power represents the proportion or percentage of power within a specific frequency band relative to the total power across all frequency bands. It provides a normalized measure of power distribution, allowing for comparisons between different frequency bands or conditions. Relative power features can indicate the relative engagement or dominance of specific frequency ranges.

- **Spectral Edge Frequency:** The spectral edge frequency is the frequency below which a certain percentage of the total power in the EEG signal is contained. For example, the 95% spectral edge frequency represents the frequency below which 95% of the total power is found. Spectral edge frequency features provide information about the dominant or cutoff frequency in the EEG spectrum.

- **Spectral Entropy:** Spectral entropy measures the complexity or irregularity of the EEG spectrum. It quantifies the amount of information or uncertainty in the power distribution across different frequencies. Higher spectral entropy indicates a more complex or diverse power distribution, while lower spectral entropy indicates a more focused or predictable distribution.

These frequency domain features are computed by analyzing the EEG signals using spectral analysis techniques such as the FFT, periodogram, or wavelet transform. They are often used to characterize specific frequency bands or assess changes in power distribution associated with different cognitive, emotional, or pathological states. Combining frequency domain features with time domain or connectivity features can provide a more comprehensive understanding of EEG signals and their underlying brain dynamics.

1.3.3.3 Wavelet Domain Features

Wavelet domain features in EEG analysis involve transforming the EEG signals from the time domain to the wavelet domain using wavelet transforms. Wavelet transforms provide a time-frequency representation of the EEG signals and allow for the extraction of relevant features at different scales. These features provide both the time and frequency domain resolutions making them a very useful set of parameters for training the classification models. Here are some common wavelet domain features used in EEG analysis:

- **Wavelet Coefficients:** Wavelet coefficients represent the contribution of different wavelet basis functions at different scales and positions in the time-frequency plane. By decomposing the EEG signal using wavelet transforms, wavelet coefficients can be obtained. These coefficients carry information about the local energy or power at specific time-frequency locations and can be used as features for further analysis.

- **Wavelet Power Spectrum:** The wavelet power spectrum represents the distribution of power or energy in the wavelet domain across different scales and positions. It provides a time-frequency representation of the EEG signals similar to the spectrogram but with better time-frequency localization. Wavelet power spectrum features capture the distribution of power across different scales and can provide insights into the temporal dynamics of oscillatory activity.

- **Wavelet Entropy:** Wavelet entropy measures the complexity or irregularity of the wavelet coefficients or wavelet power spectrum. It quantifies the amount of information or uncertainty in the distribution of power across different scales and positions. Wavelet entropy features provide insights into the complexity or variability of the EEG signals at different scales and can be used to assess changes in brain activity patterns.

- **Wavelet Phase Synchronization:** Wavelet phase synchronization measures the phase coherence or synchronization between different scales or frequencies in the wavelet domain. It quantifies the degree of phase locking between different oscillatory components. Wavelet phase synchronization features provide information about the coordination or interaction between different frequency bands and can be useful for assessing functional connectivity in EEG signals.

- **Wavelet Fractal Dimension:** Wavelet fractal dimension characterizes the self-similarity or fractal properties of the EEG signals in the wavelet domain. It measures the scaling behavior of the wavelet coefficients at different scales. Wavelet fractal dimension features provide insights into the complexity or scaling properties of the EEG signals and can be used to distinguish between different brain states or pathologies.

These wavelet domain features can be extracted using various wavelet transforms, such as the Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT), or Wavelet Packet Transform (WPT). Different wavelet families, such as the Morlet wavelet, Daubechies wavelet, or Haar wavelet, can be used depending on the specific requirements of the analysis. Wavelet domain features complement time and frequency domain features and can provide a more detailed and localized representation of the EEG signals' time-frequency characteristics.

1.3.3.4 Empirical Mode Decomposition Features

Empirical Mode Decomposition (EMD) is a data-driven decomposition method used to analyze non-stationary and nonlinear signals, such as EEG signals. EMD decomposes a signal into a finite number of intrinsic mode functions (IMFs), which are functions with well-behaved amplitude and frequency modulation properties [43]. Some common features derived from Empirical Mode Decomposition (EMD) in EEG analysis is as follows:

- **Instantaneous Frequency:** Instantaneous frequency represents the local frequency of an IMF at each point in time. It provides information about the temporal dynamics of the signal's frequency content. Instantaneous frequency can be calculated by differentiating the phase of the IMF with respect to time. It is useful for capturing changes in frequency over time, such as frequency modulation or transient events.

- **Hilbert Spectrum:** The Hilbert spectrum represents the distribution of energy or power in the time-frequency plane for each IMF. It provides a detailed time-frequency representation of the signal. Hilbert spectrum features capture the energy distribution across different frequency bands and their temporal variation. They can reveal transient events, oscillatory patterns, or changes in spectral power.

- **Hilbert-Huang Transform (HHT) Energy:** HHT energy represents the energy content of each IMF in the EMD decomposition. It quantifies the contribution of each IMF to the overall signal energy. HHT energy features can be calculated by summing the squared amplitude values of each IMF. They provide information about the relative energy distribution across different intrinsic mode functions.

- **Kurtosis:** Kurtosis measures the peakedness or flatness of the probability distribution of each IMF. It quantifies the degree of non-Gaussianity or departure from a Gaussian distribution. Higher kurtosis indicates a more peaked or spiky distribution, while lower kurtosis indicates a flatter distribution. Kurtosis features can capture the nonlinearity or complexity of each IMF.

- **Bandwidth:** The bandwidth represents the range of frequencies covered by each IMF. It provides information about the spectral width or spread of the signal at different time points. Bandwidth features can be calculated by measuring the width of the frequency spectrum within a certain fraction of the maximum amplitude. They can help identify broad or narrowband components in the signal.

- **Variance:** Variance measures the variability or spread of each IMF. It quantifies the average squared deviation from the mean value. Variance features provide information about the stability or irregularity of each IMF. Higher variance indicates greater variability or complexity, while lower variance indicates more consistent or regular behavior.

These features derived from Empirical Mode Decomposition (EMD) capture different aspects of the non-stationary and nonlinear characteristics of EEG signals. They can be used for various purposes, such as characterizing transient events, analyzing oscillatory patterns, or identifying changes in spectral content over time. Additionally, combining EMD features with other feature extraction methods, such as time domain or frequency domain features, can provide a more comprehensive analysis of EEG signals.

1.3.3.5 Spatial Features

Spatial features in EEG analysis refer to characteristics that capture the spatial distribution or topographic patterns of electrical activity across different scalp locations. These features provide insights into the spatial organization of brain activity and can help identify regional differences or abnormalities. Here are some common spatial features used in EEG analysis:

- **Topographic Maps:** Topographic maps represent the distribution of EEG activity across different scalp locations. These maps visually depict the relative amplitude or power of the EEG signal at each electrode site. They provide an overview of the spatial patterns of electrical activity and can reveal localized or widespread changes in brain activity.
- **Electrode Location-based Analysis:** Spatial features can be derived by considering the specific electrode locations on the scalp. This involves analyzing the activity at specific electrode sites or regions of interest (ROIs). For example, features can be extracted based on the mean or power of the EEG signal at frontal, central, or occipital electrode clusters. This allows for the characterization of regional differences in brain activity.
- **Scalp Potential Distribution:** Scalp potential distribution features quantify the spatial patterns of voltage or potential differences across different electrode sites. These features can be calculated by comparing the voltage values at different electrode pairs or by analyzing the spatial gradients of the voltage distribution. Scalp potential distribution features provide insights into the spatial relationships and interactions between different brain regions.
- **Connectivity Measures:** Spatial features can also be derived from measures of functional connectivity between different electrode pairs. Connectivity measures quantify the statistical dependence or interaction between pairs or groups of EEG channels. Spatial connectivity features capture the strength or coherence of the connectivity between different brain regions and can help identify functional networks or communication pathways.

These spatial features can be used to analyze EEG data in various research contexts, such as studying brain dynamics, identifying brain disorders, or investigating the effects of cognitive tasks or stimuli. Combining spatial features with other feature extraction methods, such as time domain or frequency domain features, can provide a more comprehensive analysis of EEG signals and their underlying brain networks.

1.3.4 Feature Selection

Feature selection in EEG analysis involves identifying the most informative and relevant features from the vast amount of available EEG data. By selecting a subset of features, the dimensionality of the data can be reduced, leading to improved computational efficiency and potentially enhanced classification or regression performance. Here are some commonly used approaches for feature selection in EEG analysis:

- **Univariate Feature Selection:** This approach assesses the statistical significance of individual features by measuring their relationship with the target variable. Various statistical tests, such as t-tests, analysis of variance (ANOVA), or mutual information, can be used to rank the features based on their relevance. Features with the highest scores are selected for further analysis.

- **Wrapper Methods:** Wrapper methods evaluate the performance of a specific machine learning algorithm using different subsets of features. They involve a search algorithm, such as forward selection, backward elimination, or genetic algorithms, to find the optimal subset of features that maximizes the algorithm's performance. Wrapper methods take into account the interaction between features and can lead to improved classification or regression accuracy.

- **Embedded Methods:** Embedded methods incorporate feature selection as an inherent part of the learning algorithm itself. For instance, certain machine learning algorithms, like decision trees or LASSO (Least Absolute Shrinkage and Selection Operator) [44], perform feature selection during their training process by assigning weights or importance scores to each feature. These methods simultaneously learn the model and select the most informative features.

- **Dimensionality Reduction Techniques:** Dimensionality reduction techniques, such as Principal Component Analysis (PCA) [45] or Independent Component Analysis (ICA), transform the original EEG data into a lower-dimensional space. In this reduced space, only a subset of principal or independent components that explain the majority of the variance in the data are retained as features. These techniques aim to retain the most important information while reducing the dimensionality.

- **Filter Methods:** Filter methods evaluate the relevance of features based on their intrinsic properties, without considering a specific machine learning algorithm. Common filter methods include correlation-based feature selection, mutual information-based selection, or chi-square tests [46]. Features are ranked or scored according to their relevance, and a threshold is set to select the top-ranked features.

- **Regularization Techniques:** Regularization techniques, such as L1 or L2 regularization (e.g., LASSO or Ridge regression), introduce a penalty term into the learning algorithm that encourages sparsity in the feature weights. These methods automatically select the most informative features by driving the weights of irrelevant or redundant features towards zero.

- **Genetic Algorithms:** Genetic algorithms (GAs) are optimization techniques inspired by natural selection and genetics [47]. They can be used for feature selection in machine learning. GAs start with an initial population of feature subsets, evaluate their fitness using an objective function, and then apply genetic operators like selection, crossover, and mutation to evolve better feature subsets over generations. GAs explore the search space efficiently and find optimal or near-optimal feature subsets. However, they require computational resources and careful parameter tuning for good results.

- **Fuzzy Logic:** Fuzzy logic is a mathematical framework that deals with uncertainty and imprecision in data [48]. It can also be applied to feature selection in machine learning. Fuzzy logic-based feature selection considers the degree of membership or relevance of each feature to the target variable. By assigning membership values to features based on their importance, fuzzy logic helps determine which features contribute most significantly to the classification task. Fuzzy logic-based feature selection can handle situations where features have overlapping or ambiguous boundaries, providing a more nuanced approach to feature selection.

Feature selection should be combined with proper cross-validation or resampling techniques to ensure the reliability and generalizability of the selected features.

1.3.5 Classification

The final step in the EEG processing is the classification of data into various classes using the classifiers trained by the selected features. These classifiers have many types and have evolved with the time in terms of both accuracy and flexibility. The three types of classifiers used are traditional, Neural Network based and Support Vector Machines (SVM).

1.3.5.1 Traditional Classifiers

Traditional classifiers are machine learning algorithms that are commonly used for data classification tasks. These classifiers are based on well-established algorithms and techniques and have been widely studied and applied in various domains. Traditional classifiers include algorithms such as logistic regression, support vector machines (SVM), decision trees, k-nearest neighbors (k-NN), naive Bayes, and random forests. These classifiers employ different mathematical and statistical principles to learn patterns and relationships in the data and make predictions or assign class labels to new data points. Traditional classifiers are often interpretable, easy to implement, and suitable for a wide range of classification problems. However, their performance can vary depending on the characteristics of the data, the complexity of the problem, and the availability of labeled training data. Some of the commonly used traditional classifiers are as follows:

- **Linear Discriminant Analysis (LDA):** It is a dimensionality reduction and classification technique commonly used in machine learning and pattern recognition. LDA aims to find a linear combination of features that maximizes the separation between different classes while minimizing the within-class scatter [49]. It projects the original high-dimensional data onto a lower-dimensional subspace, where the classes are well-separated. LDA assumes that the data follows a Gaussian distribution and that the classes have equal covariance matrices. LDA is particularly effective when the classes are well-separated and the within-class variance is small compared to the between-class variance. LDA can be used for both binary and multi-class classification problems and is often used as a preprocessing step before applying classification algorithms.

- **Naïve Bayes:** It is a simple yet effective probabilistic classifier that is based on Bayes' theorem with the assumption of feature independence. It calculates the probability of each class given the input features and assigns the class with the highest probability. Naive Bayes classifiers are computationally efficient and work well with high-dimensional data. They can handle both categorical and numerical features and are robust to irrelevant or redundant features. However, the assumption of feature independence may not hold in all cases, and Naive Bayes classifiers may suffer from the "naive" assumption. Despite this limitation, Naive Bayes classifiers are widely used in various applications, including text classification, spam detection, and sentiment analysis. They are easy to implement and provide good results in many practical scenarios.
- **Mahalanobis Distance Classifier**: It is a distance-based classification algorithm that takes into account the correlation between features and the variability within each class. It measures the distance between a data point and the class centroids, considering the covariance matrix of the features. The Mahalanobis distance accounts for the inter-feature correlations and scales each feature based on its variability within the class [50]. The classifier assigns the data point to the class with the shortest Mahalanobis distance. The Mahalanobis distance classifier is especially useful when dealing with data that exhibits different variances and correlations among the features in different classes. It is widely used in pattern recognition and classification tasks, particularly when dealing with high-dimensional data. However, the classifier assumes that the data follows a multivariate Gaussian distribution and requires the estimation of covariance matrices, which can be challenging in cases with limited data or when dealing with high-dimensional data.
- **Bayes Quadratic Classifier**: Also known as the Quadratic Discriminant Analysis (QDA), is a classification algorithm that assumes that each class follows a multivariate Gaussian distribution with its own mean and covariance matrix [51]. It computes the likelihood of a data point belonging to each class using the class-specific Gaussian distributions and applies Bayes' theorem to assign the data point to the class with the highest posterior probability. Unlike the Naive Bayes classifier, QDA does not assume feature independence, allowing it to capture more complex relationships between features. QDA can model non-linear decision boundaries and handle situations where classes have different covariances. However, QDA requires estimating the covariance matrices, which can be computationally expensive, especially with high-dimensional data. It is commonly used in pattern recognition

and classification tasks where the assumption of Gaussian distributions is reasonable and where capturing complex feature interactions is important.

- **K Nearest Neighbor (k-NN)**: The k-NN (k-Nearest Neighbors) classifier is a simple and intuitive non-parametric algorithm used for classification [52]. It operates based on the idea that data points with similar features tend to belong to the same class. In k-NN, the class of a new data point is determined by examining the class labels of its k nearest neighbors in the feature space. The value of k is a user-defined parameter that influences the decision-making process. The class label is determined through majority voting among the k neighbors, where each neighbor's vote carries equal weight.
- **Hidden Markov Model (HMM)**: The HMM (Hidden Markov Model) classifier is a statistical model widely used for sequence classification tasks, such as speech recognition, gesture recognition, and natural language processing. HMMs are based on the concept of a Markov process, where the underlying system transitions between a set of hidden states. The states emit observable symbols or features, which are used to make predictions or assign class labels. In the HMM classifier, the training phase involves learning the parameters of the model, including the initial state probabilities, transition probabilities between states, and emission probabilities for each state. The Viterbi algorithm is commonly used to find the most likely sequence of hidden states given the observed sequence of features. During classification, the HMM classifier calculates the likelihood of the observed sequence of features for each class using the trained HMMs. The class with the highest likelihood is assigned to the sequence. HMM classifiers are effective for problems with sequential data, where the order and dependencies between observations matter. They can handle variable-length sequences and are capable of capturing temporal dynamics. However, HMM classifiers assume that the underlying system follows a Markov process and that the observations are conditionally independent given the hidden states. Additionally, HMMs require a sufficient amount of training data to accurately estimate the model parameters.

1.3.5.2 Neural Network Based Classifiers

These classifiers as the name suggest uses the modern day neural networks for the classification of the data. A neural network consists of various layers of neurons which consists of the activation functions and summation modules. Some commonly used classifiers based on neural network is as follows:

- **MLPNN (Multilayer Perceptron Neural Network):** A type of artificial neural network used for classification tasks. Consists of multiple layers of interconnected neurons [53]. Each neuron applies a non-linear activation function to its inputs. Learns through backpropagation algorithm to adjust weights and biases. Can model complex relationships and non-linear decision boundaries. Requires sufficient training data and may be computationally expensive. Popular choice for various classification problems.

- **PNN (Probabilistic Neural Network):** A type of neural network used for classification tasks. It assigns class labels based on the highest probability calculated using a non-parametric approach. It uses Parzen window estimation to estimate the probability density function of each class. PNN classifiers are computationally efficient during the testing phase but can be memory-intensive during the training phase. They are particularly effective when dealing with high-dimensional data and can handle both numerical and categorical features.

- **RBFNN (Radial Basis Function Neural Network):** A type of neural network used for classification tasks [54]. It consists of three layers: an input layer, a hidden layer with radial basis function neurons, and an output layer. The hidden layer neurons calculate the similarity between input data and their center points using radial basis functions. The output layer performs classification based on the weighted sum of the hidden layer outputs. RBFNN classifiers can model complex relationships and non-linear decision boundaries. They require determining the number and positions of the radial basis functions and training the network using techniques like the k-means algorithm or least squares. RBFNN classifiers are effective for certain types of classification problems, particularly when dealing with separable or overlapping classes.

- **Decision Tree:** A popular machine learning algorithm used for classification tasks. It constructs a tree-like model by recursively partitioning the data based on the values of input features. Each internal node of the tree represents a decision based on a feature, and each leaf node represents a class label. The decision tree classifier uses different splitting criteria, such as information gain or Gini index, to determine the best feature for each node. Decision trees are interpretable, handle both numerical and categorical data, and can capture complex interactions between features. However, they may suffer from overfitting and can be sensitive to small changes in the data. Techniques like pruning and ensemble methods (e.g., random forest) can be applied to mitigate these issues. Decision tree classifiers are widely used in various domains due to their simplicity, interpretability, and good performance in many practical scenarios.

1.3.5.3 Support Vector Machine (SVM)

Support Vector Machine (SVM) is a powerful and versatile machine learning algorithm commonly used for classification and regression tasks. It works by finding an optimal hyperplane that maximally separates the data points of different classes in the feature space. SVMs excel in scenarios where the data is not linearly separable. They achieve this by transforming the input data into a higher-dimensional space using kernel functions. This transformation allows SVMs to capture complex relationships and find non-linear decision boundaries. The key idea behind SVMs is to find the hyperplane that has the largest margin, known as the maximum-margin hyperplane. The margin is the distance between the hyperplane and the closest data points of different classes, called support vectors. By maximizing this margin, SVMs provide a robust decision boundary that generalizes well to unseen data. SVMs can handle high-dimensional data efficiently and are effective in cases where the number of features exceeds the number of samples. They are also suitable for datasets with imbalanced class distributions. SVMs employ a soft-margin approach that allows for some misclassifications, which can help handle noisy or overlapping data. However, SVMs can be computationally expensive, especially with large datasets, as they involve solving a quadratic programming problem. Additionally, SVMs require careful selection of hyperparameters, such as the choice of kernel function and regularization parameter, which can influence the model's performance. Despite these considerations, SVMs have been successfully applied in various domains, including image classification, text categorization, bioinformatics, and many more. Their ability to handle complex decision boundaries, robust generalization performance, and versatility make them a popular choice in the machine learning community.

1.4 Scope of Thesis

The main objective of the thesis is to provide an improvised algorithm for the computation of the Common Spatial Filters (CSP) which will provide the most discriminative features thus improving the classification accuracy of the system. The current work is based in the classification of the Motor Imagery classification. The existing algorithm has been modified to make the solutions more generalized and thus making the algorithm robust and better with smaller datasets as well. The thesis in total provides two new kind of regularization matrices for the existing algorithms and new flexible technique which can be used with multiple matrices in use.

1.5 Thesis Organization

The thesis is organized as follows:

Chapter 2: The fundamentals of the Common Spatial Patterns and Literature Review

Chapter 3: Design of experiment

Chapter 4: Problem Statement and Proposed Solution

Chapter 5: Conclusion and Future Direction

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Chapter 2:

THE FUNDAMENTALS OF COMMON SPATIAL PATTERN AND LITERATURE REVIEW

This chapter deals with the explanation of traditional Common Spatial Pattern (CSP) algorithm and variants of it. The literature survey is also being dealt in this chapter.

2.1 Traditional CSP

The traditional Common Spatial Pattern (CSP) is an optimization problem which uses the following objective function:

$$J(w) = \frac{w^T \Sigma_1 w}{w^T \Sigma_2 w} \tag{1}$$

The function is optimized with the constraint $w^T \Sigma_2 w = 1$ and the filters obtained as w are then used for the optimal discrimination between the classes [1]. The symbols Σ_1 and Σ_2 are the covariance matrices of each class which are obtained using the following formula:

$$\Sigma_i = \frac{\sum X_i X_i^T}{\sum trace\left(X_i X_i^T\right)} \tag{2}$$

Where the value X_i is the sample matrix of each class containing all the samples versus the electrodes used for the measurement. In order to solve this optimization problem we use the Lagrangian multiplier method. Lagrangian multipliers are a mathematical technique used to solve optimization problems with constraints [2]. The basic idea behind Lagrangian multipliers is to convert a constrained optimization problem into an unconstrained optimization problem by adding an additional term to the objective function, called the Lagrangian function. The Lagrangian function includes a set of variables, called Lagrange multipliers, which act as weights on the constraints. By optimizing the Lagrangian function with respect to the original variables and Lagrange multipliers, one can find the optimal solution that satisfies the constraints. Lagrangian multipliers are widely used in economics, engineering, physics, and other fields to solve a variety of optimization problems. The given problem can be expressed as the following Lagrangian equation:

$$L(\lambda, w) = w^T \Sigma_1 w - \lambda (w^T \Sigma_2 w - 1) \tag{3}$$

This problem is can now be solved by taking the derivative of the Lagrangian equation above and equating it to zero.

$$\begin{aligned} &\frac{\partial L(\lambda, w)}{\partial w} = 2w\Sigma_1 - 2\lambda w\Sigma_2 = 0 \\ &\Rightarrow \lambda = \Sigma_1 {\Sigma_2}^{-1} \end{aligned} \tag{4}$$

The required filters are now the eigenvectors corresponding to the highest and lowest eigenvalues of the matrix $M = \Sigma_1 {\Sigma_2}^{-1}$ which gives the filters with highest and lowest covariance and thus provides the optimal discrimination between the classes. The pairs of filters are selected and the data is projected on the filters and the logarithm of variance of these projections are then used as features for classification of unknown test samples.

2.2 Regularized CSP

Regularization is a technique used in artificial intelligence (AI) to prevent overfitting of a machine learning model. Overfitting occurs when a model becomes too complex and fits the training data too closely, resulting in poor performance on new, unseen data. Regularization works by adding a penalty term to the objective function that the machine learning algorithm is trying to optimize. The penalty term encourages the model to have simpler weights, or to favor certain weights over others, thus reducing the complexity of the model. There are several types of regularization techniques, including L1 regularization (Lasso), L2 regularization (Ridge), and Elastic Net regularization, among others. L1 regularization penalizes the sum of the absolute values of the weights, while L2 regularization penalizes the sum of the squared weights. Elastic Net regularization combines both L1 and L2 regularization to balance the benefits of each. Regularization is an important technique in AI because it helps to prevent overfitting and improve the generalization of machine learning models. By reducing the complexity of the model, regularization can help to improve its performance on new, unseen data. To regularize the CSP algorithm a technique was presented in [3]. The regularized objective function is given as follows:

$$J_{p1}(w) = \frac{w^T \Sigma_1 w}{w^T \Sigma_2 w + \alpha P(w)} \tag{5}$$

The term $P(w)$ is the actual penalty term and the term α is the weight of the penalty term associated. The weight has to be adjusted properly in order to get the optimal solution. This problem is also similar to that of the traditional CSP and we have the Lagrangian equation as:

$$L_{p1}(\lambda, w) = w^T \Sigma_1 w - \lambda(w^T \Sigma_2 w + \alpha P(w) - 1) \tag{6}$$

In here the constraint is $w^T \Sigma_2 w + \alpha P(w) = 1$ whereas the penalty term is considered to be quadratic to make the differentiable $P(w) = w^T K w$ and the solution of the optimization problem is obtained by differentiating the Lagrangian and equating it to zero.

$$\begin{aligned} &\frac{\partial L(\lambda, w)}{\partial w} = 2w\Sigma_1 - 2\lambda w\Sigma_2 - 2\alpha K w = 0 \\ &\Rightarrow \lambda = \Sigma_1 ({\Sigma_2}^{-1} + \alpha K)^{-1} \end{aligned} \tag{7}$$

This time the optimization problem is not solved using only this objective function as this will only regularize class 2 with respect to class 1 and not the vice versa. Therefore to get the optimal solution we need to solve another objective function simultaneously which is given as:

$$J_{p2}(w) = \frac{w^T \Sigma_2 w}{w^T \Sigma_1 w + \alpha P(w)} \tag{8}$$

Now we solve this equation as well in the similar fashion and thus we get:

$$\begin{aligned} &\frac{\partial L(\lambda, w)}{\partial w} = 2w\Sigma_2 - 2\lambda w\Sigma_1 - 2\alpha K w = 0 \\ &\Rightarrow \lambda = \Sigma_2 ({\Sigma_1}^{-1} + \alpha K)^{-1} \end{aligned} \tag{9}$$

The required filters are then obtained by combining the eigenvectors corresponding to the largest eigenvalue of both $M_1 = \Sigma_1(\Sigma_2 + \alpha K)^{-1}$ and $M_2 = \Sigma_2(\Sigma_1 + \alpha K)^{-1}$ and the filter matrix size increases as well. The samples are then projected on the filters to obtain the logarithm of the variance of the projections and then used as feature for the classification of data. This technique provides a flexible approach of regularizing the functions as K can be replaced with many kinds of penalty matrices.

2.3 Regularized CSP variants

There are many types of regularized CSP variants which make use of various types of norms and penalty matrices for penalizing the objective function of the traditional CSP and also the covariance matrices of the classes in consideration.

2.3.1 L1 Regularized CSP

L1 regularization, also known as Lasso regularization, is a technique used in machine learning to prevent overfitting by adding a penalty term to the cost function that is proportional to the sum of the absolute values of the model's parameters [4]. This penalty encourages the model to have sparse coefficients, meaning that it will only select the most important features and ignore the rest. L1 regularization is particularly useful when dealing with high-dimensional datasets with many features, as it can help to reduce the complexity of the model and prevent overfitting. It is also used for feature selection, as it can identify which features are most relevant to the model. In contrast to L2 regularization, which adds a penalty proportional to the sum of the squared values of the model's parameters, L1 regularization tends to produce sparser solutions and can be more effective in situations where the data has many irrelevant or redundant features. However, it can be less computationally efficient than L2 regularization, as it does not have a closed-form solution and requires the use of optimization algorithms. The L1 regularization is applied to the CSP in [5]. Let there be two classes of data which are required to be segregated. The class A and B has the trial segments given by $\boldsymbol{A}^1, \dots, \boldsymbol{A}^{t_a} \in \mathbb{R}^{c \times l}$ and $\boldsymbol{B}^1, \dots, \boldsymbol{B}^{t_b} \in \mathbb{R}^{c \times l}$ where t_a and t_b are the total number of trials for class A and B respectively, c is the number of EEG channels used for taking the samples and l is the total number of samples per trial. Trial matrices are formed for each class by combining all the trials in of class into a single matrix and we get two new trial matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ where $\boldsymbol{A} = (\boldsymbol{A}^1, \dots, \boldsymbol{A}^{t_a})$ and $\boldsymbol{B} = (\boldsymbol{B}^1, \dots, \boldsymbol{B}^{t_b})$**.** To make the equations simple the matrices are relabeled as $\boldsymbol{A} = (\boldsymbol{a_1}, \dots, \boldsymbol{a_m})$ and $\boldsymbol{B} = (\boldsymbol{b_1}, \dots, \boldsymbol{b_n})$ where m and n are the total number of samples in all the trials for each class and are given by $m = l \times t_a$ and $n = l \times t_b$. The traditional CSP is regularized using L-1 norm and the new objective function becomes:

$$J(w) = \frac{\left\| w^T A \right\|_1}{\left\| w^T B \right\|_1} = \frac{\sum_{i=1}^{m} \left| w^T a_i \right|}{\sum_{i=1}^{n} \left| w^T b_i \right|} \tag{10}$$

The new objective function however is not differentiable and therefore iterative solution is used to obtain the solution. The iterative solution follows the following steps:

- Set $t = 0$ and $w(t)$ as a vector of dimension c and unit rescaled to unit length
- Define to polarity functions $p_i(t) = sgn(w^T(t)a_i)$ and $q_i(t) = sgn\left(w^T(t)b_j\right)$ where $sgn(.)$ is the signum function.
- Define $\boldsymbol{d}(t) = \frac{\sum_{i=1}^{m} p_i(t)a_i}{\sum_{i=1}^{m}|w^T(t)a_i|} - \frac{\sum_{j=1}^{n} q_j(t)b_j}{\sum_{j=1}^{n}|w^T(t)b_j|}$ and update $w(t+1) = w(t) + \eta d(t)$ and also set $t \leftarrow t + 1$
- If $\tilde{J}(w(t))$ do not increase further at a specified rate then the iteration is stopped and the optimal filter w^* is given by $w^* = w(t)$

This technique provides filters which are robust to the outliers.

2.3.2 Correlation based CSP

There can be a number of penalty terms used for regularizing the objective function of the traditional CSP and one of them is the correlation between the EEG signals of various classes [6]. Correlation is a statistical measure that describes the degree of association between two or more variables [7]. Correlation coefficients range from -1 to +1, with values closer to -1 indicating a strong negative correlation (inverse relationship) and values closer to +1 indicating a strong positive correlation (direct relationship). A correlation of zero indicates no relationship between the variables. Correlation analysis is commonly used in research to examine the relationship between variables and to identify patterns in data. However, it should be noted that correlation does not imply causation, meaning that just because two variables are correlated does not necessarily mean that one causes the other. Let there be two classes class 1 and class 2. Then the EEG signals for all the trials combined for each class is given by $\boldsymbol{A_1} \in \mathbb{R}^{n\times c}$ and $\boldsymbol{A_2} \in \mathbb{R}^{n\times c}$ where n is the number of time samples for all the trials of class 1 and c is the number of EEG channels used for the collection of samples. The correlation matrix is found using the correlation between the average of class 1 and class 2 trials to reduce the computational complexity. The average are found using the following expression:

$$\overline{X}_i = \frac{\sum_{j=1}^{t} X_{ij}}{t} \tag{11}$$

Where, i and j are the class and trial index and t is the number of trials per class. Now the correlation matrix is given as:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1c} \\ r_{21} & r_{22} & \cdots & r_{2c} \\ \vdots & \vdots & \ddots & \vdots \\ r_{c1} & r_{c2} & \cdots & r_{cc} \end{bmatrix} \tag{12}$$

Where $r_{ij} = corr\left(\overline{x_i^1}, \overline{x_j^2}\right), i, j = 1,2, \dots, c$ and $\overline{x_i^1}, \overline{x_j^2}$ denote the ith and jth column of matrices $\overline{\boldsymbol{X_1}}$ and $\overline{\boldsymbol{X_2}}$**.** Then a diagonal matrix is prepared as follows:

$$R_{d1} = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_c \end{bmatrix} \tag{13}$$

Where the entries are calculated as:

$$a_i = \frac{\sum_{j=1}^{c} \left| r_{ij} \right|}{c}, i, j = 1,2,\dots, c \tag{14}$$

Similarly another penalty matrix is prepared and then the objective functions become:

$$J_{p1} = \frac{w^T \Sigma_1 w}{w^T \Sigma_2 w + \alpha w^T R_{d1} w}, J_{p2} = \frac{w^T \Sigma_2 w}{w^T \Sigma_1 w + \alpha w^T R_{d2} w} \tag{15}$$

The Lagrangians are formed and the optimal filters are given by the eigenvectors corresponding to the largest eigenvalue of both $M_1 = \Sigma_1(\Sigma_2 + \alpha R_{d1})^{-1}$ and $M_2 = \Sigma_2(\Sigma_1 + \alpha R_{d2})^{-1}$.

2.3.3 CSP with Tikhonov Regularization

Tikhonov regularization, also known as ridge regression, is a technique used in machine learning and statistical modeling to prevent overfitting by adding a penalty term to the cost function that is proportional to the sum of the squared values of the model's parameters [8] [9]. This penalty encourages the model to have smaller parameter values, which helps to reduce the variance of the model and prevent overfitting. Tikhonov regularization is particularly useful when dealing with ill-conditioned or underdetermined problems, where the data may not provide enough information to uniquely determine the model parameters. It is also used to stabilize numerical computations, as it can prevent small errors in the input data from producing large changes in the output. In contrast to L1 regularization, which adds a penalty proportional to the sum of the absolute values of the model's parameters, Tikhonov regularization tends to produce more stable solutions and can be more effective in situations where all of the features are relevant to the model. However, it may not be as effective at feature selection as L1 regularization. In this regularization the objective function of the traditional CSP is penalized with the penalty matrix of $P(w) = w^T I w$ [3]. This method penalizes the high weights equally for all the channels and therefore the new objective function become:

$$J_{p1} = \frac{w^T \Sigma_1 w}{w^T \Sigma_2 w + \alpha w^T I w}, J_{p2} = \frac{w^T \Sigma_2 w}{w^T \Sigma_1 w + \alpha w^T I w} \tag{16}$$

The Lagrangians are formed and the optimal filters are given by the eigenvectors corresponding to the largest eigenvalue of both $M_1 = \Sigma_1(\Sigma_2 + \alpha I)^{-1}$ and $M_2 = \Sigma_2(\Sigma_1 + \alpha I)^{-1}$

2.3.4 CSP with Weighted Tikhonov Regularization

The Weighted Tikhonov Regularization penalizes the higher weights with unequal amount for channels as it might be so that some channels are more significant than others with context to the task [3] [10]. Here the penalty matrix becomes $P(w) = w^T D_w w$ where $D_w = diag(w_g)$ and the w_g is given by the expression:

$$w_G = \left(\frac{1}{2 \times N_f \times |\Omega|} \sum_{i \in \Omega} \sum_{f=1}^{2 \times N_f} \left| \frac{w_f^i}{\left\| w_f^i \right\|} \right| \right)^{-1} \tag{17}$$

Where w_f^i is the fth spatial filter obtained using CSP amongst the eigenvectors corresponding to the N_f largest and lowest eigenvalues of M for the ith additional subject available. In other words, the penalty level of a channel is set to the inverse of the average absolute value of the normalized weight of this channel in the CSP filters obtained from other subjects (the less important the average channel weight, the higher the penalty).

2.3.5 Spatially Regularized CSP

The spatial locations of the EEG electrodes are ignored normally in the traditional CSP and thus this algorithm aims to provide spatially smooth filters which means the nearby electrodes should have relatively similar weights [11]. The penalty function is given as $P(w) = w^T(D_G - G)w$ with $G(i,j) = \exp\left(-\frac{1}{2}\frac{\|v_i - v_j\|^2}{r^2}\right)$ where v_i contains the 3D location of the i^{th} electrode r is a hyperparameter which considers the closeness of the electrodes. D_G is the diagonal matrix such that $D_G(i,i) = \sum_j G(i,j)$.

2.3.6 CIM based CSP

Correntropy [12] induced metric based common spatial pattern is a regularized variant of the CSP which penalizes the objective function of the standard CSP [13]. Let there be two classes X and Y and the trial matrices are $\boldsymbol{X} = (\boldsymbol{x_1}, \dots, \boldsymbol{x_m})$ and $\boldsymbol{Y} = (\boldsymbol{y_1}, \dots, \boldsymbol{y_n})$ then the correntropy is defined as:

$$V(x, y) = \frac{1}{N}\sum_{i=1}^{N} \kappa(x_i, y_i) \tag{18}$$

Where $\kappa(.,.)$ is a shift-invariant kernel and the most commonly used is the Gaussian Kernel which is given by:

$$\kappa(x, y) = \kappa(x - y) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right) \tag{19}$$

The CIM between the classes is thus defined as:

$$CIM(x, y) = [\kappa(0) - V(x, y)]^{1/2} \tag{20}$$

Now the new objective function becomes:

$$\tilde{J}(w) = \frac{CIM(w^T X, 0)}{CIM(w^T Y, 0)} = \frac{[\kappa(0) - V(w^T X, 0)]^{1/2}}{[\kappa(0) - V(w^T Y, 0)]^{1/2}} \tag{21}$$

This cost function can be rewritten as:

$$\tilde{J}(w) = \left[\frac{\frac{1}{m}\sum_{i=1}^{m}\left(1 - \exp\left(-\frac{(w^T x_i)^2}{2\sigma^2}\right)\right)}{\frac{1}{n}\sum_{j=1}^{n}\left(1 - \exp\left(-\frac{(w^T y_i)^2}{2\sigma^2}\right)\right)}\right] \tag{22}$$

This cost function is not differentiable and therefore it is required to use an iterative algorithm to obtain the solution which is as follows:

- Initialize kernel bandwidth σ, $t = 0$ and $w(t)$
- Calculate the gradient of $\log \tilde{J}(w)$ with respect to $w(t)$
- Update the filter according to the equation $w(t+1) = w(t) + \eta \frac{\partial \log \tilde{J}(w)}{\partial w}$
- Check if the stopping criteria is satisfied if not then go to step 2 and if yes then stop the iteration and the optimal filter is $w(t)$.

2.3.7 Composite CSP

The traditional CSP can also be penalized through the covariance matrix using the following set of equations:

$$\begin{aligned} \tilde{C}_c &= (1-\gamma)\hat{C}_c + \gamma I \\ \hat{C}_c &= (1-\beta)s_c C_c + \beta G_c \end{aligned} \tag{23}$$

G_c is built as a weighted sum of the covariance matrices (corresponding to the same mental state) of other subjects, by de-emphasizing covariance matrices estimated from fewer trials [14]. The parameters become:

$$\begin{aligned} G_c &= \sum_{i \in \Omega} \frac{N_c^i}{N_{t,c}} C_c^i \\ s_c &= \frac{N_c}{N_{t,c}} \end{aligned} \tag{24}$$

Where Ω is the set of subjects whose data is available, C_c^i is the spatial covariance matrix for class c and subject i, N_c^i is the number of EEG trials used to estimate C_c^i , N_c is the number of EEG trials used to estimate C_c (matrix for the target subject), and $N_{t,c}$ is the total number of EEG trials for class c (from all subjects in Ω together with the target subject)

2.3.8 Regularized CSP with generic learning

This approach also regularizes the covariance matrix of the standard CSP and aims at shrinking the covariance matrix towards both the identity matrix and a generic covariance matrix G_c [15]. Similarly to CCSP, Gc is here computed from the covariance matrices of other subjects such that:

$$s_c = s_G = \frac{1}{(1-\beta)M_{c_c} + \beta\sum_{i\in\Omega} M_{c_c^i}} \qquad (25)$$
$$G_c = s_G \sum_{i\in\Omega} c_c^i$$

Where M_C is the number of trials used to compute the covariance matrix C.

2.3.9 Regularized CSP with Diagonal Loading

This approach is guided to shrink or converge the covariance matrix towards the identity matrix [16] thus leaving us with only one parameter γ and the covariance matrix is given as

$$\tilde{C}_c = (1-\gamma)\hat{C}_c + \gamma I \qquad (26)$$

The objective function will used this regularized covariance matrix and will be as follows:

$$\hat{J}(w) = \frac{w^T \tilde{C}_1 w}{w^T \tilde{C}_2 w} \qquad (27)$$

This special case is previously studies and the value of the parameter can be determined automatically using the Ledoit and Wolf's Method [17].

2.3.10 Invariant CSP

This variant of the CSP aims at regularizing the CSP objective function in order to make filters invariant to a given noise source [18].The regularization matrix K is defined as the covariance matrix of this noise source, e.g., as the covariance matrix of the changing level of occipital α-activity. To obtain this noise covariance matrix, additional EEG measurements must be performed to acquire the corresponding EEG signals and compute their covariance matrix. It is an efficient approach to make CSP robust against known noise sources.

2.4 Phase Sensitive Common Spatial Pattern

The usual approaches to make the CSP more accurate and robust doesn't consider the phase information of the EEG signals. The approach in [19] captures the phase information in the signals and use them along with the amplitude to procure better accuracy and robustness. The phase of the EEG signals is extracted using the Hilbert transform [20] as follows:

$$\mathrm{H}(X(t)) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{x(\tau)}{\tau - t}d\tau = \mathrm{Re}(t) + j\,\mathrm{Im}(t) \tag{28}$$

The phase information is obtained using the equation:

$$\Phi(t) = \tan^{-1}\left(\frac{\mathrm{Im}(t)}{\mathrm{Re}(t)}\right) \tag{29}$$

This phase matrix is then used as the data matrix. The second approach combines both the amplitude and phase information of the EEG signals and has a complex matrix as shown below:

$$Z_i^j(t) = A_i^j(t)e^{-j\Phi_i^j(t)} \tag{30}$$

This complex matrix becomes the new data matrix and the thus used for the calculations. The A_i^j and Φ_i^j are the amplitude and phase of the jth channel of the ith class. The nonlinearity in the data makes the computations a little complex and thus to fit the conjugate harmonics nonlinear principal component analysis is used and the conformal mapping is then used to make the principal components obtained in the previous step orthogonal. Then the data is segregated using the Linear Discriminant Analysis [21].

2.5 Filter Bank Common Spatial Pattern (FBCSP)

CSP typically operates on a single frequency band, which may limit its effectiveness in capturing the full range of relevant information. FBCSP overcomes this limitation by utilizing multiple frequency bands [22]. It decomposes the EEG or MEG signals into different frequency subbands using a filter bank. Each subband represents a specific frequency range, allowing for a more comprehensive analysis of the signal. The CSP algorithm is then applied to each subband independently to extract the discriminative spatial filters.

Once the spatial filters are obtained for each subband, the filtered signals are concatenated and used as features for subsequent classification or analysis. These features capture both the spatial and spectral characteristics of the EEG or MEG data, enabling more accurate classification of different mental states or brain activities.

By leveraging multiple frequency subbands, FBCSP captures both spatial and spectral information, enhancing the performance of brain-computer interfaces and neuroimaging applications.

2.6 Common Spatial Pattern with Neural Network

This approach as the name suggests uses the Neural Networks along with the previously discussed FBCSP and thus making the classification more accurate [23].

Neural Networks are machine learning models inspired by the structure and function of biological neural networks. They are composed of interconnected layers of artificial neurons that can learn from data to perform complex tasks such as pattern recognition and classification. NNs have demonstrated remarkable capabilities in learning discriminative patterns and making accurate predictions.

CSP-NN combines these two techniques by using CSP as a preprocessing step to extract discriminative spatial filters from the EEG/MEG signals. These spatially filtered signals are then fed into a neural network architecture for further analysis and classification. The NN component learns the relationships between the extracted features and the corresponding mental states or intended actions.

The advantage of using CSP-NN lies in its ability to exploit the complementary strengths of both techniques. CSP effectively captures spatial information from the brain signals, while the NN models the complex nonlinear relationships and patterns in the transformed feature space. This combined approach has shown improved performance in BCI applications, especially in tasks like motor imagery classification, where accurate detection of specific brain patterns is crucial.

By integrating CSP with NNs, CSP-NN offers a more robust and efficient solution for pattern recognition in BCIs. It enhances the interpretability and discriminative power of the extracted features, leading to higher classification accuracy and better user experience. The utilization of deep learning architectures within the NN component has also allowed for the exploration of more complex and abstract representations of brain signals, enabling further advancements in BCI research and applications.

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Chapter 3:

DESIGN OF EXPERIMENT

This chapter describes the basic structure of the experiments of the datasets used for the study of the proposed algorithms

3.1 BCI Competition III: IVa Dataset

This dataset has three classes of Motor Imagery classes namely (L) left hand, (R) right hand, (F) right foot [1] [2]. The data was recorded from five healthy subjects. Subjects sat comfortably in a chair with arms resting on armrests. The dataset contains data from 4 initial sessions without feedback. Visual cues were indicated for 3.5 s during which the subjects were told to think of the motor imagery class mentioned on the screen. The presentation of target cues were intermitted by periods of random length, 1.75 to 2.25 s, in which the subject could relax.

There were two types of visual stimulation first where the targets were indicated by letters appearing behind a fixation cross (which might nevertheless induce little target-correlated eye movements), and second where a randomly moving object indicated targets (inducing target-uncorrelated eye movements). From subjects al and aw 2 sessions of both types were recorded, while from the other subjects 3 sessions of second type and 1 session of first type were recorded. The continuous signals of 118 EEG channels and markers that indicate the time points of 280 cues for each of the 5 subjects (aa, al, av, aw, ay) are provided in the dataset. For some markers no target class information is provided (value NaN) for competition purpose. Only cues for the classes 'right' and 'foot' are provided for the competition. The following table shows the respective number of training (labelled) trials and test (unlabelled) trials for each subject.

TABLE 3-I

NO. OF TRAINING DATA AND TEST DATA IN DATASET III IVa

SUBJECT	NO. OF TRAINING DATA	NO. OF TEST DATA
aa	168	112
al	224	56
av	84	196
aw	56	224
ay	28	252

In the present work the subjects are relabeled as A1, A2, A3, A4, A5 and the number of classes in consideration is 2 which are namely (L) left hand and (R) right hand. The recording was made using BrainAmp amplifiers and a 128 channel Ag/AgCl electrode cap from ECI. 118 EEG channels were measured at positions of the extended international 10/20-system. Signals were band-pass filtered between 0.05 and 200 Hz and then digitized at 1000 Hz with

16 bit (0.1 uV) accuracy. We provide also a version of the data that is downsampled at 100 Hz (by picking each 10th sample) that we typically use for analysis.

3.2 BCI Competition III: IIIa Dataset

Multi-Class Motor imagery with 4 classes (left hand, right hand, foot, tongue) Consisting of three subjects (ranging from quite good to fair performance). The recording was made with a 64-channel EEG amplifier from Neuroscan, using the left mastoid for reference and the right mastoid as ground. The EEG was sampled with 250 Hz, it was filtered between 1 and 50Hz with Notchfilter. Sixty EEG channels were recorded according the scheme in **Fig 3.1**. The subject sat in a relaxing chair with armrests [3]. The task was to perform imagery left hand, right hand, foot or tongue movements according to a cue. The order of cues was random. The experiment consists of several runs (at least 6) with 40 trials each each; after trial begin, the first 2s were quite, at t=2s an acoustic stimulus indicated the beginning of the trial, and a cross "+" is displayed; then from t=3s an arrow to the left, right, up or down was displayed for 1 s; at the same time the subject was asked to imagine a left hand, right hand, tongue or foot movement, respectively, until the cross disappeared at t=7s. Each of the 4 cues was displayed 10 times within each run in a randomized order. Each class has 45 trials

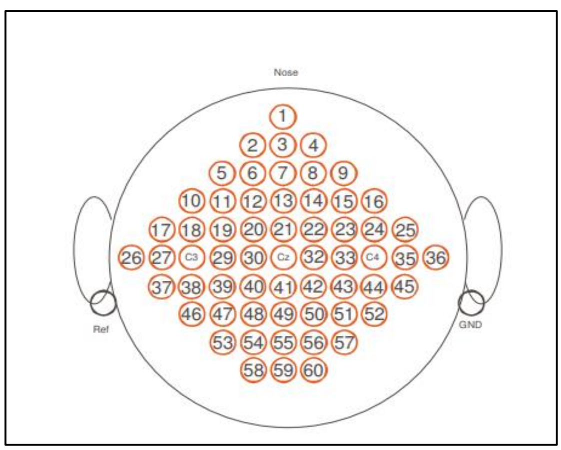

Fig. 3.1. Electrode arrangement for the recording of dataset III: IIIa

for subject K1 whereas for subjects K2 and K3 each class consists of only 30 trials each. In the present work only two classes left and right hand motor imagery were considered.

3.3 BCI Competition IV: IIa Dataset

This data set consists of EEG data from 9 healthy subjects [4]. The cue-based BCI paradigm consisted of four different motor imagery tasks, namely the imagination of movement of the left hand (class 1), right hand (class 2), both feet (class 3), and tongue (class 4). Two sessions on different days were recorded for each subject. Each session is comprised of 6 runs separated by short breaks. One run consists of 48 trials (12 for each of the four possible classes), yielding a total of 288 trials per session. At the beginning of each session, a recording of approximately 5 minutes was performed to estimate the Electrooculography (EOG) influence or the influence due to the movement of eyes. The recording was divided into 3 blocks:

- Two minutes with eyes open (looking at a fixation cross on the screen)
- One minute with eyes closed
- One minute with eye movements.

Due to some technical problems the EOG block is shorter for subject A04T and contains only the eye movement condition. The subjects were sitting in a comfortable armchair in front of a computer screen. At the beginning of a trial (t = 0 s), a fixation cross appeared on the black screen. In addition, a short acoustic warning tone was presented. After two seconds (t = 2 s), a cue in the form of an arrow pointing either to the left, right, down or up (corresponding to one of the four classes (left hand, right hand, foot or tongue) appeared and stayed on the screen for 1.25 s. This prompted the subjects to perform the desired motor imagery task. No feedback was provided. The subjects were ask to carry out the motor imagery task until the fixation cross disappeared from the screen at t = 6 s. A short break followed where the screen was black again.

Twenty-two Ag/AgCl electrodes (with inter-electrode distances of 3.5 cm) were used to record the EEG; the montage is shown in Figure 3 left. All signals were recorded monopolarly with the left mastoid serving as reference and the right mastoid as ground. The signals were sampled with 250 Hz and bandpass-filtered between 0.5 Hz and 100 Hz. The sensitivity of the amplifier was set to 100 µV. An additional 50 Hz notch filter was enabled to suppress line noise.

In addition to the 22 EEG channels, 3 monopolar EOG channels were recorded and also sampled with 250 Hz (see Figure 3 right). They were bandpass filtered between 0.5 Hz and 100 Hz (with the 50 Hz notch filter enabled), and the sensitivity of the amplifier was set to 1 mV. The EOG channels are provided for the subsequent application of artifact processing methods and are not considered for classification. Each class has 72 trials each. Individual training and test data for each subject is provided for all the 9 subjects. In the current work two classes namely class 1 and class 2 were considered and the subjects are renamed as A01, A02, A03, A04, A05, A06, A07, A08 and A09.

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Chapter 4:

PROBLEM STATEMENT AND PROPOSED SOLUTION

This chapter gives a brief description of the problem statement and the proposed solution. The proposed solution consists of two new penalty matrices for the regularization purposes and also a new flexible technique. The results obtained are also presented in the chapter.

4.1 Problem Statement

The main issue this research is aimed to solve is to make the CSP algorithm more accurate, robust and generalized in order to achieve a higher classification accuracy and also to make dataset requirements minimum so as to keep the computational cost low. The generalization of the algorithm is very much crucial and to achieve that some kind of regularization or penalization is required to be introduced in the algorithm. These penalty terms must give the algorithm some prior knowledge about the data and those data must be relevant to the used datasets.

4.2 Challenges Involved

There are numerous challenges involved with respect to the CSP and those are needed to be taken care of. The challenges are as follows:

- A limitation of CSP is that it requires a sufficient amount of training data to accurately extract the spatial filters. This can be a challenge in some applications where obtaining large amounts of labeled EEG data may be difficult or expensive.

- Another limitation of CSP is that it assumes that the data is stationary, meaning that the statistical properties of the EEG signal do not change over time. This assumption may not hold true in some applications, such as when the user's mental state changes over time, leading to changes in the EEG signal.

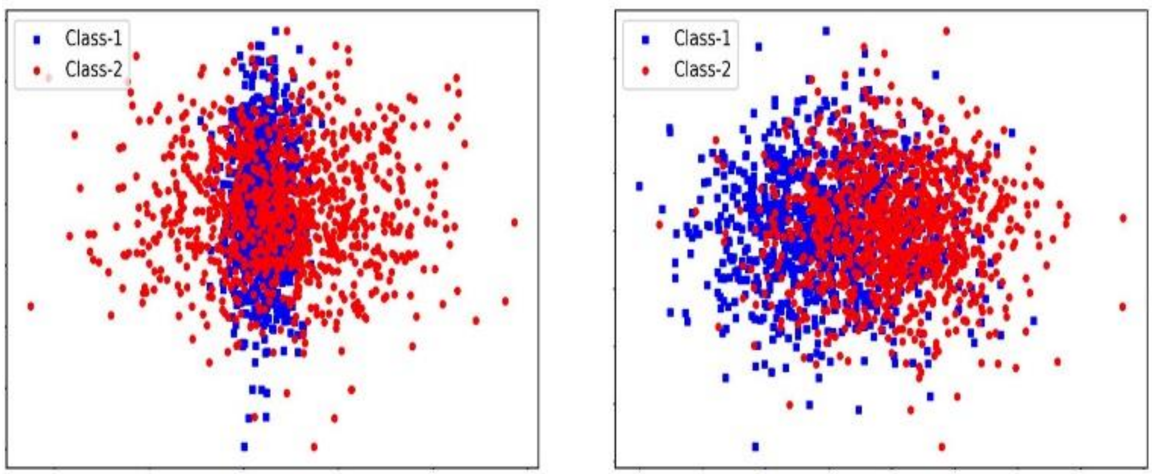
- CSP may not be effective in cases where there is significant inter-subject variability in the EEG data. In such cases, it may be necessary to develop subject-specific CSP filters, which can be time-consuming and require additional data.

4.3 Signal to Noise Ratio

The Signal to noise ratio [1]-[3] between two classes A and B is given by equation (31).

$$S2N(A,B) = \frac{\mu_{A_1} - \mu_B}{\sigma_A + \sigma_B} \tag{31}$$

Where $\mu_A, \sigma_A, \mu_B, \sigma_B$ represent the mean and standard deviation of classes A and B respectively. SNR calculated is used to define how well this variable discriminates two classes. Large values of S2N indicates strong correlation between the classes as shown for two random data in **Fig. 4.1**. This value is a statistical parameter and could be used to penalize the covariance matrix of each class and also the objective functions of conventional CSP, providing some prior knowledge about the statistical connection between each class to the model.



(a) (b)

Fig.4.1. Random data with (a) S2N=0.2 which is less correlated (b) S2N=0.5 which is more correlated.

4.4 Proposed Solutions

The proposed solutions consists of two new penalty matrices. The creation process of both the penalty matrices are bit different but they both are based on the Signal to Noise Ratio and the third solution is a new technique for flexible penalization of the covariance matrices of the classes considered in the classification. The framework the training phase of the proposed solution is shown **Fig 4.2.** The framework shows how the proposed penalty matrices are added to the algorithm and the covariance matrices and then used as the prior data for the datasets and are used for the regularization of the algorithm to provide generalized solutions which are accurate. These techniques are also computationally efficient and thus can be used with bigger data sets as well.

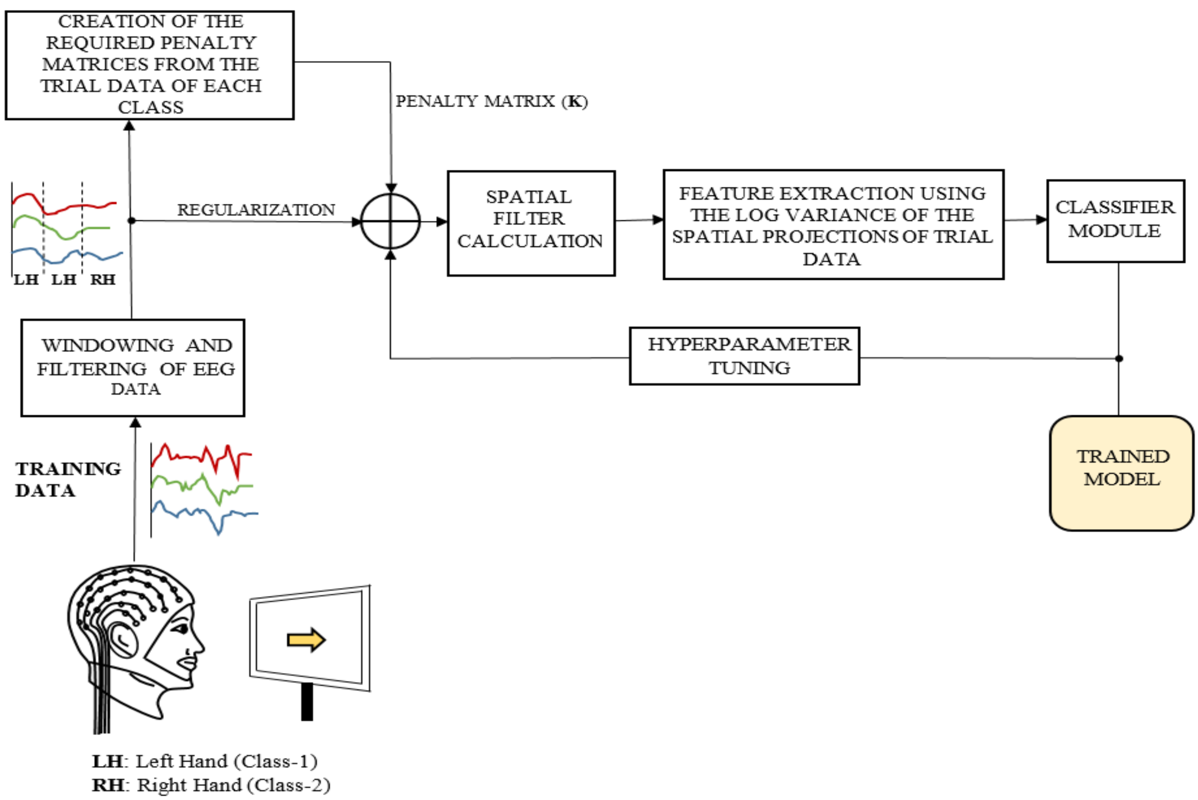


Fig.4.2. Block diagram of training phase of the proposed framework.

4.4.1 Signal to Noise Penalty for Objective Function

The first part is the creation of S2N penalty matrix which has to be used for the objective function regularization purpose. This process is described in Pseudocode. The training data of each trial corresponding to a particular class is first calculated and stored as Class1Data and Class2Data respectively. The number of channels used for measurement be m then data of each class is a matrix of dimension $(m \times t)$, where t is the total number of time points for all the trials combined. The mean and standard deviation vector of all the channels of each class is then calculated and termed as Class1mean, Class2mean, Class1stdev, and Class2stdev. The dimension of all the mean and standard deviation matrices are $(m \times 1)$. Then these vectors are used to create a matrix named as *S2N* matrix which is then used as the regularization matrix. The resulting matrix is a diagonal matrix of dimension $(m \times m)$ which is of the following form:

$$S2N = \begin{bmatrix} D_0 & 0 & \cdots & 0 \\ 0 & D_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & D_m \end{bmatrix} \tag{32}$$

Where D_i denotes i^{th} entry of *S2N* matrix and calculated and as,

$$S2N_i = \left| \frac{\mu_{A_i} - \mu_{B_i}}{\sigma_{A_i} + \sigma_{B_i}} \right| \tag{33}$$

This S2N matrix now can used for penalizing the CSP objective function.

Pseudocode for generation of S2N Matrix creation for penalizing objective function

INPUT: *m* number of EEG channels
IF dataLabel=1
  Class1data←*data*

ELSEIF *dataLabel*=2
  Class2data←*data*

FOR i=0:*m*
  Class1mean(i) ←mean(*Class1data*(i,:))
  Class2mean(i) ←mean(*Class2data*(i,:))
  Class1stdev(i) ←stdev(*Class1data*(i,:))
  Class2stdev(i) ←stdev(*Class2data*(i,:))

FOR j=0:m
  S2N(j,j)=abs(*Class1mean*(j)- *Class2mean*(j))/(*Class1stdev*(j)- *Class2stdev*(j)))

RETURN *S2N*

4.4.2 Signal to Noise Penalty for Covariance Matrix

The S2N matrix is made using the information of trial data of both classes and hence could be used to penalize the covariance matrix as well. The penalty term creation for the covariance matrix regularization is explained in Pseudocode. Here two matrices are created for the covariance matrix of both the classes. The matrices in this case does not use the modulus function as the sign of subtraction matters. The positive and negative signs that arise due to absence of modulus function ensures that covariance matrices are penalized according to the statistical dependence of the classes. The mean and standard deviations are calculated for all the channels as done in the previous case. This time the output are two matrices rather than one namely $S2N_{12}$ and $S2N_{21}$. These are two diagonal matrices of dimensions $(m \times m)$ where, m denotes the number of channels. In this case the matrices $S2N_{12}$ and $S2N_{21}$ are formulated as follows:

$$S2N_{xy} = \begin{bmatrix} D_{xy0} & 0 & \cdots & 0 \\ 0 & D_{xy1} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & D_{xym} \end{bmatrix} \tag{34}$$

Where D_{xyi} denote the i^{th} entry of the matrix $S2N_{xy}$ and is given by the equation

$$D_{xyi} = \frac{\mu_{x_i} - \mu_{y_i}}{\sigma_{x_i} + \sigma_{y_i}} \tag{35}$$

These matrices are now used to penalize the covariance matrices of both the classes.

Pseudocode for generation of S2N Matrix creation for penalizing covariance matrices

INPUT: *m* number of EEG channels
IF dataLabel=1
  Class1data←data
ELSEIF *dataLabel*=2
  Class2data←data
FOR i=0:*m*
  Class1mean(i) ←mean(*Class1data*(i,:))
  Class2mean(i) ←mean(*Class2data*(i,:))
  Class1stdev(i) ←stdev(*Class1data*(i,:))

Class2stdev(i) ←stdev(*Class2data*(i,:))

FOR j=0:m
$S2N_{12}$ (j,j)=*Class1mean*(j)- *Class2mean*(j))/(*Class1stdev*(j)- *Class2stdev*(j))
$S2N_{21}$ (j,j)=*Class2mean*(j)- *Class1mean*(j))/(*Class2stdev*(j)- *Class1stdev*(j))

RETURN S_1,S_2

4.4.3 S2N Penalty in Objective Function (S2NOCSP)

The S2N matrix obtained earlier is now used to penalize the objective function of traditional CSP and regularize it for protecting it against the overfitting. The new objective functions becomes:

$$O_1(w) = \frac{w^T \Sigma_1 w}{w^T (\Sigma_2 + \alpha S2N) w} \qquad (36)$$
$$O_2(w) = \frac{w^T \Sigma_2 w}{w^T (\Sigma_1 + \alpha S2N) w}$$

Where Σ_1 and Σ_2 are the covariance matrices of two classes which is similar to the ones discussed in the previous chapters. The corresponding Lagrangians become:

$$L_1(\lambda, w) = w^T \Sigma_1 w - \lambda[(w^T \Sigma_2 + \alpha S2N) w - 1] \qquad (37)$$
$$L_2(\lambda, w) = w^T \Sigma_2 w - \lambda[(w^T \Sigma_1 + \alpha S2N) w - 1]$$

The solution for this problem is obtained by taking the derivative of the Lagrange equation with respect to w and setting it to zero as done in (38) below.

$$\frac{\partial L_1(\lambda, w)}{\partial w} = 2\Sigma_1 w - 2\lambda(\Sigma_2 + \alpha S2N) w = 0 \qquad (38)$$
$$\Rightarrow (\Sigma_2 + \alpha S2N)^{-1} \Sigma_1 w = \lambda w$$

Similarly, undertaking the derivative: $\frac{\partial L_2(\lambda,w)}{\partial w} = 0$ and simplifying the resulting equation along with (38) we obtain the following 2 matrices:

$$\begin{aligned} N_1 &= (\Sigma_2 + \alpha S2N)^{-1}\Sigma_1 \\ N_2 &= (\Sigma_1 + \alpha S2N)^{-1}\Sigma_2 \end{aligned} \tag{39}$$

The desired filters are the combination of eigenvectors corresponding to the highest eigenvalues of N_1 and N_2.

4.4.4 S2N Penalty in Covariance Matrix (S2NCCSP)

The matrices $S2N_{12}$ and $S2N_{21}$, calculated in the previous stage, are now used to penalize the covariance matrices Σ_1, Σ_2 [4] of each class and the new penalized covariance matrices become:

$$\begin{aligned} \tilde{\Sigma}_1 &= (1-\rho_1)\Sigma_1 + \rho_1 S2N_{12} \\ \tilde{\Sigma}_2 &= (1-\rho_2)\Sigma_2 + \rho_2 S2N_{21} \end{aligned} \tag{40}$$

Here $\widetilde{\Sigma_1}$ and $\widetilde{\Sigma_2}$ represent the regularized covariance matrices of class 1 and class 2 respectively and ρ_1, ρ_2 are two user defined parameters deciding the degree of regularization for each of the covariance matrices. The regularized covariance matrices are then used in

CSP objective function, and the modified objective function becomes:

$$O(w) = \frac{w^T \tilde{\Sigma}_1 w}{w^T \tilde{\Sigma}_2 w} \tag{41}$$

The new objective function is then used to calculate the required filter as shown in (4).

4.4.5 Adjusted Penalty Covariance Matrix CSP (APCCSP)

The penalization process used in the regularization could sometimes lead to cases where a single type penalty term overpowers the data. To ensure that this doesn't occur a novel algorithm of parametrically penalizing the covariance matrix is presented here. The covariance matrix of each class is penalized using multiple penalty terms in a parameterized manner to make sure that the penalized covariance matrices prevent the over fitting of data. The penalized covariance matrix becomes:

$$\widetilde{C}_k = \left(1 - \frac{1}{M}\sum_{i=1}^{M}\alpha_i\right)\hat{C}_k + \sum_{i=1}^{M}\alpha_i f_i \tag{42}$$

Where $\widehat{C_k}$ denotes the mean covariance matrix of kth class, f_i is the i^{th} penalty function and α_i is the weight assigned to it. The new penalized covariance matrix $\widetilde{C_k}$ is then used for calculating the required spatial filters with the objective function as:

$$J(w) = \frac{w^T \widetilde{C}_1 w}{w^T \widetilde{C}_2 w} \tag{43}$$

The required filters are then calculated as shown in (4).

4.4.6 Combination of S2NOCSP and S2NCCSP

A combination of both the techniques named CS2NCSP is also used for the comparative studies to get the best out of both the techniques.

4.5 Simulation Results and Discussions

The experimental findings using the newly suggested algorithms on three publicly accessible datasets are discussed in chapter 3. There are 17 participants in total for two class motor imagery (MI) discriminating studies. In this part, a brief explanation of the datasets and the experimental setup along with the results obtained is provided. The evaluations are also carried out in this section which proves the supremacy of the newly proposed algorithms. The performance analysis and statistical evaluations are conducted for validating the results obtained.

4.5.1 Filter Development

The desired spatial filters are obtained using all the algorithms proposed. The best filter weights are then plotted for subjects A1, K2 and A02 in **Fig. 4.4**. The filter weights obtained are normalized and then plotted to get some uniformity amongst the filters. The obtained filters for standard CSP, regularized CSP with S2N matrix regularizing the objective function and then the covariance matrix (S2NOCSP, S2NCCSP), CSP with weighted penalty covariance method (APCCSP) and a combination of S2NOCSP and S2NCCSP named

CS2NCSP has been plotted for the selected subjects. The filter weights obtained shows that the proposed algorithms created much smoother weights with higher emphasis near the motor cortex region making the classification tasks more accurate. The traditional CSP filters assigned scattered weights to many regions of the brain thus causing erroneous classifications. The “red” and “blue” colored regions represent high positive and negative weights respectively. The “green” colored regions show zero weights, whereas the “light blue” and “yellow” regions represent low positive and negative weights respectively. The suggested algorithms gave electrodes close to the motor cortex more weight, as seen by the bar plot in **Fig. 4.3** in which the average weights assigned to each electrode are displayed for the conventional CSP and the combined technique CS2NCSP. The motor cortex activity is mainly captured using the electrodes C3, C4, Cz, F8, F7, T3 and T4 electrodes [5] the weights plots show that the proposed algorithm produced much higher weights at electrodes C3, C4 and Cz (marked with blue color) justifying the accuracy results achieved.

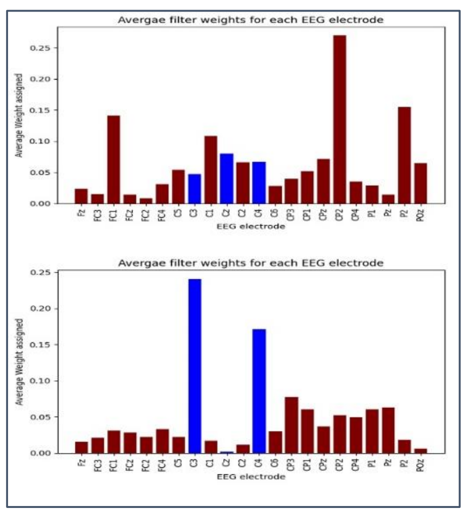


Fig.4.3. Absolute normalized filter weights for all 22 electrodes of

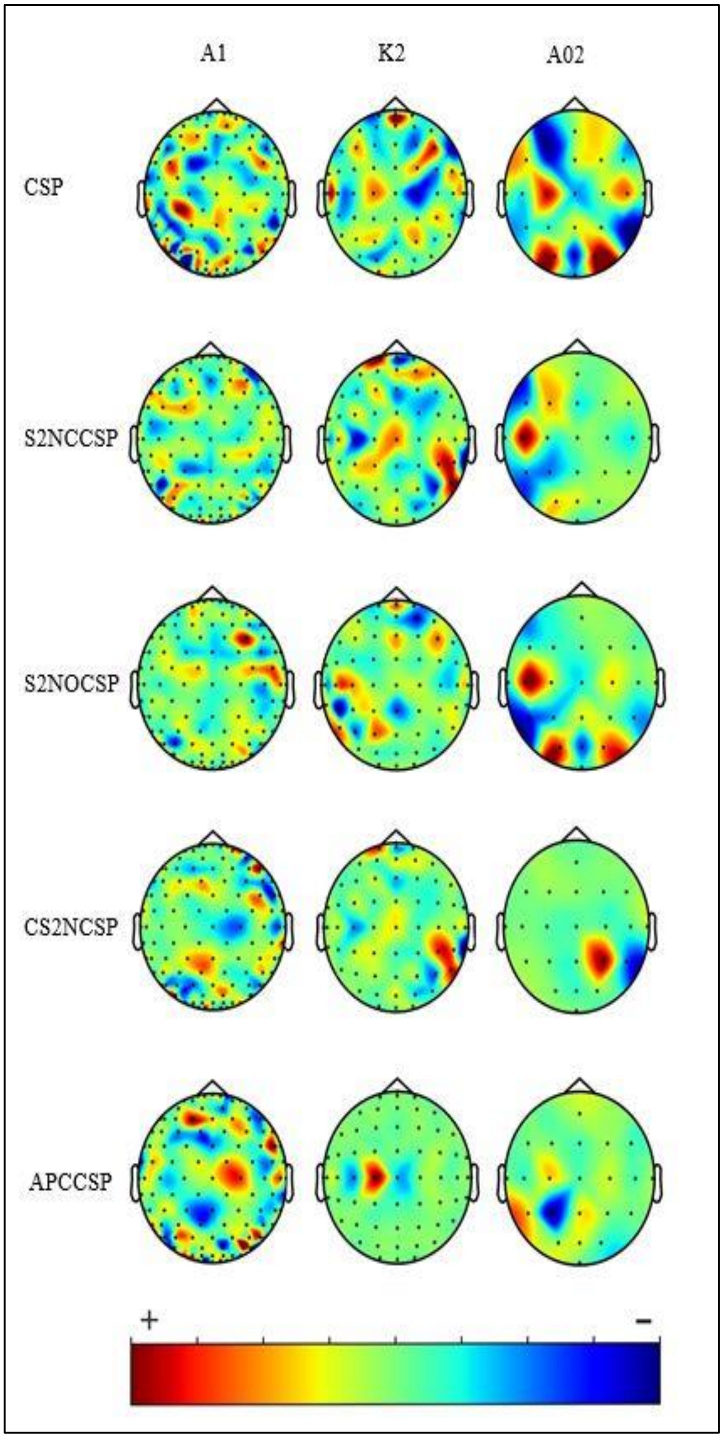


Fig.4.4. Normalized filter weights for subject A1 (118 electrodes), K2 (60 electrodes) and A02 (22 electrodes) for the proposed algorithms and one combination of them.

4.5.2 Feature extraction and Classification

The obtained filters are then used for the feature extraction stage. The trial data are projected on the filters and the log of the variance of projections are used as the features of the data. The classification is done using a LDA classifier [6] which is one of the most successful classifiers in BCI applications [7].

4.5.3 Performance Analysis of Proposed Algorithms

The proposed algorithms are used along with CSP for the classification tasks. We analyzed the performance of the classifier using five different metrics namely Classification Accuracy (CA), Sensitivity (SEN), Specificity (SPE) [8], F1 score and Kappa coefficient (κ) [9]. These metrics are formulated as:

$$CA = \frac{TP + TN}{TP + FP + FN + TN} \tag{44}$$

$$SEN = \frac{TP}{TP + FN} \tag{45}$$

$$SPE = \frac{TN}{TN + FP} \tag{46}$$

$$F1 = \frac{2TP}{2TP + FN + FP} \tag{47}$$

$$\kappa = \frac{CA - P_e}{1 - P_e} \tag{48}$$

Where Pe can be defined as $P_e = \left(\frac{TP+FN}{N}\right)\left(\frac{TP+FP}{N}\right) + \left(1 - \frac{TP+FN}{N}\right)\left(1 - \frac{TP+FP}{N}\right)$ where, $N=TP+FP+FN+TN$, and TP, TN, FP, FN denote true positive, true negative, false positive, false negative values. The higher these values better the classifier. **Table 4-I** shows the performance analysis of the proposed algorithms and the conventional CSP. The new proposed penalty matrix when used with the objective function (S2NOCSP) gave better results in comparison when the penalty term is used with covariance matrix (S2NCCSP). Although both of them outperformed the conventional CSP by a margin of ~2-3\% in terms of average classification accuracy. The parametric penalty approach on the other hand performed well than the individual proposed algorithms and surpassed the conventional CSP by a margin of ~5\%. The results show that the combination of the proposed algorithms showed the best performance and outperformed the conventional CSP with a margin of ~11\% in terms of average classification accuracy. The F1 score and kappa value also are very high for the proposed combination indicating that the performance of the algorithm is very good. The classification accuracies of all the algorithms along with others is given in **Table 4-II**. The performance of the combined algorithm surpassed the previously proposed algorithm by ~3-8\%. The best acquired result obtained on subject K1 of dataset IIIa is 99.02\% which is the highest amongst all the algorithms. The box plot representation in **Fig.4.5** shows that the median of the proposed algorithm is much higher in comparison to the other algorithms only the GLRCSP has the median near to that. The span of the box shows that the inter-quartile range (IQR) is less indicating that the scatter is less and therefore producing consistent results for all the subjects and datasets. The classification accuracies are in a very tight upper range showing the supremacy of the algorithm. The highest IQR is given by SSRCSP and the lowest is the CS2NCSP. The whiskers of the box corresponding to the proposed algorithms also doesn't have huge distance indicating a very little scatter and very good results.

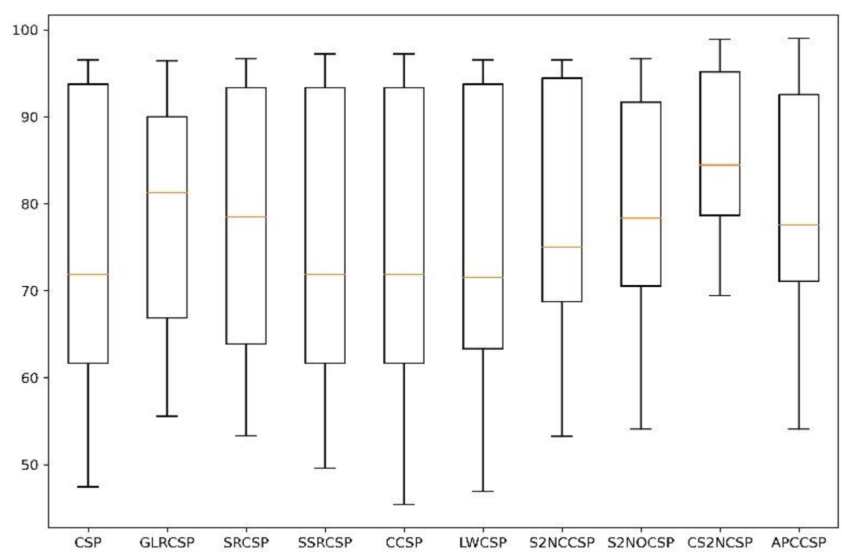


Fig.4.5. Box plot representation of existing and proposed algorithms.

TABLE 4-I:
PERFORMANCE ANALYSIS OF PROPOSED ALGORITHMS

ALGORITHMS	CA (%)	SEN	SPE	F1 (%)	κ
CSP	75.1	0.776	0.751	73.7	0.651
S2NCCSP	78.1	0.774	0.787	75.8	0.7
S2NOCSP	79.4	0.773	0.873	79.21	0.771
CS2NCSP	**84.2**	**0.85**	**0.862**	**84.21**	**0.807**
APCCSP	80	0.812	0.864	82.21	0.784

TABLE 4-II
CLASSIFICATION ACCURACIES OF PROPOSED AND EXISTING ALGORITHMS ON BCI III COMPETITION DATASET IVa

ALGORITHMS	SUBJECTS				
	A1	A2	A3	A4	A5
CSP	66.07	96.43	47.45	71.88	49.6
GLRCSP [10]	72.32	96.43	66.84	97.86	89.29
SRCSP [11]	72.32	96.43	60.2	77.68	86.51
SSRCSP[12]	70.54	96.43	53.57	71.88	75.39
CCSP [13]	65.18	96.43	45.41	71.88	49.6
LWCSP[14]	66.96	96.43	46.94	71.43	50
S2NCCSP	68.75	85.71	53.06	71.88	57.54
S2NOCSP	70.54	85.71	54.08	71.88	57.54
CS2NCSP	**74.11**	**98.89**	**53.57**	**79.17**	**90.27**
APCCSP	70.54	92.86	54.08	71.88	56.35

TABLE 4-III

CLASSIFICATION ACCURACIES OF PROPOSED AND EXISTING ALGORITHMS ON BCI III COMPETITION DATASET IIIa

ALGORITHMS	SUBJECTS		
	K1	K2	K3
CSP	95.56	61.67	93.33
GLRCSP [10]	95.56	61.67	90
SRCSP [11]	96.67	53.33	93.33
SSRCSP[12]	95.56	61.67	96.67
CCSP[13]	95.56	61.67	93.33
LWCSP[14]	94.44	63.33	95
S2NCCSP	96.67	78.34	91.67
S2NOCSP	96.67	78.34	91.67
CS2NCSP	**97.77**	**78.67**	**93.33**
APCCSP	99.02	76.67	91.67

TABLE 4-IV

CLASSIFICATION ACCURACIES OF PROPOSED AND EXISTING ALGORITHMS ON BCI IV COMPETITION DATASET IIa

ALGORITHMS	SUBJECTS								
	A01	A02	A03	A04	A05	A06	A07	A08	A09
CSP	88.89	51.39	96.53	70.14	54.86	71.53	81.25	93.75	93.75
GLRCSP [10]	86.11	58.33	93.75	67.36	55.56	65.28	81.25	93.75	88.19
SRCSP [11]	88.89	63.19	96.53	66.67	63.19	63.89	78.47	95.83	92.36
SSRCSP[12]	88.89	53.47	97.22	70.14	56.25	68.75	79.17	97.22	90.28
CCSP[13]	88.89	53.47	97.22	70.14	54.17	68.06	79.17	95.14	90.28
LWCSP[14]	88.89	51.39	96.53	70.14	56.94	71.53	81.94	93.75	93.75
S2NCCSP	90.27	70.83	96.53	68.72	70.14	75	76.19	96.53	94.44
S2NOCSP	90.27	69.44	92.36	68.72	77.08	79.86	76.19	96.53	93.75
CS2NCSP	**95.14**	**70.83**	**93.05**	**69.44**	**79.17**	**80.55**	**84.44**	**97.92**	**95.14**
APCCSP	90.97	70.14	95.83	72.22	78.47	75	70.83	97.92	95.14

TABLE 4-V
MEAN MEDIAN AND STANDARD DEVIATION OF CLASSIFICATION ACCURACIES

ALGORITHMS	MEAN	MEDIAN	STD. DEV.
CSP	75.5	71.9	18.2
GLRCSP [10]	78.2	81.3	14.3
SRCSP [11]	79.1	78.5	15.2
SSRCSP[12]	77.8	75.4	16.2
CCSP[13]	75	71.9	18.3
LWCSP[14]	75.9	71.5	18
S2NCCSP	78.1	75	14.1
S2NOCSP	79.4	78.3	13.1
CS2NCSP	**84.2**	**84.4**	**12.6**
APCCSP	80	76.7	14.3

The average ranks of algorithms are given in **Table 4- III**. The value of Friedman statistic χ_F^2=15.778 which is greater than $\chi_{6,0.05}^2$=12.592 thus the null hypothesis is rejected at 6 degrees of freedom and the algorithms are only judged on the basis of ranks. This shows that the lowest rank algorithm performed the best and therefore the proposed algorithm produced statistically significant results.

TABLE 4-VI
AVERAGE RANKS OF CLASSIFIER ALGORITHMS USING FRIEDMAN TEST

ALGORITHMS	AVERAGE CA (%)			RANKS			AVG.RANK
	IVa	IIIa	IIa	IVa	IIIa	IIa	
CSP	66.3	83.5	78	9	8	7	8
GLRCSP	78.5	82.4	76.6	3	9	10	7.33
SRCSP	78.6	81.1	78.8	2	10	5	5.67
SSRCSP	73.6	84.6	77.9	4	4	8	5.33
CCSP	65.7	83.5	77.4	10	7	9	8.67
LWCSP	66.4	84.3	78.3	8	5	6	6.33
CS2NCSP	**79.2**	**89.9**	**85.1**	**1**	**1**	**1**	**1**
APCCSP	69.1	89.1	82.9	5	2	2	3
S2NCCSP	67.4	83.7	82.2	7	6	4	5.67
S2NOCSP	68	88.9	82.7	6	3	3	4

4.5.4 Hyperparameter Tuning

The new algorithm has some hyperparameters which are required to be tuned for the optimum performance of the algorithms. The S2NOCSP algorithm has a single parameter α which has to be tuned. The range of values chosen for tuning is [10-9, 10-8, …, 10-1]. The S2NCCSP algorithm has two parameters ρ_1, ρ_2 which are chosen from a list of [10-9, 10-8, …, 10-1] The APCCSP algorithm has different number of parameters to be adjusted which is determined according to the penalties chosen. In this work we chose two penalty functions Tikhonov and S2N penalty. The hyperparameters that maximized the F1 score were chosen using the 10-fold cross validation.

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Chapter 5:

CONCLUSION AND FUTURE DIRECTION

This chapter briefly highlights the findings and contributions of the thesis. It also introduces some of the future work for interested readers that may be carried out in extension to the algorithms that have already been developed.

5.1 Conclusion

This thesis proposes a very unique way of penalizing the conventional CSP algorithm and regularizing it so that the overfitting issue could be countered. The tests undertaken on the proposed algorithms: S2NOCSP, S2NCCSP, APCCSP and their combination CS2NCSP using 3 publicly available BCI competition databases, proved their supremacy over existing algorithms with respect to mean and median classification accuracy. The robustness of the proposed algorithms is also envisaged as they take care of the overfitting issues. The proposed technique would serve as a basic tool for Motor Imagery based BCI classifiers in different scientific and engineering applications. The experiments undertaken further revealed that even for poor performers in Motor Imageries, the proposed algorithm can correctly detect them, which often were poorly classified by the existing algorithms. The boxplots presented clearly highlights low IQRs for the proposed algorithms in comparison to the relatively higher IQRs of the existing algorithms.

5.2 Future Direction

This study has demonstrated that the common spatial pattern along with the new penalization policies have shown some considerably good results in terms of accuracy and consistency as well. The outliers are very less and the algorithm is made more robust using these new methodologies. This extension of previous algorithms is very beneficial and can be easily deployed in numerous uses due to its improved accuracy and also the lower time complexity.

In future we will pursue our research in following directions:

a) Simultaneous solution of more than one objective functions

b) Addition of some new and relevant penalties

c) Addition of modern classification techniques

Appendix A:

STATISTICAL METHODS USED

This chapter briefly describes the statistical techniques used for the comparison of the algorithms

A.1 Mean

Let a series of N tests yielded results $x_n, x_2, x_3, \cdots, x_N$. Then the mean result of the tests will be given by the formula:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Mean gives the central tendency of a data set which refers to the measure that represents the "typical" or "central" value around which the data points tend to cluster. It gives an indication of the central or average value of the data set.

A.2 Variance

Let a series of N tests yielded results $x_n, x_2, x_3, \cdots, x_N$. Then the variance of the tests will be given by the formula:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \mu)^2$$

Variance gives the variability of the data set which refers to the spread or dispersion of the values within the dataset. It quantifies how much the individual data points deviate from the central tendency measures. A dataset with low variability has values that are close to each other, while a dataset with high variability has values that are more spread out.

A.3 Standard Deviation

Let a series of N tests yielded results $x_n, x_2, x_3, \cdots, x_N$. Then the standard deviation of the tests will be given by the formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (x_i - \mu)^2}$$

Standard Deviation gives the spread of a data set which refers to the extent or range of values covered by the data points. It provides information about how widely dispersed or concentrated the values are within the data set. A data set with a larger spread indicates greater variability or dispersion among the values, while a smaller spread suggests the values are more closely clustered together.

A.4 Interquartile Range (IQR)

The interquartile range (IQR) is a measure of statistical dispersion that focuses on the middle 50% of a data set. It provides information about the spread of the central portion of the data and is less influenced by extreme values or outliers. To calculate the IQR it is required to find the first quartile (Q1) and the third quartile (Q3) of the data set. The quartiles divide the data set into four equal parts, with Q1 representing the 25th percentile and Q3 representing the 75th percentile as shown in the **Fig A.1**. The IQR is then determined by subtracting Q1 from Q3:

$$IQR = Q3 - Q1$$

The IQR is expressed in the same units as the original data set and provides a measure of the range covered by the middle 50% of the data. The IQR is particularly useful when analyzing data sets that may contain outliers or skewed distributions. It gives a more robust measure of spread by focusing on the central portion of the data, which can be more representative of the overall distribution. Additionally, the IQR can be used to identify outliers by defining a range beyond which values are considered extreme.

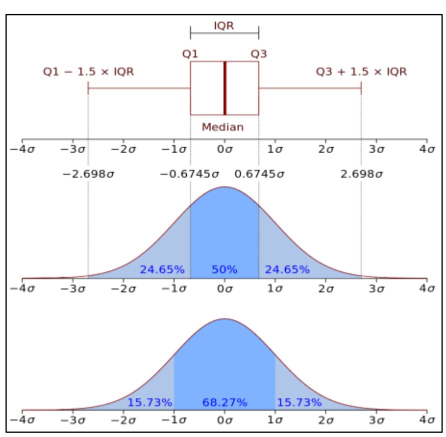

Fig. A.1. IQR of a data

A.5 Friedman Test

The Friedman test is a non-parametric statistical test used to determine if there are significant differences among multiple related groups. It is an extension of the Wilcoxon signed-rank test and is suitable for situations where the data are paired or matched. The Friedman test is often used when the data are ordinal (ranked) and not normally distributed. It allows for the comparison of three or more dependent groups or treatments. The test assesses whether there are any differences in the medians of the groups.

The step-by-step process of conducting the Friedman test is as follows:

- **Formulate the hypothesis:**
  - Null Hypothesis (H0): There is no difference among the groups.
  - Alternative Hypothesis (H1): At least two groups are significantly different.
- **Rank the observations:**
  - Combine the data from all groups and assign ranks to each observation, from 1 (lowest) to N (highest), where N is the total number of observations.
- **Calculate the sum of ranks for each group:**
  - Sum the ranks separately for each group. This gives you the sum of ranks for each treatment or condition.
- **Calculate the Friedman test statistic:**
  - Calculate the Friedman test statistic (χ^2) using the following formula:

$$\chi_F^2 = \frac{12N}{k(k+1)}\left[\sum_{i=1}^{k} R_i^2 - \frac{k(k+1)^2}{4}\right]$$

where N denotes the number of datasets used, k is number of algorithms and Ri is the rank assigned to the classifier.

- **Determine the critical value:**
  - Determine the critical value for the Friedman test statistic using the appropriate significance level and degrees of freedom. This can be found in statistical tables or calculated using statistical software.

- **Compare the test statistic and critical value:**
  - If the test statistic is greater than the critical value, reject the null hypothesis. This suggests that there are significant differences among the groups.
  - If the test statistic is smaller than the critical value, fail to reject the null hypothesis. This suggests that there is no significant difference among the groups.

Friedman test indicates significant differences among the groups, post-hoc tests (such as the Wilcoxon signed-rank test with appropriate adjustments) can be performed to identify which groups differ significantly from each other.