Impact Angle Constrained Guidance for Nonstationary Maneuvering and Nonmaneuvering Targets

This thesis is submitted in the partial fulfilment of the requirements of the degree

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Submitted by

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Certificate of Recommendation

This is to certify that **Mr. Abir Pal (002110804003)** has completed his dissertation entitled, "**Impact Angle Constrained Guidance for Nonstationary Maneuvering and Nonmaneuvering Targets**", under the direct supervision and guidance of **Dr. Smita Sadhu Ghosh**, Department of Electrical Engineering, Jadavpur University. We are satisfied with his work, which is being presented for the partial fulfilment of the degree of **Master in Control System Engineering** of Jadavpur University, Kolkata-700032.

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<u>Certificate of Approval</u>

The foregoing thesis entitled "**Impact Angle Constrained Guidance for Nonstationary Maneuvering and Nonmaneuvering Targets**" is hereby approved as a creditable study of an Engineering subject carried out and presented in a manner satisfactory to warrant its acceptance as a pre-requisite to the degree of Master in Control System Engineering for which it has been submitted. It is understood that, by this approval the undersigned does not necessarily endorse or approve any statement made, opinion expressed, or conclusion therein but approve this thesis only for the purpose for which it is submitted.

Final Examination for Evaluation of the Thesis

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Signature of the Examiners

Declaration of Originality

I hereby declare that this thesis contains a literature survey and original research work by the undersigned candidate, as part of his Master in Control System engineering curriculum. All information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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Abstract

Guidance systems with terminal impact angle constraints play a pivotal role in modern missile technology and defence systems. This thesis delves into the intricate domain of impact angle constrained guidance, focusing on its application against nonstationary nonmaneuvering and maneuvering targets. The research builds upon a foundation of proportional navigation guidance (PNG) and its adaptability for achieving precise impact angles.

Guidance laws incorporating terminal impact angle constraints have been extensively explored in prior research, with notable contributions spanning across several studies [1–7]. Proportional navigation guidance (PNG) has emerged as a prominent framework for devising impact angle constrained guidance strategies applicable to both stationary and moving targets. Lu et al. [8] introduced an adaptive guidance law that harnessed PNG for achieving hypervelocity impact angle constraints against stationary targets.

This thesis builds upon the foundational work of Ratnoo and Ghose [9], who addressed the challenge of satisfying impact angle constraints by adjusting the navigation constant N within the PNG framework. Their innovative two-stage PNG law was designed for realizing a broad spectrum of impact angles in surface-to-surface engagements with stationary targets. Kim et al. [3] further extended this concept with the introduction of a biased PNG (BPNG) law, which incorporated an additional term to nullify terminal impact angle errors while maintaining the conventional line-of-sight rate term for lateral acceleration commands. BPNG, in essence, expanded the capture region of existing guidance laws, particularly when dealing with moving targets, albeit with some limitations in tail-chase scenarios.

One of the central challenges addressed in this thesis pertains to achieving impact angles against moving targets. Notably, the PNG law generates a set of impact angles for various values of the navigation constant N when targeting moving entities. However, classical PNG law studies [10] have revealed that N must exceed a minimum threshold to ensure bounded terminal lateral acceleration demand. To bridge the gap and achieve the full spectrum of impact angles, an orientation guidance scheme is introduced for the initial phase of the interceptor trajectory.

This orientation guidance law, a derivative of the PNG framework, adjusts N as a function of the initial engagement geometry. Rigorous analysis and simulations demonstrate that following the orientation trajectory, the interceptor can seamlessly transition to $N = 3$, enabling precise control and attainment of any desired impact angle in a surface-to-surface engagement scenario.

In conclusion, this thesis offers a comprehensive exploration of impact angle constrained guidance against a spectrum of targets, encompassing both stationary and moving entities. The research contributes novel insights into the application of PNG-based guidance laws, expands the achievable set of impact angles, and provides innovative solutions for addressing the complexities of nonstationary nonmaneuvering and maneuvering targets. The findings presented herein hold significant implications for enhancing the effectiveness of missile and interceptor systems in contemporary defence scenarios.

CONTENTS

List of Abbreviation

LOS	Line-of-Sight
TPN	True Proportional Navigation
PPN	Pure Proportional Navigation
PNG	Proportional Navigation Guidance
PN	Proportional Navigation
IPN	Ideal Proportional Navigation
BPNG	Biased Proportional Navigation Guidance
UAVs	Unmanned Aerial Vehicles
IR	Infrared Sensors
INS	Inertial Navigation Systems
GNSS	Global Navigation Satellite Systems
APN	Augmented Proportional Navigation
SAR	Synthetic Aperture Radar
PIR	Passive Infrared Sensors
AIR	Active Infrared Sensors
TVC	Thrust Vector Control
RCS	Reaction Control Systems
CBC	Constant Bearing Course
GTPN	Generalized True Proportional Navigation

PID	Proportional-Integral-Derivative
MRAC	Model Reference Adaptive Control

LIST OF SYMBOLS

α_m	Heading angle of missile
α_t	Heading angle of target
$\dot{\alpha}_m$	Rate of change of heading angle of missile
α_{m0}	Initial heading angle of missile
α_{mf}	Impact angle of missile
a_m	Latax/lateral acceleration of missile
t_f	Interception time of missile
V_C	Closing velocity of missile
V_m	Velocity of missile
V_{m0}	Initial velocity of missile
V_θ	Relative velocity perpendicular to LOS
V_r	Relative velocity along LOS
V_t	Velocity of target
N	Navigation gain
N'	Effective navigation ratio
r	Range of missile
$\dot{r}$	Rate of change of range of missile
θ	LOS angle
$\dot{\theta}$	Rate of change of LOS
θ_0	Initial LOS angle
θ_f	Final LOS angle
x	Distance from the target to missile
z	Height from the ground

LIST OF FIGURES

Chapter 1

Introduction

1.1 Background and Motivation:

Guidance systems [25] are pivotal components of modern weaponry, ensuring accurate and effective engagement of targets. One critical aspect of successful target engagement is the control of the impact angle, which refers to the angle between the direction of the weapon and the line connecting the weapon to the target at the moment of impact. Controlling the impact angle is essential for optimizing the damage inflicted on the target, ensuring the desired effect, and minimizing collateral damage.

However, achieving accurate impact angles [12] becomes increasingly challenging when targets exhibit nonstationary maneuvering behaviours or when they are nonmaneuvering but still require precise impact angles for optimal engagement [25]. Nonstationary maneuvering targets [8], such as aircraft, drones, or missiles, can unpredictably change their trajectories, speeds, and accelerations to evade threats. On the other hand, nonmaneuvering targets, such as stationary ground-based objects, may still require specific impact angles to exploit their vulnerabilities effectively.

In scenarios involving impact angle constrained guidance against nonstationary maneuvering and nonmaneuvering targets, several intricate challenges arise:

1. **Target Behavior Variability:** Nonstationary targets can exhibit a wide range of complex maneuvers, making accurate prediction and interception difficult. Nonmaneuvering targets may have fixed trajectories that require precise guidance to achieve the desired impact angle.
2. **Time-Critical Decision Making:** Rapid decision-making is crucial to ensure accurate engagement, especially when targets are maneuvering. The guidance system must quickly adapt to changing target trajectories to meet impact angle constraints.
3. **Guidance Algorithm Design:** Designing guidance algorithms [8] that can accommodate both nonmaneuvering and maneuvering scenarios while adhering to impact angle constraints demands innovative approaches that balance computational complexity and real-time performance.

4. **Uncertainties and Sensor Limitations:** Real-world uncertainties in target motion prediction, sensor data, and guidance system limitations introduce complexities that must be considered for robust guidance algorithm development.
5. **Optimization Trade-offs:** Balancing impact angle constraints [6] with other performance metrics, such as miss distance, time-to-intercept, and energy efficiency, requires careful optimization.

Motivation: The motivation behind this research stems from the critical need to enhance the efficacy of guided weapon systems in engaging both nonstationary maneuvering and nonmaneuvering targets under impact angle constraints. As modern conflicts increasingly involve agile and evasive targets, it is imperative to develop advanced guidance strategies [25] that can adapt to varying target behaviours while maintaining accurate impact angles. The potential benefits of this research include:

1. **Enhanced Target Engagement:** Improved guidance strategies can lead to more accurate target engagements, thereby increasing the success rate of weapon systems.
2. **Reduced Collateral Damage:** Accurate impact angle control minimizes collateral damage by ensuring the precise delivery of the weapon's effects on the target.
3. **Versatility:** The developed guidance algorithms can be applied across a wide range of weapon systems, from anti-aircraft missiles to precision-guided munitions.
4. **Strategic Advantage:** A robust and adaptable guidance system can provide a strategic advantage in complex and dynamic warfare scenarios.
5. **Technological Advancement:** This research contributes to the advancement of guidance system technology, with potential applications in both military and civilian contexts.

1.2 Problem Statement:

The problem addressed in this thesis is the development of effective guidance strategies to achieve accurate impact angles against both nonstationary maneuvering and nonmaneuvering targets. The primary challenge lies in designing guidance algorithms that can adapt to the dynamic behaviours of targets while ensuring precise impact angles are achieved upon engagement. This problem is particularly crucial in the context of modern weapon systems, where accurate target engagement is essential for maximizing effectiveness and minimizing collateral damage.

Challenges:

1. **Nonstationary Maneuvering Targets:** Nonstationary targets, such as aircraft, drones, and maneuverable missiles, can change their trajectories, speeds, and accelerations to

evade threats. This dynamic behaviour introduces unpredictability and requires guidance strategies that can adapt rapidly to changing target positions.

2. **Nonmaneuvering Targets:** Even nonmaneuvering targets, such as stationary structures or vehicles, may require accurate impact angles to achieve specific desired effects. Guiding a weapon to impact at the desired angle becomes crucial for achieving the intended results.
3. **Impact Angle Constraints:** Adhering to impact angle constraints while considering target motion unpredictability poses a significant challenge. Traditional guidance algorithms may struggle to simultaneously account for target behaviour and impact angle requirements.
4. **Real-World Uncertainties:** Uncertainties in target motion prediction, sensor measurements, and guidance system limitations introduce complexities that can affect the accuracy of impact angle control. Devising strategies that account for these uncertainties is essential.
5. **Optimization Trade-offs:** Balancing impact angle constraints with other performance metrics, such as miss distance, time-to-intercept, and energy efficiency, requires careful consideration. Trade-offs between these metrics can impact the overall effectiveness of the guidance strategy.

Significance: The successful resolution of this problem holds significant implications for guided weapon systems:

1. **Accuracy and Efficacy:** Effective guidance strategies can lead to accurate target engagements, increasing the overall effectiveness of weapon systems.
2. **Collateral Damage Reduction:** Accurate impact angles help minimize collateral damage by ensuring the delivery of weapon effects precisely to the target.
3. **Versatile Application:** Developed strategies can be adapted to various weapon systems, enhancing their versatility and utility across different defence scenarios.
4. **Strategic Advantage:** The ability to engage both maneuvering and nonmaneuvering targets accurately provides a strategic advantage in dynamic and evolving conflict situations.
5. **Technological Advancement:** Addressing the challenges of impact angle constrained guidance contributes to the advancement of guidance system technology, benefiting both military and civilian domains.

1.3 Research Objectives: The main objectives of this research are as follows:

1. Develop novel guidance algorithms that can accommodate both nonstationary maneuvering and nonmaneuvering target scenarios while adhering to impact angle constraints.
2. Evaluate the performance of the proposed guidance strategies through simulation-based analyses, considering varying target behaviours, uncertainties, and optimization trade-offs.

3. Compare the performance of the developed guidance strategies against existing approaches, highlighting their strengths and weaknesses.
4. Provide insights into the practical implementation of the developed guidance algorithms within existing weapon systems, addressing computational efficiency and integration challenges.

By addressing these objectives, this research aims to contribute innovative solutions that enhance the accuracy and adaptability of guided weapon systems when engaging nonstationary maneuvering and nonmaneuvering targets while adhering to impact angle constraints.

1.4 Scope and Limitations

Scope: The scope of this thesis encompasses the development, analysis, and evaluation of impact angle constrained guidance systems against two distinct types of targets: nonstationary nonmaneuvering targets and maneuvering targets. The research delves into both theoretical and practical aspects, including mathematical formulations of guidance laws, target motion prediction techniques, simulation-based performance evaluations, and considerations for real-world implementation. The thesis aims to provide comprehensive insights into the challenges and methodologies involved in achieving accurate impact angles against dynamic targets.

Limitations:

1. **Simplified Target Dynamics**: The research assumes simplified models for both nonmaneuvering and maneuvering targets, which might not capture all real-world complexities. The models may not fully account for rapid changes in target behaviour or interactions with the environment.

2. **Limited Environmental Factors**: The study primarily focuses on impact angle constrained guidance and target motion, without extensively considering external factors like wind, atmospheric conditions, and terrain effects, which could influence guidance accuracy.

3. **Guidance Law Performance**: The evaluation of guidance laws primarily relies on simulation-based assessments. While these provide valuable insights, real-world implementations might introduce additional uncertainties that are not fully addressed in the simulations.

4. **Homogeneous Scenarios**: The research assumes a certain level of homogeneity in target behaviours and engagement scenarios. Complex scenarios involving multiple interacting targets or heterogeneous threat behaviour might not be fully explored.

5. **Hardware and Integration Constraints**: While the thesis briefly touches upon implementation considerations, it does not delve deeply into the intricacies of integrating the proposed guidance strategies into existing defence systems, which may involve various hardware and software constraints.
6. **Ethical and Legal Implications**: Although the thesis acknowledges safety and ethical implications, it does not provide an exhaustive analysis of the ethical and legal aspects associated with using impact angle constrained guidance systems, especially in scenarios involving civilian populations.
7. **Complex Maneuvers**: While maneuvering target models [25] are considered, the thesis might not cover the full spectrum of complex maneuvers that real-world threats might employ, potentially limiting the guidance system's effectiveness in some situations.
8. **Real-time Adaptation**: The thesis does not extensively explore real-time adaptation mechanisms of the guidance laws based on dynamic changes in target behaviour. Real-time adaptation is crucial for engaging rapidly evolving threats.
9. **Validation against Real-world Data**: The simulation-based evaluations might not fully replicate real-world conditions. Validation against real-world data, such as historical engagement scenarios, could enhance the credibility of the proposed guidance strategies.
10. **Temporal Considerations**: The thesis may not deeply address temporal considerations, such as the time required for target motion prediction and guidance algorithm execution, which could impact the overall system's responsiveness.

Chapter 2

Literature Survey

2.1 Introduction to Guidance Systems:

Guidance systems play a pivotal role in modern defence and aerospace applications, enabling precise target engagement and impact angle control. These systems are integral to various platforms, ranging from missile defence systems to precision-guided munitions, ensuring effective interception and neutralization of threats. Understanding the basics of guidance systems is essential for comprehending the intricacies of impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets.

2.1.1 Defining Guidance Systems

Guidance systems [25] are intelligent subsystems that guide vehicles, such as missiles, rockets, and drones, toward a desired target with accuracy and efficiency. They employ a combination of sensors, navigation algorithms, and control mechanisms to achieve the intended outcome. A fundamental aspect of guidance systems is to ensure that the vehicle reaches its intended target, optimizing parameters like impact angle, miss distance, and time of arrival.

2.1.2 Purpose and Significance

The primary purpose of guidance systems is to enhance mission success rates by achieving accurate target engagement. This is particularly critical in scenarios where minimizing collateral damage, maximizing lethality, or ensuring penetration through defensive layers is essential. Impact angle control, a subset of guidance strategies, focuses on directing the vehicle to strike the target at a specific angle relative to the target's orientation. This approach can significantly influence the effectiveness of the engagement, allowing for optimal damage or penetration.

2.1.3 Guided vs. Unguided Systems

Guided systems, in contrast to unguided systems, possess the ability to actively control the trajectory of the vehicle during its flight. This control empowers the vehicle to dynamically adjust its path to intercept a moving target or compensate for external influences such as wind or atmospheric conditions [22]. Unguided systems, like conventional artillery shells, lack such capabilities, relying solely on initial launch parameters for their trajectory.

2.1.4 Types of Guidance Systems

Guidance systems are categorized based on their primary objectives and mechanisms:

1. Homing Guidance: These systems guide the vehicle by tracking the target's position and maintaining the line of sight (LOS) to the target. Examples include radar-homing, infrared-homing, and passive homing systems.
2. Path Following Guidance: These systems follow a predetermined path, such as a set of waypoints or a predefined trajectory. They are common in unmanned aerial vehicles (UAVs) for surveillance and reconnaissance missions.
3. Proportional Navigation: A strategy that involves adjusting the vehicle's course based on the rate of change of the LOS angle. It is effective against targets with constant velocities.
4. Impact Angle Constrained Guidance: The focus of this thesis, impact angle constrained guidance systems ensure the vehicle strikes the target at a specific angle to achieve desired outcomes.

2.1.5 Conclusion

The introduction to guidance systems lays the foundation for understanding the intricate workings of impact angle constrained guidance against dynamic targets. These systems merge sensor data, navigation algorithms, and control mechanisms to ensure accurate and effective target engagement in the realm of defence and aerospace applications. The subsequent chapters will delve deeper into the specific challenges and methodologies associated with impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets.

2.2 Components of a Guidance System

Guidance systems are complex and integrated subsystems that involve several components working harmoniously to achieve accurate target engagement. Each component contributes a crucial role in ensuring the vehicle's trajectory aligns [22] with the intended impact angle and the target's dynamics [23]. Understanding these components is essential to grasp the intricacies of impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets.

2.2.1 Sensor Systems

Sensor systems are responsible for detecting and tracking the target's position and movement. These systems provide the essential data required to calculate the vehicle's trajectory adjustments [22]. Different types of sensors can be employed based on the characteristics of the target and engagement scenario:

- **Radar Systems:** Radar sensors [22] emit electromagnetic waves and measure the reflected signals to determine target distance, azimuth, and elevation. They are effective for detecting distant and stealthy targets.
- **Infrared (IR) Sensors:** IR sensors [22] detect the heat emitted by the target and are particularly useful for tracking hot exhaust plumes of aircraft, missiles, or vehicles.
- **Electro-Optical Systems:** These sensors use visible and infrared light to detect and track targets. They are employed in guided munitions, anti-aircraft systems, and surveillance drones.
- **Multi-Sensor Fusion:** Integrating data from multiple sensors enhances the accuracy and reliability of target tracking. Fusion algorithms combine information from different sensors to provide a comprehensive target picture.

2.2.2 Navigation Systems

Navigation systems [22] determine the vehicle's own position, velocity, and orientation. Accurate navigation is crucial for calculating the optimal course corrections needed to achieve the desired impact angle. Different navigation methods [23] are used depending on the operational environment and vehicle type:

- Inertial Navigation Systems (INS): INS [22] relies on accelerometers and gyroscopes to measure changes in velocity and orientation. It provides autonomous navigation capabilities, independent of external signals.
- Global Navigation Satellite Systems (GNSS): Systems like GPS use signals from satellites to determine position and velocity. They are widely used for aircraft, UAVs, and land-based systems.

2.2.3 Control Systems

Control systems are responsible for generating the control commands necessary to adjust the vehicle's trajectory [23]. These commands manipulate the vehicle's control surfaces, thrusters, or other control actuators to achieve the desired course corrections.

- Control Surface Actuators: These mechanisms control the vehicle's attitude by adjusting control surfaces such as ailerons, elevators, and rudders on aircraft.
- Thrusters and Propulsion Systems: Used in rockets and missiles, thrusters produce thrust in specific directions, enabling course adjustments.

2.2.4 Guidance Laws

Guidance laws determine how the vehicle's trajectory is adjusted based on sensor data, target information, and impact angle constraints. Different guidance laws [25] are tailored for specific scenarios, including nonmaneuvering and maneuvering targets. Impact angle constrained guidance laws play a crucial role in directing the vehicle toward achieving the desired angle of impact.

2.2.5 Conclusion

Understanding the components of a guidance system provides insight into the intricate interplay between sensors, navigation, control, and guidance algorithms. These components collectively enable the guidance system to calculate and execute the necessary adjustments for accurate target engagement. The subsequent chapters will delve deeper into the formulation and analysis of impact angle constrained guidance strategies against nonstationary nonmaneuvering and maneuvering targets.

2.3 Types of Guidance

Guidance systems are versatile and adaptable, designed to fulfil different objectives in defence and aerospace applications. Each type of guidance strategy has its unique characteristics and applications. Understanding these types is essential for comprehending the role of impact angle constrained guidance [25] against nonstationary nonmaneuvering and maneuvering targets.

2.3.1 Homing Guidance

Homing guidance [23] strategies focus on tracking and maintaining the line of sight (LOS) to the target throughout the engagement. These strategies are based on real-time target position updates and aim to align the vehicle's trajectory with the target's predicted path. Homing guidance can be further categorized into various types:

- **Radar Homing**: Radar-guided systems [22] use electromagnetic waves to detect and track targets. They are particularly effective against targets with reflective surfaces and can operate in adverse weather conditions.
- **Infrared Homing**: Infrared-guided systems [22] detect the heat emitted by the target's engines or exhaust plumes. They are suited for engaging heat-emitting targets like aircraft and missiles.
- **Passive Homing**: Passive homing [22] utilizes the target's emissions (such as radar signals or thermal radiation) to guide the vehicle. These systems offer reduced detectability by not emitting signals themselves.

2.3.2 Path Following Guidance

Path following guidance involves directing the vehicle along a predetermined trajectory, often composed of waypoints or a predefined flight path. These strategies are commonly used in surveillance, reconnaissance, and navigation tasks. Path following guidance ensures the vehicle remains within a designated path while adjusting for deviations caused by external factors like wind or atmospheric conditions.

2.3.3 Proportional Navigation

Proportional Navigation (PN) is a guidance strategy based on maintaining a constant rate of change of the LOS angle between the vehicle and the target. As the target maneuvers, the vehicle adjusts its trajectory to ensure a collision course [25] with the target. This strategy is effective against targets with constant velocities but may encounter challenges with rapidly maneuvering targets.

2.3.4 Impact Angle Constrained Guidance

Impact angle constrained guidance [1-7] is a specialized guidance strategy that prioritizes achieving a specific angle of impact with the target. Instead of merely reaching the target's location, this strategy focuses on controlling the direction from which the vehicle approaches the target. Impact angle control is particularly important in scenarios where the angle of impact influences penetration depth, damage optimization [22], or target neutralization.

2.3.5 Conclusion

Different types of guidance strategies cater to various mission objectives and engagement scenarios. Homing guidance, path following guidance, proportional navigation, and impact angle constrained guidance each offer unique advantages in specific contexts. In the subsequent chapters, the focus will shift to impact angle constrained guidance, exploring its methodologies, challenges, and applications against both nonstationary nonmaneuvering and maneuvering targets.

2.4 Impact Angle Constrained Guidance Fundamentals

Impact angle constrained guidance [1-7] represents a specialized approach within guidance systems that emphasizes achieving a specific angle of impact when engaging a target. This approach goes beyond reaching the target's location and focuses on directing the vehicle's trajectory to strike the target at a predetermined angle relative to the target's orientation. Understanding the fundamentals of impact angle constrained guidance [23] is crucial for comprehending its application against nonstationary nonmaneuvering and maneuvering targets.

2.4.1 Rationale for Impact Angle Control

In scenarios where the outcome of engagement is influenced by the angle of impact, impact angle control becomes paramount. This is particularly relevant when considering:

- Penetration: Achieving a specific angle of impact can influence the vehicle's ability to penetrate protective layers of the target, maximizing damage potential.
- Optimal Lethality: Certain targets exhibit different levels of vulnerability when impacted from specific angles. Impact angle control can maximize lethality by targeting these vulnerabilities.
- Avoiding Deflection: Impacting the target from unfavourable angles may cause deflection or bouncing, reducing the intended effect of the engagement.

2.4.2 Mathematical Formulation

Impact angle constrained guidance involves complex mathematical formulations to calculate the necessary adjustments to the vehicle's trajectory. These formulations consider factors such as target position, velocity, and orientation, as well as the desired impact angle.

- **Geometric Considerations**: Impact angle is defined as the angle between the relative velocity vector and the target's normal vector. Achieving the desired angle [23] requires intricate geometric calculations.
- **Constrained Optimization**: The guidance system formulates an optimization problem that incorporates the impact angle constraint. This optimization aims to

determine the required changes in the vehicle's trajectory to achieve the desired impact angle.

2.4.3 Challenges and Considerations

Impact angle constrained guidance presents unique challenges:

- Target Dynamics: Engaging moving targets introduces complexities, as their motion must be predicted to achieve the desired angle of impact.
- Limited Time-to-Go: Depending on the vehicle's speed and the target's proximity, there might be limited time to execute the required trajectory adjustments.
- Real-time Calculations: Guidance systems must perform calculations in real time to ensure timely adjustments and accurate impact.

2.4.4 Applications of Impact Angle Constrained Guidance

Impact angle constrained guidance finds applications across various defence and aerospace scenarios:

- Armoured Penetration: Guided anti-armour munitions aim to strike armoured vehicles at specific angles to penetrate their protective layers.
- Air-to-Ground Munitions: Precision-guided bombs strive to achieve optimal angles to maximize target damage or avoid collateral destruction.
- Aerial Interception: Air defence systems use impact angle control to neutralize aerial threats while minimizing the risk of deflection.

2.4.5 Conclusion

Understanding the principles of impact angle constrained guidance provides insight into its crucial role in achieving desired engagement outcomes. By controlling the angle of impact, guidance systems can optimize penetration, lethality, and overall effectiveness. The subsequent chapters will explore the methodologies, challenges, and performance evaluations of impact angle constrained guidance against both nonstationary nonmaneuvering and maneuvering targets.

2.5 Guidance Laws and Equations

Guidance laws serve as the backbone of guidance systems, determining how the vehicle's trajectory is adjusted to achieve accurate target engagement. These laws are derived from mathematical formulations and equations that consider target dynamics, sensor data, and impact angle constraints. Understanding the various guidance laws [25] and their equations is essential for comprehending their application in impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets.

2.5.1 Pure Proportional Navigation (PPN)

Pure Proportional Navigation [25] is a fundamental guidance law based on maintaining a constant proportional navigation constant, often denoted as N. The equation governing PPN is: $N = \frac{V}{R}$

Where:

- N is the proportional navigation constant.
- V is the relative velocity between the vehicle and the target.
- R is the range between the vehicle and the target.

PPN ensures that the LOS rate remains constant, directing the vehicle toward the target's future position.

2.5.2 Augmented Proportional Navigation (APN)

Augmented Proportional Navigation [23] enhances PPN by incorporating terms that account for target acceleration and maneuvering. The APN equation includes additional terms to adapt the guidance law to dynamic targets.

2.5.3 Line-of-Sight (LOS) Guidance

Line-of-Sight guidance involves adjusting the vehicle's trajectory to maintain a constant angle between the LOS and the target's position. The LOS guidance equation determines the angular rate of change required for this constant angle.

2.5.4 Impact Angle Constrained Guidance Laws

Impact angle constrained guidance laws incorporate the desired impact angle into the guidance calculations. These laws are developed based on geometric relationships and constrained optimization techniques. The equations involved are complex and depend on the specific formulation used for the impact angle constraint.

2.5.5 Combining Guidance Laws

In practice, guidance systems often use a combination of guidance laws to address various aspects of engagement. For instance, a system might utilize PPN [25] for initial target interception and then switch to an impact angle constrained guidance law as the vehicle approaches the target.

2.5.6 Conclusion

Guidance laws and equations provide the mathematical foundation for directing the vehicle's trajectory. Each guidance law is designed to address specific challenges and scenarios, from pursuing targets with constant velocities to achieving impact angle constraints. The subsequent chapters will explore the formulation and adaptation of these guidance laws to engage both nonstationary nonmaneuvering and maneuvering targets while achieving the desired impact angles.

2.6 Sensor and Tracking Systems

Sensor and tracking systems [23] are essential components of guidance systems, providing the critical data required to calculate the necessary trajectory adjustments for accurate target engagement. These systems detect, monitor, and relay information about the target's position, velocity, and other relevant parameters. Understanding the types of sensors [23] and tracking methods is crucial for comprehending their role in impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets.

2.6.1 Radar Systems

Radar (Radio Detection and Ranging) systems [23] emit electromagnetic waves that bounce off objects, including targets. By measuring the time it takes for the radar waves to return, radar systems determine the distance to the target. Different types of radar systems include:

- **Continuous Wave (CW) Radar**: Generates a continuous radio signal and measures the frequency shift in the returning signal to calculate target velocity.
- **Pulse-Doppler Radar**: Transmits pulses of radio waves and analyzes the Doppler shift in the returning signal to determine both range and target velocity.
- **Synthetic Aperture Radar (SAR)**: Provides high-resolution images by utilizing the motion of the radar platform to simulate a larger antenna.

2.6.2 Infrared (IR) Sensors

Infrared sensors [22] detect the heat emitted by objects, including targets. These sensors are particularly useful for tracking heat-emitting sources, such as aircraft engines, missiles, and vehicles. IR sensors are classified into two categories:

- **Passive Infrared (PIR) Sensors**: Detect the thermal radiation emitted by the target and are effective for tracking hot exhaust plumes.
- **Active Infrared (AIR) Sensors**: Emit infrared radiation and measure the reflected signal to determine target distance and properties.

2.6.3 Electro-Optical Systems

Electro-optical systems encompass a range of sensors that use visible light and infrared radiation to detect and track targets. These sensors provide visual and thermal imagery, enabling both daytime and night-time tracking. Different types of electro-optical systems include:

- Daylight Cameras: Capture visible light images and videos of the target, providing visual information.
- Infrared Cameras: Detect the thermal radiation emitted by the target and produce thermal images, useful for tracking heat sources.

2.6.4 Multi-Sensor Fusion

Multi-sensor fusion involves combining data from different sensors to obtain a comprehensive and accurate representation of the target's state. Fusion algorithms integrate information from radar, infrared, and electro-optical sensors to improve tracking accuracy, reduce false alarms, and enhance situational awareness.

2.6.5 Challenges and Considerations

Sensor and tracking systems encounter challenges such as:

- **Clutter**: Interference from environmental objects can obscure target signals and lead to inaccurate tracking.
- **Target Discrimination**: Distinguishing between multiple targets or decoys is crucial for accurate engagement.
- **Jamming and Stealth**: Advanced adversaries might attempt to jam sensors or use stealth technologies to avoid detection.

2.6.6 Conclusion

Sensor and tracking systems provide essential data for guidance systems to accurately calculate target engagement trajectories. Understanding the strengths and limitations of radar, infrared, and electro-optical sensors, as well as the benefits of multi-sensor fusion, is vital for developing effective impact angle constrained guidance strategies. The subsequent chapters will explore the integration of sensor data into guidance algorithms for engaging both nonstationary nonmaneuvering and maneuvering targets.

2.7 Navigation and Control Systems

Navigation and control systems [25] are integral components of guidance systems, responsible for determining the vehicle's position, velocity, and orientation, as well as generating the necessary control commands for trajectory adjustments. These systems work together to ensure the accurate execution of guidance strategies, including impact angle constrained guidance. Understanding navigation and control systems is essential for comprehending their role in engaging nonstationary nonmaneuvering and maneuvering targets with desired impact angles.

2.7.1 Inertial Navigation Systems (INS)

Inertial Navigation Systems (INS) rely on accelerometers and gyroscopes to measure changes in the vehicle's velocity and orientation. These measurements are integrated over time to determine the vehicle's position. INS [22] provides autonomous navigation capabilities and is not dependent on external signals. However, errors can accumulate over time due to sensor drift, leading to decreased accuracy over long missions.

2.7.2 Global Navigation Satellite Systems (GNSS)

Global Navigation Satellite Systems, such as GPS (Global Positioning System), utilize signals from a network of satellites to determine the vehicle's position, velocity, and time. GNSS [22] provides accurate positioning information and is widely used in various applications, including aircraft, drones, and ground vehicles. However, GNSS signals can be affected by factors like line-of-sight obstruction, atmospheric conditions, and intentional jamming.

2.7.3 Control Systems

Control systems are responsible for generating the necessary control commands to adjust the vehicle's trajectory according to the guidance strategy. These commands manipulate the vehicle's control surfaces, thrusters, or other control actuators. Different types of control systems include:

- **Aerodynamic Control**: Aircraft and missiles use aerodynamic surfaces such as ailerons, elevators, and rudders to control roll, pitch, and yaw.
- **Thrust Vector Control (TVC)**: Rockets and missiles use thrust vectoring mechanisms to change the direction of thrust, allowing for precise control of the vehicle's orientation.
- **Reaction Control Systems (RCS)**: Small thrusters are used to provide fine adjustments to the vehicle's orientation in space or in atmospheric flight.

2.7.4 Flight Control Laws

Flight control laws govern the vehicle's response to control inputs, ensuring stability and desired maneuverability. These laws are developed to optimize the vehicle's performance while adhering to safety constraints.

2.7.5 Challenges and Considerations

Navigation and control systems encounter challenges such as:

- **Dynamic Environments**: Navigational accuracy can be affected by changes in the environment, such as turbulent air currents or variable terrain.
- **Sensor Integration**: Accurate navigation requires effective integration of data from sensors, including accelerometers, gyroscopes, and GNSS receivers.
- **Real-time Adjustments**: Control systems must execute adjustments in real time to ensure accurate trajectory following.

2.7.6 Conclusion

Navigation and control systems provide the necessary data and commands for guidance systems to accurately adjust the vehicle's trajectory according to the chosen strategy. Understanding the operation of inertial navigation systems, global navigation satellite systems, and control mechanisms is crucial for developing effective impact angle constrained guidance strategies. The subsequent chapters will explore the integration of navigation and control systems into impact angle constrained guidance algorithms for both nonstationary nonmaneuvering and maneuvering targets.

2.8 Guidance System Performance Metrics

Evaluating the effectiveness of guidance systems is crucial for ensuring accurate target engagement and assessing their operational capabilities. Various performance metrics [25] are used to measure the system's accuracy, efficiency, and overall impact on achieving desired outcomes. Understanding these metrics is essential for comprehending the efficacy of impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets.

2.8.1 Miss Distance

Miss distance is a key metric that quantifies the separation between the vehicle's point of impact and the target's desired point of impact. It reflects the accuracy of the guidance system's calculations and adjustments. Smaller miss distances indicate higher precision in target engagement.

2.8.2 Time-to-Go

Time-to-Go measures the time remaining for the vehicle to reach the target's position. It indicates how quickly the vehicle is closing in on the target. Shorter time-to-go values are desirable for swift engagement.

2.8.3 Impact Angle Accuracy

Impact angle accuracy quantifies how closely the actual impact angle aligns with the desired impact angle. Achieving the desired impact angle is critical for optimizing engagement outcomes, especially in scenarios where impact angle influences penetration or damage.

2.8.4 Engagement Envelope

The engagement envelope defines the operational conditions under which the guidance system can successfully engage the target. It takes into account factors such as the vehicle's speed, altitude, and engagement range. A wider engagement envelope enhances the system's versatility and effectiveness.

2.8.5 Robustness to Uncertainties

Guidance systems must perform reliably under various uncertainties, including sensor noise, target maneuvers, and environmental conditions. Evaluating the system's robustness involves analysing its performance in the presence of these uncertainties.

2.8.6 Computational Efficiency

Guidance systems often operate in real-time environments where computational resources are limited. Computational efficiency measures the system's ability to perform complex calculations and adjustments within the available time frame.

2.8.7 Simulation Framework

To evaluate guidance system performance, simulations provide a controlled environment for testing different scenarios. Simulations enable the assessment of the system's behavior under different conditions, such as target trajectories, sensor accuracy, and engagement parameters.

2.8.8 Conclusion

Guidance system performance metrics offer insights into the system's accuracy, efficiency, and robustness. Evaluating miss distance, impact angle accuracy, time-to-go, and other relevant metrics helps gauge the system's effectiveness in achieving desired engagement outcomes. The subsequent chapters will explore the application of these performance metrics in assessing the impact angle constrained guidance strategies against both nonstationary nonmaneuvering and maneuvering targets.

2.9 Trade-offs and Challenges in Guidance Systems

Guidance systems are designed to balance multiple considerations while achieving accurate target engagement. These systems face trade-offs [25] between accuracy, computational complexity, real-time requirements, and adaptability to various scenarios. Understanding these trade-offs and the challenges associated with guidance systems is crucial for developing effective impact angle constrained guidance strategies against nonstationary nonmaneuvering and maneuvering targets.

2.9.1 Accuracy vs. Computational Complexity

One of the primary trade-offs is between achieving high accuracy in target engagement and the computational complexity of the guidance calculations. More sophisticated guidance laws that consider complex target dynamics may require extensive computational resources, potentially affecting real-time performance.

2.9.2 Real-Time Requirements

Guidance systems must operate in real time to provide timely adjustments for accurate engagement. Meeting real-time requirements can be challenging, particularly when dealing with complex guidance laws, sensor data fusion, and rapid target maneuvers.

2.9.3 Robustness vs. Specificity

Guidance systems need to be robust to uncertainties, such as sensor noise, target maneuvers, and environmental conditions. However, overly robust systems might sacrifice specificity, leading to suboptimal engagement outcomes under normal conditions.

2.9.4 Handling Nonmaneuvering vs. Maneuvering Targets

Different guidance strategies are needed to engage nonmaneuvering and maneuvering targets. While engaging nonmaneuvering targets may require simpler strategies, maneuvering targets demand advanced algorithms capable of predicting and adapting to target trajectory changes.

2.9.5 Integration with Existing Systems

Integrating guidance systems with existing platforms, such as missiles or defence systems, requires ensuring compatibility and optimization. Retrofitting or upgrading existing systems to incorporate new guidance strategies can present technical challenges.

2.9.6 Ethical and Safety Considerations

Guidance systems used in military and defence applications have ethical implications, particularly when considering collateral damage, civilian safety, and adherence to rules of engagement. Ensuring that guidance systems prioritize minimizing harm is a critical consideration.

2.9.7 Cost and Resource Constraints

Developing and implementing advanced guidance systems can incur significant costs. Balancing the benefits of improved accuracy and effectiveness against budget constraints is a common challenge.

2.9.8 Conclusion

Trade-offs and challenges inherent in guidance systems stem from the need to balance accuracy, real-time performance, robustness, and adaptability to diverse scenarios. Developing impact angle constrained guidance strategies requires addressing these challenges and optimizing guidance laws, sensor integration, and computational efficiency. The subsequent chapters will delve into the methodologies for addressing these trade-offs and challenges in the context of engaging nonstationary nonmaneuvering and maneuvering targets with desired impact angles.

2.10 Conclusion

In this chapter, we explored the foundational concepts of guidance systems, laying the groundwork for understanding the intricacies of impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets. We discussed the integral components of guidance systems, including sensor and tracking systems, navigation and control systems, and the various types of guidance strategies employed in defence and aerospace applications.

We delved into the significance of impact angle constrained guidance, highlighting its role in achieving specific angles of impact for optimized outcomes such as penetration, damage optimization, and target neutralization. Mathematical formulations and challenges associated with impact angle control were also addressed, setting the stage for deeper exploration.

We covered key guidance laws [25], equations, and performance metrics used to evaluate guidance system effectiveness. The trade-offs and challenges in guidance systems, ranging from accuracy-complexity trade-offs to real-time constraints and ethical considerations, were discussed to provide a comprehensive perspective on the design and implementation of guidance strategies.

As we move forward, the subsequent chapters will delve into the specific methodologies, strategies, and adaptations required for impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets. By building upon this foundation, we aim to provide a comprehensive understanding of the complexities and practicalities of achieving accurate target engagement with desired impact angles in dynamic scenarios.

Chapter 3

Brief Review of Proportional Navigation

3.1 Engagement Geometry of Missile-Target System

Let us consider the missile and the target to be the two-point objects denoted as ‘m’ and ‘t’ in the figure [3.1] below

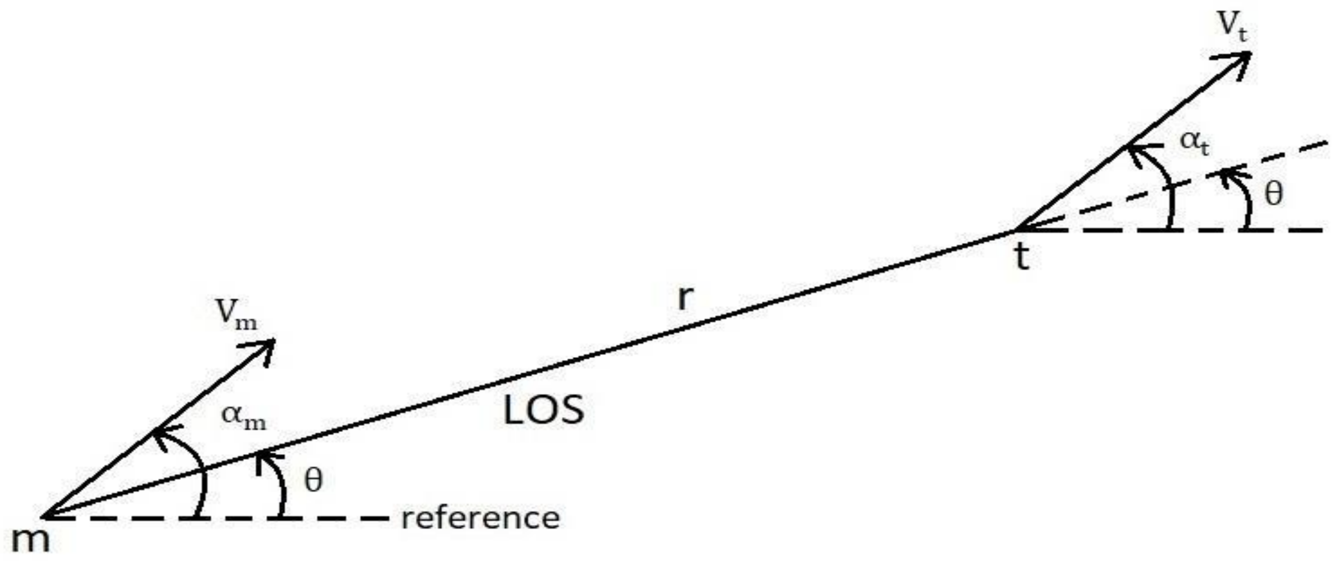

Fig. 3.1: Engagement geometry of missile-target

[Source: D. Ghose, “*Guidance of Missiles*”, NPTEL, 2012]

V_m and V_t are the constant velocities of the point objects, ‘m’ and ‘t’ which means both are moving with constant speeds in straight lines. The imaginary line joining the missile position and the target position at any instant is called the line-of-sight or LOS [25]. Here, ‘r’ denotes the distance between the two-point objects and is also called the LOS separation. The angle made by the LOS with the reference is called LOS angle, denoted by θ. The angles α_m and

α_t give the point objects 'm' and 't' the direction of movement. Here, r and θ are the only system states because they vary with time and the rest of the quantities V_m, V_t, α_m and α_t are constants. So, the equations of motion are given by,

$$V_r = \dot{r} = V_t \cos(\alpha_t - \theta) - V_m \cos(\alpha_m - \theta) \tag{3.1}$$

$$V_r = r\dot{\theta} = V_t \sin(\alpha_t - \theta) - V_m \sin(\alpha_m - \theta) \tag{3.2}$$

The components of the relative velocity of the target are V_r which is along the LOS and V_θ is normal to the LOS, with respect to the missile. There is no guidance applied here as seen in [3.1]. V_m is usually greater than V_t. The lateral acceleration (a_m) employed by the missile, commands the missile to take turn in a proper direction. The rate of change of the LOS separation is given as $\dot{r}$ and the LOS rate is $\dot{\theta}$. The closing velocity is denoted as V_c and is the negative rate of change of the LOS separation, written as:

$$V_c = -\dot{r} \tag{3.3}$$

The two equations (3.1) and (3.2) are not sufficient to call as the complete kinematic equations set. To make it a complete set, it will contain model equations showing variations in α_m, α_t, θ, V_m and V_t, which upon integrating with respect to time from some given initial conditions will give the complete trajectory of this system of equations.

3.2 Collision Triangle

The collision geometry [25] or the collision triangle is the most fundamental concept in guidance law design. Based on this concept, the very first guidance law was developed. It was called Constant Bearing Course (CBC) guidance law. This guidance law was conceptually developed and so drew attention during the initial days of guidance law development as it was the best guidance law that one could think of, but soon it revealed several drawbacks, mainly in performance and implementation. As the basic concepts were quite logical, it reveals a lot about how guidance schemes work in reality. In the later phase, it was seen that all the guidance laws try to achieve the performance of this CBC guidance law, given in [3.1].

Now let us understand the concept of collision triangle

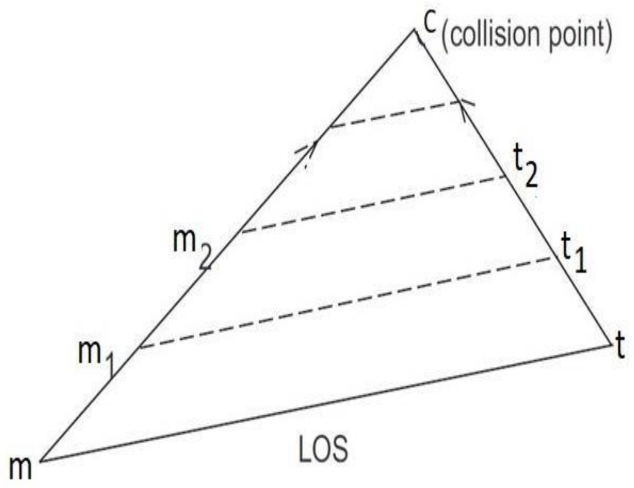

Fig. 3.2: Collision triangle

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

Consider the triangle cmt shown in Fig. 3.2 [25]. At the beginning, the missile is at 'm' and the target is at 't'. They are supposed to be moving with constant speeds in the directions shown by the arrows which implies that the time taken to cover m_c is same as the time taken to cover t_c. This means the velocity ratio of the missile and target is same as the ratio $\frac{m_c}{t_c}$ between the sides of the triangle. This is called the collision triangle. Now, when the missile is at position m_1 the target is at position t_1, when the missile is at position m_2 the target is at position t_2, and so on.

It is seen that under the given conditions the LOS [25] does not rotate at all. In other words, mt is parallel to m_1t_1, which in turn is parallel to m_2t_2 and so on. The necessary and sufficient conditions for the missile-target interception are:

- The LOS does not rotate in space (necessary condition). Mathematically, it implies that, in equation (3.2),

  $$V_\theta = 0 \tag{3.4}$$

- When the missile and the target are both flying in straight line paths and with constant speed, then in equation (3.1) V_r remains as a constant negative quantity i.e.,

  $$V_r < 0 \tag{3.5}$$

These two together ensure that the engagement ends up in a successful interception of the target by the missile. Another condition is that the length of the LOS should decrease with time.

3.3 Introduction to Proportional Navigation Guidance

Proportional Navigation Guidance (PNG) is a fundamental concept in guidance systems that plays a pivotal role in ensuring the accuracy and effectiveness of guided projectiles, such as missiles, aircraft, and even spacecraft. PNG [22] is designed to help a guided system intercept a moving target by continuously adjusting its course to maintain a collision course with the target. Here's an overview of PNG as a fundamental concept in guidance systems:

3.3.1. The Pursuit Problem:

- In guidance systems, one of the primary challenges is ensuring that a guided object (e.g., a missile) successfully intercepts a target that is in motion.
- This is known as the "pursuit problem," where the goal is to minimize the distance (miss distance) between the guided object and the target at the time of intercept.

3.3.2. The Basic Principle of PNG:

- PNG is based on the idea of continuously measuring and adjusting the Line of Sight (LOS) between the guided system and the moving target.
- The LOS is the straight line connecting the guided system to the target.
- PNG maintains a constant rate of change of the LOS angle (LOS rate) by adjusting the guided system's course.

3.3.3. Key Components of PNG:

- Line of Sight (LOS) Rate: The LOS rate is the rate at which the LOS angle between the guided system and the target is changing.
- Proportional Gain: PNG adjusts the control inputs (e.g., steering or thrust) in proportion to the LOS rate. The proportional gain factor determines how quickly the guided system responds to changes in the LOS rate.

3.3.4. Achieving Collision Course:

- The fundamental objective of PNG is to make sure the guided system's trajectory converges with the target's trajectory in a way that guarantees a collision if the target continues on its current path.
- As long as the target and guided system maintain a constant LOS rate, they are on a collision course.

3.3.5 Definition of PNG:

The definition of PNG law in [1] goes as follows:

It is a law that generates a guidance command (or a latax) which ensures that the rate of rotation of the missile velocity vector is proportional to the rate of rotation of the LOS i.e.

$$\dot{\alpha}_m = N\dot{\theta} \tag{3.6}$$

According to Newton's first law, if a body moves in a straight line at constant speed, it will continue moving in the same manner unless and until a force is acted upon it. This force is analogous to the guidance command used for making a change in the direction of the trajectory of the interceptor.

From [1], we see,

$$a_m = v_m \dot{\alpha}_m \tag{3.7}$$

In the collision triangle theory, the LOS is a constant value and the LOS rate is zero. Therefore, by combining (3.6) and (3.7) and putting the values, we get,

$$a_m = N v_m \dot{\theta}_m \tag{3.8}$$

Or, $a_m=0$

When lateral acceleration is zero, the path of the missile tends to be a straight line which is observed in case of the CBC guidance law. But in case of PNG law, it is a non-zero value, thus the path is a curvature.

3.3.6 Pure Proportional Navigation

Pure Proportional Navigation (PPN) is a simplified version of Proportional Navigation Guidance (PNG) that uses only the proportional navigation gain to guide a missile or guided system towards a moving target. PPN is a straightforward and effective method for achieving a collision course with a target, but it does not take into account some of the refinements and complexities found in more advanced guidance systems.

Here's an overview of the key characteristics of Pure Proportional Navigation:

1. Proportional Navigation Gain:

- In PPN, the guidance system relies solely on the proportional navigation gain (K) to control the trajectory of the guided system.
- The proportional gain determines how quickly the guided system responds to changes in the Line of Sight (LOS) rate, which is the rate at which the LOS angle between the guided system and the target is changing.

2. Simple Guidance Law:

- The guidance law in PPN is relatively simple. It adjusts the course of the guided system by applying a steering command proportional to the LOS rate.
- The command is typically generated as follows: Steering Command = K * LOS Rate.

3. Maintaining Constant LOS Rate:

- The fundamental goal of PPN, like PNG, is to maintain a constant LOS rate between the guided system and the target.
- When the LOS rate is constant, it ensures that the guided system is on a collision course with the target, assuming the target maintains its current trajectory.

4. Adaptation to Target Motion:

- PPN is effective in adapting to changes in the target's motion, including maneuvers and changes in velocity.
- The proportional gain allows the guided system to adjust its course in real-time, ensuring that it continues to pursue the target effectively.

5. Derivation of PPN:

For a target-centred reference frame work in [25], the position of the missile at any instant of time is given with respect to the position of the target. A given (r, θ) will be interpreted as the position of a missile that lies at an angle ($\alpha_t - \theta$) (measured clockwise), from the target velocity vector and at a distance r from the target.

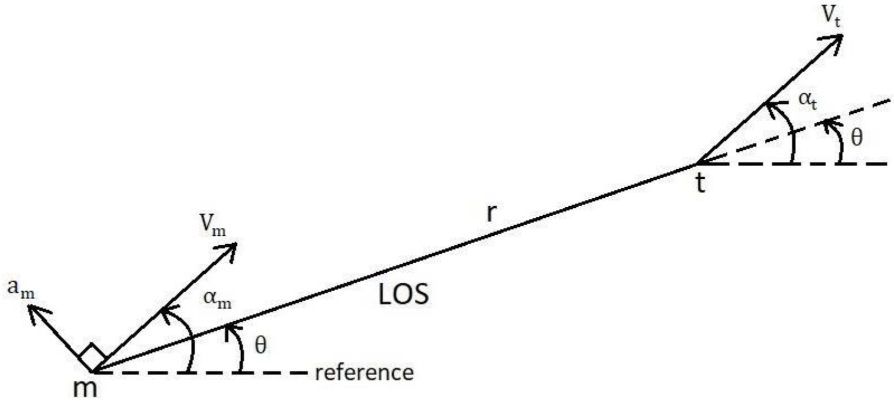

Fig. 3.3: Engagement Geometry of PPN

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

The work done here is based on stationary target. So, the equations of motion are given as,

$$V_r = \dot{r} = V_t \cos(\alpha_t - \theta) - V_m \cos(\alpha_m - \theta) \tag{3.9}$$

$$V_r = r\dot{\theta} = V_t \sin(\alpha_t - \theta) - V_m \sin(\alpha_m - \theta) \tag{3.10}$$

Here, V_r and V_θ are the relative velocities of the target, along and normal to the LOS respectively. Note that the above equations have been written using the conventional LOS angle θ. So, the engagement kinematics [25] are,

$$\dot{r} = -V_m \cos(\alpha_m - \theta) \tag{3.11}$$

$$\dot{\theta} = -\frac{V_m \sin(\alpha_m - \theta)}{r} \tag{3.12}$$

$$\dot{\alpha}_m = \frac{a_m}{v_m} \tag{3.13}$$

Combining equations (3.6) and (3.13), we get the PPN law,

$$a_m = N v_m \dot{\theta}_m \tag{3.14}$$

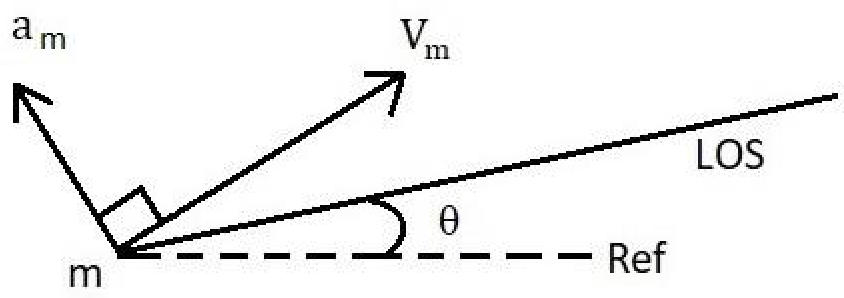

Fig. 3.4: Latax for PPN

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

6. Limitations:

- PPN is a simplified guidance method and may not provide optimal performance in all scenarios.

- It doesn't take into account factors such as acceleration guidance, lead angle, or anticipatory control, which can be considered in more advanced guidance strategies.

7. Applications:

- PPN has been used in various applications, including early generations of guided missiles and some basic guidance systems.
- While it may not be the primary guidance method in modern, high-precision systems, it can serve as a simple and effective approach for certain applications.

3.3.7 True Proportional Navigation:

Its working principle is based on the closing velocity as the LOS separation is directed to zero by the closing velocity. Also, the LOS rate is tried to drive towards zero. Thus, by exploring possibilities, the developers of guidance system were satisfied to try out the idea of making the latax of the missile perpendicular to the LOS as shown in Fig. 3.5 [25] and also proportional to the closing velocity. Again, the closing velocity was more easily available than the missile velocity.

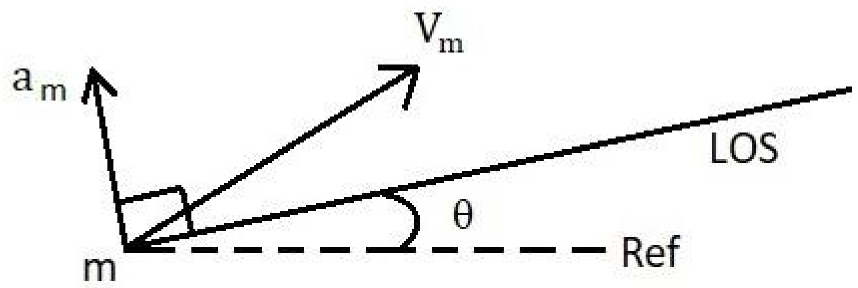

Fig. 3.5: Latax for TPN

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

TPN in [25] is defined as a law which generates a latax proportional to the closing velocity and perpendicular to the LOS. Therefore, the TPN law is obtained in the form:

$$a_m = N'V_c\dot{\theta} = -N'V_r\dot{\theta} \tag{3.15}$$

3.3.8 Generalized True Proportional Navigation

In the later years, a generalized form of the TPN law was developed where the latax direction was shifted towards the LOS by an angle from the normal as shown in Fig. 3.6 [25].

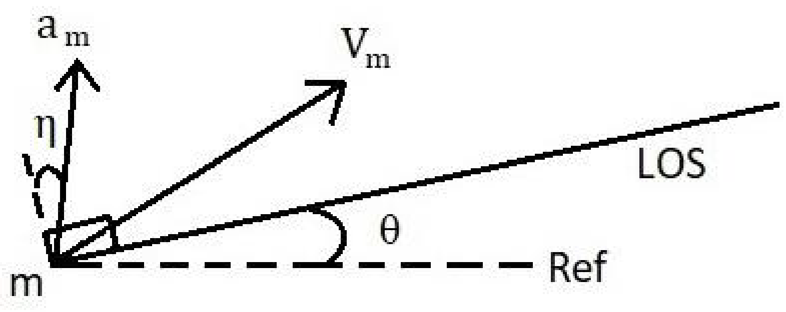

Fig. 3.6: Latax for GTPN

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

This was done to increase the performance of the guidance law in terms of capturability. It was also aimed to compare the performance of this law with that of the PPN law. Yet, the results were not up to mark when compared to the PPN law.

3.3.9 Ideal Proportional Navigation

In the ideal case, the proportional navigation law ensures that the missile's velocity vector always has a constant angle (θ) with the line of sight to the target. This means that as long as the LOS rate is nonzero (indicating that the missile is not directly on the target), the missile will make adjustments to its trajectory to intercept the target. This law is effective against targets with constant velocity.

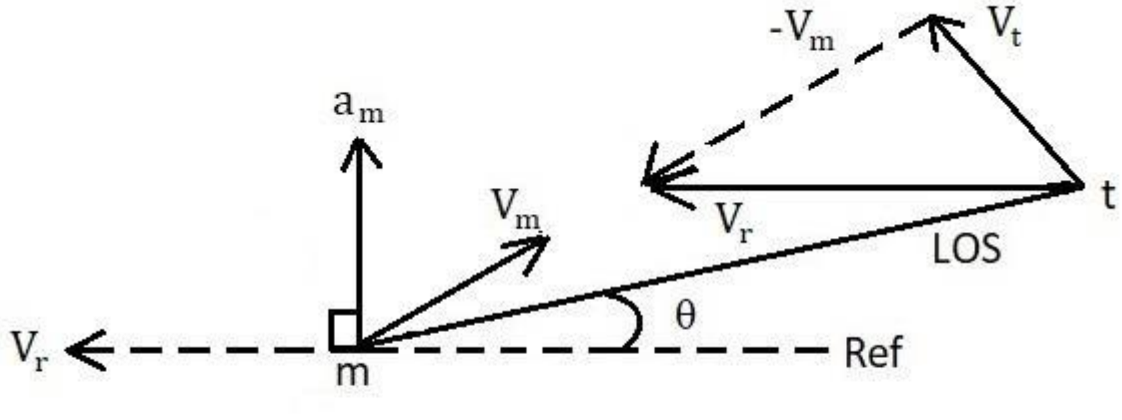

Fig. 3.7: Latax for IPN

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

3.4 Special Terminologies in Missile Guidance

3.4.1 Lateral Acceleration:

Lateral acceleration refers to the rate of change of an object's velocity in the lateral (sideways) direction. It is a measure of how quickly an object is changing its speed or direction as it moves horizontally. Lateral acceleration is a vector quantity, meaning it has both magnitude and direction.

In the context of physics and engineering, lateral acceleration is often denoted by the symbol "a_lat" and is typically measured in units of meters per second squared (m/s^2) in the International System of Units (SI).

Lateral acceleration can be experienced in various situations:

1. **Turning a Vehicle**: When you make a turn while driving a car, you experience lateral acceleration. The car changes its direction, and you feel a force pushing you to the side of the turn. This force is due to the car's lateral acceleration.
2. **Aircraft Banking**: During a turn in an aircraft, the pilot banks the plane by tilting its wings. This results in a change in the aircraft's direction, and passengers feel a force pushing them to the side. This force is due to the lateral acceleration generated by the turn.
3. **Curve in a Roller Coaster**: When riding a roller coaster and going around a curve, you may experience lateral acceleration as the coaster changes its direction quickly.

Mathematically, lateral acceleration can be calculated using the following formula:

$$a\_lat = \frac{v^2}{r}$$

Where:

- a_lat is the lateral acceleration.
- v is the velocity of the object (in the direction it's moving).
- r is the radius of the circular path the object is following (for example, the radius of a turn in a road).

Key points to note about lateral acceleration:

- Lateral acceleration is always directed toward the center of the circle or the inside of the turn in a curved path.
- The greater the speed (velocity) or the sharper the turn (smaller radius), the greater the lateral acceleration will be.
- Lateral acceleration is an important consideration in vehicle dynamics, as it affects the handling and stability of vehicles during turns.

3.4.2 Line-Of-Sight (LOS):

During a missile-target engagement, the imaginary line joining the missile and the target at any given instant in time is called the instantaneous line-of-sight or the LOS. This line changes in length and orientation as the engagement proceeds.

3.4.3 Line-Of-Sight (LOS) Angle:

The angle made by the LOS with the horizontal plane or the reference line is known as the LOS angle.

3.4.4 Line-Of-Sight (LOS) Rate:

The change in angular orientation of the LOS is given by its angular velocity or rate of turn and is usually expressed in units of radians/sec. This is called the LOS rate shown in Fig. 3.8 [22]. To ensure LOS rate is zero, the missile turn rate is made proportional to the LOS rate.

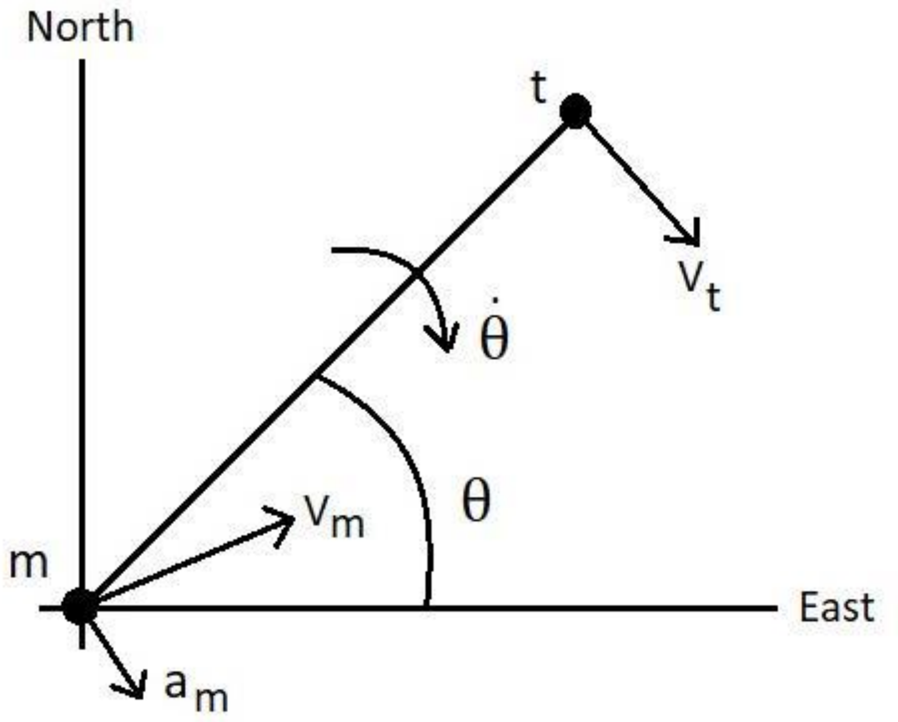

Fig. 3.8: LOS rate

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

3.4.5 Navigation Constant (Gain):

The rate of turning proportional to the angular rate of the LOS is called the Navigation Constant. For example, the missile should rotate to the right by a specific amount i.e., the factor N, faster than the LOS rate if the LOS travels slowly from north to east, shown in Fig. 3.8. N, having integer values between 3 – 5, is the dimensionless proportionality constant.

3.4.6 Heading Error:

The heading error is the difference in angle between the actual missile velocity vector and the angle required by the missile velocity vector to satisfy the collision geometry conditions. This parameter is an important performance measure for missiles that follow the mixed guidance scheme and have to transit from a command phase to a homing phase.

3.4.7 Closing Velocity:

This is the velocity with which the missile closes on to the target. Obviously, this is given by the rate at which the length of the LOS or the LOS separation shrinks. Hence, it is the negative of the rate of change of the LOS separation or range rate. It is also the Doppler relative velocity of the target with respect to the missile along the line-of-sight. Note that the Doppler relative velocity is positive when the target is approaching and negative when it is receding.

Chapter 4

Impact Angle Constrained Guidance

4.1 Importance of Impact Angle Control:

Effective guidance systems must not only intercept a target but also do so at a specific impact angle. The control of impact angle holds significant importance in various domains, and this section discusses the reasons behind its relevance.

4.1.1 Precision Targeting:

Impact angle control [1] is essential for precision targeting. By regulating the angle at which the guided entity intercepts the target, it becomes possible to direct the impact energy with precision. This is crucial in applications such as:

- **Missile Defence:** Intercepting incoming threats, such as ballistic missiles, with high precision to ensure their destruction while minimizing collateral damage.
- **Aerial Combat:** Achieving optimal firing solutions in air-to-air engagements, where hitting a specific part of an enemy aircraft can be strategically advantageous.
- **Space Exploration:** Ensuring precise rendezvous and docking procedures in space missions, where docking angles must be controlled to avoid collisions.

4.1.2 Target Vulnerabilities

Different targets [3] may have varying vulnerabilities depending on their structure and function. Impact angle control allows for exploiting specific vulnerabilities, such as targeting a weaker or less protected side of the target. This is especially relevant in:

- **Anti-Armor Missiles:** Targeting armoured vehicles by striking their less protected top or rear surfaces.

- **Naval Warfare:** Targeting specific areas on ships, like the bridge or engine room, to incapacitate the vessel effectively.

4.1.3 Minimizing Deflection and Bounce

When intercepting a target, the angle at which the guided entity collides with the target surface can affect the outcome. Impact angles that are too shallow can result in deflection or even bouncing off the target, reducing the effectiveness of the engagement [5]. Impact angle control [3] is crucial to prevent these undesirable outcomes, particularly in applications like:

- **Ground Penetrating Munitions:** Ensuring that munitions penetrate the ground or structures at an optimal angle to maximize their destructive effect.
- **Anti-ship Missiles:** Achieving a steep impact angle to minimize the chance of ricochet when targeting naval vessels.

4.1.4 Safety and Collateral Damage

In scenarios where civilian populations or friendly forces are nearby, controlling the impact angle becomes vital for minimizing collateral damage. Striking targets at specific angles can reduce the risk of stray fragments or shockwaves affecting unintended areas. Examples include:

- **Urban Warfare:** Engaging targets in urban environments where minimizing collateral damage is a top priority.
- **Counter-Terrorism Operations:** Conducting precision strikes against high-value targets in close proximity to non-combatants.

4.1.5 Environmental Considerations

In certain situations, such as space missions, planetary exploration, or environmental monitoring, impact angle control plays a role in minimizing the disturbance to the target environment. This can be vital for:

- **Planetary Exploration:** Ensuring spacecraft land at suitable angles to minimize soil disturbance and facilitate scientific research.
- **Environmental Monitoring:** Deploying sensors or scientific instruments on specific areas with minimal environmental impact.

In summary, impact angle control is a critical aspect of guidance systems as it enables precise targeting, exploitation of target vulnerabilities, and the mitigation of undesirable outcomes like deflection, bounce, and collateral damage. It also contributes to safety, environmental considerations, and the overall effectiveness of guided systems in various applications across defence, aerospace, and scientific research. Consequently, incorporating impact angle constraints into Proportional Navigation (PN) guidance systems is a valuable endeavour, as explored further in this thesis.

4.2 Strategies for Impact Angle Constrained Guidance

Effectively incorporating impact angle constraints [7] into guidance systems is a complex task that requires careful consideration of various strategies and techniques. This chapter explores the strategies used for impact angle constrained guidance, with a focus on their integration with Proportional Navigation (PN) [3] guidance.

4.2.1 Direct Control of Impact Angle

One approach to impact angle constrained guidance involves directly regulating the impact angle by modifying the guidance law's parameters. This strategy typically requires a deep understanding of the guidance law's mathematical framework and its sensitivity to changes in control parameters. Some sub-strategies within direct control include:

- **Proportional-Integral-Derivative (PID) Control:** Implementing PID controllers [7] to adjust the guidance commands based on the deviation of the impact angle from the desired value.
- **Trajectory Shaping:** Modifying the reference trajectory [2] followed by the guided entity to achieve the desired impact angle. This can involve pre-defined waypoints or adjusting the trajectory in real-time.

4.2.2 Feedback Control Systems

Feedback control systems [25] provide a dynamic approach to impact angle constrained guidance. These systems continuously measure and adjust the guidance commands to maintain the desired impact angle. Strategies in this category include:

- **State Estimation and Feedback:** Using state estimation techniques, such as Kalman filtering [7], to estimate the target's state and adjust guidance commands in real-time to achieve the desired impact angle.
- **Adaptive Control:** Implementing adaptive control algorithms [8] that adjust the control gains based on the error in impact angle, target behaviour, and other factors.

4.2.3 Two-Stage Guidance

In some cases, impact angle constraints are met by employing a two-stage guidance approach [10]. The first stage is dedicated to achieving a suitable intercept trajectory, while the second stage fine-tunes the approach to satisfy impact angle constraints. Strategies in this category include:

- **Mid-Course and Terminal Guidance:** Using mid-course guidance [9] to bring the guided entity close to the target and then transitioning to terminal guidance, which focuses on precise impact angle control.
- **Guidance Handover:** Employing different guidance laws or control strategies at different stages of the engagement to optimize impact angle control.

4.2.4 Guidance Filters and Smoothers

Guidance filters and smoothers are mathematical tools used to optimize guidance commands to satisfy impact angle constraints. These strategies involve:

- **Kalman Filters:** Employing Kalman filters [7] to estimate target states and optimize guidance commands based on estimated target behaviour and impact angle requirements.
- **Sliding Mode Control:** Using sliding mode control techniques [16] to ensure that the guided entity converges to the desired impact angle while accounting for uncertainties.

4.2.5 Monte Carlo and Numerical Optimization

Monte Carlo simulations and numerical optimization techniques [16] can be applied to identify optimal guidance commands and trajectories that meet impact angle constraints. These strategies involve:

- **Monte Carlo Simulations:** Running a large number of simulations with different guidance parameters to find the combination that satisfies impact angle constraints most effectively.
- **Numerical Optimization:** Employing numerical optimization algorithms to iteratively adjust guidance commands and parameters to meet impact angle requirements while minimizing other performance metrics.

4.2.6 Machine Learning and Artificial Intelligence

Recent advancements in machine learning and artificial intelligence have introduced novel approaches to impact angle constrained guidance. Strategies in this category include:

- **Reinforcement Learning:** Training reinforcement learning agents to optimize guidance commands in real-time to achieve the desired impact angle.
- **Neural Networks:** Employing neural networks to model and predict target behaviour and adjust guidance accordingly.

In conclusion, achieving impact angle constrained guidance requires the integration of various strategies and techniques into the guidance system. The choice of strategy depends on factors such as the specific application, the complexity of the target behaviour, and the available computational resources. The subsequent chapters of this thesis will delve into the development and analysis of impact angle constrained guidance strategies, considering the Proportional Navigation (PN) guidance framework and its application to nonstationary nonmaneuvering targets.

4.3 Integration of Constraints into PN Guidance

Integrating impact angle constraints into the Proportional Navigation (PN) guidance framework is a critical step in achieving precise targeting against nonstationary nonmaneuvering targets. This chapter explores the methodologies and techniques involved in incorporating impact angle constraints into PN guidance systems.

4.3.1 Augmented Guidance Laws

One approach to integrating impact angle constraints [2] into PN guidance is to augment the PN guidance law with additional terms or controllers responsible for regulating the impact angle. These augmentations can take the form of:

- **Angle Error Feedback:** Introducing a feedback loop that continuously monitors the difference between the desired impact angle and the actual impact angle. This error is used to modify the guidance commands in real-time.
- **Secondary Guidance Loops:** Implementing secondary guidance loops dedicated to impact angle control, running in parallel with the primary PN guidance loop.

4.3.2 Constraint Formulation

Formulating impact angle [3] constraints within the PN guidance system involves defining the desired impact angle and representing it mathematically. Key steps in this process include:

- **Defining the Desired Impact Angle:** Establishing a clear and specific definition of the desired impact angle based on mission objectives and target vulnerabilities.
- **Mathematical Representation:** Translating the desired impact angle into a mathematical form that can be integrated into the guidance system's control algorithms. This often involves trigonometric relationships and vector calculations.

4.3.3 Control Mechanisms and Algorithms

To ensure the integration of constraints into PN guidance, suitable control mechanisms and algorithms [4] must be selected or designed. These mechanisms include:

- **Controller Design:** Developing controllers that generate corrective commands to align the guided entity's trajectory with the desired impact angle.
- **Control Law Synthesis:** Synthesizing control laws that combine PN guidance commands with impact angle control commands in a coherent and effective manner.

4.3.4 Trajectory Planning

Effective trajectory planning [8] is crucial for meeting impact angle constraints within the PN guidance framework. Key considerations include:

- **Trajectory Generation:** Generating optimal trajectories that lead to the target while satisfying the impact angle constraints.
- **Dynamic Adjustments:** Dynamically adjusting the guided entity's trajectory based on real-time feedback to ensure impact angle compliance.

4.3.5 Simulation and Testing

Before deployment, impact angle constrained PN guidance systems must undergo rigorous simulation and testing to validate their performance. This involves:

- **Simulated Scenarios:** Creating diverse and realistic scenarios to assess the system's ability to meet impact angle constraints under various conditions.
- **Performance Metrics:** Defining appropriate performance metrics to quantify the system's effectiveness in achieving the desired impact angle.
- **Sensitivity Analysis:** Conducting sensitivity analysis to evaluate how changes in parameters or target behaviour affect impact angle compliance.

4.3.6 Real-world Implementation Challenges

Implementing impact angle constraints in real-world systems may pose challenges related to sensor accuracy, computational requirements, and environmental conditions. These challenges need to be addressed through:

- **Sensor Fusion:** Integrating data from multiple sensors to improve the accuracy of target tracking and impact angle estimation.
- **Hardware and Software Optimization:** Optimizing hardware and software components to meet computational demands while maintaining real-time performance.
- **Environmental Adaptation:** Developing strategies to adapt to changing environmental conditions that may affect impact angle control, such as wind or atmospheric disturbances.

In conclusion, integrating impact angle constraints into the Proportional Navigation (PN) guidance framework is a multidisciplinary process involving mathematical formulation, control system design, trajectory planning, and extensive testing. This integration is essential for achieving precise targeting against nonstationary nonmaneuvering targets and is a central focus of this thesis. Subsequent chapters will delve into specific methodologies, case studies, and performance assessments related to impact angle constrained PN guidance.

4.4 Control Mechanisms and Algorithms for Impact Angle Constrained Guidance

The integration of impact angle constraints into the Proportional Navigation (PN) guidance system necessitates the development of control mechanisms and algorithms that can regulate the guided entity's trajectory to achieve the desired impact angle. This chapter explores the key control mechanisms and algorithms employed for impact angle constrained guidance within the PN framework.

4.4.1 Impact Angle Error Feedback Control

One fundamental control mechanism for achieving impact angle constrained guidance is the feedback control system [12]. It operates by continuously monitoring the difference between the desired impact angle ($\theta_{desired}$) and the actual impact angle (θ_{actual}). The error signal (θ_{error}) is then used to generate corrective commands to adjust the trajectory:

$$\Theta_{error} = \theta_{desired} - \theta_{actual}$$

The feedback loop calculates control commands based on θ_{error} to align the guided entity's trajectory with the desired impact angle. Common control techniques used in this context include:

- **Proportional (P) Control:** Adjusting the guidance commands proportionally to the error signal. This provides a basic level of control but may result in steady-state errors.
- **Proportional-Integral (PI) Control:** In addition to proportional control, integrating the error signal over time to eliminate steady-state errors and improve convergence.
- **Proportional-Integral-Derivative (PID) Control:** Combining proportional, integral, and derivative control components to further enhance performance, particularly in response to rapid changes in impact angle.

4.4.2 Trajectory Modification

Achieving the desired impact angle [12] often involves modifying the guided entity's trajectory. Various algorithms can be employed to compute trajectory adjustments based on real-time feedback and impact angle constraints:

- **Path Planning Algorithms:** Employing path planning techniques, such as A* or D* algorithms, to generate trajectories that satisfy impact angle constraints while avoiding obstacles or disturbances.
- **Trajectory Shaping:** Pre-defining waypoints or control points along the trajectory that ensure the guided entity reaches the target at the desired impact angle.
- **Optimization Methods:** Using numerical optimization techniques, such as gradient descent or genetic algorithms, to iteratively adjust the trajectory to meet impact angle constraints.

4.4.3 Control Law Synthesis

The synthesis of control laws [14] is essential for harmoniously combining PN guidance commands with impact angle control commands. This involves developing mathematical expressions that incorporate both the PN guidance law and the impact angle control mechanisms:

Total Guidance Command=PN Guidance Command+Impact Angle Control Command

The choice of control law synthesis method depends on the specific requirements of the application, the complexity of the target's behaviour, and the desired level of precision.

4.4.4 Adaptive Control

Adaptive control mechanisms [16] offer flexibility and robustness in the face of changing target dynamics or external disturbances. These mechanisms adjust control parameters and algorithms based on real-time feedback and system identification:

- **Model Reference Adaptive Control (MRAC):** Adapting the guidance system to track a reference model that represents the desired impact angle behaviour.
- **Gain Scheduling:** Adjusting control gains based on the guided entity's state, target behaviour, or environmental conditions.

4.4.5 Guidance Filters and Smoothers

Guidance filters and smoothers [16] are mathematical tools that can optimize guidance commands to meet impact angle constraints:

- **Kalman Filters:** Employing Kalman filters for state estimation and guidance command optimization to ensure the guided entity reaches the target with the desired impact angle.
- **Sliding Mode Control:** Implementing sliding mode control techniques to enforce impact angle constraints and provide robustness against uncertainties.

4.4.6 Machine Learning and Artificial Intelligence

Recent advancements in machine learning and artificial intelligence have introduced innovative approaches to impact angle constrained guidance:

- **Reinforcement Learning:** Training reinforcement learning agents to optimize guidance commands and trajectory adjustments in real-time to achieve the desired impact angle.
- **Neural Networks:** Utilizing neural networks to model and predict target behaviour and adapt guidance accordingly for impact angle control.

In conclusion, control mechanisms and algorithms play a central role in achieving impact angle constrained guidance within the Proportional Navigation (PN) framework. The selection of the appropriate control strategy depends on factors such as the application's requirements, the nature of the target, and the available computational resources. The subsequent chapters of this thesis will delve into specific methodologies, simulations, and case studies that showcase the practical implementation and performance evaluation of these control mechanisms and algorithms in impact angle constrained PN guidance systems.

Chapter 5

Modelling of Nonstationary Nonmaneuvering Targets

5.1 Characterizing Nonstationary Target Behaviour

Characterizing the behaviour of nonstationary targets [21] is a critical step in developing effective guidance strategies against such targets. Nonstationary targets are those whose motion patterns vary over time, and understanding their behaviour is essential for predicting their future positions and velocities accurately. This chapter explores the methods and considerations for characterizing nonstationary target behaviour.

5.1.1 Nonstationary Target Types

Nonstationary targets can exhibit various dynamic behaviours, which can include:

- **Varying Speed:** Targets may change their speed, either gradually or abruptly, during their motion. These speed variations can affect their relative velocity with respect to the guided entity.
- **Changing Heading:** Targets can alter their heading or orientation, causing shifts in their motion trajectory and direction.
- **Erratic Motion:** Some nonstationary targets exhibit erratic or unpredictable motion patterns, making their behaviour challenging to predict accurately.
- **Environmental Interaction:** In specific contexts, such as underwater or atmospheric conditions, targets may be subject to external forces or disturbances that influence their motion.

5.1.2 Target Trajectory Patterns

Understanding common trajectory patterns [22] of nonstationary targets helps in predicting their future positions. Some typical trajectory patterns include:

- **Linear Paths:** Targets moving in a straight-line trajectory at varying speeds. This pattern is relatively simple but can still pose challenges due to speed variations.
- **Curved Paths:** Targets following curved or nonlinear trajectories, which may require more complex modelling.
- **Loitering:** Targets that linger in a specific area before resuming movement, which can complicate predictions.
- **Intermittent Motion:** Targets that alternate between periods of movement and rest, necessitating adaptive tracking and prediction.

5.1.3 Target Behaviour Estimation

Accurate estimation of a nonstationary target's future positions [22] and velocities is crucial for successful engagement. Methods for estimating target behaviour include:

- **Kinematic Models:** Developing mathematical models based on observed target behaviour to predict future positions and velocities. These models may consider factors like acceleration, deceleration, and turning rates.
- **Sensor Data Fusion:** Integrating data from multiple sensors, such as radar, lidar, or cameras, to enhance target tracking and motion estimation accuracy. Sensor fusion techniques aim to reduce measurement uncertainties.
- **Machine Learning:** Leveraging machine learning algorithms, such as neural networks or Bayesian networks, to predict target behaviour based on historical data and current observations. Machine learning models can adapt to complex and non-linear target behaviours.

5.1.4 Uncertainty and Noise Considerations

Nonstationary target tracking [23] is often affected by uncertainties and sensor noise[23]. Accurate characterization and quantification of these uncertainties are vital for robust predictions. Methods for handling uncertainty include:

- **Probabilistic Models:** Implementing probabilistic models, such as Kalman filters or particle filters, to account for uncertainty in target behaviour. These models provide probability distributions over possible target states.
- **Sensitivity Analysis:** Conducting sensitivity analysis to assess how variations in target behaviour estimates affect the guidance system's performance.

Understanding the system's sensitivity to modelling errors is crucial for making robust predictions.

5.1.5 Environmental Factors

Environmental factors, such as wind, terrain, or obstacles, can significantly influence the behaviour of nonstationary targets. Modelling these factors and their effects on target trajectories is essential for accurate guidance. Considerations include:

- **Wind Models:** Incorporating wind speed and direction data into target trajectory predictions, especially for applications like missile guidance, where wind can have a substantial impact.
- **Terrain Mapping:** Utilizing digital terrain models to account for ground elevation variations in target tracking. This is particularly relevant for guided entities navigating over complex landscapes.
- **Obstacle Avoidance:** Implementing algorithms to ensure that the guided entity can navigate around obstacles when engaging nonstationary targets in cluttered environments, such as urban warfare scenarios.

5.1.6 Time-Varying Behaviours

Some nonstationary targets may exhibit time-varying behaviours [22] that change over the course of an engagement. Recognizing and adapting to these time-varying patterns [21] is crucial for adaptive guidance strategies. Considerations include:

- **Behaviour Pattern Recognition:** Developing algorithms to recognize and adapt to changing target behaviours during an engagement. This may involve real-time updates to prediction models.
- **Learning Algorithms:** Employing online learning techniques that adapt to evolving target behaviours in real-time, ensuring that the guidance system can respond to changes.

5.1.7 Data Sources and Sensors

The choice of data sources and sensors for target tracking and behaviour modelling is critical for the accuracy of predictions. Considerations include:

- **Sensor Fusion:** Combining data from multiple sensors, such as radar, GPS, and visual cameras, to improve target tracking accuracy and reduce the impact of sensor limitations.

- **Data Latency:** Accounting for the time delay between sensor measurements and their integration into target behaviour models. Reducing latency is essential for real-time tracking and prediction.

In summary, characterizing nonstationary target behaviour is a foundational step in developing effective guidance strategies. Understanding the various behaviours and their potential complexities, along with modelling uncertainties and environmental factors, is essential for predicting future target positions and velocities accurately. The subsequent chapters of this thesis will build upon this understanding to develop guidance strategies that can effectively engage nonstationary nonmaneuvering targets within the Proportional Navigation (PN) framework.

5.2 Mathematical Models for Nonstationary Targets

To effectively engage nonstationary nonmaneuvering targets, it is essential to develop mathematical models that describe the behaviour and motion of these targets accurately. This chapter explores various mathematical models and approaches commonly used for characterizing nonstationary target behaviour.

5.2.1 Linear Kinematic Models

Linear kinematic models [23] are fundamental for describing the motion of nonstationary targets with constant or changing velocities. These models are relatively simple but can be effective for predicting target positions and velocities over short time horizons. Common linear kinematic models include:

- **Constant Velocity (CV) Model:** Assumes that the target moves at a constant velocity.
- **Constant Acceleration (CA) Model:** Assumes that the target's acceleration remains constant.
- **Time-Varying Velocity Model:** Allows for changing velocities over time, accommodating targets with speed variations.

5.2.2 Nonlinear Kinematic Models

Nonlinear kinematic models [22] offer more flexibility than linear models, making them suitable for describing nonstationary targets with complex or variable motion patterns. These models account for nonlinearity in target behaviour and often involve higher-order derivatives. Some examples of nonlinear kinematic models include:

- **Pursuit Curves:** Models that describe target motion patterns as pursuit curves, which are a function of time and include parameters for curvature and orientation.
- **Bezier Curves:** Parametric curves that allow for flexible modelling of target trajectories, particularly useful when targets exhibit curved or erratic motion.
- **Spline Models:** Spline-based models enable the representation of smooth and continuously differentiable trajectories, accommodating both linear and nonlinear target behaviours.

5.2.3 Dynamic Models

Dynamic models [23] consider not only the kinematics of nonstationary targets but also their dynamic behaviour, including changes in acceleration. These models are more complex but offer a more accurate representation of target motion. Dynamic models include:

- **Constant Jerk (CJ) Model:** Accounts for variations in acceleration, assuming that the target's jerk (rate of change of acceleration) remains constant.
- **Constant Turn Rate and Acceleration (CTRA) Model:** A more sophisticated dynamic model that considers variations in both turn rate and acceleration. This model is suitable for targets that exhibit more complex behaviour.

5.2.4 Probabilistic Models

Probabilistic models [22] are essential for handling uncertainty in target behaviour. These models provide probability distributions over possible target states, accounting for measurement noise and modelling errors. Common probabilistic models include:

- **Kalman Filter:** A recursive estimation algorithm that combines measurements with dynamic models to estimate target positions and velocities while taking into account measurement noise and uncertainty.
- **Particle Filter:** A Monte Carlo-based approach that represents target states using a set of particles. Particle filters can handle nonlinear and non-Gaussian uncertainty.

5.2.5 Neural Network Models

Artificial neural networks [22], particularly recurrent neural networks (RNNs) and long short-term memory networks (LSTMs), have shown promise in modelling complex and time-varying target behaviours. These models can capture patterns in historical data and adapt to changing target dynamics over time.

5.2.6 Hybrid Models

Hybrid models [22] combine multiple modelling techniques to leverage their respective strengths. For example, a hybrid model may use a linear kinematic model for short-term predictions and switch to a neural network-based model for long-term predictions when targets exhibit dynamic and nonlinear behaviour.

5.2.7 Model Adaptation and Learning

In dynamic environments, it may be necessary to adapt or update mathematical models based on real-time observations and data. Adaptive models and learning algorithms, such as online machine learning methods, allow models to continuously evolve and improve their accuracy as they receive new information.

In conclusion, selecting the appropriate mathematical model for characterizing nonstationary target behaviour depends on factors such as the complexity of the target's motion, the available data, and the computational resources. Effective modelling is crucial for predicting target positions and velocities accurately, which forms the foundation for developing guidance strategies against nonstationary nonmaneuvering targets within the Proportional Navigation (PN) framework, as explored in subsequent chapters.

5.3 Prediction and Estimation Techniques for Nonstationary Targets

Predicting and estimating the future positions and velocities of nonstationary targets are critical components of effective guidance systems. This chapter explores various prediction and estimation techniques [21] commonly employed to anticipate the behaviour of nonstationary targets within the Proportional Navigation (PN) guidance framework.

5.3.1 Extrapolation Methods

Extrapolation methods [21] involve extending the target's current trajectory into the future to predict its future positions and velocities. Common extrapolation techniques include:

- **Linear Extrapolation:** Assuming that the target will continue its current motion pattern in a straight line at a constant velocity.
- **Polynomial Extrapolation:** Fitting a polynomial curve to the target's historical positions and velocities and extrapolating it to estimate future behaviour.
- **Spline Extrapolation:** Employing spline curves to capture more complex and nonlinear target trajectories and extrapolating them into the future.

5.3.2 Kalman Filtering

The Kalman filter [16] is a recursive estimation algorithm that combines measurements with dynamic models to estimate a target's state, including position and velocity. Kalman filters are well-suited for handling noisy measurements and can adapt to changing target behavior over time. They provide not only point estimates but also covariance matrices that represent the uncertainty in the estimates.

5.3.3 Particle Filtering

Particle filtering [21], also known as Monte Carlo localization, is a probabilistic technique that represents the target's state using a set of particles or samples. These particles are propagated through a dynamic model, and their weights are updated based on measurements. Particle filters can handle nonlinear and non-Gaussian uncertainties and are particularly useful when dealing with complex target behaviours.

5.3.4 Neural Networks

Artificial neural networks [22], including recurrent neural networks (RNNs) and long short-term memory networks (LSTMs), have shown promise in predicting nonstationary target behaviour. These networks can capture temporal patterns in historical data and provide dynamic predictions based on past observations.

5.3.5 Model-Based Prediction

Model-based prediction [22]involves using mathematical models of target behaviour, such as kinematic or dynamic models, to forecast future target positions and velocities. These models can be integrated with measurement updates from sensors to refine predictions.

5.3.6 Sensor Fusion

Sensor fusion techniques [22] combine data from multiple sensors, such as radar, LIDAR, and cameras, to improve the accuracy of target predictions. Fusion algorithms use complementary information from different sensors to reduce measurement noise and increase confidence in the estimated target state.

5.3.7 Learning and Adaptation

In dynamic environments, it may be necessary to continuously adapt prediction and estimation techniques to changing target behavior. Online machine learning methods, including reinforcement learning and online parameter tuning, enable guidance systems to learn and adapt based on real-time observations.

5.3.8 Hybrid Approaches

Hybrid prediction and estimation approaches [23] combine multiple techniques to leverage their respective strengths. For example, a hybrid approach may use Kalman filtering for short-term predictions and neural networks for long-term predictions, providing accurate estimates across different time horizons.

5.3.9 Uncertainty Quantification

Quantifying uncertainty in predictions and estimations is crucial for decision-making in guidance systems. Techniques such as confidence intervals, Bayesian methods, and Monte Carlo simulations can provide insight into the uncertainty associated with predicted target states.

In summary, prediction and estimation techniques for nonstationary targets play a pivotal role in developing accurate and reliable guidance systems within the Proportional Navigation (PN) framework. The choice of technique depends on factors such as the nature of the target's behaviour, available sensor data, and computational resources. These techniques provide the foundation for making informed decisions and generating guidance commands to effectively engage nonstationary nonmaneuvering targets.

5.4 Overview of Nonmaneuvering Targets

Nonmaneuvering targets are a class of targets that exhibit limited or no intentional changes in their trajectory or motion patterns during an engagement. Understanding the characteristics and challenges associated with nonmaneuvering targets is crucial for developing guidance strategies within the Proportional Navigation (PN) framework.

5.4.1 Characteristics of Nonmaneuvering Targets

Nonmaneuvering targets are characterized by several key features:

- **Constant Velocity:** Nonmaneuvering targets typically maintain a relatively constant velocity throughout their motion. This simplifies their kinematic behaviour.
- **Linear Trajectories:** They often follow linear or near-linear trajectories, making their motion predictable over short time intervals.
- **Limited Maneuverability:** Nonmaneuvering targets lack the ability to perform abrupt or rapid maneuvers, which distinguishes them from maneuvering targets.

- **Steady Heading:** Their heading or orientation tends to remain relatively stable during the engagement, simplifying predictions of their future positions.

5.4.2 Challenges in Guiding Against Nonmaneuvering Targets

While nonmaneuvering targets have predictable behaviour compared to maneuvering targets, they still pose challenges for guidance systems:

- **Prediction Errors:** Even small errors in predicting the target's future position or velocity can lead to misses or off-target engagements.
- **Environmental Factors:** External factors like wind, terrain, or obstacles can affect the target's motion, requiring consideration in guidance strategies.
- **Sensor Limitations:** The accuracy and refresh rate of sensors used to track nonmaneuvering targets can impact prediction quality.
- **Impact Angle Control:** Achieving the desired impact angle against nonmaneuvering targets is essential for optimizing engagement effectiveness.

5.4.3 Guidance Strategies for Nonmaneuvering Targets

Guidance strategies [25] for nonmaneuvering targets within the PN framework often focus on achieving precise impact angles and minimizing prediction errors:

- **Proportional Navigation (PN):** The classic PN guidance law remains a powerful tool for intercepting nonmaneuvering targets. By regulating the line-of-sight rate, PN guidance can converge to the desired impact angle.
- **Impact Angle Control:** Special attention is given to impact angle control, ensuring that the guided entity intercepts the target at the desired angle, exploiting target vulnerabilities, and minimizing the risk of deflection or bounce.
- **Trajectory Shaping:** Guidance systems may incorporate trajectory shaping techniques to fine-tune the intercept path and ensure optimal impact angles.
- **Adaptive Approaches:** Adaptive control mechanisms and learning algorithms may be used to continuously adjust guidance commands based on real-time observations, enhancing performance against nonmaneuvering targets.

5.4.4 Sensor Requirements

Effective engagement of nonmaneuvering targets relies on accurate target tracking and prediction [22]. The choice and capabilities of sensors play a significant role in target tracking:

- **Radar Systems:** High-frequency radar systems can provide accurate range and velocity measurements for nonmaneuvering targets.
- **Lidar and Optical Systems:** Lidar and optical sensors are valuable for precise target tracking and impact angle estimation, especially in clear line-of-sight conditions.
- **Inertial Navigation Systems (INS):** INS can improve tracking accuracy and help account for sensor limitations and data latency.

5.4.5 Safety Considerations

Engaging nonmaneuvering targets may require careful consideration of safety factors [25], especially in scenarios where collateral damage or unintended consequences must be minimized. Guidance systems should incorporate safety constraints to prevent engagements that could lead to undesirable outcomes.

In conclusion, nonmaneuvering targets, although exhibiting relatively predictable behaviour, pose unique challenges for guidance systems, particularly in terms of prediction accuracy and impact angle control. Developing effective guidance strategies and ensuring accurate target tracking through appropriate sensors are essential for successful engagements against nonmaneuvering targets within the Proportional Navigation (PN) framework.

Chapter 6

Simulation for Stationary Target

6.1 Introduction:

A stationary target [9] refers to an object or entity that remains fixed in one position relative to its surroundings, with no significant motion or change in location over a specified period of time. In various fields and applications, the concept of a stationary target plays a crucial role, serving as a reference point, a benchmark, or a point of observation. The stationary nature of such targets is fundamental for numerous scientific, technological, and practical purposes.

Stationary targets are encountered in a wide range of domains, each with its specific significance and implications:

1. **Radar and Sonar Systems:** In military and civilian applications, radar and sonar systems rely on the detection and tracking of stationary targets. These systems emit electromagnetic or acoustic waves and analyse their reflections to identify objects that are not in motion. Stationary targets in this context can include fixed infrastructure, such as buildings, or objects with negligible velocity, like buoys or landmarks.
2. **Astronomy and Astrophysics:** Astronomers use stationary celestial objects, such as stars, as reference points for navigation and observation. The stability of these targets is essential for precise measurements of the position, motion, and properties of other celestial bodies.
3. **Geodetic Surveys:** In the field of geodesy, stationary targets are used as reference points for mapping and measuring the Earth's surface. These reference points, often marked with survey monuments or benchmarks, help in accurately determining distances, elevations, and land boundaries.

4. **Navigation and GPS:** Global Positioning System [9] (GPS) and navigation systems rely on stationary ground-based transmitters or satellites to provide accurate location information to users. The predictability of these stationary targets ensures reliable and consistent positioning data.
5. **Environmental Monitoring:** In environmental sciences, stationary targets, such as weather stations and monitoring equipment, are placed at specific locations to record data over time. This data is vital for tracking changes in weather patterns, air quality, and other environmental parameters.
6. **Industrial Applications:** In manufacturing and quality control, stationary targets are often used for aligning machinery, calibrating instruments, and ensuring the precision of processes. These targets act as stable reference points for maintaining product quality and consistency.
7. **Space Exploration:** In space missions, stationary targets can refer to objects like space stations or planets that are relatively stationary in the context of the mission. Understanding the motion and characteristics of these targets is crucial for mission planning and execution.

The study and utilization of stationary targets are integral to the advancement of technology, scientific research, and various applications that depend on accuracy, stability, and predictability. Whether in the realm of surveillance, measurement, navigation, or scientific observation, the concept of stationary targets plays a fundamental role in achieving precision and reliability in diverse fields.

6.2 Mathematical Model

Let us consider, the target is at (L, 0) and the missile is at (0, 0) and the main object is to hit the target.

Here,

x = Distance from the target to missile

z = Height from the ground

α = impact angle

v_m = Missile velocity

a_m = Centrifugal Acceleration

So, we can write that

$$\dot{x} = v_m \cos\alpha \tag{6.1}$$

$$\dot{z} = v_m \sin\alpha \tag{6.2}$$

$$\dot{\alpha} = -\frac{a_m}{v_m} \tag{6.3}$$

Impact time = t_f =final time

So that,

$$\alpha = \alpha_0 = -\frac{a_m}{v_m} t \tag{6.4}$$

if α_f = Target Angle

We can write that,

$$\dot{x}(t) = v_m[\cos(\alpha_0 - \frac{a_m}{v_m} t) \tag{6.5}$$

By integrating the above equation, we find that

$$x(t) = -\frac{v_m^2}{a_m}[\sin(\alpha_0 - \frac{a_m}{v_m} t) - \sin\alpha_0] \tag{6.6}$$

Similarly, we find that

$$z(t) = \frac{v_m^2}{a_m}[\cos(\alpha_0 - \frac{a_m}{v_m} t) - \cos\alpha_0] \tag{6.7}$$

Putting $z(t_f) = 0$ we get, $\cos\alpha_f = \cos\alpha_0$ and from this we said that

$\alpha_f = -\alpha_0$.

Again putting $x(t_f) = L$

We get,

$$\frac{v_m^2}{a_m} = \frac{L}{2\sin\alpha_0} \tag{6.8}$$

We assume that, t_f is known then,

$$\frac{v_m}{a_m} = \frac{t_f}{2\alpha_0} \tag{6.9}$$

$$v_m = \frac{\alpha_0 L}{t_f \sin\alpha_0} \tag{6.10}$$

$$a_m = \frac{2\sin\alpha_0\, v_m^2}{L} \tag{6.11}$$

If T= Total time and t = iteration then $T = \Delta T \cdot t$

So the equations are,

$$x_{k+1} = v_m \cos\alpha \cdot \Delta T + x_k \tag{6.12}$$

$$z_{k+1} = v_m \sin\alpha \cdot \Delta T + z_k \tag{6.13}$$

$$\alpha_{k+1} = \alpha_k - \frac{a_m \cdot \Delta T}{v_m} \tag{6.14}$$

6.3 Simulation Result:

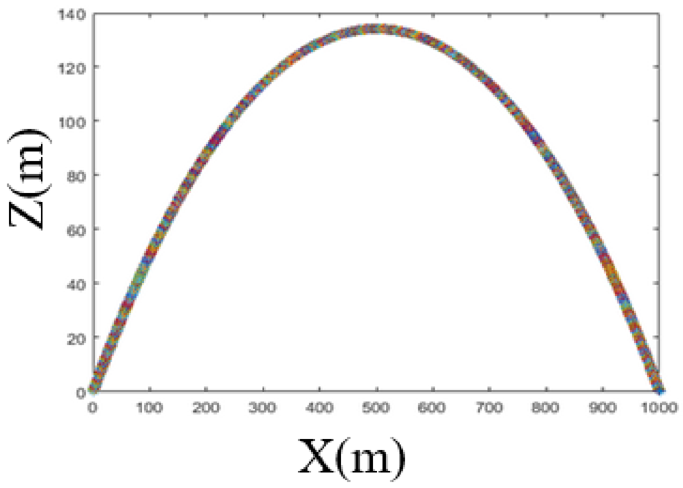

Fig 6.1 Impact angle (-30 degree)

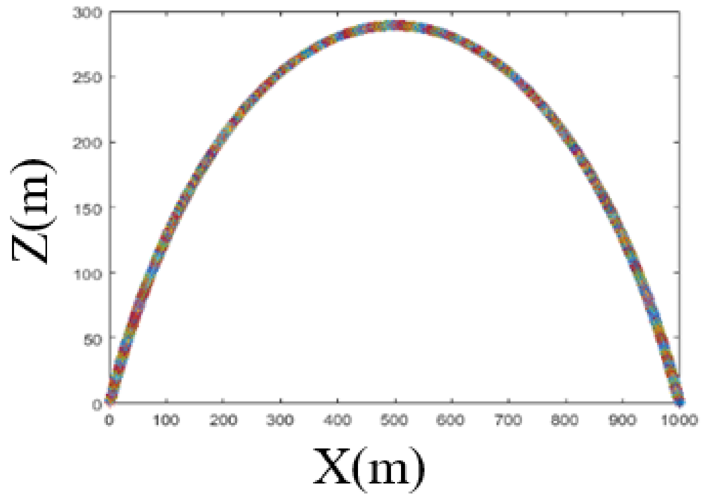

Fig 6.2 Impact angle (-60 degree)

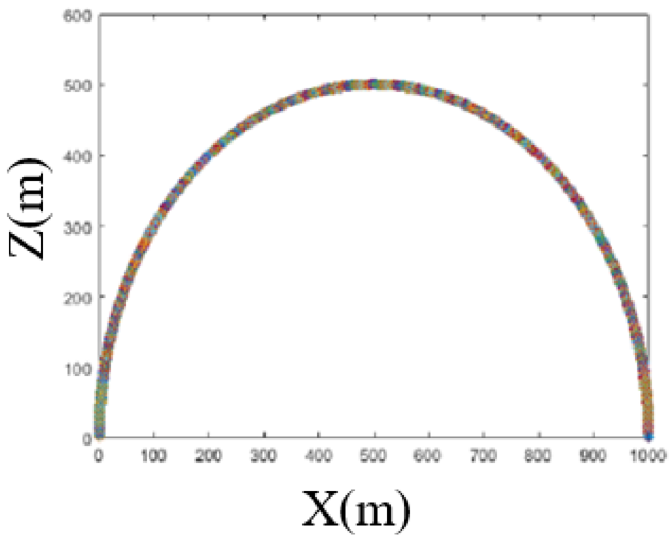

Fig 6.3 Impact angle (-90 degree)

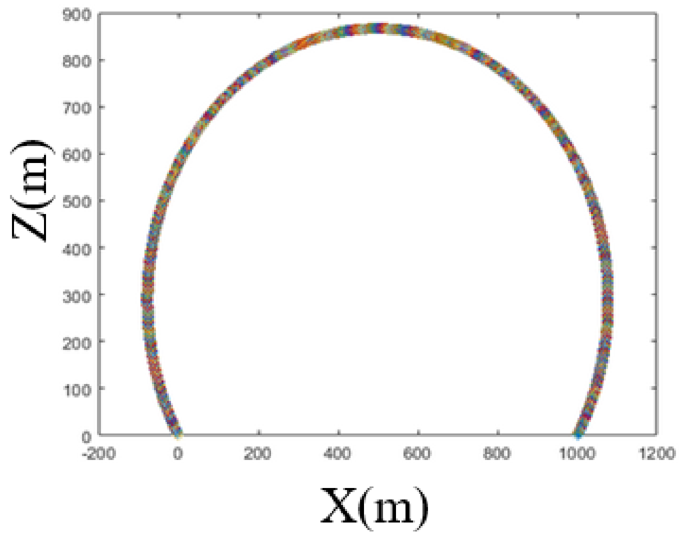

Fig 6.4 Impact angle (-120 degree)

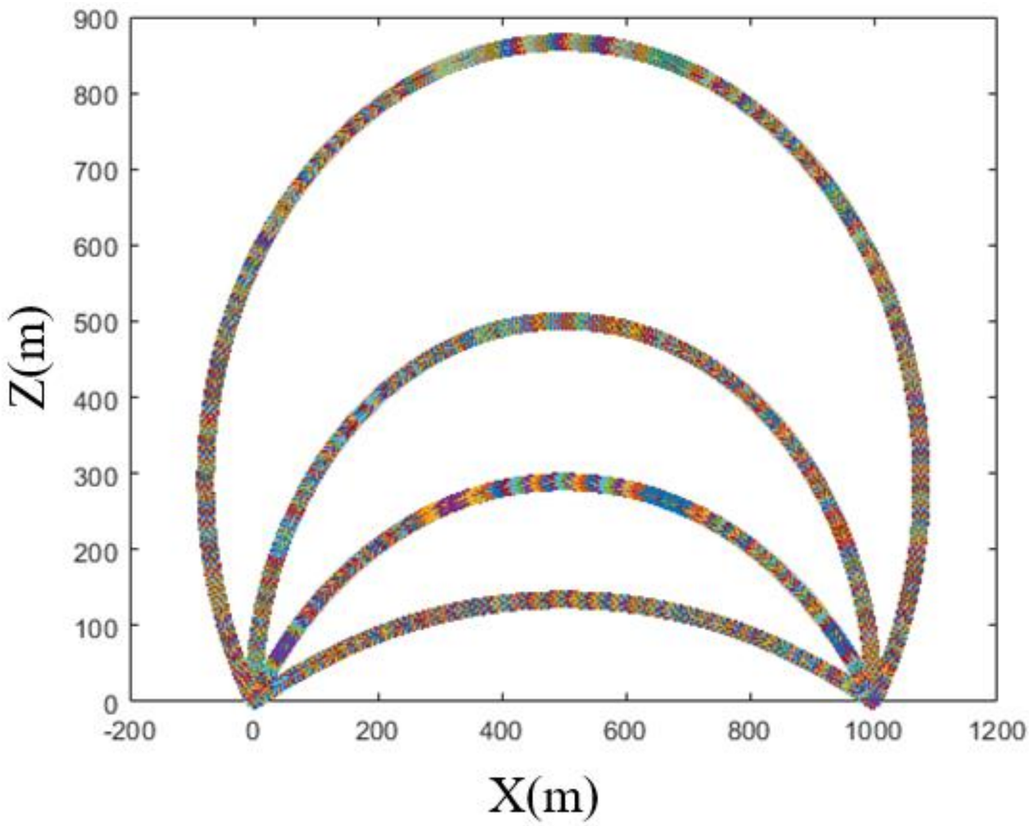

Fig 6.5 Comparison between all Impact angles

6.4 Conclusion

From the simulation results we can conclude that the missile follow a circular path to hit the target and the equation of the circular path is

$$(x-\sin\alpha\cdot\frac{v_m^2}{a_m})^2+(z+\cos\alpha\cdot\frac{v_m^2}{a_m})^2=\frac{v_m^4}{a_m^2} \tag{6.15}$$

From the Simulation Result we can conclude that,

- It is observed that with the increase of missile velocity, the interception time decreases for a particular impact angle, whereas interception time increases with increase of impact angle for a particular initial missile velocity.
- The magnitude of maximum latax as well as range of latax increase with the increase of impact angles for a particular initial missile velocity. The latax also increases with the increase of initial missile velocity for a particular impact angle.

Chapter 7

Simulation for Nonstationary Targets

7.1 Introduction

Ratnoo and Ghose handle the issue of satisfying the impact angle constraint [1] by adjusting the PNG's navigation constant N. For reaching all impact angles against stationary targets in surface-to-surface engagements, a two-stage PNG law is proposed in their study. Kim et al.'s biased PNG (BPNG) law [3] has an additional term for annulling the terminal impact angle error together with the conventional PNG rule.

The capture area of current guideline legislation against moving targets is widened by BPNG law. The effectiveness of BPNG law, however, suffers from agreements of this nature. Here, the issue of hitting moving targets from every impact angle is addressed. Ratnoo and Ghose's two-stage PNG law theory [14] is expanded upon and refined for nonstationary, non-moving targets. It should be noted that the PNG law generates a variety of impact angles for various values of N.

However, tests on the traditional PNG law show that for the final lateral acceleration demand to be bounded, N must be bigger than a minimum value. For PNG law, the set of impact angles that are feasible is determined, with N values that satisfy the aforementioned limitation.

An orientation guiding technique is suggested for the interceptor's early phase of trajectory in order to achieve the remaining impact angles. The orientation guidance law is a PNG law, where N is dependent on the geometry of the initial contact. It has been demonstrated that the interceptor may transition to N=3 and achieve any desired impact angle in a surface-to-surface engagement scenario by following the orientation trajectory.

7.2 Mathematical Derivations

Let us Consider, the target and the interceptor are constant speed point masses moving in a Plane. Here the Target is a nonstationary nonmaneuvering target and the main objective is to hit the target along a desired impact angle (α_{mf}).

Here,

α_m = interceptor heading

α_t = target heading

θ = Line of sight angle

From PNG law we can write that,

$$\dot{\alpha}_m = N\dot{\theta} \tag{7.1}$$

By integrating the (7.1) equation, we get

$$\frac{\alpha_{mf} - \alpha_{m0}}{\theta_f - \theta_0} = N \tag{7.2}$$

Where θ_f is the line of sight angle at interception.

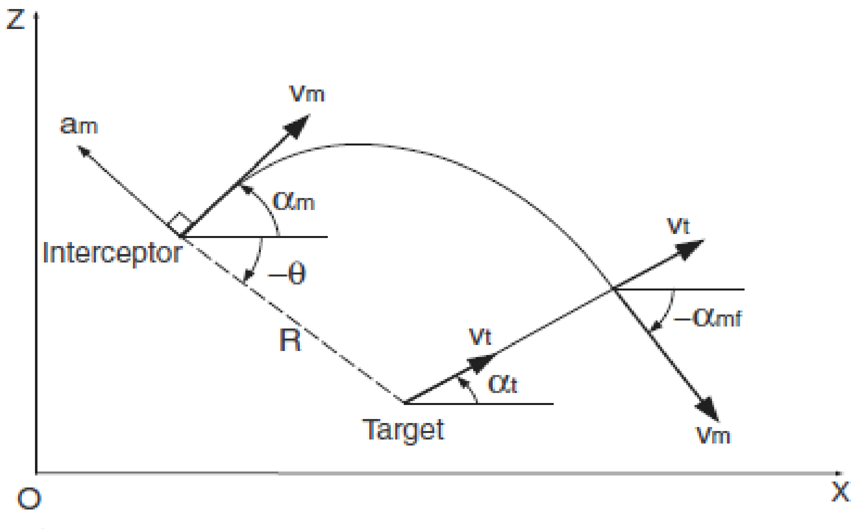

Fig 7.1 Engagement Geometry

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

We know that at the time of the interception, velocity components to the line-of-sight should be equal;

$$v_m \sin(\alpha_{mf} - \theta_f) = v_t \sin(\alpha_t - \theta_f) \tag{7.3}$$

We get,

$$\Rightarrow \theta_f = \tan^{-1}[\frac{\sin\alpha_{mf} - \beta\sin\alpha_t}{\cos\alpha_{mf} - \beta\cos\alpha}] \tag{7.4}$$

From the Equation (7.2) and (7.4) we get,

$$\Rightarrow N = (\alpha_{mf} - \alpha_{m0}) / (\tan^{-1}[\frac{\sin\alpha_{mf} - \beta\sin\alpha_t}{\cos\alpha_{mf} - \beta\cos\alpha}] - \theta_0) \tag{7.5}$$

Where β is the target to interceptor velocity ratio ($\beta = \frac{v_t}{v_m}$)

Equation (7.5) contain the desired impact angle (α_{mf}) to the navigation constant *N*. The sufficient condition [10] on *N* for the terminal line of sight rate, and hence the terminal lateral acceleration demand to be bounded, is given as

$$N \geq 2(1+\beta) \tag{7.6}$$

We know that the target move slower than the interceptor. So that, we assume $\beta \leq \frac{1}{2}$ in our domain of interest. By using the (7.6) equation we get,

$$N \geq 3 \tag{7.7}$$

Here, Equation (7.7) gives the available set of *N* for achieving different impact angles. With this bound on *N*, we now determine the set of impact angles that can be achieved by the PNG law. Equation (7.5) can be rewritten, assuming the ground as the frame of reference with $\alpha_t = 0$, as

$$\frac{\sin\alpha_{mf}}{\cos\alpha_{mf} - \beta} = \tan(\frac{\alpha_{mf} - \alpha_{m0}}{N} + \theta_0) \tag{7.8}$$

By solving the (7.8) equation for α_{mf} as $N \to \infty$,

we get,

$$\alpha_{mf} = \theta_0 + \sin^{-1}(-\beta\sin\theta_0) \tag{7.9}$$

which is the collision heading at $t = 0$.

Simplifying Eq. (7.8) with N =3, we have,

$$\frac{\sin\alpha_{\text{mf}}}{\cos\alpha_{\text{mf}}-\beta}=\tan(\frac{\alpha_{\text{mf}}-\alpha_{\text{m0}}}{3}+\theta_0) \tag{7.10}$$

Let $\alpha_{\text{mf}}=\alpha^{*}_{\text{mf}}$ be the solution of Eq. (7.10); then, the limiting impact angles using PN guidance are given as

$$\alpha_{\text{mf}}=\alpha^{*}_{\text{mf}} \qquad \text{if N=3}$$

$$\alpha_{\text{mf}}=\theta_0+\sin^{-1}(-\beta\sin\theta_0)\ \text{ if }\ \text{N}\rightarrow\infty \tag{7.11}$$

$$\alpha_{\text{mf}}\in[\alpha_{\text{mf}}^{\ *}\ \ \theta_0+\sin^{-1}(-\beta\sin\theta_0)] \qquad \text{N}\geq 3 \tag{7.12}$$

When $\text{N}\rightarrow\infty$ gives the tightest turn and smaller value of N (N=3) gives a curved trajectory, which intercept the target with an angle of less than $\alpha_{\text{mf}}<\theta_0+\sin^{-1}(-\beta\sin\theta_0)]$. So, the achievable impact angles (with $\theta_0=0$) using PN law is lie on the shaded region in fig 7.2

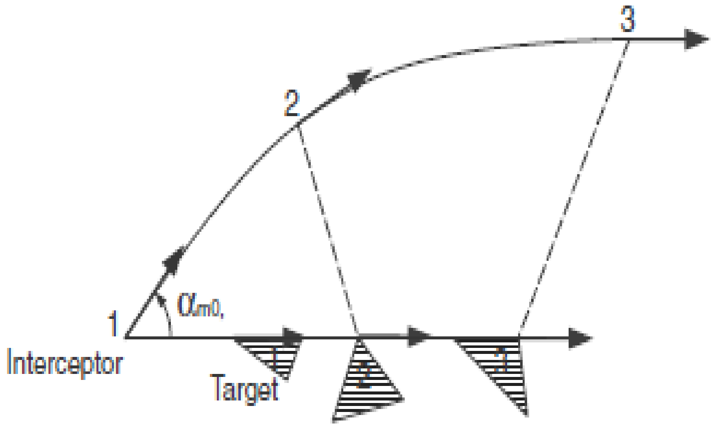

Fig 7.2 PNG impact angle zone

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

But in case of the impact angles with N < 3 in equation (7.5) cannot be achieved with the help of PN guidance against a nonstationary nonmaneuvering target, as the lateral acceleration may go to infinity at the time of interception.

7.3 Orientation Guidance:

Here we consider $\alpha_{mf} \in [\pi \quad 0]$ as the desired set of impact angles against a surface moving target. In the previous Section we found that in Classical PNG ($N \geq 3$) does not cover the desired impact angles completely. For all other impact angles which is outside the range of given by equation (7.12), an orientation guidance has proposed for the initial phase of the interceptor flight. The interceptor follows the new orientation trajectory (Fig. 7.3) until the value of *N*, satisfying the following relation, becomes equal to three:

$$\Rightarrow N = (\alpha_{mf} - \alpha_{m0}) / (\tan^{-1}[\frac{\sin\alpha_{mf} - \beta\sin\alpha_{t}}{\cos\alpha_{mf} - \beta\cos\alpha}] - \theta_{0}) \quad (7.13)$$

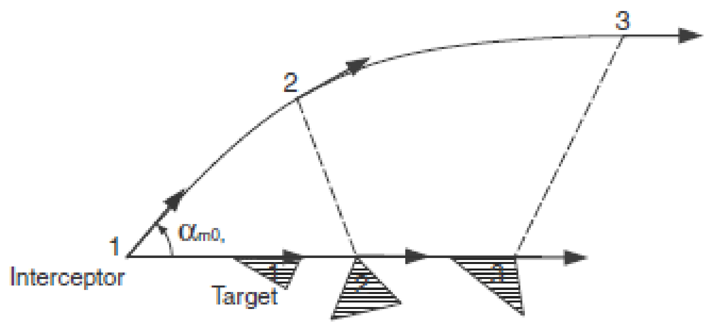

Fig 7.3 orientation trajectory

[Source: D. Ghose, "*Guidance of Missiles*", NPTEL, 2012]

After that, the interceptor follows the PN guidance with *N*=3. As shown in Fig. 7.3, the achievable impact angle band, using PN guidance at the time of firing the interceptor, is the shaded region 1.when the interceptor reaches point 2 on the orientation trajectory, the achievable band shifts to the shaded region 2.So, The purpose of the orientation guidance is to eventually take the interceptor to point 3, in Fig.7.3 .At Point 3 on the orientation trajectory is chosen such that, if the interceptor switches to *N* = 3, the resulting impact angle $\alpha_{mf} = -\pi$. So, the union of all the shaded impact angle regions formed by tracing the orientation trajectory is $\alpha_{mf} \in [0, \ -\pi]$.

7.3.1 Orientation Guidance Command:

For orientation guidance, the PN guidance law has proposed:

$$\dot{\alpha}_{m} = N\dot{\theta} \tag{7.14}$$

The Orientation trajectory takes the interceptor from point 1 to 3, shown in (fig 7.3).

At point 3, when the interceptor switches to PN guidance with *N*= 3, the resulting impact angle is $\alpha_{mf} = -\pi$. By using Equation (7.4), with $\alpha_{t} = 0$ and $\alpha_{mf} = -\pi$, we have $\theta_{f} = -\pi$ Substituting the preceding values in Equation (7.13), we have at point 3

$$\frac{-\pi - \alpha_{m}}{-\pi - \theta} = 3 \Rightarrow \alpha_{m} = 2\pi + 3\theta \tag{7.15}$$

We choose $\alpha_{m} = 0$ and $\theta = -\frac{2\pi}{3}$, satisfying Eq. (15) for the terminal point on the orientation trajectory. So, the orientation navigation constant is derived as

$$N = \frac{\alpha_{m0} - 0}{0 - [-(\frac{2\pi}{3})]} = \frac{3\alpha_{m0}}{2\pi} \tag{7.16}$$

Note that,

$$N \in (0, 1.5) \qquad \alpha_{m0} = (0, \pi) \tag{7.17}$$

Using Equations (7.14) and (7.16), the orientation guidance command is given by

$$a_{m} = (\frac{3\alpha_{m0}}{2\pi}) v_{m} \dot{\theta} \tag{7.18}$$

The orientation navigation constant is a function of α_{m0}, and the effect of the other engagement parameters like v_t is reflected in the orientation command through $\dot{\theta}$.

7.3.2 Properties of the Orientation Trajectory:

Using Equation (7.18), we have, on the orientation trajectory

$$\alpha_{m} = \frac{a_{m}}{v_{m}} = \frac{3\alpha_{m0}}{2\pi} \dot{\theta} \tag{7.19}$$

Integrating with respect to time

$$\alpha_{\mathrm{m}} = \frac{\mathrm{a_m}}{\mathrm{v_m}} = \frac{3\alpha_{\mathrm{m0}}}{2\pi}\theta + \alpha_{\mathrm{m0}} \tag{7.20}$$

Equation (7.20) relates the interceptor heading and the line-of-sight angle on the orientation trajectory.

We have two Propositions:

Proposition 1: On the orientation trajectory the line-of-sight rate $\dot{\theta} < 0$.

Proposition 2: On the orientation trajectory:

$$[\alpha^{*}_{\mathrm{mf}} \quad \theta + \sin^{-1}(-\beta \sin\theta)] = [-\pi \ 0]$$

Proof of Proposition 1:

For a moving target, with $\alpha_{\mathrm{t}} = 0$, we have

$$\dot{\theta} = \frac{\mathrm{v_t}}{\mathrm{R}}\sin(-\theta) - \frac{\mathrm{v_m}}{\mathrm{R}}\sin(\alpha_{\mathrm{m}} - \theta) \tag{7.21}$$

Using Equation (7.20) in Equation (7.21), we have

$$\dot{\theta} = \frac{\mathrm{v_t}}{\mathrm{R}}\sin(-\theta)[1 - \frac{\sin\{\alpha_{\mathrm{m0}} + [(3/2\pi)\alpha_{\mathrm{m0}} - 1]\theta\}}{\beta\sin(-\theta)}] \tag{7.22}$$

On the orientation trajectory, that is, $\theta \in [0, \ -\frac{2\pi}{3}]$, we have

$$\sin[\alpha_{\mathrm{m0}} + (\frac{3}{2\pi}\alpha_{\mathrm{m0}}) - 1)\theta] \in [\sin\alpha_{\mathrm{m0}} \quad \sin(\frac{2\pi}{3})] \tag{7.23}$$

$$\sin(\theta) \in [0 \quad \sin(\frac{2\pi}{3})] \tag{7.24}$$

Using Equation (7.23) and (7.24) with $\alpha_{\mathrm{m0}} \in (0, \ \pi)$, we have

$$\frac{\sin\{\alpha_{m0} + [(3/2\pi)\alpha_{m0} - 1]\theta\}}{\sin(-\theta)} \geq 1 \tag{7.25}$$

For all,

$$\theta \in [-\frac{2\pi}{3}, 0]$$

From Equation (7.25) with $\beta \leq \frac{1}{2}$, we have

$$\frac{\sin\{\alpha_{m0} + [(3/2\pi)\alpha_{m0} - 1]\theta\}}{\beta \sin(-\theta)} > 1 \tag{7.26}$$

For all,

$$\theta \in [-\frac{2\pi}{3}, 0]$$

Using Equation (7.26) in Equation (7.22), we have

$$\dot{\theta} < 0 \tag{7.27}$$

Proof of Proposition 2:

Let us consider,

$$q_1 = \frac{\alpha^{*}_{mf}}{3} \tag{7.28}$$

$$q_2 = \theta + \sin^{-1}(-\beta \sin\theta) \tag{7.29}$$

Using Equation (7.10), and substituting $\alpha^{*}{}_{m} = 3q_1$ into it, we have on the orientation trajectory

$$\frac{\sin(3q_1)}{[\cos(3q_1) - \beta]} = \tan(\frac{3q_1 - \alpha_m}{3} + \theta) \tag{7.30}$$

$$\Rightarrow \frac{\sin 2q_1 \cos q_1 + \cos 2q_1 \sin q_1}{\cos 2q_1 \cos q_1 - \sin 2q_1 \sin q_1 - \beta} = \tan(q_1 + a) \tag{7.31}$$

Where

$$a = \theta - \frac{\alpha}{3} \tag{7.32}$$

By Simplifying Equation (7.31), we have

$$\frac{\sin 2q_1 \cos q_1 + \cos 2q_1 \sin q_1}{\cos 2q_1 \cos q_1 - \sin 2q_1 \sin q_1 - \beta} \tag{7.33}$$

Finally we get,

$$\frac{\sin q_1 \cos a + \cos q_1 \sin a}{\cos q_1 \cos a - \sin q_1 \sin a} \tag{7.34}$$

Rearranging and simplifying the terms, after cancellation, we have

$$-\beta \sin q_1 \cos a - \beta \cos q_1 \sin a = \sin 2q_1 \cos a - \cos 2q_1 \sin a \tag{7.35}$$

$$\Rightarrow \frac{-\beta \sin q_1 - \sin 2q_1}{-\cos 2q_1 + \beta \cos q_1} = \tan a \tag{7.36}$$

$$\Rightarrow \frac{\beta \sin q_1 + \sin 2q_1}{-\cos 2q_1 + \beta \cos q_1} = \tan(-a) \tag{7.37}$$

Substituting the value of *a*, using Equation (7.32), into Equation (7.37), we have

$$\Rightarrow \frac{\sin 2q_1 + \beta \sin q_1}{-\cos 2q_1 + \beta \cos q_1} = \tan(\frac{\alpha_m}{3} - \theta) \tag{7.38}$$

Differentiating Equation (7.38) with respect to θ , we have

$$\frac{(-2 + \beta^2 + \beta \cos 3q_1)}{[-\cos 2q_1 + \beta \cos q_1]^2} \square \frac{dq_1}{d\theta} = \sec^2(\frac{\alpha_m}{3} - \theta)(\frac{1}{3}\frac{d\alpha_m}{d\theta} - 1) \tag{7.39}$$

Differentiating Eq. (20) with respect to θ, we have on the orientation trajectory

$$\frac{d\alpha_{m}}{d\theta}=\frac{3}{2\pi}\alpha_{m0} \tag{7.40}$$

Using Equation (7.40) in Equation (7.39), we have

$$\frac{(-2+\beta^{2}+\beta\cos 3q_{1})}{[-\cos 2q_{1}+\beta\cos q_{1}]^{2}}\,\frac{dq_{1}}{d\theta}=\sec^{2}(\frac{\alpha_{m}}{3}-\theta)(\frac{\alpha_{m0}}{2\pi}-1) \tag{7.41}$$

$$\Rightarrow\frac{dq_{1}}{d\theta}>0 \tag{7.42}$$

Because,

$$\frac{(-2+\beta^{2}+\beta\cos 3q_{1})}{[-\cos 2q_{1}+\beta\cos q_{1}]^{2}}<0 \tag{7.43}$$

For all,

$$\beta\leq\frac{1}{2}$$

and

$$3\sec^{2}(\frac{\alpha_{m}}{3}-\theta)(\frac{\alpha_{m0}}{2\pi}-1)<0 \tag{7.44}$$

for all

$$\alpha_{m0}\in(0,\ \pi)$$

Differentiating Equation (7.29) with respect to θ, we have

$$\frac{dq_{2}}{d\theta}=[1-\frac{\beta\cos\theta}{\sqrt{(\beta\cos\theta)^{2}+1-(\beta)^{2}}}]>0 \tag{7.45}$$

because $\beta\leq\frac{1}{2}$. Using Equations (7.42) and (7.45), we have

$$\frac{(-2+\beta^{2}+\beta\cos 3q_{1})}{[-\cos 2q_{1}+\beta\cos q_{1}]^{2}}\,\frac{dq_{1}}{d\theta}=\sec^{2}(\frac{\alpha_{m}}{3}-\theta)(\frac{\alpha_{m0}}{2\pi}-1) \tag{7.46}$$

At the initial point of the orientation trajectory (Fig.7.3) with $\theta = 0$, we have [using Equation (7.29)]

$$q_2(\theta = 0) = 0 \tag{7.47}$$

which is the maximum possible impact angle in a surface-to-surface to engagement. Therefore, using Equation (7.47), we have

$$\max[3q_1(\theta = 0), q_2(\theta = 0)] = 0 \tag{7.48}$$

At the terminal point of the orientation trajectory (Fig. 7.3), we have $\theta = -\frac{2\pi}{3}$ and $\alpha_m = 0$. The values of θ and α_m at this point satisfy Equation (7.30). Therefore, we have

$$3q_1(\theta = -\frac{2\pi}{3}) = \alpha_{mf}(\theta = -\frac{2\pi}{3}) = -\pi \tag{7.49}$$

which is the minimum possible impact angle for a surface-to-surface engagement. Using Equation (7.49), we have

$$\min[3q_1(\theta = -\frac{2\pi}{3}), q_2(\theta = -\frac{2\pi}{3})] = -\pi \tag{7.50}$$

Using Equations (7.48) and (7.50) in Equation (7.46), we have

$$[\alpha^*_{mf} \quad \theta + \sin^{-1}(-\beta \sin\theta)] = [-\pi \ 0] \tag{7.51}$$

7.4 The Proposed Guidance Law:

Proposition 2 shows that for any impact angle in $[-\pi \ 0]$, there exists a point on the orientation trajectory from which the PN guidance law with $N \geq 3$ results in the desired interception. The proposed two-stage PN guidance law follows the orientation guidance command given by Equation (7.18) if the value of *N*, satisfying Equation (7.13), is less than three until Equation (13) is satisfied with *N* =3. After which, *N* =3 is used. The proposed guidance law is given as

$$a_m = N v_m \dot{\theta} \tag{7.52}$$

For engagement geometries with

$$(\alpha_{mf} - \alpha_{m0}) / (\theta_f - \theta_0) \geq 3$$

The value of N is

$$N = \frac{\alpha_{mf} - \alpha_{m0}}{\theta_f - \theta_0} \tag{7.53}$$

For engagement geometries with

$$(\alpha_{mf} - \alpha_{m0}) / (\theta_f - \theta_0) \geq 3$$

The value of N is

$$N = \frac{3\alpha_{m0}}{2\pi} \qquad \text{if } t < t_s$$

$$N = 3 \qquad \text{if } t \geq t_s \tag{7.54}$$

where t_s is the switching time when the value of the expression $(\alpha_{mf} - \alpha_m) / (\theta_f - \theta)$ increases to a value of three.

7.5 Simulation Result:

Here, the simulation is performed with two different Interceptor Model

1. Constant Speed Interceptor
2. Realistic Interceptor

7.5.1 Constant Speed Interceptor:

To demonstrate the basic properties of the proposed guidance law, we use a constant speed interceptor model. Here we consider

$$v_m = 300 \text{ m/s}$$

$$v_t = 100 \text{ m/s}$$

$$\alpha_t = 0 \text{ (not accelerating target)}$$

The Interceptor initial position is $(x_{m0}, z_{mo}) = (0, 0)$

The Target initial position is $(x_{t0}, z_{to}) = (5000\text{m}, 0)$

The interceptor has a maximum lateral acceleration limit of ± 15 g.

We consider $\alpha_{m0} = 30$ degree and $\alpha_{mf} = 90$ for the Simulation. So, the value of

$(\alpha_{mf} - \alpha_{m0}) / (\theta_f - \theta_0)$=1.1066 which is outside the capturable impact angle set [see Eq. (7.12)] for the classical PNG ($N \geq 3$).

So, From the Simulation results the lines existing trajectory shaping guidance law [11] that, by linearization, can also be simplified to obtain the optimal impact angle constrained guidance law [5]. The trajectory shaping law is given as

$$a_m = 4V_c\dot{\theta} + 2V_c \frac{(\theta_f - \theta)}{t_{go}} \tag{7.55}$$

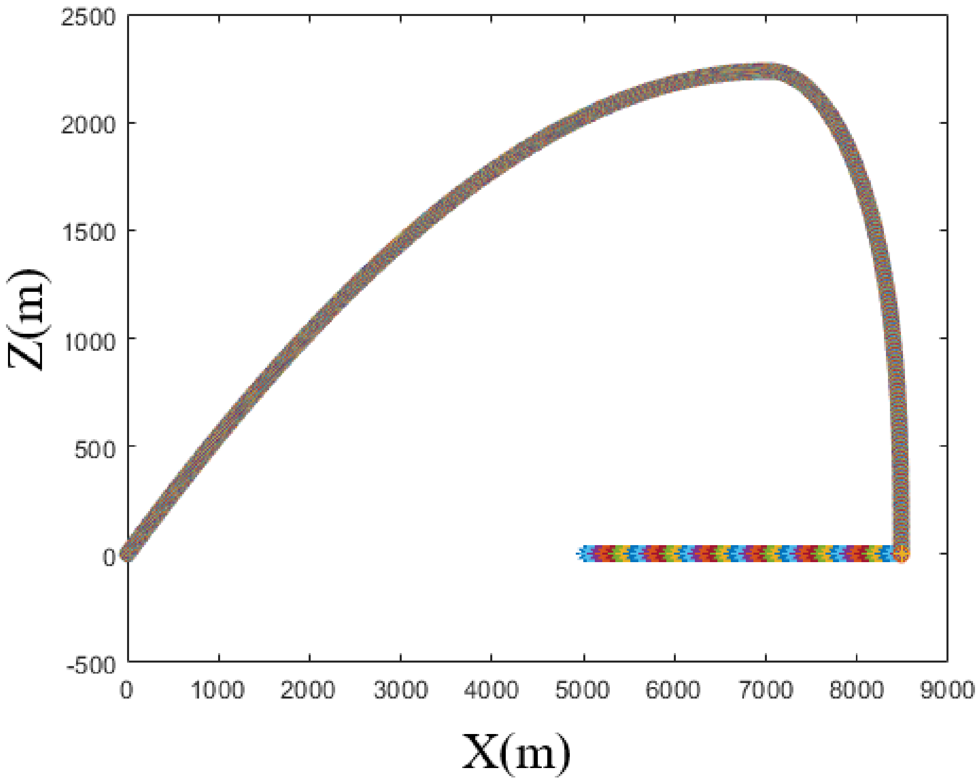

Fig 7.4 Trajectories for Constant Speed Interceptor

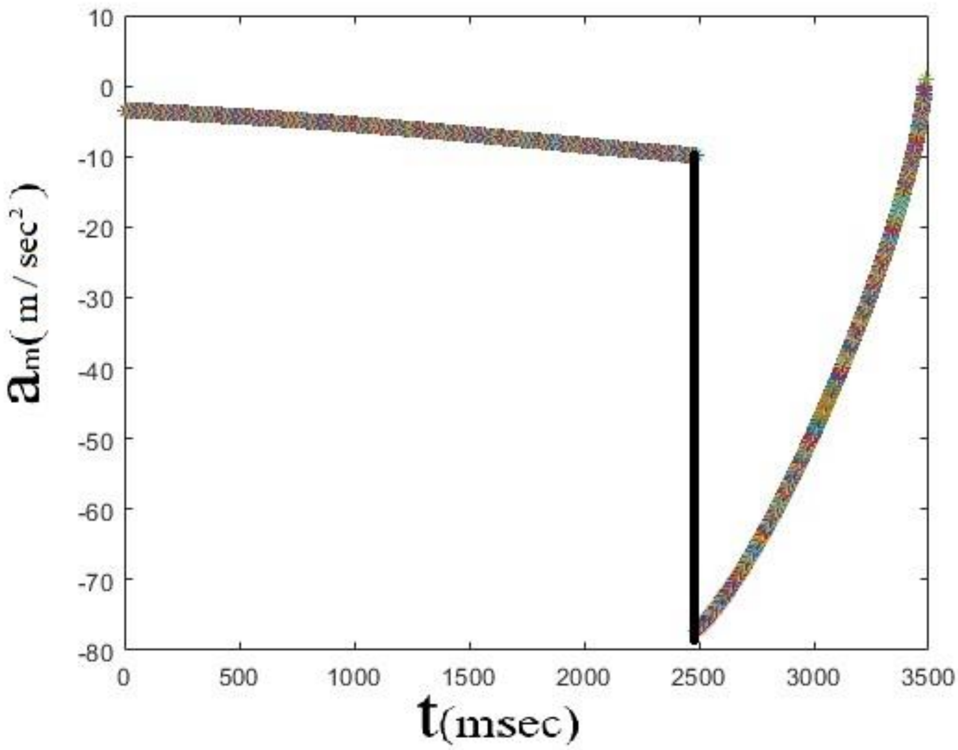

Fig 7.5 Lateral acceleration profiles

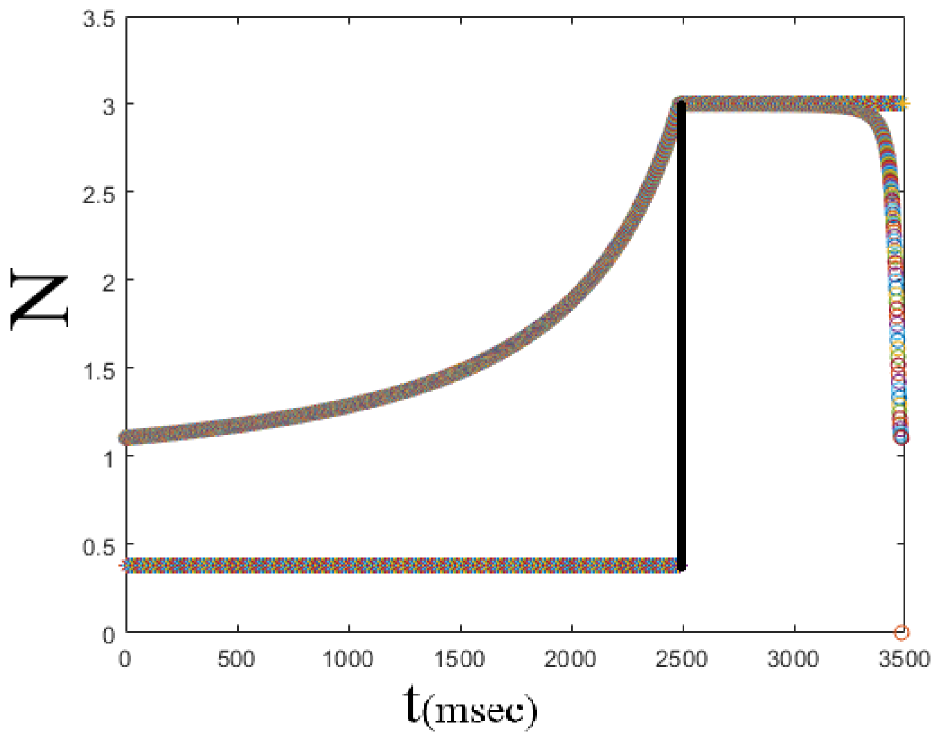

Fig 7.6 N, $\frac{\alpha_{mf} - \alpha_m}{\theta_f - \theta}$ vs time

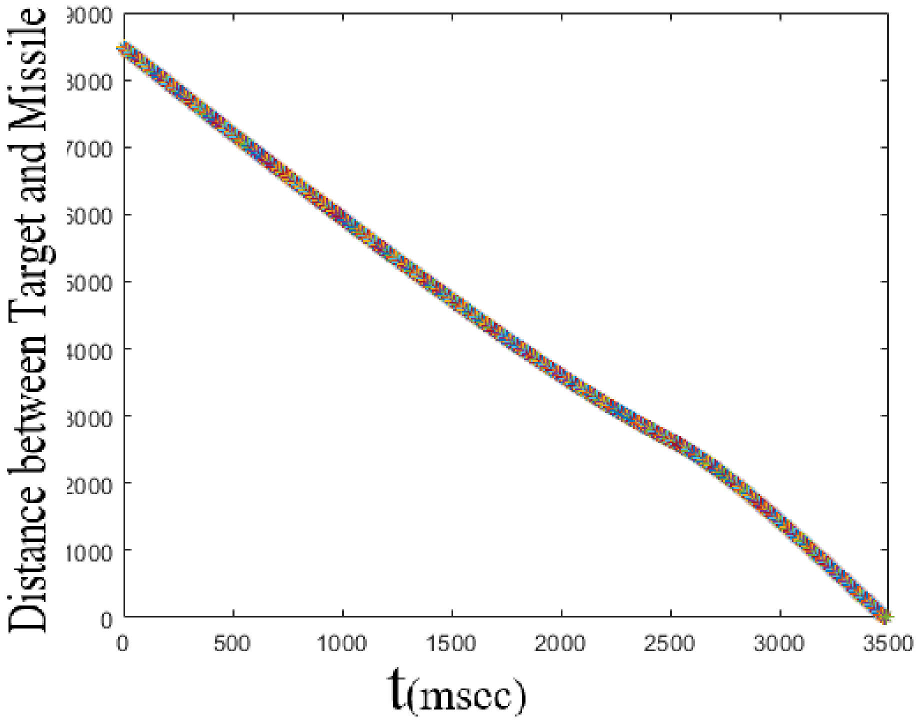

Fig 7.7 Distance between Target and Missile vs Time

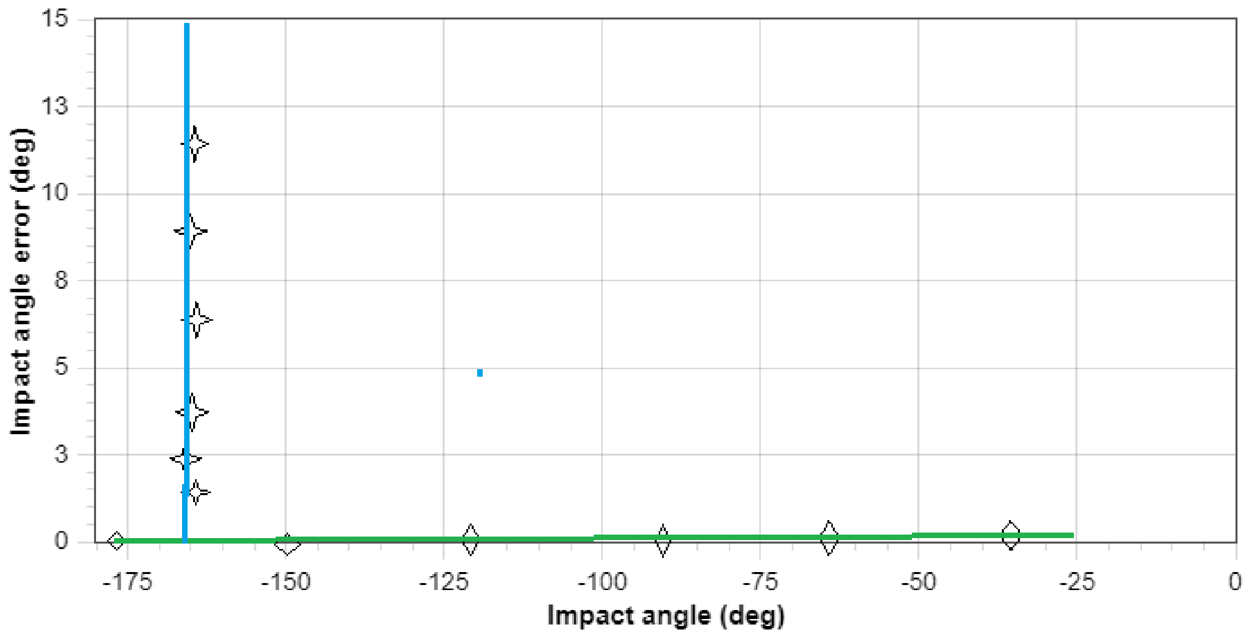

Fig 7.8 Capturability comparison

where V_c is the closing speed and t_{go} is the time to go. We vary the desired impact angle α_{mf} and compare the performance in terms of the impact angle error. The comparative results are shown in Fig. 7.8.The proposed guidance law captures all impact angles $\alpha_{mf} \in [-\pi \;\; 0]$ for which, as the trajectory shaping guidance law breaks down, nears head-on kind of desired terminal geometries.

7.5.2 Realistic Interceptor:

To validate the applicability of the proposed guidance law in realistic engagement scenarios, we carry out simulations with a realistic interceptor model. The detailed model with vehicle and aerodynamic properties is described in Kee et al. [12] and is borrowed here.

Here all the simulations performed for a closing distance of $R < 0.5\mathrm{m}$ and we take $\pm 20\mathrm{g}$ on maximum lateral acceleration. As the guidance loop is closed after the first boost phase is over, the orientation command derived from Equation (7.16) is modified (for realistic engagements) as

$$N = \frac{(-0 - \alpha_{mGLC})}{[(-2\pi/3) - \theta_{GLC}]} = \frac{\alpha_{mGLC}}{[(2\pi/3) + \theta_{GLC}]} \tag{7.56}$$

Where α_{glc} and θ_{glc} are the interceptor heading and line-of-sight angle, respectively, at the time of the guidance loop closure (GLC).

From Equation (7.4), for a predefined α_{mf}, the value of θ_f varies with interceptor speed. So, for realistic engagements, the value of $(\alpha_{mf} - \alpha_m)/(\theta_f - \theta)$ may deviate from the switching value with a variation of interceptor speed and it may fall below the minimum allowable limit of $2(1+\beta)$. We add the minimum allowable limit on the navigation constant in the terminal phase for realistic engagements. So, we get the modified guidance law, with the gravity compensation:

$$a_m = N v_m \dot{\theta} + g \cos \alpha_m \tag{7.57}$$

When the engagement geometry with $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0) \geq 3$,

$$N = \frac{(\alpha_{mf} - \alpha_{m0})}{(\theta_f - \theta)} \tag{7.58}$$

But in case of the engagement geometry with $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0) < 3$

$$N = \frac{\alpha_{mGLC}}{[(\frac{2\pi}{3}) + \theta_{GLC}]} \qquad \text{if } \frac{(\alpha_{mf} - \alpha_{m0})}{(\theta_f - \theta)} < 3,\ t < t_s$$

$$N = \frac{(\alpha_{mf} - \alpha_{m0})}{(\theta_f - \theta)} \qquad \text{if } \frac{(\alpha_{mf} - \alpha_{m0})}{(\theta_f - \theta)} > 2(1+\beta),\ t > t_s$$

$$N = 2(1+\beta) \qquad \text{if } \frac{(\alpha_{mf} - \alpha_{m0})}{(\theta_f - \theta)} \le 2(1+\beta),\ t > t_s \tag{7.59}$$

Where t_s is the switching time, which is defined as the time when the condition $(\alpha_{mf} - \alpha_m)/(\theta_f - \theta) = 3$ is satisfied first and the interceptor leaves the orientation trajectory. Note that the navigation constants in Equations (7.58) and (7.59) are no longer constants and are updated at every guidance cycle.

In realistic missile model, the desired impact angles to be outside the capture region of the classical PN guidance law ($N \ge 3$) with

v_m = 300 m/s

v_t = 50 m/s

α_t = 0 (not accelerating target)

The Interceptor initial position is $(x_{m0}, z_{mo}) = (0, 0)$

The Target initial position is $(x_{t0}, z_{to}) = (5000m, 0)$

We consider $\alpha_{m0} = 30$ degree for the Simulation and the desired impact angles are α_{mf} = -90 degree,-100 degree, -120 degree,-140 degree,-160 degree and -180 degree. The trajectories (plotted in Fig. 7.9) show successful interception of the target. The interceptor flies unguided for the first boost phase (1.5 s) and then follows the orientation command (see Fig. 7.10) before switching to attain the desired impact angle.

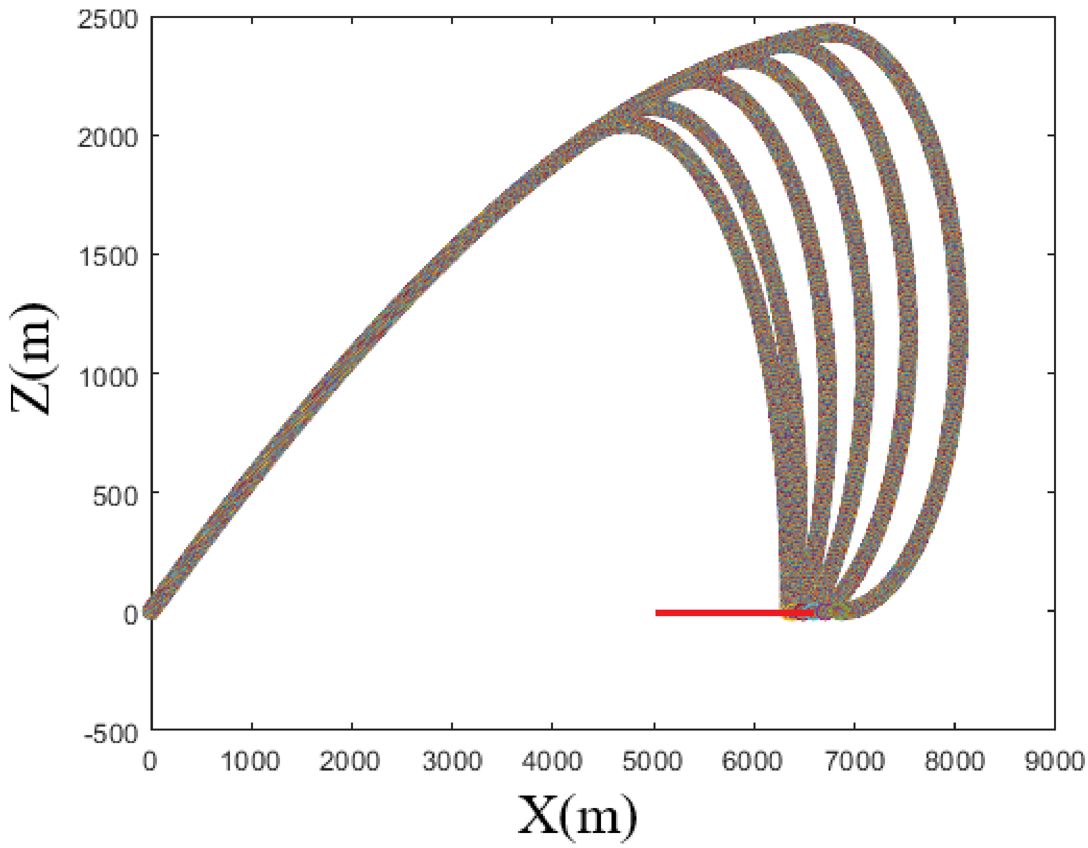

Fig 7.9 Trajectories for realistic interceptor model

The corresponding impact angle errors are less than 1 deg. The proposed guidance scheme is derived using the nonmaneuvering target model. However, Eq. (7.57) is a feedback guidance law that can be used against maneuvering targets.

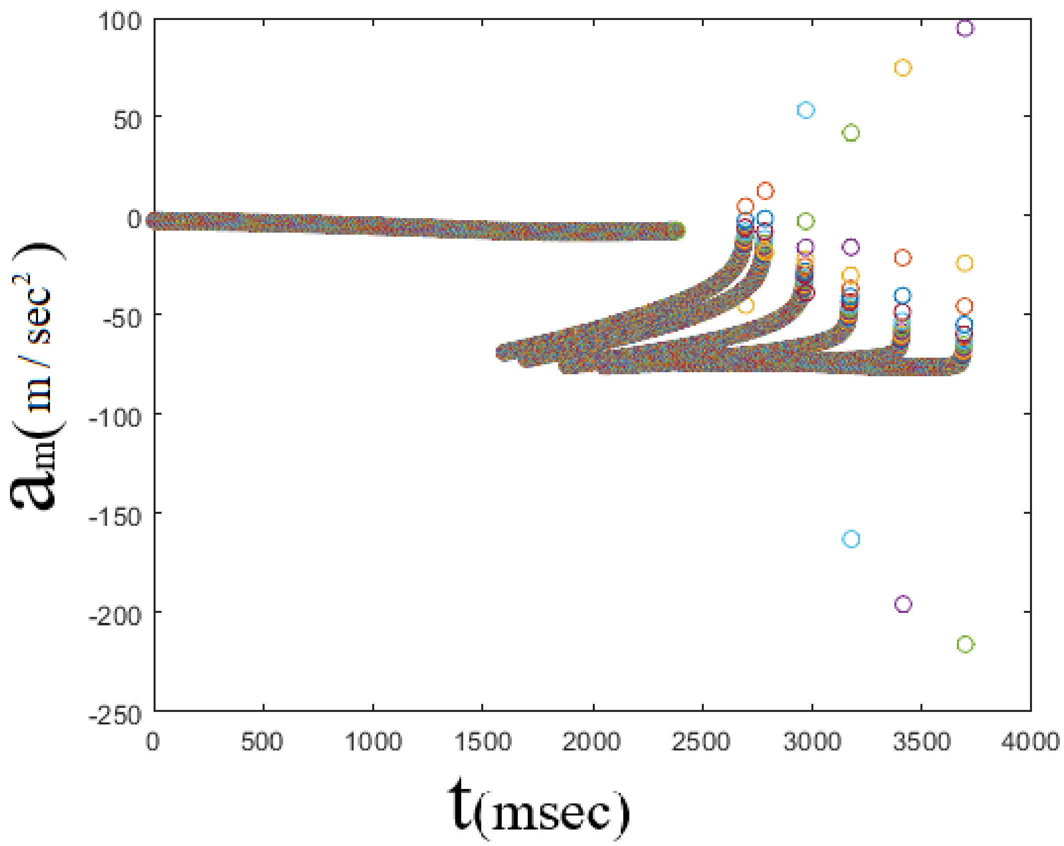

Fig 7.10 Lateral acceleration

Now, we take the desired impact angles are α_{mf} = -90 degree, -135 degree and – 180 degree with different target step acceleration a_t levels. In that case impact angle errors are plotted. In this case, we accelerated the target up to 4 m/sec^2.

For Impact angle α_{mf} = -90 degree, the target closes to the interceptor faster than the other impact angle, leading to lateral acceleration saturation, resulting in higher impact angle errors.

Here, Simulation is performed with impact angles -90 degree,-135 degree,-180 degree

and The target is accelerating from 0 to 4 m/sec^2 with a increase of 0.5.

7.5.2.1 Simulation For (-90 degree) impact angle:

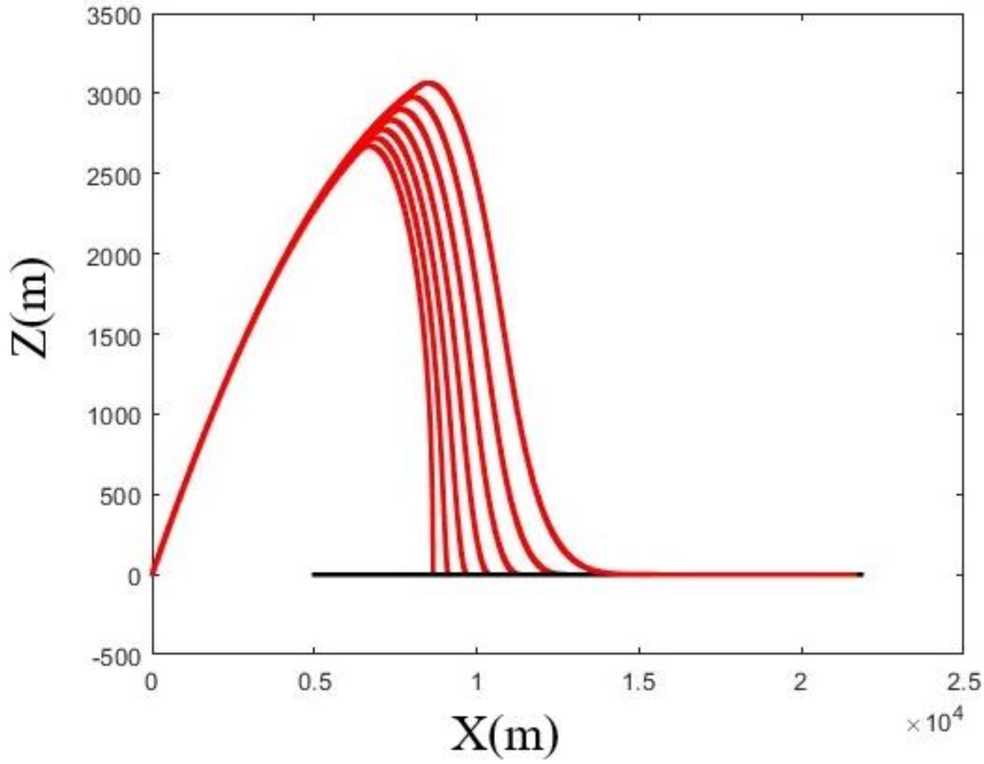

Fig 7.11 Trajectories for accelerating target

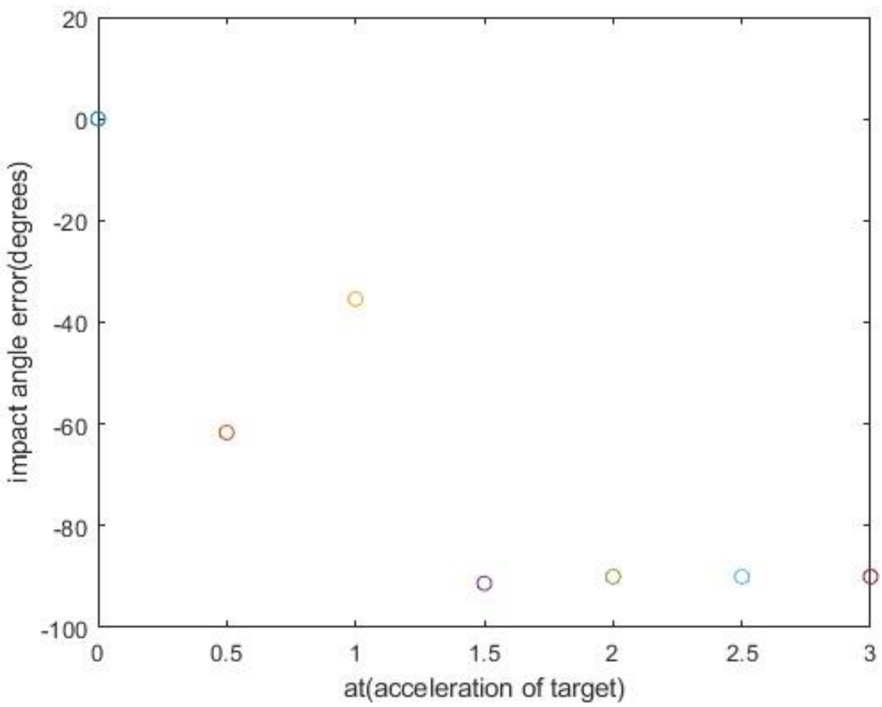

Fig 7.12 impact angle error vs a_t

7.5.2.2 Simulation For (-135 degree) impact angle:

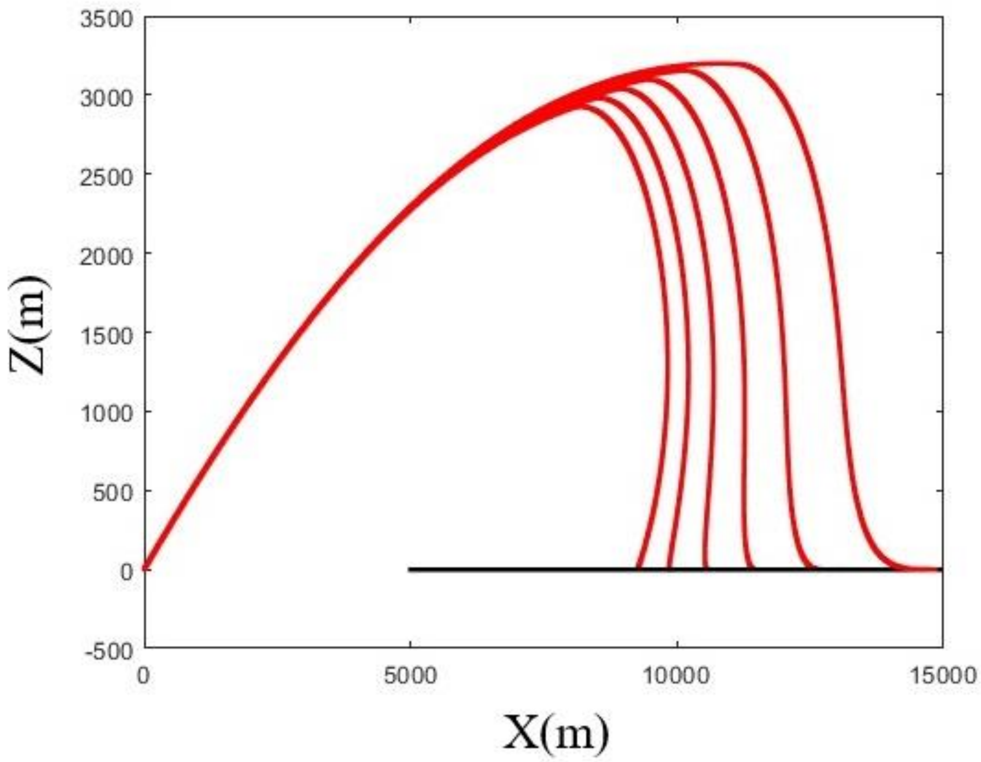

Fig 7.13 Trajectories for accelerating target

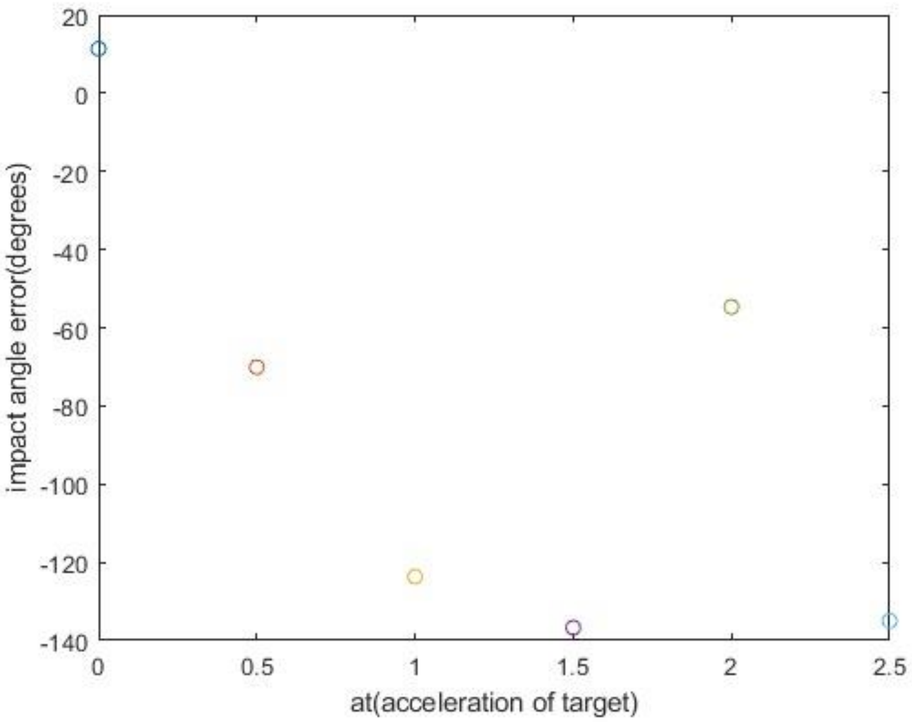

Fig 7.14 impact angle error vs a_t

7.5.2.3 Simulation For (-180 degree) impact angle:

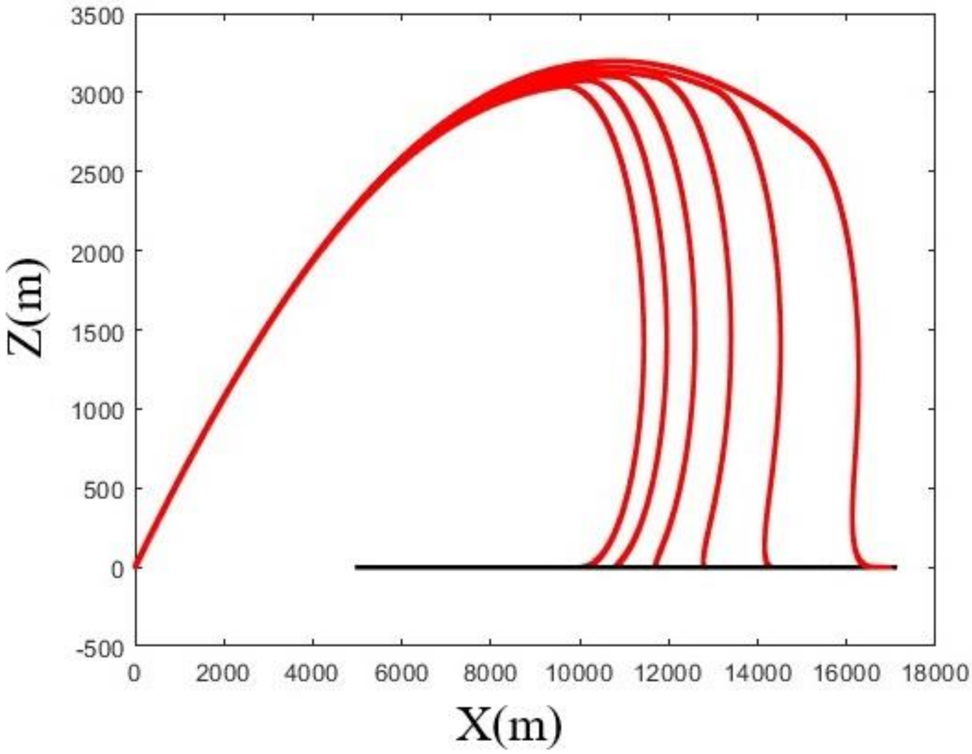

Fig 7.15 Trajectories for accelerating target

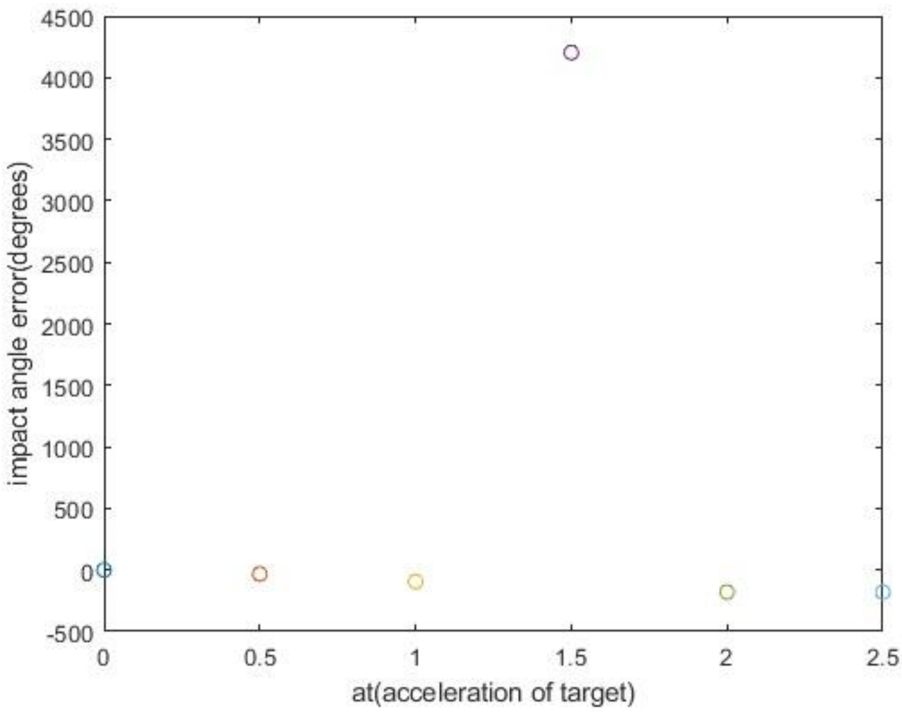

Fig 7.16 impact angle error vs a_t

To get the robustness of the proposed guidance law, simulation of trajectories for the values of the first-order autopilot lag time constant τ , up to 0.3 s. The results, as plotted in Fig. 7.17, show less than a 2 degree error in impact angle for α_{mf} = 90 and α_{mf} =180 deg.

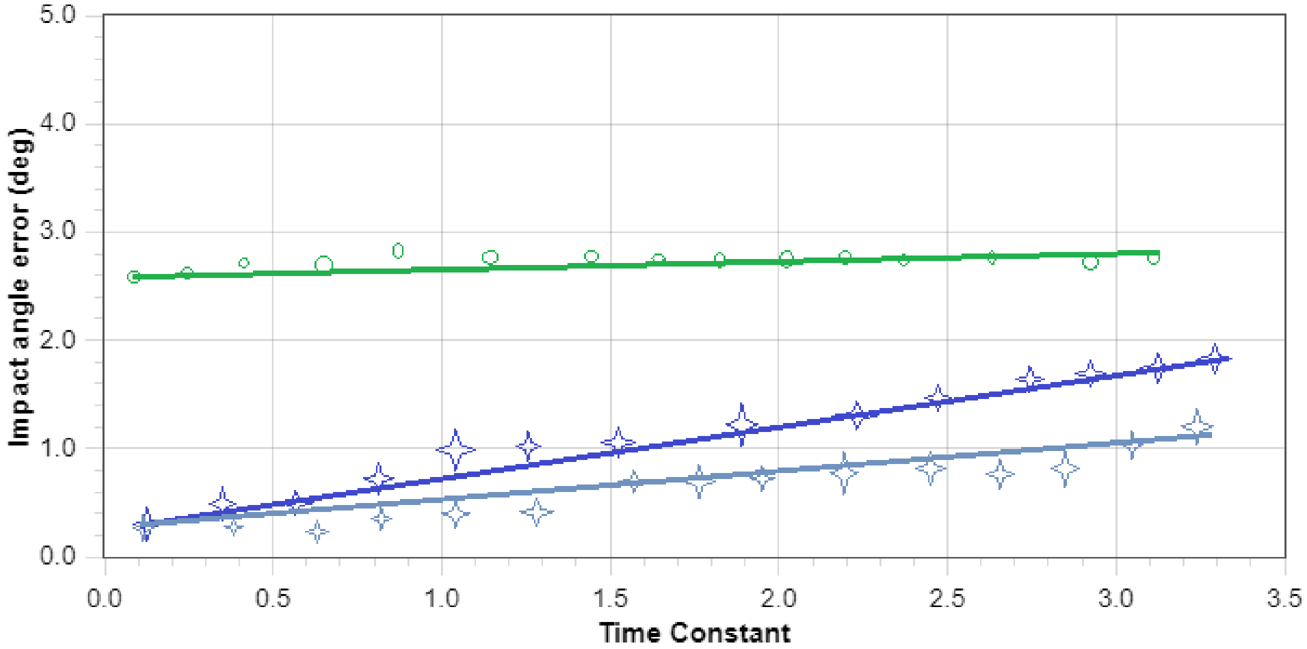

Fig 7.17 Impact angle error vs τ

In Fig 7.17 the graph is plotted between Impact angle error vs τ. Here Green Line represent the impact angle of -135 degree. Dark blue Line represent the impact angle of -90 degree and the light green line represent the impact angle of -180 degree.

For α_{mf} = 135 degree, the lateral acceleration saturation causes an error of around 2.4 degree (even with no delays), and the error increases by 0.2 degree for the considered range of τ .

7.6 Simulation for Maneuvering Target:

A maneuvering target, in the context of military and defence systems, refers to an object or entity that is actively changing its course, speed, or altitude in an attempt to evade detection, interception, or tracking by potential threats. These targets are often encountered in various military scenarios, including air defence, naval warfare, and ground-based missile defence systems.

The ability to accurately detect, track, and engage maneuvering targets is crucial for the effectiveness of modern defence systems. Such targets can include aircraft, missiles, unmanned aerial vehicles (UAVs), or even ground vehicles that are capable of evasive maneuvers. The primary goal of maneuvering targets is to disrupt the opponent's ability to engage or neutralize them, increasing their chances of survival or achieving their mission objectives.

Maneuvering targets pose significant challenges to defence systems, as they may change direction suddenly, execute evasive maneuvers such as rapid turns, dives, climbs, or unpredictable zig-zag patterns. Detecting and intercepting these targets require advanced sensor technologies, accurate tracking algorithms, and agile weapon systems capable of adjusting their trajectory to match the target's movements.

In recent years, advancements in radar, infrared sensors, artificial intelligence, and guided missile technologies have improved the ability of defence systems to deal with maneuvering targets. These developments have led to more effective countermeasures and increased the overall effectiveness of military and defence operations.

Here, Equation (7.57) is a feedback guidance law that can be used against maneuvering targets. So that, the main guidance law for the maneuvering target is

$$a_m = N v_m \dot{\theta} + g \cos \alpha_m$$

In this simulation, all the conditions are kept same and all the parameters are same as previous simulations. Here, the target is going in different path like parabolic path, circular path and Zigzag path. Simulation is performed and checked that the Missile is going to hit the target or not.

7.6.1 Target follow Parabolic Path:

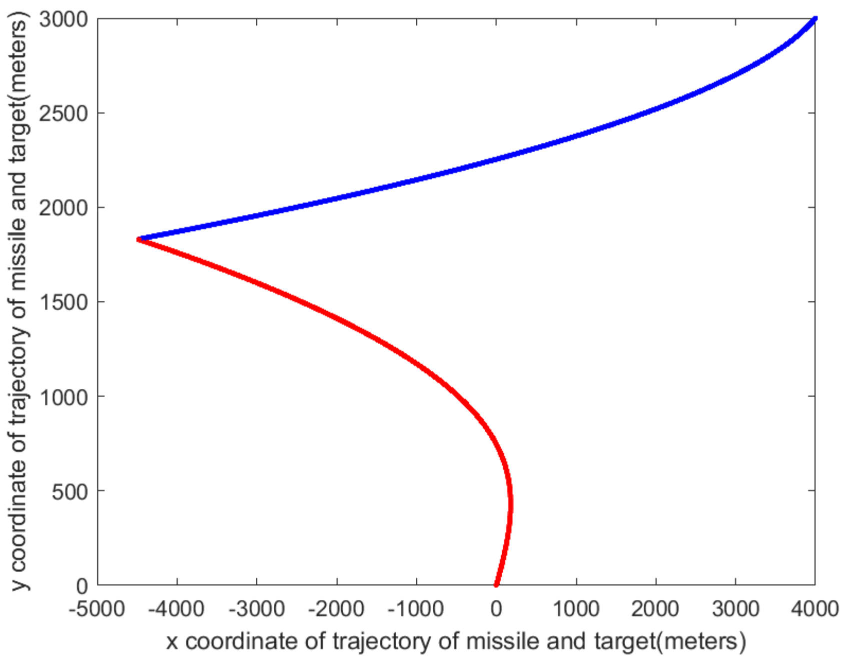

Fig 7.18 Trajectories for Parabolic Path

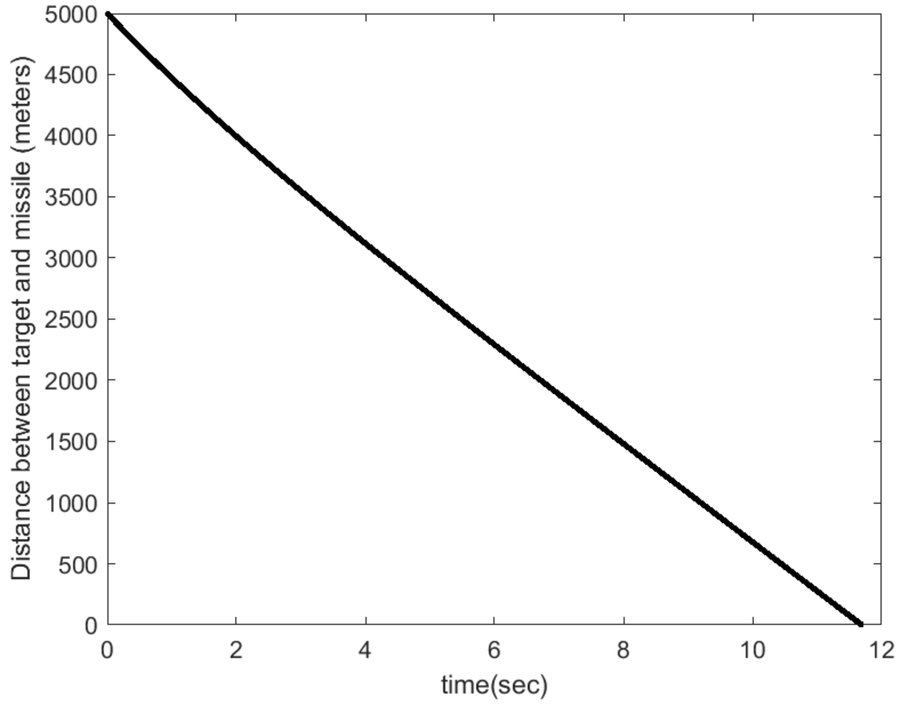

Fig 7.19 Distance between target and missile vs time

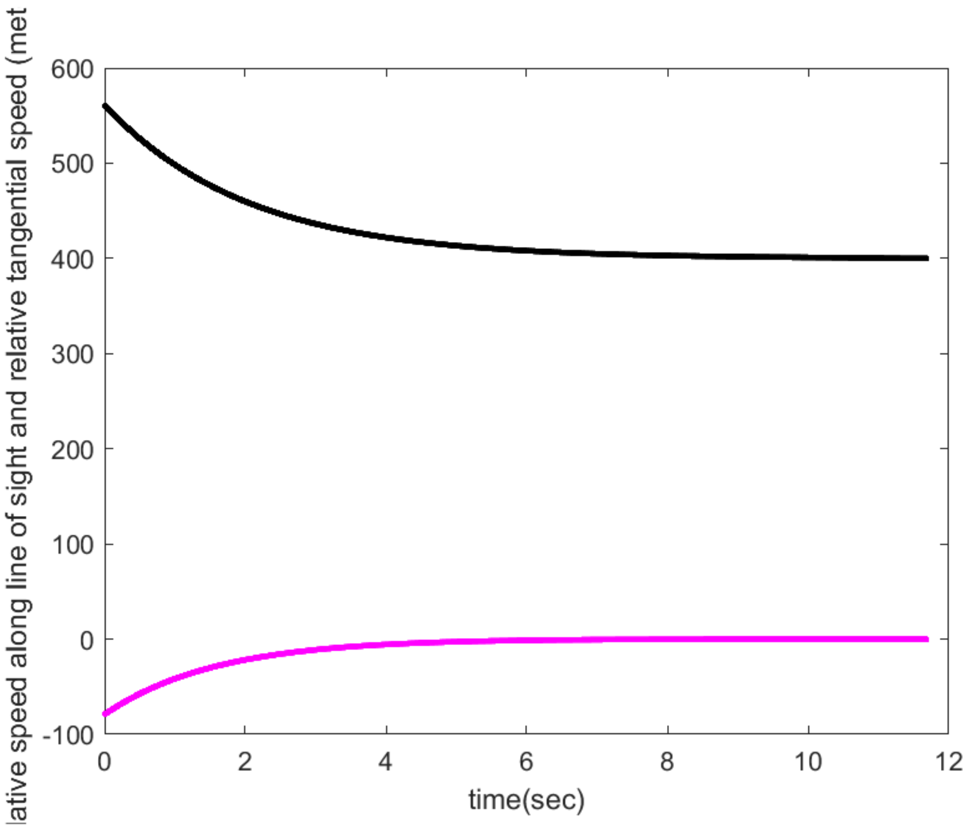

Fig 7.20 Tangential speed vs time

7.6.2 Target follow Circular Path:

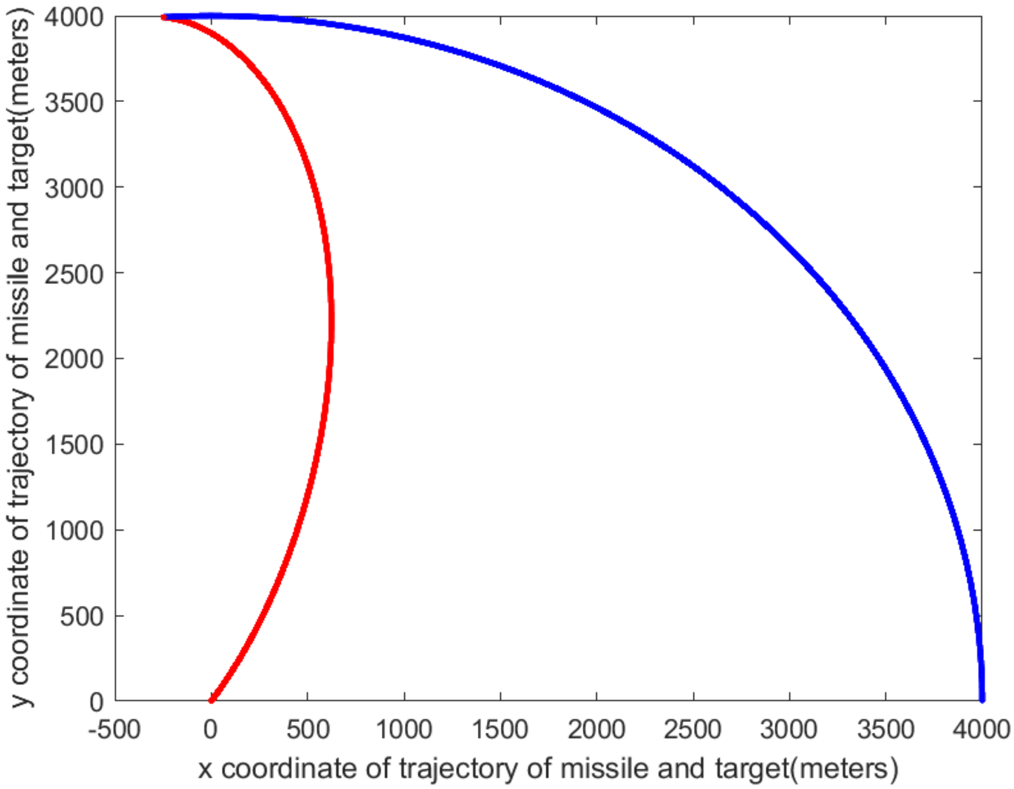

Fig 7.21 Trajectories for Circular Path

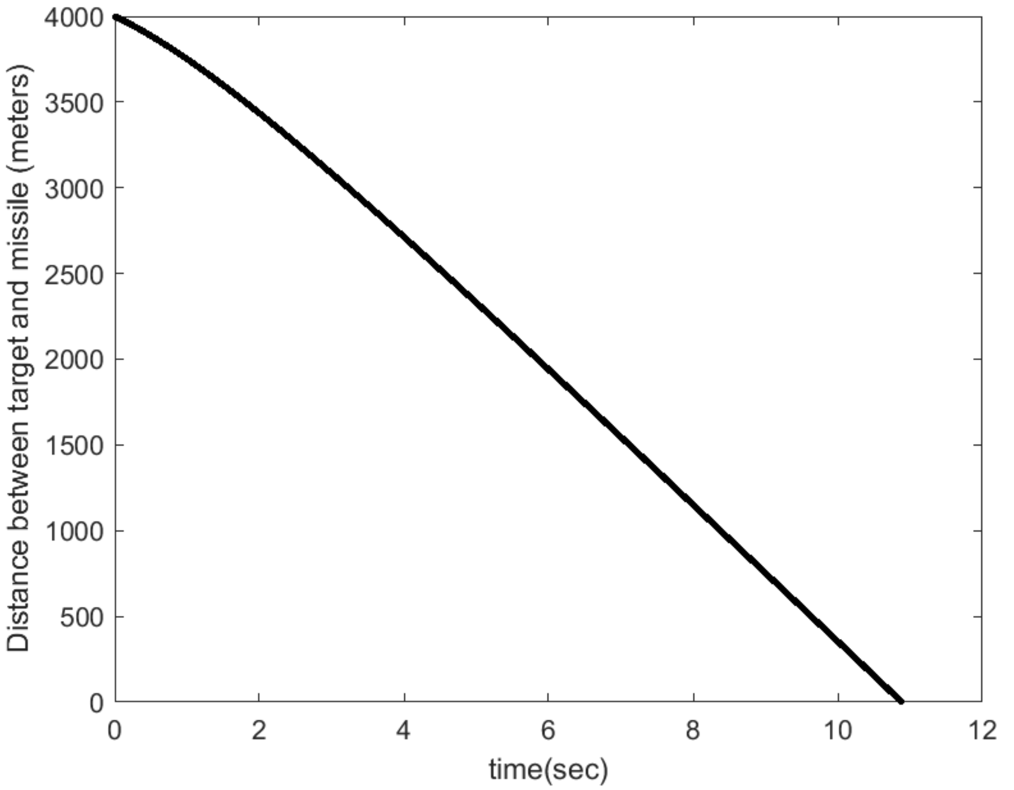

Fig 7.22 Distance between target and missile vs time

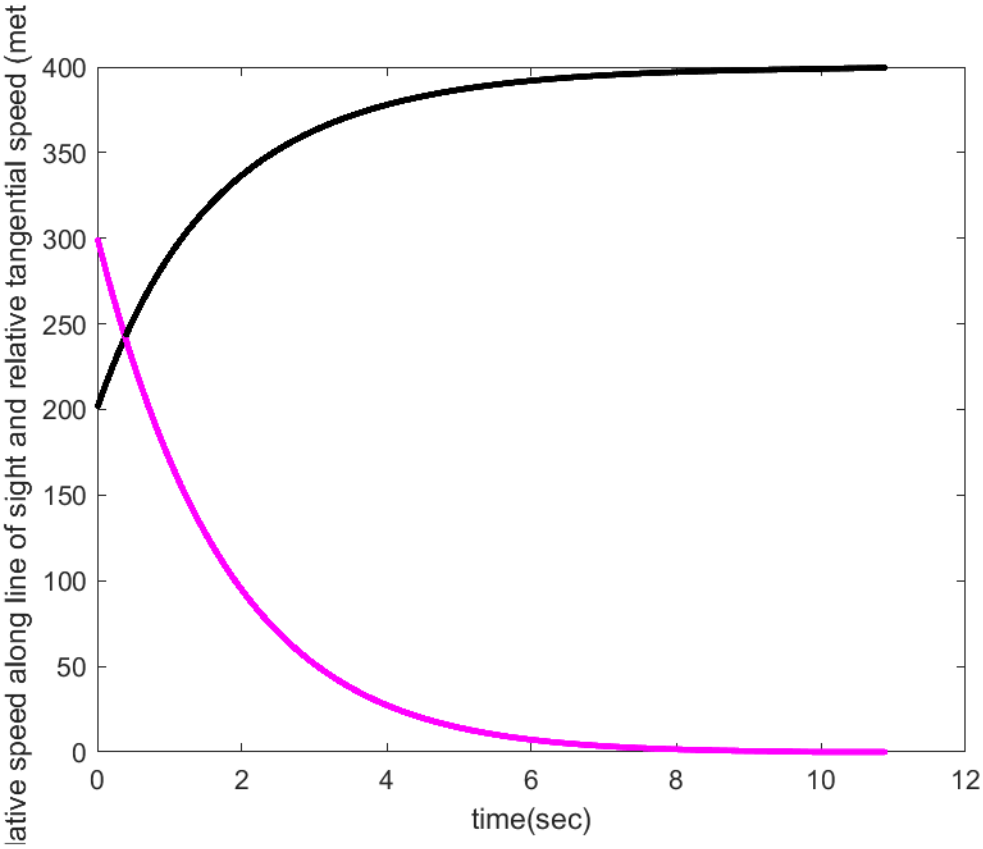

Fig 7.23 Tangential speed vs time

7.6.3 Target follow Zigzag Path:

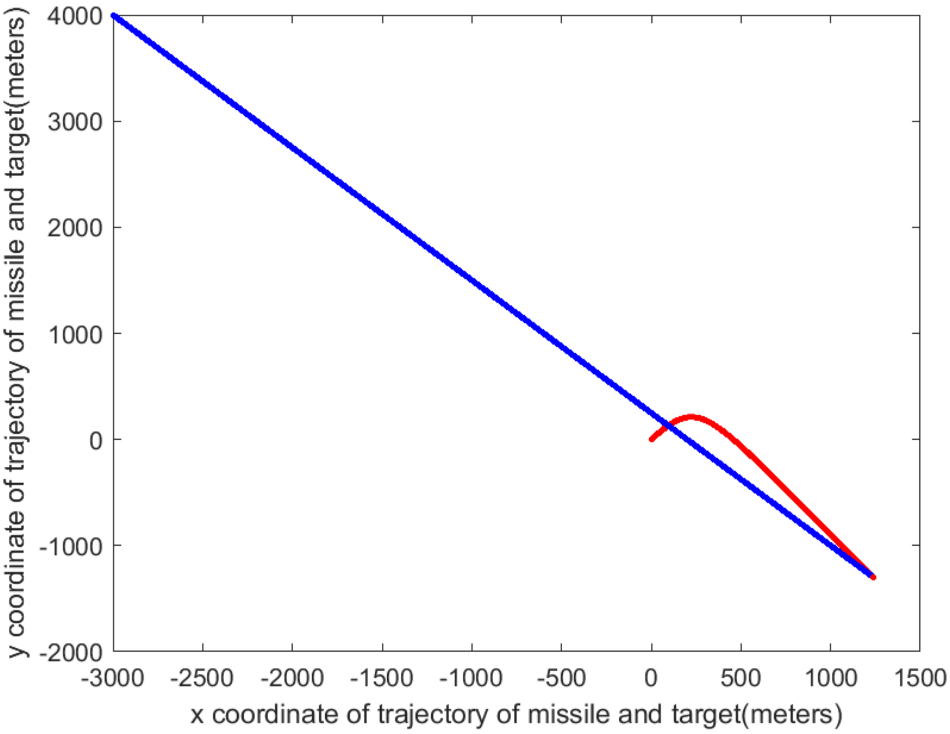

Fig 7.24 Trajectories for Zigzag Path

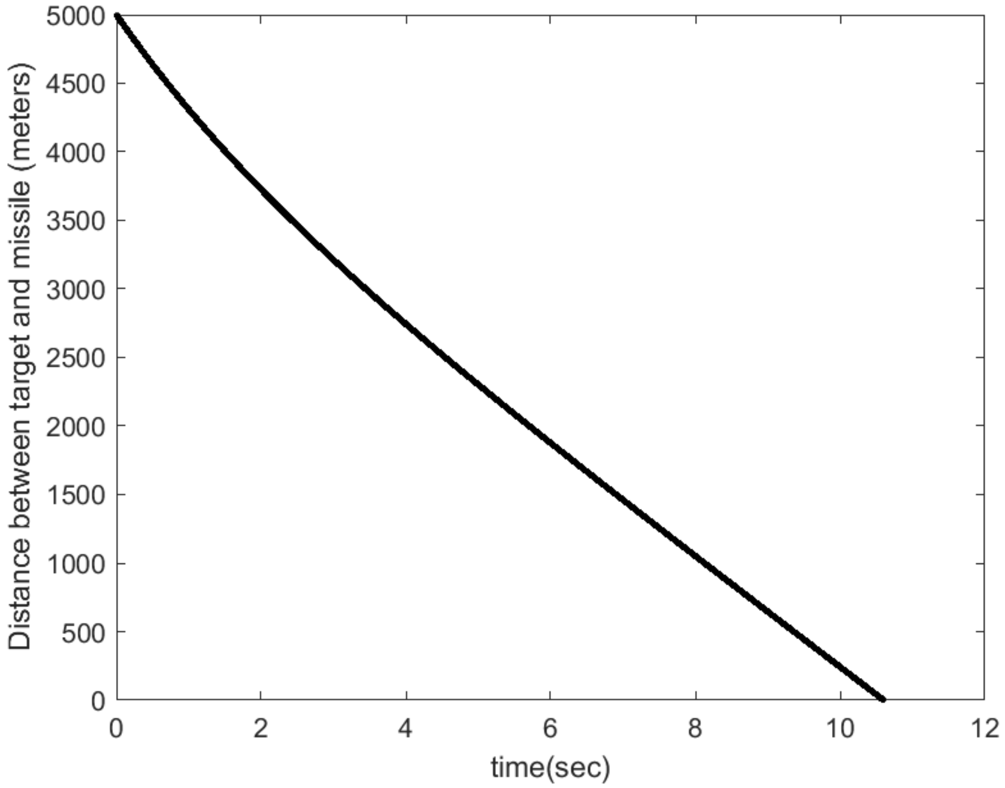

Fig 7.25 Distance between target and missile vs time

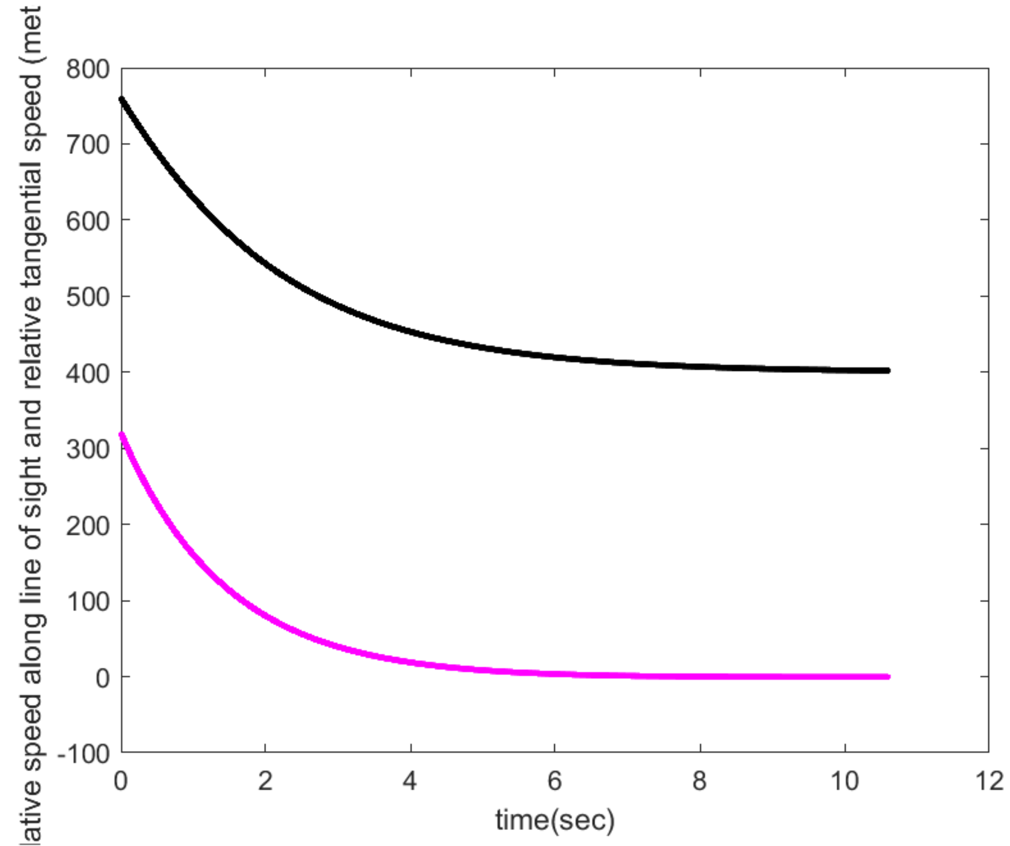

Fig 7.26 Tangential speed vs time

Discussion:

Our primary contribution to this field is the development of a two-stage PNG-based guidance law tailored explicitly for achieving impact angle-constrained interceptions in surface-to-surface engagement scenarios. This guidance strategy harnesses the power of Proportional Navigation Guidance (PNG) in a novel way, offering a unique and effective approach to address the intricacies associated with both nonstationary nonmaneuvering and maneuvering targets.

The key components of our proposed guidance scheme include an orientation guidance phase, where a lower value of the navigation constant N is employed, and a subsequent transition to N = 3, enabling the interceptor to achieve any desired impact angle. This two-stage approach allows for precise control and adaptability, making it well-suited for surface-to-surface applications where impact angle constraints are paramount.

In addition to presenting the theoretical framework, we have also provided a feedback implementable form of the guidance law, ensuring practical applicability in realistic engagement scenarios. Through extensive simulation studies, we have demonstrated the successful achievement of all impact angles, validating the effectiveness and versatility of our proposed guidance strategy.

Furthermore, we have evaluated the robustness of the guidance law by subjecting it to realistic simulations, accounting for first-order autopilot lags. This rigorous analysis underscores the resilience and reliability of our approach, even in scenarios where external factors introduce delays and uncertainties.

Importantly, we acknowledge that the performance of our proposed guidance law in maneuvering target scenarios is not without limitations, and we have highlighted the impact angle limitations inherent in such situations. This insight into the constraints of the guidance scheme underlines the need for continued research and development in this area.

In conclusion, our research represents a significant step forward in the pursuit of effective and adaptable guidance strategies for intercepting nonstationary nonmaneuvering and maneuvering targets while adhering to impact angle constraints. The inherent simplicity, robustness, and implement ability of the Proportional Navigation Guidance framework make it a valuable tool in the arsenal of missile and interceptor technology. We anticipate that the findings and insights presented in this thesis will contribute to the ongoing evolution of precision strike capabilities and defence systems, ultimately enhancing our ability to address the complex challenges posed by modern surface-to-surface engagement scenarios.

Chapter 8

Conclusion

8.1 Summary of Key Findings:

This thesis has explored the development and application of Proportional Navigation (PN) guidance strategies against nonstationary nonmaneuvering and maneuvering targets with a specific focus on impact angle constrained guidance. The key findings and contributions of this research can be summarized as follows:

8.1.1 Stationary Target:

1. From the Stationary Target, we can conclude that the missile follow a circular path to hit the target and the equation of the circular path is
$$(x-\sin\alpha\cdot\frac{v_m^2}{a_m})^2+(z+\cos\alpha\cdot\frac{v_m^2}{a_m})^2=\frac{v_m^4}{a_m^2}$$

2. It is observed that with the increase of missile velocity, the interception time decreases for a particular impact angle, whereas interception time increases with increase of impact angle for a particular initial missile velocity.

3. The magnitude of maximum latax as well as range of latax increase with the increase of impact angles for a particular initial missile velocity. The latax also increases with the increase of initial missile velocity for a particular impact angle.

8.1.2 Nonstationary Nonmaneuvering Target:

In a surface-to-surface engagement situation, a two-stage PNG-based guiding legislation is proposed for impact angle limited interception of nonstationary nonmaneuvering targets. The interceptor has switched to N=3 and achieve any desired impact angle for surface-to-surface applications. The outcomes of the simulation demonstrate that all impact angles for realistic interceptor models and constant speed interceptor were successfully achieved. Realistic simulations with first-order autopilot delays are used to demonstrate the robustness of the proposed guidance law. Proportional navigation provides the inherent simplicity, robustness, and implementation feasibility to the proposed guidance scheme.

1. **For Constant interceptor model:** In this model, we vary the desired impact angle α_{mf} and compare the performance in terms of the impact angle error. The proposed guidance law captures all impact angles $\alpha_{mf} \in [-\pi, 0]$ for which, as the trajectory shaping guidance law breaks down, nears head-on kind of desired terminal geometries.

2. **For Realistic Interceptor Model:** In this model, we use accelerating target which accelerated up to 4 m/sec^2 . Impact angle errors for $\alpha_{mf} = -90$,180 degree are less than 2 and 4 degree, respectively. For $\alpha_{mf} = -90$ degree the target closes to the interceptor early, leading to lateral acceleration saturation, resulting in higher impact angle errors. For different impact angles, the interceptor hit the target with different acceleration. To study the robustness of the proposed guidance law, we use first-order autopilot lag time constant τ , up to 0.3 s. For $\alpha_{mf} = -90$ degree, the lateral acceleration saturation causes an error of around 2.4 degree and the error increases by 0.2 degree for the considered range of τ .

8.1.3 Maneuvering Target:

In recent years, advancements in radar, infrared sensors, artificial intelligence, and guided missile technologies have improved the ability of defence systems to deal with maneuvering targets. These developments have led to more effective countermeasures and increased the overall effectiveness of military and defence operations.

Here, Equation (7.57) is a feedback guidance law that can be used against maneuvering targets. So that, the main guidance law for the maneuvering target is

$$a_m = N v_m \dot{\theta} + g \cos \alpha_m$$

In this simulation, all the conditions are kept same and all the parameters are same as previous simulations. Here, the target is going in different path like parabolic path, circular path and Zigzag path. Simulation is performed and we found that in all cases, the interceptor hit the target successfully.

8.2 Implications for Defence and Aerospace:

The research conducted in this thesis on impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets within the Proportional Navigation (PN) framework carries significant implications for the defence and aerospace sectors. These implications pertain to the enhancement of guidance system effectiveness, target engagement precision, and overall mission success in military and aerospace applications.

The following are key implications for these sectors:

1. Improved Target Engagement Precision: The emphasis on impact angle control and the exploration of various guidance strategies enable defence systems, such as missile defence and air defence systems, to achieve higher precision in targeting nonstationary nonmaneuvering and maneuvering threats. This enhancement in precision can significantly reduce the risk of target deflection and ensure that engagements result in desired outcomes.

2. Enhanced Missile Guidance: The research findings provide valuable insights into optimizing missile guidance systems. By incorporating impact angle constrained guidance techniques, missile systems can better intercept nonstationary targets, including hostile aircraft, drones, and other airborne threats. This is particularly crucial in scenarios where adversaries employ evasive tactics.

3. Autonomous Systems Advancement: The principles and methodologies discussed in this research have implications for the development of advanced autonomous systems, including unmanned aerial vehicles (UAVs) and autonomous ground vehicles. These systems can benefit from improved target tracking, prediction, and guidance capabilities when operating in dynamic and challenging environments.

4. Defence against Asymmetric Threats: Nonstationary nonmaneuvering and maneuvering targets may include asymmetric threats such as unmanned aerial systems (UAS) used by non-state actors. The research offers insights into countering these threats effectively, bolstering defence against unconventional adversaries.

5. Adaptation to Changing Target Behaviours: The exploration of real-time adaptation strategies and machine learning techniques enables defence and aerospace systems to adapt dynamically to changing target behaviours. This adaptability is critical when facing adversaries who may employ tactics to disrupt conventional tracking and engagement methods.

6. Sensor Technology Advancement

The research underscores the importance of sensor technologies in accurate target tracking and prediction. Defence and aerospace sectors can benefit from advancements in sensor technology to improve situational awareness and overall system performance.

7. Ethical and Safety Considerations

Incorporating safety considerations and ethical guidelines in the design and deployment of defence and aerospace systems is of utmost importance. These principles ensure that engagement decisions prioritize safety, minimize collateral damage, and adhere to international laws and regulations.

8. Dual-Use Applications

The research findings and strategies discussed in this thesis have dual-use applications. They can be applied not only in defence and military contexts but also in civil aerospace, search and rescue operations, disaster response, and other civilian domains where precision targeting and impact angle control are essential.

8.3 Future Research Directions:

1. Advanced Impact Angle Constrained Guidance:

A. **Dynamic Impact Angle Control:** Future research can focus on dynamic impact angle control mechanisms that adapt in real-time to target behaviour. These mechanisms would continuously assess the target's trajectory and update the guidance commands to ensure that the desired impact angle is maintained throughout the engagement. Dynamic control can enhance the precision of targeting by accounting for target deviations and disturbances.

B. **Predictive Impact Angle Control:** Developing predictive impact angle control techniques involves using predictive models to anticipate future target behavior and optimize the guidance commands accordingly. Machine learning algorithms, such as recurrent neural networks (RNNs) and long short-term memory networks (LSTMs), can be employed to forecast target trajectories and adjust guidance commands proactively. Predictive control can reduce the reliance on real-time tracking accuracy and enhance the ability to intercept challenging targets.

C. **Multi-Objective Optimization:** Consideration of multiple objectives beyond impact angle control can be integrated into guidance strategies. Future research can explore multi-objective optimization techniques that simultaneously optimize impact angle, time-to-intercept, and other mission-specific criteria. These approaches enable guidance systems to make trade-offs between different objectives and adapt to changing engagement scenarios.

D. **Constraint Relaxation Strategies:** While impact angle constraint is essential, future research can investigate constraint relaxation strategies that temporarily relax the impact angle requirement under specific conditions. This flexibility can be particularly valuable when engaging elusive or rapidly changing targets, allowing for adaptive behaviour without compromising overall mission objectives.

2. Integration with Other Guidance Strategies:

A. **Proportional Navigation (PN) with Impact Angle Control:** One avenue for integration is combining classic Proportional Navigation (PN) guidance with impact angle control. While PN provides effective navigation toward a target, integrating it with impact angle control strategies can ensure precise targeting. This hybrid approach can adapt PN's line-of-sight rate control with impact angle optimization, enhancing both guidance accuracy and convergence.

B. **Proportional Navigation and Trajectory Shaping:** Integrating PN with trajectory shaping techniques can offer advantages in terms of fine-tuning the intercept path. Trajectory shaping allows guidance systems to generate tailored trajectories that optimize impact angles while adhering to PN's navigation principles. This integration can be particularly valuable when dealing with targets exhibiting non-standard behaviours.

C. **Homing Guidance and Impact Angle Control:** Homing guidance systems, often used in missile applications, focus on homing in on a target using sensor inputs. By integrating homing guidance with impact angle control, the guidance system can actively manipulate the trajectory to achieve not only target intercept but also the desired impact angle. This integration is especially relevant when targeting nonmaneuvering threats in complex environments.

D. **Kinematic Guidance and Dynamic Control:** Some guidance strategies are kinematic in nature, considering only the target's position and velocity. Integrating these kinematic guidance approaches with dynamic control mechanisms can enhance guidance systems' adaptability to varying target behaviors. Dynamic control can adjust kinematic guidance commands to maintain impact angle constraints even when targets exhibit unpredictable motion.

3. Target Recognition and Tracking Advancements:

A. **Advanced Sensor Technologies:** Investigating and incorporating advanced sensor technologies can significantly improve target recognition and tracking. Researchers can explore the use of:

- **High-resolution Radar:** High-frequency radar systems with improved resolution can provide detailed target data, enabling precise target tracking and classification.
- **Multispectral and Hyperspectral Imaging:** Multispectral and hyperspectral sensors can capture detailed spectral information, aiding in target recognition and discrimination.
- **LiDAR and 3D Imaging:** LiDAR technology can provide three-dimensional target profiles, enhancing tracking accuracy and enabling better prediction of target motion.
- **Quantum Sensors:** Emerging quantum sensors may offer unprecedented precision in target tracking and recognition, revolutionizing guidance systems.

B. **Artificial Intelligence (AI) and Machine Learning:** Leveraging AI and machine learning algorithms can enhance target recognition and tracking capabilities. Research can include:

- **Object Detection and Classification:** Developing deep learning models for real-time object detection and classification, allowing guidance systems to recognize and categorize targets accurately.
- **Tracking Algorithms:** Advancing tracking algorithms, including particle filters and Kalman filters, with machine learning techniques to handle complex target behaviours and uncertainties.
- **Anomaly Detection:** Using AI to detect anomalous target behaviours or potential threats, improving situational awareness and response.
- **Feature Extraction:** Developing algorithms for extracting relevant features from sensor data to improve target recognition and tracking accuracy.

C. **Sensor Fusion and Multimodal Integration:** Integrating data from multiple sensors and modalities can provide a more comprehensive view of the target.
Research can explore:

- **Sensor Fusion Techniques:** Advancing sensor fusion algorithms to combine data from radar, lidar, optical, and other sensors seamlessly.

- **Sensor Calibration:** Ensuring accurate sensor calibration to minimize measurement errors and uncertainties in target tracking.
- **Cross-Modal Registration:** Developing methods to align data from different sensors, allowing for a more coherent and accurate representation of the target.

8.4 Closing Remarks:

In this thesis, we have embarked on a comprehensive exploration of impact angle constrained guidance against nonstationary nonmaneuvering and maneuvering targets within the Proportional Navigation (PN) framework. Our journey through the chapters has unveiled the importance of precise impact angle control, strategies for achieving it, modelling target behaviour, prediction techniques, and implications for defence and aerospace.

In closing, this thesis serves as a testament to the ongoing pursuit of precision and adaptability in guidance systems. As technology continues to evolve, the field of guidance systems will be at the forefront of innovation, driven by the need to engage nonstationary nonmaneuvering and maneuvering targets with unparalleled accuracy and effectiveness. Our journey may have reached its conclusion, but the quest for precision in targeting and the advancement of guidance strategies continues. It is our hope that this work inspires further research, innovation, and practical applications that contribute to the ever-evolving landscape of guidance systems.

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