REASONING WITH CLASSICAL AND TYPE-2 FUZZY LOGIC

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CERTIFICATE

This is to certify that the dissertation entitled **"Reasoning with Classical and Type-2 Fuzzy Logic"** has been carried out by AMRESH KUMAR (University Registration No.: 160212 of 2021-2022) under my guidance and supervision and be accepted in partial fulfilment of the requirement for the degree of Master of Electronics and Telecommunication Engineering. The research results presented in the thesis have not been included in any other paper submitted for the award of any degree to any other University or Institute.

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I hereby declare that the thesis entitled d **"Reasoning with Classical and Type-2 Fuzzy Logic"** contains literature and original research work by the undersigned candidate, as part of his Degree of Master of Electronics and Telecommunication Engineering. All information have been obtained and presented in accordance with academic rules and ethical conduct. It is hereby declared that, as required by these rules and conduct, all materials and results that are not original to this work have been properly cited and referenced.

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Preface

The main idea behind using Type-2 fuzzy sets and logic as it allow us to model and minimize the effects of uncertainties in rule-base fuzzy logic systems. However, they are difficult to understand for a variety of reasons which we express. In this thesis , we attempt to analysis the importance of type-2 fuzzy sets and also let us define such sets very precisely, presenting type-2 fuzzy sets, and using this to derive formulas for union, intersection and complement of type-2 fuzzy sets without having to use the Extension Principle. type-2 fuzzy sets let us model and minimize the effects of uncertainties in rule-based fuzzy logic systems (FLSs), Unfortunately, type-2 fuzzy sets are more difficult to use and understand than are type-1 fuzzy sets; hence, their use is not yet widespread. In this paper we make type-2 fuzzy sets easy to use and understand.

There are (at least) four sources of uncertainties in type-1 FLSs: (1) the meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people). (2) Consequents may have a histogram of values associated with them, especially when knowledge is extracted from a group of experts who do not all agree. (3) Measurements that activate a type-1 FLS may be noisy and therefore uncertain. (4) The data that are used to tune the parameters of a type-1 FLS may also be noisy. All of these uncertainties translate into uncertainties about fuzzy set membership functions.

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Chapter 1:

Introduction

This chapter discusses about the basics of fuzzy sets and membership function and different operations performed on it . It also deals with the various types of membership function. It also deals with concept of dimensional extension of fuzzy sets..

1.1 Conventional Set:

Set is a collection of well defined distinct objects and these distinct objects are known as members of set. We represent set by a symbol such as A, B etc.

Eg:A={1,2,4,5,6} Here symbol A denotes the set and 1,2,4,5,6 are its members, we can also say that members are subset of set A and it is denoted as members$\subseteq$A.

We can also write sets with conditions such as A{x/ conditions, x$\in$X} where ‘x’ ‘denotes members of set and X are universe of discourse.

Eg: A={x/50≤x≤60, x∈ X} and according to condition set can have members A={50,51,52……,59,60} concept of conventional set is based on binary logic that means either it completely satisfies the condition or completely does not satisfy.

We can also define conventional set in terms of membership value it will be either 1 or 0 where 1 represent members completely belongs to set and 0 means the element doesn’t belong to set completely. From this we say conventional set is based on binary logic.

1.2 Fuzzy Sets:

In case of fuzzy sets the membership value of members can vary from 0 to 1. Fuzzy logic is based on multi-valued logic, in this case we can define the degree of belongingness of members. gIn case of fuzzy logic we can define three scenario like ,0 it depicts element completely does not belongs to set,1 it depicts element completely belongs to set and any value between 0 to 1 depicts element partially belongs to set.

Fuzzy set is represented in terms of ordered pair of elements and its degree of membership value.

Eg:A={(5,.1),(6,.2),(7,.4),(8,.3)} where 5,6,7,8 represent elements and .1,.2,.4,.3 indicates corresponding membership value of elements.

•Representation of discrete fuzzy set:

$A=\mu_{A(X1)/X1+}\mu_{A(X2)/X2+............=\sum}\ \mu_{A(}x_{i)/}\ x_i$ where $x_i \in X$ and X is universe of discourse and $\mu_{A(}x_{i)}$ denotes degree of membership value, x_i represents members of fuzzy set.

.•Representation of continuous fuzzy set:

$A=\int_x \mu_{A\ (}x_{i)/}\ x_{i\ \text{where}}\ x_{i\ \text{represents members and}}\ \mu_{A(}x_{i)}$ denotes degree of membership value.

Example of fuzzy set:

If we have name of four metropolitan city and extent of air pollution that exist there in terms of membership value so that we can decide which city is better for living, we can represent this statement in terms of fuzzy set in many ways such as:

•A={(KOLKATA,.4),(DELHI,.8),(MUMBAI,.7),(CHENNAI,.5)} or

•A={(.4/Kolkata),(.8/Delhi),(.7/Mumbai),(.5/Chennai)} or\

•A={(.4/Kolkata)+(.8/Delhi)+(.7/Mumbai)+(.5/Chennai)or

•A={(Kolkata/.4)+(Delhi/.8)+(Mumbai/.7)+(Chennai/.5) or

•A={(Kolkata/.4),(Delhi/.8),(Mumbai/.7),(Chennai/.5)}

1.3 Membership Functions:

Membership function gives relation between members of fuzzy set and its corresponding membership value and the membership value varies between 0 and 1.when we observe the graph of membership function if the degree of membership value is 1 that means element is fully present or fully belongs to the fuzzy set ,if corresponding membership value of element is 0 that means element is not member of fuzzy set and if the graph of membership function shows that degree of membership value lies between 0 and 1 then element partially belongs to that fuzzy set .

we can define membership function in terms of mapping between element of fuzzy set and its membership value due to this mapping membership function can have different type of curves like the normal functions.

• $\mu_{A:}$ xi→ (0, 1), xi∈X this represents the mapping of generic variable xi (elements of fuzzy set) mapped with respect to its corresponding membership value. Where μ_A is membership value, xi is generic variable, X is the universe of discourse and A is the fuzzy set which is subset of X.

• IF a fuzzy set is represented in this form A= {μ_{A}(xi)/ xi} where xi∈X.

Then μ_{A}(xi) represents membership function of xi in set A.

•we can illustrate many problems with the help of membership function that can be seen in below example.

Example: if we have three categories of age group like young aged, middle aged, old aged then in terms of membership function can be depicted as given below.

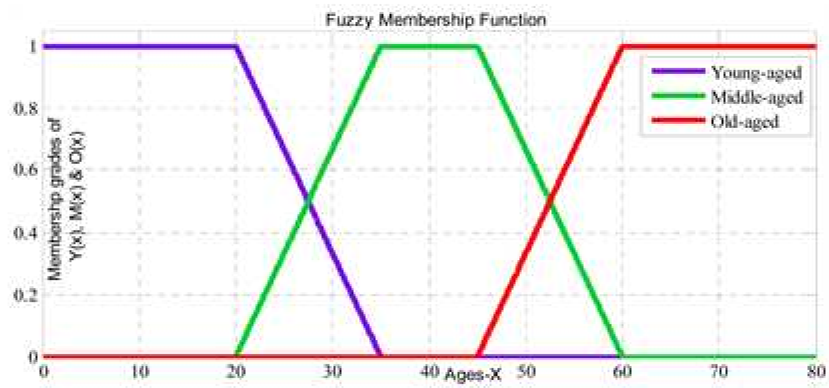


Fig.1.1 membership function represents three groups

1.4 Continuous and discrete membership function:

In case of continuous membership function the universe of discourse will be continuous in nature such as temperature variation for a particular day. Here we have to allot a range of temperature.

In the case of discrete membership function the universe of discourse will be discrete in nature such as age of a person, it will take discrete value. We can also convert continuous universe of discourse into discrete by sampling. There can be two types of sampling: uniform sampling and non-uniform sampling, in case of uniform sampling interval of generic variable will be equal whereas in case of non-uniform sampling interval of generic variable will be unequal.

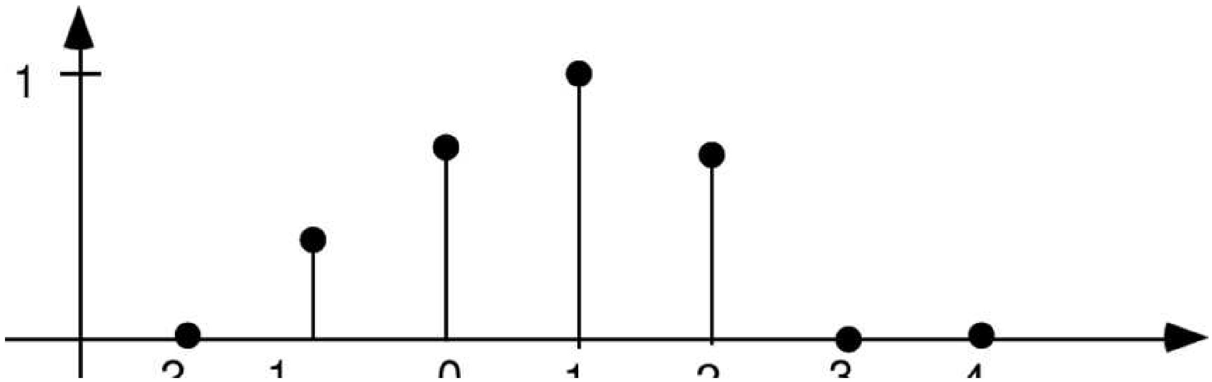


Fig.1.2 Represents discrete membership function.

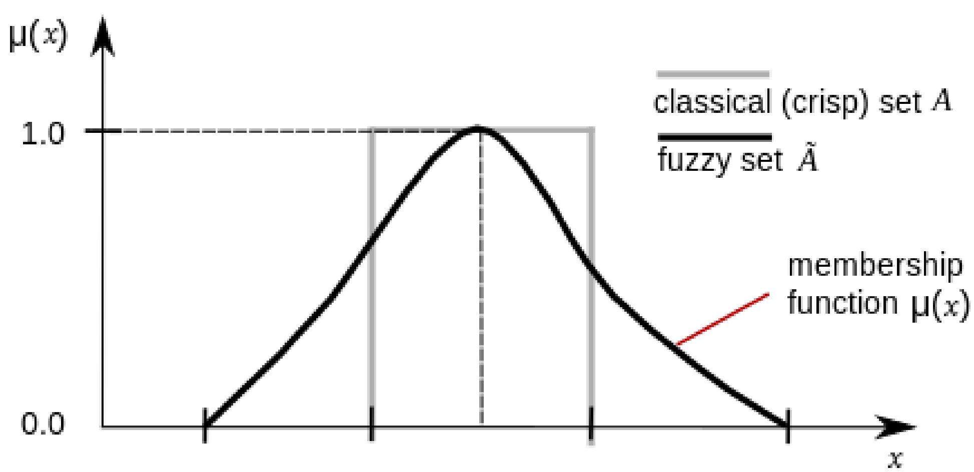


Fig.1.3

1.5 Standard membership function:

Generally membership functions can take any form but there are some function is frequently used in fuzzy logic. Those functions can be referred as standard membership function; some of them are mentioned below:

Triangular membership function
ϒ membership function
Trapezoidal membership function
Gaussian membership function
Sigmoid membership function
Left –Right Membership function
Right open membership function
π membership function

1.5.1 Triangular membership function:

In case of triangular membership function the shape of function will be triangular and three points on x –axis (p, q, and r) will represent three vertices of triangular function and mathematical expression of function is given below:

$$TRI(x; p, q, r) = \begin{cases} 0 & x \leq p \\ (X-p)/(q-p) & p \leq x \leq q \\ (r-x)/(r-q) & q \leq x \leq r \end{cases}$$

0 $x \geq r$

Triangular membership function can also be generated max-min composition and its expression is given below:

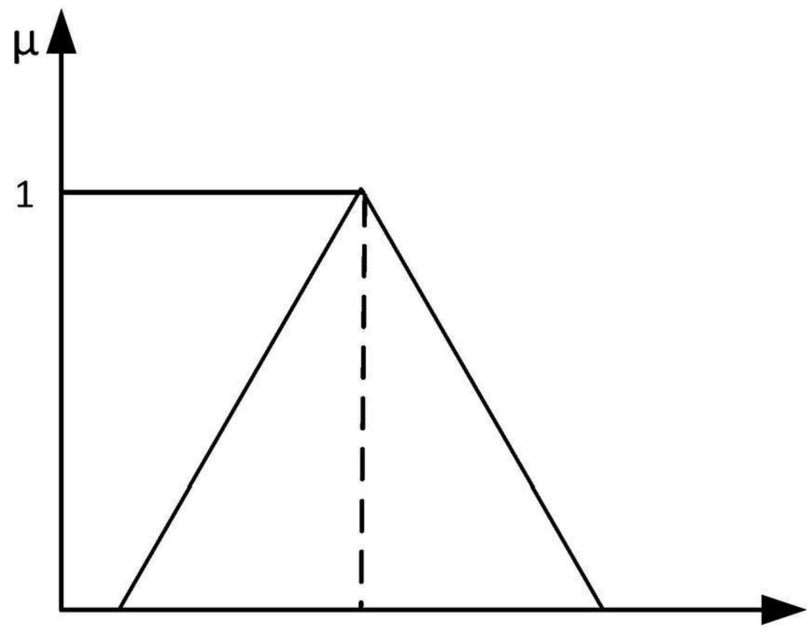
TRI(x; p, q, r) =max [min {(x-p/q-p),(r-x/r-q)},0]

•.We can generate triangular membership functions by using below given mat lab code of any shape according to the need of application or problem to solve.

Eg: mat lab code to generate triangular membership function having three vertices p=8, q=10, r=12

Mat lab Code:

```
Clear;
Close all;
 Clc;
X= (0, .2, 20);
Y=tri mf(x,[8 10 12]);
Plot (x, y,'linewidth',8.0)
 Y lim ([0 1]);
 Y label ("membership value");
Set (gca,'fontname', Times','Fontsize', 25.0);
```

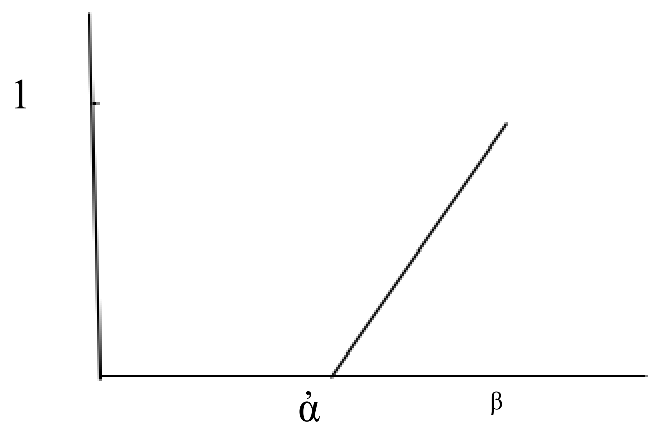


1.4 triangular membership function r.g

1.5.2 ϒ membership function:

This membership function is generally defined by two parameters ‘p’ and ‘q’ on x-axis and the shape of membership function plotted below along with mathematical expression.

$$\Upsilon(x; p, q) = \begin{cases} 0 & x \leq p \\ (x-p)/(q-p) & p \leq x \leq q \\ 1 & x \geq q \end{cases}$$



y-axis represents membership function and x-axis represents generic variable 1.5

1.5. 3 Trapezoidal membership function

It takes four parameters to represent trapezoidal membership function (p, q, r ,s) where p <q ≤ r <s .mathematical expression and figure is represented below:

0 $x \leq p$

(x- p)/ (q-p)   p ≤x ≤q

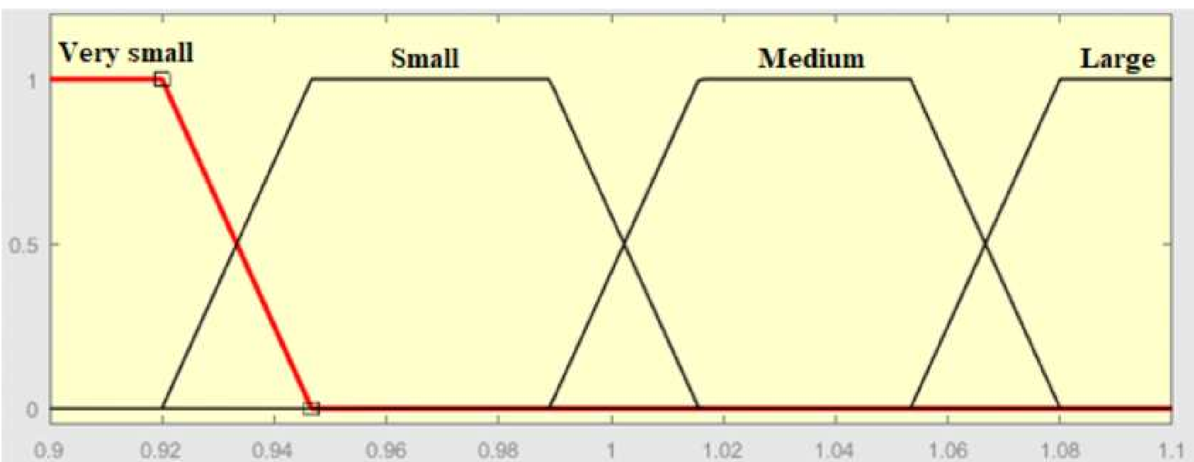
Trapezoidal(x; p, q, r, s) =   1   q≤ x ≤r

d- x /d-c   r≤ x ≤s

0   x ≥s

We can also generate trapezoidal membership function using min -max composition.

п(x; p, q, r, s) =max [min{x-p/q-x, 1, s-x/s-r}, 0]



1.6rg

1.5.4 Trapezoidal membership function

By writing the below given Mat lab code we can generate the trapezoidal membership function:

Taking respective values of parameter p=4, q=6, r=10, s=12

```
Clear;

Close all;

Clc;

X= (0:0.2:20);

Y=trapmf(x,[4,6,10,12]);

Plot (x, y,’Linewidth’, 8.0);

Ylim ([0,1]);

Ylabel (“membership value”);

Set (gca,’Fontname’,’Times’,’Fontsize’, 26.0);
```

1.5.5 Gaussian membership function:

Gaussian membership function depends on two parameters μ and σ where they are responsible for centre and width of membership function.

Gaussian(x;μ,σ)=exp $exp\ -1/2[\{(x-\mu)/\sigma\}^2]$

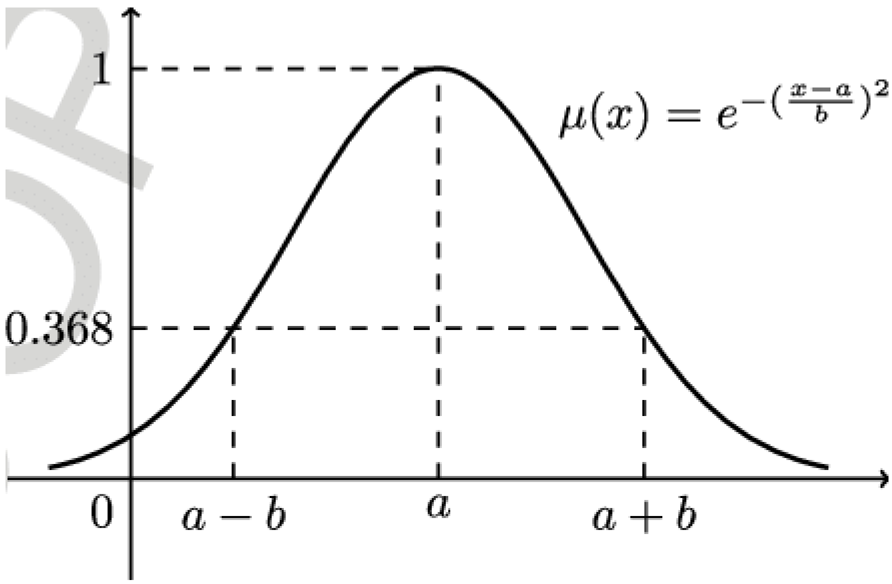


Fig.1.7rg

1.5.6 Gaussian membership function

Gaussian is one of the most widely used membership function in fuzzy logic for instance with the help of this function we can explain the population growth of anything.

We can generate Gaussian function using below given Mat alb code:

Taking μ=3 and =.8

Clear;

```
Close all;

Clc;

x= (0:0.2:20);

y=gaussmemb(x, [0.8 3]);

Plot (x, y,'Linewidth', 8.0);

Ylim ([0 1]);

Ylabel ("Membership value");

Set (gca,'Fontname','Times','Fontsize', 25.0);
```

1.5.7 Membership function:

This function is also based on two parameters 'p' and 'q' where they manipulate the width of changing area and the centre of the changing area respectively. Mathematical expression of function is given below.

$$Sig\ (x; p, q) = 1/(1+ e^{-p(x-q)})$$

This function can be generated in Mat lab using given below code:

```
  Clear;

Close all;

Clc;

x= (0:0.2:20);

y=sigmf(x, [6 7]);
```

```
Plot (x, y,'Linewidth', 8.0);
Ylim ([0 1]);
Ylabel ("Membership value");
Set (gca,'FontName', Times','FontSize', 26.0);
```

1.5.8 Left-Right membership function:

This function is based on three parameters 'p', 'q' and 'r' and its mathematical expression is given below:

$$\text{Left-Right}(x; p, q, r) = \begin{cases} F_{left}[(r-x)/p] & x \leq r \\ F_{right}[(x-r)/q] & x \geq r \end{cases}$$

Where F_{left} and F_{right} are monotonically decreasing function and defined on the domain [0 ∞] and it should satisfy two conditions:

(a) $F_{left}(0) = F_{right}(0) = 1$

(b) $\text{Lim}_{\to\infty} F_{left} = \lim_{\to\infty} F_{right} = 0$

•r is the value on the x axis corresponding to which membership value is1.

• Width of the left region is controlled by p.

- Width of right region is controlled by q.

1.5.9 Right open membership function:

This function is based on two parameters ‘p’ and ‘q’, mathematical expression is given below

$$\text{Open Right}(x; p, q) = \begin{cases} 0 & x < p \\ x-p/q-p & p \leq x \leq \\ 1 & q < x \end{cases}$$

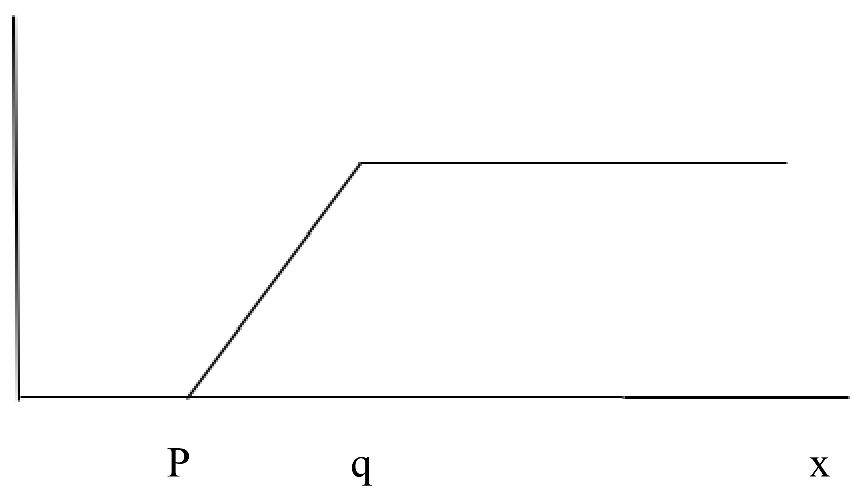


Fig.1.8

1.5.10 this membership function can be generated by using Mat lab code:

Taking p=4 and q=6

Clear;

```
Close all;
Clc;
x= (0:0.2:20);
p=4; q=6;
Open R =zeros (size(x));
For i =1: size(x, 1)
If x (i, 1) <p
Open (i, 1) =1;
Else
Open R (i, 1) ={x (i, 1)-p}/ (q-p);
  End
End
Plot (x, openR,'Linewidth', 8.0);
```

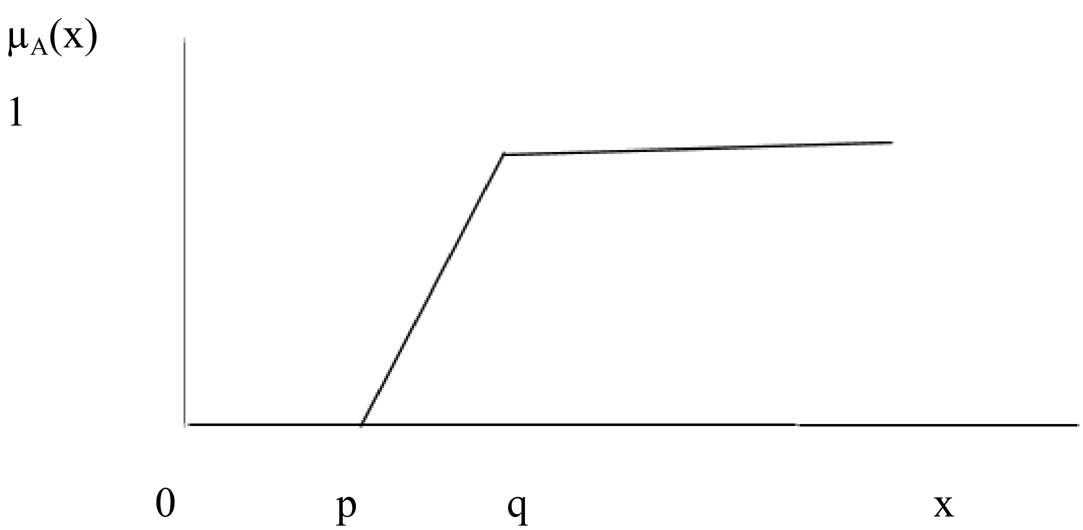


Fig 1.9

1.6 Terms used in Fuzzy Set Theory:

1.6.1Support:

Collection of all the points on the x-axis(generic variable) for which the value of membership function is greater than zero is known as support.

Mathematical expression of support is:

Support (A) = [x| μ_A(X)>0]

Where μA, x represents membership function and generic variable respectively.

$\mu A(x)$

\

Eg:

If a fuzzy set is given A[(1,.2),(3,.4),(5,.6),(7,.8), (9,0)] then find support of set A.

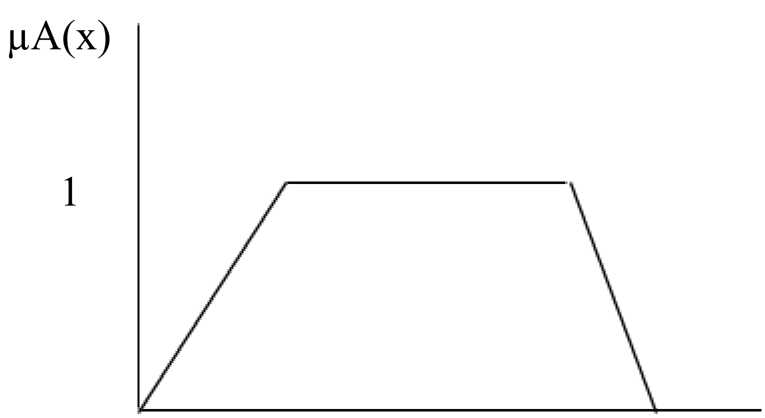
Sol: The support of set A will be (1, 3, 5, 7) and 9 is not included in support of A because its membership value is 0 to become support of A membership value of that element should be ≥0

1.6.2 Core:

For any generic variable to become core of given fuzzy set its corresponding membership value should be equal to 1 and its mathematical expression is given below:

Core (A) =[x/$\mu_{a(x)}$=1] from x_1 to x_2

And its graph will be:



x_1 x_2 x

fig1.11

Example: find the core of given fuzzy set A[(3,.5),(4,.5),(6,1),(8,1),(9,.9),(1,1)] .

Solution:

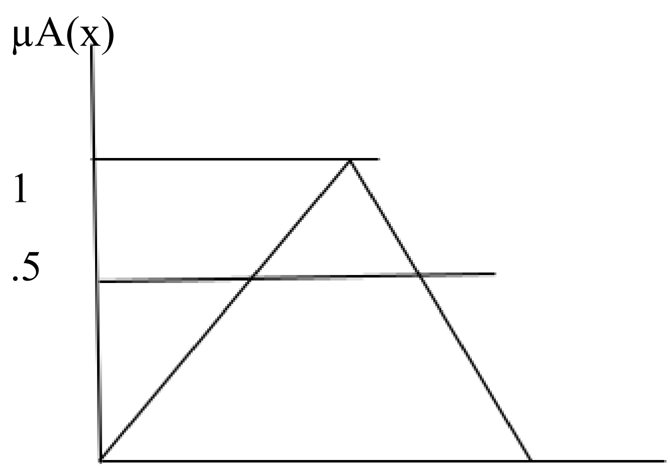
the core of fuzzy set A will be (6,8,1) as for other elements of universe of discourse X the membership value is less than 1.

1.6.3 Crossover Points:

Crossover point is the value of generic variable corresponding to which the membership value is 0.5. And the mathematical expression_is given below:

Cross-over-point (A) = [x / $\mu_A(x)$ =0.5] from x_1 to x_2

Figure of crossover point is represented below:



x_1 x_2 x

fig1.14

Example:

Find the crossover points of fuzzy set A{(1,.2),(3,.4),(5,.6),(7,.5),(8,.5),(9,.5)}

Solution:

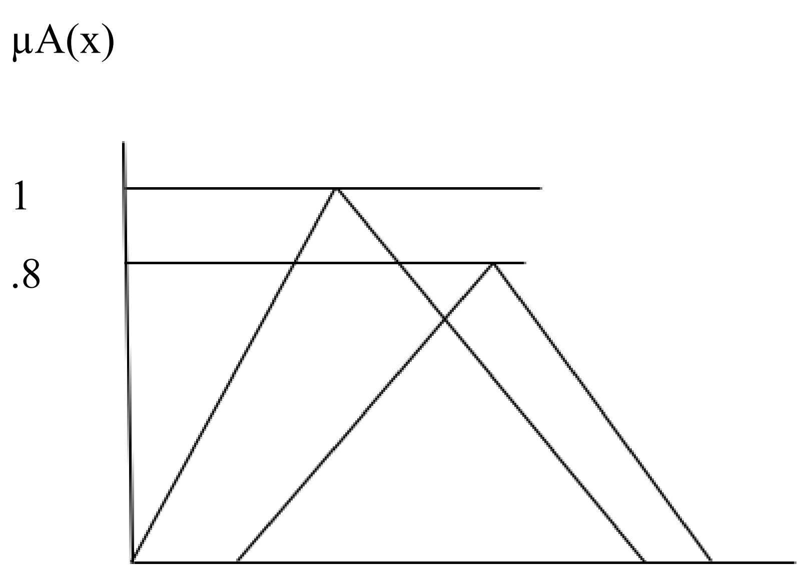
The crossover points in fuzzy set A is (7, 8, 9) and others are not because their membership value is not equal to 0.5 they have either less or greater than 0.5.

1.6.4 Height:

The generic variable which has maximum membership value within the fuzzy set is known as height of fuzzy set. If no generic variable has membership value equal to 1 than the fuzzy set is called sub-normal fuzzy set. Mathematical expression is given below.

$$\text{Height (A)} = \text{Max } \{\mu_A(x)\}$$

Figure of Height of fuzzy set will be



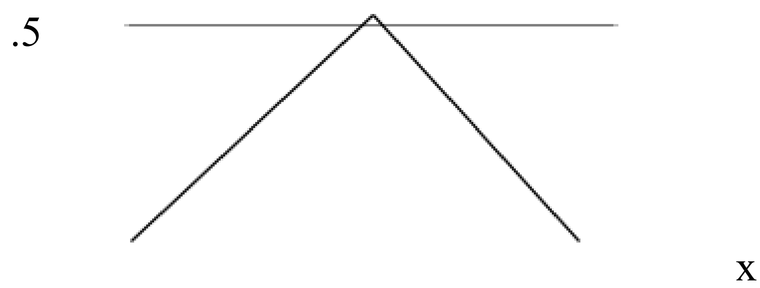


fig1.15

Example: IF given fuzzy sets are

A{(1,.2),(3,.2),(6,.8),(9,.2)}

B{(2,.3),(4,.6),(6,.2),(9,.8)} then find the height of fuzzy sets.

Solution:

In case of fuzzy set A, max [$\mu_A(x)$] is 0.8 and corresponding generic variable is 6. So height of set A will be 6

Similarly height of set B will be 9.

1.6.5 Normal fuzzy set:

Fuzzy set is called normal fuzzy set if at least one generic variable exist whose corresponding membership value is 1 then the set is called normal fuzzy set.

In other words we can say if the CORE of fuzzy set is non-empty then the set is called normal fuzzy set.

Figure of normal fuzzy set:

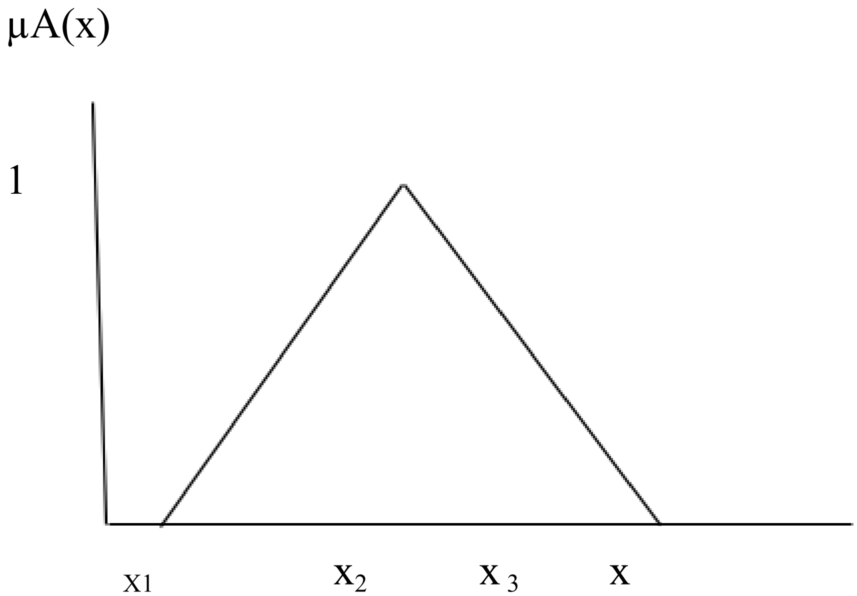


Fig1.16

Example: If two fuzzy sets are given below find which one is normal fuzzy set and which one is not

A{(2,.3),(4,.2),(6,.5),(8,.9),(5,1)}

B{(5,.2),(6,.4),(7,.9),(8,.5),(1,.1)}

Solution:

After observing the membership values corresponding to each generic variable we can see in case of set A membership value corresponding to 5 is 1 so set A is normal fuzzy set and set B is not.

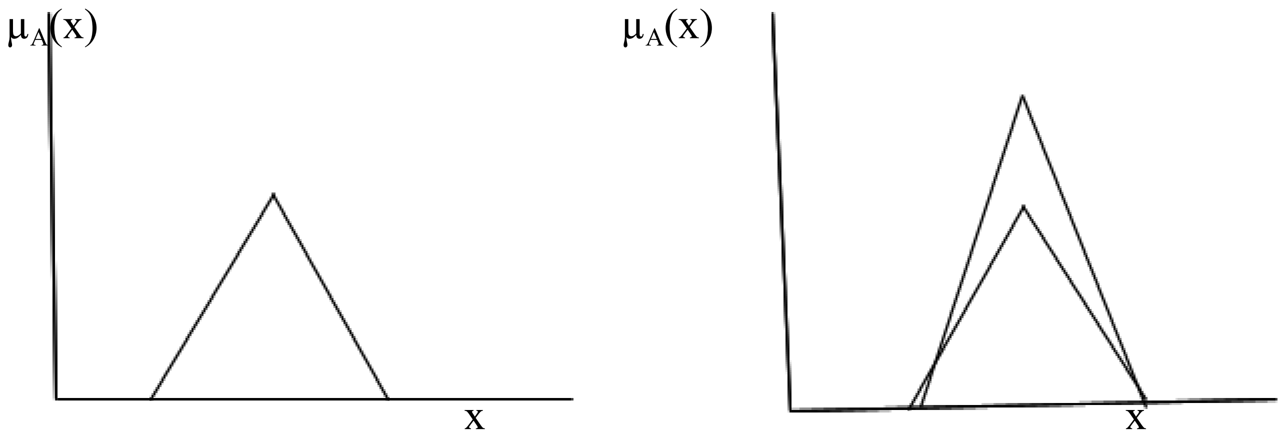
1.6.6 Normalization of a fuzzy set:

If a given fuzzy set is not normal then we can convert the fuzzy set into normal fuzzy set and the process is known as Normalization of fuzzy set. The mathematical expression for normalization is given below of sub-normal fuzzy set A, and it is represented by $A_{N.}$

$$A_N = \sum_X [\mu_A(x)/\text{height } \{A\}]$$

Figure representation of Normalization of a fuzzy set is given below:

Fig1.17



<u>Example</u>: A subnormal fuzzy set A {(1,.2),(3,.6),(5,.6),(8,.9),(10,.5),(12,.8),(13,.7)}

Solution: height of A is .9 which we can say by comparing all the membership values of fuzzy set A.

$A_N = \sum_X [\mu_A(x)/\text{height}\ \{A\}]$ this is normalization process by which we can find the normal of a fuzzy set. We will take all the membership value and divide it by the highest membership value.

For 1 ,.2/.9=.22

For 3, .6/.9=.66

For 5, .6/.9=.66

For8, .9/.9=1

For 10,.5/.9=.55

For 12,.8/.9=.88

For13,.7/.9=.77

So normalized fuzzy set will be

{(1,.22),(3,.66),(5,.66),(8,1),(10,.55),(12,.88),(13,.77)}

1.6.7 Alpha- cut of fuzzy set:

In case of alpha cut of a fuzzy set the alpha corresponds to any fixed or crisp membership value which will divide the graph plotted between generic variable and their membership value into two parts and we consider upper part of graph as alpha cut that mean $\mu_A(x) \geq \alpha$.

Mathematical expression is given below:

$$A_{\alpha =} \{x/ \mu_A(x) \geq \alpha\}$$

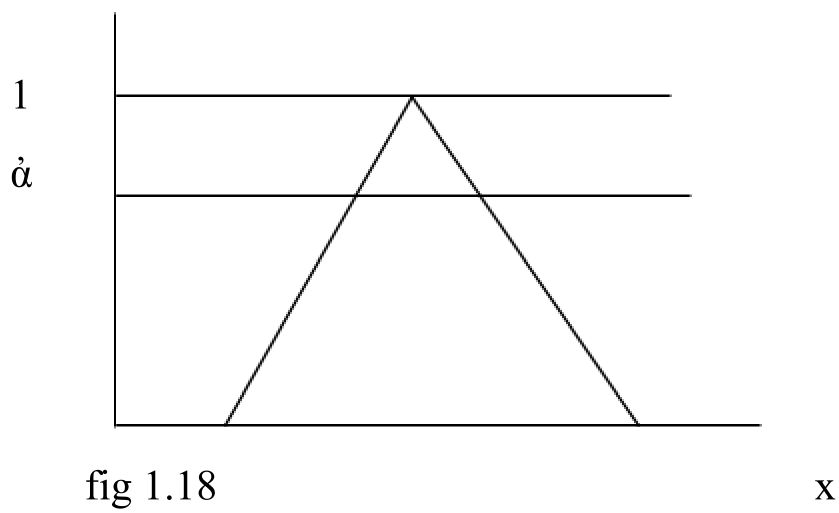


fig 1.18

Example: Determine the alpha cut of given Fuzzy set A

A{(2,.3),(5,.4),(6,.1),(8,.8),(9,.8),(12,1),(14,.9)}

IF value of α=0.4

Solution: After observing the membership value of fuzzy set and given value of alpha we can write alpha cut of set A.

$A_{0.4}$= (5, 6, 8, 9, 12, 14)

1.6.8 Strong alpha cut:

In case of strong alpha all the things are mathematical to alpha cut but the only difference between alpha and strong alpha cut is the equal to sign in mathematical equation.

$A_\alpha=\{x/\ \mu_a(x)> \alpha\}$

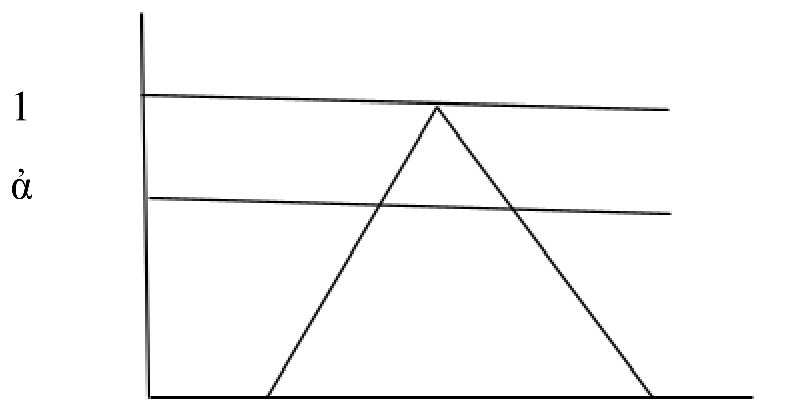


fig 1.19

Example:

Determine strong alpha cut of given fuzzy set A, where value of α= 0.5

A{(5,.4),(6,.6),(8,.9),(7,.8),(9,.5)}

Solution: After observing the membership value of fuzzy set and given value of alpha we can write strong alpha cut of set A.

STRONG $A_{0.5}$= {6, 7, 8} Where as $A_{0.5}$= {6, 7, 8, 9}

1.6.9 Convexity:

It is also very important term in fuzzy set theory, a fuzzy set is said to be convex if it satisfies following condition at any two points on generic variable x_1 and x_2 which belongs to universe of discourse X. If we have $p \in [0\ 1]$ then it should satisfy following mathematical condition.

$$\mu_A\{ px_1 + (1-p)x_2\}| \geq \text{Min}[\mu_A(x_1), \mu_A(x_2)]$$

A fuzzy set is said to be convex if the value of its membership function is strictly monotonically increasing first then monotonically decreasing for increasing values of generic variable (x) which is sub-set of universe of discourse. In mathematical expression we can write.

If the set is defined for three generic variable points x_1, x_2, x3 where x_1 x_2 x_3 then above situation of convexity can be expressed as:

$\mu_A(x_2) \geq \min[\mu_A(x_1), \mu_A(x_3)]$ where $x_{1,}\ x_{2,}\ x_3 \in x \in X$

x:generic variable represented on x-axis

X: universe of discourse

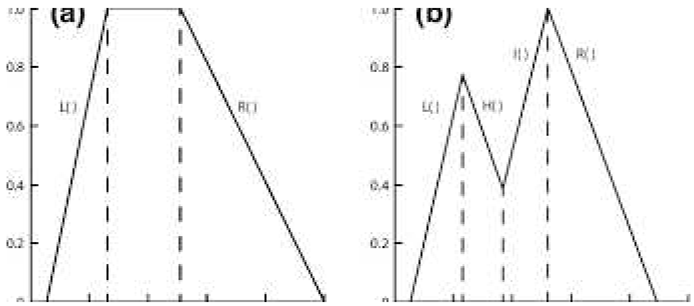


Fig 1.20

Example:

Determine whether given fuzzy set is convex or not

A={(1,.1),(3,.2),(4,.3),(5,.4),(6,.6),(8,.7)}

B ={(2,.3),(3,.4),(4,.5),(6,.7),(8,.4),(9,.8)}

Solution:

After observing the membership values of above fuzzy sets we can say set A satisfies the convexity condition where as in case set B

$\mu_B(x_5) \geq$ Min $[\mu_B(x_4), \mu_B(x_6)]$ that means $0.4 \geq$ Min [0.7, 0.8]

So, set A is convex and set B is not convex in nature.

1.6.10 Fuzzy number:

If a fuzzy set satisfies condition of normality and convexity at the same time then it is called FUZZY NUMBER.

Mathematically if it satisfies max $\{\mu(x) = 1\}$ and $\mu(x_2) \geq [\mu(x_1), \mu(x_3)]$ where $x_1 < x_2 < x_3$.

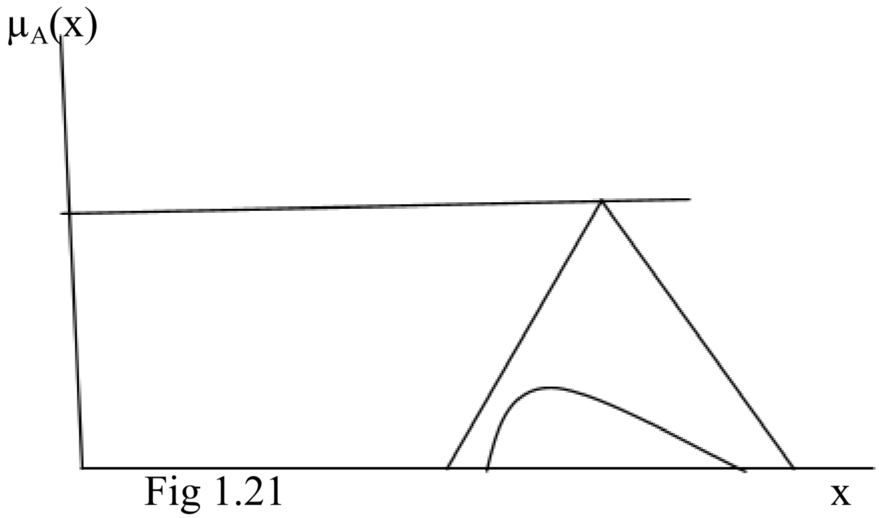


Fig 1.21

1.6.11 Cardinality of fuzzy set:

Cardinality simply counts the number of elements present in the given fuzzy sets for those sets which have discrete type of universe of discourse.In case of sets which have continuous nature of universe of discourse then its cardinality will be infinite(∞).

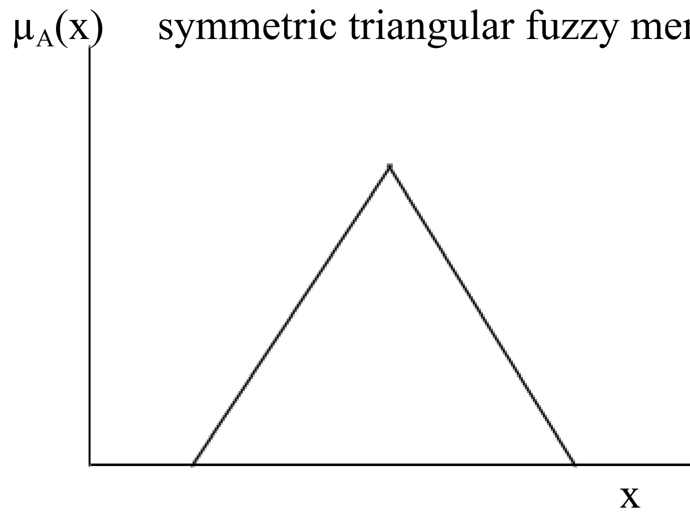
Example:

A{(2,.3),(5,.2),(6,.5),(8,.4),(12,.9)} so cardinality of set A will be five. It simply counts number of elements present in the given fuzzy set.

1.6.12 Fuzzy Symmetry:

A Fuzzy set is said to be symmetric if its membership function shows symmetry about a point (on x-axis) and mathematical expression for this is if we consider given point as p then:

$$\mu_A\,(p+x) = \mu_A\,(p-x) \quad \text{where } x \in X$$



symmetric triangular fuzzy membership function

1.6.13Equality of fuzzy sets:

Two given fuzzy sets are equal if they satisfy below given mathematical expression.

$$\mu_A(x) = \mu_B(x) \text{ where } x \in X \quad \text{and X is universe of discourse}$$

Example:

A[(2,.3),(4,.1),(6,.5),(8,.9)(9,.1)]

B[(2,.3),(4,.1),(6,.5),(8,.9),(9,.1)]

Here set A=B as it satisfies above mathematical expression.

Subset of fuzzy set:

If we two sets A and B then set A is subset of B if

$\mu_A(x) \leq \mu_B(x)$

Example (1):

Example (2): If we have two sets 'P' and 'Q'

P[(2,.3),(3,.2),(5,.6),(7,.8),(9,.2),(12,.8)]

Q[(2,.2),(3,.1),(5,.4),(7,.6), (9,.1),(12,.6)]

Solution:

$\mu_Q(x) \leq \mu_P(x)$ so after observing the above sets we can

Say Q is subset of P.

Example (3)

If we have two fuzzy sets

P [(4,.1),(5,.2),(6,.5),(7,.8)]

Q[(4,.1),(5,.3),(6,.4),(7,.6)] Solution: In above case $\mu_Q(x) \leq \mu_P(x)$ that means it is not able to satisfy the condition of equality at element $\mu_Q(5) > \mu_P(5)$.

2 Operations on sets:

2.1(1) Union

In order to implement the above mentioned operator both the set should have same universe of discourse but the mathematical expression for union operator will be different in case of classical and fuzzy set.

2.1.1(a) In case of classical set: if we have two sets P and Q then after applying union operator the output will contain elements either from set P or set Q or it can be from both the set, mathematical expression is given below.

$P \vee Q = [x / x \in P \text{ or } x \in Q]$

Example: If we have two classical sets P and Q as given below

P {1, 2, 3} and Q {2, 3, 4}

Solution:

According to mathematical expression $P \vee Q = \{1, 2, 3, 4\}$

2.1.2(b)In case of fuzzy set:

If we have two sets P and Q then mathematical expression for union operator will be as given below.

$P \vee Q = \text{Max}\ [\mu_P(x), \mu_Q(x)]$ where $x \in X$ and X is universe of discourse.

2.1.3 Example: If two fuzzy sets P and Q are given below then find $P \vee Q$.

P[(4,.3),(5,.4),(12,.5),(16,.9),(17,.8)]

Q[(4,.5),(17,..9)]

Solution: According to mathematical expression

$P \vee Q$=[(4,.5),(5,.4),(12,.6),(16,.9),(17,.9)]

Properties of union and intersection operator in case of fuzzy set:

If we have fuzzy sets P,Q and R then

(1)Commutative:

$[P \vee Q]=[Q \vee P]$

(2)Associativity:

$[P \vee Q] \vee R = P \vee [Q \vee R]$

(3)Idempotent:

$P \vee P = P$

(4)Involution:

$\overline{\overline{P}} = P$

(5)Distributive:

$P \vee (Q \vee R)=(P \vee Q) \wedge (P \wedge R)$ OR $P \wedge (Q \vee R)=(P \wedge Q) \vee (P \wedge R)$

(6)Absorption:

$P \vee (P \wedge Q) = P$

$P \wedge (P \vee Q) = P$

(7) DeMorgan's Law:

$\overline{P \vee Q} = \overline{P} \wedge \overline{Q}$

$\overline{P \wedge Q} = \overline{P} \vee \overline{Q}$

(7)Law of contradiction:

$P \wedge \overline{P} = \phi$

2.2Intersection:

In case of intersection operator also both the set should have same universe of discourse X. The mathematical expression for union operator is different in case of classical and fuzzy se

2.2.1(a) Classical set:

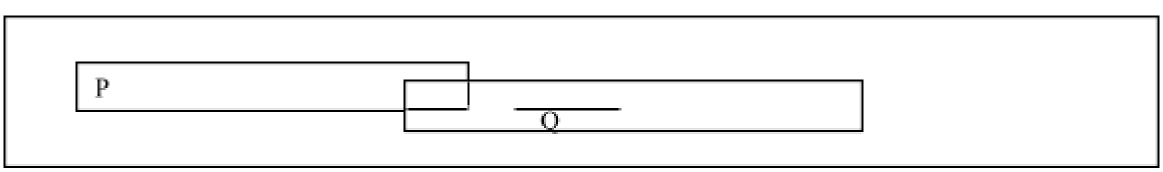
If we have two classical set P and Q then P^Q will include all the elements which are present in both the set P and Q at the same time.

Example:

If we have two sets P[1,2,3,4,5,6] and Q[3,4,5,6,7,8] then determine P^Q.

Solution:

P^Q = [3, 4, 5, 6] as these elements are present in both the sets.



(b)Fuzzy set:

Intersection operator works differently in case of fuzzy set which we can observe from mathematical expression as mentioned below.

$$P \wedge Q = Min\ [\mu_P(x), \mu_Q(x)]$$

Example: If we have two fuzzy sets P and Q then determine P∧Q.

P[(3,.1),(4,.2),(5,.3),(6,.5),(8,.8)]

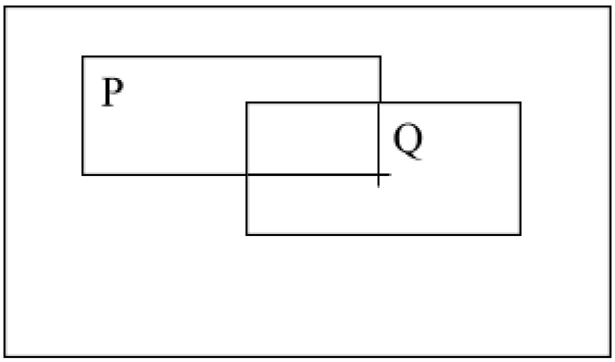
Q[(5,.6),(6,.3),(8,.7)]

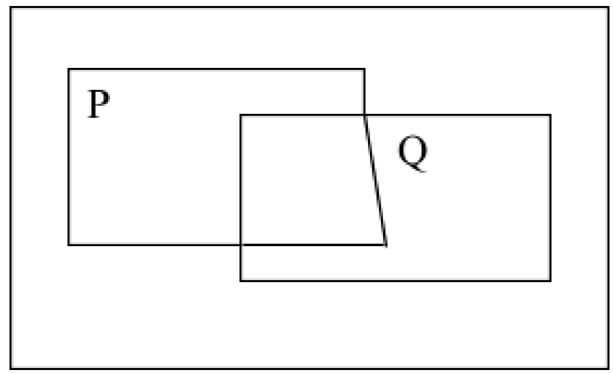
Solution:

As we know from mathematical expression $P \wedge Q = Min[\mu_P(x), \mu_Q(x)]$ then P∧Q=[(3,.1),(4,.2),(5,.3),(6,.3),(8,.7)]

2.3Difference operator:

2.3.1(a)<u>Classical Set</u>: we find difference of one set with respect to other that means if we are finding difference of set P with respect to other set Q then it will contain all the elements that are present in set P but not present in set Q.





<u>Example</u>: If we have two sets P and Q then find difference of set P with respect to Q.

P[2,3,4,5,6,7]

Q[3,4,5,6,7,9]

Solution:

Difference of set P w.r.t Q=P/Q = P/Q = [2] because other elements are present in set Q.

2.3.2(b)Fuzzy set:

In case of fuzzy set the difference of of two sets can be expressed in following way

Difference of set P with respect to set Q=P/Q=Min [$\mu_p(x)$, $\mu_Q(\bar{x})$]

Difference of set Q with respect to P=Q/P= Min [$\mu_Q(x)$, $\mu_p(\overline{x})$]

Example: If we have two sets P[(3,.4),(4,.5),(5,.6),(8,.9),(12,.5)] and

Q[(3,.3),(4,.6),(5,.9),(8,.7),(12,.8)] then find difference of P with respect to Q.

Solution:

In order to find difference of set P with respect to Q we need to find complement of membership value of set Q.

P[(1,.4),(2,.3),(4,.5),(6,.7),(8,.4),(9,.8)]

Q[(1,.3),(2,.4),(4,.6),(6,.9),(8,.6),(9,.6)]

Solution: Q [1,.3),(2,.4),(4,.6),(6,.9),(8,.6),(9,.6)

Complement of set Q [(1,.7),(2,.6),(4,.4),(6,.1),(8,.4),(9,.4)]

P[(1,.4),(2,.3),(4,.5),(6,.7),(8,.4),(9,.8)] difference of P with respect to Q=min[$\mu_p(x)$,complement of$\mu_Q(x)$]

Then answer will be P/Q=[(1,.4),(2,.3),(4,.4),(6,.1),(8,.4),(9,.4)] where P/Q represents difference of P with respect to set Q.

2.4Arithmetic Operations on Fuzzy Numbers:

When a fuzzy set satisfies both the properties normality and convexity simultaneously then the fuzzy set is known as fuzzy number. There are four types of arithmetic operations which are performed on fuzzy number.

(1)Addition

(2)Subtraction

(3)Multiplication

(4)Division

2.4.1Addition:

If we have two fuzzy numbers P and Q then R which is output of addition

If R is discrete then $R=\sum_x \mu_R(x_R)/x_R$

If R is continuous then $R=\int_x \mu_R(x_R)/x_R$

Mathematical expression for finding fuzzy number in terms of fuzzy membership function is given below.

$\mu_R(x_R) = \mu_{A+B}(x_R) = \text{Max}\,[\mu_p(x_P) \wedge \mu_Q(x_Q)]$

$XR_= x_{p+}x_Q$ and $x_P\ x_Q\ x_R \in X$

Then using addition operator we will get

R=P+Q and R can be defined differently for discrete and continuous numbers.

2.4.2Example:

If we have two fuzzy numbers P and Q then determine addition of fuzzy numbers where

P[(5,.3),(6,.5),(7,.2),(8,.1),(9,.4)]

Q[(13,.4),(14,.6),(15,.8)] and both are discrete fuzzy numbers.

Solution:

$R=P+Q = \sum\mu_R(x_R)/x_R = \sum Max[\mu_p(x_p) \wedge \mu_Q(x_Q)/ x_{p+}x_Q$

P[(5,.3),(6,.5),(7,.2),(8,.1),(9,.4)]

Q[(13,.4),(14,.6),(15,.8)]

In order to find R we have to operate on each term in following way:

Suppose if we take first term from each set (5,.3) from P and (13,.4) from set Q , we can form first term of R in following way

Min {.3, .4}/(5+13)= 0.3/18 and similarly we can generate other terms of set R.

R=[{min(.3,.4)/(5+13)},{min(.3,.6)/(5+14)},{(.3,.8)/(5+15)},{min(.4,.5)/(6+13)},{min(.5,.6)/(6+14)},{min(.5,.8)/(6+15)},{min(.2,.4)/(7+13)},{min(.2,.6)/(7+14)},{min(.2,.8)/(7+15)},{min(.1,.4)/(13+8)},{min(.1,.6)/(8+14)},{min(.1,.8)/(8+15},{min(.4,.4)/(9+13)},{min(.4,.6)/(9+14)},{min(.4,.8)/(9+15)}]

Rearranging the above expression:

R=[{.3/18}+{.3/19}+{.3/20}+{.4/19}+{.5/20}+{.5/21}+{.2/20}+{.2/21}+{.2/22}+{.1/21}+{.1/22}+{.1/23}+{.4/22}+{.4/23}+{.4/24}]

Now we will rearrange the above expression to bring the membership value of same generic variable together.

R=[{.3/18},Max{(.3/19),(.4/19)},Max{(.3/20),(.5/20),(.2/20)},Max{(.5/21),(.2/21),(.1/21)},Max{(.2/22),(.1/22),(.4/22)},Max{(.1/23),(.4/23)},Max{.4/24}]

R=.3/18+.4/19+.5/20+.5/21+.4/22+.4/23+.4/24

2.4.3Subtraction:

If we have two fuzzy numbers P and Q then R which is output of subtraction.

If R is discrete then $R=\sum_x \mu_R(x_R)/x_R$

If R is continuous then $R=\int_x \mu_R(x_R)/x_R$

Mathematical expression for finding fuzzy number in terms of fuzzy membership function is given below.

$\mu_R(x_R) = \mu_{p\text{-}Q}(x_R) = \text{Max}\,[\mu_p(x_P) \wedge \mu_Q(x_Q)]$

$x_R = x_P - x_Q$ and $x_P\ x_Q\ x_R \in X$

Then using subtraction operator we will get

R=P-Q and R can be defined differently for discrete and continuous numbers.

2.4.4Example:

If we have two fuzzy numbers P and Q then determine subtraction of fuzzy numbers where

P[(5,.3),(6,.5),(7,.2),(8,.1),(9,.4)]

Q[(13,.4),(14,.6),(15,.8)] and both are discrete fuzzy numbers.

Solution:

In order to find R we have to operate on each term in following way:

Suppose if we take first term from each set (5, .3) from P and (13,.4) from set Q , we can form first term of R in following way

Min {.3, .4}/ (5-13) = 0.3/-8 and similarly we can generate other terms of set R.

R=[{min(.3,.4)/(5-13)},{min(.3,.6)/(5-14)},{(.3,.8)/(5-15)},{min(.4,.5)/(6-13)},{min(.5,.6)/(6-14)},{min(.5,.8)/(6-15)},{min(.2,.4)/(7-13)},{min(.2,.6)/(7-14)},{min(.2,.8)/(7-15)},{min(.1,.4)/(8-13)},{min(.1,.6)/(8-14)},{min(.1,.8)/(8-15},{min(.4,.4)/(9-13)},{min(.4,.6)/(9-14)},{min(.4,.8)/(9-15)}]

Rearranging the above expression:

R=[{.3/-8}+{.3/-9}+{.3/-10}+{.4/-7}+{.5/-8}+{.5/-9}+{.2/-6}+{.2/-7}+{.2/-8}+{.1/-5}+{.1/-6}+{.1/-7}+{.4/-4}+{.4/-5}+{.4/-6}]

Now we will rearrange the above expression to bring the membership value of same generic variable together.

R=[{.3/-8},Max{(.3/-9),(.5/-9)},Max{(.5/-8),(.2/-8)},Max{(.4/-7),(.2/-7),(.1/-7)},Max{(.2/-6),(.1/-6),(.4/-6)},Max{(.1/-5),(.4/-5)},Max{.3/-10},Max{.4/-4]

R=.3/18+.5/-9+.5/-8+.4/-7+.4/-6+.4/-5+.3/-10+.4/-4

2.4.5Multiplication:

If we have two fuzzy numbers P and Q then R which is output of multiplication.

If R is discrete then $R=\sum_x \mu_R(x_R)/x_R$

If R is continuous then $R=\int_x \mu_R(x_R)/x_R$

Mathematical expression for finding fuzzy number in terms of fuzzy membership function is given below.

$\mu_R(x_R) = \mu_{p*Q}(x_R) = \text{Max}[\mu_p(x_P) \wedge \mu_Q(x_Q)]$

$XR_{=Xp*xQ}$ and $x_P\ x_Q\ xR \in X$

Then using subtraction operator we will get

R=P-Q and R can be defined differently for discrete and continuous numbers.

2.4.6Example:

If we have two fuzzy numbers P and Q then determine multiplication of fuzzy numbers where

P[(5,.3),(6,.5),(7,.2),(8,.1),(9,.4)]

Q[(13,.4),(14,.6),(15,.8)] and both are discrete fuzzy numbers And universe of discourse X∈X[-90,90]

Solution:

In order to find R we have to operate on each term in following way:

Suppose if we take first term from each set (5,.3) from P and (13,.4) from set Q , we can form first term of R in following way

Min {.3,.4}/(5*13)= 0.3/65 and similarly we can generate other terms of set R.

R=[{min(.3,.4)/(5*13)},{min(.3,.6)/(5*14)},{(.3,.8)/(5*15)},{min(.4,.5)/(6*13)},{min(.5,.6)/(6*14)},{min(.5,.8)/(6*15)},{min(.2,.4)/(7*13)},{min(.2,.6)/(7*14)},{min(.2,.8)/(7*15)},{min(.1,.4)/(8*13)},{min(.1,.6)/(8*14)},{min(.1,.8)/(8*15},{min(.4,.4)/(9*13)},{min(.4,.6)/(9*14)},{min(.4,.8)/(9*15)}]

R=[{min(.3,.4)/(65)},{min(.3,.6)/(60)},{min(.3,.8)/(75)},{min(.4,.5)/(78)},{min(.5,.6)/(84)},{min(.5,.8)/(90)},{min(.2,.4)/(91)},{min(.2,.6)/(84)},{min(.2,.8)/(105)},{min(.1,.4)/(104)},{min(.1,.6)/(112)},{min(.1,.8)/(120},{min(.4,.4)/(117)},{min(.4,.6)/(126)},{min(.4,.8)/(135)}]

R=.3/65+.3/60+.3/75+.4/78+.5/84+.5/90+.2/91+.2/84+.2/105+.1/104+.1/112+.1/120/.4/117+.4/126+.4/135

And the given range of universe of discourse is from -90 to +90, so those terms will be eliminated which doesn’t satisfy the range of universe of discourse.

So, finally

R=.3/60+.3/65+.3/75+.4/78+.5/84+.5/90

2.4.7Division of fuzzy number:

If we have two fuzzy numbers P and Q then R which is output of multiplication

If R is discrete then $R=\sum_x \mu_R(x_R)/x_R$

If R is continuous then $R=\int_x \mu_R(x_R)/x_R$

Mathematical expression for finding fuzzy number in terms of fuzzy membership function is given below.

$\mu_R(x_R) = \mu_{p-Q}(x_R) = \text{Max}\,[\mu_p(x_P) \wedge \mu_Q(x_Q)]$

$XR_{= xP}\text{-}x_Q$ $_{\text{and}}$ x_P xQ $xR \in X$

then using division operator we will get

R=P÷Q and R can be defined differently for discrete and continuous numbers.

2.4.8Example:

If we have two fuzzy numbers P and Q then determine division of fuzzy numbers where

P[(5,.3),(6,.5),(7,.2),(8,.1),(9,.4)]

Q[(13,.4),(14,.6),(15,.8)] and both are discrete fuzzy numbers.

Solution:

In order to find R we have to operate on each term in following way:

Suppose if we take first term from each set (5,.3) from P and (13,.4) from set Q , we can form first term of R in following way

Min {.3,.4}/(5÷13)= 0.3/.384 and similarly we can generate other terms of set R.

R=[{min(.3,.4)/(5÷13)},{min(.3,.6)/(5÷14)},{(.3,.8)/(5÷15)},{min(.4,.5)/(6÷13)},{min(.5,.6)/(6÷14)},{min(.5,.8)/(6÷15)},{min(.2,.4)/(7÷13)},{min(.2,.6)/(7÷14)},{min(.2,.8)/(7÷15)},{min(.1,.4)/(8÷13)},{min(.1,.6)/(8÷14)},{min(.1,.8)/(8÷15},{min(.4,.4)/(9÷13)},{min(.4,.6)/(9÷14)},{min(.4,.8)/(9÷15)}]

R=[{min(.3,.4)/(.384)},{min(.3,.6)/(.357)},{(.3,.8)/(.333)},{min(.4,.5)/(.461)},{min(.5,.6)/(.428)},{min(.5,.8)/(.4)},{min(.2,.4)/(7÷13.538)},{min(.2,.6)/(.5)},{min(.2,.8)/(.466)},{min(.1,.4)/(.615)},{min(.1,.6)/(.571)},{min(.1,.8)/(.533},{min(.4,.4)/(.692)},{min(.4,.6)/(.642)},{min(.4,.8)/(.6)}]

R=[{.3/.384},{.3/.357},{.3/.333},{.4/.461},{.5/.428},{.5/.4},{.2/.538},{.2/.5},{.2/.466},{.1/.615},{.1/.571},{.1/.533},{.4/.692},{.4/.642},{.4/.6}]

2.5 OPERATIONS ON FUZZY SETS: generally all the operations on fuzzy sets are performed with reference to membership function of fuzzy sets

(1)Complement of fuzzy set

(2)T-Norm operator on fuzzy sets

(3)S-Norm operator on fuzzy sets

2.5.1Complement of fuzzy set:

Complement is a function or operator which transforms the membership function of fuzzy set P into membership function of fuzzy set P^C (complemented version of set P) and mathematical expression is mentioned below:

Complement $[\mu_P(x)] = \mu_P{}^C(x) = 1 - \mu_P(x)$ [generally other form of functions also exist]

If we have complemented function then it should satisfy following two criteria one of them is boundary condition and other is non-increasing criteria,

(1)Boundary condition:

Complement (0) =1 & Complement (1) =0

(2)Non-increasing condition

If $\mu_P(x_1) \geq \mu_P(x_2)$ Then

Complement $\{\mu_P(x_1)\} \leq$ Complement $\{\mu_P(x_2)\}$

2.5.2T-Norm operator on fuzzy set:

It is also known as S- Co norm and uses intersection operator on membership function of given fuzzy sets and also has to satisfy some properties then only it will qualify to become a T-Norm operator.

Mathematical expression of T-Norm is given below:

Suppose we have two fuzzy sets P and Q and we are applying T-Norm on the given two sets then

$T\ [\mu_P(x),\ \mu_Q(x)] = \mu_{p\wedge Q}(x)$ where all x belongs to given universe of discourse X.

2.6.2.1Four properties which is satisfied by the T-Norm operator:

(1)Boundary condition

(2)Monotonicity or non- decreasing

(3) Commutative

(4)Associativity

Let l, m, n, o are membership values of membership function at four different generic variable points, then

(1)Boundary condition:

$T[0,0]=0$ {From this we can interpret that $\wedge[0,0]=0$ OR $0\wedge 0=0$}

&

T [m, 1] =m

{This shows that whatever will be the value of membership function if take T-Norm of membership function with 1 then it will return the value of membership function as it is.}

(2)Monotonicity or non-decreasing:

If $l \leq n$ & $m \leq o$ then $T [l, m] \leq T [n, o]$

OR

We can also represent monotonicity in terms of membership function, suppose we have four fuzzy sets P, Q, R, S and it satisfies the condition:

$\mu_p(x) \leq \mu_R(x)$ & $\mu_Q(x) \leq \mu_S(x)$ then

$T [\mu_P(x), \mu_Q(x)] \leq T [\mu_R(x), \mu_s(x)]$

(3)Commutative:

T [l, m] =T [m, l] in terms of membership value and

It can also be represented in terms of membership function

$T [\mu_p(x), \mu_Q(x)] = T [\mu_Q(x), \mu_P(x)]$

(4)Associativity:

In terms of membership value Associativity can be expressed as

$T[T\{l, m, n] = T[l, T\{m, n\}]$

And in terms of membership function it can be expressed as

$T[T\{l, m\}, n] = T[l, T\{m, n\}]$

There are many forms of T-Norm operator or function:

(1)Minimum:

$T_{MIN}[l, m] = Min[l, m]$ {in terms of membership value}

$T_{MIN}[\mu_P(x), \mu_Q(x)] = Min\{\mu_P(x), \mu_Q(x)\}$ In terms of membership function

(2)Algebraic Product:

$T_{ap}[l, m] = l*m$ {In terms of membership value}

$Tap[\mu_P(x), \mu_Q(x)] = \{\mu_P(x)*\mu_Q(x)\}$ In terms of membership function

(3)Einstein Product:

$T_{EP}[l, m] = (l*m)/\{2-(l+m-l*m)\}$ In terms of membership value

$T_{EP}[\mu_P(x), \mu_Q(x)] = [0\vee\{\mu_P(x) + \mu Q(x)-1\}$ this is in terms of Membership function

(4)Drastic Product:

$$T_{DP}[l, m] = \begin{cases} l & \text{if } m=1 \\ m & \text{if } l=1 \\ 0 & \text{otherwise} \end{cases}$$

Above expression is for membership value

And below expression is for membership function.

$$T_{DP}[\mu_P(x), \mu_Q(x)] = \begin{cases} \mu_P(x) & \text{if } \mu_Q(x) = 1 \\ \mu_Q(x) & \text{if } \mu_Q(x) = 1 \\ 0 & \text{otherwise} \end{cases}$$

2.5.3 S-Norm operator on fuzzy set:

It is also known as T Co-norm and uses union operator on membership function of given fuzzy sets and also has to satisfy some properties then only it will qualify to become a S-Norm operator.

Mathematical expression of S-Norm is given below:

Suppose we have two fuzzy sets P and Q and we are applying S-Norm on the given two sets then

$S\ [\mu_P(x),\ \mu_Q(x)] = \mu_{p\wedge Q}\ (x)$ where all x belongs to given universe of discourse X.

2.5.3.1Four properties which is satisfied by the S-Norm operator:

(1)Boundary condition

(2)Monotonicity or non- decreasing

(3) Commutative

(4)Associativity

Let l, m, n, o are membership values of membership function at four different generic variable points, then

(1)Boundary condition:

$S\ [1, 1] = 1$ {from this we can interpret that $\vee\ [1, 1] = 1$ OR $1\vee 1=1$}

&

$T\ [m, 0] = m$

{This shows that whatever will be the value of membership function if take S-Norm of membership function with 0 then it will return the value of membership function as it is.}

(2)Monotonicity or non-decreasing:

If $l \leq n$ & $m \leq o$ then $S[l, m] \leq T[n, o]$

OR

We can also represent monotonicity in terms of membership function, suppose we have four fuzzy sets P, Q, R, S and it satisfies the condition:

$\mu_p(x) \leq \mu_R(x)$ & $\mu_Q(x) \leq \mu_S(x)$ then

$S[\mu_P(x), \mu_Q(x)] \leq S[\mu_R(x), \mu_s(x)]$

(3)Commutative:

$S[l, m] = S[m, l]$ in terms of membership value and

it can also be represented in terms of membership function

$S[\mu_p(x), \mu_Q(x)] = S[\mu_Q(x), \mu_P(x)]$

(4)Associativity:

In terms of membership value Associativity can be expressed as

$S[S\{l, m\}, n] = S[l, S\{m, n\}]$

And in terms of membership function it can be expressed as

$S[S\{l, m\},n] = S[l, S\{m, n\}]$

2.5.3.2There are many forms of S-Norm operator or function:

(1)Maximum:

$S_{Max}[l, m] = Max[l, m]$ {in terms of membership value}

$S_{MAX}[\mu_P(x), \mu_Q(x)] = Max\{\mu_P(x), \mu_Q(x)\}$ In terms of membership function

(2)Algebraic Sum:

$S_{AS}[l, m] = l + m - lm$ {In terms of membership value}

$S_{AS}[\mu_P(x), \mu_Q(x)] = [\mu_P(x) + \mu_Q(x) - \{\mu_P(x) * \mu_Q(x)\}]$ In terms of membership function

(3)Einstein Sum:

$T_{ES}[l, m] = (l + m) / \{1 + l * m\}$ In terms of membership value

$T_{ES}\{\mu_P(x), \mu_Q(x)\} = [\{\mu_P(x) + \mu_Q(x)\} / \{1 + \mu_P(x) * \mu_Q(x)]$

(4)Drastic Sum:

$$T_{DP}[l, m] = \begin{cases} l & \text{if } m=0 \\ m & \text{if } l=0 \\ 0 & \text{otherwise} \end{cases}$$

Above expression is for membership value

And below expression is for membership function.

$$T_{DP}[\mu_P(x), \mu_Q(x)] = \begin{cases} \mu_P(x) & \text{if } \mu_Q(x) = 0 \\ \mu_Q(x) & \text{if } \mu_p(x) = 0 \\ 1 & \text{if } \mu_P(x), \mu_Q(x) > 0 \end{cases}$$

2.5.3.3Example: If P and Q are two fuzzy sets and universe of discourse is X {1, 2, 3, 4}, then what will be union of P and Q using S-Norm operators.

P= {(.6/1),(.5/2),(.9/3),(.8/4)}

Q= {(.9/1), (.2/2), (.4/3), (.7/4)}

Solution:

(1)Maximum:

For maximum we have S = Max $\{\mu_P(x), \mu_Q(x)\}$ that means from above given fuzzy sets we can write:

S=[{max(.6,.9)/1},{max(.5,.2)/2},{max(.9,.4)/3,max(.8,.7)/4]

S= [(.9/1), (.5/2), (.9/3),(.8/4)] so this will be the final result of maximum.

(2)Algebraic Sum:

For algebraic sum we have expression $S = [\mu_P(x) + \mu_Q(x) - \{\mu_P(x) * \mu_Q(x)\}]$ Then,

S=
[{(.6+.9-.6*.9)/1},{(.5+.2-.5*.2)/2},{(.9+.4-.9*.4)/3},{(.8+.7-.8*.7)/4}

S= [{(1.5-.54)/1}{(.7- .10)/2}{(1.3- .36)/3}{(1.5-.56)/4}]

S= [{(.96)/1}{(.6)/2}{(.94)/3}{(.94)/4}]

(3)Einstein Sum:

In case of Einstein sum we have expression:

$S=T_{ES}\{\mu_P(x), \mu_Q(x)\} = [\{\mu_P(x) + \mu_Q(x)\}/ \{1+ \mu_P(x) * \mu_Q(x)]$

S=[{.6+.9}/{1+.6*.9}]/1,[{.5+.2}/{1+.5*.2}]/2,[{.9+.4}/{1+.9*.4}]/3,[{.8+.7}/{1+.8*.7}]/4

S=[1.5/1.54]/1,[.7/1.1]/2,[1.3/1.36]/3,[1.5/1.56]/4

S=.97/1,.63/2,.95/3,.96/4 this will be final result.

Chapter-3 Fuzzy Relation

3.1PROJECTION OF FUZZY RELATIONS:

Where P and Q is defined on the universe of discourse X and Y respectively then relation 'R' will be defined as $R \subseteq P \chi Q$ on the space X cross Y .

$R= [(x, y), \mu_R(x, y)]$ where all (x, y) belongs to X cross Y.

Now we will define projection of relations on fuzzy sets between whom that relation exist.

Projection of relation R on set P is denoted by $\mu_{RP}(x) = \max^y \mu_{RP}(x, y)$ and

Projection of relation R on set Q is denoted by

$\mu_{RQ}(y) = \max^x \mu_{RQ}(x ,y)$

3.2Example: If two fuzzy sets P and Q are defined on universe of discourse X and Y .

$P= \{.4/x_1, .2/x_2,.3/x_3\}$ And $Q=\{.5/y_1,.1/y_2,.3/y_3\}$

(1) Find relation $R_{AB}(x, y)$

Solution: From definition we know that relation exist on X cross Y space , so it will be expressed as follows

$R=[\{(x_1,y_1),\mu_R(x_1,y_1)\}, \{(x_1,y_2),\mu_R(x_1,y_2)\},\{(x_1,y_3),\mu_R(x_1,y_3)\}]$

$[\{(x_2,y_1),\mu_R(x_2,y_1)\},\{(x_2,y_2),\mu_R(x_2,y_2)\},\{(x_2,y_3),\mu_R(x_2,y_3)\}]$

$[\{(x_3,y_1),\mu_R(x_3,y_1)\},\{(x_3,y_2),\mu_R(x_3,y_2)\},\{(x_3,y_3),\mu_R(x_3,y_3)\}]$

R= $R=[\{(x_1,y_1),mini(.4,.5)\}, \{(x_1,y_2),mini(.4,.1)\},\{(x_1,y_3),mini(.4,.3)\}]$

$[\{(x_2,y_1),mini(.2,.5)\},\{(x_2,y_2),mini(.2,.1)\},\{(x_2,y_3),mini(.2,.3)\}]$

$[\{(x_3,y_1),mini(.3,.5)\},\{(x_3,y_2),mini(.3,.1)\},\{(x_3,y_3),mini(.3,.3)\}]$

$R= [\{(x_1, y_1),.4\}, \{(x_1,y_2),.1\},\{(x_1,y_3),.3\}]$

$[\{(x_2, y_1),.2\},\{(x_2,y_2),.1\},\{(x_2,y_3),.2\}]$

$[\{(x_3, y_1),.3)\},\{(x_3,y_2),.1)\},\{(x_3,y_3),.3\}]$

In matrix form :

R=

	y_1	y_2	y_3
X_1	.4	.1	.3
X_2	.2	.1	.2

X_3 .3 .1 .3

(2)Find projection of relation R on set P is represented as μ_{RP}.

$\mu_{RP}= Max^y \mu_R(x,y)=$	x_1	$_{max}\{.4 , .1, .3\}$
	x_2	$_{max}\{.2, .1, .2\}$
	x_3	$_{max}\{.3 , .1 , .3\}$

$\mu_{RP}=$	x_1	.4
	x_2	.2
	x_3	.3

$\mu_{RP}= \{.4/x_1, .2/x_2 ,.3/x_3 \}$

(3)Projection of relation R on set B is represented as $\mu_{RQ.}$

$\mu_{RQ}=Max^x \mu_R(x, y)=$

y_1	y_2	y_3
max(.4,.2,.3) ,	max(.1,.1,.1),	max(.3,.2,.3)

μ_{RQ}=[.4 ,.1 ,.3]

Finally which can be represented as?

μ_{RQ}= [(.4/y_1),(.1/y_2),(.3/y_3)]

3.3CYLINDRICAL EXTENSION OF ONE DIMENSIONAL FUZZY SET:

Cylindrical extension of one dimensional fuzzy set changes it into two dimensional space or we can say that dimension of space on which cylindrical extension is defined increases by one and one extra universe of discourse is also included ,even from name extension suggests that extension of dimension takes place. As we can see fuzzy set has one generic variable but after extension the generic variable increases from one to two.

If we have a fuzzy set P which is defined on the universe of discourse X then its cylindrical extension will be represented as C (P) and it will have different representation for continuous and discrete fuzzy set.

3.3.1(a)Representation in case of continuous fuzzy set:

$C(P)=\int\mu_{CP}(x, y)/(x,y)$ And integration will be defined on the space X*Y and $P=\int\mu_P(x)/x$ and in case of fuzzy set P integration will be defined on X.

3.3.2(b)Representation in case of discrete fuzzy set:

$C(P)=\sum\mu_{CP}(x, y)/(x,y)$ here summation is defined on the space X*Y basically in case of cylindrical extension universe of discourse changes from X to X*Y. And here $P=\sum\mu_P(x)/X$ and universe of discourse will be X.

But the value of membership function remains same before and after the cylindrical extension and the expression is given below:

$\mu_{cp}(x,y)=\mu_p(x)$ for all x, y which belongs to X cross Y

3.4Cylindrical extension of fuzzy relation:

If we have two fuzzy set P and Q and their universe of discourse is X and Y respectively and a relation R defined on the space X cross Y and we can obtain a new fuzzy set C(R) using cylindrical extension and it can be expressed as :

$C(R)=[(x ,y ,z),\mu_{CR}(x, y ,z)]$ for all x ,y, z belongs to X cross Y cross Z

And even in this case of extension the membership function value remains same before and after the extension and expressed as:

$\mu_{CR}(x, y, z)=\mu_R(x, y ,z)$ for all x, y ,z which belongs to X cross y cross Z.

_And using extension principle we can add any number of dimensions . In case of projection of fuzzy set or relation decrease in dimensionality takes place but in case of cylindrical extension of fuzzy set or relation increase in dimensionality takes place and it can be of any number according to our need, and we can obtain more clarity through given below example.

3.4.1Example:

If relation R is given between set A and B as:

$\{(x_{1,}y_1),.5\}$, $\{(x_1,y_2),.3\}$, $\{(x_{1,}y_3),.5\}$

R=__ $\{(x_2,y_1),.4\}$, $\{(x_2,y_2),.1\}$, $\{(x_2,y_3),.8\}$

$\{(x_{3,}$ y1), .2}, $\{(x_3$, y2), .5}, $\{(x_3$, y3), .6}

(1)Find the projection of relation R on set P.

$\mu_{RP}(x)= \max^y \; \mu_R(x, y)=x_1 \; \max(y_1,y_2,y_3)$

x_2 max (y_1, y2, y3)

x_3 max (y_1, y2,y3)

$= x_1$ max (.5,.3,.5)

X_2 max (.4, .1, .8)

- X_3 max (.2, .5, .6)

Final result which we will get x_1=.5 ,x_2=.8,x_3 =.6

(2)Find the projection of relation R on the set Q.

$\mu_{RQ}(y)$= $\max^x \mu_R(x,y)$= y_1 y_2 y_3

Max(x_1, x2, x3) max(x_1, x2, x3)

max(x_1,x_2,x_3)

= y_1 y_2 y

Max (.2,.4,.5) max(.3,.1,.5)

max(.5,.8,.6)

= y_1 y_2 y_3

.5 .5 .8

$\mu_{RQ}(x)$= [.5/y_1,.5/y_2, .8/y_3]

(3)Find cylindrical extension of R_P in the direction of fuzzy set Q.

Solution:

If projection of relation R is given on set P =[.5/x_1, .8/X2, .6/x_3]

Then cylindrical extension of relation R in the direction of set Q will be :

$C(R_P)$=

	y	y	y
x	.5	.5	.5
x	.5	.5	.5
x	.8	.8	.8

3.5Linguistic Variables:

A linguistic is defined by a quintuple and it consists of five variables {x, T(x), X, G, M}.

x is the linguistic variable .

T(x) is the set of values of linguistic variables.

X is the universe of discourse.

G is a syntactic rule which generates the term in T(x).

M is a semantic rule which relates with each linguistic value x.

For a linguistic variable "age".

If "age" is represents linguistic variable then its term set T(age) can have following forms:

T (age)={young, middle aged, old aged}.

"age is young" then in this case linguistic value" young "will be given to linguistic variable "age".

3.6Composite Linguistic Term:

While writing linguistic variables we generally use multiple words to describe it like ‘frequency of sound’ is linguistic variable and we can assign multiple linguistic values to it. Linguistic values can be “very high” , “high” , “low” , “very low” and can be many more.

Linguistic variable consist of many terms or composite terms and these composite terms can be divided into three groups:

(a) Primary term {Low, Medium, High}

(b)Linguistic hedge {Very low, more or less medium}

(c)Complement and connectives {not low, Low but not very low}

3.6.1(a)Primary term:

The primary term of a linguistic variable can be represented by a fuzzy set. If we are talking about the speed of a bowler then primary term which can be used are “Low, Medium and High” in linguistic variable.

_Universe of discourse

3.6.2(b)Linguistic Hedges:

In case of linguistic hedges we use adjectives and adverbs before primary terms that is why they are also known as modifiers. While designing fuzzy systems we need to represent the linguistic hedges in terms of membership function .

We can represent Linguistic Hedges in terms of membership function in following way:

Eg: If we have a fuzzy set P which is defined on the universe of discourse X and membership function $\mu_P(x)$ .Then mathematical expression for discrete and continuous fuzzy set will be:

$P=\int \mu_P(x)/x$ where x belongs to universe of discourse X.

$P=\sum^{x} \mu_p(x)/x$ where x belongs to universe of discourse X.

(1) If we have linguistic term "very or too" then in terms of membership value we can express it as:

$\mu_{veryP}(x) = \mu_{tooP}(x) = [\mu_P(x)]$ ^2 we use this mathematical expression for both either the fuzzy set is discrete or continuous.

(2) Similarly if we have linguistic term "more or less" then we its mathematical form in terms of membership value will be:

μmore or less $_P(x)= [\mu_P(x)]$^.5

(3)If the form of linguistic term is "very very very or extremely" then it can be expressed in form of linguistic form in following way:

$\mu_{\text{very very very or extremely }P}(x)=[\mu_P(x)]$^8

3.6.5Example: If we have linguistic term "High" and it is define

In terms of membership value in following way:

High={(.5/1),(.6/2),(.4/3),(1/4),(.2/5),(.3/6)} Then find

(a)very high

(b)very very very high

(a)Solution:

As given High= {(.5/1),(.6/2),(.4/3),(1/4),(.2/5),(.3/6)} then

$\mu_{\text{very high}}(x)=\sum [\mu_{High}(x)]$^2

={(.5)^2/1,(.6)^2/2,(.4)^2/3,(1)^2/4,(.2)^2/5,(.3)^2/6}

= {(.25/1),(.36/2),(.16/3),(1/4),(.4/5),(.9/6)} This will be discrete fuzzy set for linguistic variable "very high".

(b) Solution:

As given in the question High={(.5/1),(.6/2),(.4/3),(1/4),(.2/5),(.3/6)} then

$\mu_{\text{very very very high}}(x)=\sum [\mu_{\text{High}}(x)]^8$

$\mu_{\text{very very very high}}(x)= =\{(.5)^8/1,(.6)^8/2,(.4)^8/3,(1)^8/4,(.2)^8/5,(.3)^8/6\}$

$\mu_{\text{very very very high}}(x) =\{(.003/1),(.016/2),(.0006/3),(1/4),(.000002/5),(.00006/6)\}$

3.6.6(c)Complement and Connectives:

For complement we use NOT function and we have two connectives ' AND' and 'OR' and its mathematical expression is mentioned below:

$NOT(P)=\int \{1-\mu_P(x)\}/x$

$P \text{ AND } Q=\int \{\mu_P(x)\wedge\mu_Q(x)\}/x$

$P \text{ OR } Q = \int \{\mu_P(x)\wedge\mu_Q(x)\}/x$

In the above expression P and Q are given fuzzy set and μ_P and μ_Q represents corresponding membership values.

3.7Concentration ,Dilation and Composite Linguistic term:

(1)Concentration:

A concentration is basically nothing but the linguistic value of fuzzy set is raised by a power 'k' where as other things remains same which we can observe in the below representation. If we have normal fuzzy set P then its concentration will be represented as $(A)^k$ that means:

$CON(P)^k = \int\{\mu_P(x)\}^k/x$ In case of continuous fuzzy set. Where $k \geq 2$

$CON(P)^k = \sum\{\mu_P(x)\}^k/x$ In case of discrete fuzzy set.

<u>3.7.1Example</u>: If we have a discrete fuzzy set P= {.4/1,.5/2,.3/3,.8/4,.7/5} Then find concentration of given discrete fuzzy set when value of k=2.

Solution: $CON(P) = (P)^k = \sum\{\mu_P(x)\}^k/x$ where k=2

$(P)^k = \sum\{\mu_P(x)\}^2/x$

$(P)^2 = \{(.4)^2/1, (.5)^2/2, (.3)^2/3, (.8)^2/4, (.7)^2/5\}$

$(P)^2 = \{.16/1, .25/2, .9/3, .64/4, .49/5\}$ This is the result of concentration of given discrete fuzzy set .

(2)Dilation:

A dilation is basically nothing but the linguistic value of fuzzy set is raised by a power 'k' but here unlike concentration value of k will be less than 1, where as other things remains same which we can observe in the below representation. If we have normal fuzzy set P then its dilation will be represented as $DIL(A)^k$ that means:

$DIL(P)^k = \int \{\mu_P(x)\}^k/x$ In case of continuous fuzzy set. Where k<1

$DIL(P)^k = \sum \{\mu_P(x)\}^k/x$ In case of discrete fuzzy set.

3.7.2Example: If we have a discrete fuzzy set P= {.4/1,.5/2,.3/3,.8/4,.7/5} Then find Dilation of fuzzy set .

Solution: $DIL(P) = (P)^k = \sum \{\mu_P(x)\}^k/x$ where k=.5

$$(P)^k = \sum \{\mu_P(x)\}^{.5}/x$$

$$(P)^2 = \{(.4)^{.5}/1,(.5)^{.5}/2,(.3)^{.5}/3,(.8)^{.5}/4,(.7)^{.5}/5\}$$

$(P)^2 = \{.63/1,.707/2,.547/3,.894/4,.836/5\}$ This is the result of DILATION of given discrete fuzzy set .

(3)Composite Linguistic Term:

Composite linguistic term is formed by combining primary term , connectives complement , and linguistic hedges. This is demonstrated in below example

3.7.3Example:

"High frequency but not very high frequency"

In above example "frequency" is linguistic variable.

Very high refers to linguistic hedge and "but" refers to connectives and "not" refers to negation or complement.

4.1FUZZY RULES AND REASONING

4.1.1Fuzzy if-Then Rule:

The above rule is also known as fuzzy implication or fuzzy conditional statement.

"IF x is P THEN y is Q" where A and B is fuzzy set with universe of discourse X and Y respectively and in some cases Q can be a crisp set as well. Fuzzy rule consist of two parts

(a)Antecedent or Premise

(b) Conclusion.

In above case 'x is P' is Antecedent part and 'y is Q' is conclusion part in the fuzzy rule.

The above fuzzy rule can be expressed in terms of Relation which can be expressed as $P \rightarrow Q$ and in terms of membership function relation R can be expressed as $\mu_R(x, y)$.

$R(x ,y)=P\rightarrow Q=$ P cross $Q=\int\mu_R(x, y)/(x, y)$ In case of continuous

$R(x , y)=P\rightarrow Q=$P cross $Q=\sum\mu_R(x, y)/(x, y)$ In case of discrete .

Fuzzy rule can be interpreted in two ways:
4.1.1P coupled with Q

4.1.2P entails Q

(1) P coupled with Q :

In this type of interpretation of fuzzy rule $P \rightarrow Q$ indicates relation R and its mathematical expression will include T-norm Operator.

$R = P \rightarrow Q = P$ cross $Q = \int [T\{\mu_P(x), \mu_Q(x)\}]/(x, y)$ for all x, y belonging to space X cross Y. (In case of continuous)

$R = P \rightarrow Q = P$ cross $Q = \sum [T\{\mu_P(x), \mu_Q(x)\}]/(x, y)$ for all x, y belonging to space X cross Y. (In case of discrete)

By using different type of T-norm this relation will have four different ways to express itself.

(a)P coupled with Q using minimum T-norm operator.

(b) P coupled with Q using algebraic product T-norm operator.

(c) P coupled with Q using bounded T-norm operator.

(d) P coupled with Q using drastic product T-norm operator.

(a) P coupled with Q using minimum T-norm operator:

$R = P \rightarrow Q = P$ cross $Q = \int [\{\mu_P(x) \wedge \mu_Q(x)\}]/(x, y)$ for all x, y belonging to space X cross Y. (In case of continuous)

$R = P \rightarrow Q = P$ cross $Q = \sum [\{\mu_P(x) \wedge \mu_Q(x)\}]/(x, y)$ for all x, y belonging to space X cross Y. (In case of discrete)

(b) P coupled with Q using algebraic product T-norm operator:

R=P→Q=P cross Q=∫ [{$\mu_P(x)$ cross $\mu_Q(x)$}]/(x, y) for all x, y belonging to space X cross Y. (In case of continuous)

R=P→Q=P cross Q=∑ [{$\mu_P(x)$ cross $\mu_Q(x)$}]/(x, y) for all x, y belonging to space X cross Y. (In case of discrete)

(c) P coupled with Q using bounded T-norm operator:

R=P→Q=P cross Q=∫ [0∨ {$\mu_P(x) + \mu_Q(x)-1$}]/(x, y) for all x, y belonging to space X cross Y. (In case of continuous)

R=P→Q=P cross Q=∑ [0∨ {$\mu_P(x) +\mu_Q(x)-1$}]/(x, y) for all x, y belonging to space X cross Y. (In case of discrete)

(d) P coupled with Q using drastic product T-norm operator:

R=P→Q=P cross Q=∫ {$\mu_R(x, y)$} /(x, y) for all x, y belonging to space X cross Y. (In case of continuous)

R=P→Q=P cross Q=∑ {$\mu_R(x , y)$} /(x, y)) for discrete

In above case membership function for relation R in drastic

product case is defined as:

$\mu_p(x)$ if $\mu_Q(y) =1$

$\mu_R(x, y) = \mu_Q(y)$ if $\mu_p(x)=1$

0 elsewhere

(2)When P entails Q

When $P \rightarrow Q$ indicates the relation between P and Q in the form of P entails Q then also it has four forms in which this relation can be depicted.

(a)Material implication:

$R_{MI} = P \rightarrow Q = \bar{P} \vee Q$ {this is also known as Zadeh's Arithmetic rule}

$R_{MI} = \int [1 \wedge \{1-\mu_P(x)+\mu_Q(y)\}/(x, y)$ for continuous fuzzy set

$R_{MI} = \sum [1 \wedge \{1-\mu_P(x)+\mu_Q(y)\}/(x, y)$ for discrete fuzzy set

(b) Propositional calculus

$R_{PC} = P \rightarrow Q = \bar{P} \vee (P \wedge Q)$ this is also known as zadeh's max-min rule

$R_{PC} = P \rightarrow Q = \int [\{1-\mu_{p(x)}\} \vee \{\mu_p(x) \wedge \mu_Q(y)\}]/(x, y)$ for continuous fuzzy set

$R_{PC} = P \rightarrow Q = \sum [\{1-\mu_{p(x)}\} \vee \{\mu_p(x) \wedge \mu_Q(y)\}]/(x, y)$ for discrete fuzzy set (c)Extended propositional calculus

R_{EPC} =P→Q = $(\overline{P}\wedge\overline{Q}) \vee Q$ this is also called Boolean fuzzy implication

$R_{EPC} = \int [\, [\{1-\mu_P(x)\} \wedge \{1-\mu_Q(y)\}]\vee\mu_Q(y)]/(x,y)$ for continuous case

$R_{EPC} = \sum [\, [\{1-\mu_P(x)\} \wedge \{1-\mu_Q(y)\}]\vee\mu_Q(y)]/(x,y)$ for discrete case

(d)Generalization of modus pones

R_{GMP} =P→Q=P≾Q this is known as "Goguen's fuzzy implication.$R_{GMP} = \int \mu_{RGMP}(x,y)/(x,y)$ For continuous case$R_{GMP} = \sum \mu_{RGMP}(x,y)/(x,y)$ For discrete case

$$\mu_{RGMP} = \begin{cases} 1 & \text{if } \mu_P(x) \le \mu_Q(y) \\ \mu_Q(y)/\mu_p(x) & \text{if } \mu_P(x) > \mu_Q(y) \end{cases}$$

4.2FUZZY INFERENCE SYSTEM

It is the heart of any fuzzy system. The fuzzy system obtains output from given fuzzy input using fuzzy logic, fuzzy reasoning and a set of "IF THEN RULE". It uses nonlinear type of mapping and input –output can be either fuzzy or crisp. Fuzzy inference system has application in various fields like data classification ,vision system etc.

Below fig: Architecture of fuzzy inference system

4.2.1FUZZIFIER:

Function of fuzzifier is to convert the crisp input into a fuzzy input with the help of membership function present in the rule base of inference system that means output of a fuzzifier will be fuzzy whether the input is fuzzy or crisp . We can see from above inference system architecture that output of a fuzzifier becomes input of the inference system .

4.2.2 INFERENCE ENGINE:

It processes the input given to it based on fuzzy rule base and gives fuzzy value with the help of membership function present in the knowledge base. Basically inference engine does the fuzzy reasoning in order to process its input. The idea of fuzzy reasoning was introduced by Prof. Zadeh.

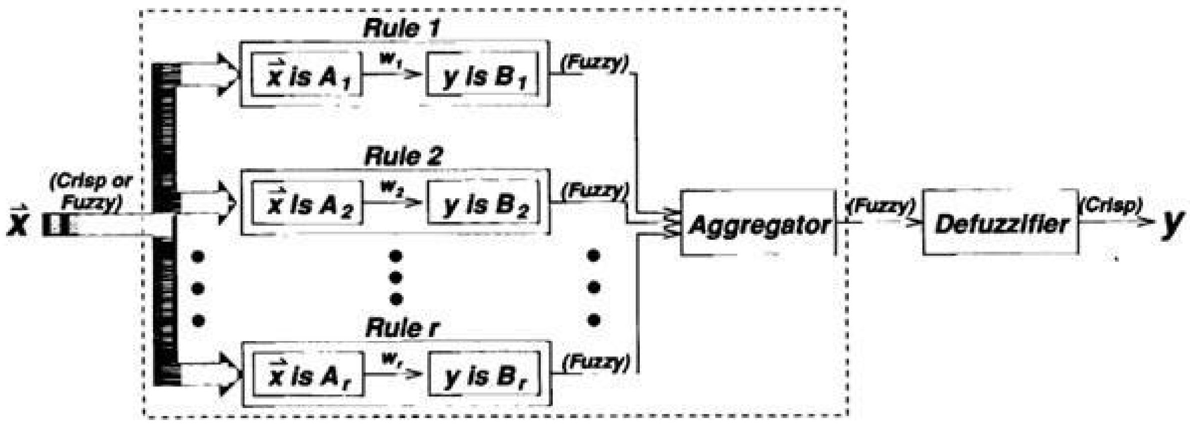


FIGURE SHOWS IF-THEN RULE ,WHERE A_1 AND B_1 ARE FUZZY SETS

4.3DEFUZZIFIER:

The work of defuzzifier is to process the fuzzy inputs and transform them into crisp value. We have many methods with the help of which we will convert fuzzy value into crisp value.

4.3(1) Centroid of Area

4.3(2)Bisector of Area

4.3(3)Mean of Maximum

4.3(4)Smallest of Maximum

4.3(5) Largest of Maximum

Fuzzy Reasoning in Fuzzy Inference System:

(1)Single Rule with Single Antecedent

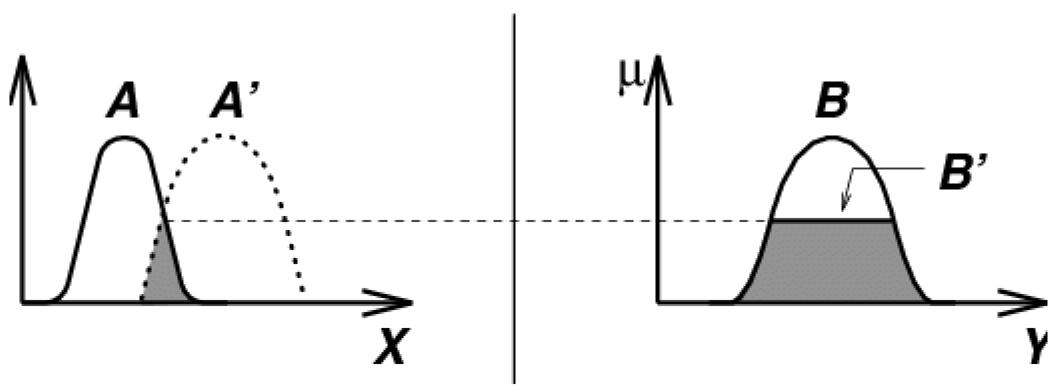
(2) Single Rule with Multiple Antecedents

(3) Multiple Rules with Multiple Antecedents

(1) Single Rule with Single Antecedent

We know fuzzy rule has two parts first part is called Antecedent and the second part is called conclusion. If in a rule it has only one first part then it is called Single Rule with Single Antecedent.

Eg: IF x is A THEN y is B.

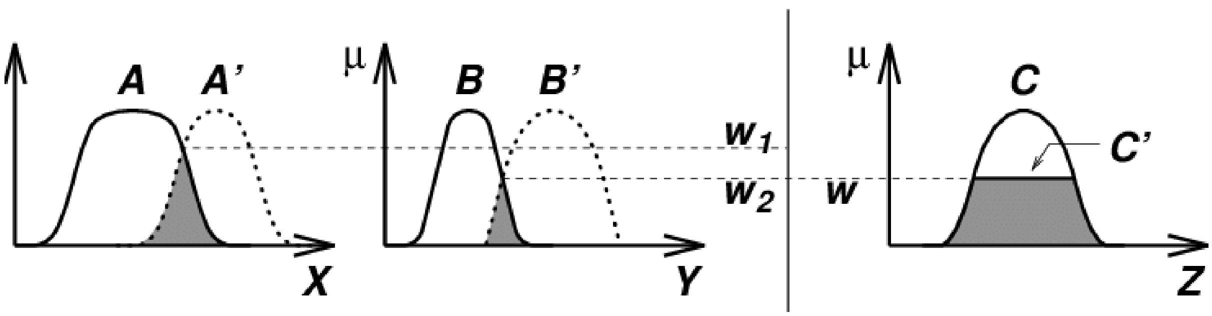


Graphical representation of above mentioned fuzzy reasoning.

(2) Single Rule with Multiple Antecedents

This reasoning uses connectives to include more than one antecedent.

Eg: "IF x is A AND y is B THEN z is C".



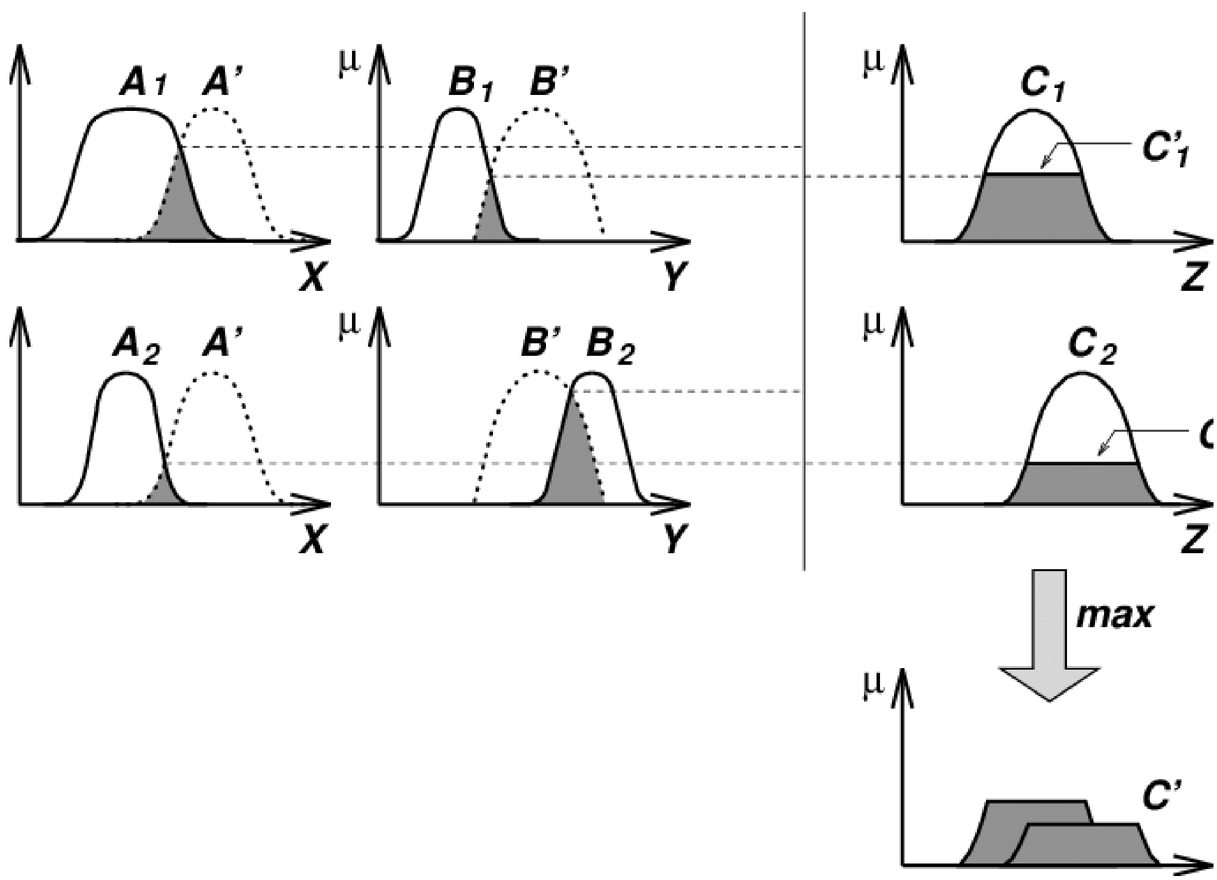
Graphical representation of above mentioned fuzzy reasoning

(3) Multiple Rules with Multiple Antecedents

This reasoning uses connectives to include more than one antecedent.

Eg: Rule (1): "IF x is A_1 AND y is B1 THEN z is C1".

Rule (2): "IF x is A_2 AND y is B2 THEN z is C2".



Graphical representation of above mentioned fuzzy reasoning

MAMDANI FUZZY MODEL

This is the first model which is based on fuzzy rules and it was developed by Prof. E .H. Mamdani. This model is used when both the parts of rule (Antecedents and Conclusion) are fuzzy in nature.

STEPS OF PROCESSING IN MAMDANI FUZZY MODEL:

(1)If the input is crisp then the Mamdani model first changes or transforms it in to fuzzy quantity.

(2)In the second step the antecedents of fuzzy rule is operated on fuzzy input it received and if the rule have multiple antecedents then it will combine them using connectives then membership function is applied on them.

(3)After second steps the system aggregates the outputs obtained from all the rules.

(4) In the last step the fuzzy output is converted into crisp value.

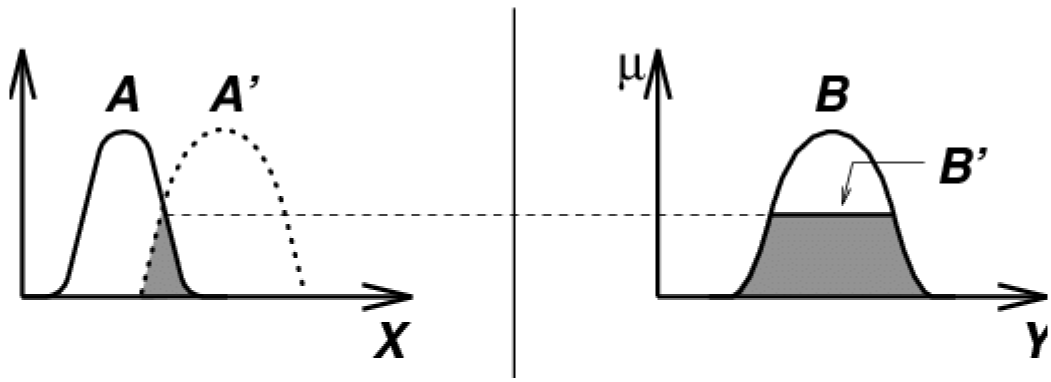
WORKING OF MAMDANI MODEL FOR FUZZY INPUT USING MAX-MIN COMPOSITION

(1) When "Single Rule with Single Antecedent"

RULE: "IF x is A THEN y is B"

INPUT: IF x is A'

CONCLUSION: y is B'



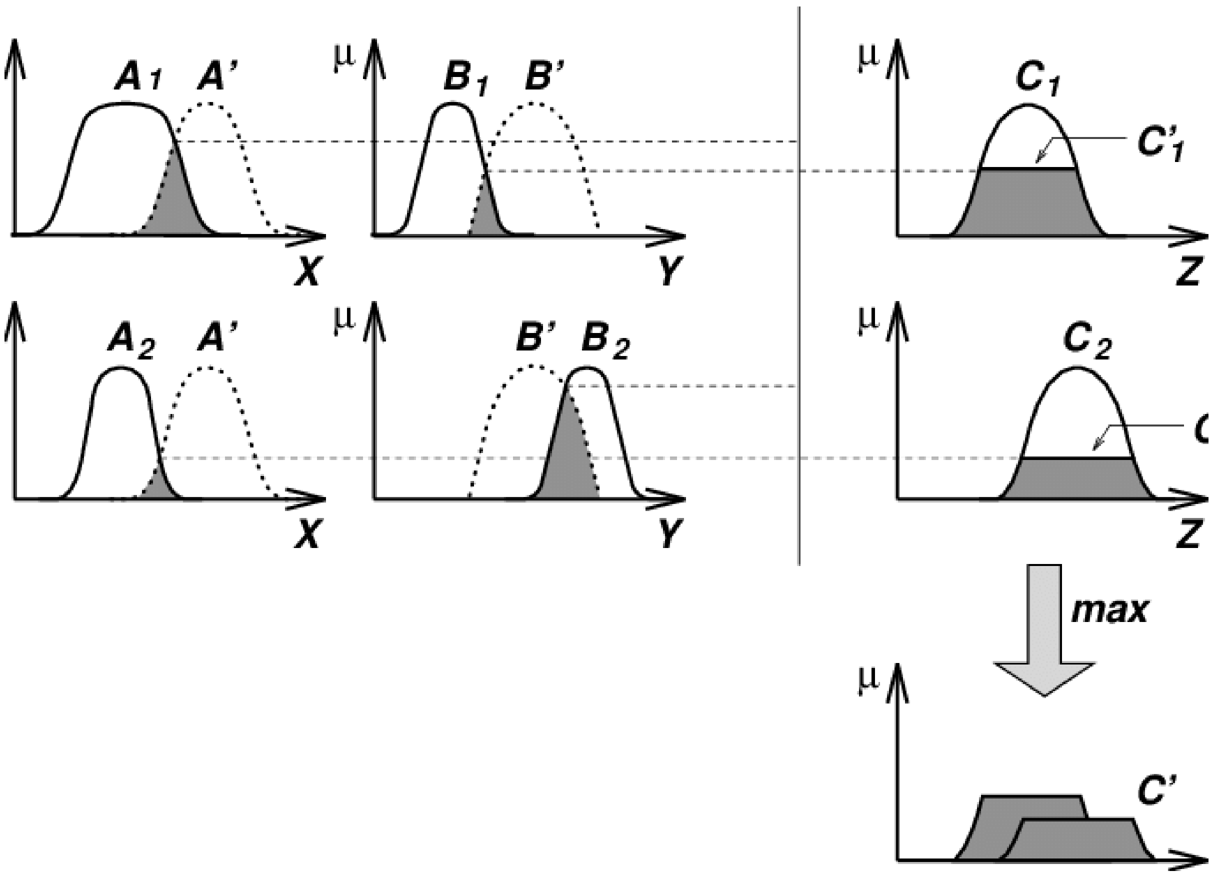
Research gate

(2)Multiple rules with multiple Antecedents:

RULE: "IF x is A AND y is B THEN z is C"

INPUT: IF x is A' and y is B'

CONCLUSION: z is C'



5.2 TYPE-2 FUZZY LOGIC

5.2.1 Type-2 fuzzy logic was introduced because it reduces the degree of uncertainties while designing the rule based systems with respect to type-1 fuzzy logic. The main difference between type-1 and type-2 fuzzy logic exist in their membership function. In case of type-1 fuzzy the membership function is crisp in nature where as in case of type-2 the membership function is fuzzy itself.

5.2.2 While designing rule based fuzzy systems based on type-1 fuzzy logic designers come across many uncertainties like in a rule the Antecedents and conclusions are linguistic in nature so meaning of words can vary person to person, measurements that are applied in case of type-1 fuzzy logic systems generally produces many types of noise and many other uncertainties are generated in case of type-1 fuzzy logic 5.2.3 system design which can be compensated by designing the rule based systems on type-2 fuzzy logic. The reason behind these kinds of uncertainties cannot be manipulated because type-1 fuzzy sets are two dimensional whereas type-2 fuzzy sets are three dimensional that is why using the effect of uncertainties can be compensated to a large extent. Due to extra dimension type-2 fuzzy sets are very complex to draw and analysis them with comparison to type-1 as they are one dimensional. We can derive complement, intersection and union related formula by implementing Prof. Zadeh's extension principle and we already know that implementing extension principle is itself not an easy thing.

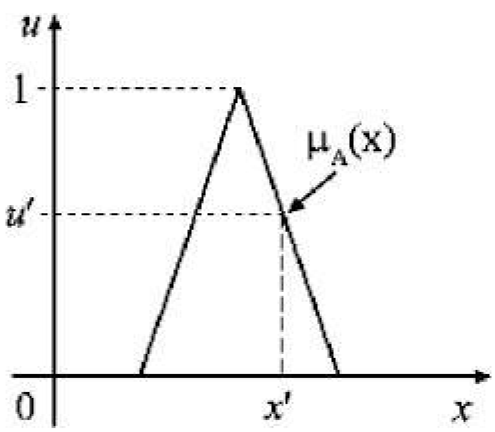


Fig.5.1

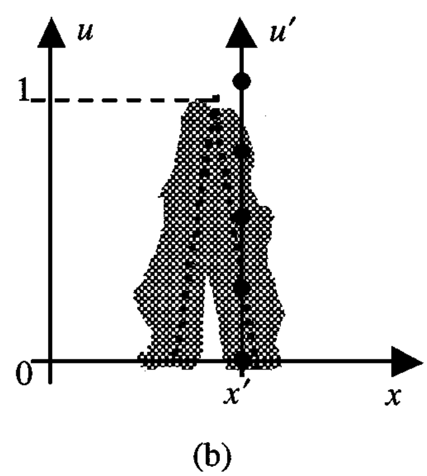


Fig5.2

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