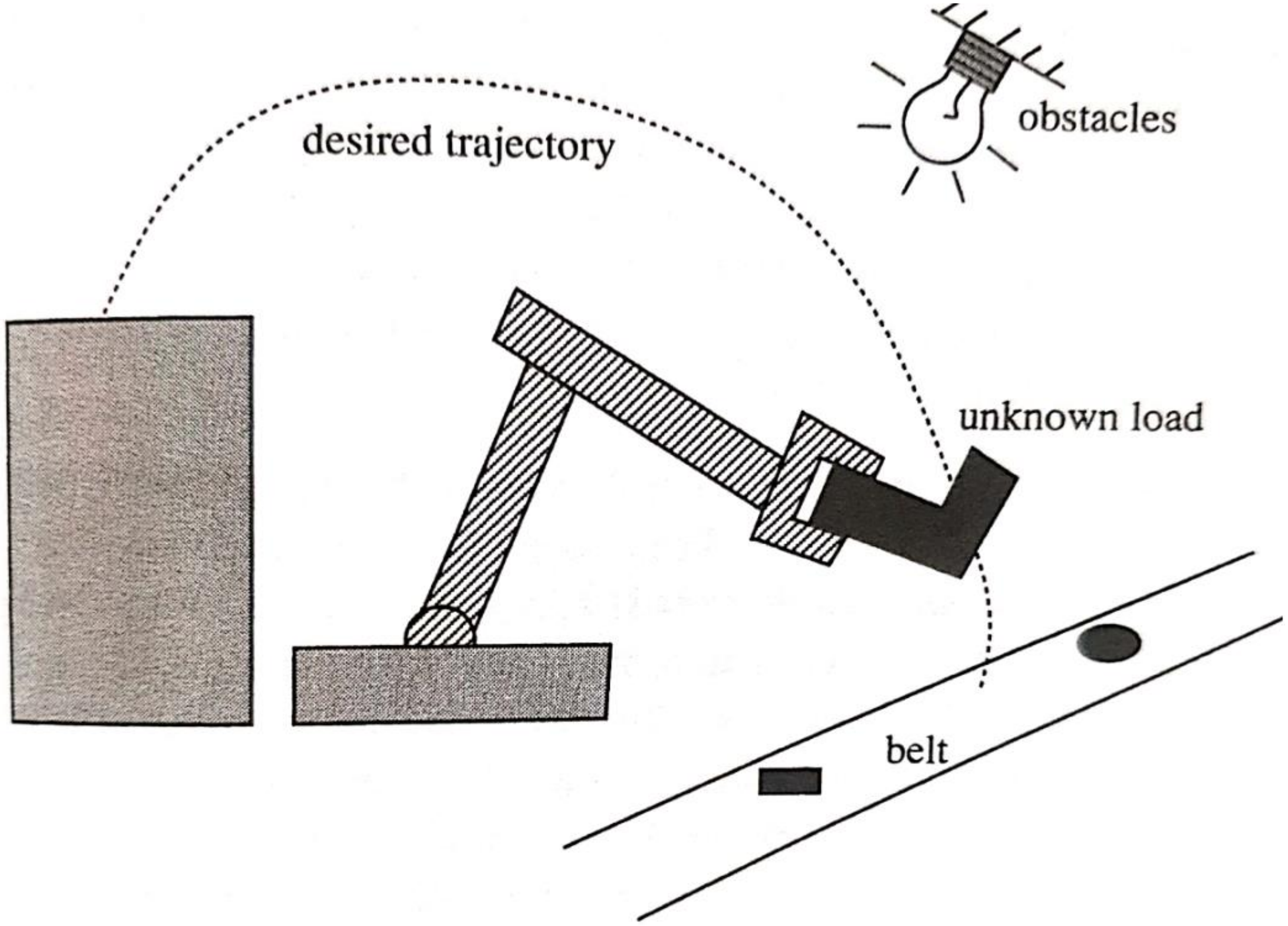
Adaptive control

- Many dynamic systems tends to have **constant** or **slowly-varying uncertain parameters**.
- Example:
  - Robot manipulator arms moving objects with unknown masses (inertial parameters).
  - Electrical power plants subjected to variations in loads.
  - Fire-fighting aircrafts experiencing considerable mass changes as they load and unload large quantities of water.
- One of the effective approaches to tackle such problem is adaptive control.
- The basic idea in adaptive control is to estimate the uncertain plant parameters “on-line” based on the measured plant (system) signals and then use the estimated values of the parameters to determine the control outputs.
- An adaptive control system ⇒ a control system with on-line parameter estimation.
- Adaptive control is inherently nonlinear – irrespective of linear plant or nonlinear plant.
- Research in adaptive control started in 1950’s - design of autopilots for aircrafts – operating in a wide range of speeds and amplitudes – experiencing a large variation in the values of the system parameters.
- Adaptive control was proposed as a way of automatically adjusting the controller parameters in the face of varying aircraft dynamics.
- Interest was lost due to lack of theoretical insight and crash of a test flight.
- The interest was rekindled in 1980’s with the development of a coherent theory and availability of cheap computation facilities.
- This lead to many practical applications – robotics, aircraft and rocket control, chemical process control, power systems control etc.

Robot manipulator arm

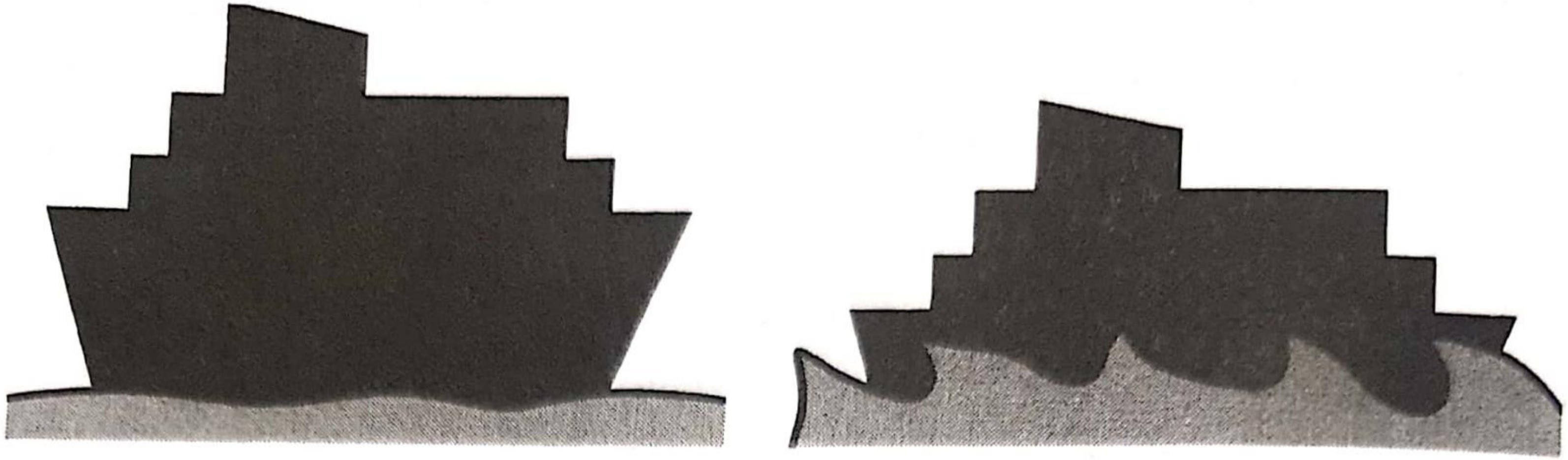
Robotic arms are required to manipulate loads of various sizes, weights and mass. It is very restrictive to assume that these parameters of the loads are well known before the robot picks them up and moves them away. If controllers with fixed gain are used with inaccurately known load parameters, robot motion can be either inaccurate or unstable. Adaptive control allows the robot to move loads of unknown parameters with high speed and accuracy.



A robot carrying a load of uncertain mass properties

Ship steering

During long sea voyages, ships are generally operated under automatic steering control. The dynamic characteristics of a ship strongly depend on many uncertain parameters like water depth, weight of the ship, wind condition, wave condition etc. Adaptive control can be applied in this case to achieve good control performance under such a varying operating condition.

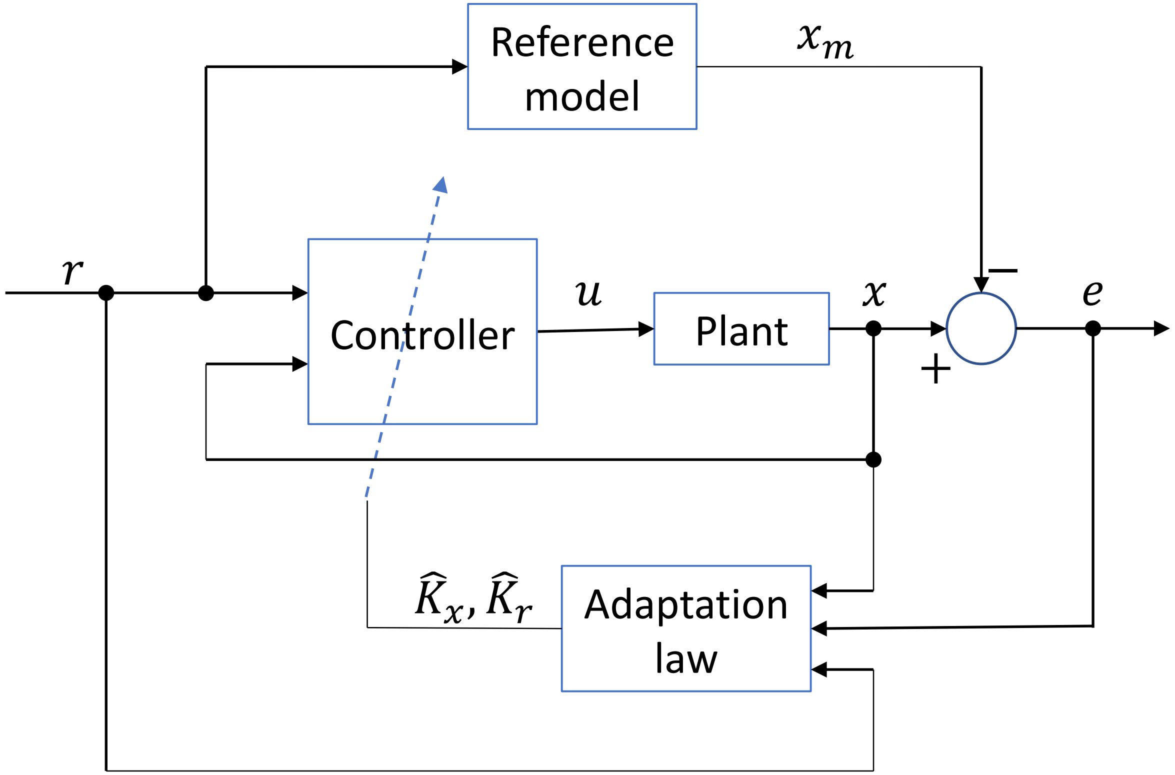


A freight ship under various loadings and sea conditions

Model Reference Adaptive Control (MRAC)

One of the main approaches of adaptive control.

- Plant:
    - Assumed to have a known structure. Parameters are unknown.
- Reference model:
    - Specifies the ideal response to the external command.
- Controller:
    - Contains a number of adjustable parameters.
- Adaptation law:
    - A way to adjust the parameters of the controller.



MRAC:

<u>The plant</u>:
The system dynamics : $\dot{x} = ax + bu$; SISO; a, b : scalars and <u>unknown</u>; $x, a, b \in \mathfrak{R}$, $b \neq 0$.
The condition on b is for the sake of controllability. If a becomes +ve, the open loop system (i.e. $b = 0$) becomes unstable.
<u>The reference model:</u>
The dynamics of the reference model: $\dot{x}_m = a_m x_m + b_m r$; r : reference input; a_m, b_m: parameters of the ref. model.
$a_m < 0$.
<u>The objective</u>:
$e(t) \to 0$ as $t \to \infty \Rightarrow$ asymptotic tracking; $e(t) \overset{\text{def}}{=} x(t) - x_m(t)$. We shall see that perfect tracking ($x(t) = x_m(t)$ for $t > 0$ with $x(0) = x_m(0)$) is not possible.
<u>The controller</u>:
$u(t) = K_x x + K_r r$; implies x is measurable i.e. it can be measured through a sensor. K_x, K_r: controller parameters or gains, which are tuneable.
<u>Error dynamics</u>:
$\dot{e} = \dot{x} - \dot{x}_m = ax + bu - a_m x_m - b_m r = ax + b(K_x x + K_r r) - a_m x_m - b_m r = (a + bK_x)x - a_m x_m + bK_r r - b_m r$.
Suppose a and b are known. Then if we set $a + bK_x = a_m$ and $bK_r = b_m$, we have the perfect tracking i.e. $\dot{e} = a_m e$.
But with unknown a, b perfect tracking is not possible. We can hope only hope to achieve asymptotic tracking.
When $a_m < 0$, we can achieve that.

The MRAC matching assumptions or conditions:

Unknown a, b implies that we cannot exactly determine the ideal controller gains K_x and K_r. But we can assume that at least K_x and K_r exist.

The matching conditions are: $a + bK_x = a_m$ and $bK_r = b_m$.

How restrictive are the matching conditions?

For our scalar case K_x and K_r always exist since we formulated our system with $b \neq 0$ e.g. $(K_x = (a_m - a)/b)$ and $K_r = b_m/b)$. For multi-dimensional case the matching conditions may or may not hold.

Conditions on the reference model:

1. Stable reference model i.e. we don't want our plant to track an unstable system
2. $r(t)$ is bounded.

These conditions imply that $x_m(t)$ is also bounded.

Controller with estimated gains:

$u(t) = \widehat{K}_x(t)x + \widehat{K}_r(t)r$; $\widehat{K}_x(t)$: time varying estimation of K_x; $\widehat{K}_r(t)$: time varying estimation of K_r.

Parameter estimation errors:

$\widetilde{K}_x(t) \stackrel{\text{def}}{=} K_x - \widehat{K}_x(t)$; $\widetilde{K}_r(t) \stackrel{\text{def}}{=} K_r - \widehat{K}_r(t)$; We can only attempt to minimize these errors; cannot eliminate them.

Error dynamics with estimated controller gains:

$\dot{e} = \dot{x} - \dot{x}_m = ax + bu - a_m x_m - b_m r = ax + b\left(\widehat{K}_x(t)x + \widehat{K}_r(t)r\right) - a_m x_m - b_m r;$

$= ax + b\left[\left(K_x - \widetilde{K}_x\right)x + \left(K_r - \widetilde{K}_r\right)r\right] - a_m x_m - b_m r;$

$= (a + bK_x)x - a_m x_m - b\widetilde{K}_x x - b\widetilde{K}_r r + bK_r r - b_m r;$

$= a_m e - b\widetilde{K}_x x - b\widetilde{K}_r r$; (Using the matching conditions. Note: knowledge of the ideal gains is not required, only there existence is needed)

If there are no errors in gain estimations, we have $\dot{e} = a_m e \Rightarrow$ the earlier case.

Use of Lyapunov candidate function to determine the estimates of controller gains:

The Lyapunov candidate function (V is globally positive definite):

$$V = \left(\frac{1}{2}\right) e^2 + \left(\frac{1}{2}\right) |b|\, \tilde{K}_x^2 + \left(\frac{1}{2}\right) |b| \tilde{K}_r^2 \,.$$

So we have the derivative of Lyapunov function:

$$\begin{aligned}\dot{V} &= e\dot{e} + |b|\tilde{K}_x\dot{\tilde{K}}_x + |b|\tilde{K}_r\dot{\tilde{K}}_r \\ &= e\left(a_m e - b\tilde{K}_x x - b\tilde{K}_r r\right) - |b|\tilde{K}_x\dot{\hat{K}}_x - |b|\tilde{K}_r\dot{\hat{K}}_r \,, (\because \tilde{K}_x = K_x - \hat{K}_x \Rightarrow \dot{\tilde{K}}_x = -\dot{\hat{K}}_x \text{ and similarly } \dot{\tilde{K}}_r = -\dot{\hat{K}}_r) \\ &= a_m e^2 - \underbrace{bxe\tilde{K}_x}_{A} - \underbrace{|b|\dot{\hat{K}}_x\tilde{K}_x}_{B} - \underbrace{bre\tilde{K}_r}_{C} - \underbrace{|b|\dot{\hat{K}}_r\tilde{K}_r}_{D}\end{aligned}$$

In the above expression we only have the definite knowledge about the sign of $a_m e^2$ which is negative. We are not at all sure about the signs of the other terms since $\tilde{K}_x, \tilde{K}_r, x, r, e$ are time varying. If we can get rid of these terms, $\dot{V}$ becomes –ve definite ($\dot{\mathrm{V}} = a_m e^2$) ⇒ the adaptive control system is globally stable .

Elimination of terms A and B: Since $bxe = |b|\,\mathrm{sgn}(b)xe$, set $\dot{\hat{K}}_x = -\,\mathrm{sgn}(b)xe$.

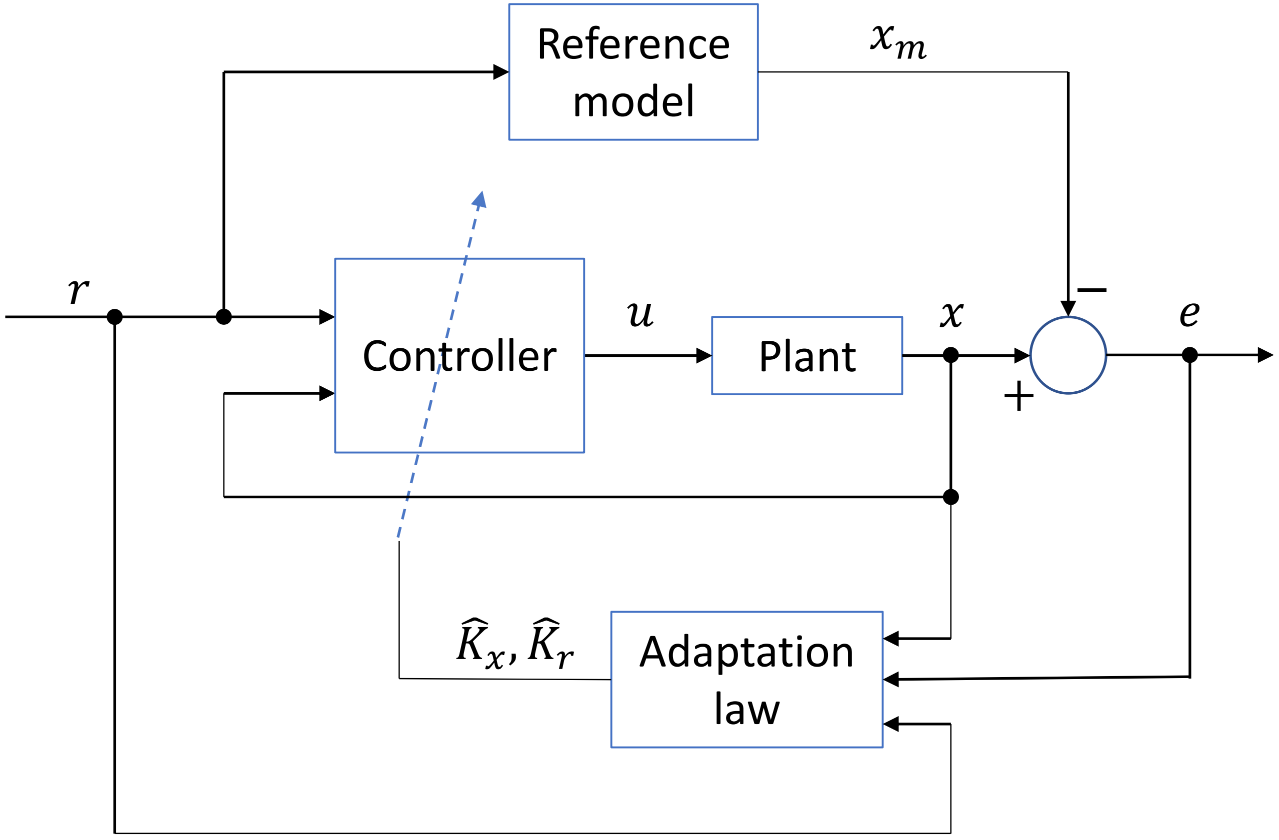
Elimination of terms C and D: Similarly set $\dot{\hat{K}}_r = -\,\mathrm{sgn}(b)re$.

So we need not know the exact value of b but we need to know only the sign of b to determine the estimates of the controller gains. This is a fairly mild condition which is often satisfied in practice. A simple example can be that if brake is applied the car stops.

Adaptation law:

$\dot{\hat{K}}_x = -\,\mathrm{sgn}(b)xe$ and $\dot{\hat{K}}_r = -\,\mathrm{sgn}(b)re$ Sometimes we include a +ve adaptation gain γ and then the adaptation law becomes

$\dot{\hat{K}}_x = -\,\mathrm{sgn}(b)\gamma xe$ and $\dot{\hat{K}}_r = -\,\mathrm{sgn}(b)\gamma re$



$$\dot{\hat{K}}_x = -\operatorname{sgn}(b)\gamma x e$$
$$\dot{\hat{K}}_r = -\operatorname{sgn}(b)\gamma r e$$

Adaptation law

Model Reference Adaptive Control

- We have already proved that this is the equilibrium point is globally stable.
- What we can further say by studying V and $\dot{V}$: If V is +ve definite and if its derivative is < 0, then we can of course say that this function is always going to be bounded.
- So if V is bounded, then of course e, $\widetilde{K}_x$ and $\widetilde{K}_r$, all have to be bounded.
- We already have $\dot{e} = a_m e$ i.e. asymptotic tracking convergence of e when $\widetilde{K}_x, \widetilde{K}_r \to 0$

MRAC for nonlinear plants:

The plant dynamic model:

$$\dot{x} = ax + bu + cf(x)$$

where the plant parameters a, b, c are unknown; $f(x)$ is a known nonlinear function.

The reference model:

$$\dot{x}_m = a_m x_m + b_m r; a_m < 0$$

Controller with estimated gains:

$$u = \widehat{K}_x x + \widehat{K}_r r + \widehat{K}_f f$$

The 3rd gain $\widehat{K}_f$ is introduced to adaptively cancel the nonlinear term.

Parameter estimation errors:

$\widetilde{K}_x(t) \stackrel{\text{def}}{=} K_x - \widehat{K}_x(t); \widetilde{K}_r(t) \stackrel{\text{def}}{=} K_r - \widehat{K}_r(t); \widetilde{K}_f(t) \stackrel{\text{def}}{=} K_f - \widehat{K}_f(t)$

Error dynamics with estimated controller gains:

$\dot{e} = \dot{x} - \dot{x}_m = ax + bu + cf - a_m x_m - b_m r = ax + b\left[\widehat{K}_x x + \widehat{K}_r r + \widehat{K}_f f\ \right] + cf - a_m x_m - b_m r$

Using the matched conditions:

$a + bK_x = a_m$, $bK_r = b_m$, and $bK_f = -c$,

and the parameter estimation errors, we have the final form of error dynamics as

$\dot{e} = a_m e - b\widetilde{K}_x x - b\widetilde{K}_r r - b\widetilde{K}_f f$

Use of Lyapunov candidate function to determine the estimates of controller gains:
Using a suitable Lyapunov candidate function and proceeding in the same manner as in the linear case we can derive the adaptation law as given below.

Adaptation law:

$$\dot{\hat{K}}_x = -\operatorname{sgn}(b)xe$$

$$\dot{\hat{K}}_r = -\operatorname{sgn}(b)re$$

$$\dot{\hat{K}}_f = -\operatorname{sgn}(b)fe$$