STABILIZATION OF CART INVERTED PENDULUM SYSTEM USING DOUBLE PID CONTROLLER WITH GENETIC ALGORITHM BASED OPTIMAL TUNING

This thesis is submitted in the partial fulfilment of the requirements of the degree

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Certificate of Recommendation

This is to certify that **Mr. Rajeev Ranjan Raj (0020110804001)** has completed his dissertation entitled, "**Stabilization of Cart Inverted Pendulum using Double PID Controller with Genetic Algorithm based Optimal Tuning**", under the direct supervision and guidance of **Dr. Sayantan Chakraborty**, Department of Electrical Engineering, Jadavpur University. We are satisfied with his work, which is being presented for the partial fulfilment of the degree of **Master in Control System Engineering** of Jadavpur University, Kolkata-700032.

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The foregoing thesis entitled "**Stabilization of Cart Inverted Pendulum using Double PID Controller with Genetic Algorithm based Optimal Tuning**" is hereby approved as a creditable study of an Engineering subject carried out and presented in a manner satisfactory to warrant its acceptance as a pre-requisite to the degree of Master in Control System Engineering for which it has been submitted. It is understood that, by this approval the undersigned does not necessarily endorse or approve any statement made, opinion expressed, or conclusion therein but approve this thesis only for the purpose for which it is submitted.

Final Examination for Evaluation of the Thesis

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Signature of the Examiners

Declaration of Originality

I hereby declare that this thesis contains a literature survey and original research work by the undersigned candidate, as part of his Master in Control System engineering curriculum. All information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

Since the 1940s, industries have utilized the proportional-integral-derivative controller, or PID Controller, extensively and it is still the most popular. The traditional tuning of PID controllers for different applications require great knowledge and experience to arrive at the optimal values. To mitigate this issue, a plethora of techniques are reported in the literature. In this research, a Genetic Algorithm (GA) based approach is used to optimize the PID Controller parameters for stabilization of a cart inverted pendulum system. The present work proposes a double PID control scheme for cart position and pendulum angle and aims to tune both the PID controllers using GA.

In this regard, first, it is necessary to acquire the stable area for the stabilization problem, which is achieved here by using a trial and error approach involving GA. Finally, the optimal K_p, K_i and K_d values for both the PID controllers are obtained inside the stable area by using GA based technique.

The thesis then compares the results obtained using different variations of GA parameters and decides the optimal one. It is further compared with results involving single-PID control schemes tuned with different traditional control techniques. It shows that the proposed GA based double PID control scheme yields superior results in terms of transient performance.

CONTENTS

	List Of Abbreviation
AGA	Adaptive Genetic Algorithm
CGA	Conventional Genetic Algorithm
CIPS	Cart Inverted Pendulum System
DOF	Degree Of Freedom
GA	Genetic Algorithm
IAE	Integral Absolute Error
ISE	Integral Square Error
ITAE	Integral Time Absolute Error
LQR	Linear Quadric Regulator
PID	Proportional-Integral-Derivative
PSO	Particle Swarm Optimization
SMC	Sliding Mode Control

Chapter 1

Introduction

1.1 History and Background

The main goal of any Control System is to ensure a desired output from any plant. We, the control engineers, must develop a controller that can meet the fundamental conditions.

The inverted pendulum on a cart, commonly referred to as the cart inverted pendulum system, has long been a difficult issue in control theory and robotics. It is a well-known and extensively studied dynamic system with intrinsic instability and nonlinearity, which makes it a perfect testbed for the creation and assessment of control algorithms. Since there are currently no techniques that are universally applicable, control design for nonlinear systems is typically carried out case-by-case. The majority of the existing nonlinear techniques are built upon conclusions from linear systems theory or its extensions. This is mostly due to the fact that there is insufficient theoretical understanding of the analysis and control of nonlinear systems, particularly the problems of stability and convergence. [1]

1.1.1 Cart Inverted Pendulum System (CIPS)

In the majority of control laboratories around the world, the swing-up and stabilisation of a cart-pendulum system is a common experiment. By applying control force on cart, this control problem involves swinging up a pendulum from its usual pendant configuration. Once the pendulum approaches the upright configuration, control is switched to a stabilising controller, which keeps it around the unstable equilibrium configuration. [2]

In the literature, a variety of techniques have been suggested for attaining swing-up and stabilisation of this system.

The primary goal of the cart inverted pendulum system is to keep the pendulum upright by regulating the cart's velocity. Due of the pendulum's instability, this seemingly straightforward activity becomes challenging. The system is highly nonlinear and dynamically unstable because, in the absence of any control, the pendulum will descend due to gravity.

Feedback control mechanisms are used to establish the ideal balance, with the aim of continuously monitoring the position or angle of the pendulum and implementing the necessary control measures to maintain it upright. There are many different control algorithms that can be used, from traditional strategies like PID control to more sophisticated methods like Model Predictive Control or Adaptive Control. [3]

A good test system for investigating control system design, stability analysis, and controller tuning is the cart inverted pendulum system. It offers a useful tool to investigate various control mechanisms and assess their effectiveness. Additionally, the system poses practical difficulties in terms of nonlinear dynamics, state estimation, disturbance rejection, and control optimization—aspects that are essential to the design of contemporary control systems.

Beyond theoretical study, the cart inverted pendulum system has significant practical applications. It is useful in fields like robotics, automation, and transportation systems. The ideas discovered from this system can be used to solve related issues, such as balancing humanoid robots, operating self-driving cars, and stabilising quadcopters.

In conclusion, the cart inverted pendulum system is an intriguing and difficult control theory topic. It is the perfect venue to investigate feedback control methods and their applications because of its innate instability and nonlinear dynamics. Researchers and engineers can learn a lot about designing and putting into practise reliable and effective control strategies for a variety of dynamic systems by studying this system.

1.1.2 PID Controller

One of the most often employed control algorithms in the fields of engineering and automation is the PID (Proportional-Integral-Derivative) controller. It is a feedback control mechanism that continually calculates the difference between a desired setpoint and the actual measured value of a system, and then modifies the control inputs to minimise that difference.

To accomplish precise and reliable control, the PID controller combines three control actions: proportional (P), integral (I), and derivative (D). The integral term accounts for the accumulated error over time, the derivative term forecasts the future behaviour of the error based on its rate of change, and the proportional term generates an output directly proportionate to the error signal. The PID can be modified by changing the parameters and weights of these three terms.[4]

A well-known illustration of feedback control's principles in control theory is the cart inverted pendulum. It is made up of a pendulum that is attached to the cart and can revolve in the vertical direction along with a cart that can drive down a track horizontally.

By regulating the cart's motion, the system aims to balance the pendulum in an upright posture. In order to accomplish this, the PID controller is essential. It estimates the error between the planned vertical position (setpoint) and the present angle by continually measuring the pendulum's angle. The PID controller modifies the control signal to move the cart properly based on this inaccuracy, trying to maintain the pendulum's balance.[3]

In an effort to return the pendulum to the appropriate position, the proportional term of the PID controller offers a quick response to the error. The integral term ensures that the system can reduce steady-state mistakes and maintain stability by accounting for any cumulative error over time. The derivative term enhances the overall system response by anticipating the pendulum's future behaviour and reducing any abrupt changes.

Even in the presence of disturbances or fluctuations in the system dynamics, the cart inverted pendulum system can achieve steady and precise control by appropriately setting the PID parameters. Due to this, the PID controller is widely used. The PID controller, in summary, is a potent control algorithm that has been widely employed in a range of control applications. It is a good option for managing trade-offs between responsiveness, stability, and precision in complicated systems like the cart inverted pendulum. The PID controller can keep the pendulum in an upright posture while demonstrating the fundamentals of control theory by making use of the proportional, integral, and derivative control actions.

1.1.3 Genetic Algorithm

Search techniques called genetic algorithms (GAs) are influenced by the principles of genetics and natural selection. They have been effectively used for search, optimisation, and machine learning tasks [9] and can be thought of as a general-purpose optimisation technique. Without any prior understanding of the system, it has already been demonstrated that GAs are capable of learning to control dynamic systems. However, certain significant issues still exist, similar to other techniques that learn to regulate through experiments, including the incomprehensibility and reliability of learnt rules, as well as resilience and learning costs.

Instead of using deterministic rules, GAs handle a population of alternative solutions known as individuals or chromosomes that iteratively evolve. The algorithm's iterations are referred to as generations. A fitness function and genetic operators including reproduction, crossover, and mutation are used to mimic the development of solutions. The most physically fit person will live for many generations while reproducing and maybe producing stronger kids. At the same time, each generation loses its weakest members. The survival of the fittest and genetic diversity concepts is used repeatedly in the genetic algorithm to iteratively refine the population until it reaches an optimal or nearly ideal answer. The technique is particularly useful for complicated optimisation tasks because to its capacity to explore a wide search space, solve non-linear and multi-modal issues, and find global or near-global solutions. [5]

Numerous issues, including the optimisation of mathematical functions, engineering design, scheduling, pattern recognition, and machine learning, have been successfully tackled using genetic algorithms. They provide a flexible and adaptable method that can deal with issues involving a variety of restrictions, non-differentiable objective functions, and high-dimensional search spaces.

1.1.4 Application of GA in CIPS Stabilization

To keep the pendulum upright, the cart inverted pendulum system poses a difficult control problem that calls for precise and efficient control solutions. Using a PID (Proportional-Integral-Derivative) controller is one common method of controlling this system. Manually adjusting the PID parameters, however, can take a lot of time and frequently results in subpar performance.

The PID parameter tuning for the cart inverted pendulum system is automated and optimised by using a GA-based method. In comparison to hand tuning, the genetic algorithm allows for a more thorough examination of a larger search space of parameter combinations and may identify better answers. [6]

1.2 Motivation

The demand for improved control methods that can successfully stabilise and govern the cart inverted pendulum system drives the development of this thesis. Despite the numerous control strategies that have been put forth, fine-tuning PID parameters continues to be quite difficult. Manual tuning frequently involves plenty of trial-and-error, necessitating specialised knowledge and substantial time investment.

This thesis seeks to alleviate the limitations of human tuning by automating the parameter optimisation process. A reliable and effective framework for determining the ideal PID parameter values is provided by the inherent ability of GA to explore and utilise the solution space. Improvements in control performance, increased stability, and decreased oscillations are anticipated when GA-based tuning techniques are used to the cart inverted pendulum system.

The advantages of this research may also go beyond the confines of the cart inverted pendulum system. The conclusions and methods established in this thesis can be used as a foundation for applying GA-based tuning techniques to additional challenging control issues. This study can enhance control theory by demonstrating how well GA performs in optimising PID parameters and offer insightful information for industrial control applications.

The overall goal of this thesis is to close the knowledge gap between manual tuning methods and automated optimisation approaches, particularly in the context of the cart inverted pendulum system. We want to improve control performance, lessen the reliance on expert knowledge, and create a foundation for future study in the field by using the potential of GA in the field of control system.

1.3 Thesis Outline

In this thesis, **Chapter 1** includes the introduction to the basic control objectives and the concept of cart inverted pendulum system. It discusses the application of PID controller with GA based tuning technique in Cart Inverted Pendulum System.

Chapter 2 provides the literature review about the different efforts and strategies developed in order to balance the CIPS as well as regarding the advancement in the field of GA. In the last section, summary in brief is provided.

Chapter 3 discusses about the GA in details. In this regard, the chapter presents the various factors which are important to be considered while using genetic algorithm process.

Chapter 4 revisits the Cart Inverted Pendulum System. The CIPS is modelled mathematically and PID controller is applied. The different existing controlling techniques is also discussed here. Application of GA in CIPS for tuning parameters of PID is shown with the necessary details required along with the flow of the process.

Chapter 5 shows the simulation done in regard to GA application in CIPS. Results of application have also been developed. Comparative analysis has also been carried out, first among the GA parameters and second with other technique used for stabilization.

Chapter 6 concludes the contributions of the thesis and points out the scope of future work.

Chapter 2

Literature Survey

2.1 Control Strategies for Cart Inverted Pendulum System

One of the most well-known benchmark problems in the discipline of control engineering is the inverted cart pendulum system. It consists of a pendulum coupled to a cart that can freely revolve in the vertical plane and a horizontal track that the cart travels along. Controlling the cart in a such that the pendulum stays balanced and in an upright posture is the goal.

The goal of this review of the literature is to give readers an overview of the numerous control schemes that have been considered and proposed for the inverted cart pendulum system. To accomplish stable and reliable control of the system, these strategies make use of several control approaches, including classical control, modern control, and intelligent control.

In 1996 [7], an energy based controller was also made for stabilization of the cart inverted pendulum system. This article introduced the idea of energy control and demonstrated how it might be used to create reliable methods for swinging up an inverted pendulum. The relationship between the greatest pivotal acceleration and the acceleration of gravity has a significant impact on the behaviour produced using the technique. Interesting insights into the robustness difficulties can be gained through a comparison with the least time solutions.

In [8], a nonlinear controller was created for inverted pendulum systems based on the approximate linearization technique, using the inverted pendulum as an example of a nonlinear system that is not exactly linearizable. The weighted least squares method was used to satisfy the requirement for first order approximate linearization in the design in an effort to reduce the influence of the higher order residual components in selecting the coordinates. Then, through trials, its efficacy was demonstrated.

Further, N. Muskinja and B. Tovornik [9] showed that the performance of a control strategy based on a simple fuzzy control algorithm was better than the energy controller for an inverted pendulum. The pendulum's energy is pushed towards a value equal to the steady-state upright position as it swings upward, and the pendulum-cart movement constraint is taken into account. The pendulum must be caught using the right technique as it gets closer to the upright position. The pendulum stops after the state-controller gains have been adjusted while balancing the pendulum in the upright position with various state controllers. The state vector's norm serves as the foundation for the adaptation approach. Even with faults active or after shifting the desired cart position, the real-time experiments have proven highly outstanding. From any beginning situation, the fuzzy swinging algorithm swings the pendulum, and the pendulum cart always stays inside the pendulum-rail restrictions.

The inverted pendulum control problem was even introduced with two sliding mode control laws, one traditional SMC and another integral SMC. In the same simulation setting, a comparison of conventional and integral SMC reveals that integral SMC is superior to traditional SMC in terms of static errors and robustness when perturbations are applied to systems. [10]

Meanwhile in 2010 [11], the innovative chaos particle swarm optimisation technique (CPSO) was created and used to tackle the well-known inverted pendulum engineering problem. Applying CPSO to the nonlinear system model allowed for the creation of the PID controller. The PID parameters are located off-line for each iteration before the system is subjected to the optimised controller. The outcome demonstrated that a smooth reaction is possible.

Prasad, L.B., Tyagi, B., Gupta and H.O [12] introduced the control of nonlinear inverted pendulum-cart system using PID control and LQR, an optimal control technique, both with and without continuous disturbance input. The PID control method has been established in order to compare the outcomes of the suggested PID+LQR control method. All the instantaneous states of the nonlinear system are deemed available for measurement and are input directly into the LQR for optimal control of the nonlinear inverted pendulum dynamical system utilising the PID controller and LQR technique. The linear state-space model of the system is used to construct the LQR. To provide an optimal control, the LQR optimal control value is added negatively to the PID optimal control value. The Matlab-Simulink models were created for performance evaluation and simulation. By applying the trial-and-error process and evaluating the responses obtained to be optimal, the PID controllers that are employed here as either PID control methods or PID+LQR control methods are tuned. The simulation findings support optimal control's comparative benefit when employing the LQR approach. The pendulum's ability to stabilise in the upright position and the cart's ability to move rapidly and smoothly to the intended position even in the presence of constant disturbances like wind force demonstrate the effectiveness and sturdiness of the control strategies. The proposed PID+LQR control approach performs better than PID control, according to the examination of the responses of control schemes. The proposed PID+LQR control strategy is a straightforward, efficient, and reliable control scheme for the optimal control of nonlinear dynamical systems, according to this comparative performance assessment for this benchmark system. Future studies may examine the effectiveness of this control strategy using PID controller parameter adjustment using GA and PSO rather than the trial-and-error method.

In the year of 2014 [13], M. Tum, G. Gyeong, J. Park, and Y. Lee used the swing up controller with feedback controller and feed forward. Here, the system is made to follow a trajectory determined by the feedforward controller using a feedback controller. The feedback gain is regenerated at each timestep using a linearization around the current state, making the feedback controller similar to a LQR controller. The mathematical model and system constraints are included as constraints in the optimisation problem that determines the reference trajectory for the feedforward controller in advance.

In paper [14] presented by R. Sbresny, A. Getler, N. Felker and C. Frederickson in 2016, they approached the classical control problem of the inverted pendulum with the help of a stepper motor to control the movement of the cart. A Proportional, Integral, and derivative (PID) control system was constructed in MATLAB to balance the inverted pendulum system utilising velocity as the system's input. A microcontroller interacting with MATLAB over serial communication received sensor data from a potentiometer attached to the pendulum. The pendulum might be balanced and centred on the track by using PID control.

It can be concluded that over time, the inverted cart pendulum system's control approaches have advanced from traditional control to contemporary control and intelligent control methods. The choice of control method is based on the particular system requirements and desired performance objectives. Each approach has advantages and limitations. To address the difficulties posed by uncertainties, nonlinearities, and disturbances in the inverted cart pendulum system, additional research is presently being conducted.

2.2 Genetic Algorithm and its application in CIPS

2.2.1 Introduction to Genetic Algorithm

The process of natural selection and genetics served as the inspiration for genetic algorithms, often known as genetic algorithms (GAs). They are frequently employed in many disciplines, including control engineering, to address challenging optimization issues. Also, due to the inherently nonlinear and unstable nature of the cart inverted pendulum system, it presents a difficult control problem. Genetic algorithms have been used by researchers to tackle various facets of this control challenge. The primary uses of genetic algorithms in the cart inverted pendulum system are covered in the sections that follow. The goal of this study of the literature is to give a general understanding about the progress of works in genetic algorithm .

In 1993 [15], a method was proposed by Varsek A., Urbancic T. and Filipic B. where for learning control, a three-stage system built on genetic algorithms was suggested. First, without having any prior knowledge of the system to be controlled, operational control rules were obtained and displayed as tables. The next step was to structure the data recorded in the tables to synthesise the rules, producing understandable control knowledge. The final step in making control knowledge operational was to fine-tune the numerical parameters that make up this information. When sufficient domain knowledge is known to predict the structure of a control rule in advance (such as the traditional PID controller, as demonstrated in Learning from Scratch), the same fine-tuning process can also be used. as two extremes, and the fine-tuning of a control rule with a given structure. Due to significant experimentation, the first scenario's learning process is costly, yet it requires prior knowledge. On the other hand, if enough is known about the system, the cost in the second scenario can be greatly decreased. The user can benefit from the potential trade-off between the amount of experimentation and the accessible domain knowledge within the suggested framework. With an emphasis on the dependability of the control rules, this framework's performance was assessed in relation to the inverted pendulum control problem. [15]

Further, Gotshall S. and Rylander B. [16] talked about its work on work on optimal population size. After analysing the preceding data, three conclusions were drawn. First, for arbitrary large population sizes, the genetic algorithm's accuracy approaches but falls short of 100%. The likelihood that the population will initially have a chromosome representing the ideal answer increases with population size. Second, as the population grows, there are more generations convergent at once. This seems to be brought on by the general rise in mutational likelihood. A larger number of generations is required to eradicate the altered chromosomes if mutation occurs in a large population. Thirdly, the point at which the advantages of quick convergence and improved accuracy as population size grows balance each other out is known as the ideal population for a specific problem implementation. The average rate of change for the number of generations until convergence is at its lowest point at this time. The best option, in other words, is when the average slope is closest to zero. This is the point of inflection for cubic graphs, where the curve switches from being concave down to concave up or vice versa. It is the global minimum in the case of quadratics. The benefits of precision and convergence are balanced at this point in time. At this stage, the average slope of the quantity of wrong answers varies for each convergence graph.

Herrero, Blasco, Martinezand Salcedo [17] showed how powerful GA tuning can be for controllers. All control parameters that can be converted to a cost index can be used because the GA is a very effective optimisation tool. Applications for various performance requirements (IAE minimization and temporal domain limits) and robustness quality improvement (model faults and input noises) are demonstrated. A nonlinear process is subject to all laws. Because it is one of the most crucial fundamental controllers, just the application for a PID industrial controller is displayed. However, both linear and non-linear controllers can use this method. By simply modifying the cost index function, it is also easy to expand the application to a multivariable control. The sole restriction is the computational expense of the optimisation process, however, this is not a significant issue for off-line tuning.

With time, genetic algorithm was drawing attention. In 2007 [18], a kind of fast genetic algorithm was introduced. The population, crossover, and mutation probabilities of straightforward genetic algorithms are improved. In order to compare the performances of fast genetic algorithms and simple algorithms, common second-order object PID parameter tweaking is simulated. The simulation's output demonstrates how quickly evolving genetic algorithms enhance the control object's dynamic performance and population convergence rate. The selection operation method is enhanced, and the faults of early convergence, slow convergence speed, fixed crossover, and mutation probability are corrected. Because they are more practical and efficient optimisation algorithms, fast genetic algorithms are superior to simple genetic algorithms.

Lin G. and Liu G. [19] applied adaptive GA to tune the PID controller’s parameters. The step responses of the closed loop system were compared with those of the current methods (ZN and classical GA) to demonstrate the efficacy of the suggested method. The suggested controller's fundamental design is comparable to that of the CGA PID controller. The initial gains of the proposed controller are calculated using the PID gains acquired by the ZN tuning methods. The suggested controller has a straightforward structural design and a manageable computational task, making online adaptation simple. The simulation results demonstrate that the AGA PID controller outperforms ZN and CGA in terms of performance.

2.2.2 Application of Genetic Algorithm in CIPS

In 2014, Yusuf, Lukman A. and Magaji observed that the performance of the two control schemes, i.e., conventional PID and GA-PID, was good in controlling the pendulum angle of an inverted pendulum However, when settling time, rising time, and overshoot are taken into account, the GA-PID controller outperforms the traditional PID controller significantly. As a result, the GA-PID controller can be a useful and efficient controller for the system. [20]

It can be concluded that the control of the cart inverted pendulum system using genetic algorithms is a potential method for addressing the complexity and nonlinearity of the control problem. In the context of the cart inverted pendulum system, genetic algorithms have been used for parameter optimisation, controller design, hybrid control techniques, and multi-objective optimisation. These strategies have the potential to enhance the control's resilience, stability, and performance. In order to improve the control of the cart inverted pendulum system, more research is required to investigate sophisticated genetic algorithm variations and their integration with other control approaches.

2.3 Summary of Literature Review

Year	**Author**	**Topic**	**Remarks**
1996	K. J. Åström and K. Furuta	Swinging Up a Pendulum by Energy Control [7]	Method proved to be reliable for Swinging Up a Pendulum
1998	T. Sugie and K. Fujimoto	Controller design for an inverted pendulum based on approximate linearization [8]	Many aspects were ignored due to linearization in second order
2006	N. Muskinja and B. Tovornik	Swinging up and stabilization of a real inverted pendulum [9]	Cart movement's constraint was also taken into account and fuzzy controller used was better than energy controller
2009	Zhiping Liu, Fan Yu and Zhi Wang	Application of Sliding Mode Control to Design of the Inverted Pendulum Control System, International Conference on Electronic Measurement & Instrument [10]	Integral SMC is superior to traditional SMC

2010	O.T. Altinoz, A.E. Yilmaz and G. W. Weber	Particle swarm optimized PID controller for the inverted pendulum system [11]	Each iteration's result can be applied to PID system
2011	Prasad, L.B., Tyagi, B. and Gupta	Optimal control of nonlinear inverted pendulum dynamical system with disturbance input using PID controller and LQR [12]	PID + LQR performs better than only PID
2014	M. Tum, G. Gyeong, J. Park, and Y. Lee	Swing-up Control of a Single Inverted Pendulum on a Cart with Input and Output Constraints [13]	System's constraints were taken into account
2016	R. Sbresny, A. Getler, N. Felker, and C. Frederickson	Implementation of an Inverted Pendulum PID Control System Using a Stepper Motor [14]	Microcontroller data could be feed into PID
1993	Varsek, A., Urbancic, T. and Filipic	Genetic algorithms in controller design and tuning [15]	Operational Control rules could be obtained without prior knowledge of control system
2000	Gotshall, S. and Rylander B	Optimal Population Size and The Genetic Algorithm [16]	Conclusions were drawn about selecting population size for GA process
2002	Herrero, J.M., Blasco, X, Martinez, M and Salcedo, J.V	Optimal PID Tuning with Genetic Algorithm for Non-Linear Process Models [17]	GA tuning can be used for both linear and non-linear system
2007	X. Meng and B. Song	Fast Genetic Algorithms Used for PID Parameter Optimization [18]	Improving the GA parameters improved can make the GA fast
2010	Lin, G., Liu, G	Tuning PID controller using adaptive genetic algorithms [19]	AGA outperformed ZN and CGA technique
2014	Yusuf, Lukman A., Magaji	GA-PID controller for position control of inverted pendulum [20]	GA PID outperforms Conventional PID

Table 2.1 Summary of Literature Review

Chapter 3

Genetic Algorithm

3.1 Introduction

A stochastic global search technique called genetic algorithms (GAs) imitates the course of natural evolution. One technique for optimisation is this one.

This approach was initially presented in the United States by John Holland in 1970 at the University of Michigan. Computational systems have improved their performance throughout time, making them desirable for various forms of optimisation.

The best answer is ultimately determined by the responses of the environment and evolution operators like reproduction, crossover, and mutation. The genetic algorithm begins with no knowledge of the optimal solution. The approach prevents local minima and convergence to suboptimal solutions by starting at multiple independent sites and conducting concurrent searches. In contrast to gradient descent approaches or methods that rely on derivative information, GAs have been demonstrated to be capable of discovering high performance areas in complicated domains without running into high dimensionality problems. [21]

3.2 Characteristics of GA

Natural selection and natural genetics are two biological principles that serve as the basis for genetic algorithms, which are search and optimisation methods. GAs play with multiple possible solutions to an issue but a group of potential answers. We call this population. Chromosomes are a possible population-level solution. All of the solution's parameters are represented by these chromosomes in encoded form. Each chromosome is evaluated in relation to the other chromosomes in the population and given a fitness grade, which reflects how well it functions with the latter.

The GA will use genetic operators or evolution operators like crossover and mutation to create new chromosomes from the population's existing ones in order to encode superior answers. Either the ones already present in the population are merged, or an existing chromosome is modified, to do this.

The fitness of the parent is considered in the selection process for parent chromosomes. This will guarantee that the superior solution has a greater probability of procreating and passing on its advantageous trait to its progeny.

The starting population of a GA is typically made up of 20–100 randomly selected individuals. A real-valued number or a binary string known as a chromosome are typically used to represent this population, commonly referred to as the mating pool. The remainder of this section uses a binary string to represent each chromosome for explanatory purposes. The objective function evaluates and measures how successfully a person executes a task. Each individual is given a corresponding number termed their fitness by the objective function. Each chromosome's fitness is evaluated, and the survival of the fittest principle is used. In this project, the size of the error will be used to rate each chromosome's fitness. [22]

A genetic algorithm goes through three primary stages: reproduction, crossover, and mutation. The next part will go into more detail on this.

3.3 Population Size

One of the crucial steps in GA is to determine the population size.

The population size of a GA is quantity of individuals (sometimes referred to as chromosomes or solutions) that make up each generation of the algorithm. The population size is a crucial factor that can have a big impact on the efficiency and performance of the GA. [16]

Here are some crucial factors to consider while using genetic algorithms for determining population size:

Exploration vs. Exploitation: More people mean better space exploration in the quest. The likelihood of discovering the global optimum rises as the number of participants in the GA increases. However, a lower population size encourages exploitation by concentrating on a fewer number of plausible solutions, which might hasten convergence but also increase the chance of being stuck.

Computational Complexity: The population size has a direct impact on how hard the GA is to compute. The method requires more memory and processing power to examine and execute each generation as the population grows. To guarantee the method is executed effectively, it is crucial to balance the population size with the available computational resources.

Population Size and Variation: The population size is a key factor in preserving variation within the population. More different sets of answers are possible with a bigger population, delaying convergence and encouraging inquiry. The variety may be reduced and convergence may be hampered by very large populations, which can result in duplicate or overlapping solutions.

Problem Complexity: Depending on how complicated the problem being solved is, the ideal population size will vary. A smaller population could be adequate for straightforward issues with a limited search space. On the other hand, a bigger population is often needed to sufficiently investigate and represent the solution space in complicated issues with a wide search space.

The topic is covered in many research publications with numerous theories and tests being reported. [23]

There is no hard-and-fast rule for choosing the optimal approach to use because there are a growing number of hypotheses and tests that are run and tested. The choice of population size has traditionally been based on trial and error.

The method used in this study to determine the population is not very scientific.

The safe population size, according to the publications referred here, is between 30 and 100. A small initial population like 20-30 can be used. Then the results should be noted. If the outcome is not encouraging, then initiatives with populations of 40, 60, 80, and 90 should be tested. It is noted that the population estimate of 80 seemed to be a good approximation. Population of 90 or more has no effect on further optimisation.

3.4 Reproduction

Each chromosome's fitness value is evaluated during the reproductive phase. This factor is utilized to create bias in favour of fitter candidates throughout the selection process. A fit chromosome is more likely to be chosen for reproduction, just like in natural evolution. The 'Roulette Wheel' selection process is an illustration of a typical selection strategy. A portion of a roulette wheel is allotted to each member of the population. The section's size varies in direct proportion to an individual's level of fitness. [24]

A pointer is spun, and the individual who it points at is chosen. This keeps happening till the selection requirement is satisfied.

Thus, the selection of an individual depends on its level of fitness, guaranteeing that fitter individuals are more likely to have children.

The same string may be replicated multiple times, and the more effective strings should start to take over. However, in some situation, it's not impossible for the weakest candidate to win the competition.

The user must decide which selection method is best for each process out of the various others that are accessible. The same underlying principle underpins all techniques of selection, giving better chromosomes a higher chance of being chosen.

Four common methods for selection are:

1. Stochastic Universal sampling
2. Rank Based selection
3. Tournament selection
4. Roulette Wheel selection

Stochastic Universal Sampling – Genetic algorithms (Gas) frequently employ this selection technique to pick the parent individuals for reproduction. It is a deterministic roulette wheel selection variation that gets around some of that game's drawbacks. SUS is especially helpful when preserving population variety and individual fitness values are important considerations.

Tournament Selection: In Tournament Selection, a subset of the population is randomly selected, and the most fit member of that subset is chosen to serve as a parent for reproduction. Until the necessary number of parents is attained, this process is repeated. The number of competitors in each tournament, or the tournament size, is a factor that affects the selection pressure. The selection pressure is increased with a larger tournament field, favouring the selection of athletes.

Selection Method	**Advantages**	**Disadvantages**
Fitness Proportional Selection (Roulette Wheel Selection)	-Reflects the relative fitness of individuals -Allows for exploration and exploitation of the search space	-Can be biased towards fitter individuals -Inefficient for problems with extreme fitness differences
Tournament Selection	- Provides better selection pressure than fitness proportional selection -Suitable for problems with large populations	-Requires the tournament size to be carefully chosen -May lead to premature convergence if the tournament size is too large
Rank Selection	- Reduces the impact of outliers in the population -Balances exploration and exploitation	-Requires sorting of individuals based on fitness -Less intuitive than other selection methods
Stochastic Universal Sampling	-Provides a balanced selection pressure -Reduces the risk of losing diversity in the population	-Requires sorting of individuals based on fitness -Can be computationally expensive
Boltzmann Selection	-Allows for a controlled exploration-exploitation trade-off -Can adaptively adjust selection pressure during optimization	-Requires the calculation of Boltzmann probabilities -May suffer from premature convergence if the temperature parameter is not properly tuned

Table 3.1 Advantages and Disadvantages of different Selection Methods

Rank-based Selection: Rather than using an individual's absolute fitness levels, rank-based Selection assigns selection probability to individuals. The rank of a person in relation to other people in the population affects the selection probability. Individuals are often ranked according to their fitness scores, with selection probabilities being negatively correlated with rankings. This strategy guarantees that even those with inferior fitness have a chance to be chosen, fostering population diversity and avoiding hasty convergence.

Roulette Wheel Selection: Also known as fitness proportionate selection, roulette wheel selection is a popular selection technique in Gas. It is built on the idea of a roulette wheel, where each person's selection probability is inversely correlated with how fit they are in relation to the population as a whole. This approach mimics the “survival of the fittest” theory by giving individuals with greater fitness values a higher probability of being chosen for reproduction.

The Roulette Wheel selection and Tournament selection approach is preferred in this project due to the complexity of the other methods.

3.5 Crossover

The crossover algorithm is started after the selection step is finished. In an effort to preserve the beneficial elements of the original chromosomes and produce improved new ones, the crossover procedures swap specific portions of the two selected strings. On the premise that particular people's gene codes, on average, make fitter persons, genetic operators directly alter the features of a chromosome. The crossover probability shows how frequently crossing occurs. [25]

With a likelihood of 0%, the 'offspring' will be identical clones of their 'parents,' and with a probability of 100%, every generation will be made up completely of brand-new offspring. The Single Point Crossover method of crossover is the simplest.

Single point crossover involves the following two stages:

1. In the mating pool, individuals from the recently reproduced strings are 'mated' (matched) at random.

2. The crossover is performed as follows for each pair of strings: A random number between one and the string's length minus one, [1,L-1], is chosen as the integer k. Two new strings are produced by switching every character between locations k+1 and L, inclusive.

Example: If the strings 0101011 and 1110010 are selected for crossover and the value of k is randomly set to 3 then the newly created strings will be 0110010 and 1101011 as shown in Figure 3.1.

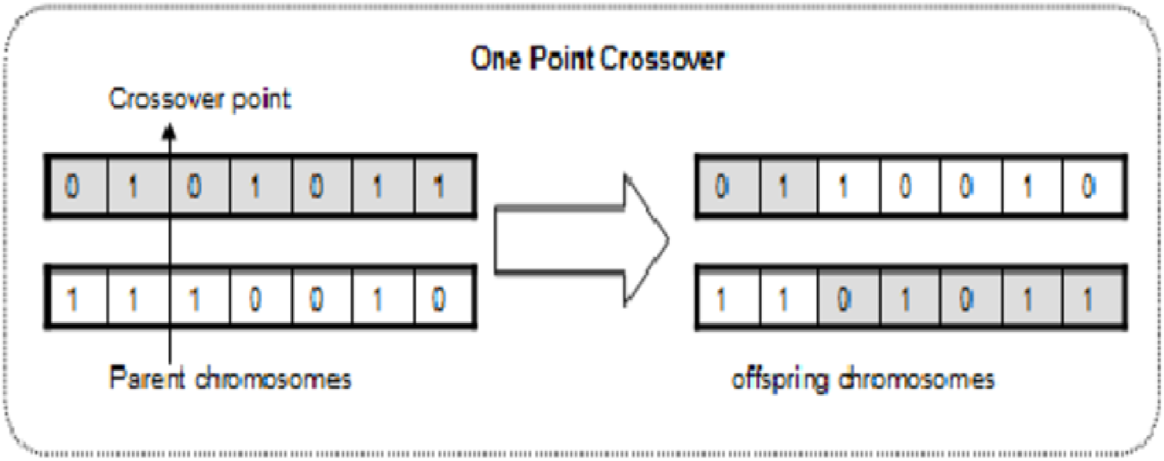

Fig 3.1 Single point Crossover [15]

Multi-point and uniform crossover algorithms are more sophisticated crossover methods. The idea behind Multi-point Crossover, which is an extension of the Single Point Crossover Algorithm, is that the majority component of its fitness-related chromosomes may not be nearby. A multi-point crossover involves three primary phases.

1. In the mating pool, individuals from the recently reproduced strings are 'mated' (matched) at random.

2. Positions in multiple categories are randomly chosen, with no duplicates, and arranged in ascending order.

3. In order to create additional offspring, bits are swapped between subsequent crossover places.

Example: If the string 00110100 and 100100010 were selected for crossover and the multipoint crossover positions were selected to be 3, 6 and 9 then the newly created strings will be 000100000 and 101101011 as shown in Figure 3.2

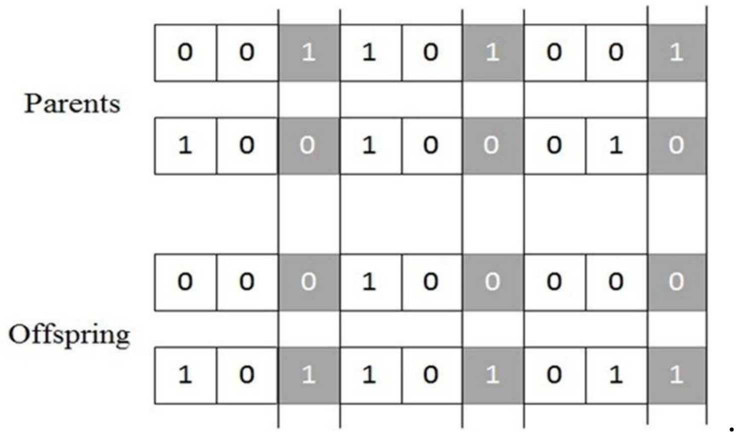

Fig. 3.2 Multi Point Crossover [30]

In a process called uniform crossover, a random mask of ones and zeros the same length as the parent strings is utilised.

1. In the mating pool, random pairings are made between members of the recently reproduced strings.

2. Each string is covered with a mask. The underlying bit is retained if the mask bit is set to one. The equivalent bit from the other string is put in this position if the mask bit is zero.

Example: If the string 1100010100 and 0110101101 were selected for crossover with the mask 1100101110 then newly created strings would be 1110000101 and 0100111100 as shown in Figure 3.3

Parent 1	1 1 0 0 0 1 0 1 0 0
Parent 2	0 1 1 0 1 0 1 1 0 1
Mask	1 1 0 0 1 0 1 1 1 0
Offspring 1	1 1 1 0 0 0 0 1 0 1
Offspring 2	0 1 0 0 1 1 1 1 0 0

Fig. 3.3 Uniform Crossover [31]

One fit string can be entirely destroyed using uniform crossover, the most disruptive crossover algorithm, rendering it unusable for the following generation. Because of this, Multi-Point Crossover will be used in this project instead of Uniform Crossover.

3.6 Mutation

By themselves, selection and crossover will produce a lot of different strings. There are, however, two significant issues with this:

1. The initial strings may not have enough variation to ensure that the Genetic Algorithm searches the whole issue space, depending on the beginning population selected.

2. Due to a poor starting population selection, the GA could converge on sub-optimal strings.

The addition of a mutation operator to the GA may be able to solve these issues. Mutation is the sporadic, random change in a string position's value. In the genetic algorithm, it is regarded as a background operator. [26]

Because a high mutation rate would eliminate fit strings and turn the genetic algorithm into a random search, the chance of mutation is typically modest.

The possibility that a particular string will be chosen for mutation is represented by mutation probability values of roughly 0.1% or 0.01%. For example, given a probability of 0.1%, one string out of 1,000 will be chosen for mutation.

A randomly picked string element is altered or "mutated" once the string has been chosen for mutation.

For example, if the GA chooses bit position 4 for mutation in the binary string 10000, the resulting string is 10010 as the fourth bit in the string is flipped as shown in Figure 3.4.

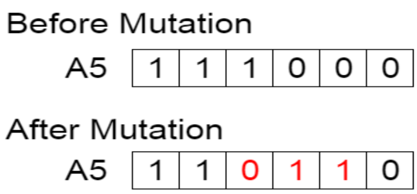

Fig. 3.4 Illustration of Mutation Operator [32]

3.7 Elitism

There is a good probability that the ideal answer will be lost throughout the crossover and mutation processes. The fittest string may not always be preserved by these operators. The elitist models are frequently applied to prevent this. In this paradigm, each of these operations is performed after the best member of a population has been saved. This model will check to see if the best structure has been kept once a new population has developed and been assessed. In that case, the population receives the saved copy once more. After then, the GA will continue as usual. [21]

3.8 Termination Criteria

Strong optimization methods called genetic algorithms (GAs) were developed as a result of natural evolution. They are frequently used to find the best answers to challenging issues across a variety of sectors. When a genetic algorithm should cease iterating is determined by the termination criterion. [27]

Here are a few typical termination standards for GA:

Maximum Number of Generations: After a certain number of generations, the GA comes to an end. This requirement guarantees that the algorithm terminates after a predetermined number of iterations, regardless of the advancement.

Convergence: When the population reaches a stable condition, the GA comes to an end. A convergence metric, such as the population's average fitness level or standard deviation, can be used to track this. The algorithm terminates if the convergence metric drops below a predetermined threshold or remains constant for a predetermined amount of generations.

Fitness Threshold: The GA ends when a person is identified whose fitness value is greater than a specified threshold. Further iterations may not be required if a good solution has been found, and the procedure can then be stopped.

Time Limit: After a predetermined period of computational time, the GA comes to an end. When there is a deadline and the algorithm needs to deliver results quickly, this criterion is helpful.

Improvement in Solution Quality: After a predetermined number of generations, the GA ends if solutions' quality does not significantly increase. This criterion guarantees that the algorithm terminates when it hits a wall and is no longer significantly progressing.

Exhaustion of Resources: The GA shuts down if it runs out of memory, processing power, or storage. When it becomes impractical to move forward owing to resource constraints, this criterion stops the algorithm from doing so.

User-Defined Criteria: The user may decide to end the GA if certain conditions or restrictions are met. For instance, the termination criterion may be based on achieving certain scheduling requirements or appeasing user preferences if the GA is being used to address a scheduling problem.

The task at hand, the available computational resources, and the intended outcome all must be taken into consideration while choosing the termination criterion. The evolutionary algorithm is stopped when it has reached the appropriate level of optimization or when it is no longer beneficial to keep looking for better solutions thanks to the careful selection of termination criteria.

3.9 Objective Function

It is possible to gauge how well people have performed in the issue domain using the objective function. The most fit people will have the lowest numerical value of the corresponding objective function in a minimization problem. This unprocessed fitness measure is typically only employed as a preliminary step in comparing participants' GA performance.

Objective Function	**Description**
Fitness Function	• The fitness or quality of an individual solution in the population is evaluated. • It directs the search for better solutions based on the criteria of the situation.
Cost Function	• The cost of a solution is calculated. • It is frequently utilized in cost-cutting optimization challenges, such as logistics or resource allocation.
Profit Function	• The profitability of a solution is measured. • It's employed in optimization issues when the goal is to maximize earnings, such portfolio optimization or revenue maximisation.
Error Function	• The error or disparity between a projected and actual value is quantified. • It is frequently used in machine learning and regression situations to direct the search towards solutions with fewer mistakes.
Distance Function	• It calculates the separation or difference between the solutions. • It is used in classification and clustering problems to locate the closest neighbors or to group similar solutions together.
Utility Function	• Evaluates how useful or satisfying a solution is. • The most preferable answer is chosen in decision-making difficulties based on a set of standards or preferences.
Constraint Function	• Determines whether a solution complies with certain requirements. • It is employed in issues involving constraint optimization to guarantee that the solutions follow the specified constraints while maximizing the objective.
Risk Function	• calculates the risk posed by a solution. • Finding solutions that balance risk and reward while taking into account elements like uncertainty and unpredictability is employed in risk management and decision-making difficulties.

Table 3.2 Different types of Objective Functions

A different function, the fitness function, is typically employed to convert the value of the objective function into a measurement of relative fitness; in this case, where f is the objective function and g converts its value to a nonnegative number, F is the resulting relative fitness.

Since lower objective function values correlate to fitter people, this mapping is always required when the objective function is to be minimised. The fitness function value frequently reflects how many offspring an individual might reasonably expect to have in the following generation. Proportional fitness assignment is a frequently used transformation. [28]

3.10 Summary of Genetic Algorithm

The flowchart that represents the procedure of the genetic algorithm in this section will be explained below.

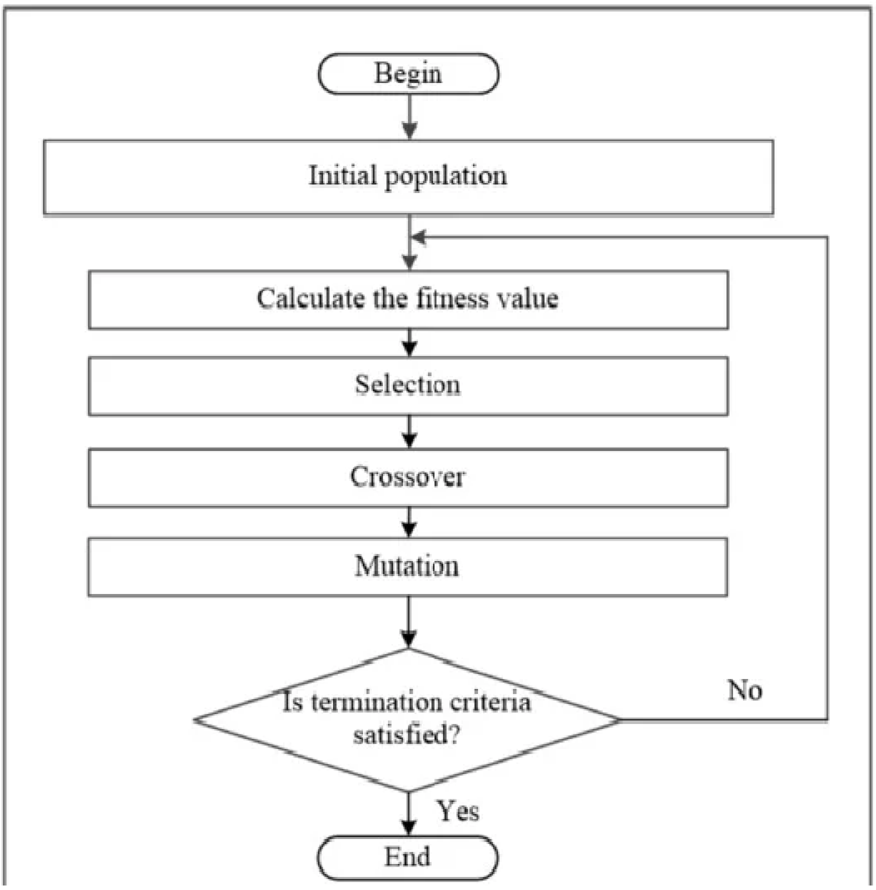

Fig. 3.5 Flowchart of Genetic Algorithm [33]

The procedures for developing and using a genetic algorithm are as follows:
1. For a fixed size, create an initial random population of people.
2. Determine how fit they are.
3. Choose the population's fittest members.
4. Repeat using a probabilistic technique, such as a roulette wheel.
5. Execute the crossover operation on the replicated chromosomes, selecting the crossover site and the "mates" based on probabilities.
6. Carry out the mutation operation with little chance.
7. Continue performing step 2 until a predetermined convergence condition is satisfied.

A user-specified condition, such as the maximum number of generations or when the string fitness value surpasses a specific threshold, is the convergence criterion of a genetic algorithm.

3.11 Application of Genetic Algorithms in Control Engineering

Currently, GA is getting a lot of attention, and more research is being done to look at its potential applications. Application has advanced significantly in field of control engineering as well. Even though in the design of control systems, problems like performance, system robustness, static and dynamic indexes, and system stability must be taken into consideration. However, controller structure and parameters have a significant impact on each of these problems. The majority of the time, a trade-off between competing performance concerns must be made in order to account for this dependence [29].

The following are some GA applications in use control engineering.

- Intelligent Control.
- Multi-objective Control.
- Optimal Control.
- PID control.
- Robust Control
- Intelligent Control.

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Application Area	Specific Use
Neural Network Design	Architecture optimization and training
Game Theory	Strategy optimization and decision-making
Manufacturing	Process optimization and control
Transportation	Route planning and traffic optimization
Medicine	Drug discovery and personalized medicine
Energy Optimization	Power grid management and renewable energy systems
Internet of Things	Networking routing and sensor placement

Table 3.3 Different Applications of GA

Chapter 4

Application of GA in Stabilization of CIPS

4.1 Cart Inverted Pendulum System

If one tries to balance a broomstick on his/her index finger or hand's palm then to maintain the object's upright position, one needs to constantly reposition his/her hand. In essence, an inverted pendulum does the same task. However, it is constrained because one’s hand can travel in all directions whereas it only moves in one.

An inverted pendulum is a system that is intrinsically unstable, just like the broomstick. For the system to remain intact, force must be applied properly. Correct control theory is needed for this. When assessing and contrasting alternative control theories, the Inverted Pendulum is crucial.

One of the most challenging systems to regulate in the discipline of control engineering is the inverted pendulum (IP). It has been a task of choice to assign Control Engineering students the duty of analyzing the model and proposing a linear compensator in accordance with the existing control rule due to its significance in the field of control engineering. As an unstable system, the inverted pendulum is a frequent control problem that a student of control system engineering is given to solve. [34]

The selection of the IP as the system was made for the following reasons:

• It is the system that is most readily available (in most academia) for laboratory use;

• It is a nonlinear system that may be regarded as linear, with little mistake, for a sizable range of variation.

• Offers potential control engineers useful training.

The following are a few notable uses of the inverted pendulum (IP):

SIMULATION OF DYNAMICS OF A ROBOTIC ARM

The robotic arm control systems are similar to the Inverted Pendulum problem. When the centre of pressure is below the center of gravity for the arm, creating an unstable system, the dynamics of an inverted pendulum replicate the dynamics of a robotic arm. Under these circumstances, a robotic arm behaves remarkably similarly to an inverted pendulum.

MODEL OF A HUMAN STANDING STILL

For humans to carry out their regular duties, they need to be able to stand upright and retain stability. The human body's stance and any changes to that pose are registered by the central nervous system (CNS), which also stimulates muscles to keep the body balanced. The inverted pendulum is frequently used as a reliable representation of a calm, silent individual.

4.1.1 Problem Definition

Without exerting any external force, it is practically impossible to balance a pendulum in the inverted position. This control force can be provided to the pendulum carriage via the cart attached.

This CIPS uses a DC servo-motor and a belt drive mechanism to transmit control force to the carriage. Cart location, cart velocity, pendulum angle, and pendulum angular velocity are the possible outputs from the CIPS rig (just pendulum angle in our case). The servo-motor is controlled by an analogue controller using the pendulum angle, ensuring steady and constant traction.

The study's aim is to stabilise the pendulum so that it can be firmly held in its inverted state throughout such motions and that the cart's position on the track can be swiftly and accurately regulated.

A cart that can go both backwards and forwards and a pendulum that is linked to the cart at the bottom of its length so that it can move in the same plane as the cart are the objects in the problem, which is illustrated below. In other words, the pendulum hung on the cart is unrestricted in its ability to fall along its axis of motion. Control of the system is required to maintain the pendulum's equilibrium, upright posture, and resistance to step disturbance.

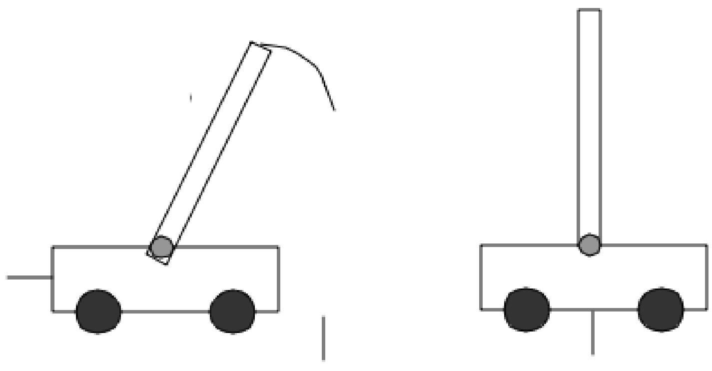

Fig. 4.1 A Simple Coupled System

A Simple Coupled System is the source of this issue. The pendulum will start to fall if it swings out of balance. The cart and pendulum are connected, and if the cart moves, the pendulum will begin to swing in the other direction, just as it would if the pendulum were uncoupled from the cart. Design a control system that keeps the pendulum balanced and tracks the cart to a consistent position because every change to one aspect of the system affects the other portion.

It is a more intricate control mechanism than it might initially appear to be. This is the reason why this issue is frequently used to illustrate fuzzy control.

A belt connected to an electric motor pulls the inverted pendulum cart along a track. A potentiometer that measures the position of the cart based on its rotation and another potentiometer that measures the pendulum's angle.

If the output is the pendulum's angle with respect to the vertical axis while it is upright, we can see that the system is unstable because even a little angle of release will cause the pendulum to descend. A feedback control mechanism must be utilised to stabilise the system, i.e., to keep the pendulum in the upright position.

In summary, a cart and a pendulum make up the Inverted Pendulum system. The controller must manoeuvre the cart to the desired location without causing the pendulum to topple over. This system is unstable in an open loop.

4.1.2 Mathematical Modelling

Here, in order to mathematically represent a cart-inverted pendulum system, equations describing the behaviour and dynamics of the system must be derived. This kind of system consists of an inverted pendulum installed on a cart that can move horizontally along a track. The goal is to manage the cart's motion so that the pendulum stays balanced and in an upright posture. [35]

The required system variables and parameters can be defined in order to start the mathematical modelling process. Let's write "x" for the cart's position and "θ" for the pendulum's angle with respect to the vertical. The other letters "M", "m", “b”, “I” and "l" stand for

M- the cart's mass,

m- the pendulum's mass,

b – the friction coefficient

I – moment of inertia of pendulum

l - the length of pendulum from centre of mass and

g – gravitational constant, respectively.

Applying Newton's second law and taking into account the forces acting on both the cart and the pendulum will allow us to determine the equations of motion for this system. The gravitational force, the reaction forces, and any control forces given to the cart are among the forces at play.

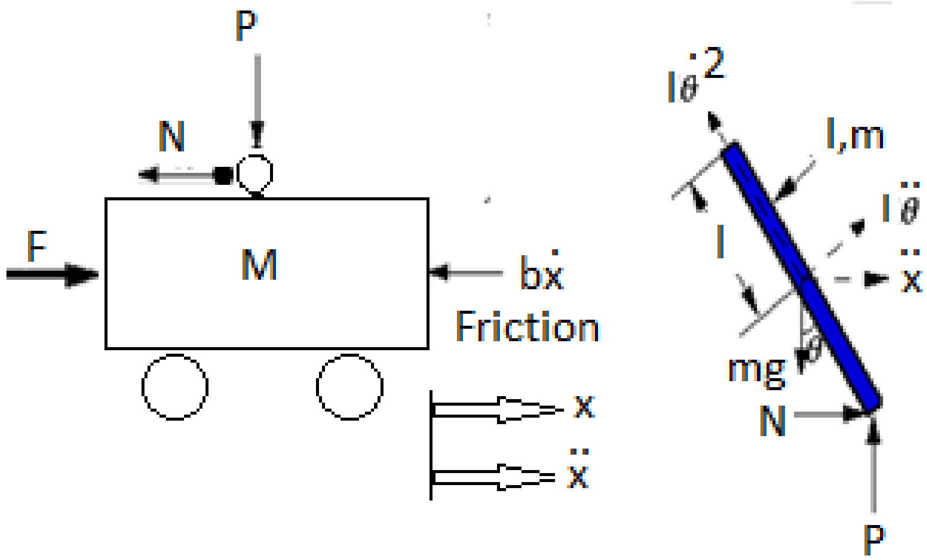

Fig.4.2 Free Body Diagram [48]

Considering the horizontal forces in the free body diagram of the cart

$$M\ddot{x} + b\dot{x} + N = F \qquad ....(1)$$

The forces in the vertical direction in the free body diagram of the cart ned not be considered as it does not yield any information.

Considering horizontal direction forces in the free body diagram of the pendulum, the expression of N is evaluated as

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \qquad ....(2)$$

By substituting equation (1) in equation (2), one of the two governing equation is derived as,

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \qquad ....(3)$$

Adding and solving the forces in the vertical direction, we get

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \qquad \text{........(4)}$$

To eliminate P and N, adding moments about the centroid of the pendulum

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \qquad \text{.........(5)}$$

Combining the last two equation (4) and (5),

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \qquad \text{.......(6)}$$

So, the two mathematical governing equation of the Cart Inverted Pendulum system is,

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \qquad \text{......(7)}$$

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \qquad \text{........(8)}$$

Linearization of the System

Classical control techniques for analysis and design is applicable only in linear system. [36]

In order to linearize the equations of Cart Inverted Pendulum system s along the vertical upright position ($\theta = \pi$), let assume that the deviation of pendulum is less than 20° from the vertically upright position.

As now ϕ is the small deviation around the equilibrium, so $\theta = 180° + \phi$ approximately,

$$\cos\theta = \cos(180° + \phi) \approx -1$$

$$\sin\theta = \sin(180° + \phi) \approx -\phi$$

$$\dot{\theta}^2 = \dot{\phi}^2 \approx 0$$

Using above calculated in equations (7) and (8), we get the linearized version of the governing equations of the Cart Inverted Pendulum system as

$$(I + ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x} \qquad \text{.......(9)}$$

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u \qquad \text{......(10)}$$

Where, u is the input force

Transfer Function Model

As we now have the linearized system equations of CIPS, Laplace Transform can be applied to system equations assuming zero initial conditions,

$$(I + ml^2)\Phi(s)s^2 - mgl\Phi(s) = mlX(s) \qquad ....(11)$$

$$(M + m)X(s)s^2 + bX(s)s - ml\Phi(s)s^2 = U(s) \qquad ....(12)$$

As transfer function representation is defined single input single output system. So, for first transfer function where output is $\Phi(s)$ and input is U(s), we need to eliminate X(s) from above equations.

So, from equation (11),

$$X(s) = \left[\frac{I+ml^2}{ml} \quad - \frac{g}{s^2}\right]\Phi(s) \qquad ....(13)$$

Substituting this in equation (12),

$$(M + m)\left[\frac{I+ml^2}{ml} \quad - \frac{g}{s^2}\right]\Phi(s)s^2 + b\left[\frac{I+ml^2}{ml} \quad - \frac{g}{s^2}\right]\Phi(s)s - ml\Phi(s)s^2 = U(s) \quad ....(14)$$

Rearranging this,

$$\frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \qquad ......(15)$$

where, $q = [(M + m)(I + ml^2) - (ml)^2]$

By cancelling the both a pole and a zero in origin, transfer function becomes

$$P_{pend}(s) = \frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \qquad [\frac{rad}{N}] \qquad .....(16)$$

Similarly, the transfer function with the cart position X(s) as input and U(s) as output is

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}} \qquad [\frac{m}{N}] \qquad ....(17)$$

State Space Model

Linearized equations of motion can rearrange to get the state space form as follow,

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{(M+m)I+Mml^2} & \frac{m^2gl^2}{(M+m)I+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{(M+m)I+Mml^2} & \frac{mgl(M+m)}{(M+m)I+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{(M+m)I+Mml^2} \\ 0 \\ \frac{ml}{(M+m)I+Mml^2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

The C matrix has 2 rows as both the cart's position and the pendulum's position are part of the output.

Symbol	Parameter	Value	Unit
M	Mass of the cart	0.5	kg
m	Mass of the pendulum	0.2	kg
l	Length of the pendulum	0.6	m
g	Gravitational constant	9.8	m/s^2
u	Applied force to cart	± 1	N
b	Cart friction	0.1	N-s/m
I	Moment of Inertia	0.006	kgm^2

Table 4.1 Parameter Values used for CIPS

4.2 Characteristics of CIPS

A system is referred to as an uncompensated cart-inverted pendulum system if no compensatory or control mechanisms are used to stabilise or govern the system. In this instance, the system only relies on its innate dynamics to function, and its behaviour can be summarised by the following salient characteristics:

Unstable Equilibrium: A system's unstable equilibrium point is represented by the pendulum's upright position at = 0. The pendulum can fall or the cart can move away from this position with the least perturbation.

Nonlinear Dynamics: Because of the trigonometric terms involved, the motion of an uncompensated cart-inverted pendulum system is controlled by nonlinear differential equations. Compared to linear systems, the nonlinearity of the system makes analysis and control more difficult.

Simulation for non-linear dynamics can be obtained as shown in fig below-

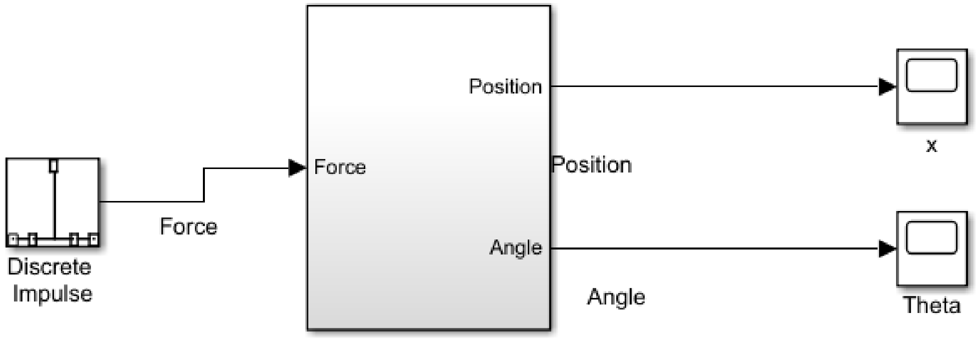

Fig. 4.3 Simulink Model for uncompensated CIPS

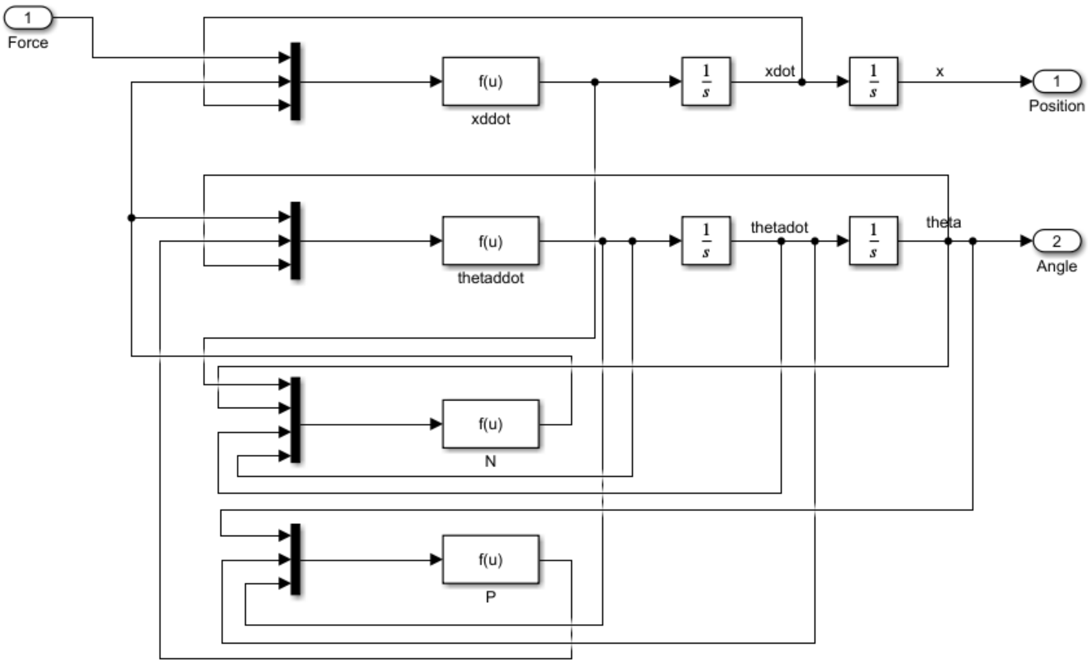

Fig 4.4 Description of Subsystem of CIPS

Impulse Response of uncompensated CIPS

The graph is obtained for position(x) of Cart Inverted Pendulum System from non-linear dynamics.

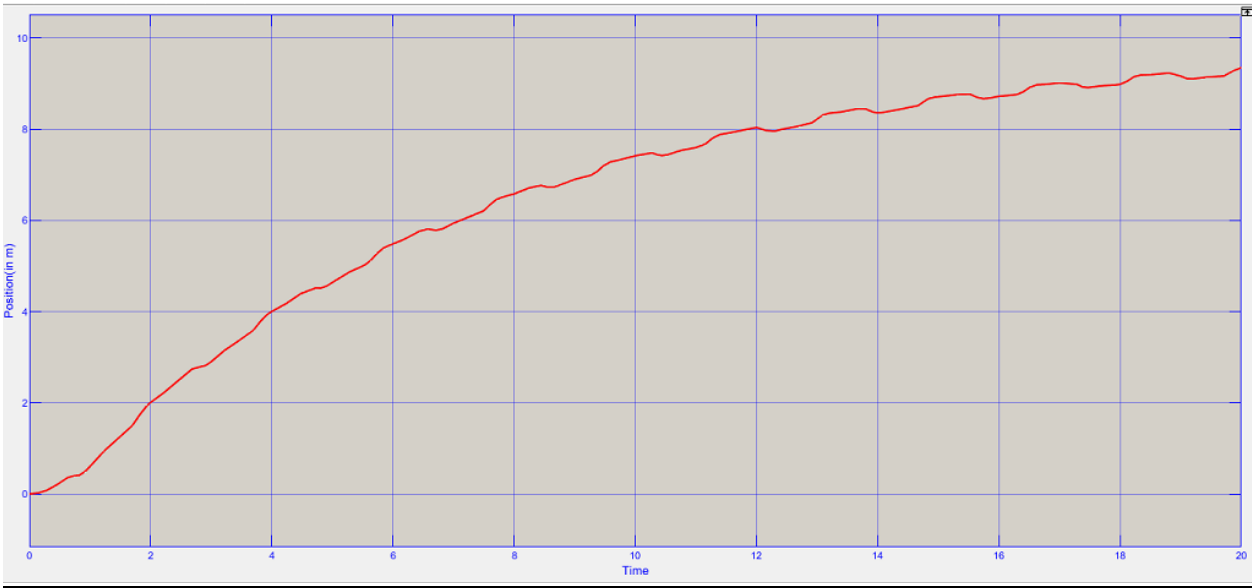

Fig. 4.5 Simulation result for cart's position of uncompensated CIPS

Here, it can be observed that the position is increasing, as system is unstable.

Similarly, if we look over the response obtained for pendulum's angle, it is oscillating.

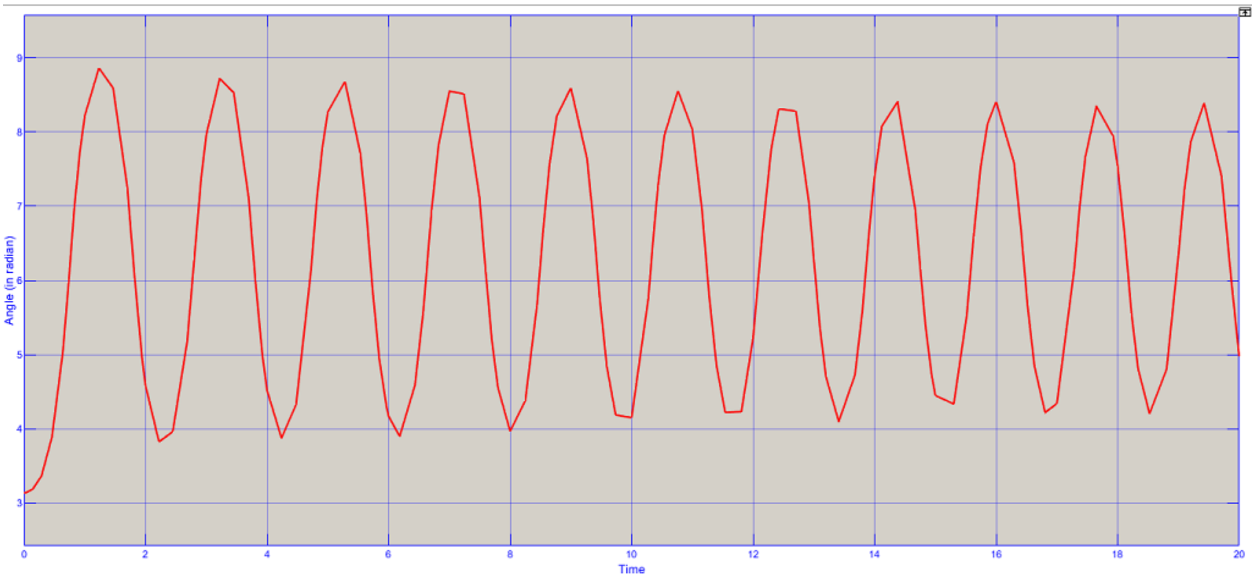

Fig. 4.6(a) Simulation result for pendulum's angle (θ) of uncompensated CIPS

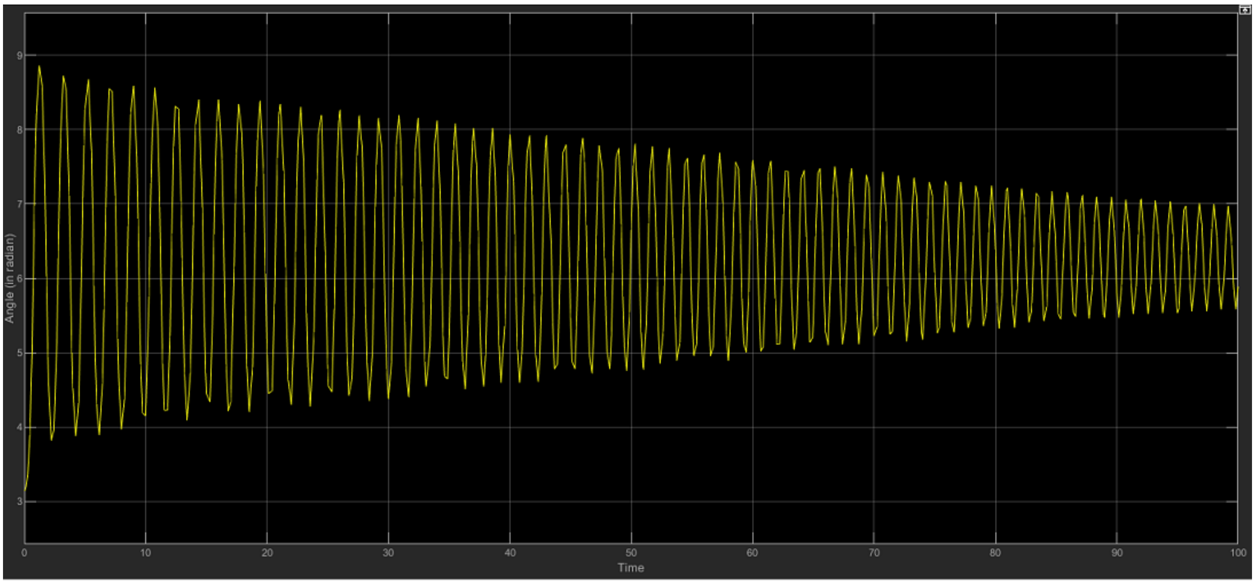

Fig. 4.6(b) Simulation result for pendulum's angle (θ) of uncompensated CIPS(50 sec)

The pendulum is oscillating between two extremes indicating that system is unstable.

So, it is important to comprehend the properties of an uncompensated cart-inverted pendulum system in order to recognise the necessity of control methods. It is possible to stabilise the system and achieve desirable behaviours, such as balancing the pendulum in an upright posture or tracing particular trajectories, by introducing control schemes like proportional-integral-derivative (PID) control, state feedback, or optimum control.

4.3 Stabilization Techniques for CIPS

4.3.1 Conventional Techniques

(1) Pole Placement Method

An inverted cart pendulum is frequently balanced using the pole placement approach. By positioning the poles of the closed-loop system at the required locations, a control system is created. The stability and performance of the system can be attained by carefully placing these poles.

Creating a mathematical model of the system is the first step in using the pole placement approach with an inverted cart pendulum.

The next stage is to choose the ideal pole positions for performance and stability once the model is available. The closed-loop system's response properties, such as settling time, overshoot, and stability, are determined by the positions of the poles, which stand in for the system's eigenvalues. Typically, the desired response criteria are used to determine the desired pole placements.

Using methods like state feedback or output feedback control, the control gains can be estimated after finding the ideal pole placements.

When using state feedback control, the control law is created using the system's complete state. In order to do this, it is necessary to gauge the cart's and the pendulum's positions and speeds. Using techniques like the Ackermann formula or pole placement techniques, the control gains are computed to place the poles at the proper locations. [37]

(2) LQR Method

LQR control is an optimum control strategy that plans system control gains while attempting to minimize a quadratic cost function. It offers a methodical approach to achieving performance and stability in the control of the pendulum-cart system.

Getting a linearized model of the system is the first step in using the LQR approach on the inverted cart pendulum. This linear model depicts a constrained area near the intended operating point where the dynamics of the system can be roughly described as linear. The pendulum-cart system's nonlinear equations are linearized around the chosen equilibrium point during the linearization procedure.

Following the availability of the linear model, the LQR control design proceeds as follows:

Creating Cost Function - Quadratic terms that penalize departures from the target states and control inputs often make up the cost function. The system's design objectives and limitations determine the cost function's precise form.

Solving the Algebraic Riccati Equation - The Algebraic Riccati equation must be solved in order to obtain the best control improvements. The gain matrix or control matrix is a symmetric positive-definite solution matrix that is determined by the Riccati equation, a matrix problem. The ideal control gains that minimize the cost function are obtained by solving this equation.

Control Input’s Calculation - A state-feedback control law can be used to determine the control input once the best control gains have been attained. To calculate the control input or force applied to the cart, the control law multiplies the system's present state by the control gain matrix. To keep the inverted cart pendulum balanced, the control input is continuously modified based on the measured or estimated states. [38]

4.3.2 Non -Conventional Technique

(1) Fuzzy Logic Controller

Its capacity to deal with nonlinearities and uncertainties makes it a useful method for balancing an inverted cart pendulum. The controller can capture and interpret human-like knowledge and behaviour in a methodical way thanks to fuzzy logic. To decide and produce control signals, the FLC uses linguistic variables, rules, and inference. [39]

The steps listed below are usually necessary to create a fuzzy logic controller for balancing an inverted cart pendulum:

Stating Input and Output Variable: Defining the input and output variables for the fuzzy logic control system is the initial stage.

Linguistic Variable Membership Functions: There are defined linguistic variable membership functions for each input and output variable. These membership functions describe the degree to which a certain value belongs to a given linguistic phrase, such as "negative large," "zero," or "positive small."

Fuzzy Rule Base: The mapping between the input variables and the output variable is defined by a set of if-then rules that make up the fuzzy rule base. Each rule selects a linguistic word for the output variable by combining terms from the input variables.

Fuzzy Inference: To find the right output, fuzzy inference applies the fuzzy rules to the input variables. The membership function values are combined using fuzzy logic operators like AND, OR, and NOT to create the output fuzzy set in fuzzy inference.

Defuzzification: The process of defuzzification, which comes to an end after defuzzification, turns the fuzzy output into a precise value that represents the force or control signal applied to the cart. Defuzzification can be accomplished using a variety of strategies, including centroid-based, maximum membership, and weighted average approaches.

(2) Particle Swarm Optimization

A population-based optimisation technique called Particle Swarm Optimisation (PSO) is inspired by the cooperative behaviour of fish schools and bird flocks. It has been widely used to solve a variety of optimisation issues, including the regulation of dynamic systems like the equilibrium of an inverted cart pendulum. [54]

The gains for the proportional, integral, and derivative (PID) controller as well as other control variables could be included in the control parameters for the cart inverted pendulum problem. The pendulum's balance and the cart's motion are measured by the fitness function, which assesses each particle's performance.

Based on their own best-known position and the collective best-known position of the entire swarm, particles update their velocities and positions at each iteration. Both the particle's own experience and the experiences of its neighbours have an impact on the velocity update. By changing the particle velocities and allowing them to converge towards promising places, the programme traverses the search space. [53]

The PSO algorithm iterates until a termination condition, such as completing a predetermined number of iterations or reaching a specific level of fitness, is satisfied. The cart inverted pendulum system's optimal set of control parameters is represented by the final, best-known position.

4.3.3 PID Controllers: A brief Overview

The most popular type of feedback currently in use is the PID controller. PID controllers are employed in most of the industrial applications, according to estimates. The PID controller family is correctly referred to be the foundation of control theory because of its simplicity and simplicity of execution.

If numerous (and frequently competing) objectives are to be accomplished, designing and adjusting a PID controller can be challenging in practise. When it comes to system complexity, a traditional PID controller with set parameters typically produces subpar control performance. The typical PID controllers produce sub-optimal corrective actions and so necessitate frequent tuning adjustments because the system's gain and time constants change with the operating conditions. As a result, engineers are encouraged to create tools that will help them achieve the optimum overall PID control across the whole operating range of a particular process.[40]

In PID, by continuously gauging the discrepancy between the desired reference value and the system's actual state, the PID controller controls the system. The following three steps are used to process the error signal:

<u>Proportional (P) Component</u> : Component that is directly proportional to the error signal is known as the proportional (P) component. It contributes to the effort being made to control the error by trying to make it smaller. The power of the proportional action is determined by the proportional gain (K_p). A stronger reaction to the present mistake is produced by raising K_p, however oscillations or overshoot may also be introduced.

<u>Integral (I) Component</u>: The integral term eliminates steady-state errors by integrating the error across time while taking into account prior errors. On the basis of the accumulated error, it adds a corrective action. The response to the integral of the error signal is determined by the integral gain (K_i). Higher values of K_i boost the integral action, but too much of a good thing might cause instability or noise amplification.

<u>Derivative (D) Component</u>: The derivative term takes into account the error signal's rate of change. It functions as a damping mechanism, lowering overshoot and bringing the response into stability. The strength of the derivative action is determined by the derivative gain (K_d). Higher K_d values improve damping, but they could also exacerbate high-frequency noise.

The role of three components in different aspects can be summarised as below :-

PID Parameters	**Rise Time**	**Overshoot**	**Settling Time**	**Steady State Error**
$\mathbf{K_p}$	Decrease	Increase	Small Change	Decrease
$\mathbf{K_i}$	Decrease	Increase	Increase	Eliminate
$\mathbf{K_d}$	Small Change	Decrease	Decrease	Small Change

Table 4.2 Effects of K_p , K_i and K_d on Closed-loop Response

These three elements are added together to provide the PID controller's control output:

Input Control = K_p * Error + Ki * Error's Integral + K_d * Error's Derivative

The error is commonly defined as the difference between the desired angle (_desired) and the actual angle (_actual) of the pendulum when using the PID controller to compensate the cart-inverted pendulum system. The control force (F control) is then applied to the cart using the PID controller's control output.

The PID controller actively stabilises and controls the pendulum by continuously adjusting the control force based on the error signal. The PID controller can offer reliable and stable control by properly tuning the gains (K_p, K_i and K_d), allowing the cart-inverted pendulum system to follow specified trajectories, maintain balance, and withstand disturbances.

The PID gains must be tuned in order to achieve satisfactory performance. To find the right gain values for the particular system, a variety of techniques can be used, including trial-and-error, Ziegler-Nichols, or sophisticated optimisation techniques. [41]

Sl No.	Tuning Technique	Advantages	Disadvantages
1.	Ziegler-Nichols Method	• Simple and widely used • Requires only a few systems test • Provides a good starting point for tuning	• Can lead to oscillations or instability • Limited applicability to certain systems • Lack of precision in tuning parameters
2.	Cohen-Coon Method	• Provides relatively quick and easy tuning • Suitable for processes with a known time delay • Moderate level of accuracy	• May not work well for complex or nonlinear systems • Limited effectiveness if time delay is unknown or varies
3.	Tyreus- Luyben Method	• Suitable for both stable and unstable- system • Provides a good balance between overshoot and settling time • Less conservative than other methods	• Requires a higher level of computational effort • May not work well for highly nonlinear systems
4.	Relay Auto-Tuning Method	• Simplicity and ease of implementation • Can be used for a wide range of systems • Provides good results for stable and linear systems	• May cause oscillations or instability if not properly implemented • Limited effectiveness for complex or nonlinear systems
5.	Genetic Algorithms	• -Suitable for complex and nonlinear systems • -Can handle multiple objectives and constraints • Offers global search capability	• Requires extensive computational resources • Longer optimization time compared to other methods • Dependent on parameter selection and fitness evaluation accuracy

6.	Particle Swarm Optimization (PSO)	• Fast convergence compared to other optimization methods • Suitable for both continuous and discrete parameter tuning • Robust performance in the presence of noise	• Can get trapped in local optima • Requires careful selection of swarm size and iteration limits • May converge to suboptimal solutions in certain cases

Table 4.3 Tuning techniques used in control system [41], [42], [43]

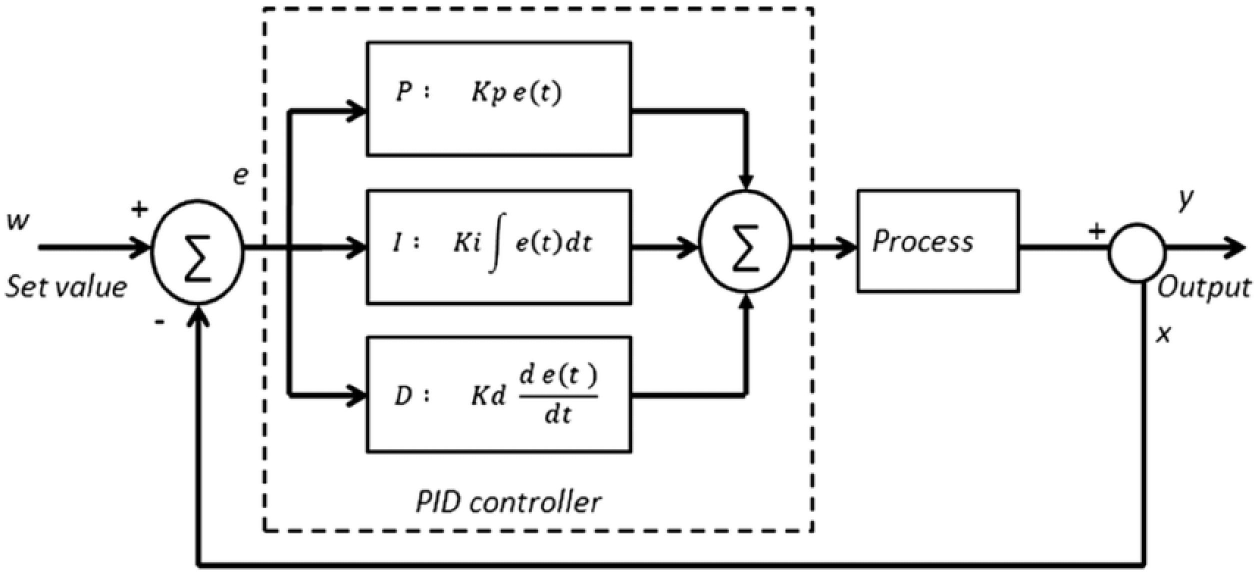

Fig. 4.7 Block diagram of PID closed loop control system [49]

4.3.4 Is Single PID Controller enough?

The double PID controller consists of two sets of PID parameters, one used to control the position of the cart (cart PID) and the other to regulate the pendulum's angle (pendulum PID). [3]

A double PID controller is required in the Cart Inverted Pendulum system for the following reasons:

Decoupling Control: In the coupled, nonlinear Cart Inverted Pendulum system, the pendulum's motion is affected by the cart's movement and vice versa. Decoupling control can be accomplished by utilising a double PID controller, one for the cart and one for the pendulum. The pendulum PID controller concentrates on balancing the pendulum, while the cart PID controller is concerned with stabilising the position of the cart. The overall control approach is made simpler and system performance is increased by separating the control jobs.

Individual Control: The dynamics and control need of the cart and the pendulum are dissimilar. While the pendulum's motion is nonlinear and unstable, the cart's motion is normally controlled by linear dynamics. The control settings can be independently modified to meet the needs of each component by utilising separate PID controllers. Better control performance and stability are made possible as a result.

Stability and Robustness: The Cart Inverted Pendulum system is more stable and robust thanks to the double PID controller. In order to stabilise each PID component, it must take into account the intrinsic dynamics and features of that component. The system's stability margins can be increased by separately operating the cart and pendulum, making it more resistant to disturbances, noise, and parameter uncertainty.

Optimal Control and Tuning: The double PID controller offers flexibility in setting the control parameters for the cart and pendulum separately for the best performance. The control settings for the cart and pendulum may need to be changed differently in order to get the best control because of their different mass, length, and dynamics. The tuning procedure can be customised to each component with individual PID controllers, improving control performance.

There are trade-offs between cart position and pendulum angle control in the Cart Inverted Pendulum system. The pendulum PID controller primarily focuses on balancing the pendulum, while the cart PID controller largely focuses on stabilising the cart's location. The trade-offs between position and angle control can be more effectively managed by using independent controllers, enabling more precise control and attaining the ideal balance between cart stability and pendulum angle control. [44]

In conclusion, the Cart Inverted Pendulum system has benefits due to the use of a double PID controller because it enables decoupling control, separate control of the cart and pendulum, increased stability and robustness, optimal control tuning, and better management of performance trade-offs. These advantages aid in obtaining efficient control and preserving the inverted pendulum’s equilibrium.

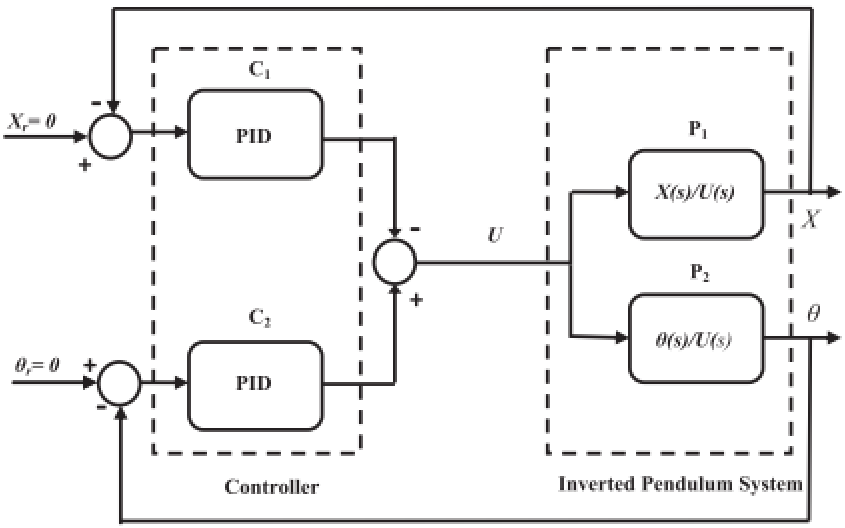

Fig. 4.8 Indicative Control Scheme for Two-Loop PID Controller [50]

4.4 PID Controller for CIPS: Genetic Algorithm approach

Although real-time implementation of neural networks is relatively challenging, they have been used in recent years to address system difficulties including nonlinearities. The scholarly community and business have both begun to pay optimisation algorithms more attention. The benefit of optimisation algorithms over neural controllers is that the former can be quickly and easily included into PID tuning. [15]

When a predetermined criterion is optimised, the control design is referred to as "optimal control". Optimality only applies to the specific criterion at hand, and the actual performance depends on how well the selected criterion fits the situation. [17]

Numerous applications in control engineering use genetic algorithms (GA), a subset of the family of evolutionary computational algorithms. They are strong optimisation algorithms that operate on a population, or collection, of candidate solutions. Through collaboration and competition among the different alternatives, GA determines the best answer. Because these algorithms can handle issues with nonlinear restrictions, numerous objectives, and dynamic components—properties that frequently emerge in real-world situations—they are extremely useful for industrial applications.

Here, GA is used to determine the best value for the PID controller's K_p, K_i, and K_d parameters.

The goal function, which in this case is the error criterion, is minimised by adjusting the values of all feasible sets of controller parameter sets, and it is thoroughly explored. It is made sure that estimated controller settings for the PID controller design provide a reliable closed-loop system.[45]

While proceeding through tuning of PID using GA we need to consider these things :-

4.4.1 Initialization of Parameters

Certain settings must be defined before using GA. These factors include population size, chromosome bit length, iterations, selection, crossover, and mutation kinds, among others. The ability of the designed system is greatly influenced by the choice of these factors.

4.4.2 Objective Function for GA

Writing the objective function is the most difficult element of designing a genetic algorithm. The objective function is necessary in this project to assess the system's best PID controller. To discover the PID controller with the smallest overshoot, quickest rise time, or quickest settling time, an objective function could be developed. But it was determined to create an objective function that will minimise the inaccuracy of the controlled system in order to combine all of these goals. When objective function is put under minimisation, then it is put under category of Cost function. There are numerous such criteria available. [46]
Below is the list of different cost functions used with reference to control system :-

Cost Function	Description
Mean Square Error (MSE)	Measures the average squared difference between the desired output and the actual output of the controlled system.
Integral Square Error (ISE)	Calculates the sum of squared errors over the entire time horizon of the control system, emphasizing both magnitude and duration of errors.
Integral Absolute Error (IAE)	Computes the sum of absolute errors over the entire time horizon, focusing on reducing the absolute error magnitude.
Integral Time-weighted Absolute Error (ITAE)	Incorporates a time weighting factor in the integral term of the IAE cost function, balancing the trade-off between magnitude and duration of errors.
Settling Time	Measures the time required for a system's response to reach and remain within a specified range around the desired setpoint.
Overshoot	Quantifies the maximum deviation of the system's response from the desired setpoint before stabilizing.

Table 4.4. Different cost functions used in control system

Performance requirements for modern complicated control systems are typically more complex than those described thus far. Both the inaccuracy and the time are crucial elements that must be taken into account. A performance index is a lone metric for a system's efficiency that emphasises the aspects of the response that are seen as crucial. There are several performance metrics like ISE, IAE, etc. but for penalising long duration transients, ITAE is considered a good choice.
So, in this study, the performance of the controller is assessed using integral time absolute error (ITAE) error criterion. The integral of time multiplied by absolute error (ITAE) equation yields the error criterion as a measure of performance index.

Mathematically, ITAE is define as:

$$\text{ITAE} = \int_0^\infty |e(t)|\ dt$$

The goal function receives one chromosome at a time from the population. The chromosome is then rated for fitness and given a number; the higher the number, the fitter the chromosome is. The genetic algorithm builds a new population of the fittest individuals using the fitness value of the chromosome. Below is the picture from Simulink how ITAE is:

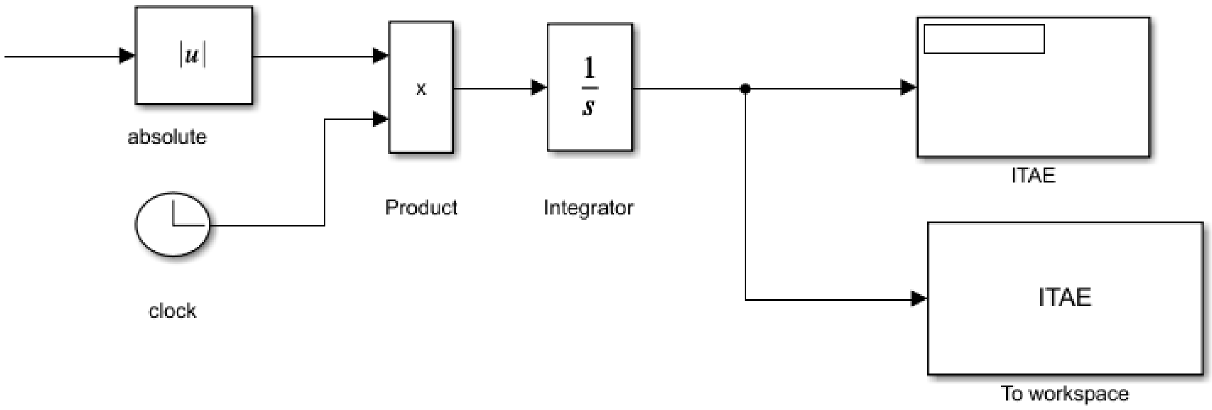

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Fig.4.9 Simulink representation of ITAE

4.4.3 Termination Criteria

Genetic algorithms (GAs) need termination criteria to know when to stop iterating and take the current result as the solution's outcome. If there were no criteria for ending iterations, the GA would keep running indefinitely, using up computational resources and possibly over-optimizing the result. Termination criteria offer a stopping condition that aids in regulating the algorithm's execution and ensuring its viability and effectiveness. [27]

The optimisation method can be stopped either when the maximum number of generations has been reached or when a suitable fitness value has been obtained. Since we are using an objective function, the fitness value in this situation is simply the objective function's magnitude. Since, we have considered about minimising the objective function. The achievement of a good fitness value, which takes place after the greatest number of generations, is regarded as the termination criterion in this case.

There are also some other termination criteria like stall generations, etc.

In this work, the maximum no of generations has been chosen between 10-70 to get better results. Large no of generations will lead to consumption of time.

4.4.4 Setting the boundary values

Setting boundary values is a crucial component of Genetic Algorithm (GA)-based PID (Proportional-Integral-Derivative) controller tuning. The boundary values specify the permissible range for the PID controller's parameters to change during the optimisation process. [47]

The conventional method for setting boundary values in a PID controller's GA tuning is described as follows:

Proportional Gain (K_p): The gain in proportion to error defines the control response. One must take into account the particular control system in use as well as the anticipated range of values for the proportional gain when establishing K_p's boundary values. The boundary values for K_p are typically selected to span a wide range of potential gains suited for the current control situation. For instance, the boundary values for K_p may be significantly larger for a control system with quick response times, compared to lower boundary values for slower systems. Thus, on the basis of response after running, the boundary set for K_p can be modulated.

Integral Gain (K_i): The integral gain helps to eliminate steady-state errors by accounting for the error that has accumulated over time. It is critical to take into account both the potential effects of integral wind-up and the time scale of the control system when determining K_i's boundary values. When the integral term builds up big values, it experiences integral wind-up, which can cause overshoot or instability. K_i's boundary values ought to be set to fall within a practical range that permits efficient error correction without resulting in integral wind-up.

Derivative Gain (K_d): Based on the error's rate of change, the derivative gain offers a control response. The amount of noise present in the system and the intended response characteristics should be taken into account when determining the boundary values for K_d. Higher K_d values could be needed if the system is subject to a lot of noise or quick changes. On the other hand, lower boundary values for K_d may be adequate for stable and low-noise systems.

The challenge at hand and the dynamics of the controlled system determine the precise range for the boundary values of each parameter. Typically, it is defined using subject expertise, prior information, or trial and error. It's crucial to make sure the bounds are established to encompass a broad enough range for the GA to efficiently examine various PID parameter value combinations.

It's also important to remember that while using the GA, the proper enforcement mechanisms must be in place so that the boundary values are upheld throughout the optimisation process. This makes sure that the algorithm does not produce people with parameter values outside the boundaries that are defined.

For effective GA tuning, the PID parameter boundaries must be set properly. It enables the algorithm to efficiently explore the search space, balance exploration and exploitation, and converge to a PID controller configuration that is optimal or nearly optimal for the specified control issue.

In this thesis, two PID controllers have been used so there are a total of 6 parameters to be optimised. Thus, there will be two boundary vectors, each with 6 set of values. The two vectors here represent lower boundary values and upper boundary values.

4.5 Choice of best GA parameters: A dilemma

Choosing the right initial set of parameters for genetic algorithms can have a big impact on the algorithm's performance and convergence. Even if there is no one solution that works for every process, following some rules can help you choose some basic parameters that are widely accepted to be efficient.

Here are some essential criteria to take into account:

<u>Population Size</u> : While a larger population allows for more solution space exploration, it could also make computing more complex. A population size of 50–100 is frequently a

reasonable place to start, although it also depends on the problem's complexity and the available processing power.[16]

Selection Strategy : Popular techniques include rank-based selection, selection through a roulette wheel, and selection through tournaments. As it achieves a balance between exploration and exploitation, tournament selection is frequently a wise first move.

Crossover Rate : While a lower crossover rate favours exploitation, a larger crossover rate encourages exploration. It is typical to start with a figure between 0.6 and 0.9, however depending on the issue, this may need to be changed.

Mutation Rate : It regulates the likelihood that a gene on a person's chromosome will experience a chance alteration. New genetic material is introduced through mutation, which also delays premature convergence. A low mutation rate, like 0.01-0.1, is frequently a good place to start.

Termination Criteria : The algorithm's termination criteria specify when it should end. A common criterion is when the improvement in solutions falls below a predetermined level, after a predetermined number of generations, or both. Setting termination standards that permit enough exploration and shield against hasty termination is crucial.

It's crucial to remember that the initial parameter values listed above are approximate. To get the most performance out of genetic algorithms, parameter adjustment is frequently necessary. This can be accomplished by conducting empirical tests and experiments, systematically adjusting the algorithm's parameters, and assessing the algorithm's effectiveness on a sample set of problem occurrences.

Consider the problem's characteristics, available computing power, and optimisation objectives carefully before choosing the appropriate parameters for a genetic algorithm. The most successful method for achieving effective parameter settings typically involves starting with suitable baseline values and iteratively fine-tuning them through testing.

A suitable baseline values to start the GA process can be summarised in table as below**:**

Parameters	**Preferred Value**
Population Size	50 - 100
Selection Strategy	Tournament Selection
Crossover Rate	0.6 – 0.9
Mutation Rate	0.01 – 0.1
Termination Criteria	Max. No. of generation (generally 80 – 100)

Table 4.5 Preferred values for Parameters of GA

There is also a trade-off between Crossover rate and Mutation rate which depends on the population size.[48]

This can be summarised in below table -

Population Size	Crossover Rate	Mutation Rate
Large (=100)	0.6	0.001
Small (=30)	0.9	0.001

Table 4.6 Trade-off between Crossover Rate and Mutation Rate

4.6 Implementing GA based PID controller for CIPS

4.6.1 Procedure of Implementation

(1) A simulink model was developed for cart inverted pendulum system (CIPS).
(2) For control balance, Double PID controller is being used (one for position control and another for pendulum's angle control).
The parameters of PIDs are kept with variable name as k(1), k(2)…..k(6).
(3) ITAE is developed in simulink which will act as objective function (cost function).
(4) An impulse force is provided
(5) MATLAB code is developed for importing the cost function.
(6) Now, with different GA parameters GA toolbox is run.
(7) Upper bounds and lower bounds are set accordingly with different run and trials.
(8) The PID values are obtained after termination of GA process.
(9) Comparison analysis is done after running with different boundary values and GA options.

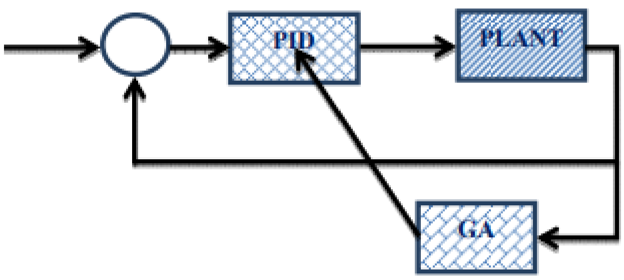

Fig.4.10 Indicative Diagram for tuning Process applied to PID [51]

Chapter 5

Simulations and Results

5.1 Simulations

As defined in the section 4.6.1, on application of GA in CIPS, the GA based tuning process for PID controller is carried out. Based on the different research papers, the tuning process is started with some well-known start parameters. After a number of trials, on the basis of observation a suitable set of upper and lower bounds are found.

Simulation results are even obtained for some different GA parameters to see how the results change.

The GA options which are kept common during all the GA process is given below

Population Size	50
Population Type	Double Vector
Creation Function	Uniform
Fitness Scales	Rank
Elite Count	0.05 * Population Size
Maximum Generation	50
Mutation Function	Uniform
Mutation Rate	0.001
Crossover Fraction	0.8
Stall Generation	10
Function Tolerance	$1*e^{-6}$

Table 5.1 Common GA options for all simulation process

5.1.1 Simulation Result for Roulette Selection Function

The other GA options are kept intact as mentioned. Only the upper and lower bound values were varied.

Less Strict Bound Values

The simulation results are obtained for boundary values with [10 10 5 10 10 5] and [200 200 200 100 100 100].

ITAE Value	For x			For Θ		
	K_p	K_i	K_d	K_p	K_i	K_d
600.8	168.44	185.75	177.31	23.13	74.69	60.71

Table 5.2 ITAE, K(x) and K(Θ) values obtained for Roulette Selection Function (less stricted bounds)

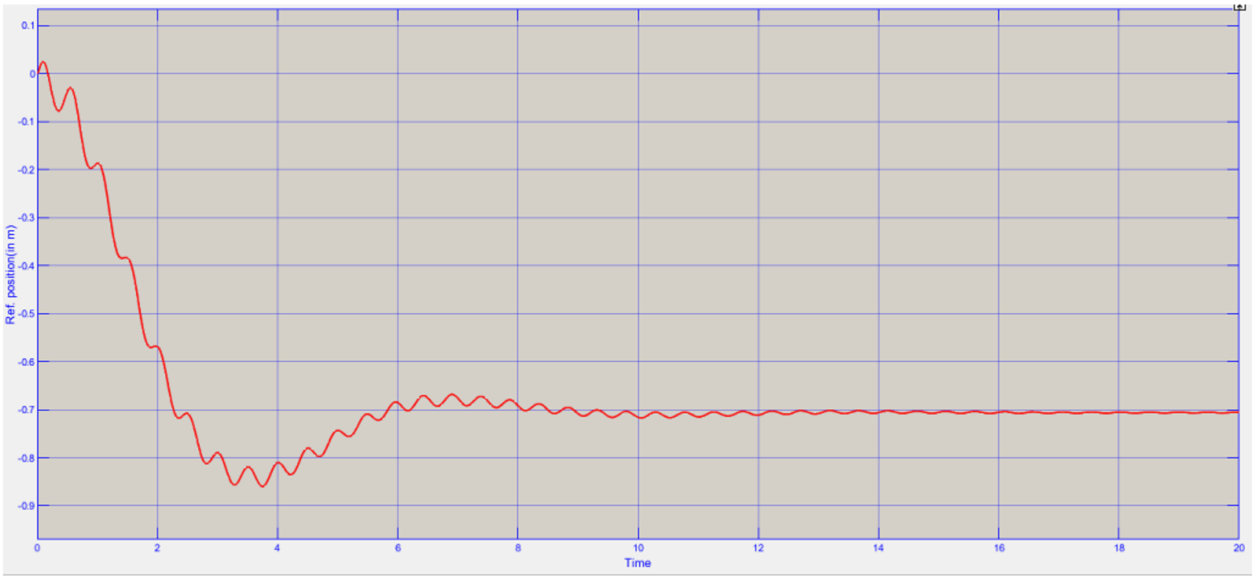

Fig. 5.1 Cart's position for Roulette Selection (less stricted bounds)

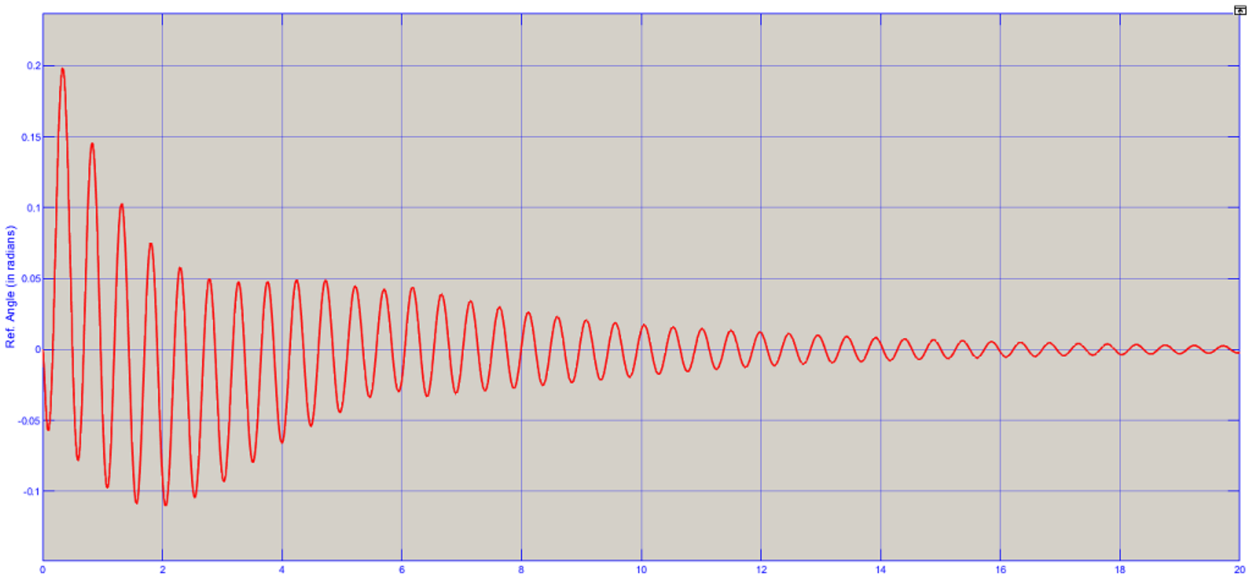

Fig. 5.2 Pendulum's angle for Roulette Selection (less stricted bounds)

From the above two, the time domain analysis can be summarized as table as below-

	Rise Time	Overshoot	Settling Value & Time
For x	2.284 s	**1st Overshoot** 0.087 at 0.02 s **1st Undershoot** 0.078 at 0.36 s	0.7 m at 14.49 sec
For ϴ	0.166 s	**1st Overshoot** 0.2 rad at 0.32 s **1st Undershoot** 0.057 rad at 0.09 s	19.5 s

Table 5.3 Simulation analysis for Roulette Selection Function (less stricted bounds)

If we look over the control effort in the above case, this could be seen as in the below graph-

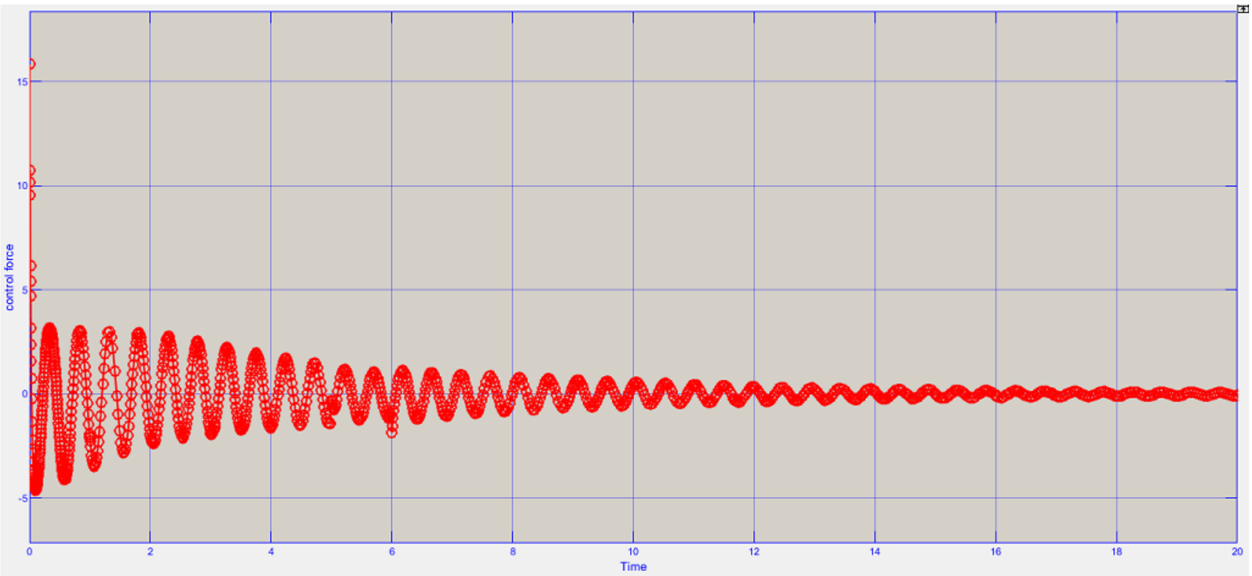

Fig. 5.3 Control force applied for Roulette Selection (less stricted bound)

Stricter Bound Values

The simulation results are obtained for GA parameters as-

Creation Function	:	Roulette
Lower Bound	:	[100 100 100 10 25 25]
Upper Bound	:	[500 500 500 100 250 250]
Crossover Function	:	Single Point

ITAE Value	For x			For Ɵ		
	K_p	K_i	K_d	K_p	K_i	K_d
310.89	396.96	296.49	106.17	36.66	206.57	37.48

Table 5.4 ITAE, K(x) and K(Ɵ) values obtained for Roulette Selection Function (stricted bounds)

The time domain response for cart's position as final output is obtained.

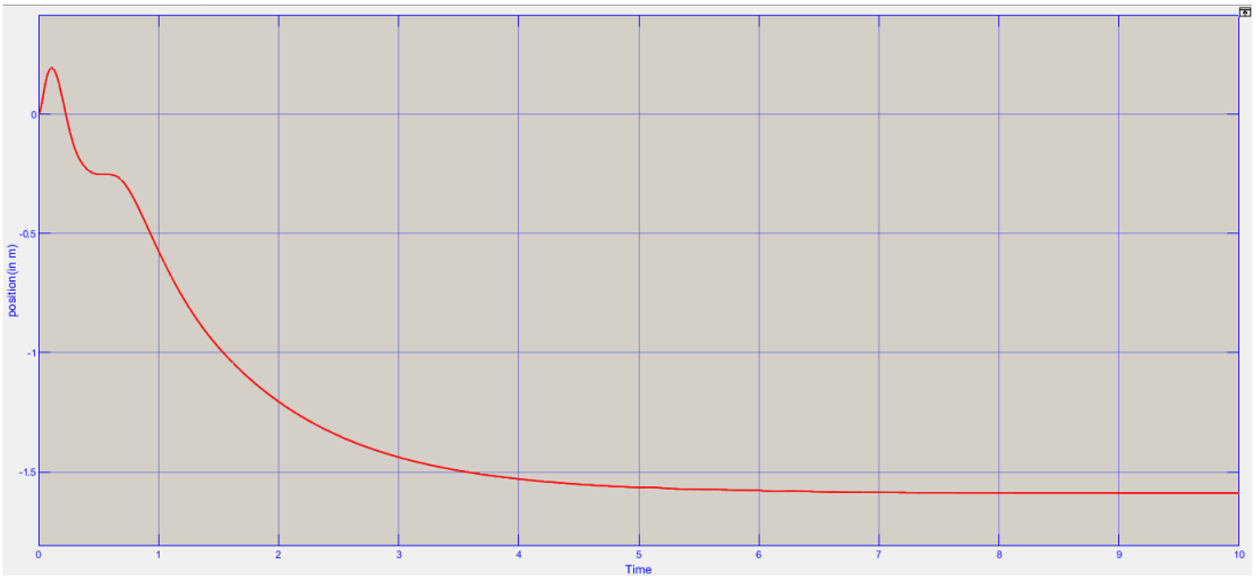

Fig. 5.4 Cart's position for Roulette Selection (stricted bounds)

Similarly, the response for pendulum's angle is obtained as-

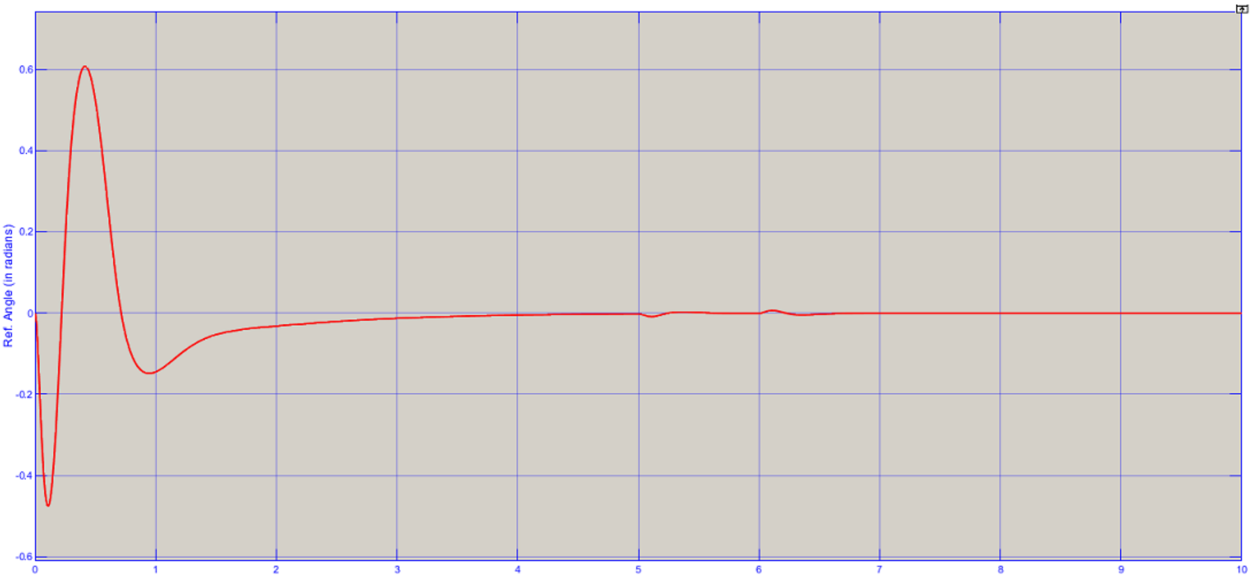

Fig. 5.5 Pendulum's angle for Roulette Selection (stricted bounds)

From the above two, the time domain analysis can be summarized as table as below-

	Rise Time	Overshoot/Undershoot	Settling Value & Time
For x	5.98 s	**Only One Overshoot** 0.1944 at 0.112 s **No Undershoot**	1.78 at 7.64 s
For ϴ	0.25 s	**1st Overshoot** 0.6 rad at 0.4 s **1st Undershoot** 0.474 at 0.112 s **2nd Undershoot** 0.14 rad at 0.9 s	0.006 radian at 6.74 s

Table 5.5 Simulation analysis for Roulette Selection Function with stricted boundary

If we see the control effort in the above case, this could be seen as in the below graph-

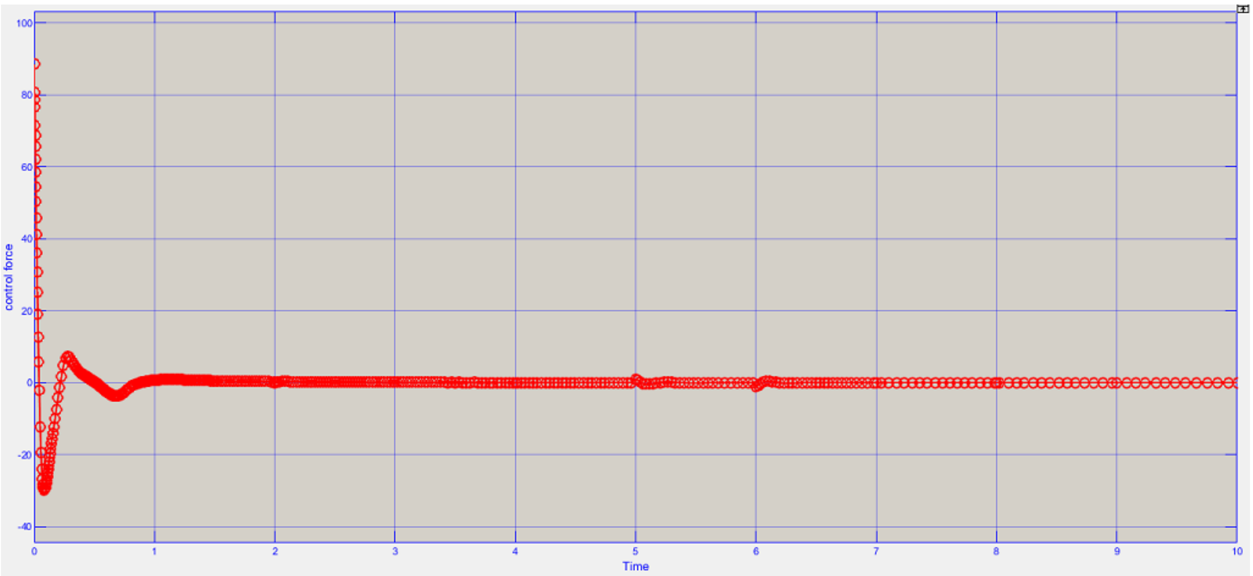

Fig 5.6 Pendulum's angle for Roulette Selection (stricted bounds)

5.1.2 Simulation Result for Tournament Selection Function

The bound values are kept the stricter one as result is better obtained in this case.

This shows our approach towards optimal tuning. Simulation result is obtained for Tournament Selection Function with two variations –

(1) **Single Point Crossover**

(2) **Two Point Crossover**

Single Point Crossover

The GA parameters for this case are as follows

Selection Function : Tournament

Lower Bound : [100 100 100 10 25 25]

Upper Bound : [500 500 500 100 250 250]

Crossover Function : Single Point

ITAE Value	For x			For Ɵ		
	K_p	K_i	K_d	K_p	K_i	K_d
415.36	426.33	377.12	127.8	83.68	61.541	44.47

Table 5.6 ITAE, K(x) and K(d) values obtained for Tournament Selection Fcn. with Single Point Crossover

The time domain response for cart's position as final output is obtained

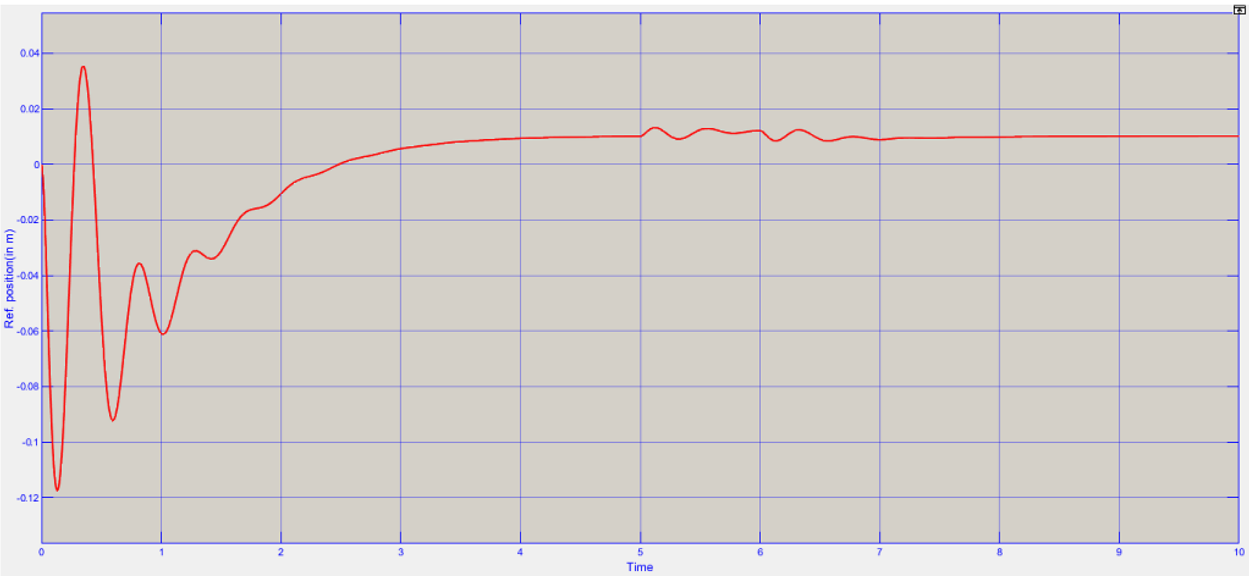

Fig. 5.7 Cart's position for Tournament Selection (Single Point Crossover)

Similarly, the response for pendulum's angle is obtained as-

Fig. 5.8 Pendulum's angle for Tournament Selection (Single Point Crossover)

From the above two, the time domain analysis can be summarized as table as below-

	Rise Time	Overshoot/Undershoot	Settling Value & Time
For x	0.27 s	1st Undershoot 0.1165 at 0.142 s 1st Overshoot 0.0353 at 0.347 s	0.01 at 8.22 s
For Ɵ	0.236 s	1st Overshoot 0.274 at 0.13 s Other Shoots are of less value	7.37

Table 5.7 Time domain analysis for Tournament Selection Fcn. with Single Point Crossover

If we look over the control effort-

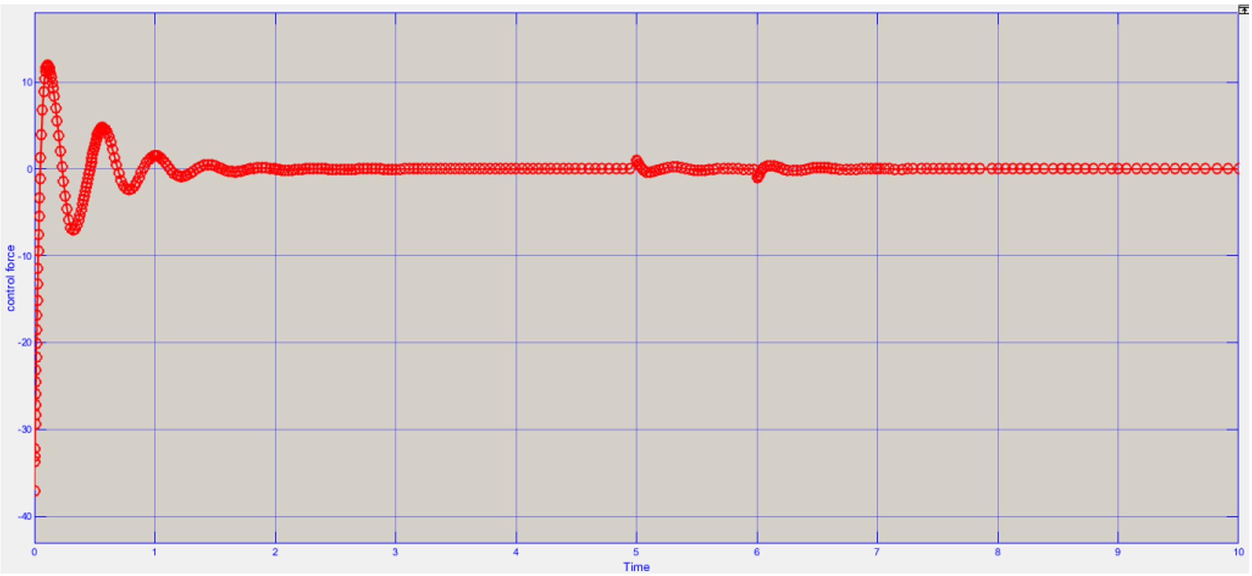

Fig. 5.9 Control Force for Tournament Selection (Single Point Crossover)

Two Point Crossover (Tournament Selection)

The GA options taken for this case are as-

Selection Function :	Tournament
Lower Bound :	[100 100 100 10 25 25]
Upper Bound :	[500 500 500 100 250 250]
Crossover Function :	Two point

ITAE Value	For x			For Ɵ		
	Kp	**Ki**	**Kd**	**Kp**	**Ki**	**Kd**
410.32	406.21	121.58	361.63	40.39	236.28	136.19

Table 5.8 ITAE, K(x) and K(d) values obtained for Tournament Selection Fcn. with Two Point Crossover

The time domain response for cart's position as final output is obtained

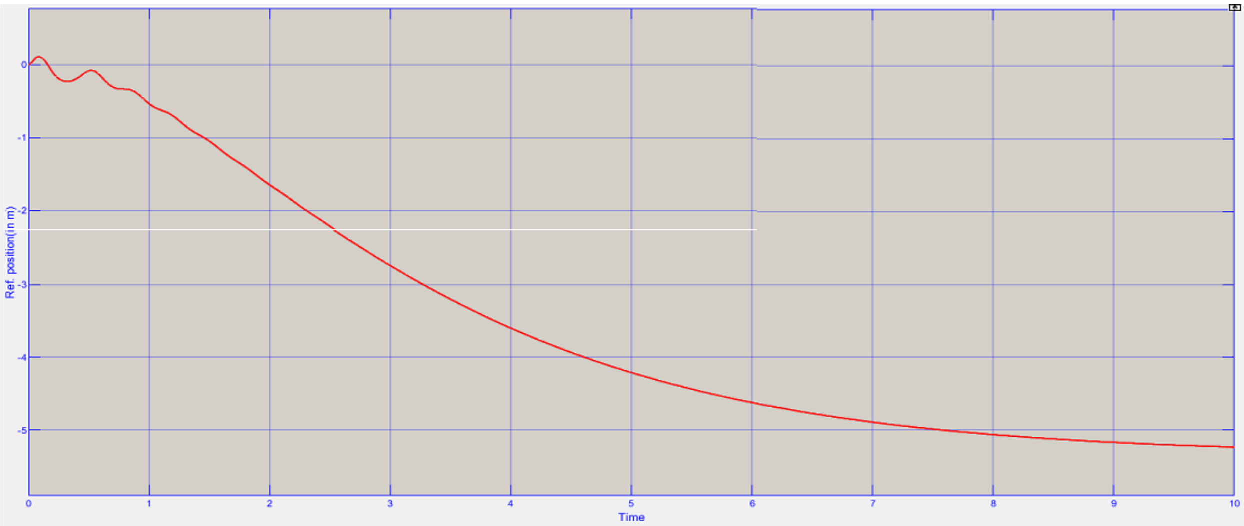

Fig. 5.10 Cart's position for Tournament Selection (two Point Crossover)

Similarly, the response for pendulum's angle is obtained as-

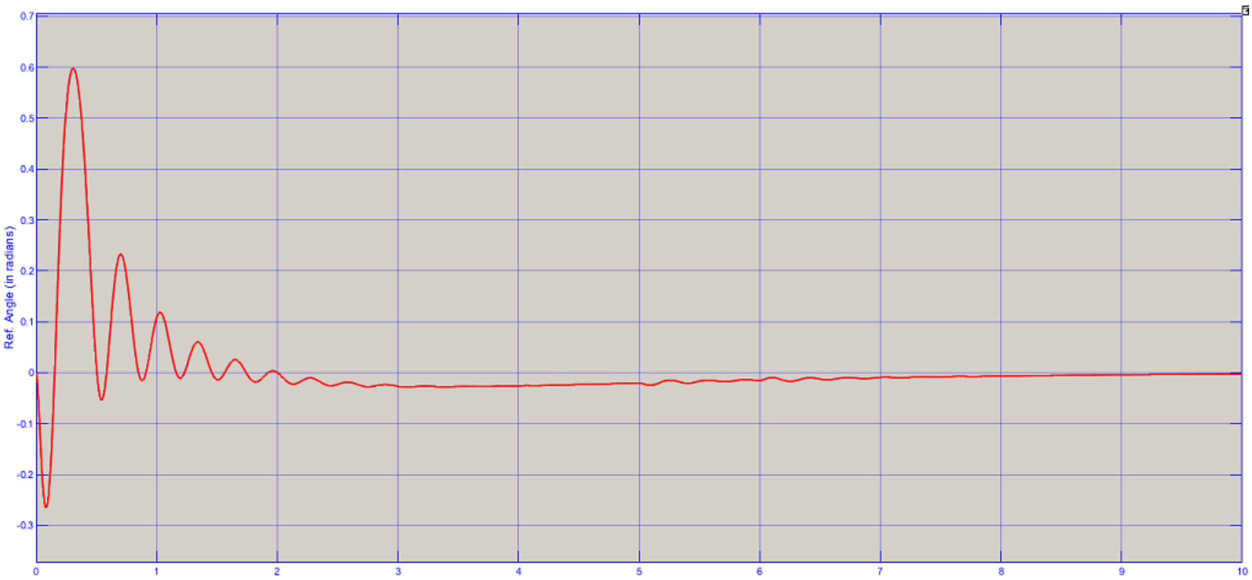

Fig. 5.11 Pendulum's angle for Tournament Selection (Two Point Crossover)

If we analyse the above two graph, we get analysis as-

	Rise Time	Overshoot/Overshoot	Settling Time & Value
For x	15.33 s	**1st Overshoot** 0.01 at 0.1 sec **1st Undershoot** 0.03 at 0.32 sec	17.64 s
For Ɵ	0.155 s	**1st Overshoot** 0.6 rad at 0.3 s **1st Undershoot** 0.02 rad at 0.08 s	9.43 s

Table 5.9 Time domain analysis for Tournament Selection Fcn. with Single Point Crossover

Looking over the control effort in this case-

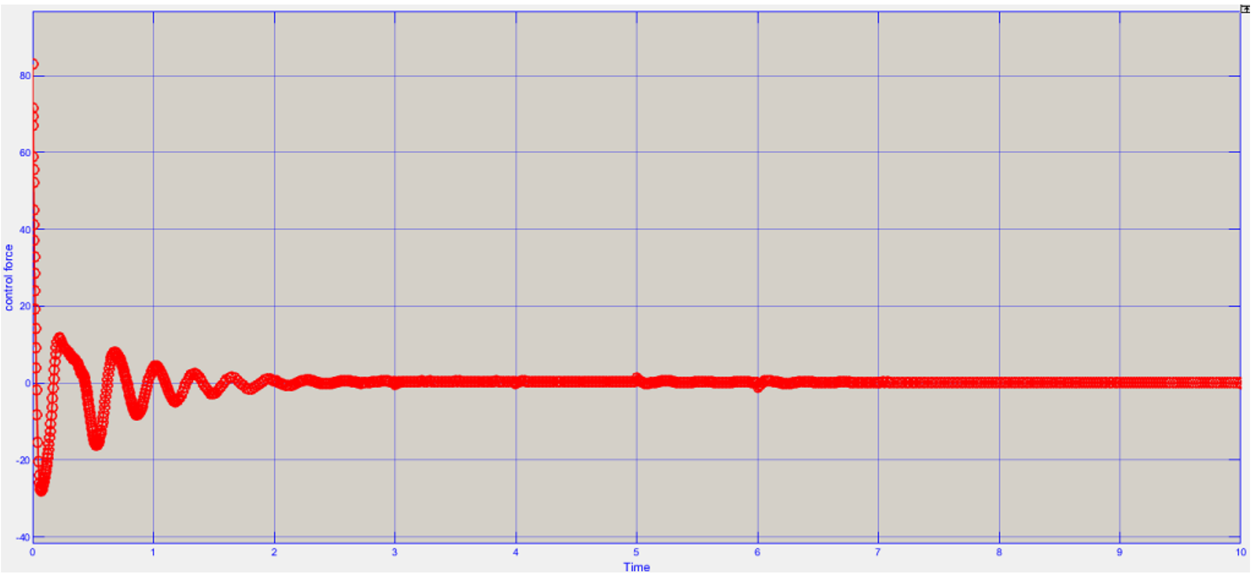

Fig. 5.12 Control Force for Tournament Selection (Two Point Crossover)

5.2 Comparative Analysis for different GA parameters

i. During process of optimal tuning, i.e, like changing selection function, changing the bound values, it is found that GA works better in case of less bound gap.
   When strict boundary was applied in case of Roulette, the better response was obtained as overshoot decreased, settling time decreased.

ii. <u>Comparison between Tournament & Roulette Selection Function</u>
    In case of tournament:
    -For pendulum's angle, settling time increased but overshoot decreased
    -Rise time decreased
    -ITAE value is more than that for Roulette Selection

iii. <u>Comparison between Single Point & Two Point Crossover Function</u>
    In case of Two point for pendulum's angle
    - Settling Time increased
    - Overshoot increased
    - Rise Time increased
    - ITAE value for Two point is less than that for Single point Crossover

5.3 Comparative Analysis of GA with LQR

Comparison between control technique by LQR is done with GA based PID for pendulum's angle, we get the following conclusions from figure below-

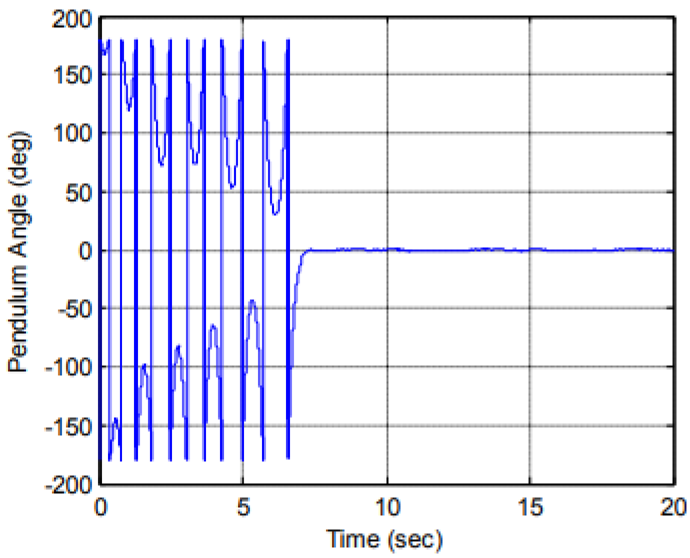

Fig. 5.13 Pendulum's angle obtained by LQR technique [52]

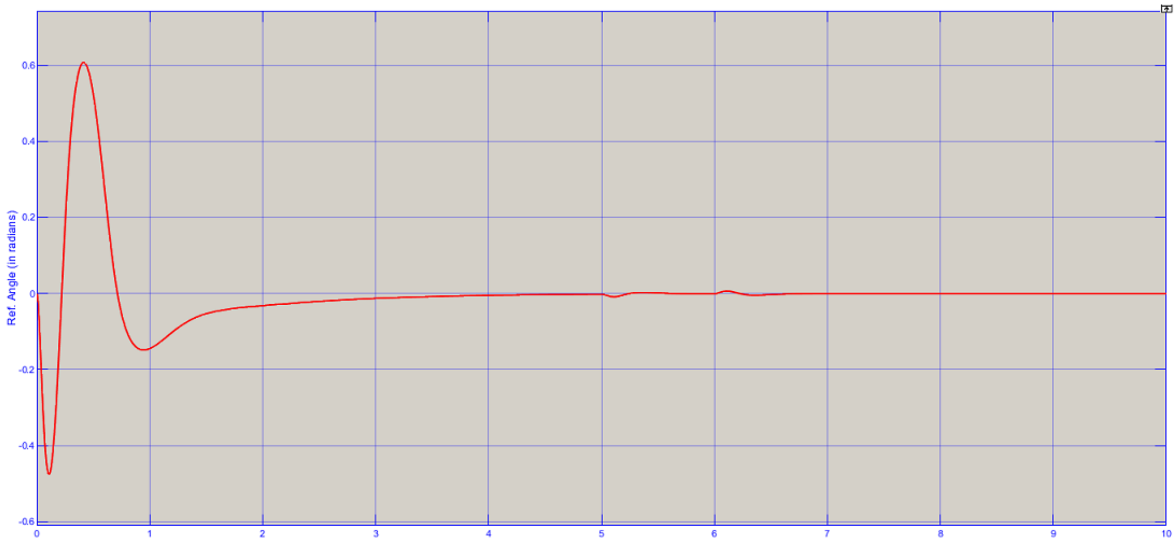

Fig. 5.14 Pendulum's angle obtained by GA based PID

It could be observed that GA based PID technique yields better result.

As GA based PID technique provides-

(1) Less overshoot
(2) Less settling time
(3) Smooth response

Chapter 6

Conclusion

6.1 Contribution of the thesis

The following topics has been explored in this:

i. Proposed double PID controller: The thesis suggests using a double PID controller to regulate an inverted pendulum system on a cart. This arrangement enables independent control of the angle and location of the pendulum, allowing for more accurate and steady control.
ii. Optimal Tuning with genetic algorithms (GAs): The thesis uses genetic algorithms as an optimization tool to fine-tune the double PID controller's parameters. GAs are effective at finding the best solutions to difficult optimization problems by efficiently scanning the parameter space.
iii. Non-linearity of the system was taken into account while designing the proposed controller. This helped to understand whether GA could be applied to non-linear system or not. It may be concluded that this scheme has the potential for implementing the optimized double PID controller in real-world applications, such as robotics, control systems, or mechatronics.

Thus, using the strength of evolutionary algorithms to fine-tune the double PID controller, the thesis hopes to offer an optimised control solution for the cart inverted pendulum system by making these contributions. By outlining a practical method for managing complicated, nonlinear systems, the study advances knowledge of control system optimisation approaches and advances the discipline of control engineering.

6.2 Scope of future work

The following topics can be further investigated and worked upon in the future:

i. Investigating the use of multi-objective genetic algorithms (MOGAs) to simultaneously optimize a number of the PID controller's performance parameters, including rise time, overshoot, settling time, and control effort. A more complete and balanced tuning solution may result from this.

ii. To utilize the advantages of various algorithms and combine them with other optimization approaches, such as particle swarm optimization (PSO) or simulated annealing, to increase the effectiveness and efficiency of the tuning procedure.

iii. Analysis of robustness: When evaluating the resilience of the GA-tuned PID controller, taking into account the system's uncertainties, disturbances, and parameter fluctuations. Conducting a sensitivity study to assess the controller's performance under various operating circumstances and pinpoint potential improvement areas.

iv. Implementing the GA-tuned PID controller in real time on an actual inverted cart pendulum system or a platform for real-time simulation to assess how well it performs in practical applications.

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