Portfolio Optimization Using Sharpe Ratio with Particle Swarm Optimization Algorithm

A Thesis submitted in partial fulfillment for the

Degree of **Master of Computer Application** of

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By

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I hereby declare that this thesis entitled "**Portfolio Optimization Using Sharpe Ratio with Particle Swarm Optimization Algorithm**" contains literature survey and original research work by the undersigned candidate, as part of his Master of Computer Application studies. All information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by thesis rules and conduct, I have fully cited and referenced all materials and results that are not original to this work.

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ABSTRACT

Portfolio Optimization is the process to select the best portfolio out of all the available portfolios according to some objective. It is a very interesting as well as important field of the financial sciences. In this report we are using a meta-heuristic algorithm named Particle Swarm Optimization (PSO) to find a portfolio with maximum Sharpe ratio.

The sharpe ratio is simple and it is a risk adjusted measure of return that is used to evaluate the performance of a portfolio. Particle Swarm Optimization (PSO) is a population based stochastic optimization technique developed by Kennedy and Eberhart in 1995, inspired by social behavior of bird flocking. This report uses PSO to solve portfolio optimization problems and we will compare the results of PSO to that of Genetic Algorithm.

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Chapter 1

Introduction

1.1 Brief of Portfolio Optimization

Portfolio Optimization is the process to select various assets in a way to make a portfolio that should maximize the profits and minimize the risk.

An optimal portfolio is that one which has the highest sharpe ratio[2]. Portfolio optimization is based on Modern Portfolio Theory(MPT)[1] which was introduced in 1952 by Harry Markowitz. The MPT[1] is based on the principle that investors want the highest return for the lowest risk.

Benefits of Portfolio Optimization:

Maximizing return: The main objective of portfolio optimization is to maximize return for a given level of risk. Managers doing this process are often able to do so for clients that results in client satisfaction.

Diversification: Diversification helps in protecting investors. If of many stocks in a portfolio some did not perform, other stocks in the portfolio will still have a chance to perform and the investor will be in a safe zone. Thus, investors generally invest in diverse manners in many different stocks of different sectors in a portfolio.

Identifying market opportunities : People involved in this process often keep updated with lots of market data and can help themselves as well as their client to get profits.

Challenges in Portfolio Optimization:

Frictionless markets : One of the assumptions MPT[1] makes is that markets are frictionless. But there are frictions like transaction cost and other constraints that complicate the process of portfolio optimization.

Normal Distribution : Another assumption under the MPT[1] is that returns are normally distributed. But returns can be skewed, kurtosis, etc. The assumptions can throwback incorrect results which could result in the loss of money.

Different models of portfolio optimization are listed in the following table:

Model	**Proposed by**	**Structure**	**Year**
Mean-Variance	Markowitz[1]	Quadratic	1952
Semi-Variance	Markowitz[1]	Quadratic	1959
Mean-Absolute-Deviation	Konno and Yamazaki[12]	Linear	1991
Minimax	Young[13]	Linear	1998

1.2 Literature Survey

Portfolio optimization simply is the process of selection of the best portfolio out of all the available portfolios according to some objective. Here our objective is to maximize sharpe ratio[2]. I have gone through a number of reports on portfolio optimization. The drawbacks in the mean-variance model were removed by Konno and Yamazaki mean absolute deviation model[12]. Fuzzy approach[4] usually lacks learning ability, neural network[5][6] approach has an overfitting problem. Also dynamic programming[14] does not allow the user to get the sub-optimal solution which is very important for some financial problems. In order to avoid these problems I have used the PSO Algorithm[3],[10],[11] as it often finds the best optimum by global search in contrast with the most common optimization algorithm. I have tried to find the portfolio having the maximum Sharpe Ratio[2] using PSO algorithm [3],[10],[11].

1.3 Scope of the thesis

In this thesis I have tried to find an optimal portfolio based on sharpe ratio[2] maximization. It is a simple technique for evaluation portfolios . A portfolio with higher sharpe ratio[2] is more risk efficient than its counterparts. For finding the maximum sharpe ratio[2] I have used Particle Swarm Optimization(PSO) algorithm[3],[10],[11].

This report can be used and implemented to further study the effect of sharpe ratio[2] in investment and a better understanding of the sharpe ratio[2] in the field of portfolio optimization can be found.

Chapter 2

2 Particle Swarm Optimization(PSO) Algorithm

Particle swarm optimization algorithm[3],[10],[11] was introduced by Kennedy and Eberhart in 1995. It is inspired by swarm behavior of flock of birds and school of fish. Suppose a group of birds want to find food available and the best strategy to get food is to follow the bird which is nearest to the food at each iteration. All of the birds can be regarded as a possible solution or particle and their group becomes swarm and the search space is the area to explore.

Initially some particle is identified as the best particle(**gbest**) in a swarm of particles based on some fitness score. All the particles then accelerate in the direction of this particle but also in the direction of their own best values(**pbest**) they have discovered previously.

Parameters of the algorithm :

Pi: Position of the particle or agent.
Vi: Velocity of the particle or agent.
A: Population of agents.
W: Inertia weight.
c1: cognitive constant.
U1, U2 : random numbers.
c2: social constant
Max_iter: maximum number of iterations

Steps of the algorithm :

1. Create a population of particles.

2. Evaluate each particle's position according to the fitness function.

3. If a particle’s current position(**pbest**) is better than its previous position, update it.

4. Determine the best particle(**gbest**)

5. Update the particles' velocities

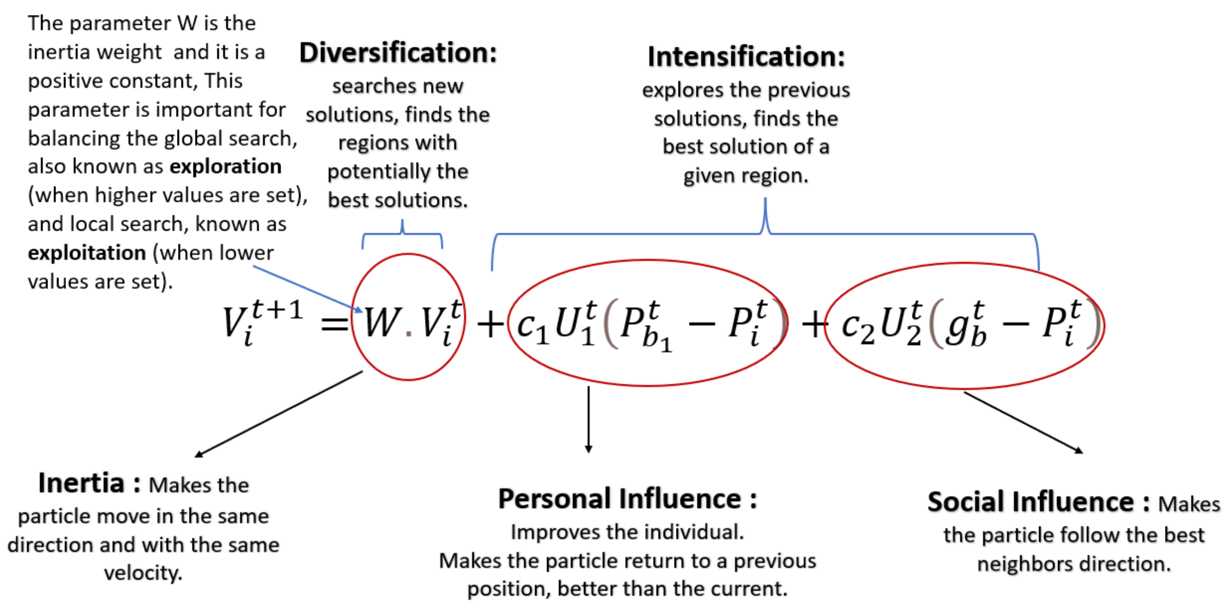
$$V_i(t+1) = W.V_i(t) + c1\ U1(t)(P_b(t) - P_i(t)) + c2\ U2(t)(g_b(t) - P_i(t))\quad [10]$$

6. Move the particles to their new positions.

$$P_i(t+1) = P_i(t) + V_i(t+1)\quad [10]$$

7. Continue step 2 until termination criteria is reached

Effects of the parameters:



↘Analytics Vidhya

W

Inertia weight W affects the particle's global and local search ability.
When W is small, the PSO algorithm hardly converges and the success rate is low. Along with the increase of W, the PSO algorithm has a better convergence and stability.

c1,c2

c1 and c2 are positive constant parameters called acceleration coefficients which are responsible for controlling maximum step size.

U1,U2

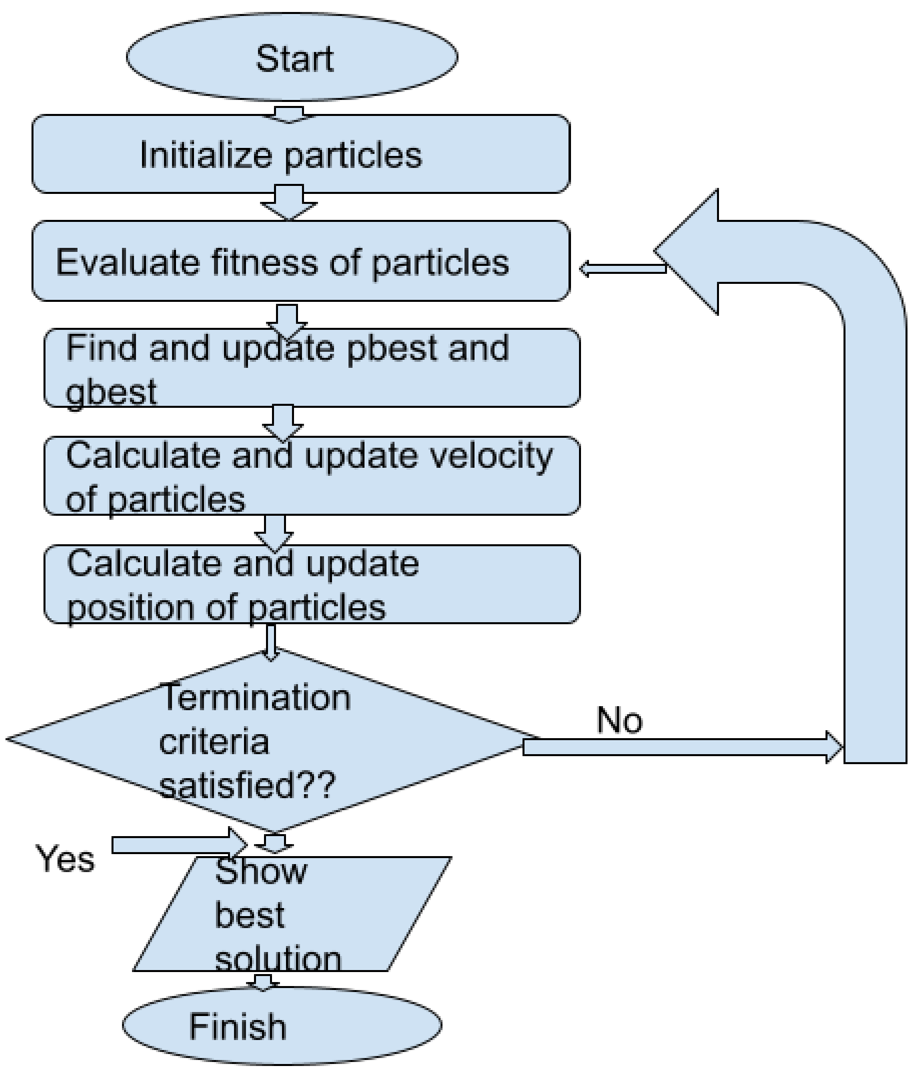
U1 and U2 are random numbers between (0,1).

Advantages Of PSO Algorithm:

1. Simple implementation.
2. Very few algorithm parameters.
3. Very efficient global search algorithm

I have used PSO[3],[10],[11] in order to maximize the sharpe ratio for portfolio optimization due to its simple implementation and efficient global search abilities and also as it removes the lack of learning ability in fuzzy based models[4], overfitting problems in neural network models[5],[6] and not getting the sub-optimal solution in dynamic programming models[14].

Flowchart of PSO Algorithm:



Chapter 3

3 Genetic Algorithm(GA):

I am comparing the results of the PSO with that of GA. Hence, a little introduction about Genetic algorithm(GA)[7].

Genetic algorithms[7] are based on ideas of natural selection and genetics. They are commonly used to generate high-quality solutions for optimization problems and search problems. They simulate “survival of the fittest “ among individuals of consecutive generations for solving a problem.

The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution.

Steps of the algorithm:

1) Randomly initialize population p

2) Determine fitness of population

3) Until convergence repeat:

    a) Select parents from population

    b) Crossover and generate new population

    c) Perform mutation on new population

    d) Calculate fitness for new population

Operators of GA:

1 Selection Operator : The idea is to give preference to the individuals with good fitness scores and allow them to pass their genes to successive generations.

2 Crossover Operator : Combine two parents to form children for the next generation

3 Mutation Operator : Apply random changes to individual parents to form children.

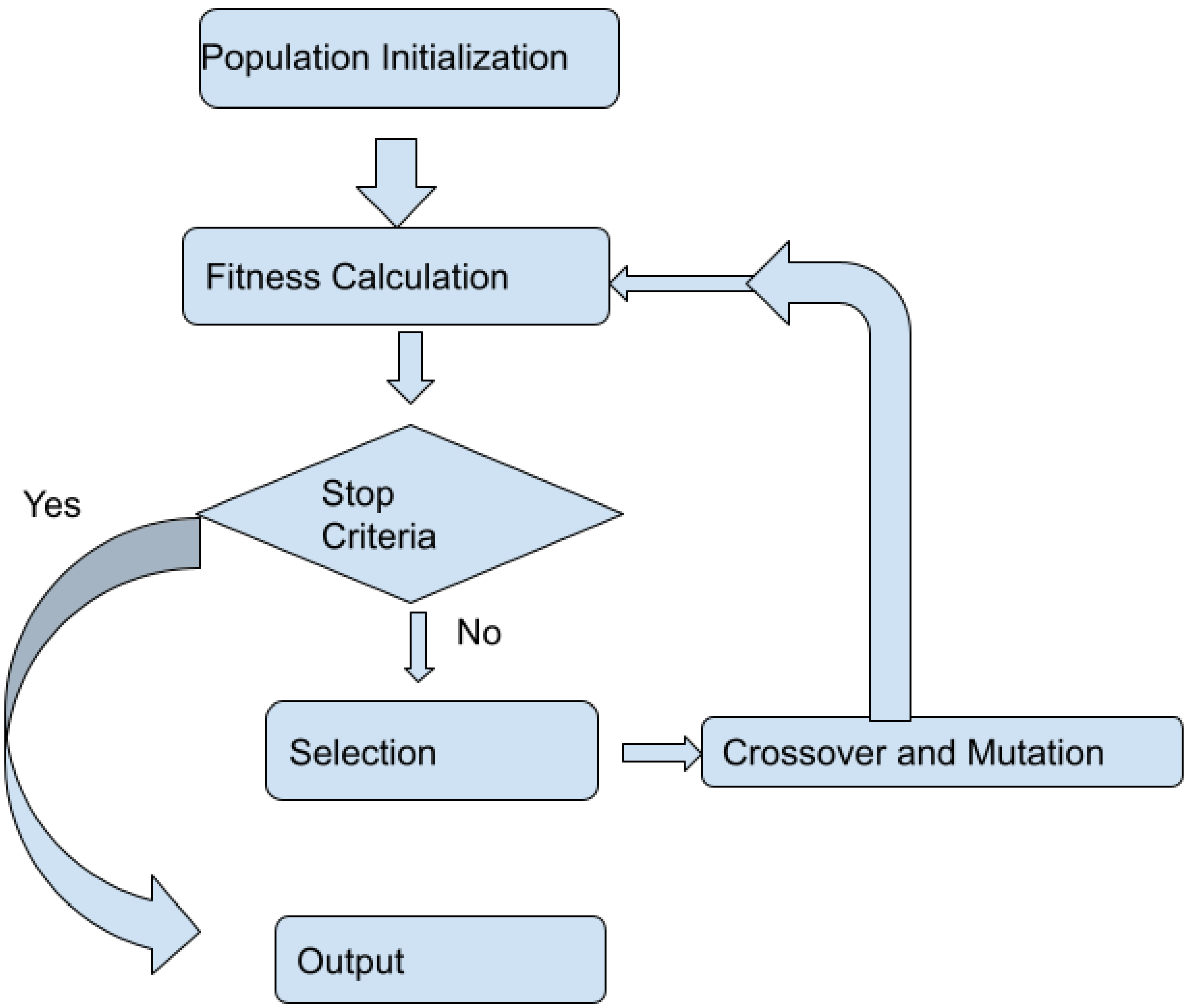
Advantages of GA:

1.Global optimization

2. A larger set of solution space

3. Provides multiple optimal solutions

Flowchart of GA :



Chapter 4

4 Sharpe Ratio optimization using PSO Algorithm

Fitness function used is : Sharpe ratio = (Rp - Rf)/Sp

where,Rp: Portfolio return , Rf: Risk free rate of return and Sp: Standard deviation of the portfolio or the risk associated with the portfolio

The Sharpe ratio[2] measures performance of a portfolio by taking risk in account. The higher it is the better the investment in terms of risk-adjusted returns. [2]

It works by giving investors a score that tells them their risk-adjusted returns. It can be used to evaluate past performance or expected future performance, but in either case this key financial ratio helps the investor understand whether returns are due to smart decisions or just taking on too much risk. [2]

Example:

Consider two portfolios: Portfolio A is expected to return 14% over the next 12 months, while Portfolio B is expected to deliver a return of 11% over the same period. If we do not consider risk, Portfolio A is clearly the better choice.

In the given example, Portfolio A has a standard deviation of 8% and Portfolio B has a standard deviation of 4% and the risk-free rate is 3%.

The Sharpe Ratio[2] for each portfolio :

- **Portfolio A:** (14 – 3) / 8 = Sharpe ratio of 1.38
- **Portfolio B:** (11 – 3) / 4 = Sharpe ratio of 2

Given the greater amount of volatility of Portfolio A, its Sharpe ratio is lower than Portfolio B’s ratio. This tells us that with a Sharpe ratio of 2, Portfolio B provides a better return on a risk-adjusted basis.

Generally speaking, a Sharpe ratio between 1 and 2 is considered good. A ratio between 2 and 3 is very good, and any result higher than 3 is excellent

This example is taken from forbes.com/sharpe-ratio.

I have used the PSO algorithm[3],[10],[11] to find the weights(composition of each stock in the portfolio) such that we maximize the returns and at the same time minimize the risk. Sharpe Ratio[2] will be used to evaluate the fitness.

I have used a dataset from kaggle which consists of 5 year historical price data of all the stocks of all companies on the S&P 500 index from 08/02/2013 – 07/02/2018. I have used 5 year historical price data of 10 stocks from this dataset for portfolio optimization.

Portfolio optimization problem is thus solved in a 10 - dimensional search space where we tried to find the optimal weights which maximizes the sharpe ratio[2].

Each point in the search space represented a possible combination of weights and therefore, a possible portfolio. Weights representing a given portfolio along with daily returns data for all stocks were used to calculate mean return, standard deviation, and hence sharpe-ratio[2] of that portfolio. Fitness of a combination of weights, represented by a particle's position in PSO [3],[10],[11] was evaluated by the sharpe-ratio[2] of the resulting portfolio.

The time-variant forms of w, c1 and c2 were incorporated by means of the following equations:

$w(t) = (w(0) - w(n)) ((n - t)/n) + w(n)$ [15]

where w(t) is the inertia weight at iteration t, w(0) is the initial inertia weight, n is the maximum number of iterations and w(n) is the final inertia weight at the end of all iterations.

$c1(t) = (c1(min) - c1(max))t/n + c1(max)$ [15]

$c2(t) = (c2(max) - c2(min))t/n + c1(min)$ [15]

where c1(t) and c2(t) are the values of c1 and c2 at iteration t, n is the maximum number of iterations, c1(min) and c2(min) are the minimum values of c1 and c2 respectively, and c1(max) and c2(max) are the maximum values of c1 and c2 respectively.

Reference for the kaggle dataset :

https://www.kaggle.com/datasets/camnugent/sandp500

Chapter 5

5 Experimental Results :

I have executed both PSO[3],[10],[11] and GA[7] with 5 different swarm sizes and population sizes 100,200,300,400,500 and 1500 iterations.

Each swarm size or population size is executed 10 times and also the average of 10 times is calculated .

Swarm Size	S/R It 1	S/R It 2	S/R It 3	S/R It 4	S/R It 5	S/R It 6	S/R It 7	S/R It 8	S/R It 9	S/R It 10	Avg.
100	**1.5420**	**1.5421**	**1.5220**	**1.5156**	**1.5386**	**1.5354**	**1.4796**	**1.5365**	**1.5447**	**1.4752**	**1.5232**
200	**1.5245**	**1.5453**	**1.5433**	**1.5223**	**1.5429**	**1.5338**	**1.5308**	**1.5222**	**1.5431**	**1.5464**	**1.5355**
300	**1.5447**	**1.5424**	**1.5330**	**1.5261**	**1.5393**	**1.5218**	**1.5091**	**1.5453**	**1.5415**	**1.5201**	**1.5323**
400	**1.5391**	**1.5459**	**1.5427**	**1.5488**	**1.5430**	**1.5403**	**1.5188**	**1.5451**	**1.5342**	**1.5452**	**1.5403**
500	**1.5452**	**1.5412**	**1.5449**	**1.5175**	**1.5453**	**1.5355**	**1.5384**	**1.5414**	**1.5406**	**1.5444**	**1.5394**

S/R : Sharpe Ratio
It : Iteration

Results of Sharpe Ratio Optimization Using PSO Algorithm

Population Size	S/R It 1	S/R It 2	S/R It 3	S/R It 4	S/R It 5	S/R It 6	S/R It 7	S/R It 8	S/R It 9	S/R It 10	Avg.
100	**1.5479**	**1.5465**	**1.5478**	**1.5462**	**1.5465**	**1.5480**	**1.5464**	**1.5468**	**1.5470**	**1.5480**	**1.5471**
200	**1.5467**	**1.5477**	**1.5478**	**1.5479**	**1.5471**	**1.5481**	**1.5473**	**1.5475**	**1.5471**	**1.5484**	**1.5476**
300	**1.5484**	**1.5486**	**1.5478**	**1.5470**	**1.5485**	**1.5479**	**1.5486**	**1.5487**	**1.5477**	**1.5471**	**1.5480**
400	**1.5480**	**1.5481**	**1.5482**	**1.5485**	**1.5486**	**1.5476**	**1.5482**	**1.5480**	**1.5486**	**1.5483**	**1.5482**
500	**1.5482**	**1.5484**	**1.5482**	**1.5482**	**1.5484**	**1.5484**	**1.5483**	**1.5485**	**1.5484**	**1.5483**	**1.5483**

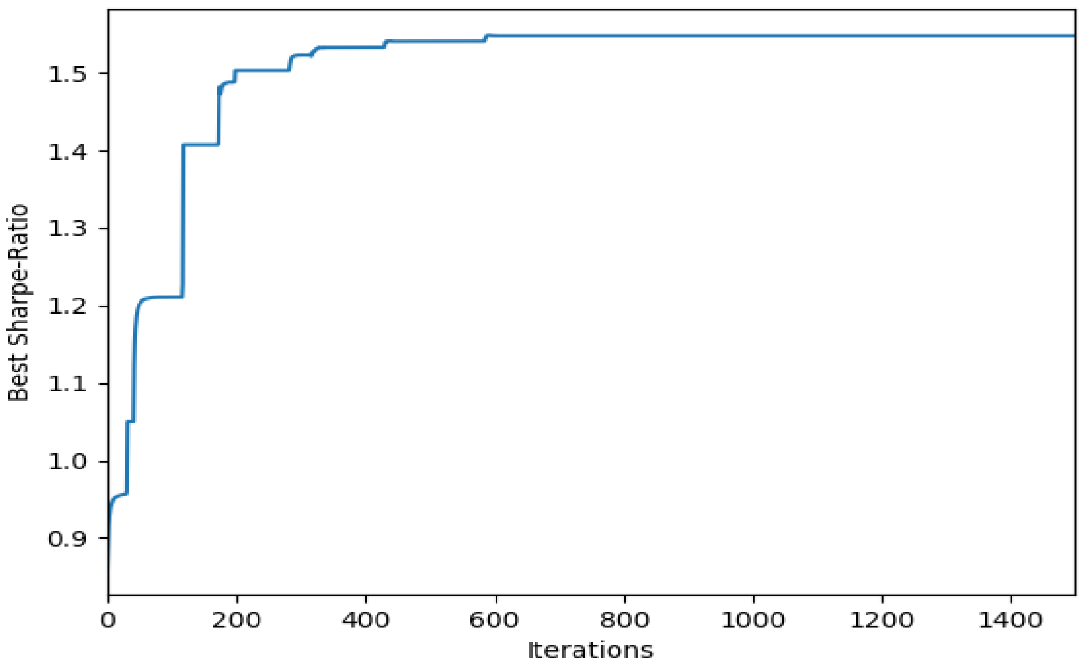
S/R : Sharpe Ratio
It : Iteration

Results of Sharpe Ratio Optimization Using GA Algorithm

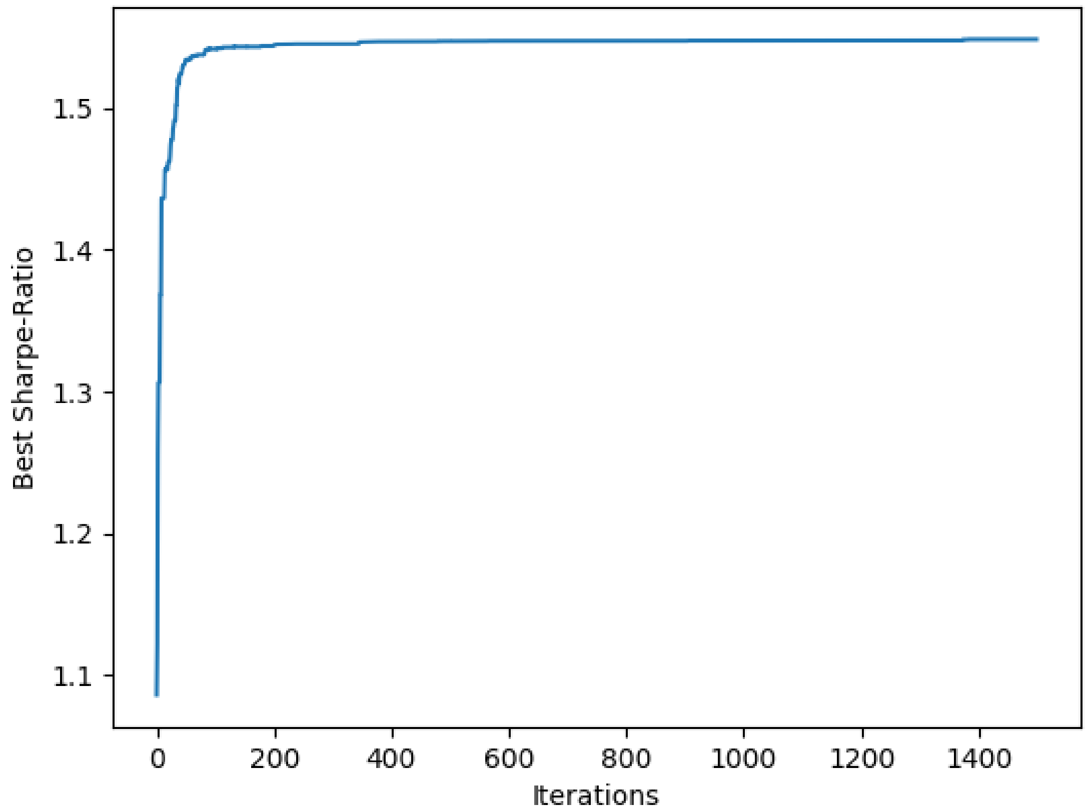
By looking at the above results generated by PSO and GA for 5 different swarm sizes and population sizes ran 10 times each we observed that the best result was given by the PSO algorithm in 400 population size and 4th iteration **1.5488** . The best result of GA was in 300 population size and in 8th iteration was almost equal to that of PSO is **1.5487.**

However upon calculating average the best average result was given by GA average of **1.5483** and the best average in PSO is **1.5403.**

The graphs of best individual result of both PSO and GA are given in the pages below:



Graph of best result of Sharpe ratio Optimization Using PSO Algorithm



Graph of best result of Sharpe ratio Optimization Using GA Algorithm

Chapter 6

6 Conclusion

I have collected data and applied PSO algorithm and GA algorithm to the dataset in order to find a portfolio which has maximum sharpe ratio.

The best result in the individual iteration was given by PSO and the best average sharpe ratio was given by GA.

In overall results including the averages and the best result the highest sharpe ratio was given by PSO.

Chapter 7

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