BRAIN MR IMAGE SEGMENTATION USING TYPE-2 FUZZY CLUSTERING ALGORITHM USING GLOBAL AND LOCAL ENTROPIES

Thesis Submitted to

FACULTY OF ENGINEERING AND TECHNOLOGY
JADAVPUR UNIVERSITY

In partial fulfillment of the requirements for the degree of

MASTER OF TECHNOLOGY IN COMPUTER TECHNOLOGY

BY

Sulagna Sarkar
Examination Roll: M6TCT22025
Registration No.: 149858 of 2019-2020

Under the guidance of

Dr. Jamuna Kanta Sing
Professor,Department of Computer Science and Engineering
Jadavpur University

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
FACULTY OF ENGINEERING AND TECHNOLOGY
JADAVPUR UNIVERSITY

I hereby recommend that the thesis entitled "Brain MR Image Segmentation Using Type-2 Fuzzy Clustering Algorithm Using Global and Local Entropies" prepared under my supervision and guidance at Jadavpur University, department of Computer Science and Engineering, Kolkata by **SULAGNA SARKAR (Reg. No. 149858 of 2019-2020,Class Roll NO. 001910504024, Examinatio Roll No. M6TCT22025)***, may be accepted in partial fulfillment for the degree of Master of Technology in Computer Technology in the Faculty of Engineering and Technology, Jadavpur University, during the academic year 2021-2022. I wish her every success in life.*

....	....
Prof. Anupam Sinha	Prof. Jamuna Kanta sing
Head of the Department	Thesis Supervisor,
Department of Computer Science and Engineering	Department of Computer Science and Engineering
Jadavpur University, Kolkata - 700032.	Jadavpur University, Kolkata - 700032.

....

Prof. Chandan Majumder

Dean, Faculty Council of Engg. & Tech.

Jadavpur University, Kolkata - 700032.

DECLARATION OF ORIGINALITY AND COMPLIANCE OF ACADEMIC PROJECT

I hereby declare that this thesis work holds literature survey and original research work by the undersigned candidate, as a student of MASTER OF TECHNOLOGY IN COMPUTER TECHNOLOGY. All the Information in this document have been obtained and presented in accordance with academic rules and ethical conduct. I am also declare that, as required by these rules and conduct, I have fully cite and referenced all the material results that are not original to this work.

NAME: SULAGNA SARKAR

EXAMINATION ROLL NUMBER: M6TCT22025

THESIS TITLE: Brain MR Image Segmentation Using Type-2 Fuzzy Clustering Algorithm Using Global and Local Entropies

SIGNATURE WITH DATE:

JADAVPUR UNIVERSITY FACULTY OF ENGINEERING AND TECHNOLOGY

CERTIFICATE OF APPROVAL

This is to certify that this thesis work entitled **"Brain MR Image Segmentation Using Type-2 Fuzzy Clustering Algorithm Using Global and Local Entropies"** is a legitimate record work done by **SULAGNA SARKAR** (Class roll Number: **001910504024**, Examination Roll Number: **M6TCT22025**, Registration Number: **149858 of 2019-2020**) in a partial fulfillment of the degree Masters of Technology in Computer Technology in the department of Computer Science and Engineering, Jadavpur University during the period August 2021 to June 2022. It is understood that by this approval the undersigned do not necessarily endorse or approve any statement made,opinion expressed or conclusion drawn therein, but approve the thesis only for purpose for which it is submitted.

....

Signature of Examiner 1 Signature of Examiner 2

ACKNOWLEDGEMENT

I would like to express my honest and sincere thanks and humble gratitude to my honourable teacher and guide *Prof.(Dr.) Jamuna Kanta Sing,* Professor of the Department of Computer Science & Engineering, Jadavpur University, for his complete guidance and full support in completing and producing this thesis successfully. I am very much obliged to him for the constant encouragement and continuous inspiration that he has given to me. These words are only a token of my deep respect towards him for all he has done to take my thesis to the present shape.

I would like to thank *Dr.Nabanita Mahata & Mrs.Madhumita Ray* for their support and valuable suggestions to the activities of the thesis at any phase.

Finally, I would like to express my sense of gratitude and thankfulness to one of my friends who helped me and motivated me. Last but not the least, I would like to thank my parents for their unconditional love and support without which I would hardly be capable of completing this huge work.

....

SULAGNA SARKAR

Examination Roll Number: M6TCT22025

Registration No: 149858 of 2019-2020

ABSTRACT

In this thesis work, we proposed a segmentation algorithm by which we can segment a noisy 3D brain MR image which also contains high Intensity Inhomogeneity(IIH). Using standard fuzzy clustering algorithm (FCM) we fail to achieve comparable segmentation accuracy. As the segmentation is not so perfect, it becomes hard to find the abnormality or the diseases in the tissue area. To mitigate this problem, we considered so many algorithms that are based on standard FCM algorithm. Among them the entropy based algorithms reduces the uncertainty of a voxel being in a cluster. Those algorithms perform well in some scenarios but appears to fail in majority of cases. In this paper we tried to improve the performance of a particular entropy based clustering algorithm by applying type-2 fuzzy. For this we have considered two types of entropy, one is local entropy and another one is Global entropy. This type-2 fuzzy clustering algorithm gives us a better segmentation result by considering the global and local entropy.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The magnetic resonance imaging (MRI) is a technique which is used for the clinical diagnosis and treatment of diseases. Due to noise, intensity inhomogeneity (IIH) and partial volume effect, sometimes tissue boundaries are blurry; resulting image segmentation becomes more difficult and challenging so if there are some abnormalities in the tissue area that becomes difficult to identify. So it is required to have a computer aided segmentation method which may give us a good result with accuracy. Main things that IIH include in an image are (i) Static field Strength, (ii) reduce radio frequency coil uniformity, (iii) Patient movement at the time of image capturing [1–4]

Brain MR images are the most frequently used as it is important for the detection of the disease, proper treatment for that disease and post treatment observation. Brain MR image basically segmented into three parts Gray matter (GM), White matter (WM) and cerebrospinal fluid (CSF). Fuzzy c-means [] and its variants [] are mostly studied for image segmentation.

1.1 *Image Segmentation:*

Image segmentation is considered as the most frequently used technique by which a digital image can be divided into various parts according to their features and properties, called image segments. This makes the image simpler and easier to process and analysis of an image [5]. Two objectives are there behind the image segmentation - (i) break the image into parts so that the image analysis becomes easier, and (ii) if we want to change the representation of the image. Image segmentation is rapidly used in medical image [6] analysis to identify the abnormality in the organs and often this task is not so easy.

1.2 *Brain MR Image Segmentation:*

Brain MR (Magnetic Resonance) images are very mostly clinically studied as it is important for a patient's treatment. By the analysis of the brain MR image segmentation we can analyze and visualize the brain's anatomical structure or the changes that are happening inside the brain structure or if there is a requirement of surgery or after surgery observation. As we know brain is one of the most sensitive parts of our body, there should be concrete image segmentation technique. There are number of application or algorithms are there based on the accuracy and the time complexity [7].

Figure 1.1: Brain MR Image

In 3D space we can represent an image like $I(i, j, k)$ where $i = 0, ..., M - 1$, $j = 0, ..., N - 1$, $k = 0, ..., l - 1$ where i, j, k are spatial coordinates. Every image is a collection of finite set of image elements, called voxel in 3D -space. The function $I(i, j, k)$ gives the intensity value of a particular voxel and that intensity value lies in between $\{0, ..., 255\}$ i.e the grey value of the brain MR(Magnetic Resonance) image. Each voxel is uniquely defined by its intensity value and the coordinates i.e (i, j, k) where i is the image-index value, j is the row number and k is the column number.

For the consideration of brain MR image, the elements of the image is Gray Matter(GM), White Matter(WM) and Cerebrospinal fluid(CSF) [8]. There are huge number of works on Brain 2D image segmentation but for the past few years for the accuracy purpose, brain 3D image segmentation is in consideration.

1.3 Image Segmentation Techniques:

One of the best uses of computer algorithms is to process an image on digital image in digital image processing. Image Segmentation is the hardest part of digital image processing for the image analysis purpose. Medical image processing and segmentation take an important role as the identification of a disease and its proper treatment is important. There are so many algorithms for image segmentation for medical image analysis. Image Segmentation techniques can be divided as Thresholding Segmentation, edge-based Segmentation, region-based Segmentation, Clustering-Based Segmentation Algorithms, neural networks for Segmentation. This techniques are frequently used for image segmentation [3, 9, 10].

The image having grey level histogram can be reviewed as the threshold level in Intensity thresholding. Calculate the optimal threshold and the spatial uncertainty as the location of the pixel is ignored- are two main disadvantages of intensity thresholding [11].

Edge based segmentation is divided into two steps; edge detection and edge linking. Edge Detection process is, to find the edge pixels of an object. Edge linking process is, link the adjacent edges to refine the edges. For low contrast and noisy image the edge based segmentation is not the best one. Also the edge linking is not a easy job [12].

Clustering-based segmentation is a method by which we can segment an image pixel wise. Fuzzy C-Means Clustering(FCM) [13] is the one of the best clustering-based algorithms. There are so many algorithms under clustering-based segmentation like k-means algorithm [14] etc. But FCM is more effective unsupervised machine learning algorithm as it relates the pixels with multiple clusters with different membership values. FCM performs well in most of the cases but not for all cases. Limitations of FCM are local optimal solution due to poor initialization. Due to these reasons there are so many modified fuzzy clustering techniques have been published in the past. The main reason is to modify the FCM algorithm is to make FCM algorithm more efficient to perform on the noisy images [15–29]. Pedrycz [15] proposed a fuzzy c-means based algorithm and introduced a new conditional variable. This method disclose the structure within a family of patterns under consideration of their vicinity in a feature space along with the similarity of the values assumed by certain conditional variable. The approach proposed by Y.A. Tolias [17] is the the adaptive fuzzy clustering / segmentation. In this method the main concern was the nonstationary nature of images using modification of the prototype vectors as methods of the sample location in the image. Qui et al. [16] proposed a modified interval type-2 fuzzy C-means algorithm with application in MR image segmentation. This paper discloses that single fuzzifier in FCM can not properly represent the pattern memberships of all clusters. In the interval type-2 FCM they introduces two fuzzifiers and a spatial contraint on the membership function. Ahmed et al. [19] introduced a modified fuzzy c-means clustering algorithm. This algorithm modified the objective function of the standard fuzzy c-means algorithm to reduce the inhomogeneities and labeled the pixels to be influenced by the labels in its immediate neighborhood. The neighbourhood effect works as a regularizer and biases the solution toward piecewise-homogeneous-labelings. Liew et al. [20] this paper present a spatial fuzzy clustering that uses the spatial contextual information in an image data. The impact of the neighbouring pixels on the central pixel in a 3X3 karnel is taken as the consideration of the objective function of their method utilises a new dissimilarity index. Pham et al. [21] proposed an image segmentation algorithm considering the multiplicative intensity inhomogeneities. The objective function of this algorithm is the objective function of the FCM in addition of the multiplier field, that allows the centroids for each class across the image. A.W.C liew [22] proposed an image segmentation algorithm that is focused on tha spatial continuity constraints with the help of dissimilarity index. Dissimilarity index allows spatial interaction between image voxels. Noise effect in an image reduces by the local spatial continuity. The intensity nonconformity artifact is formulated like a multiplicative bias field influence the true MR imaging signal. One can reduce noise and intensity ambiguity by using the local spatial constraint. Wang et al. [23] introduces a modified fuzzy c-means clustering algorithm by utilizing local contextual information and high correlation inherent between the inter-pixel. In this model at first a local similarity measure model is established and then based on local spatial similarity measure model, cluster center and membership values are initialized. Then secondly, based on the high correlation inherent between the inter-pixel, the fuzzy membership function is modified accordingly. Noordam et al. [25] describes a technique in his paper that is how to overcome the sensitivity of fuzzy c-means clustering for the different cluster sizes in multivariate images. This paper proposed a modified version of standard FCM, called conditional FCM which is used to balance the unequal sized cluster. The ratio between the cluster sizes are determined and a condition value is calculated during the clustering process. The influence of objects from larger clusters to smaller clusters is balanced by the condition value. Adhikari et al.[30] presented a method to segment a noisy MR image of brain, considering the INU(Intensity Nonuniformity) a spatial information by using fuzzy c-means clustering algorithm. with the help of fusion of Gaussian surfaces, one can correct the INU of MR image of brain. The individual Gaussian surface is evaluated independently over the different homogeneous regions under consideration of its center as the center of the respective homogeneous region. Secondly using probabilistic FCM algorithm, the IIH correcyed image is segmented.

Recently Nabanita Mahata at al. [31] suggested an algorithm where global and local entropies are considered while calculating the global membership and local membership of a particular voxel. Global membership is the belongingness of a voxel in a certain cluster and the local membership is the mean of the neighbour voxel's membership value towards a cluster for that particular voxel. In case of a noisy image because of the noise and the high IIH (Intensity Inhomogeneity), their main aim is to minimize global and spatially constrained likelihood-based local entropies.

Most of the approaches discussed above, difficulty arises in image segmentation for the presence of noise and another multiplicative factor Intensity Inhomogineity(IIH) or bias field in the medical MR images. The Intensity Inhomogineity(IIH) normally refers to the slow, non-anatomic intensity variations of the same tissue over the image domain and causes due to imperfection of the image acquisition devices, poor magnetic field, eddy current and patient movement etc.

1.4 Proposed method:

In our suggested approach we tried to incorporate Shannon entropy and Type-2 fuzzy with the conventional Fuzzy C-Means Clustering algorithm. We can successfully segment an image in the presence of noise and high intensity inhomogineity(IIH). Type-2 fuzzy logic is involved to find the better membership value of a voxel for a particular cluster and entropy is involved because of the dissimilarity between the pixels in the edge region are high.

Chapter 2

Proposed Type-2 Fuzzy Clustering Algorithm Using Global and Local Entropies

2.1 Entropy

Definition 2.1.1. Entropy can be defined as uncertainty or the amount of disorder present in an information that is being processed in machine learning. The other way we can explain that entropy is the standard approach that helps to measure the unpredictability and the impurity presents in the system. It uses in various fields like physics, information theory and other branches of science and engineering [32].

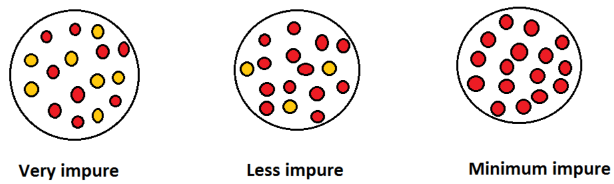


Figure 2.1: Entropy

Entropy formulated as below [33]:

$$-\sum p_i \log p_i \tag{2.1}$$

where p_i is the probability of given symbol.

There are various types of entropies for significant applications. Some of them are discussed in the following section:

a) Shannon Entropy: Shannon entropy gives us an absolute limit on the best possible lossless compression of signal under constraint. Suppose X is a random variable, then Shannon entropy can be defined as

$$H(X) = -\sum p(x_i) \log p(x_i) \tag{2.2}$$

In 2D random space p_{x_i} is probability density function [34]

b) Renyi entropy: It is generalized version of Shannon entropy. It is used for quantum information to measure the entanglement. It is defined as the entropy of order on α, where $\alpha \geq 0$ and $\alpha \neq 1$, is defined as

$$H_r(X) = \left(\frac{1}{1-\alpha}\right) \log \sum p_i^{\alpha} \tag{2.3}$$

For noisy cell image segmentation Sadek et al. [35] proposed an efficient and faster entory based method , based on generalized α - entropy by calculating the maximum structural information of image and according to desired segmentation, have to locate the optimal threshold. High rapidity and it's tolerance is the advantage of their proposed algorithm in the presence of noise in the image.

c) Kapur's entropy: This entropy can be represented as an equation of order of α and type β, is written as the following [36] :

$$H_k(X) = \left(\frac{\sum p_i^{\alpha+\beta-1}}{\sum p_i^{\beta}}\right) (2^{1-\alpha} - 1)^{-1} \tag{2.4}$$

d) Vajda entropy: This entropy is considered as the special case of Kpur's entropy where $\beta = 1$ is taken and the uncertainty is calculated. Vajda entropy tales less time as compared to the Kapur's entropy. so it is preferable than Kapur's entropy.

$$H_v(X) = \left(\frac{\sum p_i^{\alpha}}{\sum p_i} - 1\right) (2^{1-\alpha} - 1)^{-1} \tag{2.5}$$

2.2 Fuzzy Type-2:

In the rule based fuzzy logic systems, Type-2 fuzzy sets gives us the advantage of model and minimize the effects of uncertainties. Let us imagine by blurring the type-1 membership illustrated in Figure 2.2 by shifting the points on the either side of the triangle to the left or to the right and not necessarily by the same amounts, as in Figure 2.3.

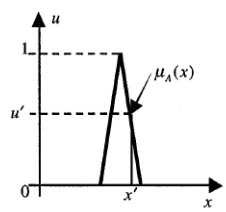


Figure 2.2: Fuzzy Type-1 membership function
[37]

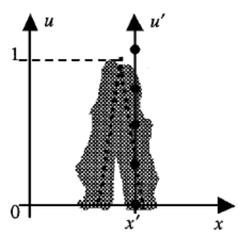


Figure 2.3: Blured Type-1 membership function
[37]

In type-1 we were getting single value of u' for a specific value of x, say x'. Now in Figure 2.3 we can see for the fuzzy type-2 membership function there no longer is a single value for the membership function u', instead the membership takes on every values wherever the vertical line intersects the blur. Those values need not to be same weighted; so we can assign an amplitude distribution to all those intersected points.

A type-2 fuzzy set can be denoted as $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$,i,e

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \tag{2.6}$$

Fuzzy type-2 membership function can be defined as

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x') \int_{u \in J_{x'}} f_{x'}(u)/u \tag{2.7}$$

where $J_{x'} \subseteq [0,1]$, $0 \leq f_{x'}(u) \geq 1$, $\mu_{\tilde{A}(x)}$ can refer a secondary membership function. Secondary membership function is referred as the type-1 fuzzy set [37].There are so many different comparative study for choosing the perfect μ from the set $j_{x'}$ for a particular data point.

It is very difficult to achieve an effective result, as MR images are sensitive to noise and intensity inhomogeneity [12]. So, to identify the diseased regions in MR images, it is really very important to segment the image accurately into different tissue regions(CSF,GM,WM). To increase the robustness of the standard FCM we incorporate Fuzzy type-2 with Shannon entropy in our proposed method. As the entropy is equivalent to uncertainty , so our aim is to minimize the entropy to reduce the uncertainty around a voxel and by using fuzzy type-2 we'll try to calculate the accurate membership value for a particular voxel.

Our proposed method is based on the algorithm which is proposed by Nabanita et al [31]. At the unsharp tissue boundaries, uncertainties is maximum. To reduce this problem they introduced two types of entropies - (i) global entropy using fuzzifier weighted global membership function and, (ii) spatially constrained likelihood based local entropy using fuzzifier weighted local membership function. The global and local memberships are obtained by minimizing the objective function. The objective function given by their algorithm(FCMGsLE) is -

$$
\begin{aligned}
J_{FCMGsLE} = \sum_{i=1}^{C}\sum_{l=1}^{Z}\sum_{j=1}^{X}\sum_{k=1}^{Y}[\alpha\mu_{i_{jkl}}^{m}(d_{ijkl})^2 + (1-\alpha)u_{ijkl}^{m}(f_{ijkl})^{-1}(\bar{d_{i_{jkl}}})^2] \\
-\alpha\sum_{l=1}^{Z}\sum_{j=1}^{X}\sum_{k=1}^{Y}\sum_{i=1}^{C}\mu_{i_{jkl}}^{m}\ln(\mu_{i_{jkl}}^{m}) \\
-(1-\alpha)\sum_{l=1}^{Z}\sum_{j=1}^{X}\sum_{k=1}^{Y}\sum_{i=1}^{C}u_{i_{jkl}}^{m}\ln(u_{i_{jkl}}^{m})
\end{aligned} \tag{2.8}
$$

where $\sum_{i=1}^{C}\mu_{i_{jkl}} = 1$ and $\sum_{i=1}^{C}u_{i_{jkl}} = 1$

μ_{ijkl} and $u_{i_{jkl}}$ are the global and local membership values for the voxel a_{jkl}. m is the fuzzifier and can be selected empirically. value of α is $(0 < \alpha \leq 1)$. $d_{i_{jkl}}$ is the euclidean distance between the voxel a_{jkl} and the cluster center of the i^{th} cluster t_i. $\bar{d_{i_{jkl}}}$ is denoted as the euclidean distance between the neighbouring voxels of a_{jkl} and the cluster center of the i^{th} cluster t_i. $f_{i_{jkl}}$ means likelihood or possibility measure of belongingness into the i^{th} cluster for the voxel a_{jkl}. $d_{i_{jkl}}$, $\bar{d_{i_{jkl}}}$, $f_{i_{jkl}}$ can be defined as-

$$(d_{i_{jkl}})^2 = \|a_{jkl} - t_i\|^2 \qquad \forall j, k, l \tag{2.9}$$

$$(\bar{d_{i_{jkl}}})^2 = \left(\frac{1}{N}\right) \sum_{x_{jkl} \in N_{jkl}} \|x_{jkl} - t_i\|^2 \tag{2.10}$$

$$f_{i_{jkl}} = \left(\frac{\sum_{x_{jkl} \in N_{jkl}} (\mu_{i_{jkl}} x_{i_{jkl}})}{\sum_{x_{jkl} \in N_{jkl}} x_{jkl}}\right) \qquad \forall i, j, k, l \tag{2.11}$$

where N and N_{jkl} denotes the total number of neighbours of a particular voxel. Main objective is to minimize the cost function. We can calculate $\mu_{i_{jkl}}$, $u_{i_{jkl}}$ and t_i by doing the partial derivative of (2.8) with respect to $\mu_{i_{jkl}}$, $u_{i_{jkl}}$, t_i respectively and equating them to zero. The final iterative equation can be determined as follows:

$$\mu_{i_{jkl}} = \left(\frac{1}{\sum_{r=1}^{C} \left(\frac{\alpha[(d_{i_{jkl}})^2 - \ln(\mu^m_{i_{jkl}}) - 1]}{\alpha[(d_{r_{jkl}})^2 - \ln(\mu^m_{r_{jkl}}) - 1]}\right)^{\left(\frac{1}{m-1}\right)}}\right) \qquad \forall i, j, k, l \tag{2.12}$$

$$u_{i_{jkl}} = \left(\frac{1}{\sum_{r=1}^{C} \left(\frac{(1-\alpha)[(f_{i_{jkl}})^{-1}(\bar{d}_{i_{jkl}})^2 - \ln(u^m_{i_{jkl}}) - 1]}{(1-\alpha)[(f_{i_{jkl}})^{-1}(\bar{d}_{r_{jkl}})^2 - \ln(u^m_{r_{jkl}}) - 1]}\right)^{\left(\frac{1}{m-1}\right)}}\right) \qquad \forall i, j, k, l \tag{2.13}$$

$$t_i = \left(\frac{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} [\alpha \mu_{i_{jkl}}^m a_{jkl} + (1-\alpha) u_{i_{jkl}}^m (f_{i_{jkl}})^{-1} \bar{a_{jkl}}]}{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} [\alpha \mu_{i_{jkl}}^m + (1-\alpha) u_{i_{jkl}}^m (f_{i_{jkl}})^{-1}]} \right) / \bar{a_{jkl}} \tag{2.14}$$

$$= \left(\frac{1}{N}\right) \sum_{x_{jkl} \in N_{jkl}} x_{jkl} \qquad \forall i$$

In our proposed method we had modified the equations of the global ($\mu_{i_{jkl}}$) and local ($u_{i_{jkl}}$) membership function. this is a trial and error approach. It can be a comparative study to get a preferable membership function by which we can segment an noisy MR image with minimum error. To mitigate the error and reach the accuracy we apply type-2 fuzzy to modify the membership equations.

We store all the global membership values we get from equation(2.12) for each voxel in a 3D matrix(μ). After that we will find the maximum membership value(μ_{max}) from that 3D matrix. Then divide all the old global membership value by μ_{max} and got a new value μ_{new} for each voxel as shown in equation(2.15). Now we converted the membership values in type-2 fuzzy as shown in equation(2.16)

$$\mu_{new_{i_{jkl}}} = \left(\frac{\mu_{i_{jkl}}}{\mu_{max}} \right) \qquad \forall i, j, k, l \tag{2.15}$$

$$\mu_{T2_{i_{jkl}}} = \left(\frac{\sum_{i=1}^{C} \sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} \mu_{i_{jkl}} \mu_{new_{i_{jkl}}}}{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} \mu_{new_{i_{jkl}}}} \right) \tag{2.16}$$

Similarly we can modify the local membership values as well. Storing all the values that we get from equation(2.13) in a 3D matrix(u). Then calculated the maximum local membership value(u_{max})) from that 3D matrix and got the new local membership value(u_{new}) for a voxel as shown is equation (2.17). Converted all the local memberships of each voxel By using new local membership value as shown in equation(2.18).

$$u_{new_{i_{jkl}}} = \left(\frac{u_{i_{jkl}}}{u_{max}} \right) \qquad \forall i, j, k, l \tag{2.17}$$

$$u_{T2_{i_{jkl}}} = \left(\frac{\sum_{i=1}^{c} \sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} u_{i_{jkl}} u_{new_{i_{jkl}}}}{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} u_{new_{i_{jkl}}}} \right) \tag{2.18}$$

After getting the type-2 values of global and local membership values, we put those values in equation(2.14) by replacing the mu by $mu_{T2_{i_{jkl}}}$ and u by $u_{T2_{i_{jkl}}}$. This is how we calculate the center(t_i). Once we get the global and local membership values we derive the final membership value $g_{i_{jkl}}$ using the following equation(2.19)

$$g_{i_{jkl}} = \left(\frac{(\mu_{T2_{i_{jkl}}})^p (u_{T2_{i_{jkl}}})^q}{\sum_{r=1}^{C} (\mu_{T2_{r_{jkl}}})^p (u_{T2_{r_{jkl}}})^q} \right) \qquad \forall i, j, k, l \tag{2.19}$$

Where p and q ($1 \leq p, q \leq 3$) can be defined as the weighting parameters to control the effect of the global and local membership values respectively.

After modifying the global membership, local membership and cluster center we can present the algorithm in the next section.

2.3 Algorithm:

Input: A noisy 3D brain MR image volume G having height X, width Y and depth Z with C different clusters.

Output: Final cluster centers, final membership matrix and the segmented 3D image volumes H_i, $i = 1, 2, ..., C$.

Steps:

1. Initialize the parameters m, p, q and error ε.

2. Initialize cluster centers t_i $\forall i$, global membership value $\mu_{i_{jkl}}$, local membership value $u_{i_{jkl}} \forall i, j, k, l$. The cluster centers, global and local membership values can be initialized according to the information available from the image histogram.

3. Set iteration $n = 0$.

4. *Repeat*

i. Find the value of global membership values using the following equation

$$\mu_{i_{jkl}} = \left(\frac{1}{\sum_{r=1}^{C} \left(\frac{\alpha[(d_{i_{jkl}})^2 - \ln(\mu^m_{i_{jkl}}) - 1]}{\alpha[(d_{r_{jkl}})^2 - \ln(\mu^m_{r_{jkl}}) - 1]} \right)^{\left(\frac{1}{m-1}\right)}} \right) \qquad \forall i, j, k, l$$

ii. Find the new global membership value by using following equation:

$$\mu_{new_{i_{jkl}}} = \left(\frac{\mu_{i_{jkl}}}{\mu_{max}} \right) \qquad \forall i, j, k, l$$

iii. Find the value of type-2 global membership values using the following equation

$$\mu_{T2_{i_{jkl}}} = \left(\frac{\sum_{i=1}^{C} \sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} \mu_{i_{jkl}} \mu_{new_{i_{jkl}}}}{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} \mu_{new_{i_{jkl}}}} \right)$$

iv. Find the local membership value by using the following equation:

$$u_{i_{jkl}} = \left(\frac{1}{\left(\frac{(1-\alpha)[(f_{i_{jkl}})^{-1}(\bar{d}_{i_{jkl}})^2 - \ln(u^m_{i_{jkl}}) - 1]}{(1-\alpha)[(f_{i_{jkl}})^{-1}(\bar{d}_{r_{jkl}})^2 - \ln(u^m_{r_{jkl}}) - 1]} \right)^{\left(\frac{1}{m-1}\right)}} \right) \qquad \forall i, j, k, l$$

v. Find the new local membership value by using following equation:

$$u_{new_{i_{jkl}}} = \left(\frac{u_{i_{jkl}}}{u_{new_{i_{jkl}}}} \right) \qquad \forall i, j, k, l$$

vi. Find the type-2 local membership value by using following equation:

$$u_{T2_{i_{jkl}}} = \left(\frac{\sum_{i=1}^{c} \sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} u_{i_{jkl}} u_{new_{i_{jkl}}}}{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} u_{new_{i_{jkl}}}} \right)$$

vii. Find the cluster centers as follows:

$$t_i = \left(\frac{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} [\alpha \mu^m_{T2i_{jkl}} a_{jkl} + (1-\alpha) u_{T2i_{jkl}} (f_{i_{jkl}})^{-1} \bar{a_{jkl}}]}{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} [\alpha \mu^m_{T2i_{jkl}} + (1-\alpha) u_{T2i_{jkl}} (f_{i_{jkl}})^{-1}]} \right) / \bar{a_{jkl}}$$

5. *Until* $\|t_i^{(n+1)} - t_i^n\| < \varepsilon \qquad \forall i$

6. Calculate the final membership values by the following weighted membership function.

$$g_{i_{jkl}} = \left(\frac{(\mu_{T2})^p (u_{T2})^q}{\sum_{r=1}^{C} (\mu_{r_{T2}})^p (u_{r_{T2}})^q} \right) \qquad \forall i, j, k, l$$

7. Return the cluster centers $T = \{t_1, t_2, ..., t_c\}$ and memberhip matrix $G = \{g_{i_{jkl}}\}$ $\forall i, j, k, l$

8. Determine the clusters of voxels $\{a_{jkl}\}$ $\forall i, j, k, l$ as follows and return the segmented 3D image volume H_i $\forall i$:

$$\text{cluster}(a_{jkl}) = \text{argmax(i)}\{g_{i_{jkl}} \text{ where } i = 1, 2, ...C$$

Chapter 3

Experimental Results

Our proposed method is extensively studied on six simulated brain MR image volumes. T1-weighted brain image volumes and the ones typically effected by IIH are most commonly used in clinical studies, so we have used this image volume. We had taken the T1 weighted image volume as our input image containing different percentage of noise and IIH collected from Brain web database site [38].The brain image volumes are segmented in four basic clusters($C = 4$), they are -(i)Cerebrospinalfluid(CSF) , (ii)Gray Matter(GM), (iii)White Matter(WM) and (iv)Background(BG).

As mentioned in the algorithm we have to initialize some parameters, they are m,p,q and c. We initialized the parameters with $m = 2.60, p = 1, p = 3, C = 4$. The neighbourhood window of size $3 \times 3 \times 3$ is selected empirically and it is used to compute Euclidean distance $\bar{d}_{i_{jkl}}$, mean voxel $\bar{a}_{jkl}$ and to measure possibility factor $f_{i_{jkl}}$ for a voxel to belong into a cluster. This investigation is performed by analyzing some performance parameters such as (i)segmentation accuracy(SA), (ii) partition coefficient(V_{pc}), (iii)similarity index(SI) and (iv)partition entropy(V_{pe}). For better understanding we'll discuss this in the next sections.

3.1 V_{pc} Calculation:

Clustering validity functions are widely considered to evaluate the performance of a fuzzy clustering algorithm. To measure the confidence of the algorithm while classifying the parameters(here voxels) V_{pc} is the important factor among them. The V_{pc} value lies in $[0.0, 1.0]$. If the V_{pc} is 1 then the algorithm is considered as the ideal clustering algorithm. High V_{pc} is considered for the better algorithms. It is defined as follows [16]:

$$V_{pc} = \left(\frac{\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} \sum_{i=1}^{C} (g_{i_{jkl}})^2}{X \times Y \times Z} \right) \tag{3.1}$$

A comparative study is given in Table 3.1 of our method and the other comparative algorithms. Here we have considered six T1-weighted simulated brain MR image volume with 5% $-$9% noise and 20%$-$40% IIH -

Noise%-IIH%	FCM	FGFCM	sFCM	ASIFC	PFCM	2sFMoF	FCMGsLE	Our Method
5_20	0.804	0.622	0.922	0.877	0.929	0.956	0.96	**0.985**
5_40	0.797	0.613	0.917	0.874	0.925	0.954	0.96	**0.97**
7_20	0.786	0.601	0.915	0.868	0.923	0.951	0.96	**0.97**
7_40	0.786	0.589	0.908	0.851	0.915	0.946	0.96	**0.98**
9_20	0.795	0.594	0.892	0.849	0.911	0.942	0.96	**0.98**
9_40	0.793	0.586	0.89	0.842	0.912	0.94	0.96	**0.98**

Table 3.1: V_{pc} of proposed and other algorithms.

We have shown the graphical representation of the Table 3.1 in the following Figure 3.1.

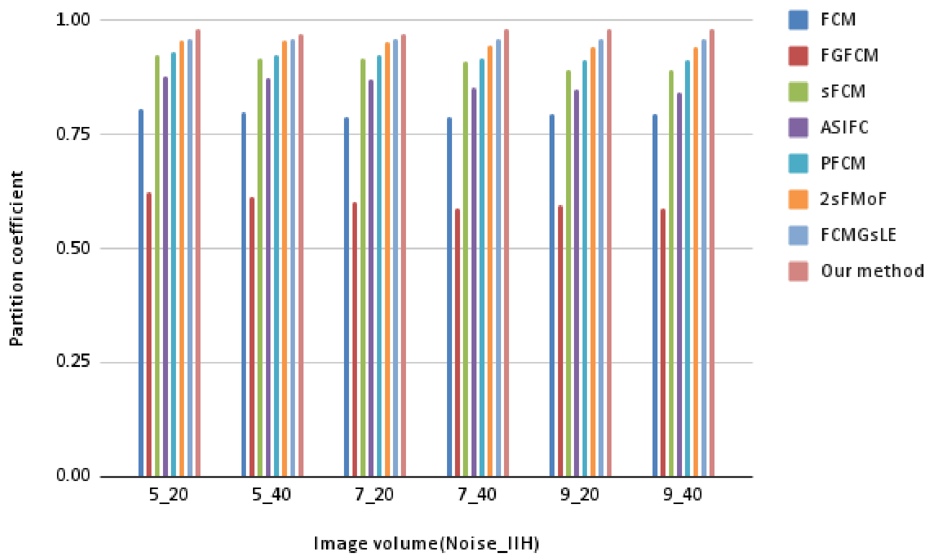


Figure 3.1: V_{pc} of proposed and other methods

3.2 V_{pe} Calculation:

V_{pe} is one the most important parameters under the clustering validity functions. While classifying the voxels into correct clusters and negating into other clusters, it measures the uncertainty. V_{pe} is 0.0 for the ideal cases and low value of V_{pe} is considered for the better algorithm. It is defined as the following:

$$V_{pe} = \left(\frac{-\sum_{l=1}^{Z} \sum_{j=1}^{X} \sum_{k=1}^{Y} \sum_{i=1}^{C} (g_{i_{jkl}} \ln g_{i_{jkl}})}{X \times Y \times Z} \right) \tag{3.2}$$

The calculated V_{pe} of our method and other algorithms are shown in Table 3.2. Here we have considered six similated brain MR image with 6 different noise and IIH.

Noise%-IIH%	FCM	FGFCM	sFCM	ASIFC	PFCM	2sFMoF	FCMGsLE	Our Method
5_20	0.307	0.621	0.143	0.284	0.0.12	**0.03**	0.043	0.04
5_40	0.317	0.628	0.151	0.288	0.119	0.042	0.043	**0.04**
7_20	0.321	0.634	0.155	0.297	0.115	0.046	0.043	**0.04**
7_40	0.314	0.645	0.148	0.285	0.098	0.946	0.043	**0.04**
9_20	0.328	0.662	0.175	0.271	0.116	0.052	0.043	**0.03**
9_40	0.327	0.672	0.171	0.265	0.123	0.054	0.043	**0.03**

Table 3.2: V_{pe} of proposed and other algorithms.

We have shown the graphical representation of the Table 3.2 in the following Figure 3.2.

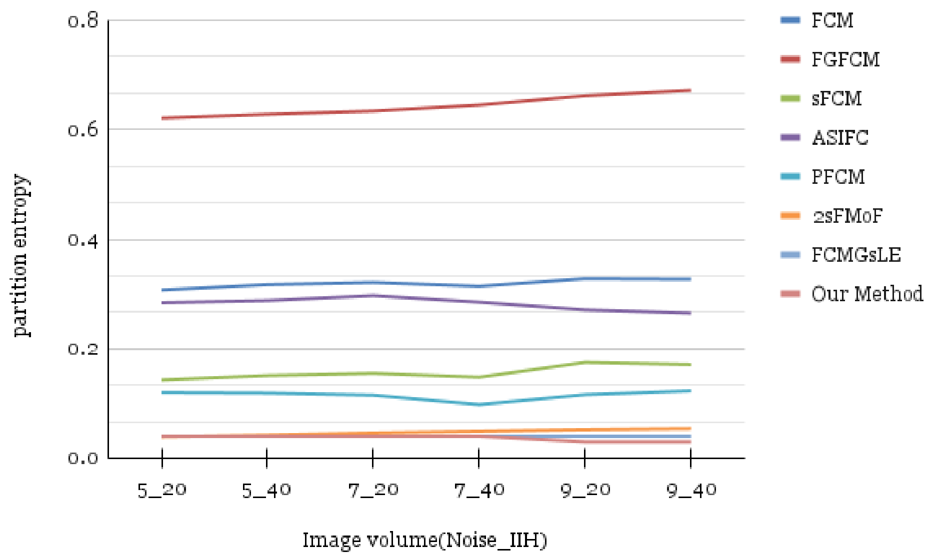


Figure 3.2: V_{pe} of proposed and other methods

3.3 Segmentation accuracy:

Segmented accuracy can defined as the ratio of total number of voxel which are correctly classified and the actual number of corresponding voxels in the ground truth. If the value of Segmented Accuracy(SA) is closer to 1.0 then the algorithm can be considered as the better algorithm. For an ideal algorithm its value is equal to 1.0. Suppose A_i is the set of voxels correctly classified by the algorithm and B_i is the set of voxels present in ground truth of the i_th cluster.Then the segmentation accuracy(SA) for cluster i can be written as $SA(i)$ and defined as follows-

$$SA(i) = \left(\frac{|A_i \cap B_i|}{B_i} \right) \tag{3.3}$$

In Table 3.3 we have shown a comparative study on SA of different algorithms. We have considered six simulated brain MR image with 5%-9% and 20%-40% IIH.

3.4 Simililarity Index:

Similarity index(SI) is a parameter by which we can measure how many pixels got it's perfect cluster as it's having in the ground truth. For the ideal algorithm the value of SI should be 100 and for the best cases it is considered to the higher value. So in the Table 3.4 we have discussed about the SI measured by the other algorithms and our method. We also represented the Table 3.4 in graphical format in Figure 3.3

Noise%-IIH%	Tissue regions	Segmentation Accuracy (SA)							
		FCM	FGFCM	sFCM	ASIFC	PFCM	2sFMoF	FCMGsLE	Our Method
5_20	CSF	0.881	0.861	0.907	0.911	0.907	0.918	0.867	**0.986**
	GM	0.834	0.828	0.916	0.919	0.879	**0.923**	**0.923**	0.752
	WM	0.848	0.941	0.938	0.946	0.959	0.968	**0.969**	0.86
5_40	CSF	0.837	0.832	0.861	0.867	0.875	0.908	0.873	**0.999**
	GM	0.825	0.821	0.909	0.911	0.837	0.919	0.911	**0.92**
	WM	0.840	0.916	0.926	0.933	0.925	**0.962**	0.932	0.89
7_20	CSF	0.819	0.816	0.852	0.859	0.836	0.901	0.876	**0.99**
	GM	0.818	0.801	0.902	0.907	0.815	0.911	**0.912**	0.825
	WM	0.829	0.909	0.912	0.918	0.949	0.954	**0.962**	0.825
7_40	CSF	0.807	0.795	.0.849	0.852	0.817	0.898	0.883	**0.99**
	GM	0.782	0.792	0.895	0.889	0.817	0.898	0.883	0.76
	WM	0.795	0.906	0.903	0.908	0.926	0.947	0.924	0.86
9_20	CSF	0.753	0.739	0.827	0.836	0.777	0.880	0.878	**0.98**
	GM	0.755	0.736	0.871	0.875	0.762	0.894	**0.902**	0.79
	WM	0.781	0.873	0.897	0.901	0.932	0.942	**0.951**	0.86
9_40	CSF	0.742	0.731	0.824	0.829	0.753	0.876	0.882	**0.995**
	GM	0.742	0.731	0.824	0.829	0.753	0.876	0.882	**0.878**
	WM	0.765	0.876	0.873	0.880	0.914	**0.923**	0.910	0.87

Table 3.3: Segmentation results of the proposed method and other algorithms

3.5 Outputs:

After applying the algorithm mentioned in section 2.3, we get the segmented images of a noisy brain MR image. From the tables Table 3.1, Table 3.2, Table 3.3, Table 3.4 we can see that in most of the cases our method performs best; but in some cases it cannot outperform the results of the previously proposed algorithms. After applying type-2 fuzzy on the algorithm that is proposed by Nabanita et al. [31], we got the following segmented images as our output in Table 3.5 and 3.6. We performed our method on six t1-weighted simulated images with different noise percentage and different IIH percentage. Among them we have presented only the results against 7_20 image volume and 9_40 image volume in Table 3.5 and 3.6.

Noise%-IIH%	FCM	FGFCM	sFCM	ASIFC	PFCM	2sFMoF	FCMGsLE	Our Method
5_20	69.39	69.17	69.03	77.84	**78.32**	78.14	75.71	75.8
5_40	68.96	69.12	68.37	77.18	77.68	77.53	75.102	**77.89**
7_20	67.27	67.79	76.52	**77.91**	77.02	77.66	75.71	74.9
7_40	66.1	66.52	75.58	75.87	76.02	**76.61**	75.831	75.7
9_20	64.21	64.54	74.29	74.78	74.86	75.12	75.52	**76.2**
9_40	63.62	63.83	73.57	73.92	73.85	74.33	75.45	**76.8**

Table 3.4: SI of proposed and other algorithms.

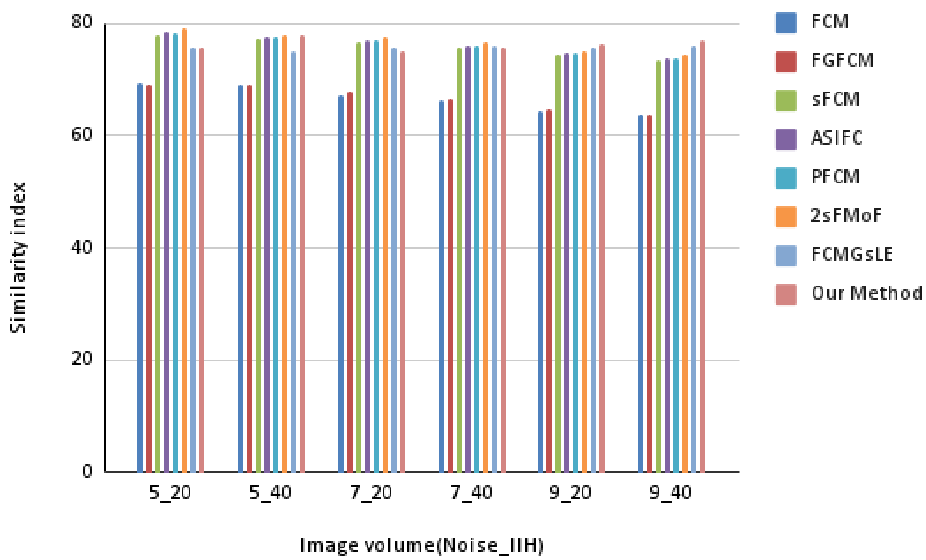
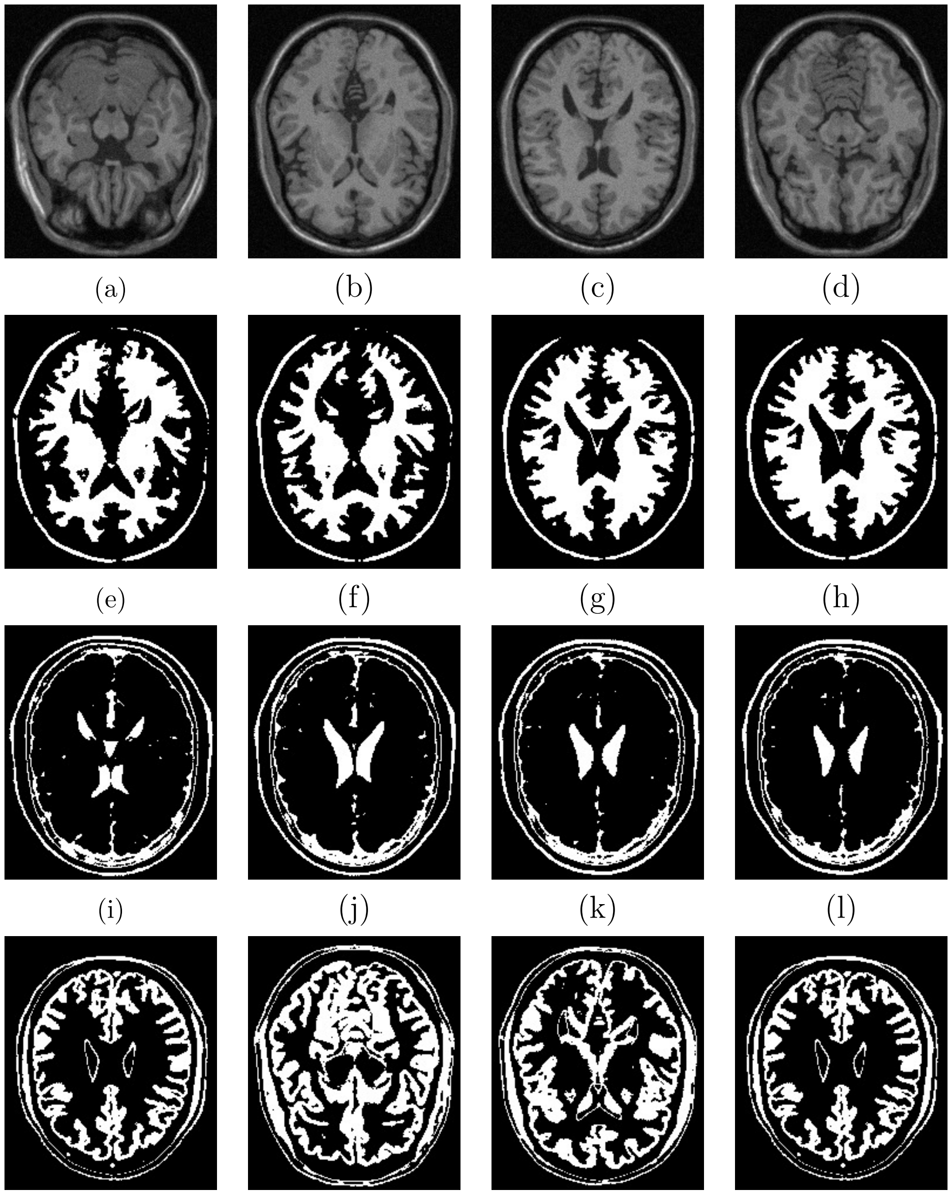


Figure 3.3: SI of proposed and other methods

3.6 Conclusion:

In conclusion, we have proposed a type-2 fuzzy clustering algorithm using global and local entropies. At the initial point our aim was to increase the efficiency of the standard fuzzy clustering method that incorporates with shannon entropy and type-2 fuzzy. Here we considered two types of entropy - (i)global entropy and (ii)local entropy. Basic idea to incorporate entropy is to minimize the uncertainties. Because, maximum uncertainty arises at the unsharp tissue boundaries. And type-2 fuzzy is applied to measure a suitable membership value for a particular cluster of a voxel. Proposed algorithm incorporates these two membership functions into the fuzzy objective function. The final membership value is obtained by the newly calculated type-2 global and local membership values. The experimental results based on the six simulated image volumes suggest that our method gives better segmentation result in many cases. In particular, in the presence of noise and high IIH ,it even yields far better results than several-sate-of-art fuzzy clustering algorithms devised reciently.



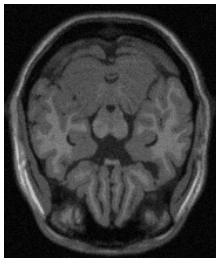
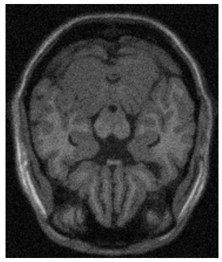
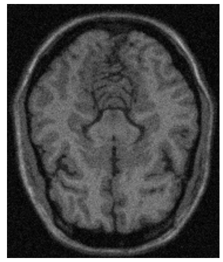
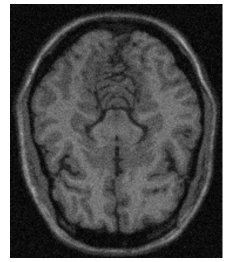
(a) (b) (c) (d)

(e) (f) (g) (h)

(i) (j) (k) (l)

(m) (n) (o) (p)

Table 3.5: (a),(b),(c),(d) are 7_20 input image volumes; (e),(f),(g),(h) are white matter(WM) segmented image; (i),(j),(k),(l) are cerebrospinal(CSF) segmented image; (m),(n),(o),(p) are gray matter(GM) fluid segmented image

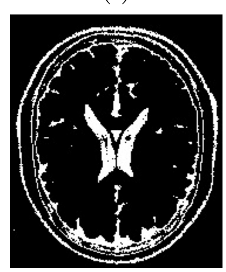
(a) (b) (c) (d)

(e) (f) (g) (h)

(i) (j) (k) (l)

(m) (n) (o) (p)

Table 3.6: (a),(b),(c),(d) are 9_40 input image volumes; (e),(f),(g),(h) are white matter(WM) segmented image; (i),(j),(k),(l) are gray matter(GM) segmented image; (m),(n),(o),(p) are cerebrospinal(CSF) fluid segmented image

Bibliography

[1] U. Vovk, F. Pernus, and B. Likar, "A review of methods for correction of intensity inhomogeneity in mri," *IEEE transactions on medical imaging*, vol. 26, no. 3, pp. 405–421, 2007.

[2] Z. Hou, "A review on mr image intensity inhomogeneity correction," *International journal of biomedical imaging*, vol. 2006, 2006.

[3] M. A. Balafar, A. R. Ramli, M. I. Saripan, and S. Mashohor, "Review of brain mri image segmentation methods," *Artificial Intelligence Review*, vol. 33, no. 3, pp. 261–274, 2010.

[4] B. Belaroussi, J. Milles, S. Carme, Y. M. Zhu, and H. Benoit-Cattin, "Intensity non-uniformity correction in mri: existing methods and their validation," *Medical image analysis*, vol. 10, no. 2, pp. 234–246, 2006.

[5] D. Kaur and Y. Kaur, "Various image segmentation techniques: a review," *International Journal of Computer Science and Mobile Computing*, vol. 3, no. 5, pp. 809–814, 2014.

[6] J. S. Duncan and N. Ayache, “Medical image analysis: Progress over two decades and the challenges ahead,” *IEEE transactions on pattern analysis and machine intelligence*, vol. 22, no. 1, pp. 85–106, 2000.

[7] I. Despotović, B. Goossens, and W. Philips, “Mri segmentation of the human brain: challenges, methods, and applications,” *Computational and mathematical methods in medicine*, vol. 2015, 2015.

[8] S. K. Adhikari, J. K. Sing, D. K. Basu, and M. Nasipuri, “Conditional spatial fuzzy c-means clustering algorithm for segmentation of mri images,” *Applied soft computing*, vol. 34, pp. 758–769, 2015.

[9] D. L. Pham, C. Xu, and J. L. Prince, “A survey of current methods in medical image segmentation,” *Annual review of biomedical engineering*, vol. 2, no. 3, pp. 315–337, 2000.

[10] S. D. Olabarriaga and A. W. Smeulders, “Interaction in the segmentation of medical images: A survey,” *Medical image analysis*, vol. 5, no. 2, pp. 127–142, 2001.

[11] L. Ma and R. C. Staunton, “A modified fuzzy c-means image segmentation algorithm for use with uneven illumination patterns,” *Pattern recognition*, vol. 40, no. 11, pp. 3005–3011, 2007.

[12] F. Tek, A. Dempster, and I. Kale, “Noise sensitivity of watershed segmentation for different connectivity: experimental study,” *Electronics Letters*, vol. 40, no. 21, pp. 1332–1333, 2004.

[13] J. C. Bezdek, R. Ehrlich, and W. Full, “Fcm: The fuzzy c-means clustering algorithm,” *Computers & geosciences*, vol. 10, no. 2-3, pp. 191–203, 1984.

[14] S. Na, L. Xumin, and G. Yong, "Research on k-means clustering algorithm: An improved k-means clustering algorithm," in *2010 Third International Symposium on intelligent information technology and security informatics*, pp. 63–67, Ieee, 2010.

[15] W. Pedrycz, "Conditional fuzzy c-means," *Pattern Recognition Letters*, vol. 17, no. 6, pp. 625–631, 1996.

[16] C. Qiu, J. Xiao, L. Yu, L. Han, and M. N. Iqbal, "A modified interval type-2 fuzzy c-means algorithm with application in mr image segmentation," *Pattern Recognition Letters*, vol. 34, no. 12, pp. 1329–1338, 2013.

[17] Y. A. Tolias and S. M. Panas, "Image segmentation by a fuzzy clustering algorithm using adaptive spatially constrained membership functions," *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, vol. 28, no. 3, pp. 359–369, 1998.

[18] N. A. Mohamed, M. N. Ahmed, and A. Farag, "Modified fuzzy c-mean in medical image segmentation," in *1999 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings. ICASSP99 (Cat. No. 99CH36258)*, vol. 6, pp. 3429–3432, IEEE, 1999.

[19] M. N. Ahmed, S. M. Yamany, N. Mohamed, A. A. Farag, and T. Moriarty, "A modified fuzzy c-means algorithm for bias field estimation and segmentation of mri data," *IEEE transactions on medical imaging*, vol. 21, no. 3, pp. 193–199, 2002.

[20] A. Liew, S. Leung, and W. Lau, "Fuzzy image clustering incorporating spatial continuity," *IEE Proceedings-Vision, Image and Signal Processing*, vol. 147, no. 2, pp. 185–192, 2000.

[21] D. L. Pham and J. L. Prince, "An adaptive fuzzy c-means algorithm for image segmentation in the presence of intensity inhomogeneities," *Pattern recognition letters*, vol. 20, no. 1, pp. 57–68, 1999.

[22] A. W.-C. Liew and H. Yan, "An adaptive spatial fuzzy clustering algorithm for 3-d mr image segmentation," *IEEE transactions on medical imaging*, vol. 22, no. 9, pp. 1063–1075, 2003.

[23] X.-Y. Wang and J. Bu, "A fast and robust image segmentation using fcm with spatial information," *Digital Signal Processing*, vol. 20, no. 4, pp. 1173–1182, 2010.

[24] S. Krinidis and V. Chatzis, "A robust fuzzy local information c-means clustering algorithm," *IEEE transactions on image processing*, vol. 19, no. 5, pp. 1328–1337, 2010.

[25] J. Noordam, W. Van Den Broek, and L. Buydens, "Multivariate image segmentation with cluster size insensitive fuzzy c-means," *Chemometrics and intelligent laboratory systems*, vol. 64, no. 1, pp. 65–78, 2002.

[26] S. Chen and D. Zhang, "Robust image segmentation using fcm with spatial constraints based on new kernel-induced distance measure," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 4, pp. 1907–1916, 2004.

[27] W. Cai, S. Chen, and D. Zhang, "Fast and robust fuzzy c-means clustering algorithms incorporating local information for image segmentation," *Pattern recognition*, vol. 40, no. 3, pp. 825–838, 2007.

[28] Z. Wang, Q. Song, Y. C. Soh, and K. Sim, "An adaptive spatial information-theoretic fuzzy clustering algorithm for image segmentation," *Computer Vision and Image Understanding*, vol. 117, no. 10, pp. 1412–1420, 2013.

[29] K.-S. Chuang, H.-L. Tzeng, S. Chen, J. Wu, and T.-J. Chen, "Fuzzy c-means clustering with spatial information for image segmentation," *computerized medical imaging and graphics*, vol. 30, no. 1, pp. 9–15, 2006.

[30] S. K. Adhikari, J. Sing, D. Basu, M. Nasipuri, and P. Saha, "Segmentation of mri brain images by incorporating intensity inhomogeneity and spatial information using probabilistic fuzzy c-means clustering algorithm," in *2012 International Conference on Communications, Devices and Intelligent Systems (CODIS)*, pp. 129–132, IEEE, 2012.

[31] N. Mahata and J. K. Sing, "A novel fuzzy clustering algorithm by minimizing global and spatially constrained likelihood-based local entropies for noisy 3d brain mr image segmentation," *Applied Soft Computing*, vol. 90, p. 106171, 2020.

[32] I. Levner and H. Zhang, "Classification-driven watershed segmentation," *IEEE Transactions on Image Processing*, vol. 16, no. 5, pp. 1437–1445, 2007.

[33] V. Pandey and V. Gupta, "Mri image segmentation using shannon and non shannon entropy measures," *International Journal of Application or Innovation in Engineering & Management*, vol. 3, no. 7, pp. 41–46, 2014.

[34] M. S. Atkins and B. T. Mackiewich, "Fully automatic segmentation of the brain in mri," *IEEE transactions on medical imaging*, vol. 17, no. 1, pp. 98–107, 1998.

[35] S. Sadek and S. Abdel-Khalek, "Generalized α-entropy based medical image segmentation," *Journal of Software Engineering and Applications*, vol. 2014, 2013.

[36] C. Lang and H. Jia, "Kapur's entropy for color image segmentation based on a hybrid whale optimization algorithm," *Entropy*, vol. 21, no. 3, p. 318, 2019.

[37] J. M. Mendel and I. Robert, "John. type-2 fuzzy sets made simple," *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp. 117–127, 2002.

[38] Brainweb_Brain_Database:, "https://brainweb.bic.mni.mcgill.ca/,"