STUDY OF RESILIENT SUPPLY CHAIN UNDER SMART MANUFACTURING ENVIRONMENT

THESIS SUBMITTED BY

SURAJIT NATH

DOCTOR OF PHILOSOPHY (ENGINEERING)

PRODUCTION ENGINEERING DEPARTMENT

FACULTY COUNCIL OF ENGINEERING & TECHNOLOGY

JADAVPUR UNIVERSITY

KOLKATA, INDIA

2021

JADAVPUR UNIVERSITY

KOLKATA – 700032, INDIA

INDEX NO.: 80/17/E

1. **Title of the thesis:** STUDY OF RESILIENT SUPPLY CHAIN UNDER SMART MANUFACTURING ENVIRONMENT.

2. **Name, Designation & Institution of the Supervisors:**

   Prof. Bijan Sarkar, Professor, Production Engg. Dept., Jadavpur University.

3. **List of Publications:**

<u>Publication in Journals:</u>

1. Nath, S., & Sarkar, B. (2021). **An Integrated 3-Phase Group Decision-Making Model for Supplier Selection in a Supply Chain Network.** *Journal of Institution of Engineers India Series C,* https://doi.org/10.1007/s40032-021-00800-w.

2. Nath, S., & Sarkar, B. (2020). **An integrated fuzzy group decision support framework for performance assessment of advanced manufacturing technologies: An eclectic comparison.** *Journal of Institution of Engineers India Series C,* 101(3):473–491.

3. Nath, S., & Sarkar, B. (2020). **An Integrated Cloud Manufacturing Model for Warehouse Selection in a Smart Supply Chain Network: A Comparative Study.** *Journal of Institution of Engineers India Series C*, 101 (1), 25-41.

4. Nath, S., & Sarkar, B. (2018). **Decision system framework for performance evaluation of advanced manufacturing technology under fuzzy environment.** *OPSEARCH*, 55, 703-720.

5. Nath, S., & Sarkar, B. (2017). **An Exploratory Analysis for the Selection and Implementation of Advanced Manufacturing Technology by Fuzzy Multi-criteria Decision Making Methods: A Comparative Study**. *Journal of Institution of Engineers India Series C.*, 98 (4), 493-506.

6. Nath, S., & Sarkar, B. (2017). **Performance Evaluation of Advanced Manufacturing Technologies: A Denovo Approach**. *Computers & Industrial Engineering*, 110, 364-378.

"Statement of Originality"

I, **SURAJIT NATH**,registered on 11th January, 2017, do hereby declare that this thesis entitled "**Study of Resilient Supply Chain under Smart Manufacturing Environment**" contains literature survey and original research work done by the undersigned candidate as part of Doctoral studies.

All information in this thesis have been obtained and presented in accordance with existing academic rules and ethical conduct. I declare that, as required by these rules and conduct, I have fully cited and referred all materials and results that are not original to this work.

I also declare that I have checked this thesis as per the "Policy on Anti Plagiarism, Jadavpur University, 2019", and the level of similarity as checked by iThenticate software is **3 %**.

Signature of Candidate: Surajit Nath

Date: 24/09/2021

Bijan Sarkar

Professor
Production Engineering Departm
Jadavpur University
Kolkata - 700 032

Certified by Supervisor(s):
24 Sept' 2021

(Signature with date, seal)

CERTIFICATE FROM THE SUPERVISORS

This is to certify that the thesis entitled **"Study of Resilient Supply Chain under Smart Manufacturing Environment."** Submitted by **Shri Surajit Nath**, who got his name registered on 11th January, 2017 for award of

Ph. D. (Engg.) degree of Jadavpur University is absolutely based upon his own work under the supervision of **Prof. Bijan Sarkar** and his thesis nor any part of the thesis has been submitted for any degree/diploma or any other academic award anywhere before.

Bijan Sarkar

Signature of the supervisor
and date with office seal

24 Sept' 2024

Professor
Production Engineering Department
Jadavpur University
Kolkata - 700 032

ACKNOWLEDGEMENTS

It is my great privilege and honour to express my profound and sincere gratitude to my respected supervisor Prof. Bijan Sarkar for his valuable guidance, support and encouragement. Working with him on the present project has been immense rewarding and pleasurable experience that has greatly benefitted me throughout the course of work. His active contribution has enhanced the scope to go deep into the subject.

My special thanks and deep regards go to the present and former Heads of the Production Engineering Department Dr. Biplab Ranjan Sarkar, Prof. Debamalya Banerjee and Prof. Arunanshu Sekhar Kuar for providing me with the laboratory and other facilities throughout my research work. I would like to pay my deep regards and thanks to all the teachers of the Production Engineering Department of Jadavpur University for several invaluable discussions, suggestions and help during my work.

I convey immeasurable thanks to my colleagues and friends along with all the technical staffs of the Production Engineering Department of Jadavpur University, who have inspired and helped me in various ways during my research period.

I am indebted to my parents and my in-laws for their encouragement and support during my research period.

I take this opportunity to convey my sincere thanks to my wife, Mrs. Surja Nath (Chakraborty) and my daughter, Shalmoli Nath, for their sacrifice, patience and understanding during the entire period of my research.

Finally, I thank the Almighty for helping me in accomplishing my research goal and pray to Him that it would contribute to the welfare of mankind.

It is needless to say that without all the above help and support, the research work would not have been possible.

Surajit Nath

TABLE OF CONTENTS PAGE NO.

ABSTRACT

Modern world is a competitive one. It's very important for any manufacturing organization to keep pace with dynamic conditions of the competitive world. The ability of any organization to make optimum decision, now-a-days, is very important in the face of increasing competition from a number of competitors. Hence, continuous quality improvement and optimum decision making are the success keys for any organization. Also, optimum utilization of time and available resources are the other main factors that contribute to the success of an organization. In a highly competitive and volatile market, supply chain management (SCM) is the main deciding factor for the growth of an organization. It is described as a chain linking each element from customer and supplier through manufacturing and services so that flow of material, money and information can be effectively managed to meet the business requirement. It is the oversight of materials, information, and finances as they move in a process from supplier to manufacturer to wholesaler to retailer to consumer. It involves coordinating and integrating these flows both within and among companies. Product design, manufacturing and distribution strategies may change frequently and rapidly for the sake of it. The challenge for a company is not only how to continue to maintain a technically advanced and competitive product but also how to reduce the design, development and manufacturing time in line with demands of the market. Quality, cost, lead-time and service level are the four performance measures in a supply chain. For a supply chain to be resilient, it has to operate under smart manufacturing environment. Smart manufacturing is a broad category of manufacturing with the goal of optimizing concept generation, production, and product transaction. While manufacturing can be defined as the multi-phase process of creating a product out of raw materials, smart manufacturing is a subset that employs computer control and high levels of adaptability. Smart manufacturing aims to take advantage of advanced information and manufacturing technologies to enable flexibility in physical processes to address a dynamic and global market. It enables all information about the manufacturing

process to be available when it is needed, where it is needed, and in the form that it is needed across entire manufacturing supply chains, complete product lifecycles, multiple industries, and small, medium and large enterprises. There is increased workforce training for such flexibility and use of the technology rather than specific tasks as is customary in traditional manufacturing. The broad definition of smart manufacturing covers many different technologies. Some of the key technologies in the smart manufacturing movement include big data processing capabilities, industrial connectivity devices and services, and advanced robotics.

Advanced Manufacturing Technology (AMT) plays a pivotal role to obtain a smart supply chain. The significant contribution of AMT is to achieve strategic objectives and improved competitiveness of manufacturing organizations. AMTs represent numerous modern technologies such as Computer Aided Design, Computer Aided Manufacturing, Flexible Manufacturing System, Computer Aided Process Planning, Artificial Intelligence, Robots, and Just-In-Time etc. Selection of the proper AMT amongst these is a very important issue for any manufacturing organization. The benefits that AMT offers to the manufacturing organizations are: Improved Productivity, Greater Flexibility, Reduced lead times, Improved Quality, Reduced Inventories, Improved Product design, Reduced Costs, Improved Competitiveness, Increased Customer Satisfaction, and sustainable green environment. The quest for all these has driven many manufacturing organizations to opt for AMT. The most important outcome of this is very evident from the fact that there has been a paradigm shift from mass production to mass customization. Although, the adoption of AMT is beneficial to manufacturing organizations, at the same time, it is very risky as well. It involves a major investment and a high degree of uncertainty. Considerable attention is needed within the organization while implementing the AMT. So, before investing on AMT, manufacturing organization must assess its strengths and weaknesses. Thus, identification of factors in selecting a particular AMT is very crucial for the organizations. The factors chosen must have long-term effects on the performance of the organization and it must make profit out of it.

The aim of this research is to develop a flexible, robust and resilient supply chain to achieve customer satisfaction and move towards a green eco-friendly environment, to contribute to the development of society and largely the mankind as a whole.

The scope of this investigation is directed towards researching the models for decision making support in smart supply chain management under uncertainties from fuzzy type. The models have to simulate human decision making by means of applying soft computing in the form of fuzzy logic, fuzzy AHP, fuzzy TOPSIS, fuzzy Dempster-Shafer theory, Design of Experiment (DOE), EVAMIX, COPRAS-G, k-Means clustering, Fuzzy Taguchi loss function, Fuzzy VIKOR and others. Smart Manufacturing aims to be an idealized practice in manufacturing. It involves the integration in all steps of the product fabrication process. The aim being a more harmonious development process utilizing data to develop intelligent technology to expedite new and higher quality goods. If adopted, intelligent networks of manufacturing will see results in influencing business both domestically and worldwide. Business models can be more easily conceptualized around the integration of every step of the development process i.e. invention, manufacturing, transportation and retailing. The eventual goal being a more flexible, adaptive and reactive approach to participating in competitive markets. Companies may be forced to adapt or adopt the practice to compete, further stirring up the market. A large expectation of the premise also resides on the collaboration of multidisciplinary professionals including scientists, engineers, statisticians, economists etc. establishing a fundamental resource for 'smart' business ventures.

The research originality lies in the fact that, different innovative methods have been used for the performance evaluation of resilient supply chain. The methods or combination of them, used here have not been reported in earlier research work.

To conclude this, a resilient supply chain has been analyzed by taking care of the integral parts of a supply chain individually. Be it selection of appropriate supplier, be it selection of modern advanced technology or be it selection of warehouse, all these aspects have been taken care of, in this research, thus, in turn, making it a purposeful research work.

1. INTRODUCTION

Supply chain management is one of the most talked about topics in today's manufacturing world. The manufacturing sector is facing a lot of challenges in today's volatile market environment. So, a supply chain has to be flexible and at the same time resilient as well. To achieve these, all the areas that contribute to a robust supply chain, has to be taken care of. The present study aims to develop a robust supply chain. And for this, the three levels of supply chain namely, selection of appropriate supplier (upstream), selection of optimum advanced technology (middle stream) and selection of warehouse location (downstream), have been taken care of, in this study. Also, the effects on environment from industrialization are as important as any. So, this aspect has also been taken care of, by taking factors related to environmental issues while making solutions to the selection and evaluation problems, such that, it could lead to a green supply chain.

1.1. Overview

Modern world is a competitive one. It's very important for any manufacturing organization to keep pace with dynamic conditions of the competitive world. The ability of any organization to make optimum decision, now-a-days, is very important in the face of increasing competition from a number of competitors. Hence, continuous quality improvement and optimum decision making are the success keys for any organization. Also, optimum utilization of time and available resources are the other main factors contributing to the success of an organization. In a highly competitive and volatile market, supply chain management (SCM) is the main deciding factor for the growth of an organization. It is a chain that links customer to supplier. The same is done through manufacturing and services. Material, money and information flow are effectively managed to meet the business requirements. A typical flow chain is shown in the following Figure 1.1.

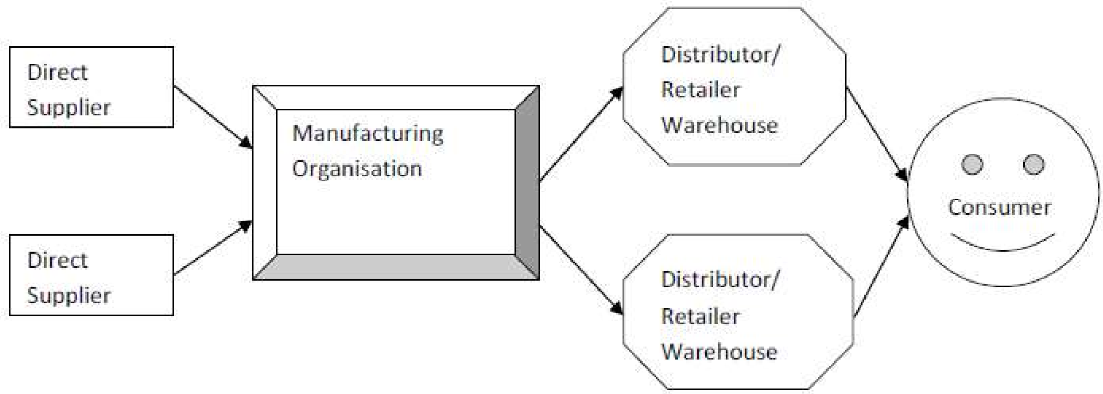


Figure 1.1. Product/service flow chain

In an organization, not only material, but, information and finances also move from supplier to manufacturer in a process. Then these go to wholesaler, retailer and lastly, the consumer. The flow chain oversights the same. It coordinates and integrates these flows within the organization. Product design, manufacturing and distribution strategies might change.

customarily due to this. In this kind of circumstances, the challenge for a company is to continue producing a technically advanced and competitive product. At the same time, design, development and manufacturing time need to be reduced in line with demands of the market. The four performance measures in a supply chain are production cost, product quality, product lead times and after sales service.

1.2. Resilient Supply Chain

Now-a-days, manufacturing systems face unrivalled challenges imposed by highly demanding constraints. They range from high product customization to the demand for lower cost to increasing product standard to significant oscillation in market demands. Traditional manufacturing systems can no longer cope with the scenario. It requires to be upgraded to new manufacturing paradigms that meet the challenges better. The monolithic, rigid structures don’t suffice to meet the requirements thrown by manufacturing environments demanding flexibility, productivity, robustness, reconfigurability and responsiveness. New and more modern manufacturing techniques like holonic manufacturing system (HMS), reconfigurable manufacturing system (RMS), multi-agent system (MAS), bionic manufacturing system (BMS), fractal manufacturing system (FMS), evolvable manufacturing system (EMS) etc. have emerged into the manufacturing space. The urgent requirement of achieving overall sustainability has aroused due to some already prevailing and some emerging causes such as environmental concerns, diminishing non-renewable resources, strict legislation, inflated energy costs, increasing customer preferences for eco-friendly products. In the midst of high turbulence in today's uncertain market, supply chain vulnerability has grown significantly in organizations. The threat of erosion for a supply chain is greater than ever due to risks that include natural disaster, terrorism, cyber attacks and so on. These could yield to a substantial loss in each and every aspect of organization, be it productivity, be it profitability, or be it competitive advantage. That is where, the resilient supply chain (Figure 1.2) works in whole kit and boodle.

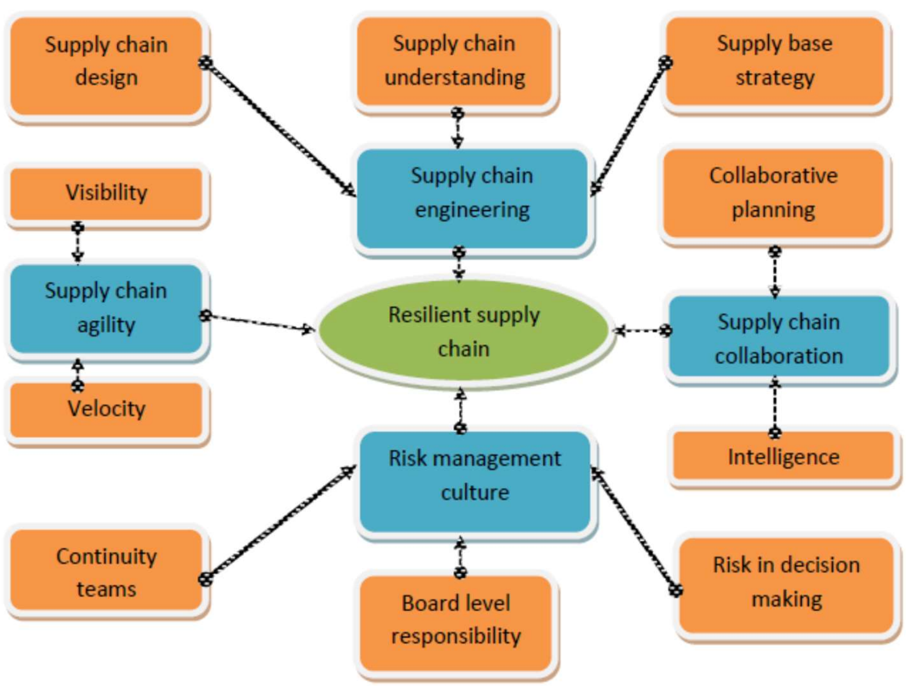


Figure 1.2. Resilient supply chain: strategic approach (M. Blos, H. M. Wee, W-H Yang,2012)

It supposes to be the ability of a substance to get back to its original form after deformation. There have been some industrial revolutions in the domain of manufacturing and supply chain in the past few years. However, Industries together with the researchers and policy makers have promoted an upcoming industrial revolution. A German strategic initiative known as industry 4.0, has been focused at generating factors that are intelligent. Here, the three key things like internet of things (IoT), cloud manufacturing, cyber-physical systems (CPS), transform the manufacturing processes such that they are able to communicate real time and take smart decisions in conjunction with humans, machines and sensors. There has been a rapid evolution of radio technologies since the inauguration of analogue cellular systems termed as 1st generation or simply 1G, back in 1980s. Now, the progress is towards fifth generation or 5G. It commits to bring the reliability, scalability, latency, edgeless computing to the existing system hands down and that is needed for several IoT applications. Stupendous success depends on ethos, pathos and logos as expounded and propounded by Socrates, a Greek philosopher, 2500 years ago. Thus, many manufacturing organizations today are presented with a number of supply chain strategies. Appropriate and lucrative decision making in industrial environment is the key to sustainable development in today's market scenario. Moreover, environmental aspects in terms of emission of harmful gases, logistic costs have forced supply chain managers to reinvent and renovate their distribution strategies. They look for a suitable decision support aid that would possibly extract the best solution out of the problem.

1.2.1. Smart Manufacturing

For a supply chain to be resilient, it has to operate under smart manufacturing environment. It is a broadened manufacturing process with a goal to optimize concept generation and product transaction. Conventional manufacturing process is the multi-phase process of creating a product from raw materials, whereas, the main feature of smart manufacturing is high levels of adaptability by commissioning computer control and advanced information. It enables flexibility in physical processes to address the globally dynamic market. All the information about the manufacturing process remains available and can be accessed anytime, from anywhere and in any form. It is available through complete product life cycle across entire manufacturing supply chains. To achieve such levels of flexibility and usage of modern technology, as opposite to customary specific tasks in traditional manufacturing, increased workforce training is highly needed.

1.2.1.1. General Description

In recent years, the paradigm of "manufacturing as an ecosystem" has emerged, which has been conceptualized as a system beyond the manufacturing plant site. The term "smart" generates data and information throughout the product lifecycle with the goal of creating a flexible manufacturing process that allows businesses to respond quickly to changing demand at low cost without harming the environment. And the company you are using is included. This concept requires a lifecycle perspective in which the product is designed for efficient production and recycling. Through Smart Manufacturing, all information about the manufacturing process is available when and when needed, in the entire manufacturing supply chain, the entire product lifecycle, and in many industries, small businesses and large enterprises. The Smart Manufacturing Leadership Coalition (SMLC) is building the technology and business infrastructure to facilitate the development and deployment of smart manufacturing systems across the manufacturing ecosystem. One of the previous definitions of advanced manufacturing companies is: "intensified application of advanced intelligence systems to enable rapid manufacturing of new products, dynamic response to product demand, and real-time optimization of manufacturing production and supply-chain networks (SMLC 2011)." This idea is represented by smart factories that rely on interoperable systems, Multiple scale dynamic modeling and simulation, intelligent automation, extensible multi-level cybersecurity, networked sensors. These companies leverage data and information throughout the life cycle of their products with the goal of creating flexible manufacturing processes that respond quickly to changing demand at low cost, not only for the enterprise but also for the environment. These processes facilitate the flow of information from all business functions within the enterprise and manage connections with external suppliers, customers and other stakeholders within the enterprise.

1.2.1.2. Current Technology

The extensive definition of smart manufacturing covers several technologies. Key technologies in the smart manufacturing movement include processing of big data, advanced robotics, connectivity devices in industrial service.

a) Processing of big data

Smart manufacturing uses big data analytics to improve complex processes and manage supply chains. Big data processing refers to a method of collecting and understanding large data sets in terms of what is known as 3 Vs, velocity, variety, and volume. Velocity refers to the frequency of data collection. This can be done concurrently with the application of previous data. Variety represents the different types of data that can be processed. Volume refers to the amount of data. Big data analytics enables businesses to move from reactive to predictive practices using smart production. This is a change aimed at improving the efficiency of the process and the performance of the product.

b) Advanced robotics

Advanced robots are also known as smart machines. They can operate autonomously and communicate directly with manufacturing systems. By evaluating sensory input and distinguishing between different product configurations, they can solve problems and make decisions independently of humans. These robots have artificial intelligence that can complete work beyond what is programmed to be the first and can be learned from experience. These machines are flexible enough to be reconfigured and repurposed. This feature provides the ability to respond quickly to design changes and innovations, which is more competitive than most traditional manufacturing processes. An area of concern though, is the safety and well-being of the workers interacting with the robot system. Measures have been taken to separate conventional robots from the human workforce, but it opens up opportunities for the development of robots' cognitive abilities, such as cobots, i.e. robots that work collaboratively with human workforce. Figure 1.3 shows the usage of advanced robotics in automotive production plant.

Figure 1.3. Advanced robotics in automotive production (source: Wikipedia.org)

c) Industrial connectivity devices and services

Internet capabilities enable businesses to increase consolidation and data storage. By adopting cloud software, businesses can access highly configurable computing resources. This allows to quickly create and release servers, networks, and other storage applications. The enterprise integration platform allows manufacturers to track statistics such as system history and workflow to collect data broadcast from their systems. Open communication between the manufacturing device and the network can be implemented via an Internet connection. This, circumscribes everything from tablets to machine automation sensors and allows machines to adjust their processes based on inputs from external devices.

1.2.1.3. Benefits and Aim

Smart manufacturing aims to become an ideal practice in the manufacturing field. This includes integration at every stage of the product manufacturing process. The goal is to leverage data and develop intelligent technologies through a more harmonious development process to rapidly develop new, higher quality products. The benefits are:

a) New and Innovative business practices

On successful adoption, manufacturing intelligent networks could see results affecting domestic and global business. Business models can be more easily conceptualized at all stages of the development process, be it Invention, be it manufacturing, be it transportation or retail. The ultimate goal is a more flexible, adaptive and responsive approach to participating in the highly competitive market. Companies need to adapt or adopt practices for competition, which can further stimulate the market. The great expectation of the premise is also the coopetition between technicians, intermediaries and consumers. Setting up an interdisciplinary network of experts (also known as the Internet of Things) of scientists, engineers, statisticians, economists, etc. is a fundamental resource for “smart” startups.

b) Elimination of workplace hazards and workforce inefficiencies

Smart manufacturing also helps to investigate workplace inefficiencies and support worker safety. Efficiency optimization is a major focus on employers of "smart" systems that are performed using data research and intellectual learning automation. One can build intelligent, interconnected "smart" systems, set performance goals, determine if it is achievable and identify failures or inefficiencies through failed performance goals. In general, automation can reduce inefficiencies due to human error. And Artificial Intelligence (AI), which generally evolves, eliminates the inefficiencies of previous models.

Worker safety is enhanced by an increasing number of integrated networks of safe and innovative designs and automation. This is under the notion that automation reduces the exposure of mature technicians to hazardous environments. If successful, the lack of human oversight and user direction for automation soars safety concerns in the workplace.

1.2.1.4. Industry 4.0

Industry 4.0 is a high-tech strategic project of the German government advocating the computerization of traditional industries such as manufacturing. The goal is an intelligent factory (Smart Factory) that features adaptability, resource efficiency and ergonomics. It also makes integration of business partners with customers in business and value processes. The technology base consists of CPS and IoT. This kind of "intelligent manufacturing" makes excellent usage of the following:

- Wireless network, during product assembly and distant communications.
- The last generation sensor, located along the same product (Internet of Things) as the supply chain.
- Refines large amounts of data, to control all stages of product construction, distribution, and use.

1.3. Levels of Supply Chain

1.3.1. Upstream: selection of appropriate supplier that grows business

Supply chain management and supplier selection are receiving great deal of attention in competitive environment that is prevailing currently in manufacturing world. The purchase function is seen as the strategic approach in different sectors. The performance of a manufacturing organization now-a-days is largely dependent on the buyer and supplier relationship. A long term buyer-supplier relationship makes way for a resilient supply chain which makes it hard for the competitors to topple. Suppliers play an important role in growing a business. Organizations need to work hand-in-hand with logistic partner to give best service to their customers and achieve excellence. This includes converting natural resources, raw materials and parts into finished products that are delivered to end customers. A resilient flow chain should have the desired agility, risk management culture, strategic advancement and collaboration to go with it. Smart manufacturing also enables data and information about the whole supplier evaluation procedure to be available whenever it’s required, wherever it’s required and in the form it’s required across the entire value chain.

1.3.2. Middle stream: selection of advanced technology which is flexible and robust in all situations

In today's manufacturing world, Advanced Manufacturing Technologies (AMT) play a pivotal role in organization’s growth. Implementation of AMT offers the advantage of producing and delivering products as demanded by the customer, in a shorter lead time with the help of latest advanced technologies, so that efficiency can be maintained. These technologies include the following: Computer-Aided Design (CAD), Computer Aided Manufacturing (CAM), industrial robotics, automated material handling systems, group technology, Flexible Manufacturing Systems (FMS), Rapid Prototyping (RP) processes and Computer Numerically Controlled (CNC) machines. These not only provide flexibility but at the same time yield greater productivity as well. In terms of flexibility and productivity, AMTs are in a whole different league. These are capable of adapting to changes in product variety with a very short lead time while maintaining the efficiency as well as the cost effectiveness. This is where AMT scores ahead of conventional manufacturing technologies and by a considerable margin. With the advancement in civilization, the traditional production technologies are increasingly being replaced by advanced technologies. The outcomes of this supplement are high productivity, improved reliability, greater flexibility and increased efficiency. This transition is very prominent from the fact that there has been a paradigm shift of the customer order decoupling point (CODP) towards highly customized product from mass production as depicted in figure 1.4.

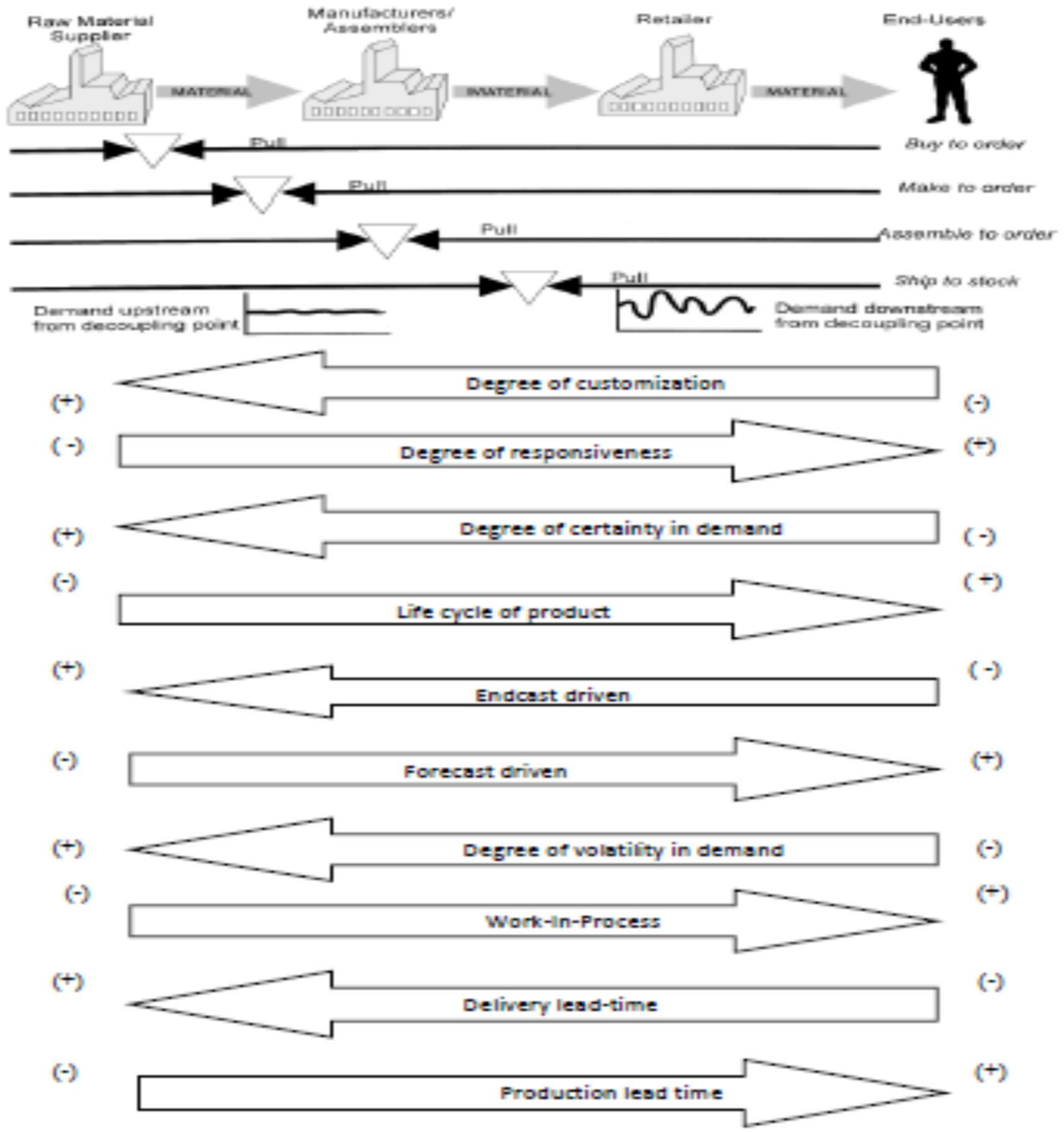


Figure 1.4. CODP shift: Modern manufacturing concept

Again, CODP is an important factor in manufacturing design as well as supply chain. It is a point in the material flow where a particular product is links to a specified order from customer, which can typically be buy-to-order (BTO), make-to-order (MTO), assemble-to- order (ATO) and make-to-stock (MTS), having different ratios of production lead time (P) and delivery lead time (D). CODP is also known as order penetration point (OPP). Make-to-stock (MTS) and Buy-to-order (BTO) are diametrically opposite in the present scenario. In case of MTS, the product is least customized and easily available as in case of a finished product. The responsiveness is very high. On the other hand, it is evident that BTO is highly customized product as customer comes down to the selection level of raw material to be used in the finished product. The responsiveness of procurement of that exotic material from different sources in proper amount poses a big challenge for the organizations to meet up the demand.

1.3.3. Downstream: selection of appropriate warehouse location

Stupendous success depends on ethos, pathos and logos as expounded and propounded by Socrates, a Greek philosopher, 2500 years ago. Thus, many manufacturing organizations today are presented with a number of supply chain strategies. Appropriate and lucrative decision making in industrial environment is the key to sustainable development in today's market scenario. Moreover, environmental aspects in terms of emission of harmful gases, logistic costs have forced supply chain managers to reinvent and renovate their distribution strategies. They look for a suitable decision support aid that would possibly extract the best solution out of the problem. In a supply chain design, the location of warehouse is one of the most critical and fundamental decisions to be made. This contributes enormously to the performance of the supply chain. The problem has received considerable attention in the past few years in manufacturing world. It focuses on a number of warehouses and selecting the optimum one with respect to the various criteria suitable for the manufacturing unit such that they could minimize the cost associated with manufacturing. The decision process is highly complex and involves incomplete information. That leads to uncertainty and fuzziness in the given problem. That is where, fuzzy multi-criteria decision making (MCDM) has a huge part to play and choose the optimum warehouse location.

2. REVIEW OF THE PAST RESEARCH WORK

The objective of this section is as follows:

a) to depict the past works and findings in the area of supply chain management.

b) to analyse the areas covered in these past researches.

c) To analyse the areas that are yet to be addressed in the past researches.

There have been many researches in the past that put emphasis on the evaluation of resilient supply chain and selection problems in different areas, by multi-criteria decision making (MCDM) and outranking approaches that deal with uncertainties, and also different contextual comparative studies amongst them.

Supplier selection in today's volatile market environment has gained a lot of attention in literature.

Chen et. al. suggested a supplier selection problem on fuzzy MCDM. They used fuzzy technique for ordered preference by similarity to ideal solution (TOPSIS) to measure the closeness co-efficient and subsequent ranking of the suppliers.

An MCDM model was developed by **Buyukozkan et. al.** for the evaluation of strategic alliance partner in logistic value chain.

A supplier selection model was developed by **Deng & Chan** with the applications of Dempster-Shafer theory of evidence (DST) and fuzzy set theory (FST).

A multi-criteria evaluation model was proposed **Buyukozkan & Cipci** for the evaluation of green supplier for an organization. They used a hybrid fuzzy MCDM tool comprising of analytic network process (ANP), decision making trial and evaluation laboratory (DEMATEL) and TOPSIS to evaluate suppliers relevant to the green supply chain network.

An integrated fuzzy MCDM model was developed by **Liao & Kao** for the evaluation of supplier in a watch firm. Integration of TOPSIS and MCGP made way for evaluators in setting different aspiration levels in the problem.

There have been a handful of researches undertaken in recent times for the performance assessment of AMTs as well as other decision making problems. Different MCDM and outranking approaches have found relevance to the likes of selection and evaluation models.

Yurdakul developed a model based on AHP and goal programming for the evaluation of CIM (Computer Integrated Manufacturing) technologies.

Al-Ahmari developed a multi- criteria approach for the evaluation of modern technologies by using fuzzy set theory and AHP.

Mohanty & Desmukh proposed a framework in solving a firm's investment justification problem in advanced technologies. Their integrated model was based on AHP and Nominal Group Technique (NGT). **Kengpol & O'Brien** developed a tool for decision aid in the selection of AMT eying product development at a rapid pace. They utilized AHP, cost/benefit and statistical analysis in achieving the goal.

A technology selection algorithm was proposed by **Chan et. al.** by integrating AHP, fuzzy multi- criteria decision making (FMCDM) and fuzzy cash flow analysis. They incorporated a factor namely fuzzy appropriate index in the problem.

An extensive and potential usage of AHP was shown by **Yusuff et. al.** in predicting advanced technology implementation. They have proven the ability of AHP method in structuring a complex decision problem that includes multi- person, multi- attribute, multi- period in a hierarchy. The whole implementation process was segregated into four main modules namely the institutionalization, acceptance, reutilization and infusion modules. **Safei et. al.** proposed a combined AHP, K-means clustering algorithm in preserving a proper ranking order. The clustering result proved to be more accurate than that of the normal weighted K- Means. **Panapakidis & Christoforidis** developed a methodology using TOPSIS, multi- criteria decision analysis (MCDA) for selecting optimal clustering algorithm for load profiling applications. **Ince et. al.** utilised a combination of AHP and TOPSIS in evaluating learning objects (LOs) and their descriptive metadata from web- based intelligent learning object framework (LOR).

Nath and Sarkar exhibited a distance based TOPSIS methodology in fuzzy environment for the assessment of AMT.

Fernandez and Perez analyzed and brought out a model of occupational risk measurement pertaining to advanced technology innovation.

Teti proposed a model for manufacturing pertaining to zero defect in machining with a unique solution of signal processing and decision theory.

Rohrmus et. al. came up with a model of advanced carbon-based manufacturing with environment friendly green raw materials and green production to contribute to a safer and greener future.

Efthymiou et. al. introduced a semantic technology approach that could facilitate the knowledge storage and extraction in terms of past production processes configuration in manufacturing, product design and process planning.

Nath & Sarkar developed a Denovo perspective for the performance evaluation of AMTs. They made a comparative study of two MCDM tools namely PROMETHEE and DST of evidence based on the basic ideas of TOPSIS.

Chuu proposed two models for the implementation of AMT. He utilized fuzzy multi-attribute analysis in group decision making for the same. The mathematical frameworks involved a fusion method of fuzzy information that was performed by MEOWA (maximum entropy ordered weighted averaging) operators. A scientific approach involving analytic hierarchy process (AHP) and fuzzy AHP as MCDM tools was presented by **Al-Ahmari** for the selection and evaluation of AMTs. The suggested methodology combined two databases namely manufacturing organization database and AMTs database for the upliftment of the model.

Ahmed et. al. addressed a multi-period investment problem in selection, allocation of modern manufacturing technology that could replace traditional ones over a long-range planning horizon. They applied linear programming (LP) relaxation solution to a multi-period mixed-integer programming, capacity shifting heuristic and probability analysis.

A multi-criteria mathematical model based on data envelopment analysis (DEA) and assurance region (AR) was formed by **Liu** for the selection of FMS.

A model on group decision support, based on consensus, was developed by **Choudhury et. al.** for the selection of advanced technology. They used proximity measure, consensus measure and multi-agent system (MAS) based negotiation to resolve the problem.

A distance-based fuzzy MCDM approach was proposed by **Karsak** to evaluate alternatives to flexible production systems. Both economic performance indices and strategic performance variables are at a strong decision-making stage integrated into the above approach.

A fuzzy multipurpose programming approach for choosing an FMS was presented by **Karsak** and **Kuzgunkaya**. They incorporated fuzzy set theory in the proposed approach to cope with the vagueness and uncertainty in production environment that could affect investments in future.

Karsak and Tolga developed an algorithm based on fuzzy MCDM for the implementation of AMT. Preference ratings of experts for economic, strategic criteria and alternatives were aggregated in measuring fuzzy suitability indices and subsequent ranking of alternatives.

A fuzzy ANP-VIKOR model was proposed by **Demirel and Yucenur** for the selection of cruise port site. The study also compared the fuzzy ANP and VIKOR method results.

Fouladger et. al. developed model on selection of project portfolio, based on the implementation of an organized framework in fuzzy VIKOR platform.

A group MCDM model using fuzzy VIKOR was proposed by **Mirahmadi and Teimoury** for the selection and evaluation of suppliers.

Ramachandran and Alagumurthi proposed fuzzy VIKOR approach for lean manufacturing facilitator selection problem. This method provided the advantage of taking decision which was closer to the ideal solutions**.**

Samantra et. al. developed a vendor selection model based on the application of fuzzy logic combining with the VIKOR method.

Brans and Vincke proposed principles for a new family of outranking methods. They considered six possible extensions of PROMETHEE based on extensions of notion of criteria.

A new approach based on PROMETHEE II mathematical method multi-criteria evaluation was developed by **Tomic et. al.** for supply chain logistic evaluation.

Macharis et. al. discussed the strengths and weaknesses of PROMETHEE and AHP methods regarding design of decision hierarchy and determination of weights.

An outranking-based multi-factor-sorting evaluation modeling and robustness analysis framework was developed by **Kadzinski and Ciomek.** This general framework was based on mixed integer linear programming (MILP). They implemented the same in the likes of outranking models specific for Electre and PROMETHEE.

An approach based on PROMETHEE and MCDA was developed by **Jedrkiewicz et. al.** in order to perform ranking of analytical procedures for determination of chloropropanols in soy sauces. The analysis was performed in three different scenarios namely metrological, economic and environmental.

An integrated data envelopment analysis-multi-criteria decision aid (DEA-MCDA) model was developed by **Bagherikahvarin and Smet** to restrict weight values of a classic DEA model by using tools from MCDA and increase the discrimination power of DEA. The stability intervals based on PROMETHEE II was used in this approach as weight constraints in DEA.

A mathematical model based on PROMETHEE and multi-criteria analysis was developed by **Stamatakis et. al.** to facilitate Photovoltaic integration in buildings, which combined benefits from shading, power generation and esthetics. This model evaluates the most proper shading device when the photovoltaic system is mounted on it.

A mathematical model combining ANP and PROMETHEE was developed by **Kilic et. al.** for selection of the optimum Enterprise Resource Planning (ERP) system in enterprises. ANP was used to determine the criteria weights and these were utilized by PROMETHEE in optimal ranking of alternative system choices.

A new hybrid fuzzy MCDM method based on fuzzy ANP, fuzzy DEMATEL and fuzzy PROMETHEE was developed by **Khorasaninejad et. al.** to choose the best Prime Mover (PM) in a thermal power plant.

Development and use of work recovery expander in place of expansion valve in trans-critical CO_2 refrigeration and air conditioning system was proposed by **Singh & Dasgupta .** They put the qualitative data from literature survey through selective multi-attribute decision making (MADM) techniques in the likes of AHP, TOPSIS, and PROMETHEE for the evaluation of work recovery expander.

Chen devised a new MCDM model based on PROMETHEE utilizing a likelihood-Based outranking comparison approach in the interval type 2 fuzzy set environment containing trapezoidal fuzzy numbers. The applicability of the developed model was illustrated in the two practical applications of vehicle evaluation issues and site for landfill selection.

An approach based on AHP-PROMETHEE methods was developed by **Kazan et. al.** for electing the deputy candidates for nomination in the grand national assembly of Turkey.

Murat et. al. devised a model based on PROMETHEE outranking method for measuring performance quality of schools.

A new MCDM method in fuzzy environment was evolved by **Deng & Chan** for supplier selection problem. They used FST and DST based on the main ideas of TOPSIS, to deal with the problem. **Cholvy** studied the relations existing between Dempster-Shafer theory and one of its extensions that considered frames of discernment with non-exclusive hypothesis.

Boudaren et. al. proposed a unifying general formalism that opened new possibilities to achieve Dempster-Shafer fusion in Markov fields context.

Aggarwal et. al. developed an Inertial Navigation System (INS) assisted by Global Positioning System (GPS) for land vehicle navigation application with reduced uncertainty and ambiguity.

Chai et. al. devised an extended ranking method for fuzzy numbers that synthesize the fuzzy targets and DST. They aggregated the ranking results using combination rule of Murphy.

A handwriting recognition system based on Hidden Markov Model (HMM) was proposed by **Kessentini et. al.** by the use of DST.

Wang et. al. proposed an approach based on ambiguity measure of fuzzy soft sets and DST. They applied the proposed approach in medical diagnosis.

A mathematical model discovering user preferences using Dempster-Shafer theory was formulated by **Troiano et. al.**

Yue et. al. developed a reliability-maximizing model based on multi-software-based architecture, using DST and enhanced differential development.

An approach using the Dempster-Shafer theory was proposed by **Wang & Jing** for ranking and selection of wireless network in a very much complicated scenario.

A fusion based Wireless Sensor Networks (WSN) surveillance application was deployed by **Senouci et. al.** to achieve high detectivity rate along with a lowering the false alarm rate in WSN.

Gruyer et. al. applied the DST to the Multi Hypotheses Tracking (MHT) method. This was able to resolve the ambiguity that arises in how to connect objects and tracks in a highly volatile vehicle environment. In decision making, a novel approach based on fuzzy set was proposed by **Tang** by incorporating DST and the grey relational analysis (GRA).

Lepskiya introduced estimation of conflict index and decreasing of ignorance index configuring the basic DST. After the application of combining rule, it is seen that, the adequate condition to decrease the ignorance is to heighten the correlation between the bodies of evidence. **Wang et. al.** proposed a novel approach model based on TOPSIS and RSM with interval numbers. They also demonstrated three illustrative MCDM problems with interval numbers to showcase the effectiveness of the proposed method.

Simsek et. al. proposed the TOPSIS-based Taguchi optimization problem of the optimum compounding ratio of high-strength non-segregating concrete.

An integrated TOPSIS- DOE method was developed by **Tansel Ic.** to solve different real-life CIM evaluation problems of industrial applications.

A hybrid DOE-TOPSIS model was proposed by **Sabaghi et. al.** in an MCDM problem.

Tansel Ic. proposed a TOPSIS based DOE approach for the assessment of company ranking.

Selection of warehouse is of supreme importance in a resilient supply chain. It has emerged into the scene during the last few decades. Some amount of research has been done in the past regarding the problem.

Huang presented a two-stage network model for warehouse site selection problem. **Wutthisirisart et. al.** proposed a material location selection problem allocating material to two warehouses- owned and rented, while minimizing the total storage and transportation costs. This shows a cost saving opportunity between 20%-40% due to reassigning materials between the rented and owned warehouses.

Hofstra et. al. contributed a case study to the literature of warehouse safety limitation, in which the factors governing safety are obscure. It provides valuable insights in safety aspects of logistics service providers (LSPs).

A group MCDM model was proposed by **Dey et. al.** for selection and evaluation of warehouse in a supply chain. The importance of heterogeneity in expertise degree is set through the pairwise comparison matrix. Analysis of variance (ANOVA) and sensitivity analysis (SA) find the proposed approach as a robust decision-making aid in the supply chain network.

Lin & Wang developed a two-stage model consisting of genetic algorithm (GA) and gradient method for optimal warehouse location selection.

Makaci et. al. provided an empirical study about the main specifications of a pooled warehouse, examined from the perspective of both literature review and explanatory qualitative study built on seven cases in France.

Jacyna-Golda & Izdebski presented multi-factor warehouse selection problem in the supply chain network based on GA.

An integrated simulation model has been built by **Fichtinger et. al.** to examine the interaction of inventory management-warehouse management and its environmental impact.

Garcia et. al. proposed a mathematical approach based on AHP as multi-attribute problem for a warehouse site selection. They included a case study of the site selection for a new banana distribution warehouse.

Atieh et. al. proposed an automated warehouse management inventory system for enhanced workflow, timely and efficient data handling, resulting in better space utilization and optimization of warehouse. **Vasiljevic et. al.** developed a model based on localization algorithm to fill the gap between the latest scientific advances in the localization of self-governed cars and autonomous warehouse storage.

So, therefore, the review of past researches leads to the next chapter gap analysis, that shows a tabular from of past research, the findings and issues that are yet to be addressed.

3. GAP ANALYSIS

This section depicts some of the past researches on related field, the areas covered and the areas which are unexplored keeping a scope of future exploration. It is presented in tabular form as follows:

Sl. No.	Reference	Areas Covered	Issues Not Addressed
1	[16]	i) Agent based framework ii) Multi agent system iii) Consensus group decision making iv) Multi-person, multi-preference, multi-criteria group decision making v) Soft consensus vi) Degree of proximity vii) Degree of consensus	i) Lesser alternatives. ii) Triangular fuzzy nos. iii) Linguistic terms. iv) Sensitivity analysis. v) Capital and operating costs of alternatives are not considered.
2	[44]	i) Both the economic evaluation criteria as well as strategic criteria have been considered ii) Triangular fuzzy no.s throughout iii) Fuzzy discounted cash flow analysis iv) Cost effective as well as customized	i) Loss in precision since fuzzy models provide only best and worst case analysis. ii) Possible errors don't get compensated. iii) Sensitivity analysis is not done, Robustness is not assured.
3	[99]	i) Fuzzy set theory ii) Membership function iii) Algebraic operations on fuzzy sets iv) Separation theorem	i) Triangular fuzzy numbers. ii) Trapezoidal fuzzy numbers.
4	[100]	i) Selection of construction project managers ii) Application of COPRAS-G method iii) The parameters of the alternatives determined by grey relational grade and expressed in intervals	i) COPRAS-G may be applied to the solution of wide range of problems by using discrete multi-attribute assessments technique.

Sl. No.	Reference	Areas Covered	Issues Not Addressed
5	[12]	i) Flexible Manufacturing System design ii) System simulation iii) MCDM iv) Artificial Intelligence (AI) v) FST vi) AHP. vii) Neural network	i) Full automation of interface between simulation models and intelligent decision tools.
6	[42]	i) Fuzzy set theory ii) Distance-based fuzzy MCDM approach iii) Linguistic terms. iv) Evaluation of Flexible Manufacturing System (FMS) alternatives	i) Uncertainty treatment can be incorporated for robustness.
7	[50]	i) Fuzzy axiomatic design approach based on hierarchical structure ii) AHP iii) Sensitivity analysis for defuzzification method iv) Illustrative example of teaching assistant selection problem	i) Defuzzification process can lead to loss of information. Instead α-cut method can be used to cope with that.
8	[103]	i) MADM ii) Application of COPRAS-G method iii) Simulation to reflect fuzzy inputs	i) COPRAS-G integrated with discrete multi-attribute assessments technique could be applied.
9	[34]	i) A comparison of five MCDA tools ii) Decision making iii) Water resource management	i) Unavailability or little availability of guidance to help decision analyst structuring the MCA problem. ii) Further work is required on MCA problem structuring and development of decision support tools.

Sl. No.	Reference	Areas Covered	Issues Not Addressed
10	[19]	i) Performance measures of five MCDA methods. ii) Assessment of Sustainability assessment. iii) Uncertainty management	i) Analysis of the MCDA methods on experts' judgments, which could be the outcome of the study of the method.
11	[38]	i) AHP Ii) Learning object iii) TOPSIS Iv) Metadata	i) Learning objects in large data set ii) Cost of operation iii) Smart manufacturing
12	[72]	i) Load data processing ii) Multi- criteria decision analysis iii) TOPSIS iv) Comparison of algorithms	i) Cost of software implementation
13	[76]	i) AHP ii) K-means iii) Clustering on ranking consideration	i) Integration of other ranking algorithms into clustering algorithm
14	[63]	i) Fuzzy multi- criteria decision making	i) Post optimality check may be done.
15	[21]	i) Industrial decision making. ii) Combination of EVAMIX and AHP	i) Post optimality check has not been performed. ii) The approach may be used in other selection problems with more numbers of attributers.
16	[17]	i) Fuzzy sets. ii) Multi-attribute analysis. iii) Evaluation of advanced technology. iv) Linguistic fuzzy quantifier. v) Fusion of fuzzy information.	i) Post optimality check may be done. ii) Fuzzy linguistic qualifier could be processed until a congruous decision is reached.
17	[13]	i) FMCDM ii) TOPSIS iii) Linguistic variable iv) Trapezoidal fuzzy number.	i) Group decision support system ii) Sensitivity analysis.

Sl. No.	Reference	Areas Covered	Issues Not Addressed
18	[8]	i) Fuzzy AHP ii) Fuzzy TOPSIS	i) Group decision making environment ii) Adaptation of different aggregation techniques.
19	[24]	i) DST ii) FST iii) Fuzzy TOPSIS	i) Conflict data fusion algorithm.
20	[9]	i) Fuzzy ANP ii) Fuzzy DEMATEL iii) Fuzzy TOPSIS	i) Post optimality check may be done.
21	[55]	i) Multi-choice goal programming (MCGP) ii) TOPSIS iii) Group decision making	i) Post optimality check may be done. ii) The model can be used in other management and marketing problems.

The study of the gap analysis has exhibited the areas somewhat covered in the past research works. But it also revealed a few areas, where, there are some scope for work and improvement. Some of the areas would include post-optimality check or uncertainty treatment integrated with different fuzzy MCDM techniques, for the robustness of the research findings; group decision making in fuzzy environment; consideration of the environment friendly factors while taking selection decision and so on. We have tried to cover a few of these areas in our current research for the purpose of bridging the gap to some extent.

4. AIMS, OBJECTIVES & SCOPE OF THE RESEARCH WORK

4.1. Aims of the Work

1. To move to a direction such as to achieve customer satisfaction.

2. To have the cutting edge and flexibility for achieving competitive advantage and sustainable development.

3. To contribute to the development of the society, the country, the mankind, to have a much more improved civilization and life style.

4.2. Objective of the Work

1. To study the role of supply chain in the environment of uncertain/unstructured information regarding demand pattern of the product in global scenario

2. To study and analyze the resilient supply chain under smart manufacturing environment in manufacturing organization.

3. To understand and analyze various marketing strategies and their correlations with volatility in demand prevalent in the market.

4. To study and analyze the shifting of marketing strategies from mass production to mass customization.

4.3. Scope of the Work

The scope of this investigation is oriented towards finding decision support models in multi-criteria problems under fuzzy type vagueness or uncertainty. The model should (roughly) simulate human decision making by applying one of the fundamentals of soft computing: fuzzy logic, or more precisely, fuzzy set theory. Cases of ambiguity present in the initial information and multi-criteria ambiguous decision-making problems are also considered in the troubleshooting steps. It has got ample applications in manufacturing industries, in various engineering fields, in vast areas of decision support.

5. FUZZY MULTI-CRITERIA DECISION MAKING APPROACH

The fuzzy multi-criteria decision making or MCDM approach combines the fuzzy decision matrix, linguistic variable, triangular fuzzy number (TFN), trapezoidal fuzzy number (TrFN), fuzzy scales, defuzzification process, arithmetic operations on fuzzy numbers, distance between two fuzzy numbers etc.

5.1. Fuzzy Scales

The fuzzy data are expressed in linguistic terms. They are converted into fuzzy numbers first. Then all the fuzzy numbers (or fuzzy sets) are defuzzified to obtain crisp values. In the scenario of fuzzy problem, the information about the system and design range are incomplete. The ranges for a certain criterion can be expressed over a number, around a number or between two numbers. TFNs or TrFNs can represent these kinds of expressions in a convenient manner. “The common area between the design range and the system range is the intersection area of triangular or trapezoidal fuzzy numbers” (Celik et. al. ,2007). The same is shown in Figure 5.1.

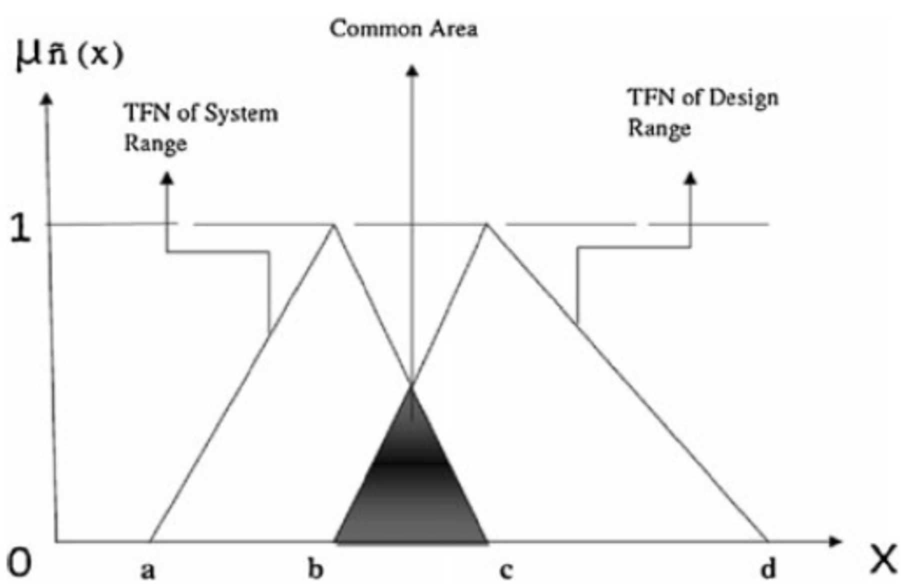


Figure 5.1. Representation of system and design range

Various types of rating scales or measurement formats can be used to measure the strength of a concept or attitude (for example, semantic differential, Stapel scale, Likert scale, Thurston scale, and direct rating scale). The proposed approach contains five conversion scales namely very low, low, medium, high, very high for evaluating weights of criteria and six scales namely very poor, poor, fair, good, very good, extremely good for the conversion of criteria values of alternatives.

5.2. Fuzzy Sets

A fuzzy set is denoted by, $A = \{(x, f_A(x)) | x \in U,$ where U is the universe of discourse, x is an element in U; A is a fuzzy set in U, $f_A(x)$ is the membership function of A at x . The larger $f_A(x)$, the stronger the grade of membership for x in A.

5.3. Linguistic Variable and Triangular Fuzzy Number

Linguistic variables are used to represent fuzzy data. They need to be represented in linguistic terms like very poor, fair, good, etc. for a subjective attribute, e.g. condition; and low, medium, high, etc. for a subjective attribute, e.g. importance. These are very useful approach where situations are complex and not so clearly defined to be expressed by traditional quantitative expressions. These variables are further converted into triangular fuzzy numbers (TFNs) as shown in figure 5.2.

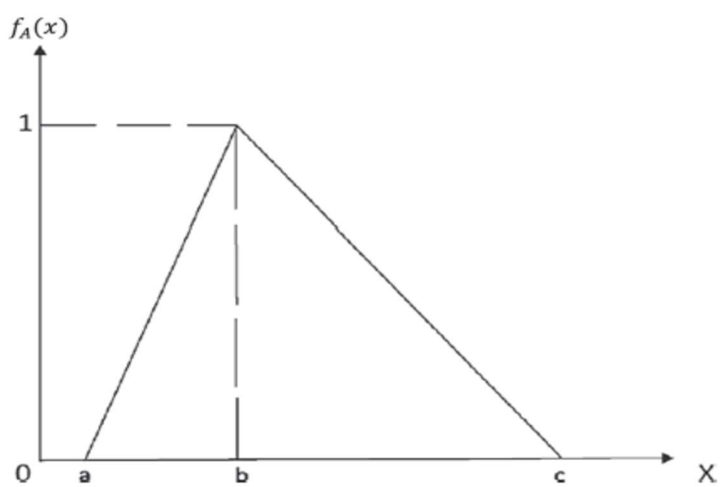


Figure 5.2. Representation of triangular fuzzy number

These TFNs provide crisp numbers to a linguistic term by following a certain scale and try to eliminate the fuzziness present in the problem. A TFN "A" is expressed by a triplet as in figure 3 and is denoted by A= (a, b, c), where, a is the lower limit and c is the upper limit. The membership function $f_A(x)$ of this TFN is given by the following conditions (Zimmermann, 1991):

$$f_A(x) = \begin{cases} 0; \ x < a, x > c \\ x - a/b - a\,; \ a \leq x \leq b \\ c - x/c - b; \ b \leq x \leq c \end{cases}$$

A linguistic variable can be converted into fuzzy set (Kauffmann & Gupta, 1985) comprising of a TFN by using a fuzzy scale. A fuzzy scale is used in a situation information about the system and design range is vague and not complete. Many conversion scales e.g. Semantic differential, Stapel scale, Likert scale etc. could convert cognitive attitudes of the organizational experts in the form of linguistic variables.

5.4. Linguistic Variable and Trapezoidal Fuzzy Number

In general, multi-criteria problems are associated with uncertainty and vagueness. Linguistic variables could eliminate the fuzziness and uncertainty in the problem. An expert in the field has to deal with subjective and objective factors that could possibly influence a selection decision. While, the objective factors can be represented by crisp values, it is not the case for the subjective factors. Linguistic variable such as ‘significance’ or ‘importance’ plays a pivotal role in adapting to the situation. These variables could further be processed by trapezoidal fuzzy numbers (TrFNs) in representing opinion of experts. There are some ordinal approaches as well, which are not based on TrFNs. Algorithms based on sentiment analysis with multi-granular fuzzy linguistic modelling, unbalanced fuzzy linguistic information have been developed in the recent times for the representation of user information. A TrFN can be expressed as a quadruplet $\psi = (a, b, c, d)$ as given in Fig. 5.3.

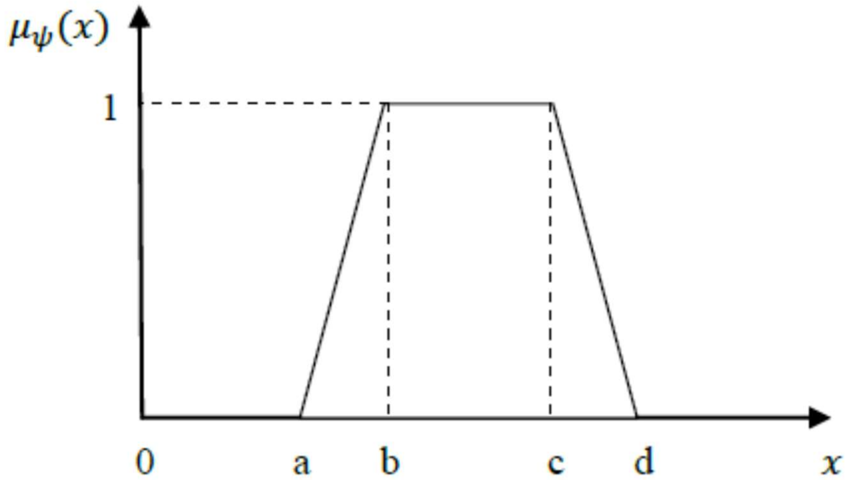


Figure 5.3. Representation of trapezoidal fuzzy number

A fuzzy set ψ in the universe of discourse X is defined as $\psi = \{\left(x, \mu_\psi\ (x)\right): x \in X\}$. The membership function $\mu_\psi\ (x): X \to [0,1]$ is defined as follows:

$$\mu_\psi\ (x) = \begin{cases} 0, & x < a \\ \frac{(x-\mathrm{a})}{(\mathrm{b}-\mathrm{a})}, & \mathrm{a} \le x \le b \\ 1, & \mathrm{b} \le x \le c \\ \frac{(\mathrm{d}-x)}{(\mathrm{d}-\mathrm{c})}, & \mathrm{c} \le x \le d \\ 0, & x > d \end{cases}$$

Thus, variables in the linguistic weight set can be represented by trapezoidal fuzzy scaling. The upper and lower limits of such a weight set could be extreme significance = (0.7, 0.8, 0.9, 1) and extreme insignificance = (0, 0, 0.1, 0.2). This is a conversion scale defined by experts that could take values between 0 and 1. Some other conversion scale could be defined to take the values between 0 and 10. The upper and lower limits of such a weight set could be extremely high importance = (9, 9, 10, 10) and extremely low importance = (1, 2, 3, 4).

5.5. Fuzzy Decision Matrix

Fuzzy data provides obscure information about the decision problem. Because of these uncertainties present in the problem scenario, providing crisp data is a tedious task. That is where fuzzy decision matrix comes very handy. If a fuzzy multi-criteria comprises of m numbers of alternatives and n numbers of criteria, then that is expressed by decision matrix as given below:

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ : \\ A_m \\ \end{array} \begin{bmatrix} C_1 & C_2 & \cdots\cdots & C_n \\ \tilde{\theta}_{11} & \tilde{\theta}_{12} & & \tilde{\theta}_{1n} \\ \tilde{\theta}_{21} & \tilde{\theta}_{22} & \cdots\cdots & \tilde{\theta}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\theta}_{m1} & \tilde{\theta}_{m2} & \cdots\cdots & \tilde{\theta}_{mn} \end{bmatrix}$$

Where, $\tilde{\theta}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ are the TFNs .

$A = A_1, A_2, \ldots, A_m$ are the alternatives and $C = C_1, C_2, \ldots, C_n$ are the criteria.

The weight matrix is given by: $W = [w_1 \quad w_2 \quad w_3 \ldots w_{n-1} \quad w_n]$

Where, $w_j = (w_{1j}, w_{3j}, w_{2j})$ are the triangular fuzzy weights of criteria.

The five elements of this matrix are: 1) Criteria.2) No. of criteria.3) Alternatives.4) No. of alternatives.5) Criteria values of the alternatives.

5.6. Defuzzification and α-cut

The fuzzy numbers need to be defuzzified for crisp estimation of experts at some later stage of the problem solving after overcoming the uncertainty present in the initial stage. The α-cut, $\alpha \in (0,1)$ is a very handful method in doing so, as it minimizes the loss of information going forward. It is a crisp set defined as $\psi(\alpha) = x \in$ R: $\psi(x) \geq \alpha$, where, $\psi(\alpha)$ is a closed interval of the form$[\psi_L(\alpha), \psi_U(\alpha)]$, R is a real line and TrFN ψ is a subset R; $\psi: R \rightarrow [0,1]$.

There are other methods such as the graded mean integration representation for defuzzification of TFN as follows: $P(N) = (a + 4 * b + c)/6$, where, $P(N)$ is the crisp defuzzified number.

Then, there is arithmetic mean as follows: $P(N) = (a + b + c)/3$.

5.7. Arithmetic operations on fuzzy numbers

Two given fuzzy numbers A and B; $A, B \in R$, the α-cuts of A and B are $A^{\alpha} = [A_l^{\alpha}, A_u^{\alpha}], B^{\alpha} = [B_l^{\alpha}, B_u^{\alpha}]$ respectively. Some operations of interval arithmetic on A, B are expressed as follows:

$$(A + B)^{\boldsymbol{\alpha}} = [\, A_l^\alpha + B_l^\alpha, A_u^\alpha + B_u^\alpha]$$

$$(A - B)^{\,\alpha} = [\, A_l^\alpha - B_u^\alpha \,, A_u^\alpha - B_l^\alpha \,]$$

$$(A * B)^{\,\alpha} = [\, A_l^\alpha . B_l^\alpha, A_u^\alpha . B_u^\alpha]$$

$$(A/B)^{\,\alpha} = [\, A_l^\alpha / B_u^\alpha \,, A_u^\alpha / B_l^\alpha]$$

$$(A * r)^{\,\alpha} = [\, A_l^\alpha . r, A_u^\alpha . r], where,\ r \in R.$$

6. ANALYSIS OF DIFFERENT MCDM METHODS IN PERFORMANCE ASSESSMENT OF ADVANCED MANUFACTURING TECHNOLOGIES

6.1. Case Study I: Decision System Framework for Performance Evaluation of Advanced Manufacturing Technology under Fuzzy Environment

6.1.1. Fuzzy TOPSIS Methodology Analysis

Fuzzy Technique for Ordered Preference by Similarity to Ideal Solution or Fuzzy TOPSIS method is a part of fuzzy MCDM tool. The classical TOPSIS was developed by Hwang and Yoon (1981) for solving MCDM problems. According to the methodology, the best choice is nearest to the positive ideal solution and at the same time, it is farthest from the negative ideal solution. It is a method for cardinal preference to attributes. Assuming that the utility of each attribute monotonically increases (or decreases), it is easy to find the ideal solution consisting of all the best achievable attribute values and the negative ideal solution consisting of all the worst achievable attributes. This method uses an alternative with the geometrically smallest (weighted) Euclidean distance to ideal solution. In many conditions, inadequacy of crisp data in a decision-making problem makes way for fuzzy linguistic terms and triangular fuzzy numbers, introduced by Zadeh (1965, 1974). Extended TOPSIS method is constituted by linguistic terms and triangular fuzzy numbers instead of exact numerical value.

Following are the steps involved in this methodology:

Step 1. Formation of committee of k no. of experts. They could identify selection criteria and available choices.

Step 2. Formation of Decision Matrix and weight matrix. Representation of criteria values of available choices and weights of criteria by linguistic variables and in turn, conversion of them into TFNs.

Step 2a. As there are k persons in the selection committee, aggregate values for alternatives and criteria weights are measured as:

$$x_{ij} = [x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^k]/k, i = 1,2, \dots, m: no. of\ alternatives \qquad \dots\dots\dots\dots\dots [6.1.1]$$

$w_j = [w_j^1 + w_j^2 + \cdots + w_j^k]\frac{1}{k},\ j = 1,2,\ldots,n: no.\,of\ criteria$ …………… [6.1.2]

Step 3. Normalizing the decision matrix.

For a TFN represented as (k_{ij}, l_{ij}, m_{ij}), normalization for beneficial and non-beneficial criteria, can be done, as in the following equation:

$$\left.\begin{array}{ll} Beneficial: & r_{ij} = (k_{ij}/m_j^*, l_{ij}/m_j^*, m_{ij}/m_j^*) \\ Non-beneficial: & r_{ij} = (k_j^-/m_{ij}, k_j^-/l_{ij}, k_j^-/k_{ij}) \end{array}\right\} \quad \text{…………… [6.1.3]}$$

where, $m_j^* = \max\ (m_{ij});\ k_j^- = \min\ (k_{ij})$

Step 4. Determination of the positive ideal solution $A^* = (r^*, r^*, r^*)$ and the negative-ideal solution $A^- = (r^-, r^-, r^-)$;

Step 5. Calculating the weighted distances of alternatives from the positive and negative-ideal solution (D_i^* and D_i^- respectively). The distance between two triangular fuzzy numbers $A_1 = (k_1, l_1, m_1)$ and $A_2 = (k_2, l_2, m_2)$ can be calculated as

$$D\ (A_1, A_2) = \ ½\ \{max\ (|k_1 - k_2|, |m_1 - m_2|) + (|l_1 - l_2|)\} \quad \text{…………….. [6.1.4]}$$

Since $r^* = 1$ and $r^- = 0$; the weighted distances from the positive and the negative-ideal solution can be calculated respectively as

$$D_i^* = \sum [½\ \{max\ (w_{kj}|k_{ij} - 1|, w_{mj}|m_{ij} - 1|) + w_{lj}|l_{ij} - 1|\}]; \quad i = 1, 2\ ..., m \quad \text{…………… [6.1.5]}$$

$$D_i^{-=} \sum [½\ \{max\ (w_{kj}|k_{ij} - 0|, w_{mj}|m_{ij} - 0|) + w_{lj}|l_{ij} - 0|\}]; \quad j = 1, 2, ..., n \quad \text{…………… [6.1.6]}$$

where, $w_j = (w_{kj}, w_{lj}, w_{mj})$ is the aggregate weight of *j*th criterion.

Step 6. The proximity index is pbtained by the following equation,

$$P_i^* = D_i^- / (D_i^* + D_i^-); \quad \text{where, } i = 1, 2, ..., m. \quad \text{……………. [6.1.7]}$$

Step 7.Objective Factor Measurement (OFM_i) by taking into account the cost factor as:

$$OFM_i = [OFC_i * \sum(1/OFC_i)]^{-1} \text{ ; where, } i = 1,2,\ldots,m. \qquad \ldots\ldots\ldots\ldots\ldots.. [6.1.8]$$

Where, OFC_i = Objective Factor Cost.

Step 8.Measurement of Suitability Index (SI) as:

$$SI_i = \alpha\,(P_i^*) + \;(1-\alpha)(OFM_i) \qquad \ldots\ldots\ldots\ldots\ldots.. [6.1.9]$$

Where, α = Co-efficient of attitude, $0 \le \alpha \le 1$.

Step 9. Obtaining a comprehensive ranking of alternatives on the values of Suitability Index (SI).

6.1.2. A Mathematical Model

The flowchart of the model is given in figure 6.1.

A manufacturing organization desires to select the best advanced manufacturing technology from six alternatives. For this, they engage four experts to take optimum decision. Five benefit criteria are considered:

1) Quality(C1)
2) Productivity(C2)
3) Flexibility(C3)
4) Customer Satisfaction (C4)
5) Eco-friendliness(C5)

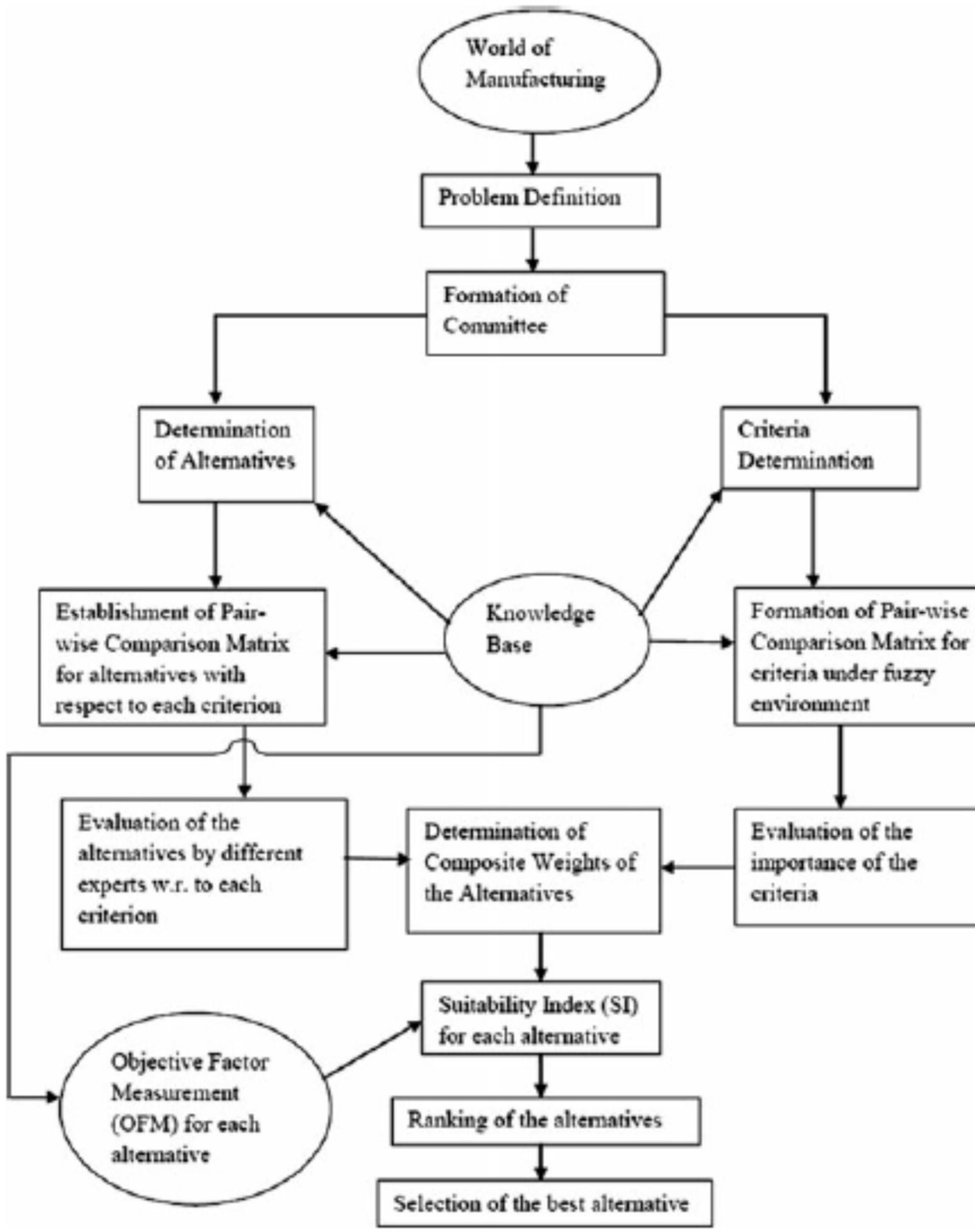


Figure 6.1. Flowchart of the case study

The problem is assumed to be homogeneous in nature. Experts' judgments are of equal importance. Table 6.1.1 and Table 6.1.2 represent linguistic variables and corresponding TFNs for criteria weights and values of alternatives.

Table 6.1.1: Linguistic variables for criteria weights

Linguistic Terms	**Fuzzy Numbers**
Very Less Weightage (VLW)	(0,0,0.2)
Less Weightage (WL)	(0,0.2,0.4)
Medium Weightage (MW)	(0.4,0.6,0.8)
High Weightage (HW)	(0.6,0.8,1)
Very High Weightage (VHW)	(0.8,1,1)

Table 6.1.2: Linguistic variables for values of alternatives

Linguistic Values	**Fuzzy Numbers**
Extremely Important (EI)	(9,10,10)
Very Important (VI)	(7,9,10)
Important (I)	(5,7,9)
Less Important (LI)	(3,5,7)
Very Less Important (VLI)	(1,3,5)
Extremely Less Important (ELI)	(0,1,3)

The Experts use the Fuzzy Linguistic terms (shown in Table 6.1.1 & Table 6.1.2) for weights of criteria and values of the alternatives presented in Table 6.1.3 & Table 6.1.4 respectively.

Table 6.1.3: Weights of the criteria by Experts

	Experts (E)			
Criteria	**E1**	**E2**	**E3**	**E4**
C1	VHW	VHW	HW	VHW
C2	HW	HW	VHW	HW
C3	HW	MW	HW	VHW
C4	VHW	HW	HW	VHW
C5	HW	HW	MW	VHW

Table 6.1.4: Values of alternatives by Experts

		Experts (E)			
Criteria	**Alternatives**	**E1**	**E2**	**E3**	**E4**
C1	AMT1	I	VI	I	LI
	AMT2	VI	VI	I	LI
	AMT3	I	LI	VI	VI
	AMT4	EI	VI	EI	I
	AMT5	EI	EI	VI	I
	AMT6	EI	VI	VI	I
C2	AMT1	I	VI	I	EI
	AMT2	VI	VI	VI	VI
	AMT3	EI	EI	VI	EI
	AMT4	LI	LI	I	I
	AMT5	I	LI	VI	I
	AMT6	EI	VI	VI	EI
C3	AMT1	I	I	I	LI
	AMT2	VI	VI	VI	VI
	AMT3	VI	VI	EI	VI
	AMT4	EI	VI	VI	VI
	AMT5	I	LI	LI	I
	AMT6	LI	LI	I	I
C4	AMT1	VI	VI	EI	VI
	AMT2	EI	EI	EI	EI
	AMT3	I	I	LI	LI
	AMT4	VI	VI	VI	VI
	AMT5	LI	LI	LI	FLI
	AMT6	VI	VI	VI	EI
C5	AMT1	I	I	I	I
	AMT2	I	LI	LI	LI
	AMT3	EI	EI	VI	EI
	AMT4	I	I	I	LI
	AMT5	VI	VI	EI	EI
	AMT6	EI	VI	VI	VI

The linguistic evaluations by Experts (Table 6.1.3, Table 6.1.4) are represented by TFNs to construct the fuzzy decision matrix as in Table 6.1.5. Weight matrix is also presented there. In the next step, the decision matrix is normalized and presented in Table 6.1.6.

Table 6.1.5: Fuzzy Decision Matrix and criteria weights

	C1	C2	C3	C4	C5
AMT1	(5,7,8.75)	(6.5,8.25,9.5)	(4.5,6.5,8.5)	(7.5,9.25,10)	(5,7,9)
AMT2	(5.5,7.5,9)	(7,9,10)	(7,9,10)	(9,10,10)	(3.5,5.5,7.5)
AMT3	(5.5,7.5,9)	(8.5,9.75,10)	(7.5,9.25,10)	(4,6,8)	(8.5,9.75,10)
AMT4	(7.5,9,9.75)	(4,6,8)	(7.5,9.25,10)	(7,9,10)	(4.5,6.5,8.5)
AMT5	(7.5,9,9.75)	(5,6.5,8.75)	(4,6,8)	(3,5,7)	(8,9.5,10)
AMT6	(7,8.75,9.75)	(8,9.5,10)	(4,6,8)	(7.5,9.25,10)	(7.5,9.25,10)
Weight	(0.75,0.95,1)	(0.65,0.85,1)	(0.6,0.8,0.95)	(0.7,0.9,1)	(0.6,0.8,0.95)

Table 6.1.6: Fuzzy Normalized Decision Matrix

	C1	C2	C3	C4	C5
AMT1	(0.5,0.7,0.89)	(0.65,0.83,0.95)	(0.45,0.65,0.85)	(0.75,0.9,1)	(0.5,0.7,0.9)
AMT2	(0.56,0.76,0.9)	(0.7,0.9,1)	(0.7,0.9,1)	((0.9,1,1))	(0.35,0.55,0.75)
AMT3	(0.56,0.76,0.9)	(0.85,0.97,1)	(0.75,0.9,1)	(0.4,0.6,0.8)	(0.85,0.97,1)
AMT4	(0.76,0.9,1)	(0.4,0.6,0.8)	(0.75,0.9,1)	(0.7,0.9,1)	(0.45,0.65,0.85)
AMT5	(0.76,0.9,1)	(0.5,0.65,0.87)	(0.4,0.6,0.8)	(0.3,0.5,0.7)	(0.8,0.95,1)
AMT6	(0.7,0.89,1)	(0.8,0.95,1)	(0.4,0.6,0.8)	(0.75,0.9,1)	(0.75,0.9,1)
Weight	(0.75,0.95,1)	(0.65,0.85,1)	(0.6,0.8,0.95)	(0.7,0.9,1)	(0.6,0.8,0.95)
FPIS	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)
FNIS	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

The distance measurements from FPIS (1, 1, 1) and FNIS (0, 0, 0) are shown in Table 6.1.7.

Table 6.1.7: The Distance Measurement

	D*	D⁻
AMT1	1.2350	3.8815
AMT2	0.9590	4.0547
AMT3	0.9025	4.0912
AMT4	1.0725	3.9862
AMT5	1.3387	3.6887
AMT6	0.8385	4.1865

The proximity index values of the AMTs are shown in Fig. 6.2.

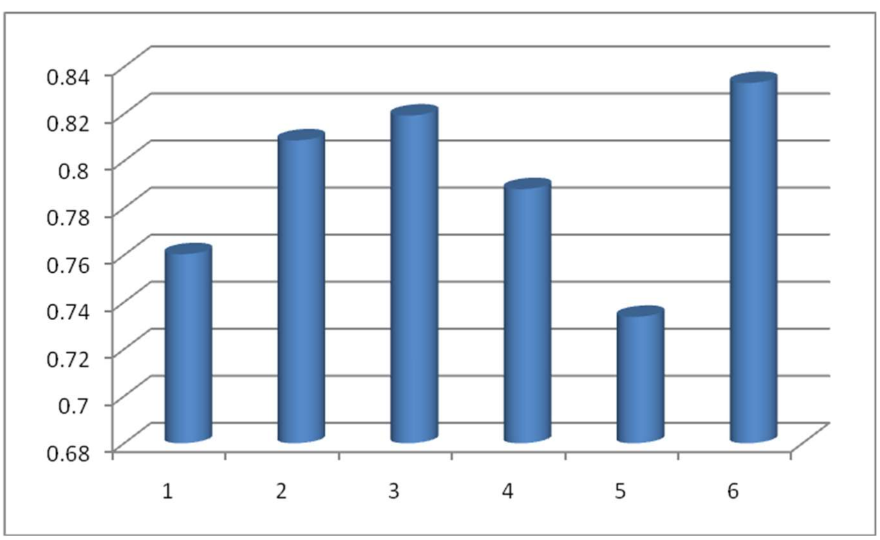


Figure 6.2. Proximity Index (P_i^*)

The capital and operating costs (in millions of $) of the six AMTs are shown in Table 6.1.8.

Table 6.1.8: Capital and Operating Cost (OFC_i)

Alternatives	**Capital and Operating Cost(millions of $)**
AMT1	3.93
AMT2	5.78
AMT3	4.56
AMT4	8.42
AMT5	6.39
AMT6	5.97

The objective factor measurement from the capital and operating costs of the alternatives are calculated and presented in Fig. 6.3.

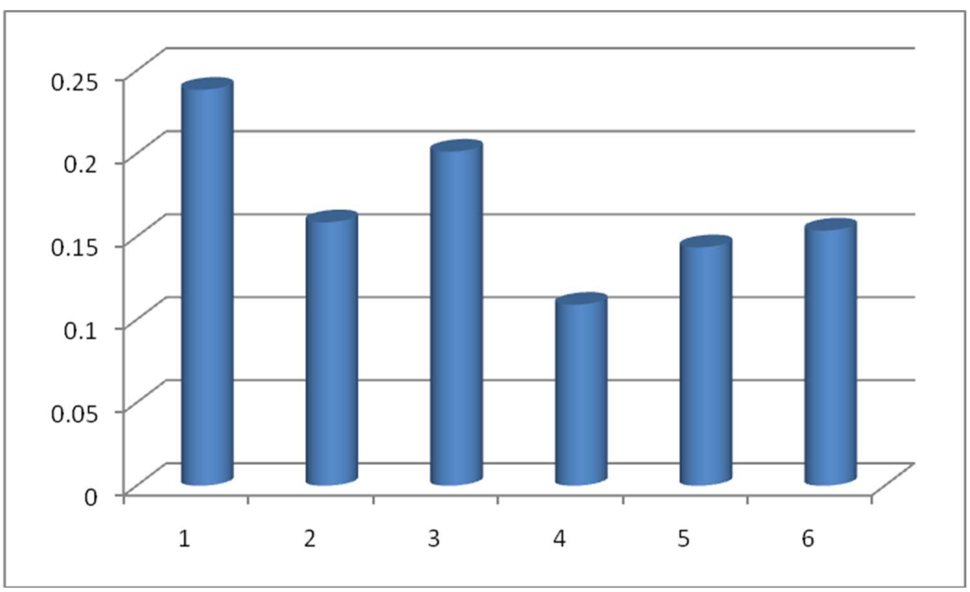


Figure 6.3. Objective Factor Measurement

To calculate the Suitability Index (SI), the co-efficient of attitude α is taken as 0.67 (by consensus of Decision Makers).

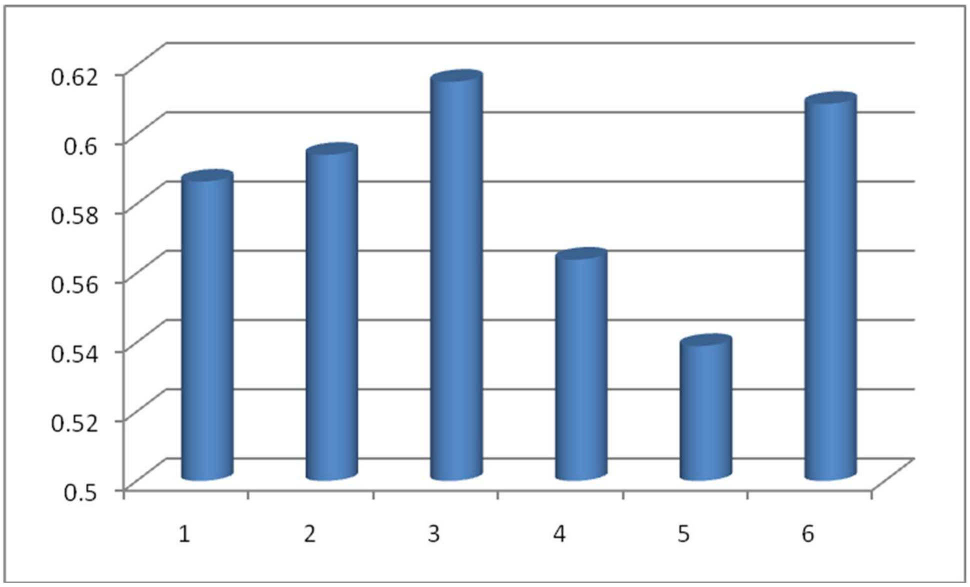


Figure 6.4. Suitability Index

According to the proximity to ideal solution, as shown in Fig. 6.4, the ranking order of the AMTs is AMT3> AMT6> AMT2>AMT1> AMT4> AMT5. So, the selected alternative is AMT3.

Where, '>' = superior to.

6.1.2.1. Sensitivity Analysis

Sensitivity Analysis or Post-optimality Analysis is done to get optimum decisions. It is the measure of sensitivity of a model to changes in its parametric values. This explores the variability (uncertainty) in the output of a mathematical model, which could be qualitatively or quantitatively disaggregated into different sources of variation in the input of the model. More generally, uncertainty and sensitivity analysis examine the reliability of a study that includes a mathematical model. It is generally used for:

1) model simplification 2) investigating model robustness 3) measuring impact of varying input assumptions and scenarios. 4) searching for errors in the model (by encountering unexpected relationships between inputs and outputs).

Modern manufacturing organizations are confronted with many challenges due to LPG environment where LPG stands for liberalization, privatization and globalization. It is very clear and evident that the market is highly dynamic and volatile. The organizations have been working under highly competitive surroundings. Now, any change in the flexibility, productivity, quality, customer satisfaction or capital and operating cost of the equipment under consideration will change the selection decision.

If the group decision makers are positive in attitude (the numerical sum of attitude is 100) i.e. they see the opportunities in the problems, or in other words, optimist group decision makers, the value of the co-efficient of attitude α will tend to move to the higher side. On the other hand, if the experts are of pessimistic in attitude i.e. they see the problems in the opportunities, the value of α will move to the lower side. That is why, sensitivity analysis is important for making an eclectic decision under the fuzzy situations.

In this case study, sensitivity analysis (figure 6.5) has been done to get an optimum range of α, for which a particular AMT is selected. The range of α has been shown in Table 6.1.9 for optimum decision.

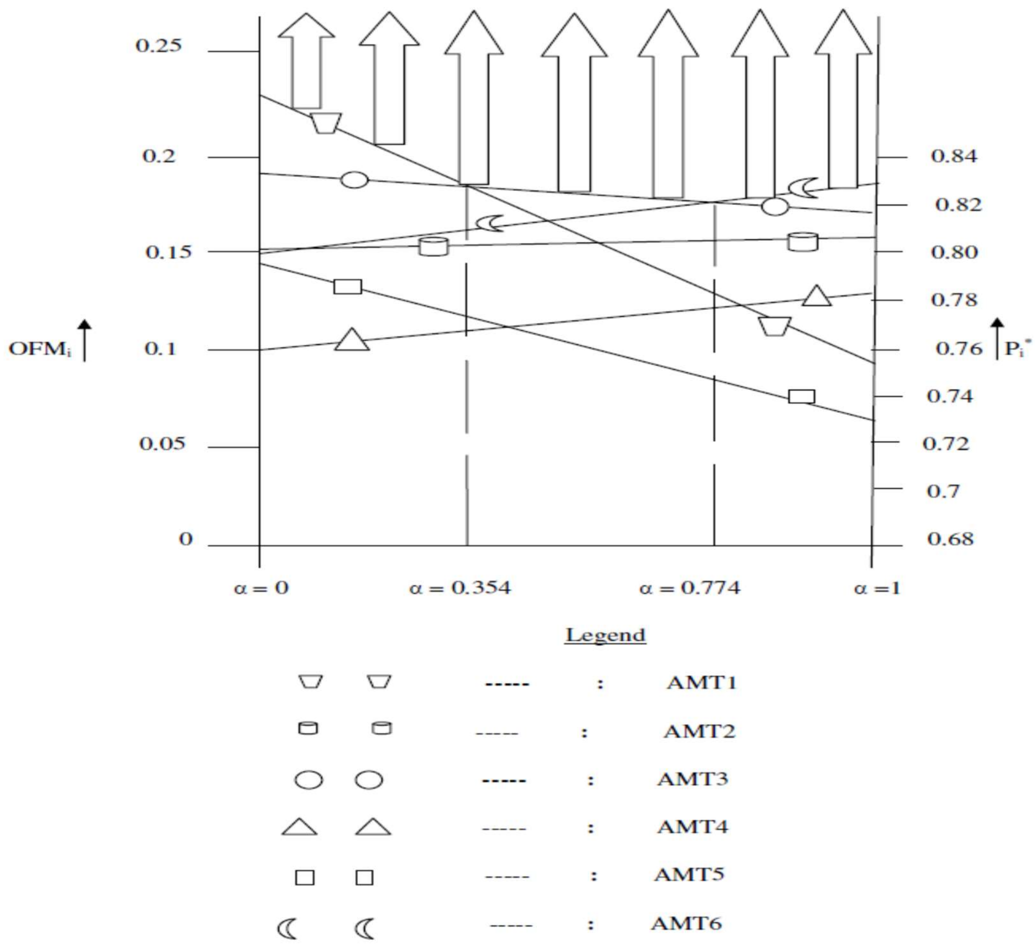


Figure 6.5. Sensitivity Analysis

Table 6.1.9: Optimum Decision

Co-efficient of attitude(α)	**Optimum Decision**
α<0.354	AMT1
0.354<α<0.774	AMT3
α >0.774	AMT6

6.1.3. Conclusion

The present model has been used to select the best advanced manufacturing technology. The factors used in the proposed method are 1) Quality 2) Productivity 3) Flexibility 4) Customer Satisfaction 5) Eco-friendliness. These are perceptual, conditional and somewhat subjective attributes. In general, greater quality is the driving force of greater productivity. Greater productivity brings in greater revenues, employment opportunities and technological advancement. Flexibility in selecting advanced manufacturing technology is also very important. One of the key elements of flexibility is volatile demand. It creates both risks and opportunities. A flexibility engineering system can easily respond to volatility, towards sustainable development. Customer satisfaction is also a key element in any organization. It is to meet the customers' expectations. In a competitive marketplace, it is a key differentiator amongst companies and a key element of company strategy. The factor eco-friendliness has its own importance in selecting an advanced manufacturing technology. The goal of any technology is to make our lives better. As companies and consumers have developed a sense of eco responsibility over the years, the need of ecofriendly technology has made its way into the organizations.

The study includes the use of a distance-based fuzzy TOPSIS approach to evaluate a set of advanced manufacturing technologies in order to reach the ultimate alternative that could satisfy the need of both the customer and the organization. Also, the objective factor measurement and the suitability index have been taken into account to rank the alternatives. Lastly, sensitivity analysis was done in the model to get the optimum decision in case of varying values of co-efficient of attitude. The result of sensitivity analysis shows that AMT1 or AMT3 or AMT6 can be the optimum decision for varying values of co-efficient of attitude, α. For α value less than 0.354, AMT1 is the optimum decision. For α value in between 0.354 and 0.774, AMT3 is selected as optimum alternative. And for α value greater than 0.774, AMT6 is the optimum decision.

The proposed approach illustrated in this chapter has some limitations. The end result of this model is largely dependent on experts' opinion. Possibility of inclination of an expert towards a particular alternative can’t be ruled out. Inconsistency may also occur in the pair wise comparison of matrices, which may lead to incorrect results. Future scopes include the usage of the model in other areas of decision problems as well.

6.2. Case Study II: A comparative exploratory analysis for performance assessment of Advanced Manufacturing Technologies by Fuzzy MCDM Methods

6.2.1. COPRAS-G & EVAMIX Methodology Analysis

6.2.2. COPRAS-G Approach

Most of the multi-attribute decision problems should be determined with interval values, and not with exact values of attributes. Zavadskas et. al. came out with the main ideas of CPPRAS-G i.e. Complex proportional assessment with grey relations method. The idea of interval numbers is to negotiate the real-life problem scenario. The procedural steps of COPRAS-G method are given below:

Step 1: Selecting attributes important for the problem.

Step 2: Construction of inter-valued decision matrix.

Step 3: Determining the attribute weights i.e. q_j.

Step 4: Decision matrix normalizing by the following formula:

$$\overline{w}_{ij} = w_{ij}/\left(\frac{1}{2}\right)\left(\sum_{i=1}^{m} w_{ij} + \sum_{i=1}^{m} b_{ij}\right) \quad \text{………. [6.2.1]}$$

$$\overline{b}_{ij} = b_{ij}/\left(\frac{1}{2}\right)\left(\sum_{i=1}^{m} w_{ij} + \sum_{i=1}^{m} b_{ij}\right) \quad \text{………. [6.2.2]}$$

$$Where, i = 1,2, \dots, m: no. of\ alternatives;\ j = 1,2, \dots, n: no. of\ criteria$$

$$w_{ij} = lower\ value\ of\ the\ jth\ criterion\ of\ the\ ith\ alternative$$

$$b_{ij} = higher\ value\ of\ the\ jth\ criterion\ of\ the\ ith\ alternative$$

Step 5: Calculating the weighted normalized decision matrix as follows:

$$\breve{w}_{ij} = \overline{w}_{ij} * q_j \quad \text{………. [6.2.3]}$$

$$\breve{b}_{ij} = \overline{b}_{ij} * q_j \quad \text{………. [6.2.4]}$$

Step 6: Determining the sums P_i of the attribute values (higher the better)

$$P_i = (1/2)\sum_{j=1}^{k}(\breve{w}_{ij} + \breve{b}_{ij}) \quad \text{……….. [6.2.5]}$$

Where, k is the number of maximizing criteria.

Step 7: Determining the sums R_i of the attribute values (lower the better)

$R_i = (1/2)\sum_{j=k+1}^{n}(\breve{w}_{ij} + \breve{b}_{ij})$ ……….. [6.2.6]

Where, $(n-k)$: no. of minimizing criteria.

Step 8: Selecting minimum R_i.

Step 9: Finding out relative weight of alternatives i.e. Q_i.

$Q_i = P_i + (\sum_{i=1}^{m} R_i / R_i \sum_{i=1}^{m} {}^{1}/_{R_i})$ ……….. [6.2.7]

Step 10: Selecting criterion K for optimality:

$K = \max(Q_i)$ ……….. [6.2.8]

Step 11: Calculation of the degree of utility (Higher the better) for alternatives:

$N_i = (Q_i/Q_{max}) * 100\%.$ ………. [6.2.9]

Step 12: Introduction of cost factor: Capital and Operating Cost (OFC_i). Calculating the Objective Factor Measure (OFM_i) as follows: $OFM_i = [OFC_i * \sum(1/OFC_i)]^{-1}$ .... [6.2.10]

Step 13: Calculating Sustainability Index (Higher the value, better the ranking):

$A_i SI = \alpha\,(Q_i) + (1-\alpha)(OFM_i)$ ………. [6.2.11]

Where, α = Co-efficient of Cognitive Attitude

6.2.3. EVAMIX Approach

Evaluation of mixed data (EVAMIX) method was first established by H. Voogd in the year 1983. After many years, in 2005, it was emphasized by J. M. Martel & B. Matarazzo. The uniqueness lies in the fact that the EVAMIX could process qualitative and quantitative data simultaneously.

It consists of the following procedural steps as furnished below:

Step 1: Differentiation of cardinal and ordinal criteria in the decision matrix.

Step 2: Decision matrix normalization. For beneficial attributes (higher the better), the following equation could be used for normalization:

$r_{ij} = [x_{ij} - min(x_{ij})]/[\max(x_{ij}) - \min(x_{ij})]$ ………. [6.2.12]

For non-beneficial attributes (lower the better), the equation is as follows:

$r_{ij} = [\max(x_{ij}) - x_{ij}]/[\max(x_{ij}) - \min(x_{ij})]$ ……… [6.2.13]

Step 3: Calculating the difference in attribute values between different alternative-pairs. This step involves evaluating differences of $i - th$ alternative with the others, for each attribute.

Step 4: Dominance score compilation of each alternative pair, (i, i') for all ordinal and cardinal attributes. The following equations hold good: $\alpha_{ii'} = [\sum_{j\epsilon O} \{wjsgn(r_{ij} - r_{i'j})\}^{c}]^{\ 1/c}$ ……… [6.2.14]

Where, $wjsgn(r_{ij} - r_{i'j}) = \begin{cases} +1, & if\ r_{ij} > r_{i'j} \\ 0, & if\ r_{ij} = r_{i'j} \\ -1, & if\ r_{ij} < r_{i'j} \end{cases}$

$\gamma_{ii'} = [\sum_{j\epsilon C} \{wjsgn(r_{ij} - r_{i'j})\}^{c}]^{\ 1/c}$ ……… [6.2.15]

Where, *c:* a scaling parameter, any arbitrary positive odd number i.e. 1,3,5 etc.

O, C: the sets of ordinal and cardinal criteria respectively,

$\alpha_{ii'}$ $\gamma_{ii'}$: the dominance scores for alternative pair, (i, i') with respect to ordinal and cardinal criteria respectively,

w_j: relative importance of $j - th$ criterion.

Step 5: Calculating standardized dominance scores for the alternative pair, (i, i') as follows:

Ordinal score $(\delta_{ii'}) = (\alpha_{ii'} - \alpha^{-})/(\alpha^{+} - \alpha^{-})$ ……… [6.2.16]

Where, α+, α- are the highest and the lowest ordinal dominance scores for the alternative pair, (i, i').

Cardinal score $(d_{ii'}) = (\gamma_{ii'} - \gamma^{-})/(\gamma^{+} - \gamma^{-})$ ……… [6.2.17]

Where γ+, γ - are the highest and the lowest cardinal dominance score for the alternative pair, (i, i').

Step 6: Determining the overall dominance score.

The degree of dominance of alternative i over i' is measured by the overall dominance score, $(D_{ii'})$ for each alternative pair (i, i') in the following equation:

$D_{ii'} = w_O \delta_{ii'} + w_C d_{ii'}$ ……… [6.2.18]

Where, $w_O = \sum_{j\epsilon O} w_j$ (i.e. summation of ordinal attribute weights) and $w_C = \sum_{j\epsilon C} w_j$ (i.e. summation of cardinal attribute weights)

Step 7: Calculating the alternatives' appraisal scores (S_i) by the following equation**:**

$$(S_i) = \sum_{i'} (D_{i'i}/D_{i'})^{-1} \quad \text{………. [6.2.19]}$$

The higher is the score, the optimum is the alternative.

Step 8: Introduction of cost factor: Capital and Operating Cost (OFC_i). Calculating the Objective Factor Measure (OFM_i) by following equation 4.

Step 9: Calculating Selection Index (Higher the value, better the ranking):

$$SI = \alpha\,(S_i) + (1-\alpha)(OFM_i) \quad \text{………. [6.2.20]}$$

6.2.4. A Numerical Example

The flowchart of the given problem is given in figure 6.6.

Today's competitive world has compelled organizations to customize their product and that too at a competitive price. Proper selection of AMT offers great productivity and profitability. But these can be hampered if the selection is wrong. Here, in this chapter, a numerical example is shown comparing the two methods namely COPRAS-G and EVAMIX for the proper evaluation and selection of AMT. Three Evaluators (E) having different fields of expertise in manufacturing organizations have been engaged for the same. Their job demands picking up the best AMT from a number of choices. We have considered some criteria which were very rarely used by the past researchers while evaluating the AMTs.

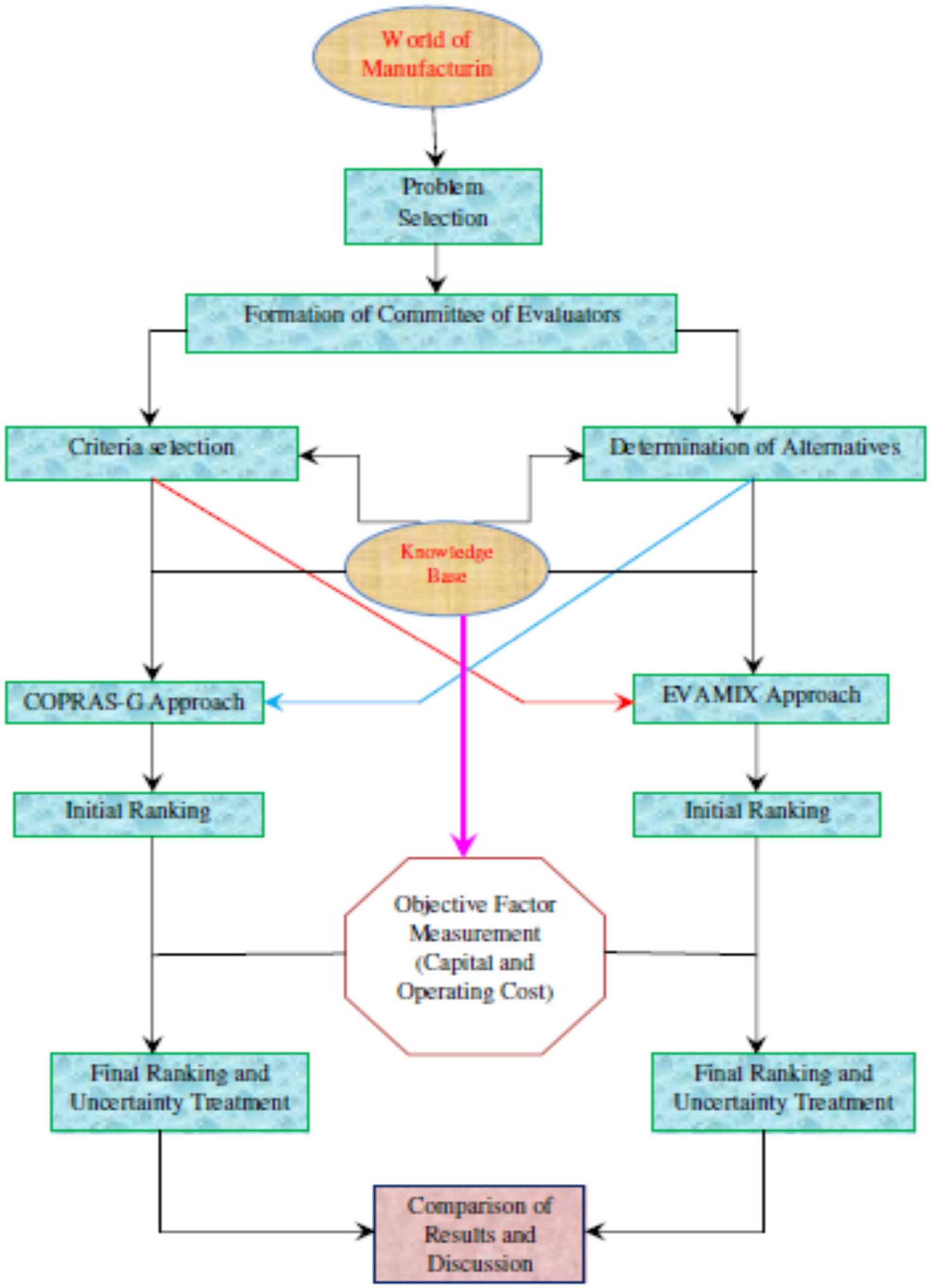


Figure 6.6. Flowchart of the case study

Three benefit criteria and two non-benefit criteria have been considered for the selection of the same. These are introduced in Table 6.2.1 as follows:

Table 6.2.1: Selected Criteria, the Indicators

Criteria	Indicators	Beneficial/ Non-beneficial	Cardinal/ Ordinal
Yield Rate (C1)	quantity of machinable material available after the manufacturing process completion. It is enunciated as a percentage of produced quantity.	Beneficial	Cardinal
Quickness of Delivery (C2)	Promptness of delivery of order.	Beneficial	Ordinal
Volume Flexibility (C3)	Ability to produce above/below the installed capacity for a product.	Beneficial	Ordinal
Environ cognitive Hazards (C4)	The things or events likely to threaten the natural environment and negatively influences people's health.	Non-beneficial	Ordinal
Dissemination of Material (C5)	Wastage of material.	Non-beneficial	Ordinal

Each evaluator has equal weightage on his/her decision making. The attribute weights and weights of AMTs values are presented in terms of linguistic variables and TFNs as in Table 6.2.2 and Table 6.2.3.

Table 6.2.2: Linguistic variables for importance weights of each criterion

Linguistic Variable	TFN
Extreme Less Important (ELI)	(0,0,0.2)
Less Important (LI)	(0,0.2,0.4)
Important (I)	(0.2,0.4,0.6)
Very Important (VI)	(0.4,0.6,0.8)
Extremely Important (EI)	(0.6,0.8,1)

Table 6.2.3: Linguistic variables for the criteria values of alternatives

Linguistic Variable	TFN
Extremely Less Importance (ELI)	(0,0,1)
Very Less Importance (VLI)	(0,1,3)
Less Importance (LI)	(1,3,5)
Medium Importance (MI)	(3,5,7)
Heavy Importance (HI)	(5,7,9)
Very Heavy Importance (VHI)	(7,9,10)
Extremely Heavy Importance (EHI)	(9,10,10)

As uncertainty and vagueness are present in the problem, providing crisp data for the weights of criteria as well as the alternatives is a very tedious task. So, the Evaluators (E) utilized linguistic variables (in Table 6.2.2 & Table 6.2.3) for attribute weights and values of the alternatives as in Table 6.2.4 & Table 6.2.5 respectively.

Table 6.2.4: Attribute Weightage by Evaluators

	Evaluators (E)		
Criteria	**E1**	**E2**	**E3**
C1 (Beneficial)	VI	EI	I
C2 (Beneficial)	VI	VI	EI
C3 (Beneficial)	VI	VI	VI
C4 (Non-Beneficial)	EI	EI	EI
C5 (Non-Beneficial)	VI	I	VI

Table 6.2.5: Attribute values of alternatives by Evaluators

		Evaluators (E)		
Criteria	**Alternatives**	**E1**	**E2**	**E3**
C1	A1	VHI	VHI	HI
	A2	MI	HI	VHI
	A3	VHI	HI	MI
	A4	VHI	EHI	EHI
	A5	VHI	EHI	VHI
C2	A1	HI	HI	MI
	A2	EHI	EHI	EHI
	A3	HI	VHI	EHI
	A4	HI	HI	VHI
	A5	HI	HI	MI
C3	A1	EHI	VHI	HI
	A2	VHI	VHI	EHI
	A3	VHI	VHI	HI
	A4	EHI	VHI	VHI
	A5	HI	EHI	VHI
C4	A1	HI	MI	MI
	A2	HI	HI	MI
	A3	EHI	EHI	VHI
	A4	EHI	EHI	EHI
	A5	LI	MI	MI
C5	A1	LI	MI	MI
	A2	HI	MI	HI
	A3	HI	HI	VHI
	A4	MI	LI	MI
	A5	EHI	EHI	VHI

The linguistic terms given by the evaluators are transformed into TFNs to form the initial decision matrix and as given in Table 6.2.6 as follows:

Table 6.2.6: Initial decision matrix

	Criteria				
Alternatives	**C1 (+ve)**	**C2 (+ve)**	**C3 (+ve)**	**C4 (-ve)**	**C5 (-ve)**
A1	(6.33,8.33,9.67)	(4.33,6.33,8.33)	(7,8.67,9.67)	(3.67,5.67,7.67)	(2.33,4.33,6.33)
A2	(5,7,8.67)	(9,10,10)	(7.67,9.33,10)	(4.33,6.33,8.33)	(4.33,6.33,8.33)
A3	(5,7,8.67)	(7,8.67,9.67)	(6.33,8.33,9.67)	(8.33,9.67,10)	(5.67,7.67,9.33)
A4	(8.33,9.67,10)	(5.67,7.67,9.33)	(7.67,9.33,10)	(9,10,10)	(2.33,4.33,6.33)
A5	(7.67,9.33,10)	(4.33,6.33,8.33)	(7,8.67,9.67)	(2.33,4.33,6.33)	(8.33,9.67,10)
Weights (w_j)	(0.4,0.6,0.8)	(0.47,0.67,0.87)	(0.4,0.6,0.8)	(0.6,0.8,1)	(0.33,0.53,0.73)

Table 6.2.2 to Table 6.2.6 are common to the two methodologies namely COPRAS-G and EVAMIX. Table 6.2.7 enters in the territory of COPRAS-G method and provides the values of criteria describing the compared alternatives in intervals, which is a specialty of COPRAS-G method.

COPRAS-G Approach

Table 6.2.7: Values of criteria describing the compared alternatives in intervals

	Criteria									
	C1		**C2**		**C3**		**C4**		**C5**	
Optimization Decision	Max (+ve)		Max (+ve)		Max (+ve)		Min (-ve)		Min (-ve)	
Criteria Weights (q_j)	0.188		0.209		0.188		0.25		0.165	
Alternatives	W_1	B_1	W_2	B_2	W_3	B_3	W_4	B_4	W_5	B_5
A1	6.33	9.67	4.33	8.33	7	9.67	3.67	7.67	2.33	6.33
A2	5	8.67	9	10	7.67	10	4.33	8.33	4.33	8.33
A3	5	8.67	7	9.67	6.33	9.67	8.33	10	5.67	9.33
A4	8.33	10	5.67	9.33	7.67	10	9	10	2.33	6.33
A5	7.67	10	4.33	8.33	7	9.67	2.33	6.33	8.33	10

Weighted normalized decision matrix, Normalizing Indices and Utility Degree are calculated according to the algorithm and provided in subsequent Tables 6.2.8, 6.2.9 and 6.2.10.

Table 6.2.8: Weighted normalized decision matrix

Alternatives	**Criteria (values in intervals)**									
	C1		**C2**		**C3**		**C4**		**C5**	
	W_1	B_1	W_2	B_2	W_3	B_3	W_4	B_4	W_5	B_5
A1	0.03	0.046	0.024	0.046	0.031	0.043	0.026	0.055	0.045	0.033
A2	0.024	0.04	0.050	0.054	0.034	0.045	0.031	0.06	0.023	0.043
A3	0.024	0.04	0.038	0.053	0.028	0.043	0.06	0.073	0.03	0.049
A4	0.039	0.047	0.031	0.051	0.034	0.045	0.065	0.073	0.045	0.033
A5	0.036	0.047	0.024	0.046	0.031	0.043	0.017	0.045	0.043	0.053

Table 6.2.9: Normalizing Indices

Alternative	P_i **(Maximizing)**	R_i **(Minimizing)**
A1	.110	.080
A2	.124	.079
A3	.113	.106
A4	.124	.108
A5	.114	.079

Table 6.2.10: Relative Weight and Utility Degree

Alternative	Relative Weight (Q_i)	Utility Degree (N_i) (%)	Initial Position
A1	0.211	93.78	3rd
A2	0.225	100	1st
A3	0.188	83.56	5th
A4	0.198	88	4th
A5	0.215	95.56	2nd

According to the strategic criteria, the alternative with highest Utility Degree is alternative A2.The economic evaluation criteria are taken in the form of Capital and Operating Costs (OFC_i) as given in Table 6.2.11. The same is used to calculate the Sustainability Index (A_iSI), as in Table 6.2.12, by taking the Co-efficient of cognitive attitude (α) value equal to 0.7. The higher is the Sustainability Index value, the better is the ranking. Again, the value of α is dependent on how much optimistic or pessimistic an evaluator is. An optimistic evaluator sets higher value of α. While a pessimistic one is bound to set a lower value of α.

Table 6.2.11: Capital and Operating Cost (OFC_i)

Alternative	OFC_i (Millions of $)	OFM_i
A1	4.2	0.215
A2	5.1	0.177
A3	4.1	0.220
A4	4.3	0.210
A5	4.9	0.184

Table 6.2.12: Sustainability Index and Ranking

Alternative	Sustainability Index (A_iSI)	Final Ranking
A1	0.2122	1st
A2	0.2106	2nd
A3	0.1976	5th
A4	0.2016	4th
A5	0.2057	3rd

It is seen that the introduction of Capital and Operating Cost factor changes the ranking. And the final selection is alternative A1.

EVAMIX Approach

Decision Matrix containing crisp data from Table 6.2.6 are calculated and shown in Table 6.2.13 as follows:

Table 6.2.13: Decision Matrix (Crisp Data)

Alternatives	Criteria				
	C1(+ve)	C2(+ve)	C3(+ve)	C4(-ve)	C5(-ve)
A1	8.22	6.33	8.56	5.67	4.33
A2	6.945	9.83	9.165	6.33	6.33
A3	6.945	8.56	8.22	9.5	7.6
A4	9.5	7.6	9.165	9.83	4.33
A5	9.165	6.33	8.56	4.33	9.5
Weight	0.6	0.67	0.6	0.8	0.53

Normalized decision matrix is shown in Table 6.2.14 as follows:

Table 6.2.14: Normalized Decision Matrix

Alternatives	Criteria				
	C1(+ve)	C2(+ve)	C3(+ve)	C4(-ve)	C5(-ve)
A1	0.499	0	0.36	0.756	1
A2	0	1	1	0.636	0.613
A3	0	0.637	0	0.06	0.368
A4	1	0.363	1	0	1
A5	0.869	0	0.36	1	0
Weight	0.188	0.209	0.188	0.25	0.165

Dominance scores for alternative pairs are calculated for cardinal and ordinal criteria and given in Table 6.2.15 as follows:

Table 6.2.15: Dominance Scores of each Alternative Pair

Pair	$\gamma_{ii'}$	$\alpha_{ii'}$	**Pair**	$\gamma_{ii'}$	$\alpha_{ii'}$
(1,2)	0.188	0.018	(3,4)	-0.188	0.106
(1,3)	0.188	0.394	(3,5)	-0.188	-0.064
(1,4)	-0.188	-0.147	(4,1)	0.188	0.147
(1,5)	-0.188	-0.085	(4,2)	0.188	-0.294
(2,1)	-0.188	-0.018	(4,3)	0.188	-0.106
(2,3)	0	0.812	(4,5)	0.188	0.312
(2,4)	-0.188	0.294	(5,1)	0.188	0.085
(2,5)	-0.188	0.312	(5,2)	0.188	-0.312
(3,1)	-0.188	-0.394	(5,3)	0.188	0.064
(3,2)	0	-0.812	(5,4)	-0.188	-0.312

Standardized dominance scores are calculated as in Table 6.2.16 as follows:

Table 6.2.16: Standardized Dominance Scores

Pair	$d_{ii'}$	$\delta_{ii'}$	**Pair**	$d_{ii'}$	$\delta_{ii'}$
(1,2)	1	0.511	(3,4)	0	0.565
(1,3)	1	0.743	(3,5)	0	0.46
(1,4)	0	0.409	(4,1)	1	0.591
(1,5)	0	0.448	(4,2)	1	0.32
(2,1)	0	0.489	(4,3)	1	0.435
(2,3)	0.5	1	(4,5)	1	0.692
(2,4)	0	0.68	(5,1)	1	0.552
(2,5)	0	0.692	(5,2)	1	0.308
(3,1)	0	0.257	(5,3)	1	0.54
(3,2)	0.5	0	(5,4)	0	0.308

Overall dominance scores for alternative pairs are calculated and shown in Table 6.2.17 as follows:

Table 6.2.17: Overall Dominance Score

Pair	$D_{ii'}$	**Pair**	$D_{ii'}$	**Pair**	$D_{ii'}$
(1,2)	0.603	(2,5)	0.562	(4,3)	0.541
(1,3)	0.791	(3,1)	0.209	(4,5)	0.75
(1,4)	0.332	(3,2)	0.094	(5,1)	0.636
(1,5)	0.364	(3,4)	0.459	(5,2)	0.438
(2,1)	0.397	(3,5)	0.374	(5,3)	0.626
(2,3)	0.906	(4,1)	0.668	(5,4)	0.25
(2,4)	0.552	(4,2)	0.448		

Appraisal score for AMTs is calculated by the method of additive interval as proposed in the algorithm and shown in Table 6.2.18:

Table 6.2.18: Appraisal Score

Alternative (i)	Appraisal Score (S_i)	Preliminary Result
A1	0.2136	3rd
A2	0.3112	2nd
A3	0.0614	5th
A4	0.3435	1st
A5	0.1834	4th

These Appraisal Scores are used against the Capital and Operating Costs of alternatives, as provided in Table 6.2.11, to get the final ranking. It is calculated in accordance with the Selection Index by taking α as 0.7, by consensus of evaluators as the same in the previous COPRAS-G method. The same is presented in Table 6.2.19 as follows:

Table 6.2.19: Selection Index and Final Ranking

Alternative (i)	Selection Index (SI)	Final Ranking
A1	0.2140	3rd
A2	0.2709	2nd
A3	0.1090	5th
A4	0.3035	1st
A5	0.1836	4th

So, it is seen that, introduction of cost factor does not change the ranking as in case of COPRAS-G method. Hence the final selection is alternative A4.

6.2.4.1. Uncertain treatment

Uncertain treatment or Post-Optimality Analysis has the ability of handling imprecise information. It is done to check the robustness of the model. Robustness, again, is the influence of addition or deletion of alternatives on the assessment results [Cinelli, M., Coles, S. R., & Kirwan, K. (2014)]. It is the test sensitivity of a model to variation in the input data. Here, uncertain treatment is done to get the range of α, for which the selection of a particular AMT is justified and an optimum one. Figure 6.7 and figure 6.8 reflect the same in case of COPRAS-G and EVAMIX method respectively.

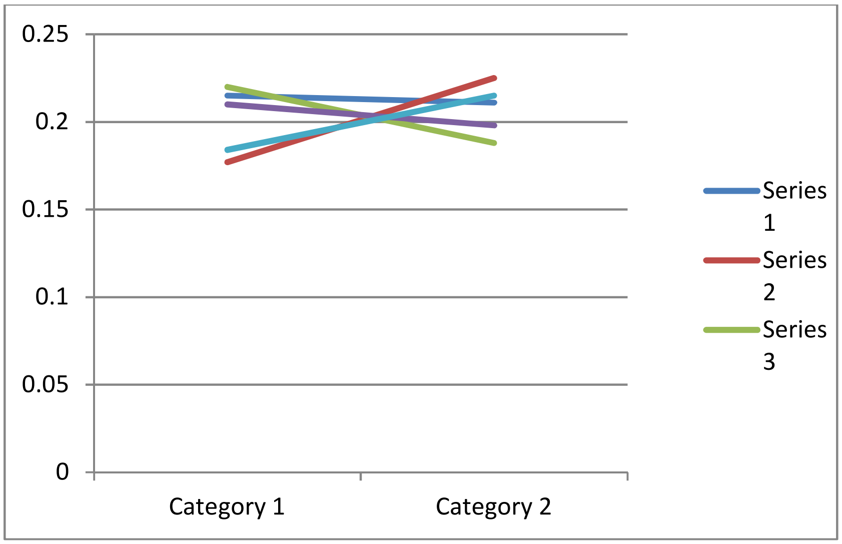


Figure 6.7. Uncertain treatment (COPRAS-G)

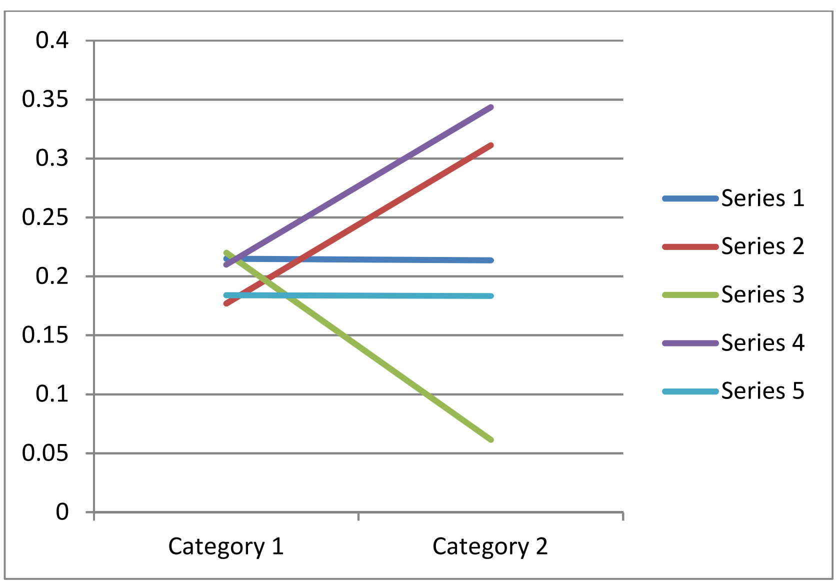


Figure 6.8. Uncertain treatment (EVAMIX)

On the basis of the uncertain treatment of the parameters, the results of optimum selection decision are given in Table 6.2.20 as follows:

Table 20: Uncertain treatment Result

COPRAS-G		**EVAMIX**	
Value of α	**Selection Decision**	**Value of α**	**Selection Decision**
α <= 0.178	A3	α <= 0.034	A3
0.178 <= α <=0.73	A1	0.034 <= α <= 1	A4
α >= 0.73	A2		

6.2.4.2. Results and Discussion

The result of this comparative study shows that alternative A4 comes out as the distant winner in case of EVAMIX approach. But, in COPRAS-G approach, alternative A1 gets the highest rank although the difference with the other alternatives is too nominal. In COPRAS-G method, the final ranking changes with the cost factor coming into play. Initially, the alternative A2 got the best ranking. But, the ranking changes with the introduction of the cost factor and alternative A1 emerges out as the optimal selection decision. On the other hand, the EVAMIX approach keeps the ranking same before and after the introduction of the cost factor. The result of the uncertain treatment reveals that alternative A1, in CPRAS-G, is the optimum selection decision for a long range of α value which matches with the highest ranked alternative. In case of EVAMIX also, the optimum selection decision is alternative A4 for a very long span of α value which again matches with the best ranked alternative. So, in both the cases, the optimum selection decisions are same with the best results in the final rankings. But in case of EVAMIX approach, the selection is valid for almost the entire range of α. That goes to show that, the selection made by this approach is more accurate one than that by COPRAS-G method.

6.2.5. Conclusion

In general, the Multi-Criteria problems are associated with uncertainties. So it is not always suitable to use numerical values. In the proposed method, linguistic variables and TFNs have been used in dealing with uncertainties. Here, in this investigation, we have taken two MCDM tools i.e. COPRAS-G, EVAMIX and provided a similarity study by taking a real-life example of selection of AMT. We have also used the linguistic variables and TFNs to capture preferences given by the Evaluators for different criteria as well as criteria values of alternatives. The decision matrix is formed and normalized. Then according to the procedural steps of two methods used here, a comprehensive ranking of the alternatives have been derived for the two. Also, the uncertain treatment has been done in both the cases for robustness testing of the study. EVAMIX method gives considerable attention to cardinal and ordinal criteria separately besides considering the beneficial or non-beneficial criteria as well. So, between the two, EVAMIX is more accurate and acceptable one than COPRAS-G. Although the comparative study has been used to select the suitable AMT, it can solve other selection and evaluation problems as well, from real manufacturing world. Future scope would include applying other MCDM and preference ranking methods namely ARAS (Additive Ratio Assessment), PSI (Preference Selection Index), OCRA (Operational Competitiveness Rating Analysis) etc. to solve complex selection and decision problems prevailing in the manufacturing world.

6.3. **Case Study III:** A De novo approach for performance assessment of advanced technologies

6.3.1. PROMETHEE & Dempster- Shafer Theory of Evidence (DST) Methodology Analysis

6.3.2. PROMETHEE

Preference ranking organization method for enrichment evaluations was first introduced by Jean-Pierre Brans (1982). It was later developed by Brans & Vincke (1985). It is an MCDM method that can handle information in the form of qualitative and quantitative data with the qualitative being reduced to point scales. It has specific application in decision making and can be used in a wide variety of decision situations, in areas such as manufacturing, transportation, healthcare and education. Instead of indicating a single right decision, the PROMETHEE approach helps experts find the alternative that best fits their goals. It provides a logical and comprehensive framework for structuring a decision-making problem, identifying and quantifying its uncertainties and conflicts, and highlighting key alternatives and reasoning structure behind the scene. It provides experts with complete and partial grading of actions. It enforces a strong sustainability perspective to the problem. The outranking flows are very important aspects in PROMETHEE. The positive outranking flow $\varphi^{+}(a)$ presents how much each alternative is outperforming all other choices (higher the better) and negative outranking flow $\varphi^{-}(a)$ presents how much each alternative is outperformed by them (lower the better). The net outranking flow score $\varphi(a)$ exhibits the variation between the two.

The steps of PROMETHEE are the following:

Step 1: Linguistic variables assigned by DMs for weights of criteria as well as for performance of alternatives on each criterion.

Step 2: Conversion of linguistic variables into TFNs by following a said conversion scale.

Step 3: Formation of initial decision matrix by TFNs.

Step 4: Defuzzification i.e. Conversion of TFNs in decision matrix into crisp data as in equation 6.3.1:

$$x = \frac{a+4b+c}{6},\ x = \text{crisp member of decision matrix} \qquad \text{.... [6.3.1]}$$

Step 5: Normalizing the decision matrix as mentioned below in equation 6.3.2:

$$\left.\begin{aligned} r_{ij} &= [x_{ij} - \min(x_{ij})]/[\max(x_{ij}) - \min(x_{ij})] \text{ (For beneficial criteria)} \\ r_{ij} &= [max(x_{ij}) - (x_{ij})]/[\max(x_{ij}) - \min(x_{ij})] \text{ (For cost criteria)} \end{aligned}\right\} \quad \text{.... [6.3.2]}$$

Where, x_{ij} is the importance weight of i^th alternative for j^th criteria in the decision matrix, i = no. of alternatives, j = no. of criteria.

Step 6: Obtaining deviations $d_{j(a,b)}$ showing the difference evaluations of alternatives a and b with respect to criteria j as in equation 6.3.3: $d_{j(a,b)} = g_j(a) - g_j(b)$ ..... [6.3.3]

Step 7: Selection of preference for usual criterion by following equation 6.3.4:

$$P(a,b) = \begin{cases} 0, & d_{j(a,b)} \leq 0 \\ 1, & d_{j(a,b)} > 0 \end{cases} \quad \text{.... [6.3.4]}$$

Step 8: Measuring the aggregate preference indices as in equation 6.3.5:

$$\pi(a,b) = \sum_{j=1}^{n} P(a,b) \times w_j, \forall a, b \in A \quad \text{..... [6.3.5]}$$

It is the weighted sum of P (a, b) i.e. the measure of how preferable alternative a is over alternative b for a particular factor.

Step 9: Compute the outranking flows i.e. $\varphi^+(a)$ and $\varphi^-(a)$ following equation 6.3.6:

$$\varphi^+(a) = \frac{1}{m-1}\sum_{x \in A} \pi(a,x), \varphi^-(a) = \frac{1}{m-1}\sum_{x \in A} \pi(x,a) \quad \text{... [6.3.6]}$$

Where, m: no. of alternatives.

Step 10: Calculate the net outranking flows i.e. the performance of each AMT as follows in equation 6.3.7:

$$\varphi(a) = \varphi^+(a) - \varphi^-(a) \quad \text{.... [6.3.7]}$$

Step 11: Introduction of objective criteria in the form of investment costs (IC_i) of AMTs. Calculating the objective factor measure (OFM_i) by taking the cost factor into consideration as in equation 6.3.8:

$$OFM_i = [IC_i \times \sum \frac{1}{IC_i}]^{-1} \quad \text{.... [6.3.8]}$$

Step 12: Calculating the selection index as follows in equation 6.3.9:

$$AMT_iSI = \beta\{\varphi(a)\} + (1-\beta)OFM_i \quad \text{.... [6.3.9]}$$

Where β is the index of cognitive mind of decision maker.

Step 13: Final ranking of the AMTs according to the results of the selection index (Higher the better).

6.3.3. Dempster Shafer Theory of evidence (DST) method: - The classical TOPSIS was evolved (C. L. Hwang & K. Yoon, 1981) as a solution to fuzzy multi-criteria problems prevailing in the manufacturing organizations. It projects the optimum alternative as being closest to the ideal solution and at the same time, farthest from the anti-ideal solution. The ideal solution comprises of best criteria values. The anti-ideal one contains worst criteria values. The solutions are found out assuming each criterion taking repetitiously increasing or decreasing utility. It is a method of cardinal preference of attributes. Dempster-Shafer Theory (A. P. Dempster, 1967; G. Shafer, 1976), in general is contemplated as the extension of Bayesian theory which can deal with imprecise data set efficiently. The benefits of DST include measuring the probability and attaching it to the frame of discernment. It is a set of hypothesis Ĥ defined as follows:

$\hat{H} = \{H_1, H_2 \ldots H_N\}$.

It is a finite non-empty set composed of N mutually exclusive hypothesis. The power set of Ĥ consists of all the subsets of Ĥ and is defined as follows:

$P(\hat{H}) = [\emptyset, \{H_1\}, \{H_2\} \ldots \{H_n\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\} \ldots \hat{H}]$,

Where, Ø: the empty set; n: no. of subsets with only single element.

The basic probability assignment or BPA is the main element of evidence theory. It is a function from $P(\hat{H})$ to [0, 1] interpreted as: m: $P(\hat{H}) \rightarrow [0, 1]$, satisfying the conditions: $\sum_{A \in P(\hat{H})} m(A) = 1$ and $m(\emptyset) = 0$.

Example. For an element, the BPA is: m {IS} = 0.7; m {NS} = 0.2; m {IS, NS} = 0.1.

This signifying the BPA supporting the following hypothesis:

i) "the alternative is an ideal solution with belief degree of 0.7".

ii) "the alternative is a negative ideal solution with belief degree of 0.2".

iii) "We know nothing about the alternative with a belief degree of 0.1".

In cases of problems containing imprecise data, fusion could be the solution for generating dataset. Evidence theory offers fusion tools as well. It is possible to use Dempster rule of combination i.e. orthogonal sum in a basic belief assignment, m, for information source S, and is noted by $m = m_1 \oplus \mathrm{m}_2$. It can combine two BPAs to yield a new BPA. The same is represented in equations 6.3.10 and 6.3.11.

$$m\ (\mathrm{A}) = \sum_{B \cap C = A} m_1(B) m_2(C)/(1-k) \qquad \text{… [6.3.10]}$$

$$\mathrm{k} = \sum_{B \cap C = \emptyset} m_1(B)\ m_2(C) \qquad \text{.... [6.3.11]}$$

Where, k is the normalizing constant known as 'conflict'. It measures the degree of conflict between m1 & m2. k= 0 means absence of conflict between m1 & m2; k= 1 means absolute contradiction of m1, m2.

The belief function m is denoted in equation 6.3.12 as:

$$m = m_1 \oplus m_2 \oplus \ldots \oplus m_n \qquad \text{….. [6.3.12]}$$

To handle conflict, a discounting rule (Shafer, 1976) is introduced in DST given by the equations 6.3.13 and 6.3.14 as follows: $BEL^{\alpha}(\hat{\mathrm{H}}) = 1$ …. [6.3.13]

$$BEL^{\alpha}(A) = (1-\alpha) * BEL(A)\ ; \forall\ \mathrm{A} \subset \hat{\mathrm{H}} \text{ and } \mathrm{A} \neq \emptyset \qquad \text{…. [6.3.14]}$$

Where, BEL: $2^{\hat{\mathrm{H}}} \rightarrow [0, 1]$ is a belief function and $BEL^{\hat{\mathrm{H}}}$: $2^{\hat{\mathrm{H}}} \rightarrow [0, 1]$ is a discounted belief function.

α (0 ≤ α ≤ 1) is the discounting co-efficient qualifying for the strength of reliability of the evidence.

The BPA m^{α}corresponding to the discounted belief function $BEL^{\alpha}$is further modified (Shafer, 1976) in equations 6.3.15 and 6.3.16 in the following manner:

$$m^{\alpha}(\hat{\mathrm{H}}) = (1-\alpha) m(\hat{\mathrm{H}}) + \alpha \qquad \text{…. [6.3.15]}$$

$$m^{\alpha}(A) = (1-\alpha) m(A), \forall\ \mathrm{A} \subset \hat{\mathrm{H}} \text{ and } \mathrm{A} \neq \emptyset \qquad \text{…. [6.3.16]}$$

Beliefs react at the following: 1) Credal level, where belief is contemplated 2) Pignistic level, where belief is used in taking decisions. The term 'pignistic' originates from 'pignus' having the meaning 'bet' in Latin. ‘Credal’ comes from credibility. ‘Pignistic probability’ is utilised to make decisions and is derived from BPA. It is a crisp estimation in a belief interval and is determined by:

$$bet\ (A_i) = \sum_{A_i \subset \mathrm{A_k}} \frac{m(A_k)}{|A_k|} \qquad \text{…. [6.3.17]}$$

The equation 6.3.17 is also known as Pignistic Probability Transformation (PPT).

The steps in extended DST method are the following:

Step 1: Linguistic variables assigned by DMs for weights of criteria as well as for performance of alternatives on each criterion.

Step 2: Conversion of linguistic variables into TFNs by following a said conversion scale.

Step3: Defuzzification by graded mean integration representation in equation 6.3.18 as follows:

$$P\ (A) = \frac{a+4b+c}{6}, \text{ Where, A is any TFN.} \qquad \ldots [6.3.18]$$

Step 4: Weights of criteria are transformed into discounting co-efficient.

Step 5: Distance measurement {d (IS), d (NS), d (IS, NS)} of AMTs from ideal and anti-ideal solution.

Step 6: Determination of BPA for each alternative by following the equation 6.3.19:

$$m\ (IS) = \frac{d\ (NS)}{d(IS)+d(NS)+d(IS,NS)};\ m\ (NS) = \frac{d\ (IS)}{d(IS)+d(NS)+d(IS,NS)};\ m\ (IS,NS) = \frac{d\ (IS,NS)}{d(IS)+d(NS)+d(IS,NS)} \qquad \ldots [6.3.19]$$

Step 7: Discounting the BPA of performance using the discounting co-efficient as follows in equation 6.3.20:

$$m^{\alpha}(IS) = \alpha \times m(IS);\ m^{\alpha}(NS) = \alpha \times m(NS);\ m^{\alpha}(IS, NS) = \alpha \times m(IS, NS) + (1-\alpha) \qquad \ldots [6.3.20]$$

Step 8: Combining the BPAs of all the criteria to get a compendious evaluation of an AMT by the following equation 6.3.21: $m_{DM}^{i} = \sum BPA_{Cj}^{\alpha_{Cj}}$; where, *i* = no. of AMTs, *j* = no. of criteria. … [6.3.21]

Step 9: Combining the BPA of all DMs to set the combined result $\{m^{i}(IS), m^{i}(NS), m^{i}(IS, NS)\}$.

Step 10: The assessment of each AMT based on PPT by following equation 6.3.22:

$$bet^{i}(IS) = m^{i}(IS) + m^{i}(IS, NS)/2 \qquad \ldots [6.3.22]$$

Step 11: Introduction of investment costs of AMTs and calculating the objective factor measure by following equation 6.3.8.

Step 12: Calculating the suitability index as in equation 6.3.23:

$$AMT_{i}SI = \beta\{bet(IS)\} + (1-\beta)OFM_{i}. \qquad \ldots [6.3.23]$$

Step 13: Final ranking of the AMTs according to the results of the suitability index (Higher the better).

6.3.4. A Numerical Example

The flowchart of the problem is given in figure 6.9.

A leading manufacturing firm wants to shift from traditional manufacturing technologies to advanced ones considering the leading edge of AMTs in today's manufacturing world. The tough competition amongst superior organizations has forced them to achieve eclectic decision making at any cost while looking into the alternatives. Considering the present scenario, they engage four Experts (EX) for the selection and implementation of AMT, suitable in a given manufacturing environment. The EXs are having equal weightage making the model homogeneous. The overall profile of EXs is given in table 6.3.1. They shortlisted five alternatives namely AL1, AL2, AL3, AL4 and AL5. The optimum selection will be made from amongst them. They also set five criteria namely general productivity, product flexibility, Overall equipment effectiveness, environmental hazard, manufacturing scrap dissemination. These are detailed as follows:

C1: General productivity- The average measure of the efficiency of production is to be maximized.

C2: Product flexibility- Adaptability for any change in future product design is to be maximized.

C3: Overall equipment effectiveness- Effective utilization of manufacturing operation is to be maximized.

C4: Environmental menace- A substance/event that threatens natural environment and affects life on earth deleteriously is to be minimized.

C5: Manufacturing scrap dissemination- Wastage of scrap materials produced in manufacturing is to be minimized.

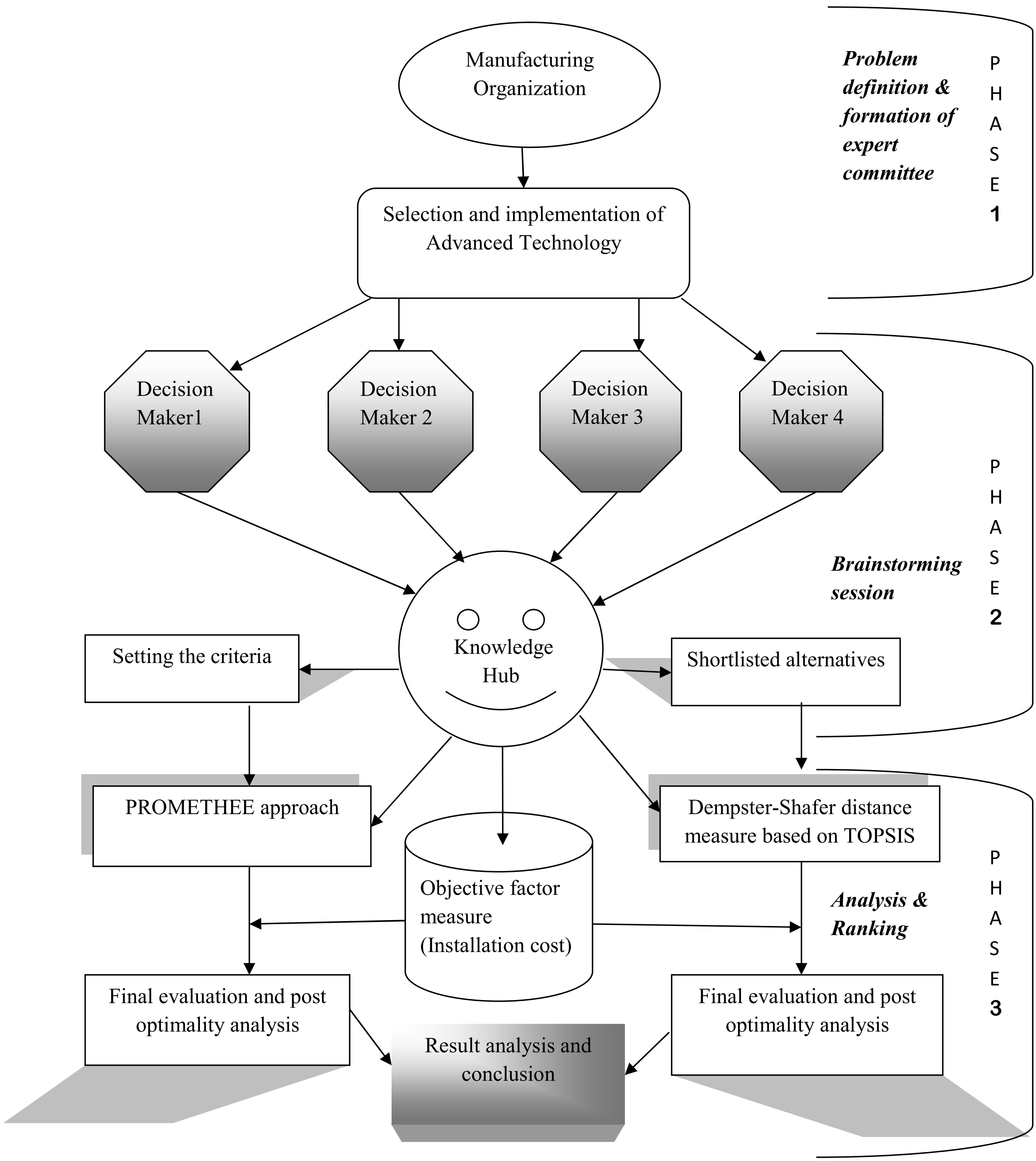


Figure 6.9. Flow chart of the case study

6.3.1: Profile of Experts (EX)

	Experts			
	EX1	EX2	EX3	EX4
				
Age	48 yrs.	57 yrs.	39 yrs.	63
Academic Degree	BE	PhD	ME	BE
Experience	23 yrs.	36 yrs.	14 yrs.	39 yrs.
Expert Skill	Design & Intelligent Manufacturing Systems	Finance & Strategic Management	Quality Control & Logistics	Advanced Manufacturing Planning & Control

The fuzziness present in the problem scenario is eliminated by using TFNs as the replacement of linguistic variables. The EXs set conversion scales as presented in table 6.3.2 and table 6.3.3 for the weights of criteria and values of AMTs.

Table 6.3.2: Linguistic weight set for criteria weights

Linguistic Variable	**TFN**
Extremely Less Important (ELI)	(0,0,0.2)
Less Important (LI)	(0,0.2,0.4)
Important (I)	(0.2,0.4,0.6)
Very Important (VI)	(0.4,0.6,0.8)
Extremely Important (EI)	(0.6,0.8,1)

Table 6.3.3: Linguistic weight set for AMTs

Linguistic Variable	**Triangular Fuzzy No.**
Extremely Heavy Importance (EHI)	(9,10,10)
Very Heavy Importance (VHI)	(7,9,10)
Heavy Importance (HI)	(5,7,9)
Medium Importance (MI)	(3,5,7)
Less Importance (LI)	(1,3,5)
Very Less Importance (VLI)	(0,1,3)
Extremely Less Importance (ELI)	(0,1,1)

The problem is homogeneous in nature as the EXs are given equal importance. Table 6.3.4 gives the criteria weights by linguistic weight set, whereas, table 6.3.5 presents the fuzzy decision matrix. The same is converted into TFNs by using suitable scales as in table 6.3.2 and table 6.3.3, and presented in table 6.3.6.

Table 6.3.4: Importance Weights of criteria by Experts (EX)

Criteria	**Decision Makers (DM)**			
	EX1	**EX2**	**EX3**	**EX4**
C1	EI	EI	EI	VI
C2	VI	EI	VI	VI
C3	I	VI	EI	VI
C4	VI	VI	EI	EI
C5	EI	VI	VI	VI

Table 6.3.5: Values of alternatives with respect to criteria given by Experts (EX)

Criteria	**AMTs**	**Decision Makers (DM)**			
		EX1	**EX2**	**EX3**	**EX4**
C1	AL1	VHI	VHI	MI	HI
	AL2	MI	VHI	VHI	HI
	AL3	VHI	HI	MI	VHI
	AL4	VHI	EHI	EHI	HI
	AL5	HI	VHI	EHI	EHI
C2	AL1	EHI	HI	VHI	HI
	AL2	VHI	VHI	VHI	HI
	AL3	EHI	VHI	EHI	EHI
	AL4	HI	HI	MI	MI
	AL5	HI	VHI	MI	HI
C3	AL1	MI	HI	HI	HI
	AL2	VHI	VHI	VHI	VHI
	AL3	VHI	EHI	VHI	VHI
	AL4	MI	MI	HI	HI
	AL5	HI	MI	MI	HI
C4	AL1	VHI	EHI	VHI	EHI
	AL2	MI	VHI	MI	MI
	AL3	VHI	VHI	VHI	VHI
	AL4	HI	MI	MI	MI
	AL5	EHI	VHI	EHI	EHI
C5	AL1	EHI	VHI	HI	HI
	AL2	VHI	VHI	MI	MI
	AL3	HI	HI	EHI	VHI
	AL4	EHI	EHI	MI	HI
	AL5	HI	MI	VHI	EHI

Table 6.3.6: Initial Decision Matrix

AMTs	Criteria				
	C1 (+ve)	C2 (+ve)	C3 (+ve)	C4 (-ve)	C5 (-ve)
AL1	(6.33,8.33,9.67)	(4.33,6.33,8.33)	(7,8.67,9.67)	(3.67,5.67,7.67)	(2.33,4.33,6.33)
AL2	(5,7,8.67)	(9,10,10)	(7.67,9.33,10)	(4.33,6.33,8.33)	(4.33,6.33,8.33)
AL3	(5,7,8.67)	(7,8.67,9.67)	(6.33,8.33,9.67)	(8.33,9.67,10)	(5.67,7.67,9.33)
AL4	(8.33,9.67,10)	(5.67,7.67,9.33)	(7.67,9.33,10)	(9,10,10)	(2.33,4.33,6.33)
AL5	(7.67,9.33,10)	(4.33,6.33,8.33)	(7,8.67,9.67)	(2.33,4.33,6.33)	(8.33,9.67,10)
Weight of Criteria (w_j)	(0.4,0.6,0.8)	(0.47,0.67,0.87)	(0.4,0.6,0.8)	(0.6,0.8,1)	(0.33,0.53,0.73)

Table 6.3.7 gives the crisp decision matrix with weights of criteria in crisp data form after defuzzification by following equation 6.3.1. The values are normalized to get unit independent values of criteria for each alternative following equation 6.3.22 and the same is represented in table 6.3.8.

Table 6.3.7: Decision Matrix (Crisp Data)

AMTs	Criteria				
	C1 (+ve)	C2 (+ve)	C3 (+ve)	C4 (-ve)	C5 (-ve)
AL1	8.220	6.330	8.560	5.670	4.330
AL2	6.945	9.830	9.165	6.330	6.330
AL3	6.945	8.560	8.220	9.500	7.600
AL4	9.500	7.600	9.165	9.830	4.330
AL5	9.165	6.330	8.560	4.330	9.500
Weight of Criteria (w_j)	0.60	0.67	0.60	0.80	0.53

Table 6.3.8: Normalized Matrix

AMTs	Criteria				
	C1 (+ve)	C2 (+ve)	C3 (+ve)	C4 (-ve)	C5 (-ve)
AL1	0.499	0	0.360	0.756	1.000
AL2	0	1.000	1.000	0.636	0.613
AL3	0	0.637	0	0.060	0.368
AL4	1.000	0.363	1.000	0	1.000
AL5	0.869	0	0.360	1.000	0
Weight of Criteria (w_j)	0.188	0.209	0.188	0.25	0.165

Table 6.3.9 and table 6.3.10 represent alternative difference values and aggregate preference indices of PROMETHEE method respectively. Primary ranking in PROMETHEE technique is devised on the basis of net outranking flow of alternatives as presented in table 6.3.11.

Table 6.3.9: Alternative Difference Values

AMTs	**Criteria**				
	C1 (+ve)	C2 (+ve)	C3 (+ve)	C4 (-ve)	C5 (-ve)
AL1-AL2	0.499	-1.000	-0.640	0.120	0.387
AL1-AL3	0.499	-0.637	0.360	0.696	0.632
AL1-AL4	-0.501	-0.363	-0.640	0.756	0
AL1-AL5	-0.370	0	0	-0.244	1.000
AL2-AL1	-0.499	1.000	0.640	-0.120	-0.387
AL2-AL3	0	0.363	1.000	0.576	0.245
AL2-AL4	-1.000	0.637	0	0.636	-0.387
AL2-AL5	-0.869	1.000	0.640	-0.364	0.613
AL3-AL1	-0.499	0.637	-0.360	-0.696	-0.632
AL3-AL2	0	-0.363	-1.000	-0.574	-0.245
AL3-AL4	-1.000	0.274	-1.000	0.060	-0.632
AL3-AL5	-0.869	0.637	-0.360	-0.940	0.368
AL4-AL1	0.501	0.363	0.640	-0.756	0
AL4-AL2	1.000	-0.637	0	-0.636	0.387
AL4-AL3	1.000	-0.274	1.000	-0.060	0.632
AL4-AL5	0.131	0.363	0.640	-1.000	1.000
AL5-AL1	0.370	0	0	0.244	-1.000
AL5-AL2	0.869	-1.000	-0.640	0.364	-0.613
AL5-AL3	0.869	-0.637	0.36	0.940	-0.368
AL5-AL4	-0.131	-0.363	-0.640	1.000	-1.000
Weight of Criteria (w_j)	0.188	0.209	0.188	0.25	0.165

Table 6.3.10: Preference Function & Aggregate Preference Indices

AMTs	**Preference Function of Criteria [P(d)]**					**Aggregate Preference Indices [π (a,b)]**
	C1 (+ve)	C2 (+ve)	C3 (+ve)	C4 (-ve)	C5 (-ve)	
AL1-AL2	1	0	0	1	1	.603
AL1-AL3	1	0	1	1	1	.791
AL1-AL4	0	0	0	1	0	.250
AL1-AL5	0	0	0	0	1	.165
AL2-AL1	0	1	1	0	0	.397
AL2-AL3	0	1	1	1	1	.812
AL2-AL4	0	1	0	1	0	.459
AL2-AL5	0	1	1	0	1	.562
AL3-AL1	0	1	0	0	0	.209
AL3-AL2	0	0	0	0	0	0
AL3-AL4	0	1	0	1	0	.459
AL3-AL5	0	1	0	0	1	.374
AL4-AL1	1	1	1	0	0	.585
AL4-AL2	1	0	0	0	1	.353
AL4-AL3	1	0	1	0	1	.541
AL4-AL5	1	1	1	0	1	.750
AL5-AL1	1	0	0	1	0	.438
AL5-AL2	1	0	0	1	0	.438
AL5-AL3	1	0	1	1	0	.626
AL5-AL4	0	0	0	1	0	.250

Table 6.3.11: Net outranking flow & initial ranking

AMTs	Outranking Flow		Net Outranking Flow [φ (a)]	Primary Ranking
	Positive [φ^{+} (a)]	Negative [φ^{-} (a)]		
AL1	1.809	1.629	0.180	3
AL2	2.230	1.394	0.836	1
AL3	1.042	2.77	-1.728	5
AL4	2.229	1.418	0.811	2
AL5	1.752	1.851	-0.099	4

Table 6.3.12 enters the territory of extended distance based DST method. It represents the defuzzified values of criteria weights and weights of alternatives by using graded mean integration representation technique.

Table 6.3.12: Graded mean integration representation for criteria weights and weights of alternatives

Criteria Weights		Weights of Alternatives	
Linguistic Variable	Graded Mean Integration (GMI)	Linguistic Variable	Graded Mean Integration (GMI)
ELI	0.03	EHI	9.8
LI	0.2	VHI	8.8
I	0.4	HI	7
VI	0.6	MI	5
EI	0.8	LI	3
		VLI	1.17
		ELI	0.17

The criteria weights, for the four EXs engaged, are given in table 6.3.13. Table 6.3.14 exhibits the decision matrix in the crisp data form with the values of discounting coefficients which are nothing but the aggregated criteria weights emerging from table 6.3.13. Distances from ideal and anti-ideal solution are measured to the likes of TOPSIS and are presented in table 6.3.15. Generating basic probability assignment (BPA) and discounting BPA of AMTs are calculated by distance function. The same are represented in table 6.3.16 and table 6.3.17 respectively. Table 6.3.18 presents the fuse multi-criteria data of AMTs using discounting coefficient of each criterion as in table 6.3.17. A primary ranking of AMTs based on the values of pignistic probability transformation is given in table 6.3.19.

Table 6.3.13: Importance weights of criteria according to Graded mean integration (crisp data)

Criteria	**Experts (EX)**			
	EX1	**EX2**	**EX3**	**EX4**
C1	.80	.80	.80	.60
C2	.60	.80	.60	.60
C3	.40	.60	.80	.60
C4	.60	.60	.80	.80
C5	.80	.60	.60	.60

Table 6.3.14: Criteria values of alternatives according to Graded mean integration (crisp data)

Experts (EX)	AMTs	Criteria				
		C1	C2	C3	C4	C5
EX1	AL1	8.8	9.8	5.0	8.8	9.8
	AL2	5.0	8.8	8.8	5.0	8.8
	AL3	8.8	9.8	8.8	8.8	7.0
	AL4	8.8	7.0	5.0	7.0	9.8
	AL5	7.0	7.0	7.0	9.8	7.0
	Discounting Co-efficient (α)	1.0	0.752	0.50	0.752	1.0
EX2	AL1	8.8	7.0	7.0	9.8	8.8
	AL2	8.8	8.8	8.8	8.8	8.8
	AL3	7.0	8.8	9.8	8.8	7.0
	AL4	9.8	7.0	5.0	5.0	9.8
	AL5	8.8	8.8	5.0	8.8	5.0
	Discounting Co-efficient (α)	1.0	1.0	0.749	0.749	0.749
EX3	AL1	7.0	8.8	7.0	8.8	7.0
	AL2	8.8	8.8	8.8	5.0	5.0
	AL3	7.0	9.8	8.8	8.8	9.8
	AL4	9.8	7.0	7.0	5.0	5.0
	AL5	9.8	7.0	5.0	9.8	8.8
	Discounting Co-efficient (α)	1.0	0.76	1.0	1.0	0.76
EX4	AL1	7.0	7.0	7.0	9.8	7.0
	AL2	7.0	7.0	8.8	5.0	5.0
	AL3	8.8	9.8	8.8	8.8	8.8
	AL4	7.0	5.0	7.0	5.0	7.0
	AL5	9.8	7.0	7.0	9.8	9.8
	Discounting Co-efficient (α)	0.75	0.75	0.75	1.0	0.75

Table 6.3.15: Distance measures from ideal and anti-ideal solution

AMTs	**Criteria**				
	C1 (+ve) d{(IS),(NS),(IS,NS)}	C2 (+ve) d{(IS),(NS),(IS,NS)}	C3 (+ve) d{(IS),(NS),(IS,NS)}	C4 (-ve) d{(IS),(NS),(IS,NS)}	C5 (-ve) d{(IS),(NS),(IS,NS)}
EX1					
AL1	(0,3.8,1.9)	(0,2.8,1.4)	(3.8,0,1.9)	(3.8,1.0,1.4)	(2.8,0,1.4)
AL2	(3.8,0,1.9)	(1.0,1.8,0.4)	(0,3.8,1.9)	(0,4.8,2.4)	(1.8,1.0,0.4)
AL3	(0,3.8,1.9)	(0,2.8,1.4)	(0,3.8,1.9)	(3.8,1.0,1.4)	(0,2.8,1.4)
AL4	(0,3.8,1.9)	(2.8,0,1.4)	(3.8,0,1.9)	(2.0,2.8,0.4)	(2.8,0,1.4)
AL5	(1.8,2.0,0.1)	(2.8,0,1.4)	(1.8,2.0,0.1)	(4.8,0,2.4)	(0,2.8,1.4)
EX2					
AL1	(1.0,1.8,0.4)	(1.8,0,0.9)	(2.8,2.0,0.4)	(4.8,0,2.4)	(3.8,1.0,1.4)
AL2	(1.0,1.8,0.4)	(0,1.8,0.9)	(1.0,3.8,1.4)	(3.8,1.0,1.4)	(3.8,1.0,1.4)
AL3	(2.8,0,1.4)	(0,1.8,0.9)	(0,4.8,2.4)	(3.8,1.0,1.4)	(2.0,2.8,0.4)
AL4	(0,2.8,1.4)	(1.8,0,0.9)	(4.8,0,2.4)	(0,4.8,2.4)	(4.8,0,2.4)
AL5	(1.0,1.8,0.4)	(0,1.8,0.9)	(4.8,0,2.4)	(3.8,1.0,1.4)	(0,4.8,2.4)
EX3					
AL1	(2.8,0,1.4)	(1.0,1.8,0.4)	(1.8,2.0,0.1)	(3.8,1.0,1.4)	(2.0,2.8,0.4)
AL2	(1.0,1.8,0.4)	(1.0,1.8,0.4)	(0,3.8,1.9)	(0,4.8,2.4)	(0,4.8,2.4)
AL3	(2.8,0,1.4)	(0,2.8,1.4)	(0,3.8,1.9)	(3.8,1.0,1.4)	(4.8,0,2.4)
AL4	(0,2.8,1.4)	(2.8,0,1.4)	(1.8,2.0,0.1)	(0,4.8,2.4)	(0,4.8,2.4)
AL5	(0,2.8,1.4)	(2.8,0,1.4)	(3.8,0,1.9)	(4.8,0,2.4)	(3.8,1.0,1.4)
EX4					
AL1	(2.8,0,1.4)	(2.8,2.0,0.4)	(1.8,0,0.9)	(4.8,0,2.4)	(2.0,2.8,0.4)
AL2	(2.8,0,1.4)	(2.8,2.0,0.4)	(0,1.8,0.9)	(0,4.8,2.4)	(0,4.8,2.4)
AL3	(1.0,1.8,0.4)	(0,4.8,2.4)	(0,1.8,0.9)	(3.8,1.0,1.4)	(3.8,1.0,1.4)
AL4	(2.8,0,1.4)	(4.8,0,2.4)	(1.8,0,0.9)	(0,4.8,2.4)	(2.0,2.8,0.4)
AL5	(0,2.8,1.4)	(2.8,2.0,0.4)	(1.8,0,0.9)	(4.8,0,2.4)	(4.8,0,2.4)

Table 6.3.16: BPA according to distance function

AMTs	Criteria				
	C1 (+ve) m{(IS),(NS),(IS,NS)}	**C2 (+ve) m{(IS),(NS),(IS,NS)}**	**C3 (+ve) m{(IS),(NS),(IS,NS)}**	**C4 (-ve) m{(IS),(NS),(IS,NS)}**	**C5 (-ve) m{(IS),(NS),(IS,NS)}**
EX1					
AL1	(0.67,0,0.33)	(0.67,0,0.33)	(0,0.67,0.33)	(0.16,0.6,0.23)	(0,0.67,0.33)
AL2	(0,0.67,0.33)	(0.56,0.3,0.125)	(0.67,0,0.33)	(0.67,0,0.33)	(0.3,0.56,0.125)
AL3	(0.67,0,0.33)	(0.67,0,0.33)	(0.67,0,0.33)	(0.16,0.6,0.23)	(0.67,0,0.33)
AL4	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0.54,0.38,0.08)	(0,0.67,0.33)
AL5	(0.5,0.46,0.03)	(0,0.67,0.33)	(0.5,0.46,0.03)	(0,0.67,0.33)	(0.67,0,0.33)
EX2					
AL1	(0.56,0.3,0.125)	(0,0.67,0.33)	(0.38,0.54,0.08)	(0,0.67,0.33)	(0.16,0.6,0.23)
AL2	(0.56,0.3,0.125)	(0.67,0,0.33)	(0.6,0.16,0.23)	(0.16,0.6,0.23)	(0.16,0.6,0.23)
AL3	(0,0.67,0.33)	(0.67,0,0.33)	(0.67,0,0.33)	(0.16,0.6,0.23)	(0.54,0.38,0.08)
AL4	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0.67,0,0.33)	(0,0.67,0.33)
AL5	(0.56,0.3,0.125)	(0.67,0,0.33)	(0,0.67,0.33)	(0.16,0.6,0.23)	(0.67,0,0.33)
EX3					
AL1	(0,0.67,0.33)	(0.56,0.3,0.125)	(0.5,0.46,0.03)	(0.16,0.6,0.23)	(0.54,0.38,0.08)
AL2	(0.56,0.3,0.125)	(0.56,0.3,0.125)	(0.67,0,0.33)	(0.67,0,0.33)	(0.67,0,0.33)
AL3	(0,0.67,0.33)	(0.67,0,0.33)	(0.67,0,0.33)	(0.16,0.6,0.23)	(0,0.67,0.33)
AL4	(0.67,0,0.33)	(0,0.67,0.33)	(0.5,0.46,0.03)	(0.67,0,0.33)	(0.67,0,0.33)
AL5	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0.16,0.6,0.23)
EX4					
AL1	(0,0.67,0.33)	(0.38,0.54,0.08)	(0,0.67,0.33)	(0,0.67,0.33)	(0.54,0.38,0.08)
AL2	(0,0.67,0.33)	(0.38,0.54,0.08)	(0.67,0,0.33)	(0.67,0,0.33)	(0.67,0,0.33)
AL3	(0.56,0.3,0.125)	(0.67,0,0.33)	(0.67,0,0.33)	(0.16,0.6,0.23)	(0.16,0.6,0.23)
AL4	(0,0.67,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0.67,0,0.33)	(0.54,0.38,0.08)
AL5	(0.67,0,0.33)	(0.38,0.54,0.08)	(0,0.67,0.33)	(0,0.67,0.33)	(0,0.67,0.33)

Table 6.3.17: Assessment by discounted BPA of AMTs

AMTs	Criteria				
	C1 (+ve) m^{α}\{(IS),(NS),(IS,NS)\}	**C2 (+ve) m^{α}\{(IS),(NS),(IS,NS)\}**	**C3 (+ve) m^{α}\{(IS),(NS),(IS,NS)\}**	**C4 (-ve) m^{α}\{(IS),(NS),(IS,NS)\}**	**C5 (-ve) m^{α}\{(IS),(NS),(IS,NS)\}**
EX1					
AL1	(0.67,0,0.33)	(0.504,0,0.496)	(0,0.335,0.665)	(0.12,0.45,0.43)	(0,0.67,0.33)
AL2	(0,0.67,0.33)	(0.42,0.226,0.342)	(0.335,0,0.665)	(0.504,0,0.496)	(0.3,0.56,0.125)
AL3	(0.67,0,0.33)	(0.504,0,0.496)	(0.335,0,0.665)	(0.12,0.45,0.43)	(0.67,0,0.33)
AL4	(0.67,0,0.33)	(0,0.504,0.496)	(0,0.335,0.665)	(0.41,0.286,0.308)	(0,0.67,0.33)
AL5	(0.5,0.46,0.03)	(0,0.504,0.496)	(0.25,0.23,0.52)	(0,0.504,0.496)	(0.67,0,0.33)
EX2					
AL1	(0.56,0.3,0.125)	(0,0.67,0.33)	(0.285,0.404,0.311)	(0,0.5,0.498)	(0.12,0.45,0.43)
AL2	(0.56,0.3,0.125)	(0.67,0,0.33)	(0.45,0.12,0.43)	(0.12,0.45,0.43)	(0.12,0.45,0.43)
AL3	(0,0.67,0.33)	(0.67,0,0.33)	(0.5,0,0.498)	(0.12,0.45,0.43)	(0.404,0.285,0.31)
AL4	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.5,0.498)	(0.5,0,0.498)	(0,0.5,0.498)
AL5	(0.56,0.3,0.125)	(0.67,0,0.33)	(0,0.5,0.498)	(0.12,0.45,0.43)	(0.5,0,0.498)
EX3					
AL1	(0,0.67,0.33)	(0.43,0.23,0.335)	(0.5,0.46,0.03)	(0.16,0.6,0.23)	(0.41,0.29,0.3)
AL2	(0.56,0.3,0.125)	(0.43,0.23,0.335)	(0.67,0,0.33)	(0.67,0,0.33)	(0.51,0,0.49)
AL3	(0,0.67,0.33)	(0.51,0,0.49)	(0.67,0,0.33)	(0.16,0.6,0.23)	(0,0.51,0.49)
AL4	(0.67,0,0.33)	(0,0.51,0.49)	(0.5,0.46,0.03)	(0.67,0,0.33)	(0.51,0,0.49)
AL5	(0.67,0,0.33)	(0,0.51,0.49)	(0,0.67,0.33)	(0,0.67,0.33)	(0.122,0.46,0.415)
EX4					
AL1	(0,0.503,0.498)	(0.285,0.41,0.31)	(0,0.503,0.498)	(0,0.67,0.33)	(0.41,0.285,0.31)
AL2	(0,0.503,0.498)	(0.285,0.41,0.31)	(0.503,0,0.498)	(0.67,0,0.33)	(0.503,0,0.498)
AL3	(0.42,0.23,0.344)	(0.503,0,0.498)	(0.503,0,0.498)	(0.16,0.6,0.23)	(0.12,0.45,0.423)
AL4	(0,0.503,0.498)	(0,0.503,0.498)	(0,0.503,0.498)	(0.67,0,0.33)	(0.41,0.285,0.31)
AL5	(0.503,0,0.498)	(0.285,0.41,0.31)	(0,0.503,0.498)	(0,0.67,0.33)	(0,0.503,0.498)

Table 6.3.18: Fuse multi-criteria data using discounting co-efficient

{(IS),(NS),(IS,NS)}	AMTs	Experts (EX)			
		EX1 (1.0)	**EX2** (1.0)	**EX3** (1.0)	**EX3** (1.0)
	AL1	(0.8564,0.101,0.043)	(0.723,0.12,0.157)	(0.8586,0.122,0.0194)	(0.578,0.1672,0.2548)
	AL2	(0.8026,0.085,0.1124)	(0.8524,0.078,0.0696)	(0.6693,0.069,0.2617)	(0.94,0,0.06)
	AL3	(0.7416,0,0.2584)	(0.91,0.086,0.004)	(0.8,0.2,0)	(0.898,0.0621,0.0399)
	AL4	(0.8053,0.1932,0)	(0.835,0.165,0)	(0.9705,0,0.0295)	(0.8053,0.036,0.1587)
	AL5	(0.7925,0.2016,0.006)	(0.93,0.0675,0.00285)	(0.71,0.105,0.185)	(0.6446,0.0695,0.286)

Table 6.3.19: Pignistic Probability Transformation (bet/PPT) and initial ranking

AMTs	Combined Result {(IS),(NS),(IS,NS)}	bet (IS)	Primary Result
AL1	(0.995,0.000247,0.0047)	0.9973	2
AL2	(0.997,0.00045,0.00255)	0.9983	1
AL3	(0.345,0.3141,0.341)	0.5155	5
AL4	(0.868,0.00134,0.131)	0.9335	3
AL5	(0.571,0.387,0.042)	0.5920	4

The EXs complete a market survey for the investment costs of AMTs. They apply their knowledge and survey results to set investment cost of AMTs as objective factor in the problem. The same is showcased in table 6.3.20. Table 6.3.21 gives the measures of mental attitude of EXs. Index of cognitive mind is a measure of how optimistic or pessimistic is a person's thoughts. Higher value of this indicates an optimistic mind whereas; lower values indicate a pessimistic one. The final rankings for both the methods are formulated by taking into account the subjective factors as well as the objective ones coming into the problem. The comparison result is presented in table 6.3.22.

Table 6.3.20: AMT investment cost (IC_i) [from market survey by experts]

AMTs	**IC_i (millions of $)**	**OFM_i**
AL1	4.20	0.215
AL2	5.10	0.177
AL3	4.10	0.220
AL4	4.30	0.210
AL5	4.90	0.184

Table 6.3.21: Index of cognitive mind of experts

Index of cognitive mind (β)	**Experts (EX)**			
	EX1	EX2	EX3	EX4
	0.75	0.79	0.61	0.65

Table 6.3.22: Final ranking and comparison of result

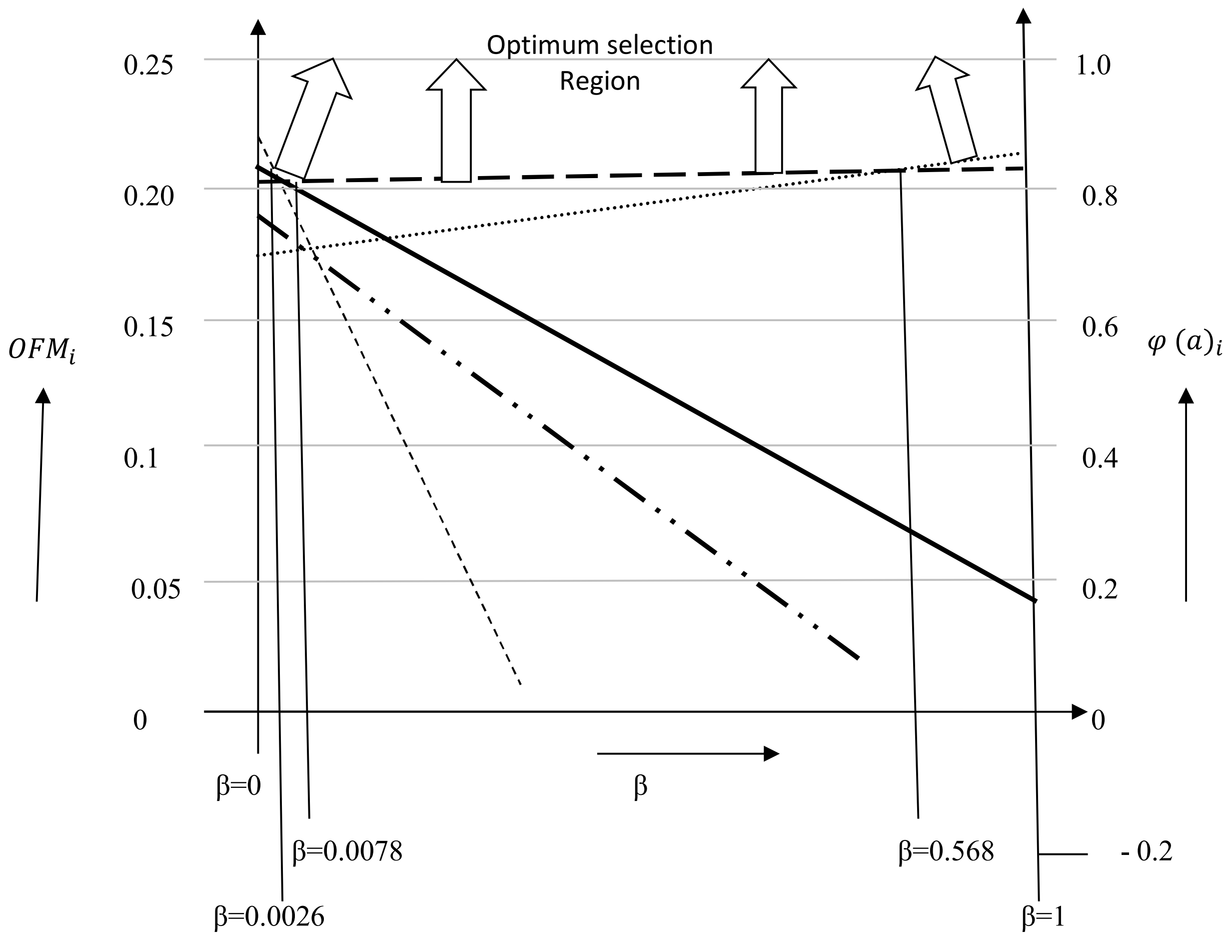
AMTs	**PROMETHEE**			**Dempster-Shafer Theory based distance model**		
	Primary Ranking	Selection Index (SI)	Final Ranking	Primary Result	Suitability Index (AMT_iSI)	Final Ranking
AL1	3	0.1905	3	2	0.7626	1
AL2	1	0.6383	1	1	0.7519	2
AL3	5	-1.1436	5	5	0.4269	5
AL4	2	0.6307	2	3	0.7165	3
AL5	4	-0.0141	4	4	0.4696	4

6.3.4.1. Post optimality analysis

The post optimality analysis is the other name of uncertainty treatment. In general, decision problems are prone to fuzziness and uncertainty. These uncertainties in the input data level are managed by linguistic weight set and TFNs. Next step is the defuzzification according to the given problem scenario. But, uncertainties associated in the design level are handled with the help of post optimality analysis. As the name suggests, it is carried out after the optimum result is found out. Basically, it tests the robustness of the model. It can also be called as sensitivity analysis. It gives a measure of how sensitive a model is with the alteration of alternatives or criteria or attitude of mind of decision makers. This chapter presents a unified approach for post optimality analysis for the two MCDM methods presented. In the given selection problem, post optimality analysis is carried out to find the feasible range of index of cognitive mind (β) of decision maker in which the optimally selected AMT is functional. Figure 6.10 and figure 6.11 pictures the of post optimality analysis on PROMETHEE and extended DST methods. The analysis of the alternatives in case of PROMETHEE reveals that AMTs AL3, AL1, AL4 and AL2 are qualified in the optimum selection region for different ranges of β whereas, the analysis on DST based distance method chooses AMTs AL3, AL1 and AL2 as the optimum selections for different ranges of β. The results of the analysis along with the ranges of β are presented in table 6.3.23.

Table 6.3.23: Optimum selection decision by post optimality analysis

PROMETHEE		**Dempster-Shafer Theory based distance model**	
Range of β	Optimum decision	Range of β	Optimum decision
$\beta \leq 0.0026$	AL3	$\beta \leq 0.010$	AL3
$0.0026 \leq \beta \leq 0.0078$	AL1	$0.010 \leq \beta \leq 0.97$	AL1
$0.0078 \leq \beta \leq 0.568$	AL4	$\beta \geq 0.97$	AL2
$\beta \geq 0.568$	AL2		



Legends

AL1; AL2; AL3; AL4; AL5

Figure 6.10: Post optimality analysis (PROMETHEE)

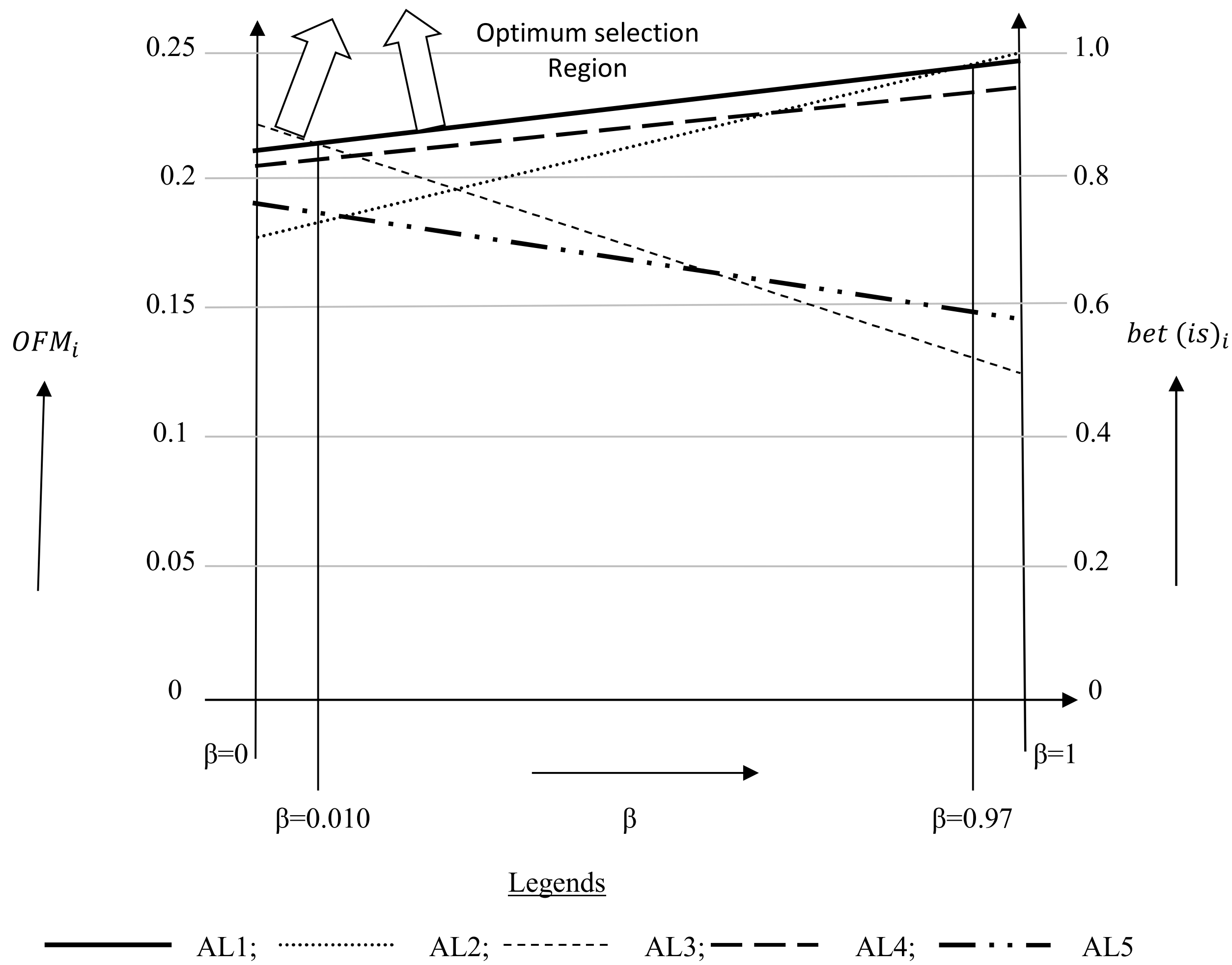


Figure 6.11: Post optimality analysis (Extended DST)

6.3.4.2. Result analysis

The experimental results of PROMETHEE reveals that alternative AL2 is getting the highest rank until the introduction of investment cost in the problem. Even after the impact of OFM, AL2 comes out as a clear winner although the same is carrying the tag of investment cost. The results of post optimality analysis also confirm the optimum selection of AMT in the operational range of β, i.e. AL2 emerges as the optimum selection decision for the values of β equal to 0.568 onwards until it reaches the maximum optimism value of 1. In this way, the robustness of the proposed method is hereby established.

The results of DST based distance method also publishes alternative AL2 as the highest ranked one till the introduction of cost factor. The scenario changes with the commencement of the same. That brings out the best in alternative AL1 while AL2 gets the second-best position although the difference is too nominal. The post optimality analysis also confirms the selection for almost entire range of β up to the value of 0.97. So, this method also establishes its stability and robustness in its own way. But the catch is that, investment of AL1 is much cheaper (almost a million $) than that of AL2 as surveyed by the EXs and mentioned in table 6.3.20. According to that survey, AL1 is supposed to be the optimum solution for the organization. The extended DST method successfully establishes the same. It is more mature and as good as you can get. In this face off, extended DST method is having the edge over PROMETHEE.

6.3.5. Conclusion

In this chapter, a new multi-criteria decision aid on the basis of extended DST method, is put up against another MCDM outranking method namely PROMETHEE. An AMT selection problem has been taken up to demonstrate the application of the suggested approach. The two methods discussed here are having their own strengths and weaknesses. Both the methods can simultaneously handle qualitative and quantitative data. They both can handle uncertainty very judiciously. But, the fact is that, PROMETHEE suffers from rank reversal problem with the inclusion of new alternative. It delivers a ranking of alternatives but is less suitable for implementation and evaluation of the same. Also the information processing is very complicated and hard to perceive for a non-expert. By-the-by, DST method gets incorrect results with the collection of highly conflicting evidences. The future scopes of the proposed methods include combining PROMETHEE with other MCDA methods. Another interesting future direction of this extended DST method is the addition of conflict data fusion algorithm and efficiently handling the highly conflicting information in the process of selecting decision. Besides the selection of AMT, this comparison model can also be applied to other multi-criteria selection and evaluation problems.

6.4. Case Study IV: A comparative study for evaluation of advanced technologies by fuzzy Taguchi Loss Function and fuzzy VIKOR

6.4.1. Taguchi Loss Function & Fuzzy VIKOR Methodology Analysis

6.4.2. Taguchi loss functions

The concept of Taguchi loss function is somewhat different from the traditional loss function popularly known as goal post view (Figure 6.12).

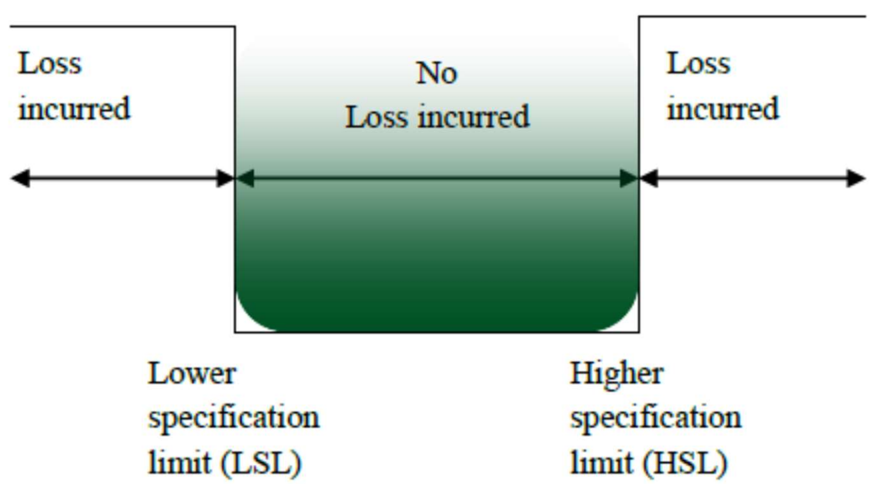


Figure 6.12: Goalpost view of traditional loss function

The goal posts are the specification limits i.e. the upper and the lower, in traditional loss function. If the product feature falls within the limit of the designed specifications, it is taken to be of acceptable quality no matter what the deviation is from the centre. On the contrary, the same gets rejected if it doesn't meet the designed specifications. "Taguchi suggested a restricted and more focused perspective of characteristic acceptability and indicated that any departure from a preset target value resulted in a loss" [Pi & Low (2005)]. According to him, Quality can only be defined in terms of the amount of financial loss incurred to the society. It can be shown graphically that an increase in variation in the specified quality limits, can lead to exponential decrease in customer satisfaction. A characteristic measurement equal to the target value incurs zero loss. The loss, otherwise, is measured by quadratic functions and measures ought to be taken to minimize the divergence from the targeted zone. The formulation of Taguchi identifies the losses incurred even before a product is shipped.

Three types of loss functions (Ross, 1996) could be assimilated depending on the variation of product characteristic. The first one i.e. nominal is better approach, fixes the target region, either at the centre (two-sided equal specification loss function) (figure 6.13) or allows for nominal shift in both directions from the centre (two-sided with specification preference loss function) (figure 6.14).

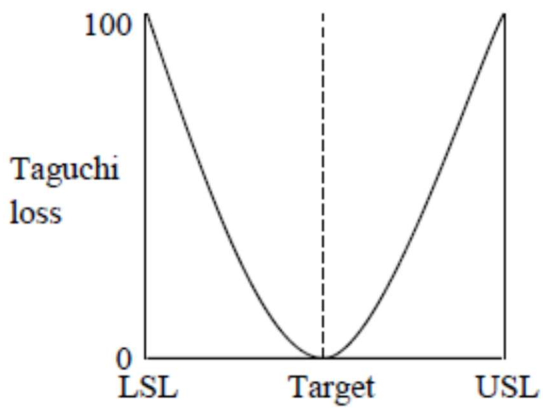


Figure 6.13: Two-sided equal specification loss function- target at the centre

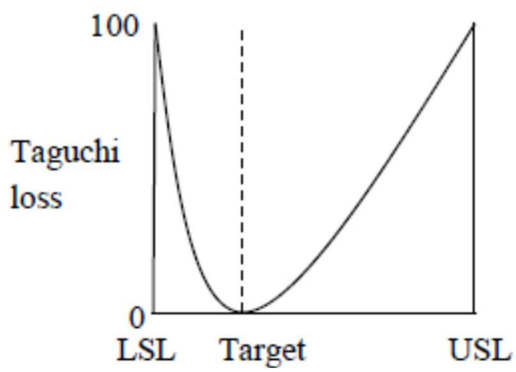


Figure 6.14: Two-sided with specification preference loss function- nominal target shift from the centre

The loss is depicted by a parabolic loss function as in equation 6.4.1 as follows:

$$f_l = l(k - y)^2, \qquad \text{......... [6.4.1]}$$

Where, f_l corresponds to the loss incurred, l is the loss coefficient, k is product size and y be the nominal value of the specification.

If $(k - y)$ is large, loss is more, no matter what the tolerance limits are. The above loss function (equation 6.4.1) holds good for a single product. But for multiple products, there is slight variation in the loss function and is given by equation 6.4.2:

$$f_l = l\,[V^2 + (\bar{k} - y)^2] \qquad \text{……… [6.4.2]}$$

Where, V^2 represents product size variance and $\bar{k}$ be mean product size, other variables remain same as in equation 6.4.1.

The second one i.e. smaller is better approach (figure 6.15), corresponds to one-sided LSL and the third one i.e. higher is better approach (figure 6.16), corresponds to one-sided USL.

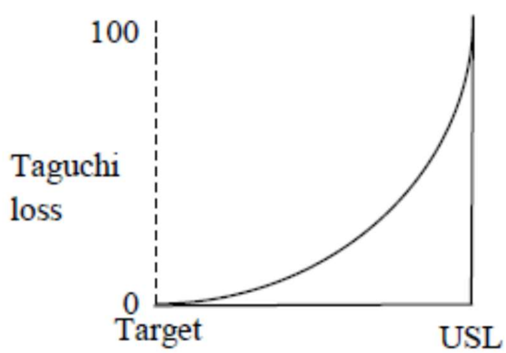


Figure 6.15: Smaller the better approach

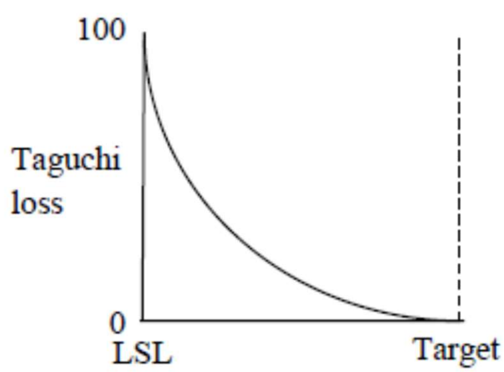


Figure 6.16: Higher the better approach

They confront to equation 6.4.3 and equation 6.4.4 respectively as follows:

$$f_l = l(k)^2 \qquad \text{……… [6.4.3]}$$

$$f_l = {}^{l}/_{k^2} \qquad \text{………. [6.4.4]}$$

Taguchi loss function can be used for non-manufacturing applications as well. It can be used for selection of supplier, solving non-linear optimization problem, multivariate multi-response problem etc.

6.4.3. The proposed methodology

A unified framework for the model is depicted in figure 6.17. The mathematical stages are as follows:

Stage 1. Formation of decision council having k_n number of Decision Experts (DEx).

Stage 2. Ascertainment of the alternatives and criteria according to the organization's need.

Stage 3. Construction of Decision Matrix by defining the criteria values as linguistic values and subsequently, converting to TFNs. Conversion of aggregated criteria weights into TFNs by a scale bearing values between 0 and 1.

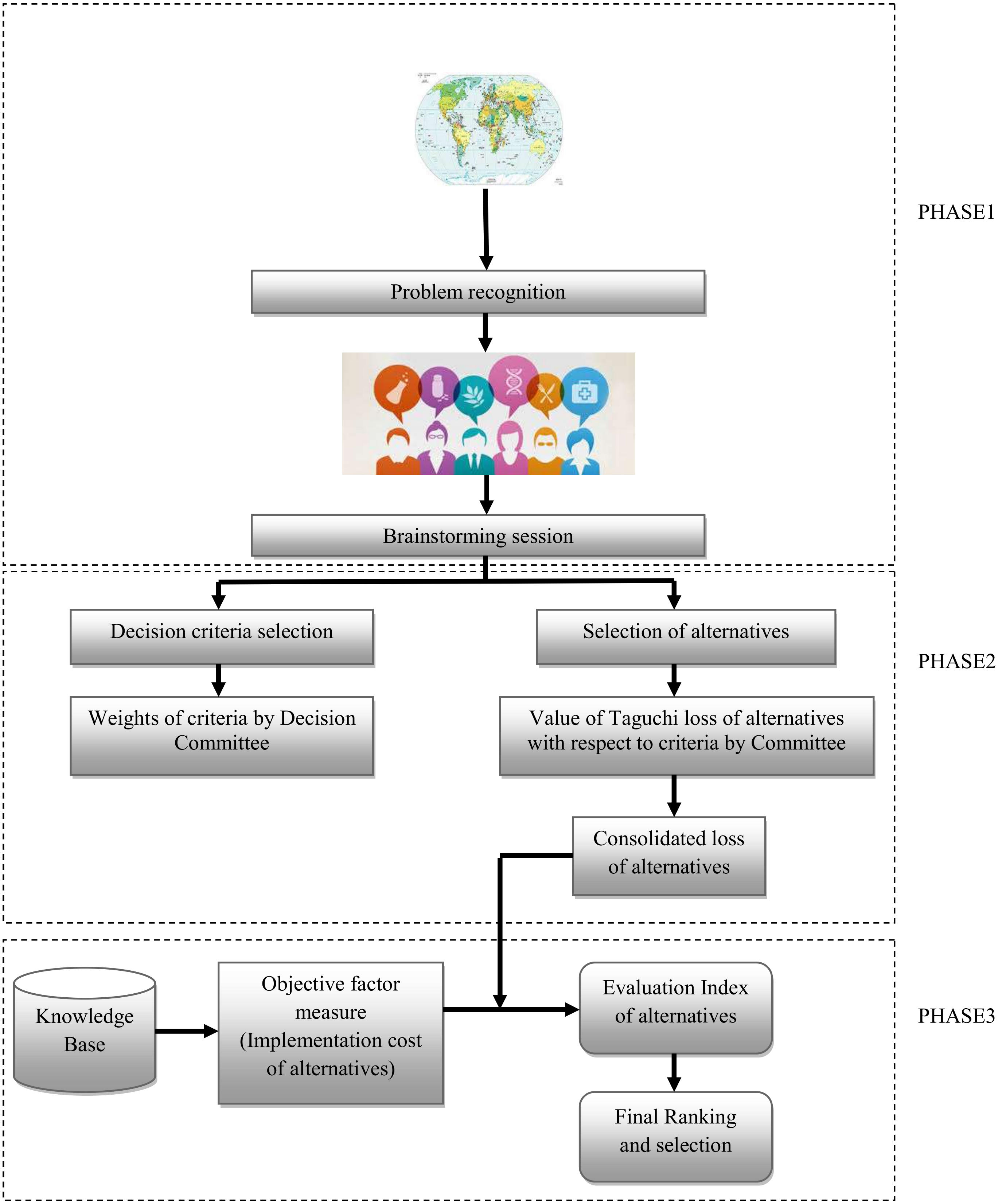


Figure 6.17: A unified decision support framework for the proposed methodology

Stage 4. Measure of criteria values of alternatives and the importance weights of criteria by simple arithmetic mean as in equation 6.4.5 and equation 6.4.6 respectively:

$\emptyset_{ij}=[\emptyset_{ij}{}^{1}+\emptyset_{ij}{}^{2}+\cdots+\emptyset_{ij}{}^{k_n}]/k_n,$ …..... [6.4.5]

Where, $\mathrm{i} = \text{no. of alternatives};\ \mathrm{j} = \text{no. of criteria}, k_n = \text{no. of members.}$

$\breve{\omega}_j = [\breve{\omega}_j{}^{1}+\breve{\omega}_j{}^{2}+\cdots+\breve{\omega}_j{}^{k}]/k_n, \breve{\omega}_j = (\breve{\omega}_{aj,}\breve{\omega}_{bj}, \breve{\omega}_{cj})$ is the TFN for weight vector ……. [6.4.6]

Stage 5. Normalized TFNs for weight vectors are calculated and converted into crisp values as in equation 6.4.7 and equation 6.4.8:

$\omega_j = \breve{\omega}_j/\sum\breve{\omega}_{j\,,}\ \omega_j = (\omega_{aj}, \omega_{bj}, \omega_{cj})$ is the normalised TFN for weight vector ……. [6.4.7]

$w_j = (\omega_{aj}+\omega_{bj}+\omega_{cj})/3$, w_j= crisp value of weight vector …… [6.4.8]

Stage 6. Decision matrix normalization; the corresponding TFNs are represented by,

$\emptyset_{ij}{}^{N} = (a_{ij}{}^{N}, b_{ij}{}^{N}, c_{ij}{}^{N})$. The generating equations for beneficial and non-beneficial criteria are presented in equations 6.4.9, 6.4.10 respectively:

$\emptyset_{ij}{}^{N} = (a_{ij}{}^{N}, b_{ij}{}^{N}, c_{ij}{}^{N}) = \left({}^{a_{ij}}/_{c_j{}^*}, {}^{b_{ij}}/_{c_j{}^*}, {}^{c_{ij}}/_{c_j{}^*}\right);\ where\ c_j{}^* = \max c_{ij}$ ……. [6.4.9]

$\emptyset_{ij}{}^{N} = (a_{ij}{}^{N}, b_{ij}{}^{N}, c_{ij}{}^{N}) = \left({}^{a_j{}^-}/_{c_{ij}}, {}^{a_j{}^-}/_{b_{ij}}, {}^{a_j{}^-}/_{a_{ij}}\right);\ where\ a_j{}^- = \min a_{ij}$ …….. [6.4.10]

Stage 7. Combining the fuzzy normalized values with the loss function. Calculating Taguchi loss value (L_{ij}) of the alternatives in fuzzy form, thereby not losing any information that was contained in the problem at the beginning. Specification limits of decision criteria and fuzzy values of alternatives are integrated to achieve the same.

Stage 8. Calculating the fuzzy weighted Taguchi loss value (WL_i) for each alternative. The generating equation is provided in equation 6.4.11 as follows:

$WL_{\mathrm{i}} = \sum_{j=1}^{n}(L_{ij} * w_j),\ \ WL_i = (WL_{ai}, WL_{bi}, WL_{ci})$ is the TFN for weighted Taguchi loss. …. [6.4.11]

Stage 9. Defuzzification (Zimmermann,1991) of weighted Taguchi loss value as in equation 6.4.12 and ranking of alternatives based on the same. The lower the value, the higher the ranking.

Crisp $WL_i = (WL_{ai} + 4 * WL_{bi} + WL_{ci})/6$ …… [6.4.12]

Stage 10. The assistance of knowledge base and market survey is extracted by the DAs for the Installation Costs (R_i) of the AMTs. The same is introduced as the objective factor in the mathematics. Calculation of the objective factor measure (OFM_i) is carried out by equation 6.4.13 stated below:

$$OFM_i = [R_i \times \sum \frac{1}{R_i}]^{-1} \qquad \text{……… [6.4.13]}$$

Stage 11. Calculating the Evaluation Index (EI_i) for each alternative as in equation 6.4.14 as proposed by Bhattacharya et. al. Subsequent ranking of the AMTs based on the same. Higher values of EI_i produce better ranking of alternatives.

$$EI_i = (\text{ɣ} * \text{SFM}_\text{i}) + (1 - \text{ɣ})(\text{OFM}_\text{i}); \text{SFM}_\text{i} = \text{subjective factor measure} = {WL_i}^{-1} \qquad \text{………. [6.4.14]}$$

Where, ɣ = co-efficient of cognition; $\text{SFM}_\text{i} = \text{Subjective factor measure}$ for the AMTs.

These above-mentioned steps are utilized by the DAs of three individual decision committees separately to find out three different solutions to the given problem.

6.4.4. A Mathematical Problem

The section establishes a real-life case study through a mathematical problem.

6.4.4.1. Experimental setting

The present chapter represents a formulation of performance assessment problem of five pre-selected AMTs. For the purpose of assessment, three decision committees namely K1, K2 and K3 are formed. They involve certain number of DAs in the decision committees. All of them are from different virtuosity having considerable experience and expertise to deal with any problem scenario. The overall spectrum of the DAs included in the decision committees is presented in table 6.4.1. The DAs of each decision committee are assigned with the responsibility of the whole assessment process. They go through a number of brainstorming sessions for several hours to get the desired outcome. They construct the decision matrix by defining the criteria values as linguistic values and subsequently, converting to TFNs.

Table 6.4.1: Spectrum of Decision Associates (DAs)

Decision Committee	Decision Associate (DA)		Age	Academic Qualification	Working Experience	Expertise
K1	DA1		39 yrs.	BE	18 yrs.	Advanced Manufacturing Planning & Control
	DA2		55 yrs.	PhD	28 yrs.	Supply Chain Management
	DA3		63 yrs.	MBA	38 yrs.	Marketing Management
	DA4		45 yrs.	ME	20 yrs.	Artificial Intelligence
K2	DA1		58 yrs.	PhD	26 yrs.	Sustainability Engineering & Science
	DA2		35 yrs.	BE	13 yrs.	Enterprise Resource Planning
	DA3		49 yrs.	MBA	23 yrs.	Operation Management
K3	DA1		57 yrs.	PhD	25 yrs.	Finance & Strategic Management
	DA2		62 yrs.	MBA	36 yrs.	Human Resource
	DA3		47 yrs.	ME	22 yrs.	Productivity & Quality Management
	DA4		38 yrs.	BE	16 yrs.	Decision Theory

A conversion scale in the range of 0 to 10 is used here as given in table 6.4.2.

Table 6.4.2: Linguistic weight set for values of alternatives

Linguistic Variable	Triangular Fuzzy Number
Very High Significance (VHS)	(8,10,10)
High Significance (HS)	(6,8,10)
Medium Significance (MS)	(4,6,8)
Low Significance (LS)	(2,4,6)
Very Low Significance (VLS)	(0,2,4)

Conversion of aggregated criteria weights into TFNs is done by a scale bearing values between 0 and 1 as given in table 6.4.3.

They initially choose criteria like quality loss, delay in order delivery, operational flexibility and environ safety. At a later stage, they incorporate the installation costs of the AMTs in the form of objective factor measure (OFM) and make ranking as per the values of Evaluation Indices (EIs).

Table 6.4.3: Linguistic values for weights of criteria

Linguistic Variable	Triangular Fuzzy Number
Extremely Highly Important (EHI)	(0.8,0.9,1)
Very Highly Importance (VHI)	(0.7,0.8,0.9)
Highly Importance (HI)	(0.5,0.6,0.7)
Less Important (LI)	(0.3,0.4,0.5)
Very Less Important (VLI)	(0.1,0.2,0.3)

Three different ranking sequences are obtained for the three decision committees. Post optimality treatment of the parameters is also accomplished for the decision committees by instituting a new factor i.e. co-efficient of cognition, ɣ. The value of the same has to be set between 0 and 1. The cognitive minds of the DAs play a pivotal role in taking comprehensive decision. Explicit knowledge of all the DAs is known and certain. But what's about their tacit knowledge? Submerged iceberg could represent the tacit knowledge that includes attitude, emotion, commitment, empathy of individual DA. These are the things we can't judge from outside. That's why, variation is found in decision making amongst the DAs. An optimistic DA sets a high value of ɣ. On the other hand, the value of the same is less for a pessimistic approach. The cognitive mind of the DA is like that iceberg comprising of explicit and tacit knowledge as shown in figure 6.18.

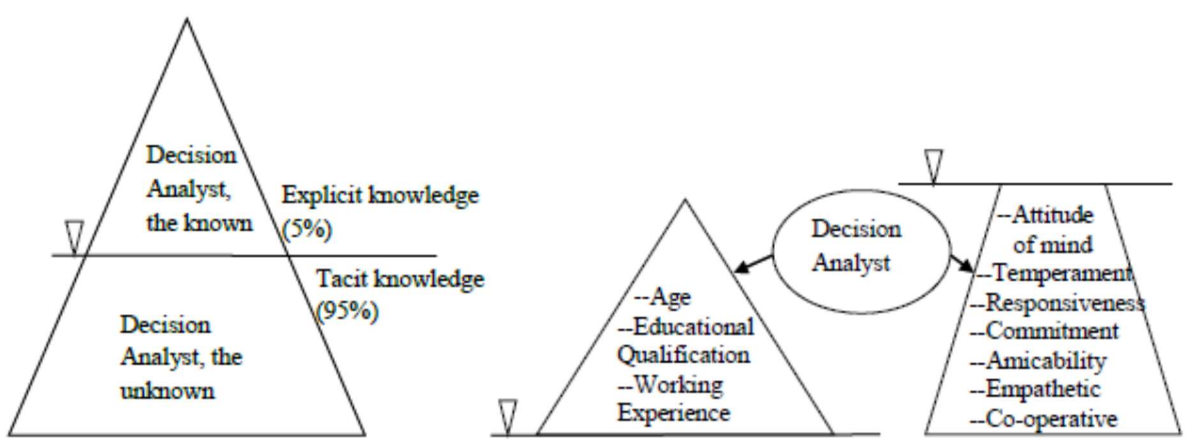


Figure 6.18: Iceberg- Cognitive mind of Decision Associate representing explicit and tacit knowledge

The higher officials of a manufacturing firm would like to implement AMT as it has the leading edge in manufacturing environment worldwide at this day and age. They are forced to efficiently customize their products in a cost-effective manner while keeping the customer satisfaction intact. A successful implementation of AMT offers great productivity, flexibility and profitability. But, on the other hand, implementation of AMT leads to replacing a good amount of manual labor with automated systems requiring large capital investment. It could be a nightmare for the firm if the project goes wrong. So, considering the scenario, the officials formed three decision making committees. The three decision committees involved four, three and four DAs respectively having different profiles and varied fields of expertise. They have given the responsibility to come to a solution individually although the chosen decision criteria and alternatives are same for all of them. There are five alternatives namely AMT1, AMT2, AMT3, AMT4, AMT5, amongst which the optimum selection should be implemented in the firm. They DAs choose four selection criteria which are the most important ones in the given scenario. Two of them are non-beneficial criteria, supposed to be minimized, namely product quality loss (CR1) and delay in order delivery(CR3). The other two are beneficial ones, ought to be maximized, namely operational flexibility (CR2) and environ safety (CR4). The range, target value and specification limits of the decision criteria are given in table 6.4.4 based on past literature by Pi and Low (2005).

Table 6.4.4: Decision criteria for selected alternatives

Decision Criteria	**Range**	**Target Value**	**Specification Limit**	**Nature**
Quality loss (CR1)	0% to 2.5%	0%	2.5%	Lower the better
Delay in order delivery (CR2)	0-4 working days	No time delay	4 working days	Lower the better
Operational Flexibility (CR3)	100% to 65%	100%	65%	Higher the better
Environ safety (CR4)	100% to 80%	100%	80%	Higher the better

From quality perspective, the DAs could set the values according to the convenience and requirement of the firm. They set the percentage target loss at zero and USL could be set at 2.5%. On time delivery or no delay is one of the most important aspects of AMT implementation. The firm could incur huge loss if there is delay in order delivery. So, the specification limit is set to a maximum of four working days delay i.e. four working days delay will incur 100% loss. For flexibility, the loss will be zero if flexibility is 100%. The specification limit is set to 65% i.e. loss will be 100% if the flexibility goes down to 65%. The fourth criteria, environ safety, gets a lot of attention in changing global environment. The outcomes of the AMTs have to be environment friendly and could save natural environment of surroundings without creating any health hazard to the people. The specification limit, in this case, is set to 80%, at which the loss will be 100%.

6.4.4.2. Operational steps

The weights for the decision criteria and values of alternatives are given by DEx(s) of three councils distinctively, as in table 6.4.5 and table 6.4.6.

Table 6.4.5: Criteria weights by DAs in linguistic values

	Decision Council K1				**Decision Council K2**			**Decision Council K3**			
Decision Criteria	DEx1	DEx2	DEx3	DEx4	DEx1	DEx2	DEx3	DEx1	DEx2	DEx3	DEx4
CR1	EHI	VHI	VHI	EHI	VHI	EHI	HI	VHI	VHI	EHI	HI
CR2	VHI	VHI	HI	VHI	VHI	HI	HI	HI	VHI	VHI	EHI
CR3	HI	HI	VHI	VHI	VHI	HI	VHI	HI	HI	VHI	HI
CR4	EHI	EHI	EHI	VHI	VHI	VHI	VHI	EHI	EHI	VHI	EHI

The uncertainty and fuzziness associated with the subjective factors can be coped with the introduction of linguistic variables. Thus, the matrix is formed with the values. This is, then, converted into TFNs, as in table 6.4.7 and normalized to get unit free values, as in table 6.4.8. Weights of decision criteria are converted into crisp numerical values corresponding to the DAs of different committees. Taguchi loss function is then put to use according to the nature of individual criterion. The values of loss co-efficient are identified as 160000, 6.25, 42.25, 64 for the decision criteria, following equation 6.4.3 or equation 6.4.4, depending on the beneficial or non-beneficial nature. The values are integrated with decision matrix to measure fuzzy loss values of alternatives, exhibited in table 6.4.9. Weights of decision criteria are incorporated with the loss values to find out fuzzy weighted loss of alternatives given in table 6.4.10. The fuzzy attributes values till the later stage of the problem solving phase helped in keeping the much required information initially contained in the problem. The values are lastly defuzzified to get a comprehensive evaluation of choices of alternatives. This is presented in table 6.4.11. The lower the weighted loss value, the higher the ranking of the alternative.

Table 6.4.6: Linguistic values of alternatives by DAs in correspondence with decision criteria

Decision Criteria	Alternatives	**Decision Council K1**				**Decision Council K2**			**Decision Council K3**			
		DEx1	DEx 2	DEx 3	DEx 4	DEx 1	DEx 2	DEx 3	DEx 1	DEx 2	DEx 3	DEx 4
CR1	AMT1	HS	MS	MS	HS	HS	HS	MS	HS	HS	MS	MS
	AMT2	MS	HS	MS	HS	MS	HS	VHS	VHS	MS	MS	HS
	AMT3	HS	HS	HS	HS	VHS	HS	MS	HS	MS	MS	VHS
	AMT4	MS	MS	HS	HS	VHS	HS	MS	MS	MS	VHS	VHS
	AMT5	MS	HS	HS	HS	VHS	HS	HS	HS	HS	VHS	VHS
CR2	AMT1	HS	HS	VHS	MS	MS	MS	VHS	VHS	VHS	HS	MS
	AMT2	VHS	VHS	VHS	MS	HS	HS	HS	HS	HS	MS	VHS
	AMT3	VHS	VHS	VHS	VHS	HS	HS	VHS	HS	VHS	VHS	HS
	AMT4	LS	LS	MS	MS	MS	MS	MS	HS	MS	MS	MS
	AMT5	LS	MS	LS	MS	HS	MS	LS	HS	HS	MS	MS
CR3	AMT1	HS	HS	VHS	HS	VHS	VHS	MS	MS	MS	HS	VHS
	AMT2	MS	HS	LS	MS	HS	MS	MS	HS	MS	MS	MS
	AMT3	LS	MS	MS	LS	MS	MS	MS	MS	MS	LS	MS
	AMT4	HS	MS	HS	VHS	HS	HS	VHS	HS	VHS	VHS	MS
	AMT5	HS	HS	MS	HS	MS	MS	HS	VHS	VHS	MS	HS
CR4	AMT1	MS	MS	HS	MS	LS	MS	MS	HS	MS	MS	MS
	AMT2	HS	MS	HS	MS	HS	HS	MS	MS	MS	HS	HS
	AMT3	VHS	HS	HS	VHS	VHS	HS	VHS	VHS	HS	HS	VHS
	AMT4	VHS	VHS	MS	MS	HS	HS	HS	MS	MS	HS	VHS
	AMT5	HS	MS	HS	MS	HS	HS	VHS	HS	VHS	VHS	HS

Table 6.4.7: Decision matrix in the form of TFNs combining the values given by DAs

Decision Criteria	Alternative	Decision Council		
		K1	K2	K3
CR1	AMT1	(4.5,6.5,8.5)	(5.33,7.33,9.33)	(5,7,9)
	AMT2	(5,7,9)	(6,8,9.33)	(5.5,7.5,9)
	AMT3	(6,8,10)	(6,8,9.33)	(5.5,7.5,9)
	AMT4	(5,7,9)	(6,8,9.33)	(6,8,9)
	AMT5	(6,8,10)	(6.67,8.67,10)	(7,9,10)
CR2	AMT1	(6,8,9.5)	(5.33,7.33,8.67)	(6.5,8.5,9.5)
	AMT2	(7,9,9.5)	(6,8,10)	(6,8,9.5)
	AMT3	(8,10,10)	(6.67,8.67,10)	(7,9,10)
	AMT4	(3,5,7)	(4,6,8)	(4.5,6.5,8.5)
	AMT5	(3,5,7)	(4,6,8)	(5,7,9)
CR3	AMT1	(6.5,8.5,10)	(6.67,8.67,9.33)	(5.5,7.5,9)
	AMT2	(4,6,8)	(4.67,6.67,8.67)	(5,7,9)
	AMT3	(3,5,7)	(4,6,8)	(3.5,5.5,7.5)
	AMT4	(6,8,9.5)	(6.67,8.67,10)	(6.5,8.5,9.5)
	AMT5	(5.5,7.5,9.5)	(4.67,6.67,8.67)	(6.5,8.5,9.5)
CR4	AMT1	(4.5,6.5,8.5)	(3.33,5.33,7.33)	(4.5,6.5,8.5)
	AMT2	(5,7,9)	(3.33,5.33,7.33)	(5,7,9)
	AMT3	(7,9,10)	(7.33,9.33,10)	(7,9,10)
	AMT4	(6,8,9)	(6,8,10)	(5.5,7.5,9)
	AMT5	(5,7,9)	(6.67,8.67,10)	(7,9,10)

Table 6.4.8 Normalized decision matrix

Decision Council	Alternative	Criteria			
K1		CR1	CR2	CR3	CR4
	AMT1	(0.53,0.69,1)	(0.32,0.38,0.50)	(0.65,0.85,1)	(0.45,0.65,0.85)
	AMT2	(0.5,0.64,0.9)	(0.32,0.33,0.43)	(0.4,0.6,0.8)	(0.5,0.7,0.9)
	AMT3	(0.45,0.56,0.75)	(0.3,0.3,0.38)	(0.3,0.5,0.7)	(0.7,0.9,1)
	AMT4	(0.5,0.64,0.9)	(0.43,0.6,1)	(0.6,0.8,0.95)	(0.6,0.8,0.9)
	AMT5	(0.45,0.56,0.75)	(0.43,0.6,1)	(0.55,0.75,0.95)	(0.5,0.7,0.9)
	criteria Weight (w_j)	0.27	0.236	0.22	0.275
K2	AMT1	(0.57,0.73,1)	(0.46,0.55,0.75)	(0.67.0.87,0.93)	(0.33,0.53,0.73)
	AMT2	(0.57,0.67,0.89)	(0.4,0.5,0.67)	(0.47,0.67,0.87)	(0.53,0.73,0.93)
	AMT3	(0.57,0.67,0.89)	(0.4,0.46,0.6)	(0.4,0.6,0.8)	(0.73,0.93,1)
	AMT4	(0.57,0.67,0.89)	(0.5,0.67,1)	(0.67,0.87,1)	(0.6,0.8,1)
	AMT5	(0.53,0.61,0.86)	(0.5,0.67,1)	(0.47,0.67,0.87)	(0.67,0.87,1)
	criteria Weight (w_j)	0.26	0.224	0.25	0.27
K3	AMT1	(0.56,0.71,1)	(0.47,0.53,0.69)	(0.58,0.79,0.95)	(0.45,0.65,0.85)
	AMT2	(0.56,0.67,0.9)	(0.47,0.56,0.75)	(0.53,0.74,0.95)	(0.5,0.7,0.9)
	AMT3	(0.56,0.67,0.9)	(0.45,0.5,0.64)	(0.37,0.58,0.79)	(0.7,0.9,1)
	AMT4	(0.56,0.63,0.83)	(0.53,0.69,1)	(0.68,0.89,1)	(0.55,0.75,0.9)
	AMT5	(0.5,0.56,0.71)	(0.5,0.64,0.9)	(0.68,0.89,1)	(0.7,0.9,1)
	criteria Weight (w_j)	0.254	0.254	0.213	0.28

Table 6.4.9: Taguchi loss values

Decision Council	Alternative	Criteria			
		CR1	CR2	CR3	CR4
K1	AMT1	(44944,76176,160000)	(0.64,0.9,1.56)	(100,58.48,42.25)	(316,151.4,88.6)
	AMT2	(40000,65536,129600)	(0.64,0.68,1.16)	(264.1,117.36,66.02)	(256,130.6,79)
	AMT3	(32400,50176,90000)	(0.56,0.56,0.9)	(469.4,169,86.2)	(130.6,79,64)
	AMT4	(40000,65536,129600)	(1.16,2.25,6.25)	(117.4,66,46.8)	(177.8,100,79)
	AMT5	(32400,50176,90000)	(1.16,2.25,6.25)	(189.7,75.1,46.8)	(256,130.6,79)
	criteria Weight (w_j)	0.27	0.236	0.22	0.275
K2	AMT1	(51984,85264,160000)	(1.32,1.89,3.52)	(94.12,55.82,48.85)	(587.7,227.8,120.1)
	AMT2	(51984,71824,126736)	(1,1.56,2.81)	(191.26,94.12,55.82)	(227.8,120.1,74)
	AMT3	(51984,71824,126736)	(1,1.32,2.25)	(264.1,117.36,68)	(120.1,74,64)
	AMT4	(51984,71824,126736)	(1.56,2.81,6.25)	(94.12,55.82,42.25)	(177.78,100,64)
	AMT5	(44944,59536,118336)	(1.56,2.81,6.25)	(191.26,94.12,55.82)	(142.57,84.56,64)
	criteria Weight (w_j)	0.26	0.224	0.25	0.27
K3	AMT1	(50176,80656,160000)	(1.38,1.76,2.98)	(125.6,67.7,46.8)	(316,151.48,88.58)
	AMT2	(50176,71824,129600)	(1.38,1.96,3.52)	(150.41,77.15,46.8)	(256,130.6,79)
	AMT3	(50176,71824,129600)	(1.27,1.56,2.56)	(308.62,125.6,67.7)	(130.6,79,64)
	AMT4	(50176,63504,110224)	(1.76,2.93,6.25)	(91.37,53.34,42.25)	(211.57,113.78,79)
	AMT5	(40000,50176,80656)	(1.56,2.56,5.06)	(91.37,53.34,42.25)	(130.6,79,64)
	criteria Weight (w_j)	0.254	0.254	0.213	0.28

Table 6.4.10: Weighted Taguchi loss values

Alternative	Decision Council		
	K1	K2	K3
	Weighted loss (WL_i)	Weighted loss (WL_i)	Weighted loss (WL_i)
AMT1	(12245.11,20622.23,43233.76)	(13698.34,22244.79,41645.44)	(12859.91,20543.70,40675.86)
AMT2	(10928.88,17756.61,35028.39)	(13625.38,18730.55,32985.94)	(12848.32,18296.56,32951.71)
AMT3	(8886.09,13606.56,24336.80)	(13614.51,18723.86,32986.15)	(12846.40,18292.19,32951.86)
AMT4	(10875.41,17737.27,35025.30)	(13587.71,18715.83,32980.63)	(12823.57,16173.82,28029.90)
AMT5	(8860.93,13600.49,24333.30)	(11772.09,15526.35,30800.02)	(10216.15,12778.68,20515.12)

Table 6.4.11: Weighted Taguchi loss (crisp value) and preliminary ranking

Alternatives	**Council** K1		**Council** K2		**Council**	
	crisp WL_i	**Preliminary Ranking**	**Council**	**Preliminary ranking**	crisp WL_i	**Preliminary ranking**
AMT1	22994.563	5	24053.82	5	22618.43	5
AMT2	19497.29	4	20255.59	4	19831.05	4
AMT3	14608.19	2	20249.35	3	19827.84	3
AMT4	19474.97	3	20238.61	2	17591.46	2
AMT5	14599.37	1	17446.25	1	13641.00	1

Although the basic reasons to implement AMT system is to enhance productivity, quality, flexibility etc., the ultimate rationale has to be established with economy. The DEx(s) go through market survey and their knowledge base to fix the installation costs of the AMTs as given in table 6.4.12.

Table 6.4.12: Implementation cost of alternatives and objective factor measure

Alternatives	**AMT Installation Cost (R_i) (millions of $)**	**Objective factor measure (OFM_i)**
AMT1	4.087	0.223
AMT2	5.178	0.176
AMT3	4.256	0.215
AMT4	4.339	0.210
AMT5	5.203	0.176

A mathematical model combining cost-factor components with weighted loss values is established, the basic of which is proposed by Bhattacharya et. al**.** as mentioned earlier. As weighted loss is a minimization function, the inverse of the same is considered as subjective factor measure, to be maximized. In doing so, the cognitive minds of DEx(s) are also analyzed and co-efficient of cognition (ɣ) value is set as 0.67. ɣ is the measure of positivity in a DEx. It makes a balance between the subjective and the objective factor associated in the problem. An optimistic DEx sets high value of ɣ. So, the value 0.67 is regarded as the mean of the optimism expressed by the DEx(s) across each decision council. The final deciding factor i.e. the evaluation indices are calculated for the AMTs by three decision councils following equation 6.4.14. The corresponding values and finalized rankings are presented in table 6.4.13.

Table 6.4.13 Evaluation index and finalized ranking

	council K1		council K2		council K3	
Alternatives	Evaluation Index (EI_i)	**Finalized Ranking**	Evaluation Index (EI_i)	**Finalized Ranking**	Evaluation Index (EI_i)	**Finalized Ranking**
AMT1	3.015	5	2.867	5	3.089	5
AMT2	3.530	4	3.375	4	3.428	4
AMT3	4.680	2	3.394	3	3.448	3
AMT4	3.573	3	3.406	2	3.868	2
AMT5	4.687	1	4.031	1	4.835	1

The optimum selection decision is same throughout the councils and the selection is AMT5.

6.4.4.3. Post-optimality treatment

Post-optimality treatment, as the name suggests, is carried out only after the optimum solution to the problem is reached. It is done to establish the robustness of the proposed model and also termed as sensitivity or uncertainty analysis. Any vagueness at input level data is managed by TFNs. At the design level, this is carried out by post-optimality analysis. In the presented model, post-optimality treatment is carried out owing to establish a design range for co-efficient of cognition (ɣ), over which we can have optimum selection. This has something to do with the cognitive mind of DEx(s). This expresses their positivity or the negativity. If the group of DEx(s) is optimistic in nature, they see opportunities in challenges. And, the ɣ value tends to move to the higher side, getting close to 1. On the other hand, a pessimistic group could find out problems even in opportunities and the ɣ value could move to the lower side close to 0. The value of ɣ in the present problem is set to 0.67. Post-optimality treatments for three different decision councils are portrayed in figures 6.19, 6.20, 6.21 respectively. The analysis of the same is presented in table 6.4.14.

Table 6.4.14: Post-optimality analysis

Committee K1		Committee K2		Committee K3	
Value of ɣ	**Optimum Selection**	**Value of ɣ**	**Optimum Selection**	**Value of ɣ**	**Optimum Selection**
$ɣ \leq 0.0032$	AMT1	$ɣ \leq 0.01$	AMT1	$ɣ \leq 0.011$	AMT1
$0.0032 \leq ɣ \leq 0.565$	AMT3	$0.01 \leq ɣ \leq 0.038$	AMT3	$0.011 \leq ɣ \leq 0.023$	AMT4
$ɣ \geq 0.565$	AMT5	$ɣ \geq 0.038$	AMT5	$ɣ \geq 0.023$	AMT5

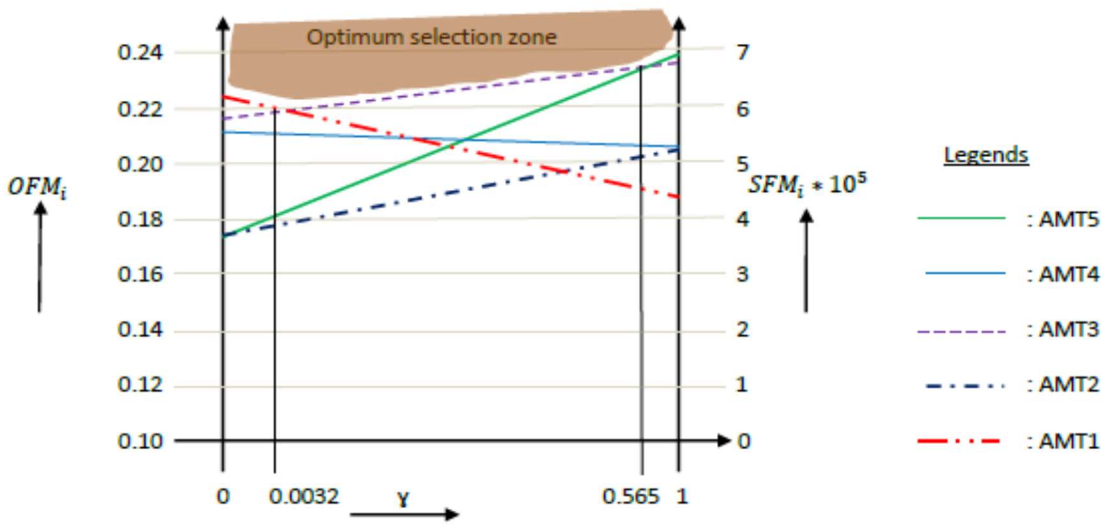


Figure 6.19: Post-optimality treatment (Decision committee K1)

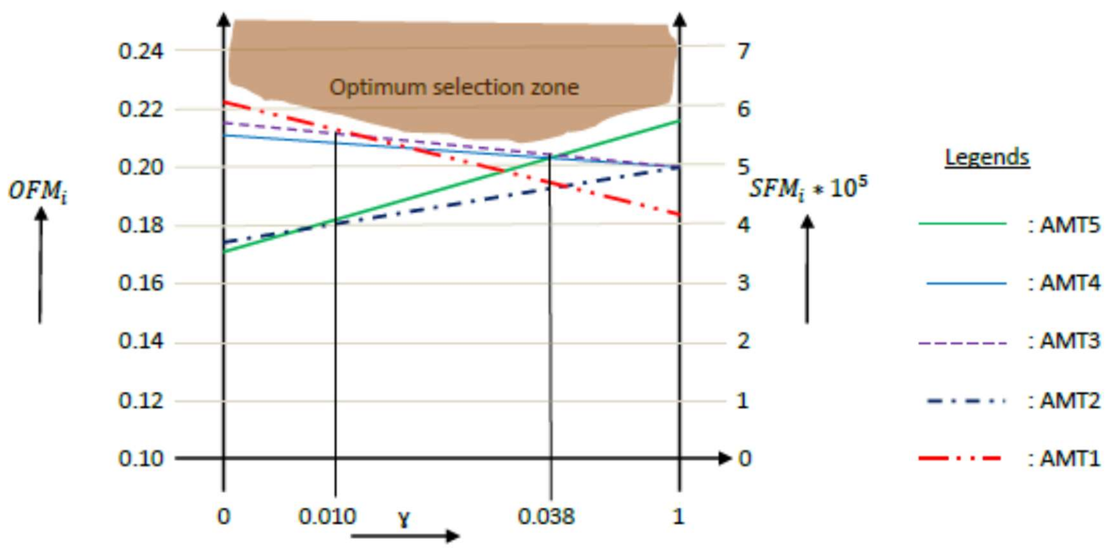


Figure 6.20: Post-optimality treatment (Decision committee K2)

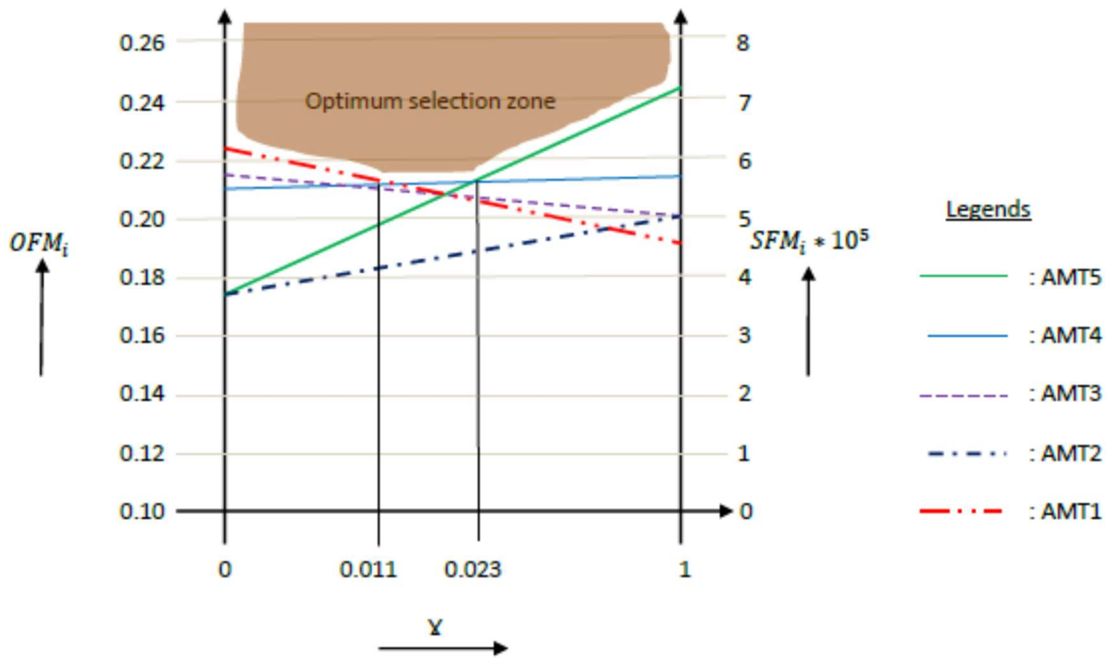


Figure 6.21: Post-optimality treatment (Decision committee K3)

6.4.4.4. Application and result analysis

The current section presents some applications of the model and analysis of result thereafter. It is a bit tedious task to make sure that a new kid on the block is having more competitive advantage than the other big guns and that, it has got robustness. To prove the same, we have pitted the proposed method with a well established fuzzy VIKOR method and exhibited the comparison result. We have compared the findings of decision committee K3, of the proposed method, with fuzzy VIKOR, to get an overall understanding about the applicability and practicality of the same.

An application of VIKOR was published in 1980 (Duckstein & Opricovic) with a view to solve decision problems with conflicting criteria with acceptable agreement for conflict resolution. VIKOR brings in compromised solution based on closest distance to the utopian condition providing ranking of selected choices. Compromise solution in MCDM was first introduced by Yu and Zeleny. The real applications of VIKOR were presented in 1998 (Opricovic, 1998) and eventually it was internationally recognized (Opricovic & Tzeng, 2004).

The steps of fuzzy VIKOR include the following:

Step 1. Aggregation of fuzzy weights of criteria into TFNs (table 6.4.15) and criteria values of alternatives by following equation 6.4.5 and equation.6.4.6.

Step 2. Formation of fuzzy decision matrix as given in table 6.4.7 earlier, same as that of our proposed method.

Step 3. Determination of best fuzzy values f_i^* and worst fuzzy values f_i^- for all the selected criteria, where, $f_j^* = \max \emptyset_{ij}$, if the j^{th} criterion is beneficial in nature; $f_j^- = \min \emptyset_{ij}$, if the j^{th} criterion is non-beneficial in nature. The same is presented in table 6.4.15.

Step 4. Computation of weighted and normalized Manhattan distance (M_i), weighted and normalized Chebyshev distance (C_i), by following equation 6.4.15 and equation 6.4.16 as follows:

$$M_i = \sum w_j (f_j^* - \emptyset_{ij}) / (f_j^* - f_j^-) \qquad \text{...... [6.4.15]}$$

$$C_i = \max[\sum w_j (f_j^* - \emptyset_{ij}) / (f_j^* - f_j^-)] \qquad \text{...... [6.4.16]}$$

The same is displayed in Table 6.4.16.

Step 6. Calculating the index values of AMTs ($\breve{Q}_i$) for the three limits, by following equation 6.4.17 as given below:

$$\breve{Q}_i = \nabla\, (M_i - m^*)/(m^- - m^*) + (1 - \nabla)(C_i - c^*)/(c^- - c^*) \qquad \text{....... [6.4.17]}$$

Where, $m^* = \min M_i, m^- = \max M_i, c^* = \min C_i,\ c^- = \max C_i$;

Calculating the average index value, $Q_i = \breve{Q}_i/3$, and, inverse of Q_i i.e., $Q_i^* = Q_i^{-1}$. These values are presented in table 6.4.17.

∇ is the maximum group utility i.e. strategic weight for the majority of criteria. The strategies arrive to a compromise solution by taking ∇ as 0.5.

Step 7. The index value in VIKOR follows the lower the better principle. So, the inverse of the average index values (Q_i^*) is taken as the subjective factor measure. On the contrary, the implementation cost of alternative (table 6.4.12) is taken into account to get the measure of objective factor (OFM_i) by equation 6.4. 13 presented earlier.

Step 8. Calculating the VIKOR selection Index (VSI_i) for each alternative by following equation 6.4.18 as follows: $VSI_i = (\gamma * \mathrm{SFM_i}) + (1 - \gamma)(\mathrm{OFM_i})$ ...... [6.4.18]

Where, $\mathrm{SFM_i}$= subjective factor measure = $Q_i{}^*$, ɣ = co-efficient of cognition mentioned earlier in section 6.4.3.

Subsequent ranking of the AMTs are determined. Higher value of VSI_i betters the ranking of the same. Table 6.4.18 shows the values of VSI_i and the ranking result as well.

Table 6.4.15: Weights of criteria in TFNs, best fuzzy value and worst fuzzy value (VIKOR) (Committee K3)

Criteria	**Decision Council K3**						
	Criteria weights	best fuzzy value (f^*)			Worst fuzzy value (f^-)		
CR1	(0.675, 0.775, 0.875)	5	7	9	7	9	10
CR2	(0.675, 0.775, 0.875)	4.5	6.5	8.5	7	9	10
CR3	(0.55, 0.65, 0.75)	6.5	8.5	9.5	3.5	5.5	7.5
CR4	(0.75, 0.85, 0.95)	7	9	10	4.5	6.5	8.5

Table 6.4.16: Values of S_i and R_i (Council K3)

Alternatives	$\boldsymbol{S_i}$			$\boldsymbol{R_i}$		
AMT1	1.470	1.590	1.720	0.750	0.850	0.950
AMT2	1.450	1.655	1.400	0.600	0.680	0.580
AMT3	1.395	1.619	1.625	0.675	0.775	0.875
AMT4	0.788	0.898	0.633	0.450	0.510	0.633
AMT5	1.177	1.360	1.542	0.675	0.775	0.875

Table 6.4.17: Index values (Q_i) (Council K3)

Alternatives	$\breve{Q}_i$			Q_i	Q_i^*
AMT1	1.000	0.955	1.000	0.985	1.015
AMT2	0.736	0.750	0.355	0.610	1.639
AMT3	0.820	0.864	0.854	0.846	0.639
AMT4	0.370	0.680	0.720	0.590	1.695
AMT5	0.660	0.695	0.820	0.725	1.379

Table 6.4.18: VIKOR selection index and Final ranking

Alternatives	**VIKOR selection index (VSI_i)**	**Final Ranking**
AMT1	0.75364	5
AMT2	1.15621	2
AMT3	0.86289	4
AMT4	1.20400	1
AMT5	0.98201	3

6.4.4.5. Experimental result

The results of the model exhibit that, values of evaluation indices for AMT3 and AMT5, measured by council K1, are pretty close. Although AMT5 retains its winning place carrying forward from the Taguchi weighted loss value, the difference is too marginal. But for the other councils, AMT5 emerges out as a clear and distant winner. The inclusion of cost factor doesn't change the ranking of the alternatives previously made out of the values of weighted loss. So, the final result is as clear as daylight. The optimum selection is AMT5 although the cost of implementation is a little on the higher side. If an organization is tight on budget, it can choose between AMT3 and AMT4, as they are occupying the 2nd or the 3rd place of priority in the rankings of all the decision councils. Also, the implementation costs of both of them are much lower than that of AMT5. Having said that, it excels highly in all other departments. For this obvious reason, it should be the automatic choice for long run, for any manufacturing organization trying to shift to AMT from traditional ones and get immense benefit out of the same. The post-optimality treatment reveals the desired robustness present in the design of the model. The selection of alternative AMT5 on the values of evaluation index is proving to be the most optimum one in post-optimality treatment too. Almost for the entire range of co-efficient of cognition, ɣ, AMT5 remains the only choice. If the value of ɣ is zero i.e. the most pessimistic approach, the DAs would make the selection decision based only on cost factor. But, a unit value of ɣ, the most optimistic one, would therefore nullify the cost factor and make the decision only on the basis of Taguchi weighted loss of alternatives for which the lowest value is the most preferable one. The comparison with fuzzy VIKOR clears that, second highest ranked alternative by council K3 in loss function is the top scorer here in VIKOR i.e. AMT4. Though the AMT5, the first choice by council K3 in loss function, is the third choice by VIKOR method, the difference in overall score is too nominal. So, according to that, AMT5 comes out as the best choice overall in the proposed case study.

6.4.5. Conclusion

Several conclusions have been drawn from the proposed model and the same has been profoundly discussed in this section. Implementation of AMT by the manufacturing organizations is the need of the hour for achieving sustainable development. Otherwise, gradual decay of the organizations is quite obvious. A framework is proposed in this chapter for performance assessment of AMTs using Taguchi loss function to a fuzzy decision model. Some experts in decision making played a pivotal role in realizing the model. They chose the criteria that could best expound the alternatives from the perspective of the manufacturing firm. Finally, they analyzed the results of proposed model and rendered their verdict.

The proposed methodology combines Taguchi loss function with fuzzy decision model, consolidated gain as subjective factor measure of alternatives and the installation cost as objective factor measure of alternatives. It can be implemented successfully in supplier selection, robot selection and many more MCDM problems prevailing in today's manufacturing scenario. It upped the novelty by drawing a comparison with traditional VIKOR method.

Also, in Taguchi loss function, fuzziness present in the initial stage is carried forward deep into the problem. We get the Taguchi loss values in terms of TFNs only, during the final legs of the problem solving. Defuzzification is done at a quite later stage. So, the loss of information is significantly less as compared to some traditional decision tools available. The comparison with VIKOR establishes that.

The numbers of DAs in decision committees are varying. Expertise of DAs in the committees are not uniform nor their experience. So, there might be slight variation in the end result. But it's in the hand of manufacturing firm, to choose the outcome of a particular committee depending on its own optimized requirements.

Then also, we have the outcome of the post-optimality treatment that could yield a considerable range of ɣ, where the optimum selections for all the committees are same, i.e. AMT5.

On the contrary, it depicts no mathematical comparison between results obtained from the manuscript and the traditional loss function.

Future scope would include undertaking another form of post-optimality treatment where the weights of criteria would be interchanged and analyzing the final result for the robustness of the problem. This particular research could also be channelized towards investigating group decision making models in multi attribute problems under uncertainty and fuzziness. These are considered in cases of fuzziness persistent, be it preliminary stage or be it problem solving stage.

Lastly, we can say that, fuzzy Taguchi loss method with consolidated gain and consolidated loss concept stands right up there and is very much suited for various kinds of critical problems that manufacturing establishments pass through. So, this method can be applied to solve various selection and evaluation problems in manufacturing industry.

7. CASE STUDY V: ANALYSIS OF DIFFERENT MCDM METHODS FOR WAREHOUSE LOCATION SELECTION IN SUPPLY CHAIN

7.1. Methodology Analysis: Dempster-Shafer Theory (DST), Analysis of Variance (ANOVA) and Regression Analysis.

7.1.1 Problem Scenario

Supply chain management is a major aspect in manufacturing industry now-a-days. It involves strategy, market planning, supply-demand ratio and many other significant criteria. But the ultimate goal is to increase the value for shareholders and investors. A supply chain framework is shown in figure 7.1.

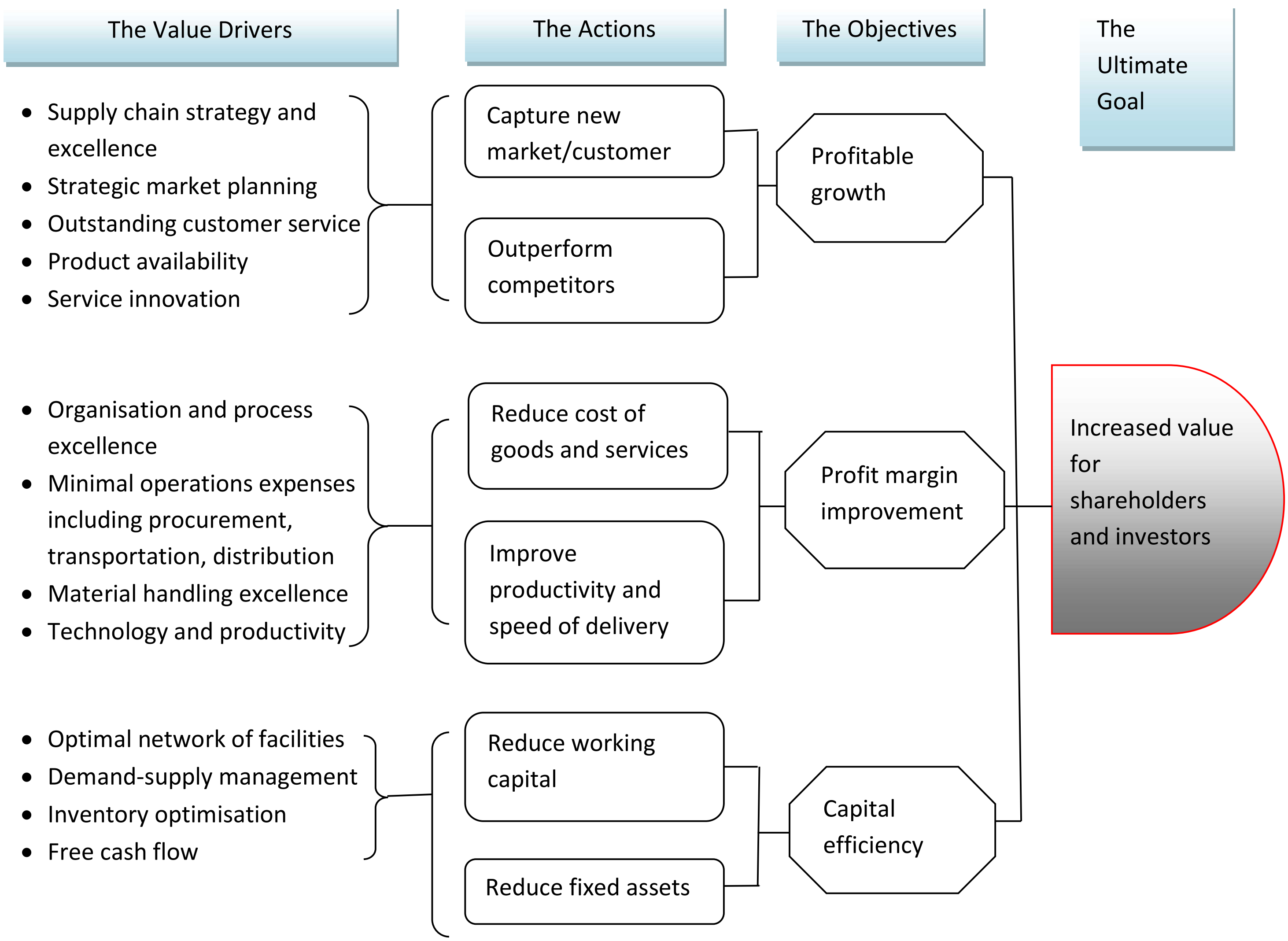


Figure 7.1: The supply chain framework

Innovation, continuous improvement and competitive advantage are the keys for a supply chain to be sustainable and resilient. The same is shown in figure 7.2.

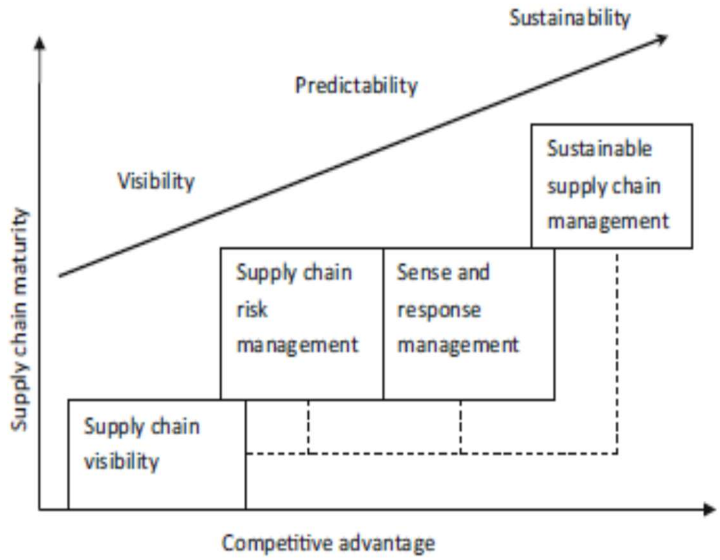


Figure 7.2: Competitive advantage: Sustainable supply chain

The upstream level of supply chain consists of supplier selection for the supply of raw materials. The middle stream level consists of technology selection for producing products conforming to organizational standards, keeping the productivity and flexibility on a high. The downstream of a supply chain network deals with selection of warehouses for transportation and proper inventory of the products such that the same can flow to the end customer in a smooth manner. There are considerable amount of political, economic, environmental, social and technological influences (PEEST analysis) on the selection and implementation of warehouses, as shown in figure 7.3.

So, as far as supply chain is considered, selection of a proper warehouse is extremely influential and of great importance, in the face of extreme challenges from a highly competitive market. That is to say, the products ought to reach the end customer at a minimum time and cost, setting the quality of the prior as the benchmark in the segment.

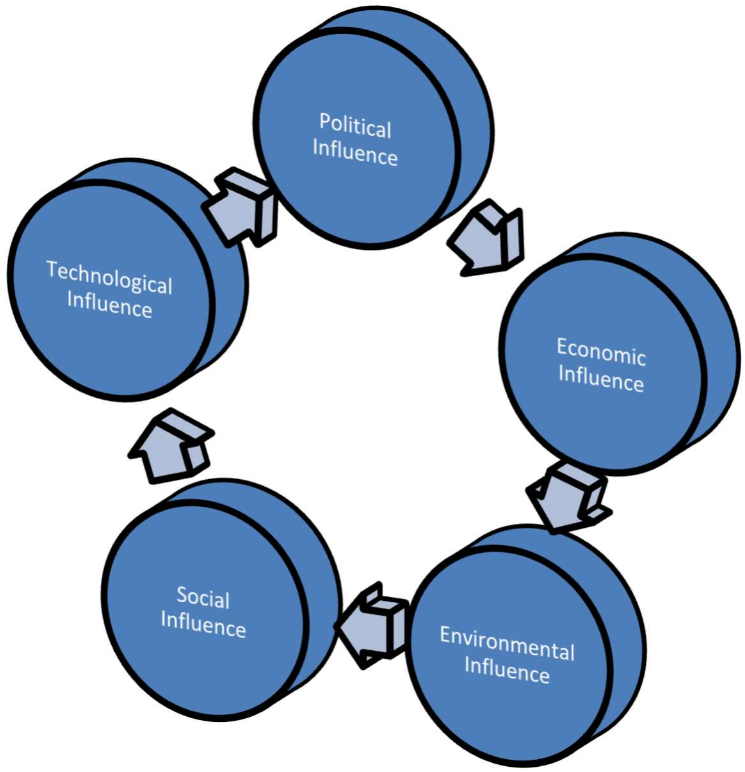


Figure 7.3: The PEEST analysis

7.1.2 Background research

7.1.2.1 Fuzzy decision matrix

A fuzzy MCDM problem is represented in terms of decision matrix in the following form:

$$D = \begin{array}{c} \\ W1 \\ W2 \\ \vdots \\ Wm \end{array} \begin{array}{c} \begin{array}{cccc} SC1 & SC2 & \dots & SCn \end{array} \\ \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & & \phi_{2n} \\ \vdots & & \ddots & \vdots \\ \phi_{m1} & \phi_{m2} & \cdots & \phi_{mn} \end{bmatrix} \end{array}$$

Where, $W1, W_2, \dots, W_m$ are the warehouse alternatives and $SC1, SC2, \dots, SCn$ are the selection criteria for the problem;

$$m = no.of\ alternatives, n = no.of\ criteria.$$

$$\phi_{ij} = \left(x_{ij}, y_{ij}, z_{ij}\right) are\ the\ triangular\ fuzzy\ no.s\ (TFNs).$$

7.1.2.2 Linguistic weight set and TFNs

A linguistic weight set is presented by linguistic variable. It involves coping with uncertainty and fuzziness inherent in the problem scenario. There are some subjective criteria which can't be represented in exact crisp numerical numbers. But the same can be accomplished by linguistic variable such as extremely significant, very significant, and so on. This further can be presented by TFNs, for example, extremely significant = (9, 10, 10), very significant = (7, 9, 10) and so on. This is a user defined scale of 0 to 10. Some other scale can be defined in the range 0 and 1 as well, to define some other linguistic variable. If A = (m1, m2, m3) and B = (n1, n2, n3) are the two TFNs, then the distance between them can be calculated by equation 7.1:

$$d\,(A, B) = \frac{1}{2}[\max(|\ m1 - n1\ |, |\ m3 - n3\ |) + (|\ m2 - n2\ |)] \qquad \text{........ [7.1]}$$

7.1.2.3 Interval number

It is another way of dealing with uncertainty and vagueness that a fuzzy MCDM problem has to offer. "Interval numbers are a set of real numbers with a property that any number lying between two numbers is also included in the set", **Kaucher, 1980**. If $a = [a^-, a^+]$, $b = [b^-, b^+]$ are the two positive interval numbers, then the distance between them is as follows as in equation 7.2:

$$D(a, b) = {\sqrt{2}}/{2}\sqrt{(a^- - b^-)^2 + (a^+ - b^+)^2} \qquad \text{........ [7.2]}$$

7.1.2.4 Defuzzification process

It's simply the conversion of TFN into crisp ones. It is the backbone step in decision making for extracting information out of the problem. There are many processes available **(Zimmermann, 1991)** for defuzzification in manufacturing and decision making world. The present chapter follows graded mean integration representation on TFNs as the defuzzification process. If ϕ_{ij} be a TFN defined by a triplet (x_{ij}, y_{ij}, z_{ij}), then the defuzzification of this TFN is done as in equation 7.3:

$$P(\phi_{ij}) = (x_{ij} + 4 \times y_{ij} + z_{ij})/6. \qquad \text{........ [7.3]}$$

7.1.2.5 Dempster- Shafer theory of evidence (DST)

TOPSIS came into picture in the early 80's by **Hwang & Yoon (1981)**. The effort was to counter the effects of fuzzy MCDM problems existing in the manufacturing organizations.

Dempster-Shafer Theory (**A. P. Dempster, 1967; G. Shafer, 1976**), in general is contemplated as the extension of Bayesian theory which can deal with imprecise data set. It is a set of hypothesis h defined as follows: P (h) = [Q, $\{h_1\}$, $\{h_2\}$,…,$\{h_n\}$,$\{h_1 \cup h_2\}$,$\{h_1 \cup h_3\}$,….,h]; where Q denotes the empty set.

The basic probability assignment or BPA is the main element of evidence theory. The mass of belief in an element of h is like probability distribution.

Evidence theory offers aggregation in the form of Dempster rule of combination or the orthogonal sum in *m*, for information source S, and is noted by $m = m_1 \oplus m_2$.

It fuses two BPAs to yield an all new BPA. The same is presented in equation 7.4 and equation 7. 5.

$$m\,(\mathrm{A}) = \sum_{B \cap C = A} \mathrm{m}_1(B) m_2(C)/1 - f \qquad \text{........ [7.4]}$$

$$f = \sum_{B \cap C = \mathrm{Q}} m_1(B)\, m_2(C) \qquad \text{........ [7.5]}$$

f: degree of conflict between *m* 1 & *m* 2. f = 0 signifies zero conflict between *m* 1 & *m* 2; f = 1 signifies absolute contradiction between *m* 1 & *m* 2.

The belief function m is denoted in equation 7.6 as follows:

$$m = m_1 \oplus m_2 \oplus \dots \oplus m_n \qquad \text{........ [7.6]}$$

To handle conflict, a discounting rule (Shafer, 1976) is introduced in DST given by equation 7.7 and equation 7.8:

$$BLF^{\gamma}(h) = 1 \qquad \text{........ [7.7]}$$

$$BLF^{\gamma}(A) = (1 - \alpha).BLF(A)\,;\ \forall\ \mathrm{A} \subset \mathrm{h} \text{ and } \mathrm{A} \neq \mathrm{Q} \qquad \text{........ [7.8]}$$

Here, BLF: $2^h \rightarrow$ [0, 1] is belief function and BLF^h: $2^h \rightarrow$ [0, 1] is discounted belief function.

γ ($0 \leq \gamma \leq 1$) is the discounting index that signifies the reliability strength of the evidence.

The BPA m^{γ} is congruous with the discounted belief function BLF^{γ} and is further modified **(Shafer, 1976)** in equation 7.9 and equation 7.10 as follows:

$$m^{\gamma}(h) = (1-\gamma)m(h) + \gamma \qquad \text{........ [7.9]}$$

$$m^{\gamma}(A) = (1-\gamma)m(A), \forall \text{ A} \subset \text{h and A} \neq \text{Q} \qquad \text{........ [7.10]}$$

The crisp estimation in a belief interval is determined by equation 7.11 as follows:

$$bet\ (A_i) = \sum_{A_i \subset \mathrm{A_k}} \frac{m(A_k)}{|A_k|} \qquad \text{........ [7.11]}$$

The expression 'bet' in Latin was proposed by **Smets (2000)**. In decision theory, it is a probability that a rational person assigns to an option when required to make a decision.

The equation 7.11 is also known as Pignistic Probability Transformation (PPT).

7.1.2.6 Design of experiment and regression analysis

Design of experiment (DoE) is also known as experimental design. It is a statistical tool used to evaluate the effect of individual factor and factor interactions, on the response of a particular system. It is a very helpful concept to make a robust design. The input factors in DoE are the experimental variables that can be changed independently and the response is the measure of experimental results. It is of prime importance to judiciously choose the most appropriate DoE method for the application in hand. The simplest method existing in DoE uses two levels and n factors i.e. 2^n factorial design. It reduces the number of experimental conditions significantly, thus, saving precious calculation time. The regression analysis (RA) approach can generate a mathematical model to analyse the effects of the factors and their interactions. The said model collects mathematical as well as statistical data and is purely based on the experimental results. It involves a dependent variable, R, known as response variable and a number of independent variables, such as, $y_1, y_2, \ldots, y_n.$ The correlation between regression approach and experimental results for four criteria model up to 2-way interactions is presented by equation 7.12 as follows:

$$Y_i = \gamma_0 + \gamma_1 y_1 + \gamma_2 y_2 + \gamma_3 y_3 + \gamma_4 y_4 + \gamma_{12} y_1 y_2 + \gamma_{13} y_1 y_3 + \gamma_{14} y_1 y_4 + \gamma_{23} y_2 y_3 + \gamma_{24} y_2 y_4 + \gamma_{34} y_3 y_4 + \epsilon. \quad \text{...... [7.12]}$$

Here, Y_i is the predicted response, $n =$ no. of experimental variables, γ_0 the constant regression co-efficient, $\gamma_i (i = 1, .., 4)$ the linear regression co-efficient, $\gamma_{ij} (j = 2, .., 4)$ the interaction regression co-efficient, y_i the experimental variables, ϵ the statistical error.

7.2. Illustration of the Proposed Method

7.2.1. Framework

The integrated framework for the illustrated approach is given in figure 7.4. It has got three distinct phases. Phase I establishes relationship of cloud manufacturing with supply chain network. All the informations of manufacturing world are coming from that source. All the data about the same are in the cloud, so that, these can be accessed anytime, anywhere, using any device and in any format. The feedback of the end outcome is also available with the cloud. Phase II is all about selecting the appropriate problem for the organisation from the world of manufacturing. The problem of warehouse location selection is taken up in this illustrated model, as lection of warehouse location plays a pivotal role in the sustainable competitive advantages of an organisation. Phase III comes up with the formation of committee of Decision Experts (DEs) and application of fuzzy MCDM, DST of evidence, DOE on the problem.

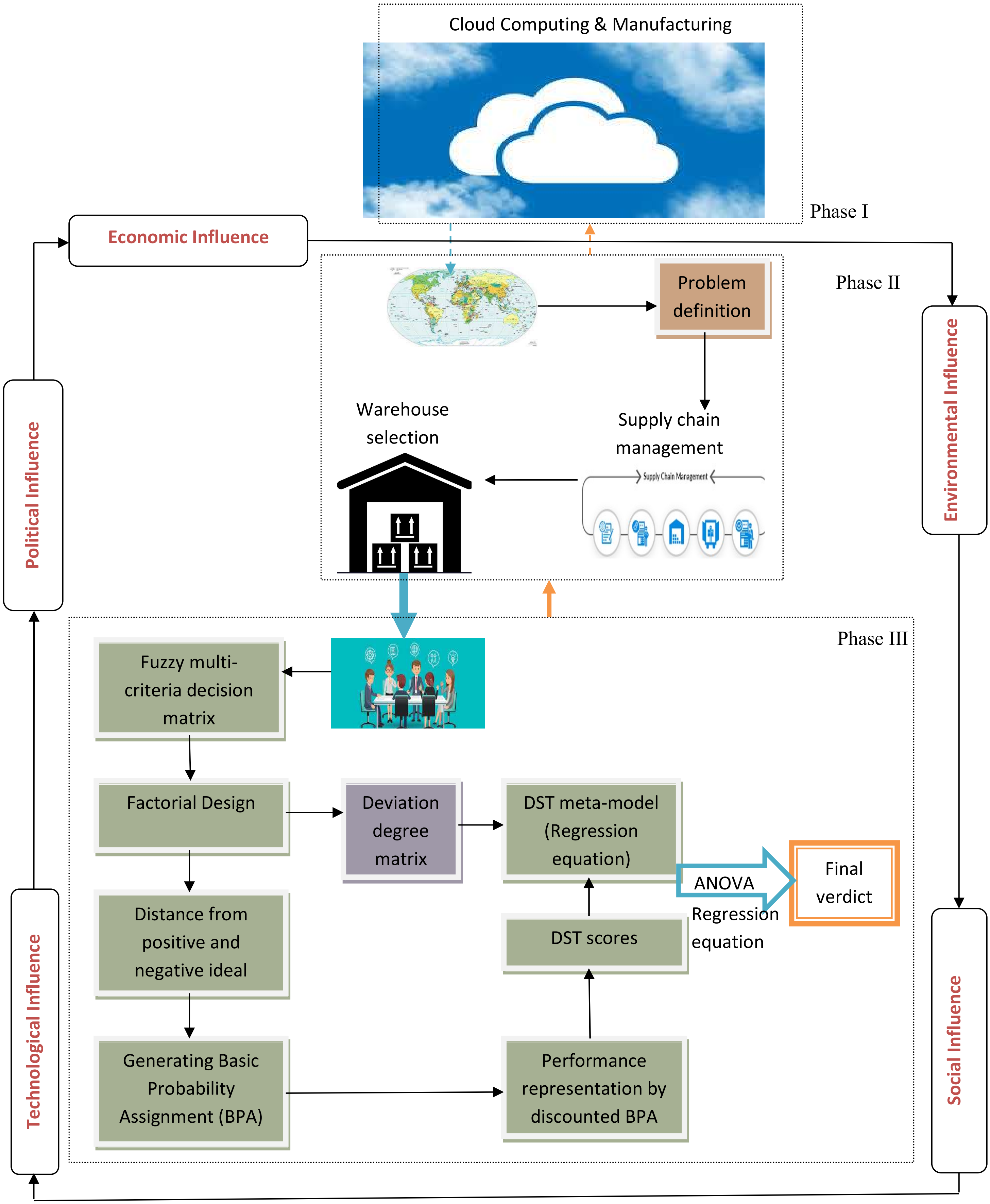


Figure 7.4: Integrated framework for the proposed model

Therefore, DEs have to form the decision matrix and deviation degree matrix of experimental design. They eventually have to find out the final DST scores and use that as the input response to ANOVA in DOE. The DEs form the regression Eq. that, in turn, becomes the main stem to give the final verdict. Phase II and phase III are under some influence from real world. Factors, such as political biasness, if any, economic condition, environmental impact, social responsibilities, technological up gradations and innovations, are associated with the selected warehouse location.

7.2.2 Numerical analysis

In this section, a simple but eloquent problem of practical importance is presented, in order to simplify the detailed procedure of DST-RA methodology. A leading manufacturing organisation seeks for the solution of optimum warehouse evaluation from a set of four options namely W1, W2, W3, and W4. A decision team combining four DAs is given the responsibility of the same. The DAs set four selection criteria namely, shipment cost (SC1), reconfigurability (SC2), surveillance (SC3) and environmental evidence (SC4). The details are as follows:

SC1: Shipment cost- to be minimized.

SC2: Reconfigurability- to be maximized.

SC3: Surveillance- to be maximized.

SC4: Environmental evidence- to be minimized.

The detailed procedural steps associated with the particular problem are given:

Stage 1(decision matrix formation with interval numbers):

a) Linguistic weight set assigned by DEs for weights of selection criteria and performance of warehouse alternatives on a particular selection criterion (the fuzzy decision matrix), as in table 7.1 and table 7.2 respectively.
b) Conversion of linguistic variables into TFNs by following a user defined conversion scale (0 - 1 for criteria weights and 0- 10 for weights of alternatives) as given in table 7.3 and table 7.4.
c) Defuzzification of criteria weights by graded mean integration representation as in equation 7.1.

d) Weights of selection criteria are transformed into discounting co-efficient as in table 7.5 by following equation 7.13 as follows: $\alpha SC_j = P(\phi_{ij})/\max P(\phi_{ij})$, …... [7.13]

Where i= no. of warehouse alternatives, j= no. of selection criteria.

Eventually we do the arithmetic mean, to get the combined weight of criteria by given DAs.

Then we get the finalized importance weight of selection criteria (α_x) by equation 7.14 as follows:

$$\alpha_{xj} = \text{Com}(\alpha SC_j)/\sum \text{Com}(\alpha SC_j) \qquad \text{…... [7.14]}$$

e) Conversion of the decision matrix (table 7.2) into TFNs. We combine the values of DEs by simple arithmetic mean and representing the matrix in interval numbers by getting the lower limit values and upper limit values of TFNs as in table 7.6. According to the table, SC1 has the best interval value of [5.5, 9] and worst interval value of [7.5, 9.75] as the criteria is minimising in nature i.e. non-beneficial. SC2 gets the best value of [8.5, 10] and worst value of [4, 8] as the criteria is maximising in nature i.e. beneficial. Similarly SC3 is a beneficial criteria with best interval value of [7.5, 10], worst interval value of [4, 8] and SC4 is a non-beneficial criteria with best interval value of [3.5, 7.5], worst interval value of [8, 10]. All of these are determined as factor levels affecting the selection of warehouse alternative.

Table 7.1: Importance level of selection criteria (SC) as suggested by Decision Expert (DE)

Selection Criteria (SC)	**DEs**			
	DE1	**DE2**	**DE3**	**DE4**
SC1 (non-beneficial)	VHS	VHS	VHS	HS
SC2 (beneficial)	HS	VHS	HS	HS
SC3 (beneficial)	MS	HS	VHS	HS
SC4 (non-beneficial)	HS	HS	VHS	VHS

Table 7.2: Weights of warehouse alternatives (The decision matrix)

DEs		SC															
		SC1 {Minimising(-)}				**SC2 {Maximising(+)}**				**SC3 {Maximising(+)}**				**SC4 {Minimising(-)}**			
		DE1	**DE2**	**DE3**	**DE4**	**DE1**	**DE2**	**DE3**	**DE4**	**DE1**	**DE2**	**DE3**	**DE4**	**DE1**	**DE2**	**DE3**	**DE4**
Warehouse alternatives	W1	VHI	VHI	AI	HI	EHI	HI	VHI	HI	AI	HI	HI	HI	VHI	EHI	VHI	EHI
	W2	AI	VHI	VHI	HI	VHI	VHI	VHI	HI	VHI	VHI	VHI	VHI	AI	VHI	AI	AI
	W3	VHI	HI	AI	VHI	EHI	VHI	EHI	EHI	VHI	EHI	VHI	VHI	VHI	VHI	VHI	VHI
	W4	VHI	EHI	EHI	HI	HI	AI	AI	HI	HI	AI	HI	AI	HI	AI	AI	AI

Table 7.3: Linguistic weight set for criteria weights

Linguistic variable	Triangular fuzzy number (TFN)
Very Less Significance (VLS)	(0,0,0.2)
Less Significance (LS)	(0,0.2,0.4)
Medium Significance (MS)	(0.2,0.4,0.6)
Heavy Significance (HS)	(0.4,0.6,0.8)
Very Heavy Significance (VHS)	(0.6,0.8,1)

Table 7.4: Linguistic value for warehouse alternatives

Linguistic Value	TFN
Extremely High Importance (EHI)	(9,10,10)
Very High Importance (VHI)	(7,9,10)
High Importance (HI)	(5,7,9)
Average Importance (AI)	(3,5,7)
Less Importance (LI)	(1,3,5)
Very Less Importance (VLI)	(0,1,3)
Extremely Low Importance (ELI)	(0,0,1)

Table 7.5: Weights of SC as discounting co-efficient

Discounting co-efficient (γSC_j)	**DEs**				**Com** (γSC_j)	**Finalised importance weights of SC** (γ_{xj})
	DE1	**DE2**	**DE3**	**DE4**		
γSC_1	1	1	1	0.75	0.9375	0.28
γSC_2	0.752	1	0.76	0.75	0.8155	0.24
γSC_3	0.5	0.749	1	0.75	0.7497	0.22
γSC_4	0.752	0.749	1	1	0.8752	0.26

Table 7.6: Decision matrix of warehouse alternatives expressed in intervals (DEs combined)

Warehouse alternatives	**SC**			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
W1	[5.5,9]	[6.5,9.5]	[4.5,8.5]	[8,10]
W2	[5.5,9]	[6.5,9.75]	[7,10]	[4,7.75]
W3	[5.5,9]	[8.5,10]	[7.5,10]	[7,10]
W4	[7.5,9.75]	[4,8]	[4,8]	[3.5,7.5]
Best (W_j, B_j)	[5.5,9]	[8.5,10]	[7.5,10]	[3.5,7.5]
Worst (W_j, B_j)	[7.5,9.75]	[4,8]	[4,8]	[8,10]
($\sum W_j$, $\sum B_j$)	[24, 36.75]	[25.5, 37.25]	[23, 36.5]	[22.5, 35.25]

Stage 2 (experimental design of attribute criteria):

We utilize the DoE to assess the influence of each factor on the final ranking results. A 2-level factorial design is used to create a data set with all the possible combinations of best interval value and worst interval value of selection criteria. This is a 2^n factorial design where n denotes the no. of factors associated with the problem. So, it's a 2-level factorial design with 2^4 or 16 number of experiments. The same is presented in table 7.7.

Table 7.7: Experimental design results of selection criteria (DOE)

Experiment No.	**SC**			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
1	[5.5,9]	[8.5,10]	[7.5,10]	[3.5,7.5]
2	[5.5,9]	[8.5,10]	[7.5,10]	[8,10]
3	[5.5,9]	[8.5,10]	[4,8]	[8,10]
4	[5.5,9]	[8.5,10]	[4,8]	[8,10]
5	[5.5,9]	[4,8]	[7.5,10]	[3.5,7.5]
6	[5.5,9]	[4,8]	[7.5,10]	[8,10]
7	[5.5,9]	[4,8]	[4,8]	[3.5,7.5]
8	[5.5,9]	[4,8]	[4,8]	[8,10]
9	[7.5,9.75]	[8.5,10]	[7.5,10]	[3.5,7.5]
10	[7.5,9.75]	[8.5,10]	[7.5,10]	[8,10]
11	[7.5,9.75]	[8.5,10]	[4,8]	[3.5,7.5]
12	[7.5,9.75]	[8.5,10]	[4,8]	[8,10]
13	[7.5,9.75]	[4,8]	[7.5,10]	[3.5,7.5]
14	[7.5,9.75]	[4,8]	[7.5,10]	[8,10]
15	[7.5,9.75]	[4,8]	[4,8]	[3.5,7.5]
16	[7.5,9.75]	[4,8]	[4,8]	[8,10]

Stage 3 (transforming the decision matrix into deviation degree matrix):

A decision matrix with interval numbers is not an easy task to manage. That is where, the deviation degree matrix comes handy. It turns the interval matrix into normal matrix with crisp numbers by calculating the distance between the intervals of the same criteria. First, we determine the best limits as [5.5, 10, 10, 3.5] that can be reached to the four alternatives. So,

$I = [5.5, 5.5], [10, 10], [10, 10], [3.5, 3.5]$ is chosen as the ideal alternative. Next, we calculate the distance between each factor (table 7.7), and exhibit the same in table 7.8. Distances are calculated following equation 7.15 as shown below:

$$d_{ij} = (\sqrt{2}/2)\sqrt{(y_{ij}^{-} - B_j^{-})^2 + (y_{ij}^{+} - B_j^{+})^2} \qquad \text{…..... [7.15]}$$

Where, (y_{ij}^{-}, y_{ij}^{+}) are the minimum and maximum bounds of experimental design and, (B_j^{-}, B_j^{+}) is the ideal alternative i.e. B_j^{-} and B_j^{+}bear the same value. Hence, $I = [B_1, B_1], [B_2, B_2], \dots, [B_j, B_j]$, is the ideal alternative.

Table 7.8: Deviation degree matrix

Experiment No.	**SC**			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
1	2.475	1.060	1.768	2.828
2	2.475	1.060	1.768	5.590
3	2.475	1.060	4.472	5.590
4	2.475	1.060	4.472	5.590
5	2.475	4.472	1.768	2.828
6	2.475	4.472	1.768	5.590
7	2.475	4.472	4.472	2.828
8	2.475	4.472	4.472	5.590
9	3.321	1.060	1.768	2.828
10	3.321	1.060	1.768	5.590
11	3.321	1.060	4.472	2.828
12	3.321	1.060	4.472	5.590
13	3.321	4.472	1.768	2.828
14	3.321	4.472	1.768	5.590
15	3.321	4.472	4.472	2.828
16	3.321	4.472	4.472	5.590

Stage 4 (calculation of DST scores):

a) Distance measurement {d (IS), d (AIS), d (IS, AIS)} of possible combinations (the no. of experiments) from ideal solution (IS) and anti-ideal solution (AIS) for both beneficial and non beneficial criteria by following equation 7.16 and equation 7.17 respectively:

For beneficial selection criteria:

$$d(IS) = \left|\max(d_{ij}) - d_{ij}\right|, d(AIS) = |d_{ij} - \min(d_{ij}), d(IS, AIS) = |d_{ij} - \frac{\max(d_{ij})+\min(d_{ij})}{2}| \quad .. [7.16]$$

For non beneficial criteria:

$$d(IS) = \left|d_{ij} - \min(d_{ij})\right|, d(AIS) = \left|\max(d_{ij}) - d_{ij}, d(IS, AIS) = \ |d_{ij} - \frac{\max(d_{ij})+\min(d_{ij})}{2}| \quad .. [7.17]$$

The same results are presented in table 7.9.

b) Generation of BPA for each alternative corresponding to distance measures by equation 7.18 as follows and the same is shown in table 7.10:

$$m\ (IS) = \frac{d\ (AIS)}{d(IS)+d(AIS)+d(IS,AIS)}; m\ (AIS) = \frac{d\ (IS)}{d(IS)+d(AIS)+d(IS,AIS)};$$

$$m\ (IS, AIS) = \frac{d\ (IS,AIS)}{d(IS)+d(AIS)+d(IS,AIS)} \quad \ldots\ldots [7.18]$$

c) Discounting the BPA of performance by equation 7.19, using the finalised importance weights of selection criteria, as obtained from the discounting co-efficient in table 5:

$$m^{\alpha}(IS) = \alpha_{xj} \times m(IS);\ m^{\alpha}(AIS) = \alpha_{xj} \times m(AIS);$$

$$m^{\alpha}(IS, AIS) = \alpha_{xj} \times m(IS, AIS) + (1 - \alpha_{xj}) \quad \ldots\ldots [7.19]$$

The outcome of the same is furnished in table 7.11.

d) Combining the BPAs of all the criteria to get a compendious evaluation of an AMT by the following equation: $m^{i} = \sum BPA_{SC_j}^{\alpha_{xj}}$; …….. [7.20]

Where, i = no. of experiments, j = no. of criteria.

e) Determine the final DST scores [$bet\ (i)$] based on equation 7.21 as stated below:

$$bet\ (i) = m^i(PIS) + m^i(PIS, NIS)/2 \quad \text{…..... [7.21]}$$

The experimental results of fuse multi-criteria data by Dempster combination rule as well as final DST scores are showcased in table 7.12.

Table 7.9: Distance from positive and negative ideal solution

Experiment No.	**SC**			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
1	(0,0.846,0.423)	(3.412,0,1.706)	(2.704,0,1.352)	(0,2.762,1.381)
2	(0,0.846,0.423)	(3.412,0,1.706)	(2.704,0,1.352)	(2.762,0,1.381)
3	(0,0.846,0.423)	(3.412,0,1.706)	(0,2.704,1.352)	(2.762,0,1.381)
4	(0,0.846,0.423)	(3.412,0,1.706)	(0,2.704,1.352)	(2.762,0,1.381)
5	(0,0.846,0.423)	(0,3.412,1.706)	(2.704,0,1.352)	(0,2.762,1.381)
6	(0,0.846,0.423)	(0,3.412,1.706)	(2.704,0,1.352)	(2.762,0,1.381)
7	(0,0.846,0.423)	(0,3.412,1.706)	(0,2.704,1.352)	(0,2.762,1.381)
8	(0,0.846,0.423)	(0,3.412,1.706)	(0,2.704,1.352)	(2.762,0,1.381)
9	(0.846,0,0.423)	(3.412,0,1.706)	(2.704,0,1.352)	(0,2.762,1.381)
10	(0.846,0,0.423)	(3.412,0,1.706)	(2.704,0,1.352)	(2.762,0,1.381)
11	(0.846,0,0.423)	(3.412,0,1.706)	(0,2.704,1.352)	(0,2.762,1.381)
12	(0.846,0,0.423)	(3.412,0,1.706)	(0,2.704,1.352)	(2.762,0,1.381)
13	(0.846,0,0.423)	(0,3.412,1.706)	(2.704,0,1.352)	(0,2.762,1.381)
14	(0.846,0,0.423)	(0,3.412,1.706)	(2.704,0,1.352)	(2.762,0,1.381)
15	(0.846,0,0.423)	(0,3.412,1.706)	(0,2.704,1.352)	(0,2.762,1.381)
16	(0.846,0,0.423)	(0,3.412,1.706)	(0,2.704,1.352)	(2.762,0,1.381)

Table 7.10: Generating Basic probability assignment (BPA) corresponding to distance measure

Experiment No.	**SC**			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
1	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0.67,0,0.33)
2	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0,0.67,0.33)
3	(0.67,0,0.33)	(0,0.67,0.33)	(0.67,0,0.33)	(0,0.67,0.33)
4	(0.67,0,0.33)	(0,0.67,0.33)	(0.67,0,0.33)	(0,0.67,0.33)
5	(0.67,0,0.33)	(0.67,0,0.33)	(0,0.67,0.33)	(0.67,0,0.33)
6	(0.67,0,0.33)	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.67,0.33)
7	(0.67,0,0.33)	(0.67,0,0.33)	(0.67,0,0.33)	(0.67,0,0.33)
8	(0.67,0,0.33)	(0.67,0,0.33)	(0.67,0,0.33)	(0,0.67,0.33)
9	(0,0.67,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0.67,0,0.33)
10	(0,0.67,0.33)	(0,0.67,0.33)	(0,0.67,0.33)	(0,0.67,0.33)
11	(0,0.67,0.33)	(0,0.67,0.33)	(0.67,0,0.33)	(0.67,0,0.33)
12	(0,0.67,0.33)	(0,0.67,0.33)	(0.67,0,0.33)	(0,0.67,0.33)
13	(0,0.67,0.33)	(0.67,0,0.33)	(0,0.67,0.33)	(0.67,0,0.33)
14	(0,0.67,0.33)	(0.67,0,0.33)	(0,0.67,0.33)	(0,0.67,0.33)
15	(0,0.67,0.33)	(0.67,0,0.33)	(0.67,0,0.33)	(0.67,0,0.33)
16	(0,0.67,0.33)	(0.67,0,0.33)	(0.67,0,0.33)	(0,0.67,0.33)
α_x	0.28	0.24	0.22	0.26

Table 7.11: Performance representation by discounted BPA

Experiment No.	SC			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
1	(0.188,0,0.09)	(0,0.161,0.119)	(0,0.147,0.133)	(0.174,0,0.106)
2	(0.188,0,0.09)	(0,0.161,0.119)	(0,0.147,0.133)	(0,0.174,0.106)
3	(0.188,0,0.09)	(0,0.161,0.119)	(0.147,0,0.133)	(0,0.174,0.106)
4	(0.188,0,0.09)	(0,0.161,0.119)	(0.147,0,0.133)	(0,0.174,0.106)
5	(0.188,0,0.09)	(0.161,0,0.119)	(0,0.147,0.133)	(0.174,0,0.106)
6	(0.188,0,0.09)	(0.161,0,0.119)	(0,0.147,0.133)	(0,0.174,0.106)
7	(0.188,0,0.09)	(0.161,0,0.119)	(0.147,0,0.133)	(0.174,0,0.106)
8	(0.188,0,0.09)	(0.161,0,0.119)	(0.147,0,0.133)	(0,0.174,0.106)
9	(0,0.188,0.09)	(0,0.161,0.119)	(0,0.147,0.133)	(0.174,0,0.106)
10	(0,0.188,0.09)	(0,0.161,0.119)	(0,0.147,0.133)	(0,0.174,0.106)
11	(0,0.188,0.09)	(0,0.161,0.119)	(0.147,0,0.133)	(0.174,0,0.106)
12	(0,0.188,0.09)	(0,0.161,0.119)	(0.147,0,0.133)	(0,0.174,0.106)
13	(0,0.188,0.09)	(0.161,0,0.119)	(0,0.147,0.133)	(0.174,0,0.106)
14	(0,0.188,0.09)	(0.161,0,0.119)	(0,0.147,0.133)	(0,0.174,0.106)
15	(0,0.188,0.09)	(0.161,0,0.119)	(0.147,0,0.133)	(0.174,0,0.106)
16	(0,0.188,0.09)	(0.161,0,0.119)	(0.147,0,0.133)	(0,0.174,0.106)

Table 7.12: Fuse multi-criteria data

Experiment No.	**{PIS, NIS, (PIS, NIS)}**	$\boldsymbol{bet(i)}$
1	(0.329,0.024,0.647)	0.6525
2	(0.188,0.073,0.739)	0.5575
3	(0.188,0.073,0.739)	0.5575
4	(0.188,0.073,0.739)	0.5575
5	(0.437,0,0.563)	0.7185
6	(0.319,0.026,0.655)	0.6465
7	(0.520,0,0.480)	0.7600
8	(0.419,0,0.581)	0.7095
9	(0.174,0.077,0.749)	0.5485
10	(0,0.150,0.850)	0.4250
11	(0.295,0.030,0.675)	0.6325
12	(0.147,0.086,0.767)	0.5305
13	(0.307,0.028,0.665)	0.6395
14	(0.161,0.081,0.758)	0.5400
15	(0.409,0,0.591)	0.7045
16	(0.284,0.033,0.683)	0.6255

Stage 5 (construction of regression model (RM) and final ranking result):

a) We establish the RM by considering the effects of four main criteria and interaction between any two criteria, as in equation 7.22.

$$Y_i = \gamma_0 + \sum_{i=1}^{4} \gamma_i \quad y_i + \sum_{i=1}^{4} \sum_{j=1}^{4} \gamma_{ij} \quad y_i \quad y_j + \epsilon \qquad \text{........ [7.22]}$$

Here, in equation 7.22, Y_i is the relation between RA and relevant experimental results y_i, γ_0 the co-efficient of intercept, γ_i is the first order effect of main factors, γ_{ij} represents two-factor interaction between factor i and j, ϵ is the statistical error as stated beforehand.

b) In order to validate the established RA model, it is analysed with analysis of variance (ANOVA). This can be achieved by Minitab statistical software. The input data to the software are given in table 7.13. The factor levels are the experimental values achieved from deviation degree matrix and the responses are the final DST scores achieved from the experiments. The results of ANOVA are shown in table 7.14. In the table, p-values less than 0.0500 specify significant contributions. The (Adj) R-squared and (Pred) R-squared values are measured as 0.9609 and 0.8640 respectively, which are in reasonable agreement with each other. So, the model is highly suitable to find out the final ranking results of warehouse alternatives for the organisation.

c) Formation of regression equation for final selection of the warehouse as follows in equation 7.23:

$$Y = 0.61861 + 0.03786\,SC1 - 0.04939\,SC2 - 0.02761\,SC3 + 0.04575\,SC4 + 0.01489\,(SC1 * SC3) \quad \ldots\ldots [7.23]$$

The final ranking is exhibited in table 7.15.

Table 7.13: ANOVA Inputs

Experiment No.	Factor levels				Response
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)	DST Scores
1	2.475	1.060	1.768	2.828	0.6525
2	2.475	1.060	1.768	5.590	0.5575
3	2.475	1.060	4.472	5.590	0.5575
4	2.475	1.060	4.472	5.590	0.5575
5	2.475	4.472	1.768	2.828	0.7185
6	2.475	4.472	1.768	5.590	0.6465
7	2.475	4.472	4.472	2.828	0.7600
8	2.475	4.472	4.472	5.590	0.7095
9	3.321	1.060	1.768	2.828	0.5485
10	3.321	1.060	1.768	5.590	0.4250
11	3.321	1.060	4.472	2.828	0.6325
12	3.321	1.060	4.472	5.590	0.5305
13	3.321	4.472	1.768	2.828	0.6395
14	3.321	4.472	1.768	5.590	0.5400
15	3.321	4.472	4.472	2.828	0.7045
16	3.321	4.472	4.472	5.590	0.6255

Table 7.14: ANOVA for Regression Analysis (RA)

Source	Co-efficient	p-value
Intercept	.61861	0
SC 1	.03786	0
SC 2	-.04939	0
SC 3	-.02761	.002
SC 4	.04575	.000
SC1* SC2	-.00276	.572
SC1* SC3	.01489	.022
SC1* SC4	-.00475	.362
SC2* SC3	.00426	.394
SC2* SC4	.00813	.147
SC3* SC4	.00300	.555
(Adj) R-squared	0.9609	
(Pred) R-squared	0.8640	

Table 7.15: The final ranking

Warehouse alternatives	Score	RA-DST Ranking
W1	1.43	4
W2	1.44	3
W3	1.52	1
W4	1.51	2

7.2.3 Experimental results

The proposed approach is pitted with another MCDM tool namely TOPSIS, a universally widely accepted tool by the manufacturing world. So, it will be appropriate to make the final selection decision on the basis of the comparison. The same warehouse selection problem is taken up with identical values of criteria and alternatives. The steps are the following:

Step 1 Decision Matrix construction (table 7.16) by establishing the criteria values (from table 7.1) as TFNs and derivation of comprehensive weights of selection criteria (from table 7.2). As the decision team has ħ persons, the weights of criteria as well as values of warehouse choices are calculated as given in equation 7.24 and equation 7.25.

$$q_{ij} = (q_{ij}^1 + q_{ij}^2 + \cdots + q_{ij}^k)/ħ \qquad \text{........ [7.24]}$$

$$w_j = (w_j^1 + w_j^2 + \cdots + w_j^k)/ħ \qquad \text{........ [7.25]}$$

Where, i=no. of warehouse alternatives, j= no. of selection criteria.

Step 2. Normalising the decision matrix to ensure data integrity and eliminating data redundancy and presented in table 7.17.

In case of fuzzy data represented by triangular fuzzy numbers say, (a_{ij}, b, c_{ij}), the normalization measures for benefit and cost criteria are calculated in equation 7.26 and equation 7.27, respectively.

$$r_{ij} = (a_{ij}/c_j^*, b_{ij}/c_j^*, c_{ij}/c_j^*) \qquad \text{........ [7.26]}$$

$$r_{ij} = (a_j^-/c_{ij}, a_j^-/b_{ij}, a_j^-/a_{ij}) \qquad \text{........ [7.27]}$$

Where, c_j^*= max (c_{ij}), a_j^-= min (a_{ij}).

Step 3. Determining the ideal solution as $r_j^* = (1,1,1)$ and negative ideal solution as $r_j^- = (0,0,0)$.

Step 4. The weighted distances of each warehouse alternative from ideal and anti-ideal solution are presented in table 7.18. These can be calculated as given in equation 7.28 and equation 7.29 as follows:

$$d_i^+ = \sum[\frac{1}{2}\{\max\ (w_{lj}\mid a_{ij}-1\mid, w_{nj}\mid c_{ij}-1\mid) + w_{mj}\mid b_{ij}-1\mid\}] \quad \text{........ [7.28]}$$

$$d_i^- = \sum[\frac{1}{2}\{\max\ (w_{lj}\mid a_{ij}-0\mid, w_{nj}\mid c_{ij}-0\mid) + w_{mj}\mid b_{ij}-0\mid\}] \quad \text{........ [7.29]}$$

Where, $w_j = (w_{aj}, w_{bj}, w_{cj})$ is the aggregate fuzzy criteria weight.

Step 5. The proxomity co-efficient is measured as in equation 7.30 as follows:

$$PN_i = d_i^- / (d_i^+ + d_i^-) \quad \text{........ [7.30]}$$

Table 7.16: Fuzzy decision matrix (TOPSIS)

Warehouse alternatives	**SC**			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
W1	(5.5, 7.5, 9)	(6.5, 8.25, 9.5)	(4.5, 6.5, 8.5)	(8, 9.5, 10)
W2	(5.5, 7.5, 9)	(6.5, 8.5, 9.75)	(7, 9, 10)	(4, 6, 7.75)
W3	(5.5, 7.5, 9)	(8.5, 9.75, 10)	(7.5, 9.25, 10)	(7, 9, 10)
W4	(7.5, 9, 9.75)	(4, 6, 8)	(4, 6, 8)	(3.5, 5.5, 7.5)
Weight	(0.55, 0.75, 0.95)	(0.45, 0.65, 0.85)	(0.4, 0.6, 0.8)	(0.5, 0.7, 0.9)

Table 7.17: Fuzzy normalized decision matrix (TOPSIS)

Warehouse alternatives	**SC**			
	SC1 (-)	SC2 (+)	SC3 (+)	SC4 (-)
W1	(0.61,0.73,1)	(0.65,0.83,0.95)	(0.45,0.65,0.85)	(0.35,0.37,0.44)
W2	(0.61,0.73,1)	(0.65,0.85,0.98)	(0.7,0.9,1)	(0.45,0.58,0.88)
W3	(0.61,0.73,1)	(0.85,0.98,1)	(0.75,0.93,1)	(0.35,0.39,0.5)
W4	(0.56,0.61,0.73)	(0.4,0.6,0.8)	(0.4,0.6,0.8)	(0.47,0.64,1)
Weight	(0.55, 0.75, 0.95)	(0.45, 0.65, 0.85)	(0.4, 0.6, 0.8)	(0.5, 0.7, 0.9)

Table 7.18: Distance measurements of alternatives and ranking (TOPSIS)

Warehouse alternatives	d_i^+	d_i^-	PN_i	Ranking
W1	1.08	2.29	0.6795	4
W2	0.741	2.67	0.7827	1
W3	0.735	2.542	0.7757	2
W4	1.04	2.285	0.6872	3

The final ranking result of TOPSIS is based on propinquity coefficient (PN_i), and, is given in table 7.18. The final ranking is slightly different in TOPSIS than that in DST-DOE method. In DST-DOE method, warehouse alternative W3 gets the highest ranking. In TOPSIS, the same gets the second place, although, the difference with the first place is too marginal. The first ranked alternative in TOPSIS i.e. W2 gets the third rank in DST-DOE method. This little difference in ranking is due to the fact that, DST-DOE method considers the influence of interactions between the criteria. According to the p-values in ANOVA, the interaction between SC1 and SC3 has significant influence on the final selection decision. On the other hand, TOPSIS is a simpler method considering the individual factors alone and not the influences originating from the interactions of more than one factor. The final outcome of the comparison and logic behind it proves that, the DST-DOE method proposed here is a rational, robust and efficient one and can extensively be used for making different selection decisions in the world of design and manufacturing.

7.3. Conclusion

In this chapter, a hybrid DST-DOE approach has been proposed to solve a complex MCDM problem associated with fuzziness and uncertainty. The integration of DST- DOE and comparing the results with TOPSIS has made the model a distinguished one. The advantages of this model are simplicity and ease of use. It can handle the ambiguity and vagueness inherent in these kinds of MCDM problems in a smooth manner. It uses TFNs and defuzzify them to generate crisp data. It uses interval valued numbers and transforms the same into deviation degree matrix to get a simpler form of the problem for the sake of calculation. On adding or removing any alternative, the DAs don't need to take up the calculation from scratch. Instead, they could use the developed RM to obtain the final outcome, thereby, saving precious time and design cost as well.

The end result goes to verify the capability and robustness of our proposed method. So, it can be concluded that, the proposed DST-DOE method is very much applicable to solve complex fuzzy MCDM problems associated with uncertainty in real world applications. It provides manufacturing organizations an edge in the competitive world. Future scopes include use of other DOE as 2^n factorial design got the inability to distinguish between linear and higher order factor effects, solving different types of engineering problems involving optimum decision making, integration of DOE with some other well established MCDM tool, the effects of addition or removal of particular selection criteria on the outcome as well.

8. Case Study: ANALYSIS OF DIFFERENT MCDM METHODS FOR SUPPLIER EVALUATION IN A SUPPLY CHAIN MANAGEMENT

8.1. Methodology: Fuzzy Analytic Hierarchy Process (FAHP), K-means Clustering.

8.1.1 Problem Definition

Current world of manufacturing works efficiently on a resilient supply chain (figure 8.1.).

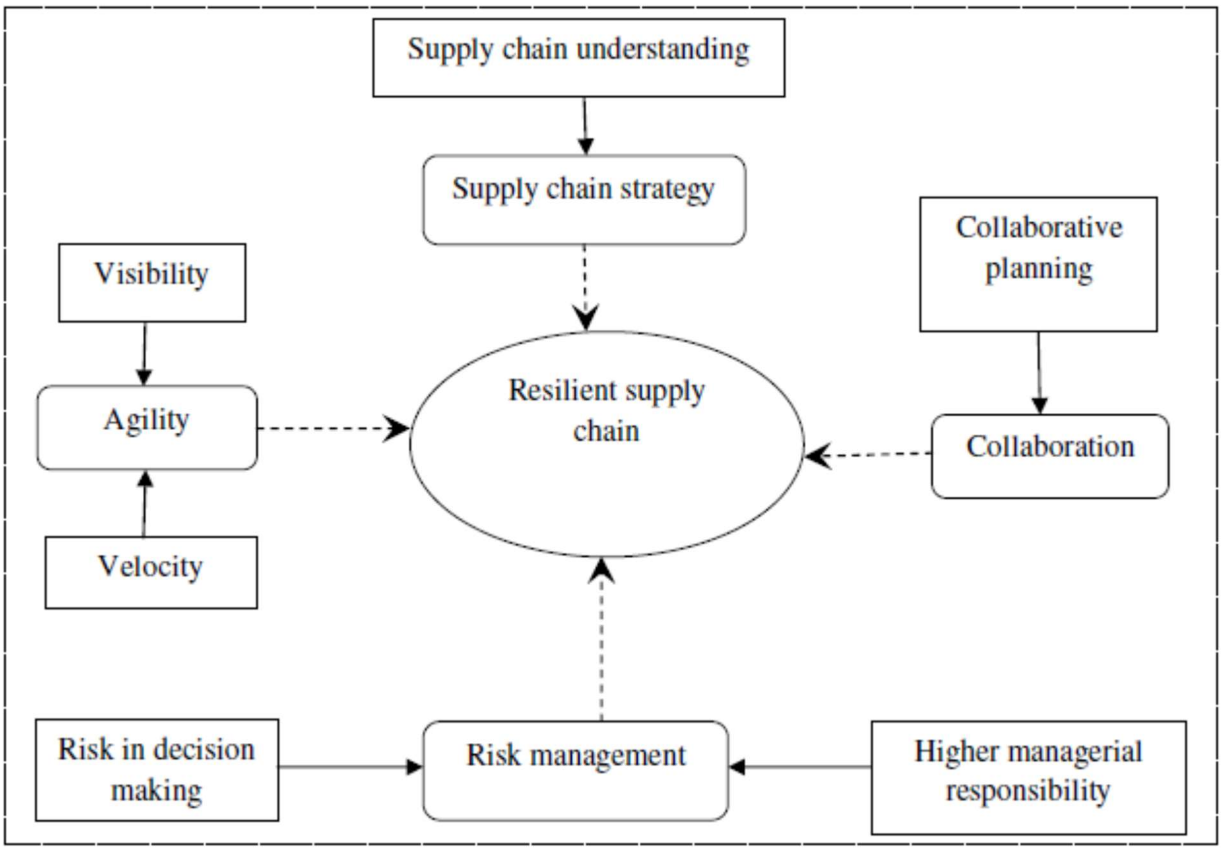


Figure 8.1: Resilient supply chain framework

Choice of appropriate supplier can grow business to a great extent. Otherwise vulnerability in the supply chain occurs. This is not desirable for any manufacturing organization. A manufacturing organization might excel or might go down owing to the performance of supplier. That leads to the problem of appropriate supplier selection for an organization.

The current chapter presents a paraphrasing of supplier selection problem in volatile market environment. Three experts in a group are given the responsibility in selecting the most appropriate supplier amongst the five possible choices. They have the experience and expertise in their respective fields. The profile of experts is shown in figure 8.2. They look for factors such as technological soundness, market awareness, goodwill, responsiveness, adherence to quality in the suppliers. They make use of their judgments in linguistic forms to cope with the uncertainty and fuzziness. Some suitable scale is defined to convert the same into trapezoidal fuzzy numbers (TrFN). Linguistic values like extreme insignificance, insignificance, equal significance, moderate significance and extreme significance are used in determining factor weights.

Expert (E)

			
Age	45 years	56 years	52 years
Qualification	Doctorate in Philosophy	Master of Business Administration	Master of Engineering
Experience	20 years in industry	32 years in industry	26 years in education & industry combined
Adroitness	 Supply Chain Management	 Finance Management	 Production Management

Figure 8.2: Profile of Experts

A conversion scale in the range from 0 to 1 is borne by the experts. Similarly, importance of alternatives takes values from 1 to 10 in the form of linguistic. Multi-criteria methods include fuzzy AHP in factor weight compilation. Subsequently K-means is used to process the supplier choices against the factors to form clustering in ranking cycle. Lastly, cost of supplier goodwill is considered as the objective factor measure (OFM) to find out the supplier selection index for the most optimum supplier. The cognitive mind of individual expert plays a keen role here. It is quite similar to a tip of iceberg as discussed in earlier chapter. The experts introduce cognition co-efficient (C) to balance between subjective factor and objective factor. An optimistic expert may choose a higher value of C, whereas, a pessimistic one likes to put a lower value. The cognitive navigation **(Dey et. al., 2017)** of an individual expert is given in figure 8.3.

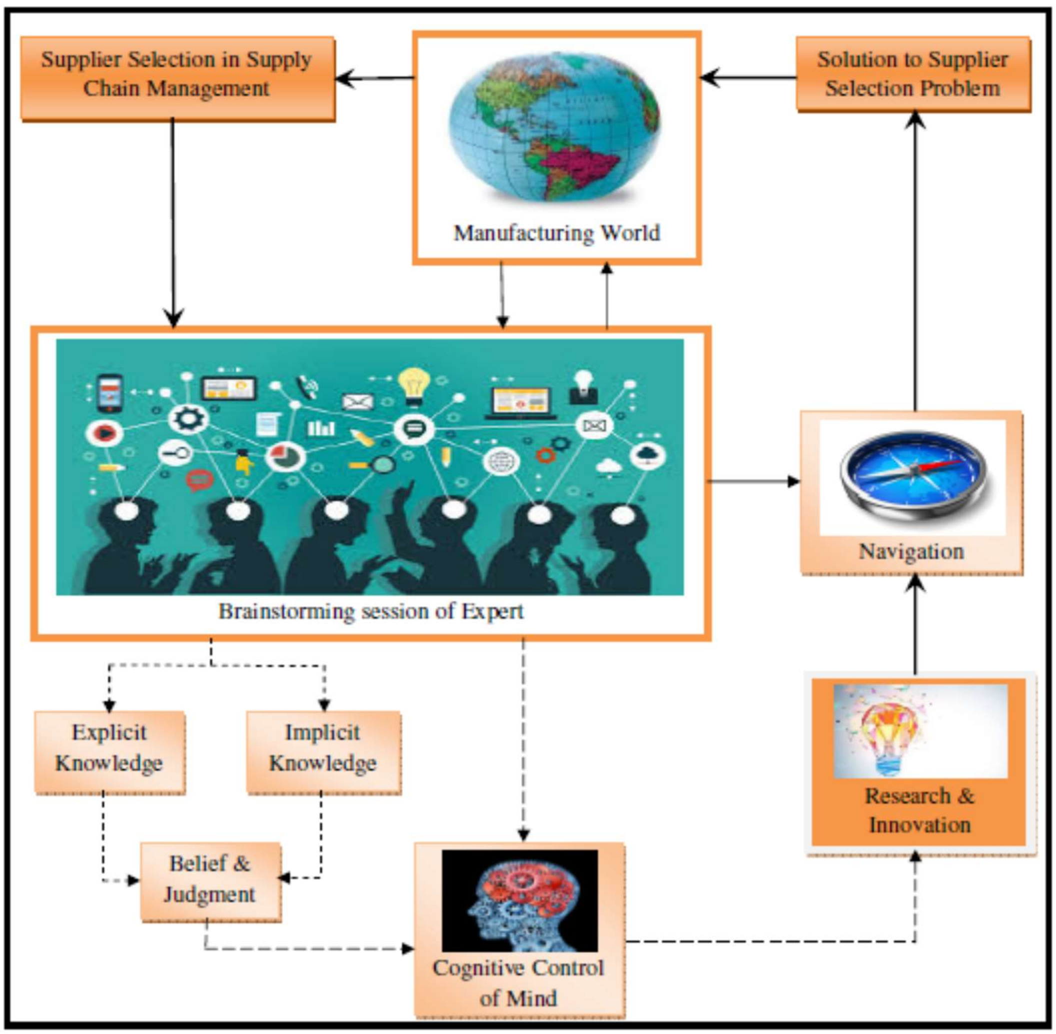


Figure 8.3: Cognitive navigation of individual expert

Novelty in the proposed approach lies in the fact that, it makes an integration of FAHP and K-means clustering. It incorporates the cognitive mind of decision maker into the model and post optimality check comes into play for the validation and robustness of the chosen supplier.

8.1.2 Background research

This section depicts the background research leading to the present model. It gives some insight on the likes of fuzzy decision matrix, linguistic weight set, TrFNs, defuzzification and α-cut, FAHP, K- means respectively that helps developing a thorough understanding about the background research of the proposed model.

8.1.2.1 Fuzzy decision matrix

A fuzzy preference of an expert is presented in a decision matrix in a supplier evaluation problem. It provides an overview about the preference values of suppliers with respect to the factors influencing the selection. The preference values are in linguistic weight set in the first place and are transformed into TrFNs eventually. The matrix takes the following form:

$$\tilde{A} = \begin{array}{c c} \mathit{Supplier\ Alternative/Factor} & \begin{matrix} \mathrm{F1} & \mathrm{F2} & \dots & \mathrm{F}n \end{matrix} \\ \begin{matrix} \mathrm{S1} \\ \mathrm{S2} \\ \vdots \\ Sm \end{matrix} & \begin{bmatrix} \psi_{11} & \psi_{12} & \dots & \psi_{1n} \\ \psi_{21} & \psi_{22} & & \psi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{m1} & \psi_{m2} & \cdots & \psi_{mn} \end{bmatrix} \end{array}$$

$where, \psi_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ are the TrFNs; $m{:}\ number\ of\ suppliers$; $n{:}\ number\ of\ criteria.$

8.1.2.2 Linguistic weight set and TrFNs

In general, multi-criteria problems are associated with uncertainty and vagueness. Linguistic weight set could eliminate the fuzziness and uncertainty by the use of linguistic variables. An expert in the field has to deal with subjective and objective factors that could possibly influence a selection decision. While, the objective factors can be represented by crisp values, it is not the case for the subjective factors. Linguistic variable such as 'significance' or 'importance' plays a pivotal role in adapting to the situation. These variables could further be processed by TrFNs **(Chan et. al., 2000)** in representing opinion of experts. There are some ordinal approaches as well, which are not based on TrFNs. Algorithms based on sentiment analysis with multi-granular fuzzy linguistic modeling **(Morente-Molinera et. al., 2018)**, unbalanced fuzzy linguistic information **(Cabrerizo et. al., 2015)** have been developed in the recent times for the representation of user information.

The present paper expressed a TrFN as a quadruplet $\psi = (a, b, c, d)$ as shown in figure 8.4.

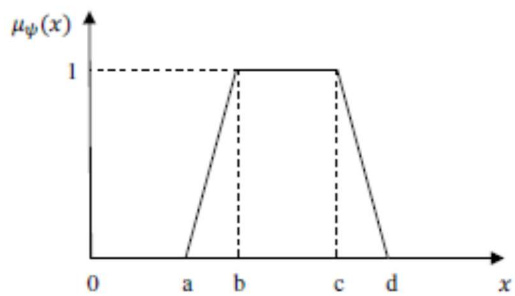


Figure 8.4: Representation of TrFN

A fuzzy set ψ is defined as $\psi = \{(x, \mu_\psi(x)) : x \in X\}$; where, X:Universe of discourse. The membership function $\mu_\psi(x): X \to [0,1]$ is defined as in equation 8.1 **(Kauffman & Gupta, 1985)**:

$$\mu_\psi(x) = \begin{cases} 0, & x < a \\ \frac{(x-a)}{(b-a)}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{(d-x)}{(d-c)}, & c \le x \le d \\ 0, & x > d \end{cases} \qquad \text{........ [8.1]}$$

Thus variables in the linguistic weight set can be represented by trapezoidal fuzzy scaling. The upper and lower limits of such a weight set could be extreme significance = (0.7, 0.8, 0.9, 1) and extreme insignificance = (0, 0, 0.1, 0.2). This is a conversion scale defined by experts that could take values between 0 and 1. Some other conversion scale could be defined to take the values between 0 and 10. The upper and lower limits of such a weight set could be extremely high importance = (9, 9, 10, 10) and extremely low importance = (1, 2, 3, 4).

8.1.2.3 Defuzzification and α-cut

The TrFNs need to be defuzzified for crisp estimation of experts at some later stage of the problem solving after overcoming the uncertainty present in the initial stage. The α-cut, $\alpha \in (0,1)$ is a very handful method in doing so, as it minimizes the loss of information going forward. It is a crisp set defined as $\psi\ (\alpha) = x \in$ R: $\psi(x) \geq \alpha$, where, $\psi\ (\alpha)$ is a closed interval of the form$[\psi_L(\alpha), \psi_U(\alpha)]$, R is a real line and TrFN ψ is a subset R; $\psi: R \rightarrow [0,1]$ **(Savitha & George, 2017)**. Some operations on the TrFNs are presented in the following section **(Vahidi & Rezvani, 2013)**.

If $\psi_1 = (a, b, c, d)$ and $\psi_2 = (p, q, r, s)$are the two TrFNs, then

a) Addition of $\psi_1$and ψ_2 is by following equation 8.2:

$$\psi_1 \oplus \psi_2 = [a + p, b + q, c + r, d + s] \qquad \text{......... [8.2]}$$

b) Subtraction of $\psi_1$and $\psi_2$is by equation 8.3:

$$\psi_1 \ominus \psi_2 = [a - s, b - r, c - q, d - p] \qquad \text{......... [8.3]}$$

c) The value of a TrFN $\psi = (a, b, c, d)$ is given by equation 8.4:

$$val\ (\psi) = (\frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}) \qquad \text{......... [8.4]}$$

d) The α-cut interval of a TrFN $\psi = (a, b, c, d)$ is given by equation 8.5:

$$\psi\ (\alpha) = [\psi_L(\alpha), \psi_U(\alpha)] = [a + \alpha(b - a), d - \alpha(d - c)] \qquad \text{......... [8.5]}$$

e) The multiplication is as follows in equation 8.6:

$$\psi_1(\alpha) \otimes \psi_2(\alpha) = [\psi_{1L}(\alpha)\psi_{2L}(\alpha), \psi_{1U}(\alpha)\psi_{2U}(\alpha)] \qquad \text{......... [8.6]}$$

$where, \psi_1\ (\alpha) = [\psi_{1L}(\alpha), \psi_{1U}(\alpha)\ and\ \psi_2\ (\alpha) = [\psi_{2L}(\alpha), \psi_{2U}(\alpha)$ are the α-cuts of the TrFNs.

When, $\alpha = 0$, i.e. $\psi_1(0) \otimes \psi_2(0) = [ap, ds]$;

When, $\alpha = 1$, i.e. $\psi_1(1) \otimes \psi_2(1) = [bq, cr]$;

The approximated multiplication of $\psi_1$and $\psi_2$is as in equation 8.7:

$$\psi_1 \otimes \psi_2 = [ap, bq, cr, ds] \qquad \text{......... [8.7]}$$

f) The division of $\psi_1$and ψ_2 is as in equation 8.8:

$$\psi_1 \oslash \psi_2 = [\frac{a}{p}, \frac{b}{q}, \frac{c}{r}, \frac{d}{s}] \qquad \text{......... [8.8]}$$

8.1.2.4 Analytic hierarchy process (AHP) and FAHP

AHP was first introduced in 1980 as an aid in decision science to decipher complex unstructured problems in different areas of science, engineering, economics and management. It can handle both qualitative and quantitative predictions given by experts. It helps analyse and organize a decision problem into a hierarchy much like a family tree. The main stem of AHP is pair wise comparison matrix which is a $n \times n$ square matrix. It takes the following form:

$$\begin{array}{cc} Factor/Factor & \begin{array}{cccc} \mathrm{F1} & \mathrm{F2} & \ldots & \mathrm{F}n \end{array} \\ \mathrm{P} = \begin{array}{c} \mathrm{F1} \\ \mathrm{F2} \\ \vdots \\ \mathrm{F}n \end{array} & \begin{bmatrix} 1 & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & 1 & & \beta_{2n} \\ \vdots & \vdots & 1 & \vdots \\ \beta_{n1} & \beta_{n2} & \cdots & 1 \end{bmatrix} \end{array}$$

Where, n = number of factors evaluated, β_{ij} = pairwise preference values of factors

The FAHP is the advancement over traditional AHP, where the pair wise preference values are by means of variables in the linguistic weight set. These are converted into TrFNs by means of trapezoidal fuzzy scaling predefined by experts. Here, in the proposed approach, FAHP has been utilized to obtain the fuzzy pair wise comparison matrix and elimination of uncertainty in expert judgment. Then, these values are processed by arithmetic operations to calculate the weight vector of factors in a crisp form.

8.1.2.5 K-means clustering

The idea of K-means clustering was first conceptualised by **Steinhaus (1957)**. Although the term 'K-means' was introduced much later **(MacQueen, 1967)**. It was proposed by **Lloyd (1957)** and officially published by **Forgy (1965)**.

It is a method of vector quantization, taken originally from pulse-code modulation in signal processing. It is famous for analysis of cluster in data mining. It aims to break-up n observations into K numbers of clusters. Each of the observations is assigned to a cluster with the closest cluster centroid.

This iterative K-means algorithm is also known as Lloyd's algorithm. It follows two steps alternatively. The first one assigns observation to cluster, according to minimum Euclidean distance measurement. And this, intuitively, is the nearest mean. Since, the square root is a monotone function, this is a minimum Euclidean distance assignment.

The second step checks the appropriateness of assignment of observations into clusters by taking the minimum Euclidean distances of observations from different clusters.

The algorithm converges when the assignment of observations into clusters no longer differ.

8.2 The proposed model

The 3-phase framework of the proposed model is shown in figure 8.5.

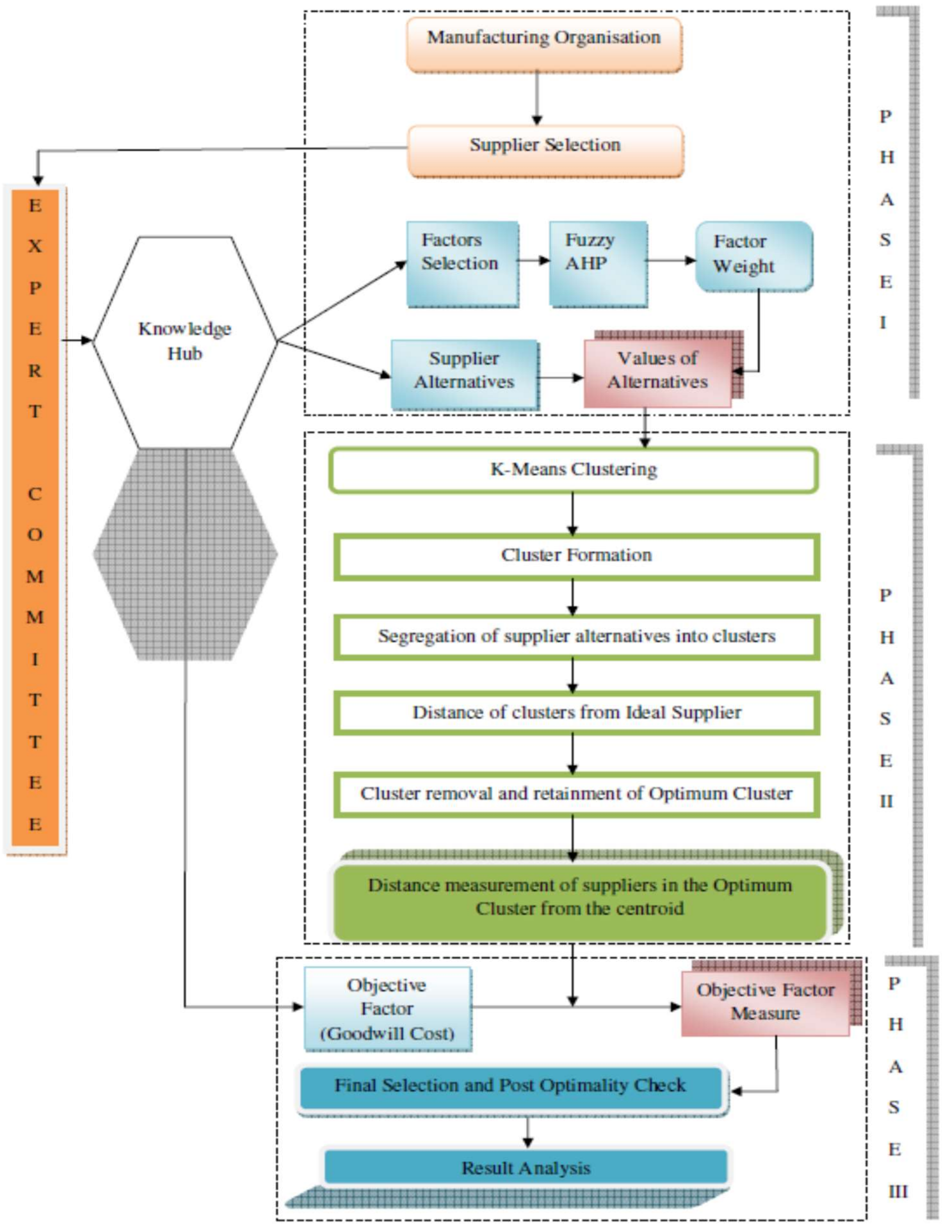


Figure 8.5: Flowchart of the proposed model

Phase I is dedicated to determining the factor weights by FAHP and weights of supplier alternatives by means of fuzzy decision matrix. Phase II works on clustering of suppliers, removal of bad clusters and retainment of optimal clusters. The objective factor is introduced in phase III and final results are analysed after a post optimality check. The operational procedure is as follows:

Step1. Denotation of linguistic weight set by experts in terms of TrFNs.

Step2. Formation of pair wise comparison matrix by FAHP and conversion of the same into crisp data by following equation 8.4.

Step3. Determination of factor weights from the comparison matrix.

Step4. Formation of principal decision matrix in terms of linguistic weight set. Conversion of the same into TrFNs.

Step5. Normalization of the matrix by following equation 8.19 and equation 8.10 for beneficial and non-beneficial factors respectively, as follows:

$$\widetilde{\psi_{ij}} = ({}^{p_{ij}}/_{s_j^*}, {}^{q_{ij}}/_{s_j^*}, {}^{r_{ij}}/_{s_j^*}, {}^{s_{ij}}/_{s_j^*}); j \in Y \qquad \text{......... [8.9]}$$

$$\widetilde{\psi_{ij}} = \left({}^{p_j^-}/_{s_{ij}}, {}^{p_j^-}/_{r_{ij}}, {}^{p_j^-}/_{q_{ij}}, {}^{p_j^-}/_{p_{ij}}\right); j \in Z \qquad \text{......... [8.10]}$$

Where, $s_j^* : \max(s_{ij}), p_j^- : \min p_{ij},\ i = no.of\ alternatives, j = no.of\ criteria, Y = set\ of\ beneficial\ criteria, Z = set\ of\ non-beneficial\ criteria.$

Step6. Conversion of normalised decision matrix into 2-level data by α-cut method following equation 8.5 and subsequently finding weighted normalised defuzzified data.

Step7. Formation of clusters by determining the clustering points from the crisp decision matrix in previous step. Assignment of alternatives to clusters by minimum Euclidean distance measurement.

Step8. Calculation of new means of the clusters to be the centroids.

Step9. Euclidean distance measurement of alternatives from cluster centroids. A particular alternative belongs to that cluster from which the distance is minimum. This is to check the assignments of alternatives into clusters in the previous step.

Step10. Distance measurement of clusters (K_i) from the best alternative (1, 1, 1, 1, 1). Minimum distance corresponds to the best cluster. Alternatives in the best cluster are processed further.

Step11. Introduction of Goodwill Cost (GC_i) of suppliers as objective factor in the problem. Calculation of objective factor measure (OFM_i) of alternatives by equation 8.11 as follows:

$$OFM_i = [GC_i \times \Sigma(^1/_{GC_i})]^{-1} \qquad \text{......... [8.11]}$$

Step12. Final selection according to the supplier selection index (SSI_i) as in equation 8.12. The higher the value, the better the ranking.

$$SSI_i = \mathrm{C}\left(K_i^{-1}\right) + (1 - \mathrm{C})(OFM_i) \qquad \text{......... [8.12]}$$

Where C is the co-efficient of cognition as mentioned earlier.

8.2.1 A numerical analysis of the proposed model

A manufacturing organization is looking for selection and evaluation of its supplier from a set of alternatives. Considering the market competitiveness in the present scenario, they engage three experts of repute with vast expertise and experience. Their observations are given equal weightage. They choose five factors namely technological soundness (TS), market awareness (MA), goodwill (G), responsiveness (R) and adherence to quality (AQ). Five suppliers' alternatives are shortlisted at the first place. The fuzziness present in the problem is eliminated by the use of linguistic weight set and corresponding trapezoiodal fuzzy scaling, devised by judgments of experts. It is nothing but a conversion scale comprising of TrFNs. Conversion scale for factor weights and weights of alternatives are shown in table 8.1 and table 8.2 respectively. Pair wise comparison matrix in AHP is given by table 8.3 in terms of linguistic weight set. Table 8.4 gives the compiled data for factor weights in fuzzy form.

Table 8.1: Presentation of TrFNs for factor weights (judgment by experts)

Linguistic weight set	**Trapezoidal fuzzy scaling**
Extreme Insignificance (EI)	(0,0,0.1,0.2)
Insignificance (I)	(0.1,0.2,0.3,0.4)
Equal Significance (ES)	(0.3,0.4,0.5,0.6)
Moderate Significance (MS)	(0.5,0.6,0.7,0.8)
Extreme Significance (ExS)	(0.7,0.8,0.9,1)

Table 8.2: Presentation of TrFNs for weights of alternatives (judgments by experts)

Linguistic weight set	**Trapezoidal fuzzy scaling**
Extremely High Importance (EHI)	(9,9,10,10)
Very High Importance (VHI)	(8,9,10,10)
High Importance (HI)	(7,8,9,9)
Medium Importance (MI)	(6,6,7,8)
Low Importance (LI)	(5,5,6,7)
Very Low Importance (VLI)	(3,4,5,6)
Extremely Low Importance (ELI)	(1,2,3,4)

Table 8.3: AHP Pair wise comparison matrix of factors by experts

Expert	E1					E2					E3				
Factor	TS	MA	G	R	AQ	TS	MA	G	R	AQ	TS	MA	G	R	AQ
TS	ES	EI	I	ExS	ExS	ES	I	I	MS	MS	ES	EI	I	ExS	EI
MA	ExS	ES	ES	I	MS	MS	ES	ExS	EI	ES	ExS	ES	MS	I	MS
G	MS	ES	ES	EI	EI	MS	EI	ES	I	EI	MS	I	ES	EI	EI
R	EI	MS	ExS	ES	I	I	ExS	MS	ES	I	EI	MS	ExS	ES	EI
AQ	EI	I	ExS	MS	ES	I	ES	ExS	MS	ES	ExS	I	ExS	ExS	ES

Table 8.4: Fuzzy compiled data for factor weights

Factor	TS	MA	G	R	AQ
TS	(.3,.4,.5,.6)	(.033,.067,.167,.267)	(.1,.2,.3,.4)	(.633,.733,.833,.933)	(.4,.467,.567,.667)
MA	(.633,.733,.833,.933)	(.3,.4,.5,.6)	(.5,.6,.7,.8)	(.067,.133,.233,.333)	(.433,.533,.633,.733)
G	(.5,.6,.7,.8)	(.133,.2,.3,.4)	(.3,.4,.5,.6)	(.033,.067,.167,.267)	(0,0,.1,.2)
R	(.033,.067,.167,.267)	(.567,.667,.767,.867)	(.633,.733,.833,.933)	(0.3,0.4,0.5,0.6)	(.067,.133,.233,.333)
AQ	(.267,.333,.433,.533)	(.167,.267,.367,.467)	(.7,.8,.9,1)	(.567,.667,.767,.867)	(0.3,0.4,0.5,0.6)

Table 8.5 gives a crisp estimation of factor weights by following equation 8.4. Principal decision matrix of supplier evaluation is presented in table 8.6 in linguistic weight set. It exhibits the judgment of all individual experts. Table 8.7 exhibits the judgment of experts in the form of TrFNs.

Table 8.5: Crisp estimation for factor weights

Factor	TS	MA	G	R	AQ
TS	.45	.128	.25	.783	.523
MA	.783	.45	.65	.189	.583
G	.65	.256	.45	.128	.067
R	.128	.717	.783	.45	.189
AQ	.389	.317	.85	.717	.45
Factor Weights (W_F)	0.175	0.284	0.104	0.198	0.301

Table 8.6: Principal decision matrix (linguistic)

		Expert (E)		
Factor	**Supplier**	**E1**	**E2**	**E3**
TS (+ve)	S1	HI	HI	HI
	S2	VHI	VHI	VHI
	S3	EHI	EHI	VHI
	S4	VHI	VHI	VHI
	S5	HI	HI	HI
MA (+ve)	S1	HI	HI	EHI
	S2	EHI	EHI	EHI
	S3	EHI	VHI	VHI
	S4	VHI	VHI	HI
	S5	HI	VHI	VHI
G (+ve)	S1	VHI	VHI	VHI
	S2	EHI	EHI	EHI
	S3	EHI	EHI	VHI
	S4	HI	HI	VHI
	S5	HI	HI	HI
R (+ve)	S1	VHI	VHI	VHI
	S2	VHI	EHI	EHI
	S3	EHI	EHI	EHI
	S4	VHI	VHI	VHI
	S5	HI	HI	VHI
AQ (+ve)	S1	VHI	VHI	VHI
	S2	EHI	EHI	HI
	S3	HI	HI	EHI
	S4	VHI	VHI	EHI
	S5	HI	HI	HI

Table 8.7: Decision matrix combining expert judgments

Supplier	Factor				
	TS	MA	G	R	AQ
S1	(7,8,9,9)	(7.67,8.33,9.33,9.33)	(8,9,10,10)	(8,9,10,10)	(8,9,10,10)
S2	(8,9,10,10)	(9,9,10,10)	(9,9,10,10)	(8.67,9,10,10)	(8.33,8.67,9.67,9.67)
S3	(8.67,9,10,10)	(8.33,9,10,10)	(8.67,9,10,10)	(9,9,10,10)	(7.637,8.33,9.33,9.33)
S4	(8,9,10,10)	(7.67,8.67,9.67,9.67)	(7.33,8.33,9.33,9.33)	(8,9,10,10)	(8.33,9,10,10)
S5	(7,8,9,9)	(7.67,8.67,9.67,9.67)	(7,8,9,9)	(7.33,8.33,9.33,9.33)	(7,8,9,9)

The matrix in table 8.7 is normalized by following equation 8.9 and equation 8.10 for maximized and minimized factors respectively. The same is presented in table 8.8. One α-cut method given in equation 8.5 is used to defuzzify the matrix and it is converted into 2-levelk data as in table 8.9. The data is then converted into crisp ones by simple arithmetic mean and weighted crisp estimation is presented in table 8.10.

Table 8.8: Normalized matrix

Supplier	Factor				
	TS	MA	G	R	AQ
S1	(.7,.8,.9,.9)	(.767,.833,.933,.933)	(.8,.9,1,1)	(.8,.9,1,1)	(.8,.9,1,1.0)
S2	(.8,.9,1,1)	(.9,.9,1,1)	(.9,.9,1,1)	(.867,.9,1,1)	(.833,.867,.967,.967)
S3	(.867,.9,1,1)	(.833,.9,1,1)	(.867,.9,1,1)	(.9,.9,1,1)	(.767,.833,.933,.933)
S4	(.8,.9,1,1)	(.767,.867,.967,.967)	(.733,.833,.933,.933)	(.8,.9,1,1)	(.833,.9,1,1)
S5	(.7,.8,.9,.9)	(.767,.867,.967,.967)	(.7,.8,.9,.9)	(.733,.833,.933,.933)	(.7,.8,.9,.9)

Table 8.9: Defuzzification into 2-level data by α-cut

Supplier	Factor				
	TS	MA	G	R	AQ
S1	(0.72,0.9)	(0.78,0.933)	(0.82,1)	(0.82,1)	(0.82,1)
S2	(0.82,1)	(0.9,1)	(0.9,1)	(0.874,1)	(0.84,0.967)
S3	(0.874,1)	(0.846,1)	(0.874,1)	(0.9,1)	(0.78,0.933)
S4	(0.82,1)	(0.787,0.967)	(0.753,0.933)	(0.82,1)	(0.846,1)
S5	(0.72,0.9)	(0.787,0.967)	(0.72,0.9)	(0.753,0.933)	(0.72,0.9)
Factor Weights (W_F)	0.175	0.284	0.104	0.198	0.301

Table 8.10: Weighted normalized data

Supplier	Factor				
	TS	MA	G	R	AQ
S1	.142	.243	.095	.180	.274
S2	.159	.270	.099	.186	.272
S3	.164	.262	.097	.188	.258
S4	.159	.249	.088	.180	.278
S5	.142	.249	.084	.167	.244

The experts then divide the alternatives into 2 clusters as shown in figure 8.6.

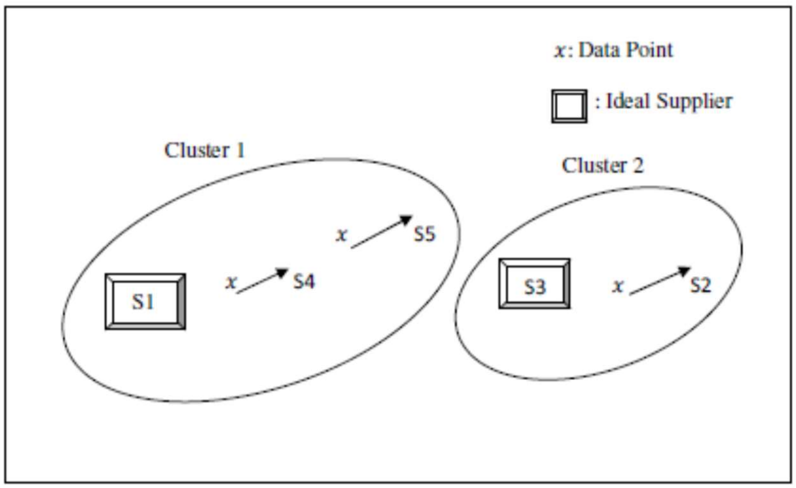


Figure 8.6: Segregation of supplier alternatives into clusters according to distance measurement

For cluster 1, they take S1 as the reference supplier. For cluster 2, S3 is taken as the reference supplier. The clustering results after distance measurement is shown in table 8.11.

Table 8.11: Clustering result

Supplier	Clustering Point	Distance Measurement		Cluster Selection
		From cluster 1	From cluster 2	
S1	(.142, .243, .095, .180, .274)	0	.03419	1
S2	(.159, .270, .099, .186, .272)	.03277	.01918	2
S3	(.164, .262, .097, .188, .258)	.03419	0	2
S4	(.159, .249, .088, .180, .278)	.01975	.02853	1
S5	(0.142,0.249,0.084,0.167,0.244)	0.03501	0.03819	1

The results of table 8.11 reveal that suppliers S1, S2, S4 belong to cluster 1 and suppliers S2, S3 belong to cluster 2. Next, centroid points of the clusters are found out by taking the mean of all points in each cluster. They are found to be (0.148, 0.247, 0.089, 0.176, 0.265) and (0.162, 0.266, 0.098, 0.187, 0.265) for cluster 1 and cluster 2 respectively. The distances of all the suppliers from cluster centroids are measured and presented in table 8.12.

Table 8.12: Verification of clustering result

Supplier	**Distance Measurement**		Cluster Selection
	From centroidal point of cluster 1	From centroidal point of cluster 2	
S1	0.01360	0.03268	1
S2	0.02998	0.00872	2
S3	0.02717	0.00843	2
S4	0.01764	0.02482	1
S5	0.02423	0.04155	1

Results show that suppliers S1, S4, S5 are having minimum distances from centroid of cluster 1 and suppliers S2, S3 are nearer to the centroid point of cluster 2. This exactly matches with the result in table 8.11. So, allocations of suppliers in the clusters are optimum (figure 8.7).

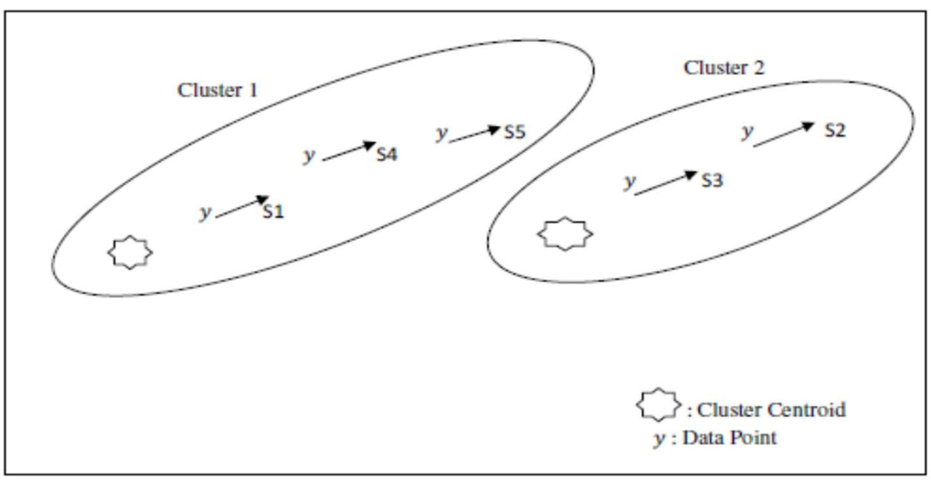


Figure 8.7: Cluster checking result

Next, the distances of the clusters centroids from the ideal solution (1, 1, 1, 1, 1) are found out. These are found to be 1.8281 and 1.8043 for cluster 1 and cluster 2 respectively. The values reveal that cluster 2 is the closest to ideal solution and hence, the best cluster. We have suppliers S2 and S3 in cluster 2. Their distances from cluster centroiod are measured and an initial ranking is established (table 8.13) based on that.

Table 8.13: Initial result

Supplier	Distance from the optimum cluster centre (K_i)	Ranking based on clustering result
S2	0.00872	2nd
S3	0.00843	1st

According to the initial ranking, S3 is the highest ranked supplier followed by S2. The experts complete a survey to find out the goodwill cost of supplier (GC) and introduce the same as an objective factor in the problem as in table 8.14.

Table 8.14: Final Ranking

Supplier	Subjective Factor Measure $(K_i)^{-1}$	Goodwill Cost of Supplier (GC) (thousands of $)	Objective Factor Measure (OFM_i)	Supplier Selection Index (SSI_i)	**Final Ranking Result**
S2	114.679	8.43	0.5058	89.330	2nd
S3	118.624	8.64	0.4936	92.399	1st

They utilize their knowledge and experience to set the value ofC, the co-efficient of cognition, at 0.778 unanimously. The co-efficient of cognition is a measure of positivity or optimistic mind. The higher the value, the optimistic mind a person possesses. It also establishes a relationship between subjective factor measure and objective factor measure in the problem as in equation 8.17. A supplier selection index is measured by the equation. The final ranking is found out by integrating objective factor and subjective factor of supplier selection. The ranking establishes S3 as the optimum supplier in the context of the present problem scenario.

8.2.1.1 Post optimality check

Post optimality check, as the name suggests, is carried out to measure the robustness of the problem with the alterations in alternative, selection factors, influence of objective factor. In general, multi-criteria decision problems are associated with uncertainty and fuzziness. Uncertainty present in the input data level is managed by using linguistic weight set and TrFNs. But the same present in the design level is managed by post optimality check. In the present problem, post optimality check is carried out to get the influence of objective factor on selection of supplier alternatives. It finds a feasible range of C, the co-efficient of cognition, for which the selection of optimum supplier is viable (figure 8.8).

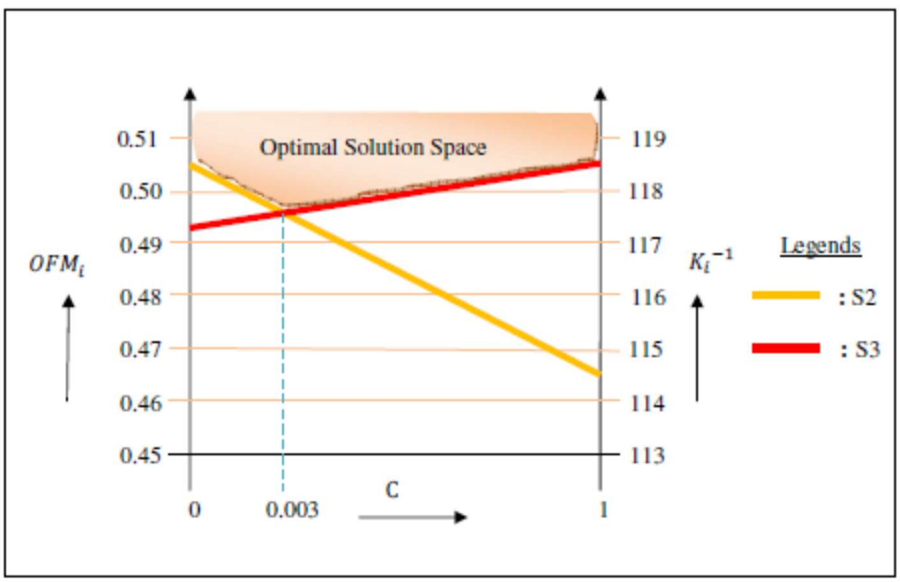


Figure 8.8: Post optimality check

The analysis reveals that selection of supplier S2 is valid for C value less than 0.003. But, selection of supplier S3 is valid for almost entire range of C, which complements the final ranking. Therefore, the post optimality check proves the high robustness of the model. The result of the check is presented in table 8.15 as follows:

Table 8.15: Post optimality check

Range of C	Optimum supplier
$C < 0.003$	S2
$C \geq 0.003$	S3

8.3 Conclusion and future scope

The present chapter consists of a novel group decision making model on selection of supplier for a manufacturing organization. It makes use of FAHP and K-means clustering techniques in doing so. The K-means clustering with FAHP focus on closest distance and produce better initial cluster centers, thus, improving the accuracy of clustering result. The proposed method efficaciously fulfils the objective of developing a decision model. It incorporates the cognitive navigation of individual expert by means of co-efficient of cognition. Thus, it maintains an integrative balance between subjective and objective factor in the problem. The outcome of the investigation provides a comprehensive selection of supplier for almost entire solution space. So, it becomes very evident from the result, that there is only one potentially optimal supplier for the manufacturing organization.

Novelty aspects of the study include the integration of FAHP, K-means with post optimality check on the final selection. The inclusion of goodwill cost carries a heavy weightage and influences the final outcome to a great deal. Also, the cognitive navigations of decision makers are considered in the present work.

K-means integrated with classification and regression trees (CART) can be a future research direction in machine learning. Also, comparison of other ranking algorithms with clustering algorithm can be an interesting scope in future. The model in the present form can also be implemented to other MCDM problems prevalent in real world of manufacturing.

9. CONCLUSION

Modern world is a competitive one. It's very important for any manufacturing organization to keep pace with dynamic conditions of the competitive world. The ability of any organization to make optimum decision, now-a-days, is very important in the face of increasing competition from a number of competitors. Hence, continuous quality improvement and optimum decision making are the success keys for any organization. Also, optimum utilization of time and available resources are the other main factors contributing to the success of any organization. In a highly competitive and volatile market, **the role of supply chain** management (SCM) is the main deciding factor for the growth of an organization. It is a chain that links customer to supplier. The same is done through manufacturing and services. Material, money and information flow are effectively managed to meet the business requirements. In an organization, not only material, but, information and finances also move from supplier to manufacturer in a process. Then these go to wholesaler, retailer and lastly, the consumer. The flow chain oversights the same. It coordinates and integrates these flows within the organization. Product design, manufacturing and distribution strategies might change customarily due to this. In this kind of circumstances, the challenge for a company is to continue producing a technically advanced and competitive product. At the same time, design, development and manufacturing time need to be reduced in line with demands of the market. The four performance measures in a supply chain are production cost, product quality, product lead times and after sales service.

For a **supply chain to be resilient**, it has to operate under smart environment. Smart manufacturing broadens the goal of manufacturing with optimized concept generation and product transaction. A resilient supply chain makes good optimization at different levels of supply chain. At the upstream level, it is to evaluate appropriate supplier to grow business. Supplier selection is one of the most critical phases of a resilient supply chain. A positive relationship with supplier makes a good impact in business. Also, the supplier reputation is of great importance. So, supplier selection for company needs to be utmost taken care of. In the middle stream level, selection of advanced optimum technology helps a supply chain to be resilient. At this day and age, modern technology helps business to grow at a rapid pace. It also helps to achieve competitive advantage in today's volatile market environment. If evaluated carefully, modern technologies can do wonders for a company. It helps achieve flexibility, profitability and superior quality products. Resilient supply chain and smart manufacturing are directly linked with the downstream level of a supply chain i.e. selection of appropriate warehouse location. Selection of warehouse location can be influenced by political, social, economic, environmental aspects. In a supply chain, the location of warehouse is one of the most fundamental and critical decisions to be made. This contributes enormously towards the resilience of a supply chain. The most important factor while selecting warehouse location is to minimize the logistics costs. And, that contributes fairly towards profitability. So, for a supply chain to be resilient, this broad aspect needs to be taken care of.

The performance of a manufacturing organization in today's **volatile** environment is largely dependent on the **marketing strategy** adopted. It should be involved in producing in masses as demanded by the customer, using latest innovative technologies while setting a new benchmark as well. As far as customization is concerned, that is also the need of the hour. Again, CODP is an important factor in manufacturing design as well as supply chain. It is a point in the material flow where a particular product links to a specified order from customer. Instead of consolidating orders to stock products for later sale, organizations can take a different approach. It is built for immediate or near-instant sale. This does not mean that the product being produced is out of the box months or years ago. Rather, they can be days or weeks old, depending on how far and how long they travel to reach their destinations. The marketing strategies corresponding to this can typically be buy-to-order, make-to-order, assemble-to- order and make-to-stock having different ratios of production lead time (P) and delivery lead time (D). CODP is also known as order penetration point (OPP). The buy-to-order approach is highly customized wherein, make-to-stock approach is highly responsive. That is where, the need of smart supply chain arises.

Advanced Manufacturing Technology (AMT) plays a pivotal role to obtain a smart supply chain. The significant contribution of AMT is to achieve strategic objectives and improved competitiveness of manufacturing organizations. AMTs represent numerous modern technologies such as Computer Aided Design, Computer Aided Manufacturing, Flexible Manufacturing System, Computer Aided Process Planning, Artificial Intelligence, Robots, and Just-In-Time etc. Selection of the proper AMT amongst these is one of the most paramount issues for any manufacturing organization. Benefits offered by AMT to the manufacturing organizations are like improved productivity, awe-inspiring flexibility, shortened lead times, improved quality, lowered inventories, innovative product design, reduced costs, improved competitiveness, increased customer satisfaction, sustainable green environment and many more. The quest for all these has driven many manufacturing organizations to opt for AMT. The most important outcome of this is very evident from the fact that there has been a **shifting of strategic pattern to mass customized product from large scale productions.** Although, the adoption of AMT is beneficial to manufacturing organizations, at the same time, it is very risky as well. It involves a major investment and a high degree of uncertainty. Considerable attention is needed within the organization while implementing the AMT. So, before investing on AMT, manufacturing organization must assess its strengths and weaknesses. Thus, identification of factors in selecting a particular AMT is very crucial for the organizations. The factors chosen must have lasting effects on the performance of a company.

The aim of this study is to focus on the research of models for decision support in smart supply chain management under uncertainties from fuzzy type. The experimentations in the study include the use of techniques like Fuzzy Multi Criteria Decision Making, Fuzzy AHP, Fuzzy TOPSIS, Fuzzy Set Theory, COPRAS-G, EVAMIX, PROMETHEE, Fuzzy Dempster- Shafer theory, Taguchi loss function, fuzzy VIKOR, regression analysis, K- means clustering technique etc. The objective is to create a customized methodology for dealing with qualitative and quantitative characteristics in the preliminary stages of the design process. The model needs to apply soft computing in the form of fuzzy logic to simulate human decision making. The method proposed here not only solves the problem of not being able to measure the blur and uncertainty of the conventional method, but also avoids the drawback that the calculated value becoming unstable by other methods.

This research work has got ample real time applications in the field of manufacturing decision making. It could be selection of appropriate supplier of raw materials or evaluation of advanced technology for a manufacturing organization to achieve competitive advantage or selection of proper warehouse location for optimized usage of resources and cost minimization. So, this present analysis can be widely used in various fields of engineering and management.

The originality of the research lies in the fact that, it contributes innovative methods for performance measure of resilient supply chain at different levels of it. So, in a way, this research suggested means of improvement to a supply chain that could make it a resilient one.

10. REFERENCES

1. Aggarwal, P., Bhatt, D., Devabhaktuni, V., & Bhattacharya, P. (2013). Dempster Shafer neural network algorithm for land vehicle navigation application. Information Sciences, 253, 26-33.
2. Ahmed, S. & Sahinidis, N. V. (2008). Selection, acquisition and allocation of manufacturing technology in a multi-period environment. *European Journal of Operational Research*, 189, 807-821.
3. Al-Ahmari, A.M.A. (2008). A methodology for selection and evaluation of advanced manufacturing technologies. *International Journal of Computer Integrated Manufacturing,* 21(7), 778-789.
4. Atieh, A. M., Kaylani, H., Al-abdallat, Y., Qaderi, A., Ghoul, L., Jaradat, L., & Hdairis, I. (2016). Performance improvement of inventory management system processes by an automated warehouse management system, *Procedia CIRP* , 41, 568 – 572.
5. Bagherikahvarin, M., & Smet, Y. D. (2016). A ranking method based on DEA and PROMETHEE II. *Measurement*, 89, 333-342.
6. Boudaren, M. E. Y., An, L., & Pieczynski, W. (2016). Dempster-Shafer fusion of evidential parwise Markov fields. *International Journal of Approximate Reasoning*, 74, 13-29.
7. Brans, J. P., & Vincke, Ph. (1985). A preference ranking organization method (The PROMETHEE method for multiple criteria decision making). *Institute for Operations Research and the Management Sciences*, 31(6), 647-656.
8. Buyukozkan, G., Feyzioglu, O., & Nebol, E. (2008). Selection of strategic alliance partner in logistics value chain. *International Journal of Production Economics,* 113, 148-158.
9. Buyukozkan, G., & Cifci, G. (2012). A novel fuzzy MCDM approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. *Expert Systems with Applications*, 39, 3000-3011.
10. Chai, K. C., Tay, K. M., & Lim, C. P. (2016). A new method to rank fuzzy numbers using Dempster Shafer theory with fuzzy targets. *Information Sciences*, 346-347, 302-317.

11. Chan, F.T.S., Chan, M.H., & Tang, N.K.H. (2000). Evaluation methodologies for technology selection. *Journal of Materials Processing Technology*, 107, 330-337.
12. Chan, F.T.S., Jiang, B., & Tang, N.K.H. (2000). The development of intelligent decision support tools to aid the design of flexible manufacturing systems. *International Journal of Production Economics*, 65(1), 73-84.
13. Chen, C-T., Lin, C-T. & Huang, S-F. (2006). A fuzzy approach for supplier evaluation and selection in supply chain management. *International Journal of Production Economics,* 102, 289-301.
14. Chen, T-Y. (2015). An interval type-2 fuzzy PROMETHEE method using a likelihood-based outranking comparison approach. *Information Fusion*, 25, 105-120.
15. Cholvy, L. (2012). Non-exclusive hypothesis in Dempster-Shafer Theory. *International Journal of Approximate Reasoning*, 53, 493-501.
16. Choudhury, A. K., Shankar, R. & Tiwari, M. K., (2006). Consensus-based group decision-making model for the selection of advanced technology. *Decision Support Systems*, 42, 1776-1799.
17. Chuu, S-J. (2009). Group decision-making model using fuzzy multiple attributes analysis for the evaluation of advanced manufacturing technology. *Fuzzy Sets and Systems*, 160, 586–602.
18. Chuu,S-J. (2009). Selecting the advanced manufacturing technology using fuzzy multiple attributes group decision making with multiple fuzzy information. *Computers & Industrial Engineering*, 57, 1033–1042.
19. Cinelli, M., Coles, S. R., & Kirwan, K. (2014). Analysis of the potentials of multi criteria decision analysis methods to conduct sustainability assessment. *Ecologocal Indicators*, 46, 138-148.
20. Dabaghi, M., Mascle, C., & Baptiste, P. (2015). Application of DOE-TOPSIS Technique in Decision-Making Problems. *IFAC-PapersOnLine*, 48 (3), 773-777.

21. Darji, V.P., & Rao, R.V. (2013). Application of AHP/EVAMIX Method for Decision Making in the Industrial Environment. *American Journal of Operations Research*, 3, 542-569.
22. Demirel, N.C. & Yucenur, G.N. (2011). The Cruise Port Place Selection Problem with Extended VIKOR and ANP Methodologies under Fuzzy Environment. *Proceedings of the World Congress on Engineering*, 2011, Vol II, London, U.K.
23. Dempster, A. P. (1967). Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics,* 38, 325–328.
24. Deng, Y., & Chan, F.T.S. (2011). A new fuzzy dempster MCDM method and its application in supplier selection. *Expert Systems with Applications*, 38, 9854-9861.
25. Dey, B., Bairagi, B., Sarkar, B., & Sanyal, S. K. (2017). Group heterogeneity in multi member decision making model with an application to warehouse location selection in a supply chain, *Computers & Industrial Engineering,* 105, 101–122.
26. Duckstein, L., & Opricovic, S. (1980). Multiobjective optimization in river basin development. *Water Resour. Res*. 16(1), 14–20.
27. Efthymiou, K., Sipdas, K., Mourtzis, D. & Chryssolouris, G. (2015). On knowledge reuse for manufacturing systems design and planning: a semantic technology approach. *CIRP Journal of Manufacturing Science and Technology*, 8, 1-11.
28. Fernandez, F.B. & Perez, M. A. S. (2015). Analysis and Modeling of New and Emerging Occupational Risks in the Context of Advanced Manufacturing Processes. *Procedia Engineering*, 100, 1150-1159.
29. Fichtinger, J., Ries, J. M., Grosse, E. H., & Baker, P. (2015). Assessing the environmental impact of integrated inventory and warehouse management, *Int. J. Production Economics*, 170, 717–729.
30. Forgy, E.W. (1965). Cluster analysis of multivariate data: efficiency versus interpretability of classifications. *Biometrics,* 21 (3), 768-769.

31. Fouladgar, M.M., Yazdani-Chamzini, A., Yakhchali, S.H., Ghasempourabadi, M.H. & Badri, N. (2011). Project portfolio Selection Using VIKOR Technique under Fuzzy Environment. *2011 2nd International Conference on Construction and Project Managemen*t IPEDR vol.15.

32. Garcia, J. L., Alvarado, A., Blanco, J., Jimenez, E., Maldonado, A. A., & Cortes, G. (2014). Multi-attribute evaluation and selection of sites for agricultural product warehouses based on an Analytic Hierarchy Process, *Computers and Electronics in Agriculture,* 100, 60–69.

33. Gruyer, D., Demmel, S., Magnier, V., & Belaroussi, R. (2016). Multi-Hypotheses tracking using the Dempster–Shafer Theory, application to ambiguous road context. *Information Fusion*, 29, 40-56.

34. Hajkowicz, S., & Higgins, A. (2008). A comparison of multiple criteria analysis techniques for water resource management. *European Journal of Operational Research*, 184(1), 255-265.

35. Hofstra, N., Petkova, B., Dullaert, W., Reniers, G., & Leeuw, S. D. (2018). Assessing and facilitating warehouse safety, *Safety Science*, 105, 134–148.

36. Huang, S., Wang, Q., Batta, R., & Nagi, R. (2015). An integrated model for site selection and space determination of warehouses, *Computers & Operations Research*, 62, 169–176.

37. Hwang, C. L., & Yoon, K. (1981). Multiple attribute decision making: Methods and applications. New York: Springer.

38. Ince, M., Yigit, T., & Isik, A.H. (2017). AHP-TOPSIS Method for Learning Object Metadata Evaluation. *International Journal of Information and Education Technology*, 7 (12), 884-887.

39. Jacyna-Golda, I., & Izdebski, M. (2017). The Multi-criteria Decision Support in Choosing the Efficient Location of Warehouses in the Logistic Network, Procedia *Engineering*, 187, 635 – 640.

40. Jedrkiewicz, R., Orlowski, A., Namiesnik, J., & Tobiszewski, M. (2016). Green analytical chemistry introduction to chloropropanols determination at no economic and analytical performance costs? *Talanta*, 147, 282-288.

41. Kengpol, A., & O'Brien, C. (2001). The development of a decision support tool for the selection of advanced technology to achieve rapid product development. *International Journal of Production Economics,* 69, 177-191.

42. Karsak, E. E. (2002). Distance-based fuzzy MCDM approach for evaluating flexible manufacturing system alternatives. *International Journal of Production Research*, 4**0** (13), 3167-3181.

43. Karsak, E.E. & Kuzgunkaya, O. (2002). A fuzzy multiple objective programming approach for the selection of a flexible manufacturing system. *International Journal of Production Economics*, 79, 101-111.

44. Karsak, E. E., & Tolga, E.(2001). Fuzzy multi-criteria decision-making procedure for evaluating advanced manufacturing system investments. *International Journal of Production Ec0nomics*, 69, 49-64.

45. Kadzinski, M., & Ciomek, K. (2016). Integrated framework for preference modeling and robustness analysis for outranking-based multiple criteria sorting with ELECTRE and PROMETHEE. *Information Sciences*, 352-353, 167-187.

46. Kilic, H. S., Zaim, S., & Delen, D. (2015). Selecting ''The Best'' ERP system for SMEs using a combination of ANP and PROMETHEE methods. *Expert Systems with Applications*, 42, 2343-2352.

47. Khorasaninejad, E., Fetanat, A., & Hajabdollahi, H. (2016). Prime mover selection in thermal power plant integrated with organic Rankine cycle for waste heat recovery using a novel multi criteria decision making approach. *Applied Thermal Engineering*, 102, 1262-1279.

48. Kazan, H., Ozcelik, S., & Hobikoglu, E. H. (2015). Election of Deputy Candidates for Nomination with AHP-Promethee Methods. *Procedia - Social and Behavioral* Sciences, 195, 603-613.

49. Kessentini, Y., Burger, T., & Paquet, T. (2015). A Dempster–Shafer Theory based combination of handwriting recognition systems with multiple rejection strategies. *Pattern Recognition*, 48, 534-544.

50. Kahraman, C., & Cebi, S. (2009). A new multi-attribute decision making method: Hierarchical fuzzy axiomatic design. *Expert Systems with Applications*, 36, 4848-4861.

51. Kaucher, E. (1980). Interval analysis in the extended interval space IR, in *Fundamentals of Numerical Computation (Computer-Oriented Numerical Analysis)*, ed. by G. Alefeld, R.D. Grigorieff, vol. 2 (Springer, Vienna, 1980), pp. 33–49.

52. Lin, Y-S., & Wang, K-J. (2018). A two-stage stochastic optimization model for warehouse configuration and inventory policy of deteriorating items, *Computers & Industrial Engineering*, 120, 83–93.

53. Lepskiya, A. (2013). Estimation of Conflict and Decreasing of Ignorance in Dempster-Shafer Theory. *Information Technology and Quantitative Management*, 17, 1113-1120.

54. Liu, S-T. (2008). A fuzzy DEA/AR approach to the selection of flexible manufacturing systems. *Computers & Industrial Engineering*, 54, 66-76.

55. Liao, C-N., & Kao, H-P. (2011). An integrated fuzzy TOPSIS and MCGP approach to supplier selection in supply chain management. *Expert Systems with Applications,* 38, 10803-10811.

56. Lloyd, S.P. (1957). Least square quantization in PCM. *Bell Telephone Laboratories*. Published: Lloyd, S.P. (1982). Least squares quantization in PCM. *IEEE Transactions on Information Theory*, 28 (2), 129-137.

57. MacQueen, J.B. (1967). Some Methods for Classification and Analysis of Multivariate Observations. *Proceedings of 5th Berkeley Symposium on Mathematical Statistics and Probability* (pp. 281–297). University of California Press.

58. Martel, J. M. & Matarrazo, B. (2005). Other Outranking Approaches, In: F. J. Salvatore and G. M. Ehrgott, Eds., *Multiple Criteria Decision Analysis*: *State of the Art Surveys*, Springer, New York, 197-262.

59. Mohanty, R. P., & Deshmukh, S. G. (1998). Advanced manufacturing technology selection: A strategic model for learning and evaluation. *International Journal of Production Economics*, 55, 295–307.

60. Mirahmadi, N. & Teimoury, E. (2012). A Fuzzy VIKOR Model for Supplier Selection and Evaluation: Case of EMERSUN Company. *Journal of Basic and Applied Scientific Research*, 2 (5), 5272-5287.

61. Macharis, C., Springael, J., Brucker, K. D., & Verbeke, A. (2004). PROMETHEE and AHP: The design of operational synergies in multicriteria analysis. Strengthening PROMETHEE with ideas of AHP. *European Journal of Operational Research*, 153, 307-317.

62. Murat, S., Kazan, H., & Coskun, S. S. (2015). An Application for Measuring Performance Quality of Schools by Using the PROMETHEE Multi-Criteria Decision Making Method. *Procedia - Social and Behavioral* Sciences, 195, 729-738.

63. Makaci, M., Reaidy, P., Evrard-Samuel, K., Botta-Genoulaz, V., & Monteiro, T. (2017). Pooled warehouse management: An empirical study, *Computers & Industrial Engineering*, 112, 526–536.

64. Mahmoodzadeh, S., Shahrabi, J., Pariazar, M., & Zaeri, M.S. (2007). Project Selection by Using Fuzzy AHP and TOPSIS Technique. *International Scholarly and Scientific Research & Innovation*, 1 (6), 270-275.

65. Nath, S., & Sarkar, B. (2017). Performance Evaluation of Advanced Manufacturing Technologies: A Denovo Approach. *Computers & Industrial Engineering*, 110, 364-378.

66. Nath, S., & Sarkar, B. (2018). Decision system framework for performance evaluation of advanced manufacturing technology under fuzzy environment. *OPSEARCH*, 55, 703-720.

67. Nath, S., & Sarkar, B. (2020). An integrated fuzzy group decision support framework for performance assessment of advanced manufacturing technologies: An eclectic comparison. *Journal of Institution of Engineers India Series C*, 101(3):473–491.

68. Nath, S., & Sarkar, B. (2019). An Integrated Cloud Manufacturing Model for Warehouse Selection in a Smart Supply Chain Network: A Comparative Study. *Journal of Institution of Engineers India Series C*, 101 (1), 25-41.

69. Nath, S., & Sarkar, B. (2017). An Exploratory Analysis for the Selection and Implementation of Advanced Manufacturing Technology by Fuzzy Multi-criteria Decision Making Methods: A Comparative Study. *Journal of Institution of Engineers India Series C*., 98 (4), 493-506.

70. Opricovic, S. (1998). Multicriteria Optimization in Civil Engineering (in Serbian) (Faculty of Civil Engineering, Belgrade, 1998), p. 302, ISBN 86-80049-82-4.

71. Opricovic, S., & Tzeng, G.-H. (2004). The Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. *Eur. J. Oper. Res.*, 156(2), 445–455.

72. Panpakidis, I.P., & Christoforidis, G.C. (2018). Optimal Selection of Clustering Algorithm via Multi-Criteria Decision Analysis (MCDA) for Load Profiling Applications. *Applied Sciences*, 8 (2): 237. DOI: 10.3390/app8020237.

73. Pi, W-N., & Low, C. (2005). Supplier evaluation and selection using Taguchi loss functions. *Int. J. Adv. Manuf. Technol.* 26, 155–160.

74. Ramachandran, L. & Alagumurthi, N. (2013). Lean Manufacturing Facilitator Selection with VIKOR under Fuzzy Environment. *International Journal of Current Engineering and Technology*, 3 (2), 356-359.

75. Rohrmus, D., Döricht, V. & Weinert, N. (2015). Green Factory Supported by Advanced Carbon-based Manufacturing, *Procedia CIRP*, 29, 28-33.

76. Safei, S., Shibghatullah, A.S., Burhanuddin, MA., & Yusof, M.K. (2015). Targeted Ranking-Based Clustering Using AHP K-Means. *International Journal of Advances in Soft Computing and its Application*, 7 (3), 100-113.

77. Samantra, C., Datta, S. & Mahapatra, S. S. (2012). Application of Fuzzy Based VIKOR Approach for Multi-Attribute Group Decision Making (MAGDM): A Case Study in Supplier Selection. *Decision Making in Manufacturing and Services*, 6 (1), 25-39.
78. Shafer, G. (1976). A mathematical theory of evidence. Princeton, NJ: Princeton University Press.
79. Stamatakis, A., Mandalaki, B., & Tsoutsos, T. (2016). Multi-criteria analysis for PV integrated in shading devicesfor Mediterranean region. *Energy and Buildings*, 117, 128-137.
80. Singh, S., & Dasgupta, MS. (2016). Evaluation of research on CO2 trans-critical work recovery expander using multi attribute decision making methods. *Renewable and Sustainable Energy Reviews*, 59, 119-129.
81. Senouci, M. R., Mellouk, A., Aitsaadi, N., & Oukhellou, L. (2016). Fusion-based surveillance WSN deployment using Dempster–Shafer theory. *Journal of Network and Computer Applications*, 64, 154-166.
82. Simsek, B., Tansel Ic, Y., & Simsek, E. H. (2013) A TOPSIS-based Taguchi optimization to determine optimal mixture proportions of the high strength self-compacting concrete. *Chemometrics and Intelligent Laboratory Systems*, 125, 18-32.
83. Smets, P. (2000). Data fusion in the transferable belief model, in *Proceedings of 3rd International Conference On Information Fusion*, Fusion, Paris, France, July 10–13, 2000, pp. PS21–PS33.
84. Steinhaus, H. (1957). Sur la division des corps matériels en parties. *Bull. Acad. Polon. Sci.* (in French), 4 (12), 801-804.
85. Teti, R. (2015). Advanced IT Methods of Signal Processing and Decision Making for Zero Defect Manufacturing in Machining, *Procedia CIRP*, 28, 3-15.
86. Tomic, V., Maronkovic, Z., & Janosevic, D. (2011). PROMETHEE method implementation with multi-criteria decisions. *Facta Universitatis, Mechanical Engineering*, 9(2), 193-202.
87. Troiano, L., Rodriguez-Muniz, L. J., & Diaz, I. (2015). Discovering user preference using Dempster-Shafer theory. *Fuzzy Sets and Systems*, 278, 98-117.

88. Tang, H. (2015). A novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence. *Applied Soft Computing*, 31, 317-325.

89. Tansel Ic., Y. (2012). An experimental design approach using TOPSIS method for the selection of computer-integrated manufacturing technologies. *Robotics and Computer-Integrated Manufracturing*, 28, 245-256.

90. Tansel Ic., Y. (2014). A TOPSIS based design of experiment approach to assess company ranking. *Applied Mathematics and Computation*, 227, 630-647.

91. Vasiljevic, G., Miklic, D., Draganjac, I., Kovacic, Z., & Lista, P. (2016). High-accuracy vehicle localization for autonomous warehousing, *Robotics and Computer-Integrated Manufacturing*, 42, 1–16.

92. Voogd, H. (1982). Multicriteria Evaluation with Mixed Qualitative and Quantitative Data, *Environment and Planning Bulletin*, 9 (2), 221-236.

93. Wutthisirisart, P., Sir, M. Y., & Noble, J. S. (2015). The two-warehouse material location selection problem, *Int. J. Production Economics,* 170, 780–789.

94. Wang, Y-M., & Jing (Tiffany), LI. (2012). Application of Dempster-Shafer theory for network selection in heterogeneous wireless networks. *The Journal of China Universities of Posts and Telecommunications*, 19(2), 86-91.

95. Wang, P., Li, Y., Wang, Y-H., & Zhu, Z-Q. (2015). A New Method Based on TOPSIS and Response Surface Method for MCDM Problems with Interval Numbers. *Mathematical Problems in Engineering*, http://dx.doi.org/10.1155/2015/938535.

96. Wang, J., Hu, Y., Xiao, F., Deng, X., & Deng, Y. (2016). A novel method to use fuzzy soft sets in decision making based on ambiguity measure and Dempster-Shafer theory of evidence: An application in medical diagnosis. *Artificial Intelligence in Medicine*, DOI: 10.1016/ j.artmed.2016.04.004.

97. Yurdakul, M. (2004). Selection of computer-integrated manufacturing technologies using a combined analysis hierarchy process and goal programming model. *Robotics and Computer-Integrated Manufacturing,* 20, 329-340.

98. Yusuf, R.M., Yee, K.P., & Hashmi, M.S.J. (2001). A preliminary study on the potential use of the analytical hierarchical process (AHP) to predict advanced manufacturing technology (AMT) implementation. *Robotics and Computer Integrated Manufacturing*, 17, 421-427.

99. Yue, F., Zhang, G., Su, Z., Lu, Y., & Zhang, T. (2015). Multi-software reliability allocation in multimedia systems with budget constraints using Dempster–Shafer theory and improved differential evolution. *Neurocomputing*, 169, 13-22.

100. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control,* 8, 338-353.

101. Zavadskas, E.K., Turskis, Z., &Tamošaitienė, J. and Marina, V. (2008). Selection of construction project managers by applying COPRAS-G method. *RelStat'08*, *Riga, Latavia.*

102. Zavadskas, E.K., Kaklauskas, A., Turskis, Z., &Tamošaitienė, J. (2009). Multi-Attribute Decision-Making Model by Applying Grey Numbers. *INFORMATICA*, 202 (2), 305–320.

103. Zimmermann, H. Z. (1991). Fuzzy set theory and its applications (Kluwer Academic Publishers, Boston, 1991)

---------------------x--------------------