

# **Circular Antenna Array for direction finding applications at 2.45 GHz**

**Thesis Submitted in Partial Fulfilment of the  
Requirement For the Degree of**

**Master of Engineering  
in  
Electronics and Tele-Communication Engineering**

**Thesis Submitted By**

**SAURAV KUMAR ROY**

**University Registration No.:154085 of 2020-2021**

**Class Roll No.: 002010702016**

**Exam Roll No.: M4ETC22016**

**UNDER THE GUIDANCE OF**

**PROFESSOR SAYAN CHATTERJEE**

**Department of Electronics and Tele-Communication Engineering**

**Jadavpur University, Kolkata – 700 032, India**

**2022**

## CERTIFICATE OF RECOMMENDATION

This is to certify that the thesis entitled “Circular Antenna Array for direction finding applications at 2.45 GHz” submitted by Saurav Kumar Roy under my guidance and supervision and be accepted in partial fulfilment of the requirement for awarding the degree of **Master of Engineering in Electronics and Telecommunication Engineering** of Jadavpur University. The research results presented in this thesis have not been included in other paper submitted for the award of any degree to any other Institute or University.

*Sayan Chatterjee 23/8/22*

**Prof. Sayan Chatterjee**

**Thesis Supervisor**

Dept. Electronics and Telecommunication Engineering  
Jadavpur University  
Kolkata 700 032, West Bengal, India

**Prof. Sayan Chatterjee**  
Electronics & Telecomm Engg. Dept.  
Jadavpur University  
Kolkata-700032

*Manotosh Biswas 23/08/22*

**Head of the Department**

Dept. Electronics and Telecommunication Engineering  
Jadavpur University  
Kolkata 700 032, West Bengal, India

**MANOTOSH BISWAS**  
Professor and Head  
Electronics and Telecommunication Engineering  
Jadavpur University, Kolkata - 32

*Manotosh Biswas 23/8/22*

**Dean**

Faculty of Engineering and Technology  
Jadavpur University  
Kolkata 700 032, West Bengal, India



**DEAN**  
Faculty of Engineering & Technology  
JADAVPUR UNIVERSITY  
KOLKATA-700 032

## **CERTIFICATE OF APPROVAL<sup>#</sup>**

The foregoing thesis is hereby approved as a credible study of an Engineering Subject carried out and presented in a manner of satisfactory to warrant its acceptance as a pre-requisite to the degree for which it has been submitted. It is to be understood that by this approval, the undersigned do not necessarily endorse or approve any statement made, opinion expressed, or conclusion drawn therein but approve the thesis only for the purpose for which it has been submitted.

Committee on final examination  
for the evaluation of the Thesis

---

(Signature of the Supervisor)

---

(Signature of the Examiner)

<sup>#</sup> Only in case the thesis is approved

## **DECLARATION OF ORIGINALITY AND COMPLIANCE OF ACADEMIC ETHICS**

I hereby declare that this thesis contains literature survey and original research work done by the undersigned candidate, as a part of his degree of **Master of Engineering in Electronics and Telecommunication Engineering** of Jadavpur University. All the information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that as required by these rules and conduct, I have fully cited and referenced all materials and results that are not original to this work.

Date: 23/08/2022

Saurav Kumar Roy

(Saurav Kumar Roy)

University Reg. No. – 154085 of 2020-2021

Class Roll No. – 002010702016

Exam Roll No. – M4ETC22016

# *Acknowledgement*

---

First, I wish to take opportunity to express gratitude to Prof. Sayan Chatterjee, my supervisor of Master degree thesis for his advice and invaluable guidance throughout my research. This work would have been impossible without his precious support and encouragement. I would also like to thank everybody who came in touch during my work.

I am grateful to all the faculty members of the Electronics and Tele-Communication Engineering Department, Jadavpur University, for getting some valuable advice to gather knowledge in different subjects included in syllabus of Master of Engineering and get success in each semester exams. Suggestions from Prof. B. Gupta, Prof. M. Biswas, Prof. S. Ray, Prof. A. Konar and Prof. C. Bose have helped me in all the time.

This thesis would not have been completed without help and co - operation of all my seniors of JU Microwave Lab. I also take advantage to acknowledge Mr. Joydeep Pal, Mr. Soham Ghosh, Dr. Ayona Chakraborty, Mr. Ardhendu Kundu, Mr. Sayan Sarkar and all other members of Microstrip Lab, Jadavpur University for their technical discussions which was really encouraging to enhance my interest in this topic.

Lastly, but more importantly, I would like to thank all my family members and well-wishers who always encourage me to pursue my studies, stay with me and

hold my hands in all ups and downs up to this journey in my life and motivate me to overcome all difficulties. I also like to acknowledge AICTE (All India Council for Technical Education) for providing me PG Scholarship to pursue my Post Graduate Degree course in duration of 2020-2022. Without their blessings, I could not complete this work successfully with little smile in my face.

Date: 23/08/2022

Saurav Kumar Roy

(Saurav Kumar Roy)

# Contents

---

---

	Page No.
Acknowledgement	i
Contents	iii
List of Figures	vi
List of Tables	ix
Abstract	xi
<b>Chapter 1 Introduction</b>	<b>1 - 4</b>
1.1 Preface	1 – 2
1.2 Motivation of the Thesis	2
1.3 Objective of the Thesis	3
1.4 Thesis Outline	3 - 4
References	4
<b>Chapter 2 Literature Review</b>	<b>5 – 18</b>
2.1 Introduction	5
2.2 Linear Arrays	5 – 9
2.3 Circular Arrays	9 – 14
References	14 - 18
<b>Chapter 3 Linear Array</b>	<b>19 - 36</b>
3.1 Introduction	19 – 20

3.2 N-Element Linear Array	20 – 23
3.3 Broadside Arrays	23
3.4 End-Fire Arrays	23
3.5 Phased Scanning Array	24
3.6 Non- Uniform Amplitude Linear Array	24 - 25
3.7 Calculation of Array Factor for Binomial and Chebyshev Arrays	25
3.8 Array Factor Plot	26 – 35
3.9 Conclusion	36
References	36
<b>Chapter 4 Circular Array</b>	<b>37 - 48</b>
4.1 Introduction	37
4.2 Array Factor Calculation	38 – 41
4.3 Array Factor Plot	41 – 46
4.4 Conclusion	46
References	46 - 48
<b>Chapter 5 Directive Antennas in Uniform Circular Arrays</b>	<b>49 - 68</b>
5.1 Introduction	49
5.2 Design Specifications for a Single Element Patch Antenna	50 – 53
5.2 Uniform Circular Array Design	53 – 59
5.3 Feed Network Design	60 – 65
5.4 Circular Array with Feed Network	66 – 67



5.5 Conclusion	67
References	67 - 68
<b>Chapter 11 Conclusion and Future Scope</b>	<b>69 – 70</b>
6.1 Conclusion	69 - 70
6.2 Future Scope	70
<b>Appendix</b>	<b>71-79</b>

# List of Figures

Figure 3.1: N-element Linear Array of point sources positioned along the x-axis.....	21
Figure 3.2 Array Factor pattern of a 10-element uniform amplitude broadside array for $d = \lambda/4$ .....	26
Figure 3.3 Array Factor pattern of a 10-element uniform amplitude broadside array for $d = \lambda$ .....	27
Figure 3.4 Array Factor pattern of a 10-element uniform amplitude end-fire array for $d = \lambda/4, \beta = -kd$ .....	28
Figure 3.5 Array Factor pattern of a 10-element uniform amplitude end-fire array for $d = \lambda/4, \beta = kd$ .....	29
Figure 3.6 Array Factor pattern of a 10-element uniform amplitude end-fire array for $d = \lambda/4, \beta = -kd \cos 60^\circ$ .....	30
Figure 3.7 Pascal Triangle used for finding the amplitude coefficients for the array.....	31
Figure 3.8 Pascal Triangle row corresponding to $m=6$ .....	31
Figure 3.8 Pascal Triangle row corresponding to $m=7$ .....	32
Figure 3.10 Code for generation of Pascal triangle in MATLAB.....	32
Figure 3.11 Array Factor pattern of a 10-element binomial array for $d = \lambda/4, N=10$ .....	33
Figure 3.12 Array Factor pattern of a 10-element binomial array for $d = \lambda, N=10$ .....	33
Figure 3.13 Array Factor pattern of a 10-element Dolph-Tschebyscheff array for $d = \lambda/4, N=10$ using Barbieri calculations.....	35

Figure 3.14 Array Factor pattern of a 10-element Dolph-Tschebyscheff array for $d=\lambda/4$ , $N=10$ using Chebwin function.....	35
Figure 4.1 Geometry of an N-Element Circular Array.....	37
Figure 4.2 A infinitesimal linear dipole positioned along the z-axis.....	38
Figure 4.3 Radiation patterns for a uniform circular array for $\theta=\theta_0=\pi/2$ , $\phi_0=0$ for $N=8$ for zero order Bessel function.....	42
Figure 4.4 Radiation patterns for a uniform circular array for $\theta=\theta_0=\pi/2$ , $\phi_0=0$ for $N=8$ for zero order Bessel function.....	43
Figure 4.5 Radiation patterns for a uniform circular array for $\theta_0=0$ , $\phi_0=0$ for $N=8$ for zero order Bessel function.....	43
Figure 4.6 Radiation pattern for a uniform circular array for $\theta_0=0$ , $\phi_0=0$ for $N=8$ for zero order Bessel function.....	44
Figure 4.7 Radiation pattern for a uniform circular array for $\theta=\theta_0=\pi/2$ , $\phi_0=0$ or xz plane for $N=8$ for Equation (4.9) .....	44
Figure 4.8 Radiation pattern for a uniform circular array for $\theta=\theta_0=\pi/2$ , $\phi_0=0$ or x-z plane for $N=8$ for Equation (4.9) .....	45
Figure 4.9 Radiation patterns for a uniform circular array for $\theta_0=0$ , $\phi_0=0$ for $N=8$ for Equation (4.9) .....	45
Figure 4.10 Radiation patterns for a uniform circular array for $\theta_0=0$ , $\phi_0=0$ for $N=8$ for Equation (4.9) .....	46
Figure 5.1 Matching Technique: $\lambda/4$ impedance transformer.....	50
Figure 5.2 Top View of a single patch element.....	51
Figure 5.3 Return Loss Plot of single element patch antenna at 2.45 GHz.....	52
Figure 5.4 Four-Element UCA.....	53
Figure 5.5 Return Loss Plot for the Four Element Circular Array.....	54
Figure 5.6 Radiation Pattern for the combined four patch elements for $R=50$ mm.....	55

Figure 5.7 Radiation Pattern of the four element circular array for radius (a) R=70 mm (b) R=80 mm (c) R=90 mm (d) R=100 mm.....	55 - 57
Figure 5.8 Radiation Pattern of the four element circular array for radius (a) R=120 mm (b) R=150 mm.....	57 - 58
Figure 5.9 Eight-Element UCA.....	59
Figure 5.10 Radiation Pattern of the four element circular array for radius 120 mm.....	59
Figure 5.11 An equal-split Wilkinson power divider in microstrip Line form.....	60
Figure 5.12 Return Loss plot for the 2:1 power divider.....	61
Figure 5.13 Insertion Loss plot for the 2:1 power divider.....	61
Figure 5.14 A four way Wilkinson power divider in microstrip Line form.....	62
Figure 5.15 Return Loss and Insertion Loss plot for the 4:1 power divider.....	62
Figure 5.16 An eight way Wilkinson power divider in microstrip Line form.....	63
Figure 5.17 Return Loss and Insertion Loss plot for the 8:1 power divider.....	63
Figure 5.18 A Symmetrical Feed network design for 8 Element circular array.....	64
Figure 5.19 Return Loss and Insertion Loss plot for the Symmetrical Feed network design of an eight power divider.....	65
Figure 5.20 Eight-Element Circular array with feed network design.....	65
Figure 5.21 Return Loss plot for the eight element circular array with symmetrical feed network design.....	66
Figure 5.22 Radiation pattern for eight element circular array with symmetrical feed network design at $\theta=90^\circ$ plane at 2.5 GHz.....	66

# List of Tables

Table 5.1 Design specifications for single patch element at 2.45 GHz.....52

Table 5.2. MICROSTRIP LINE IMPEDANCE DIMENSION.....64



# Abstract

Microstrip Patch Antennas are widely used in modern wireless communication systems due to their compact size and low cost. These patch antennas have planar structure and are low-profile and thus are very easy to fabricate and so are very popular.

The objective of this thesis is to present a new single fed uniform circular array microstrip antenna. To obtain this new antenna design a study is performed to design the antenna array using microstrip antenna with four elements and eight elements. This thesis is concerned with the design of a circular array microstrip antenna that would operate in the 2.45 GHz range. This range is commonly used by wireless local area devices and wireless personal area devices such as the 802.11 WIFI and mainly used for Wi-Fi and phone or Bluetooth applications.

Ring arrays or circular arrays have been used in radio direction finding, radar, sonar, and many other system applications. The uniform circular arrays (UCA) have a major advantage because of the symmetry property of the UCA. It has the ability to scan a beam azimuthally through  $360^{\circ}$  with little change in either the beamwidth or the sidelobe level. The Root-MUSIC algorithm is used for 2-D direction-of-arrival (DOA) estimation of azimuth and elevation angle with uniform circular arrays for direction finding.

This thesis presented focused on designing circular array of microstrip antenna with feed network operating at 2.45 GHz. This thesis presented a design of symmetrical feed network of eight way power divider for the eight element circular array. The circular array is designed and simulated with microstrip antenna in CST Microwave Studio 2019.

The microstrip antenna is designed with microstrip line feed. In microstrip feed, the patch is fed by a microstrip line that is located on the same plane as the patch. A

matching network between the feed line and the patch such as quarter wavelength transformer.

The parametric study of circular array by varying the radius of the circular array is done. It has been found that the radiation pattern of the eight element uniform circular array shows  $360^\circ$  coverage with almost no side lobes for the radius (equal to  $= \lambda_0$ ). The circular array of four microstrip antenna is first designed and the results show that the circular array provides  $360^\circ$  scanning but has significant side lobes. The feed network for the eight element circular array is then designed in microstrip line form. A symmetrical feed network design for eight way power divider microstrip line form is also designed. Lastly, the combined circular array and the feed network design is done to obtain a single fed microstrip circular antenna array.



# Chapter 1

## Introduction

---

### 1.1 PREFACE

An antenna array is group of antennas connected and arranged in a regular structure to form a single antenna that can produce radiation patterns not produced by individual antennas. A single element antenna has relatively low directivity characteristics. In many applications, there is a need for antennas with high directivity or gain to fulfil the requirements of long-range communication systems.

One way of achieving the long-distance communication problem is to make the electrical size of the antenna large. If the dimensions of the antenna are enlarged, then higher gain and directive properties for the antenna design will be achieved. Researchers have tackled this problem by giving a situation that the directive properties can be achieved by not only increasing the size but also by considering group of individual similar antennas called Antenna arrays.

An antenna array is a radiating system, which consists of two or more antenna elements. Each of these elements while functioning, has its own field. The elements can be placed with equal distance or unequal distance between them. Therefore, the radiation pattern produced by it would be the vector sum of the individual ones.

The radiation pattern will depend upon the distance between the elements, excitation phase and the excitation amplitude of the individual elements. To provide very directive patterns, it is important that the fields from the individual elements of the antenna array interfere constructively in the desired direction and

cancel each other in the remaining space [1]. The antenna array in which the elements are placed in a line are called linear antenna array.

A uniform circular Array is an array in which the elements are placed together in a circular ring fashion or in a circular ring geometry. The study of uniform circular arrays (UCA) is done because of its main principal advantages such as to get a  $360^0$  beam coverage around the azimuthal plane. Because a UCA does not have edge elements, directional patterns synthesized by this geometry can be electronically scanned in the azimuthal plane without a significant change in beam shape.

There have not been many applications for circular arrays since then despite technological advancements. Although there has been some now signs of increase of interest in such arrays for applications to DF (Direction Finding) and ESM (Electronic Support Measures) systems.

The lack of applications in the past is probably due to the fact that the basic problems of exciting circular arrays with the correct values of amplitude and phase are, in general, more complex than for linear arrays. Furthermore, electronic scanning of directional patterns for circular arrays may be difficult to implement and can require both the amplitude and phase of each element of the array to be changed [2].

Over the years the application of circular array antenna includes radio direction finding, air and space navigation, underground propagation, radar, sonar, wireless communication systems and smart antennas [3,4].

## **1.2 MOTIVATION OF THE THESIS**

A uniform circular array has a unique advantage because of its symmetry property. It can scan a beam azimuthally through  $360^0$ . But designing a circular array antenna is challenging and it requires combination of design steps. The circular array is designed using patch antennas.

### **1.3 Objective of the Thesis**

- Array Factor Pattern plot of different types of linear arrays.
- Array Factor Pattern plot of a uniform circular array
- The objective of this thesis is to present a new single fed uniform circular array using directive elements.
- Design of symmetrical feed network of eight-way power divider for the eight-element circular array in microstrip line form.

### **1.4 THESIS OUTLINE**

This thesis presents the design of single fed uniform circular array microstrip antenna at 2.45 GHz. Chapter 1 gives the brief introduction related to circular array.

Chapter 2 gives chronological literature survey related to linear and circular array. Applications related to circular array and direction of arrival estimation for direction finding using circular array are discussed in this chapter.

Chapter 3 gives the brief concepts of linear array and array factor pattern plot of broadside array, end-fire array, phased (scanning) array, binomial array and Dolph Chebyshev array.

Chapter 4 gives the concepts of uniform circular array. The array has uniform spacing and the elements have uniform excitation. The individual elements are isotropic in nature. The Array Factor Pattern plot of a uniform circular array is studied and plotted using MATLAB software.

Chapter 5 concentrates on the work done related to the thesis. A new single fed uniform circular array microstrip antenna and symmetrical feed

network of eight-way power divider for the eight-element circular array using patch antennas is designed and discussed in this chapter.

In the last chapter, conclusions and future study are discussed.

The MATLAB codes of the linear arrays and uniform circular array is given in Appendix.

## **REFERENCES**

- [1] C. A. Balanis, “*Antenna Theory Analysis and Design*”, John Wiley and Sons, 2016.
- [2] D. E. N. Davies, “Circular Arrays,” of “*The Handbook of Antenna Design*”, vol. 2, no. 12, pp. 299-329, Steven Peregrinus, Stevenage, 1983.
- [3] Page, H. Ring aerial systems, *Wireless Engineers*, Vol. 25, No. 301, pp. 308-314, October 1948.
- [4] Wild, J. P. Circular aerial arrays for radio astronomy, *Proc. Royal Soc.* (London) Ser. A Vol. 262, No. 1308, pp. 84-99, June 1961.

# Chapter 2

## Literature Survey

---

### 2.1 Introduction

An antenna array (or array antenna) is a set of multiple connected antennas which work together as a single antenna, to transmit or receive radio waves. An antenna array can achieve higher gain, that is a narrower beam of radio waves, than could be achieved by a single element. Arrays can be arranged in different geometries such as Linear (straight line), rectangular or planar, circular ring, cylindrical, concentric circles or rings, etc. So, in this section some previous works done in the field of linear arrays and circular arrays are mentioned.

### 2.2 Linear Arrays:

Enormous amount of work has been carried out on Antenna arrays for different applications. Godara in his article [1] described the applications of antenna array in mobile communication. This paper brings together almost all aspects of array signal processing. Array processing involves manipulation of signals induced on various antenna elements. Array processing is expected to play an important role in fulfilling the increased demands of various mobile communications services. This paper shows how an array could be utilized in different configurations to improve the performance of mobile communications systems. This paper provides comprehensive and detailed studies of different beam-forming schemes, adaptive algorithms to adjust the required weighting on antennas, direction-of-arrival estimation methods. The effects of errors on the performance of an array system, as well as schemes to alleviate them are also described.

Further, Shafai et.al [2] proposed a dual-band dual-polarized perforated microstrip antennas for synthetic aperture radar (SAR) applications. The operating frequencies are in the L and C-bands, centered about 1.275 and 5.3 GHz, respectively. The array elements and the interelement spacing are adjusted accordingly to the L and C-bands. The L-band elements are selected as perforated patches to enable the placement of C-band elements within them. In L-band a balanced transmission line feed was used to minimize cross polarization and in C-band symmetric parasitic slots are used to minimize cross polarization.

Razavilar et.al [3] proposed a paper on improving traffic in wireless communication using adaptive arrays. The paper highlights the use of adaptive antenna arrays for improving the traffic characteristics. The paper combines the effects of the digital signal processing at the physical layer and the traffic policies at the network layer on the overall queuing model of a cell. Each cell is modelled by a multiserver service facility, where each server is a beam formed channel formed by the cell's base station. Then from the cell model, the solutions for blocking probabilities of the calls and total carried traffic in a wireless network are found out with the adaptive arrays.

Patnaik et.al [4] have given us a method using artificial neural networks to find faults in a linear antenna array. The network takes samples of radiation pattern of the array with fault elements and maps it to the location of the faulty element in that array.

Hossein et.al [5] proposed a paper which talks about a linear array ultra-wideband (uwb) antenna for radar target detection applications. A single and a 4 element array compact microstrip fed antenna array is designed here basically. Also, from the observations made by the authors there is a very good similarities of parameters of the designed prototype and the simulated one. They have tried to achieve a high gain and they have increased the array gain to about 6 dBi as in comparison to the single element. This paper talks about constructing a  $1 \times 4$

linear antenna arrays in order to achieve a better more directive pattern with high gain and low SLL. First a single element is designed. The antenna contains a tapered microstrip line, a semi-elliptical radiating patch, and a defected ground structure. The substrate chosen here is TACONIC TLC-30. the antenna can give a nearly omnidirectional characteristic in the H-plane and quasi omnidirectional pattern in the E-plane. The antenna exhibits stable radiation patterns and a perfect cross polar isolation in the entire band of operation. The measured S11 for both simulated and measured design is under -10 dB line in the frequency range of 3.1 GHz to 10.6 GHz, required for UWB systems. Also, by applying DolphChebyshev method to the array, a 10 dB SLL improvement is obtained in comparison to the case of uniform distribution. The radiation pattern intensity is increased noticeably for both E-plane and H-plane, for array configuration in comparison to the single element design in the whole UWB band. So, in general the directivity increases. Also, it is found out that directivity in H-plane is enhanced than E-plane.

For improving the wide-angle scanning performance of the phased array antennas, a wide beam microstrip antenna with metal walls is proposed in this paper by Guangwei et.al [6]. The beamwidth of a microstrip antenna is broadened by adding the metal walls. In this paper, they have proposed a novel method to achieve excellent wide-angle scan performance without using complex antenna element structure. In this paper, they have proposed a novel method to achieve excellent wide-angle scan performance without using complex antenna element structure. The wide beam and small size antenna element are designed, and the performance of the antenna element is studied. The wide-angle scanning performance of the array antenna is analysed, which demonstrates that the wide beam and small size antenna element achieves a wide beam radiation pattern and contributes to the wide scan coverage of the phased array. The half-power beamwidth (HPBW) of the E- and H-planes are  $221^\circ$  and  $168^\circ$  at 4.0 GHz. Furthermore, the wide beam antenna element is employed in a 9-element E-plane

linear array antenna. The main beam of the E-plane scanning linear array antenna can scan from  $-70^\circ$  to  $+70^\circ$  in the frequency band from 3.7 to 4.3 GHz with a gain fluctuation less than 2.7 dB and variation in maximum side lobe level (SLL) less than  $-5.8$  dB. A linear array antenna with nine elements is fabricated and tested. The measured results achieve a good agreement with the simulated one. HFSS software is used for simulation.

Here, two wide-angle scanning linear array antennas (E and H-plane scanning linear array antenna) are studied by Guangwei et.al [7]. A wide beamwidth U-shaped microstrip antenna with the electric walls is designed so as to improve the wide angle scanning performance of the phased array system. These wide-angle scanning linear array antennas were studied in the frequency band from 3.2 to 3.8 GHz. The bandwidth of the antenna is broadened by the U-shaped slot. The electric walls are used to broaden the beamwidth of the Page 12 of 36 antenna. An excellent wide-angle scanning performance in the H- and E-plane scanning linear array is obtained. The H- and E-planes scanning linear array antennas with nine elements are fabricated and tested. The measured results have a good agreement with the simulation results. The inter-element spacing is about  $0.4\lambda$  at 3.5 GHz.

The design of an electronically steered antenna array at X band frequency was presented by Jaroš et.al [8]. The work is focused on the development of a new type of RADAR able to detect the unmanaged aerial vehicles (UAVs). The proposed array was designed for maximum 64 number of elements. Sixth order Taylor distribution is chosen to synthesis of the array elements.

In this paper, a wideband antenna array is proposed with low gain reduction. Here, the authors, Yang et.al [9] have designed a low-profile phased antenna array which has high or wide scanning ability. The operating bandwidth is also made wide with this approach. Basically, they have improved the wide-angle scanning impedance matching (WAIM). The air cavity has been embedded



in the dielectric substrate and they have then tried to change the height of the air-cavity in the substrate for realizing better wide-angle scanning impedance matching (WAIM). This has also made. The wide-angle scanning capability is analysed and verified by designing and testing a linear array antenna and a planar antenna array. This wideband array with large scanning coverage and large bandwidth is promising for wireless communications or 5G mobile communications.

### **2.3 Circular Arrays**

Ring arrays have been used in radio direction finding, radar, sonar, and many other system applications [10,11]. Early significant contributions on this subject were made by DuHamel [12], who synthesized a single-ring array, with or without a concentric cylindrical reflector, to produce a Dolph-Chebyshev type of pattern.

Ramirez et.al [13] proposed a single-feed circularly polarized ring antenna array. The antenna is fed by a 50- feedline embedded in the middle of a 62 mil substrate with dielectric constant of 2.2. Ring antenna with inner stubs produces circular polarization utilizing just one microstrip feedline. The length of the inner stubs controls the quality of the circular polarization. Polarization of either sense is controlled by the angle between the inner stubs and the embedded feedline. The axial ratio bandwidth is 1.0% and the VSWR bandwidth is 6.1%. The frequency response of a four element array was shown to be similar to the single element. It was shown that the sequential rotation feeding technique increased the AR and VSWR bandwidths to 6.1% and 18% for a four-element array. Dual polarization with one antenna element using two independent orthogonal feeds was also presented.

Michael in his article [14] proposed the application of space-time adaptive processing (STAP) for airborne early warning (AEW) radar which can be applied

to circular arrays as electronically scanned circular arrays have advantages in terms of angular and temporal coverage and mechanical simplicity because it does not need to rotate. This paper details the performance of STAP when applied to a circular array.

The design and development of a dividing/phasing network for a compact switched-beam array antenna for land-vehicle mobile satellite communications was reported in [15]. The device is formed by a switched radial divider/combiner and 1-bit phase shifters and generates a sufficient number of beams for the proper satellite tracking. It uses inexpensive substrates and low-cost UHF switching diodes and hence its manufacturing cost is very low.

An approach called global matched filter was proposed in [16] by Fuchs for direction of arrival (DOA) estimation with uniform circular arrays that uses the input values of a small number of uniformly spaced beams and apply a model-fitting approach taking into account the statistical properties of the beams. It further applies when the number of sources exceeds the number of sensors: a situation that cannot be handled by standard high resolution (HR) techniques.

Krairiksh et.al [17] designed a compact and low cost phased array antenna using a circular array of four circular microstrip antennas and four 1-bit phase shifters and a four to one Wilkinson power combiner. The main beam of the antenna can be switched in four directions with the gain of about 4 dBi in each main-beam direction.

Chung et.al [18] designed a four element circularly polarized high- $T_c$  superconducting (HTS) microstrip antenna array with corporate feed at 11.67 GHz. The system was designed for satellite broadcasting system built on a 0.5 mm thick MgO substrate. The results showed that high-temperature superconductors, when used in microstrip arrays, improved the efficiency of the circularly polarized HTS antenna array. The measured return loss of our HTS antenna array was 35.79 dB at the resonant frequency of 11.67 GHz.

A novel wavelength selective reflector for planar lightwave technology based on a circular array of coupled microring resonators was proposed by Joyce [19]. A circular array of ring resonators, when coupled to a waveguide, offers a new alternative to a wavelength-selective reflector for planar lightwave circuits. The width of the reflection peaks can be narrowed by decreasing inter-resonator coupling or by using resonators of varying sizes. The advantage of this reflector is that it can be readily fabricated in an integrated optical circuit without any additional processing steps. Moreover, the reflector can be made tunable by using the thermal-optic or electrooptic effect.

Ioannides et.al [20] conducted studies on the performance of smart antennas with uniform circular arrays (UCA) because of the symmetry property of the UCA. It is observed that UCA has a major advantage, the ability to scan a beam azimuthally through 360 with little change in either the beamwidth or the sidelobe level. The adaptive beamforming property of the smart antennas was presented in this letter.

A new hybrid algorithm of RANk REDuction (RARE) and Root-MUSIC algorithm for 2-D direction-of-arrival (DOA) estimation of azimuth and elevation angle for uniform circular arrays in the presence of mutual coupling proposed in [21]. The UCA-RARE algorithm is applied to estimate the azimuth angle independent from the elevation angle. Next, for each azimuth angle a new search-free rooting algorithm is performed based on the expansion of the array manifold into a double Fourier series.

A planar microstrip antenna array design was proposed in [22] with a Butler matrix that has narrow beamwidth, circular polarization, and polarization diversity. A circularly polarized microstrip antenna array is designed such that it consists of four identical linearly polarized patches. A 2x2 planar microstrip antenna array and a 4x4 Butler matrix have been designed and simulated using advanced design system and MATLAB software. A 4x4 Butler matrix creates

four beams; two of these beams have right-hand circular polarization and the other two have left-hand circular polarization.

The work on pulsed circular arrays (PCA) was presented by Marrocco [23] which are attractive in precise radar applications such as indoor radionavigation, automotive radars and homeland surveillance. PCA are recognized to provide better angle of arrival estimation than linear array and planar rectangular arrays.

Concentric ring array is basically a circular planar array structure. As we know that reduction in the side lobe level also increases first null beam width (FNBW). So, in this paper Chatterjee et.al [24] have tried to reduce the SLL significantly while keeping the FNBW constant. Here basically evolutionary algorithms such as Particle Swarm Optimization (PSO) algorithm and Differential Evolution (DE) algorithm are applied by optimizing both ring spacing and number of elements in each ring.

Askari et.al [25] have used Phase-mode transformation which are in beamforming with uniform circular arrays which converts circular array to a virtual uniform linear array. As, steering vector of UCA does not have vandermonde structure, so we need to convert the UCA to a virtual uniform linear array using phase mode transformation for beam steering. Also there is a limitation that we have to use all the sensors of the array. But, in certain conditions we have to use only a sector of sensors. Then in such a case phase mode transformation cannot be used. So, an optimum beamformer is used with which we can use a sector of sensors instead of selecting all the sensors of the array and it is seen that the performance of the beam former is enhanced slightly.

Ram et.al [26] have designed and obtained an optimal radiation pattern using nonuniform circular arrays and thus reduce the side-lobe level and first null beam width (FNBW) using firefly Algorithm technique.

Jackson et.al [27] studied the effect of directional antenna elements in uniform circular arrays (UCAs) for direction of arrival (DOA) estimation. The Cramer–Rao Lower Bound (CRLB) is derived for UCAs with directional antennas and is compared to isotropic antennas for 4- and 8-element arrays using a theoretical radiation pattern. Simulation results show improved DOA estimation accuracy and robustness using microstrip patch antennas as opposed to conventional dipoles.

The direction-of-arrival (DOA) estimation was done for a conformal antenna array with directive elements in the paper [28]. The two-dimension Cramer–Rao lower bound (CRLB) is derived for a conformal antenna array by using the active directive radiation patterns of the array elements and compared with the isotropic ones. The CRLB of a truncated hexagonal pyramid conformal array with seven patches is investigated which verify the significant effect of the directive elements in the DOA estimation accuracy rather than that of the isotropic ones. The simulation results prove that this conformal array achieves better DOA estimation performance rather than that of the planar array antenna especially at the horizon angles. Moreover, the conformal antenna array tilt angle is studied to achieve the optimum conformal array structure which depicts a trade-off between the DOA estimation accuracy at low and high  $\theta$  incident angles.

A circular conformal array antenna (CAA) is presented in [29] with beam-steering capabilities as well as omnidirectional performance at 3.5 GHz (S-band). The single radiating elements are formed by double-stacked microstrip patches placed on the planar faces of an octagonal holding structure. Also, an eight-way tunable feeding network (TFN) is designed, manufactured, and integrated into the conformal structure to excite the array elements. The proposed antenna is suitable for future 5G terrestrial communications in the 3.5-GHz International Mobile Telecommunication (IMT) range.

A virtual phased array (VPA) antenna is proposed in [30] which will be used in the place of uniformed linear array for unmanned aerial vehicles (UAVs). In the proposed VPA, a single antenna installed on a moving UAV will achieve the desire results. The proposed system mitigates the existing challenges of physical ULA and improves system throughput. Furthermore, due to its virtualness, VPA can adjust the number of antenna elements and interelement spacing, which adds multifrequency support and adaptive precision mechanism.

A pattern reconfigurable conformal mm Wave antenna was proposed by the authors [31], designed at 28 GHz for 5G applications using a T-junction power divider and PIN diodes. An eight 2×2 sub-arrays of microstrip patches are configured to radiate independently creating separate states capable of 360° of discrete coverage.

A lot of work has been done in the past by the researchers in the study of antenna arrays. Researchers have done various works on designing the arrays of different geometries such as linear, square shaped, rectangular, circular, concentric circular rings, etc. Also work on applications of these arrays on various sectors such as in radar applications, wearable antenna, traffic signals, wireless communications, 5G mobile communications have also been presented. Also work on conformal antennas or wrap-around antennas has also been presented in the field of missile-radar and drone applications.

## REFERENCES

1. L. C. Godara, "Application of antenna arrays to mobile communications. II. Beamforming and direction-of-arrival considerations," in *Proceedings of the IEEE*, vol. 85, no. 8, pp. 1195- 1245, Aug. 1997.
2. L. L. Shafai, W. A. Chamma, M. Barakat, P. C. Strickland and G. Seguin, "Dual-band dual-polarized perforated microstrip antennas for SAR applications," in

- IEEE Transactions on Antennas and Propagation*, vol. 48, no. 1, pp. 58-66, Jan. 2000, doi: 10.1109/8.827386.
3. J. Razavilar, F. Rashid-Farrokhi and K. J. R. Liu, "Traffic improvements in wireless communication networks using antenna arrays," in *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 458-471, March 2000, doi: 10.1109/49.840204.
  4. A. Patnaik, B. Choudhury, P. Pradhan, R. K. Mishra and C. Christodoulou, "An ANN Application for Fault Finding in Antenna Arrays," in *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 3, pp. 775-777, March 2007, doi: 10.1109/TAP.2007.891557.
  5. H. Mehrpour Bernety, R. Gholami, B. Zakeri and M. Rostamian, "Linear antenna array design for UWB radar," *2013 IEEE Radar Conference (RadarCon13)*, 2013, pp. 1-4, doi: 10.1109/RADAR.2013.6585992.
  6. G. Yang, J. Li, S. G. Zhou and Y. Qi, "A Wide-Angle E-Plane Scanning Linear Array Antenna with Wide Beam Elements," in *IEEE Antennas and Wireless Propagation Letters*, vol. 16, pp. 2923-2926, 2017, doi: 10.1109/LAWP.2017.2752713.
  7. G. Yang, J. Li, D. Wei and R. Xu, "Study on Wide-Angle Scanning Linear Phased Array Antenna," in *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 1, pp. 450-455, Jan. 2018, doi: 10.1109/TAP.2017.2761999.
  8. O. Jaroš and L. Beran, "Design of an Electronically Steered Antenna Array in the X band," *2019 Conference on Microwave Techniques (COMITE), Pardubice, Czech Republic, 2019*, pp. 1-6, doi: 10.1109/COMITE.2019.8733593.
  9. G. Yang, Y. Zhang and S. Zhang, "Wide-Band and Wide-Angle Scanning Phased Array Antenna for Mobile Communication System," in *IEEE Open Journal of Antennas and Propagation*, vol. 2, pp. 203- 212, 2021, doi: 10.1109/OJAP.2021.3057062.
  10. Page, H. Ring aerial systems, *Wireless Engineers*, Vol. 25, No. 301, pp. 308-314, October 1948.
  11. Wild, J. P. Circular aerial arrays for radio astronomy, *Proc. Royal Soc. (London)* Ser. A Vol. 262, No. 1308, pp. 84-99, June 1961.

12. DuHamel, R. H. Pattern synthesis for antenna arrays on circular, elliptical, and spherical surfaces, *Technical Report* No. 16, EE Research Lab., University of Illinois, Urbana, 1952.
13. R. R. Ramirez, F. De Flaviis and N. G. Alexopoulos, "Single-feed circularly polarized microstrip ring antenna and arrays," in *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 7, pp. 1040-1047, July 2000.
14. M. Zatman, "Circular array STAP," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, no. 2, pp. 510-517, April 2000.
15. N. C. Karmakar and M. E. Bialkowski, "A beam-forming network for a circular switched-beam phased array antenna," in *IEEE Microwave and Wireless Components Letters*, vol. 11, no. 1, pp. 7-9, Jan. 2001.
16. Dong-Chul Chung, Sung-Yul Choi, Young-Ho Ko, Jong-Ha Lee and Min-Hwan Kwak, "Circularly polarized HTS microstrip antenna array," in *IEEE Transactions on Applied Superconductivity*, vol. 13, no. 2, pp. 301-304, June 2003.
17. M. Krairiksh, P. Ngamjanyaporn and C. Kessuwan, "A flat four-beam compact phased array antenna," in *IEEE Microwave and Wireless Components Letters*, vol. 12, no. 5, pp. 184-186, May 2002.
18. Dong-Chul Chung, Sung-Yul Choi, Young-Ho Ko, Jong-Ha Lee and Min-Hwan Kwak, "Circularly polarized HTS microstrip antenna array," in *IEEE Transactions on Applied Superconductivity*, vol. 13, no. 2, pp. 301-304, June 2003.
19. J. K. S. Poon, J. Scheuer and A. Yariv, "Wavelength-selective reflector based on a circular array of coupled microring resonators," in *IEEE Photonics Technology Letters*, vol. 16, no. 5, pp. 1331-1333, May 2004.
20. P. Ioannides and C. A. Balanis, "Uniform circular and rectangular arrays for adaptive beamforming applications," in *IEEE Antennas and Wireless Propagation Letters*, vol. 4, pp. 351-354, 2005.
21. R. Goossens and H. Rogier, "A Hybrid UCA-RARE/Root-MUSIC Approach for 2-D Direction of Arrival Estimation in Uniform Circular Arrays in the Presence of Mutual Coupling," in *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 3, pp. 841-849, March 2007.



22. M. Elhefnawy and W. Ismail, "A Microstrip Antenna Array for Indoor Wireless Dynamic Environments," in *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 12, pp. 3998-4002, Dec. 2009.
23. G. Marrocco and G. Galletta, "Hermite-Rodriguez UWB Circular Arrays," in *IEEE Transactions on Antennas and Propagation*, vol. 58, no. 2, pp. 381-390, Feb. 2010.
24. A. Chatterjee, G.K. Mahanti, "Side Lobe Reduction Of a Uniformly Excited Concentric Ring Array Antenna Using Evolutionary Algorithms", *2010 ICTACT Journal on Communication Technology*, 2010.
25. M. Askari and M. Karimi, "Sector beamforming with uniform circular array antennas using phase mode transformation," *2013 21st Iranian Conference on Electrical Engineering (ICEE)*, 2013.
26. G. Ram, D. Mandal, R. Kar, S. P. Ghoshal, "Design of Non-uniform Circular Antenna Arrays Using Firefly algorithm for Side Lobe Level Reduction", *International Journal of Electrical and Electronics Engineering*, Vol. 8, No.1, pp. 33-38, 2014.
27. B. R. Jackson, S. Rajan, B. J. Liao and S. Wang, "Direction of Arrival Estimation Using Directive Antennas in Uniform Circular Arrays," in *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 2, pp. 736-747, Feb. 2015.
28. S. Mohammadi, A. Ghani and S. H. Sedighy, "Direction-of-Arrival Estimation in Conformal Microstrip Patch Array Antenna," in *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 1, pp. 511-515, Jan. 2018.
29. P. Sanchez-Olivares, P. Sanchez-Dancausa, J. L. Masa-Campos, M. Iglesias-Menendez-de-la-Vega and E. Garcia-Marin, "Circular Conformal Array Antenna With Omnidirectional and Beamsteering Capabilities for 5G Communications in the 3.5-GHz Range [Wireless Corner]," in *IEEE Antennas and Propagation Magazine*, vol. 61, no. 4, pp. 97-108, Aug. 2019.
30. Mohammad Ammad Uddin, Muhammad Ayaz, Ali Mansour, El-Hadi M. Aggoune, Ahmad Hani El Fawal, and Imran Razzak, "Ground target finding mechanism for unmanned aerial vehicles to secure crop field data". *Trans. Emerg. Telecommun. Technol.* 32, 3 (March 2021).

31. A. Stroh et al., "A Pattern Reconfigurable Conformal mmWave Antenna for 5G Applications," *2021 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (APS/URSI)*, 2021, pp. 1059-1060.

# Chapter 3

## Linear Array

---

### 3.1 Introduction

Arrays can be arranged in different geometries such as Linear (straight line), rectangular or planar, circular ring, cylindrical, concentric circles or rings, etc. Small antennas around one wavelength in size, such as quarter-wave monopoles and half-wave dipoles, don't have much directivity (gain). To create a directional antenna (high gain antenna), which radiates radio waves in a narrow beam, two general techniques can be used. First is to use reflection by large metal surfaces such as parabolic reflectors or horns, or refraction by dielectric lenses to change the direction of the radio waves, to focus the radio waves from a single low gain antenna into a beam. A parabolic dish is an example of this type of antenna. The next technique is to use multiple antennas fed from the same transmitter called an antenna array.

A Linear antenna array is a radiating system, which consists of two or more antenna elements in a straight line. Each of these elements while functioning, has its own field. The elements can be placed with equal distance or unequal distance between them. Therefore, the radiation pattern produced by it would be the vector sum of the individual ones.

The radiation pattern will depend upon the distance between the elements, excitation phase and the excitation amplitude of the individual elements. To provide very directive patterns, it is important that the fields from the individual elements of the antenna array interfere constructively in the desired direction and cancel each

other in the remaining space. The overall pattern of the antenna array depends on these following five conditions [1]:

- geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
- relative displacement between the elements
- excitation amplitude of the individual elements
- excitation phase of the individual elements
- relative pattern of the individual elements

### **3.2 N-ELEMENT LINEAR ARRAY**

The total electric field radiated by a N element array at a far field is equal to the product of electric field due to single element multiplied by the array factor. This concept is also known to as pattern multiplication. The array factor is a function of the no of elements, geometry, relative magnitude, phase and spacing between the elements.

The geometry of a N element linear antenna array is shown the Fig. 3.1. In the geometry, the individual elements are placed along the x-axis. Assuming that all the elements have identical amplitudes, but each succeeding element has a  $\beta$  progressive phase lead current excitation relative to the preceding one where  $\beta$  represents the phase by which the current in each element leads the current of the preceding element.

This arrangement of an array of identical elements having identical magnitudes and each with a progressive phase is referred to as a Uniform array. The array factor is obtained by considering the elements to be point sources.

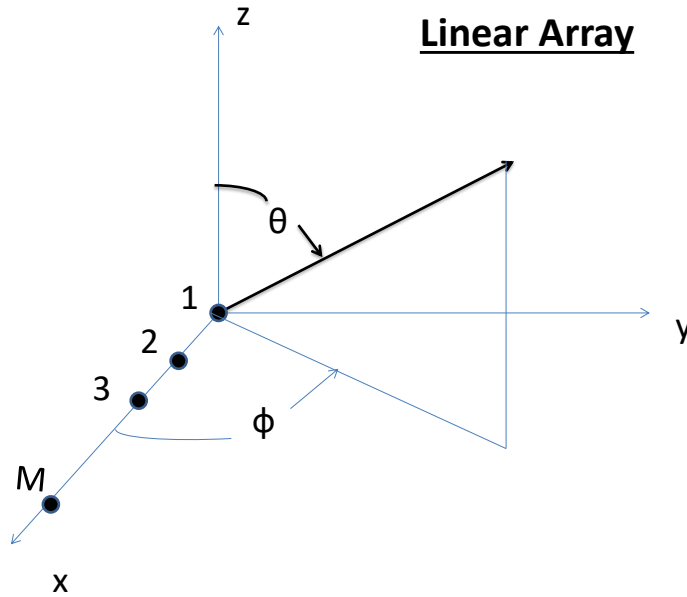


Fig 3.1: N-element Linear Array of point sources positioned along the x-axis

So, Total Field =  $E_T = [\text{Field due to single element}] * [\text{Array Factor}]$

$$\text{Array Factor} = AF = \frac{\sin(N * \psi/2)}{\sin(\frac{\psi}{2})} \quad (3.1)$$

$$\text{Normalized AF} = (AF)_{\text{norm}} = AF / N \quad (3.2)$$

where,

$$\psi = kd \cos\theta + \beta \quad (3.3)$$

A N-element linear array which has uniform spacing and uniform amplitude are termed as Uniform Arrays and if the amplitude is non-uniform, then it is classified as Non uniform Arrays such as Binomial arrays and Chebyshev arrays. In this section the focus will be on to getting the array factor pattern of above the above mentioned arrays.

The concept of Array Factor, nulls, SLL, HPBW will be discussed and then the patterns of the arrays will be plotted in MATLAB.

**Null Calculation:**

For finding the nulls ( $\theta=\theta_n$ ) of the array, (1) or (2) is set to zero

$$\Rightarrow \sin (N^* (kdcos \theta + \beta)/2) = 0$$

where,  $n = 1,2,3... \& n \neq N,2N,3N...$

The maximum values of Array Factor occurs when  $\psi/2 = \pm m\pi$

The array factor of (1) has only one maximum and occurs when  $m = 0$

So,

$$\begin{aligned} \frac{\psi}{2} &= \frac{1}{2} (kdcos\theta + \beta) |_{\theta=\theta_m} = \pm m\pi \\ \theta_m &= \cos^{-1} \left[ \frac{\lambda}{2\pi d} (-\beta \pm \frac{2n}{N} \pi) \right] \end{aligned} \quad (3.4)$$

The array factor of (3.4) has only one maximum and occurs for  $m=0$ . That is,

$$\theta_m = \cos^{-1} \left[ \frac{\lambda\beta}{2\pi d} \right] \quad (3.5)$$

The half power point or the 3-dB point of the array factor is given by

$$\theta_h = \frac{\pi}{2} - \sin^{-1} \left[ \frac{\lambda}{2\pi d} (-\beta \pm \frac{2.782}{N}) \right] \quad (3.6)$$

**HPBW:**

The half power beamwidth  $\Theta_h$  can be found once the angles of the first maximum ( $\theta_m$ ) in (3.4) and the half power point ( $\theta_h$ ) are known.

$$\Theta_h = 2|\theta_m - \theta_h| \quad (3.7)$$

where,  $\theta_m$  = maxima of Array Factor

$\theta_h$  = 3-dB point or the half power point of the array factor

**SLL = Sidelobe Level** is information on how large (or small) side lobes in the radiation pattern of an antenna are. Usually, they are compared to the maximum level of the main beam, and the difference is expressed in dB.

**FNBW** = The angular span between the first pattern nulls adjacent to the main lobe, is called as the First Null Beam Width.

Arrays can be further classified according to where the maxima or null of the radiation pattern:

- Broadside Arrays
- End -Fire Arrays
- Phased Scanning Arrays

### 3.3 Broadside Arrays:

In many applications it is desirable to have the maximum radiation of an array directed normal to the axis of the array. So, for broadside arrays:  $\theta_0 = 90^\circ$ . It should be observed that to have the maximum of the array factor of a uniform linear array directed broadside to the axis of the array, it is necessary that all the elements have the same phase excitation (in addition to the same amplitude excitation).

Also, to mention that for no principal maxima in other directions, which are referred to as grating lobes, the separation between the elements should not be equal to multiples of a wavelength, i.e.,  $d \neq n\lambda$ ,  $n = 1, 2, 3 \dots$ . It is often required to select the largest spacing between the elements but with no grating lobes. Thus,  $d_{\max} < \lambda$ .

### 3.4 End-Fire Arrays:

Now, if it is desirable to have the maximum radiation of an array directed towards the axis of the array. Then it is called as an End-Fire Array. So, for End-Fire arrays:  $\theta_0 = 0^\circ$  or  $180^\circ$ . Here, also to have only one end-fire maximum and to avoid any grating lobes, the maximum spacing between the elements should be less than  $d_{\max} < \lambda/2$ . The concept overall is same as seen in broadside.

### 3.5 Phased Scanning Array:

Assume that the maximum radiation of the array is required to be oriented at an angle  $\theta^0$  ( $0^0 \leq \theta^0 \leq 180^0$ ). Then it will be called as phased array.

### 3.6 Non- Uniform Amplitude Linear Array:

These linear arrays are of uniform spacing and uniform amplitude. The excitations of the elements of a non-uniform amplitude linear array are not uniform in nature. Binomial and Dolph-Chebyshev are the two examples of non-uniform amplitude linear array.

There are number of observations can be made from the uniform arrays, binomial arrays and a dolph-tchebyscheff array [2,3]. Of the three distributions (uniform, binomial, and Tschbyscheff), a uniform amplitude array yields the smallest half-power beamwidth, followed by the Dolph-Tschbyscheff and binomial arrays.

Binomial arrays usually possess the smallest side lobes followed, in order, by the Dolph-Tschbyscheff and uniform arrays. Binomial arrays with element spacing equal or less than  $\lambda/2$  have no side lobes. It has been shown analytically that for a given side lobe level the Dolph-Tschbyscheff array produces the smallest beamwidth between the first nulls and vice-versa. Uniform arrays usually possess the largest directivity.

#### 3.6.1 Binomial Array:

The excitation coefficients can be found with the help of Pascal triangle.



### 3.6.2 Dolph-Chebyshev Array:

The excitation coefficients of a Dolph-Tschebyscheff array can be derived or calculated using calculations given by Barbieri [4]. The coefficients using this method can be obtained using

$$a_n = \begin{cases} \sum_{q=n}^M \frac{(q+M-2)!(2M-1)}{(q-n)!(q+n-1)!(M-q)!} (-1)^{M-q} (z_0)^{2q-1} & (3.7a) \\ \text{for even } 2M \text{ elements and } n = 1, 2, \dots, M & (3.8b) \\ \sum_{q=n}^{M+1} \frac{(q+M-2)!(2M)}{(q-n)!(q+n-2)!(M-q+1)! \epsilon_n} (-1)^{M-q+1} (z_0)^{2(q-1)} & \\ \text{for odd } 2M+1 \text{ elements and } n = 1, 2, \dots, (M+1) & \\ \epsilon_n = \begin{cases} 2 & n = 1 \\ 1 & n \neq 1 \end{cases} & \end{cases}$$

### 3.7 Calculation of Array Factor for Binomial and Chebyshev Arrays:

$$AF_{2M}(\text{even}) = \sum_{n=1}^M a_n \cos [(2n-1)u] \quad (3.9)$$

$$AF_{2M+1}(\text{odd}) = \sum_{n=1}^{M+1} a_n \cos [2(n-1)u] \quad (3.10)$$

where,  $u = \frac{\pi d}{\lambda} \cos \theta$

m= the number of elements of the array

### 3.8 Array Factor Plot

Now going to design part using MATLAB. Let's see the Array Factor pattern for various uniform and Non-uniform Arrays. The MATLAB codes for the Array Factor pattern are given in the Appendix section.

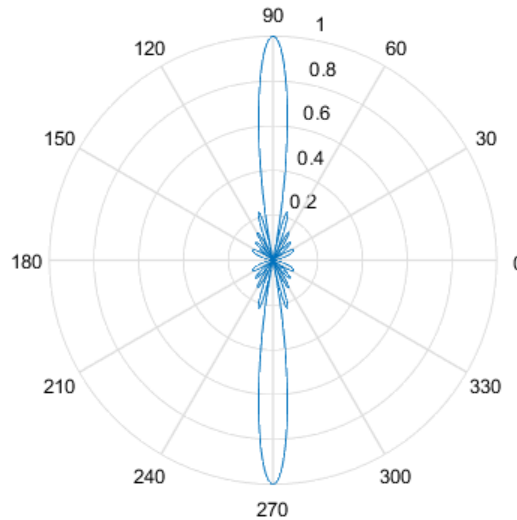
#### 3.8.2 Broadside Arrays:

MATLAB program to compute Array Factor pattern of N-element Linear Array which has Uniform Amplitude and Spacing. In this program computation of the Array Factor pattern of Linear Broadside Arrays is done and the array factor pattern is plotted in the Figure 3.2 and 3.3. For Broadside Arrays, we know that the maximum radiation is directed towards normal to the axis of the array, i.e.,  $\theta_{\max} = 90$  degrees.

Number of elements =  $N = 10$

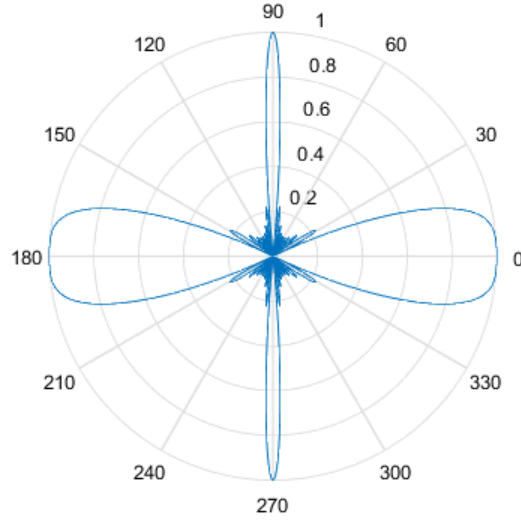
Distance between the individual elements =  $d = \lambda/4$

Array Factor pattern of a 10-element uniform amplitude Broadside array



**Figure 3.2** Array Factor pattern of a 10-element uniform amplitude broadside array for  $d = \lambda/4$

Array Factor pattern of a 10-element uniform amplitude Broadside array



**Figure 3.3** Array Factor pattern of a 10-element uniform amplitude broadside array for  $d = \lambda$

### 3.8.2 End-Fire Arrays:

MATLAB program to compute Array Factor pattern of N-element Linear Array which has Uniform Amplitude and Spacing. In this program computation of the Array Factor pattern of Linear end-fire Arrays is done and the array factor pattern is plotted in the Figure 3.4 and 3.5. For end-fire Arrays, we know that the maximum radiation is directed towards the axis of the array, i.e.,  $\theta_{\max} = 0$  degrees or 180 degrees.

Number of elements =  $N = 10$

Distance between the individual elements =  $d = \lambda/4$

To direct the first maximum towards  $\theta_0 = 0^\circ$

$$\psi = kd \cos \theta + \beta|_{\theta=\theta_0} = kd + \beta = 0 \Rightarrow \beta = -kd \quad (3.11)$$

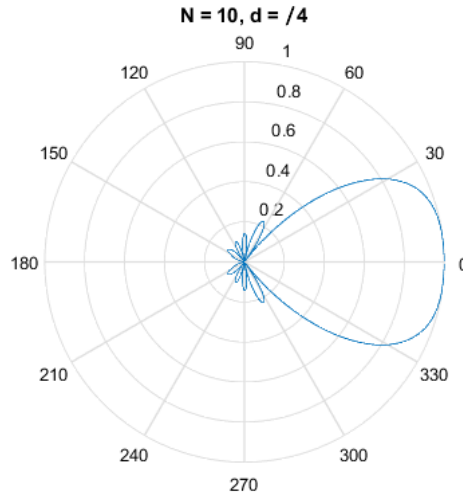
To direct the first maximum towards  $\theta_0=180^\circ$

$$\psi = kd\cos\theta + \beta|_{\theta=\theta_0} = -kd + \beta = 0 \Rightarrow \beta = kd \quad (3.12)$$

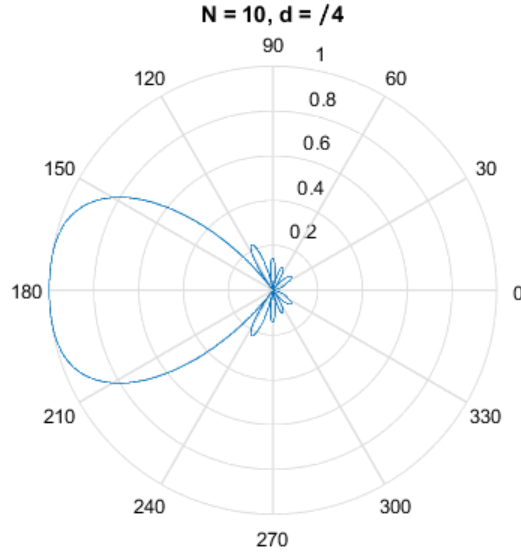
Thus end-fire radiation is accomplished when  $\beta = -kd$  (for  $\theta_0 = 0^\circ$ ) or  $\beta = kd$  (for  $\theta_0 = 180^\circ$ ). If the element separation is  $d = \lambda/2$ , end-fire radiation exists simultaneously in both directions (for  $\theta_0 = 0^\circ$  and  $\theta_0 = 180^\circ$ ). If the element spacing is a multiple of a wavelength ( $d = n\lambda$ ,  $n = 1, 2, 3, \dots$ ), then in addition to having end-fire radiation in both directions, there also exist maxima in the broadside directions. Thus, for  $d = n\lambda$ ,  $n = 1, 2, 3, \dots$  there exist four maxima: two in the broadside directions and two along the axis of the array.

To have only one end-fire maximum and to avoid any grating lobes, the maximum spacing between the elements should be less than  $d_{\max} < \lambda/2$ .

The two-dimensional pattern or the array factor of the same array ( $N = 10$ ) but with  $d = \lambda/4$  is shown in the Figure 3.4 for  $\theta_0 = 0^\circ$  and in the Figure 3.5 for  $\theta_0 = 180^\circ$ .



**Figure 3.4** Array Factor pattern of a 10-element uniform amplitude end-fire array for  $d = \lambda/4$ ,  $\beta = -kd$



**Figure 3.5** Array Factor pattern of a 10-element uniform amplitude end-fire array  
for  $d = \lambda/4$ ,  $\beta = kd$

### 3.8.3 Phased (Scanning) Arrays:

MATLAB program to compute Array Factor pattern of N-element Linear Array which has Uniform Amplitude and Spacing. In this program computation of the Array Factor pattern of phased Arrays is done and the array factor pattern is plotted in the Figure 3.6. For phased Arrays, we know that the maximum radiation is directed towards any direction. Let assume that the maximum radiation of the array is required to be oriented at an angle  $\theta_0$  ( $0^\circ \leq \theta_0 \leq 180^\circ$ ). To accomplish this, the phase excitation  $\beta$  between the elements must be

$$\psi = kdcos\theta + \beta|_{\theta=\theta_0} = kdcos\theta_0 + \beta = 0 \Rightarrow \beta = -kdcos\theta_0 \quad (3.13)$$

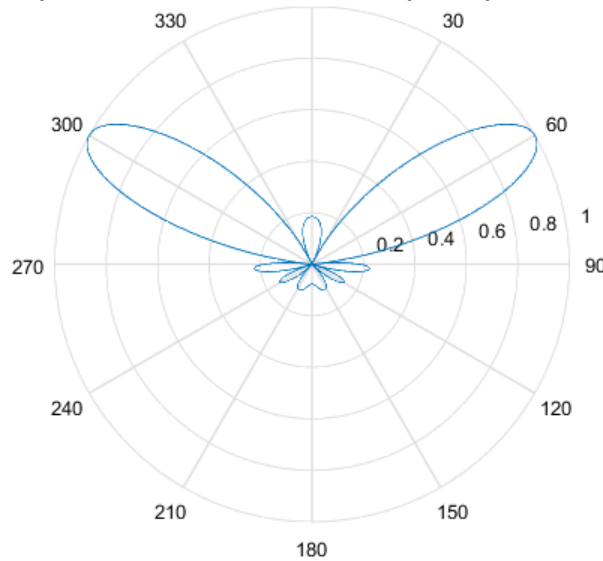
Thus, by controlling the progressive phase difference between the elements, the maximum radiation can be squinted in any desired direction to form a scanning array. This is the basic principle of electronic scanning phased array operation

To direct the first maximum towards  $\theta_0=60^\circ$

Number of elements =  $N=10$

Distance between the individual elements =  $d = \lambda/4$

Array Factor pattern of a 10-element uniform amplitude phased Scanning array



**Figure 3.6** Array Factor pattern of a 10-element uniform amplitude end-fire array  
for  $d = \lambda/4$ ,  $\beta = -kd \cos 60^\circ$

### 3.8.4 Binomial Arrays:

MATLAB program to compute Array Factor pattern of N-element Linear Array which has non-Uniform Amplitude but uniform spacing. In this program computation of the Array Factor pattern of binomial Arrays is done and the array factor pattern is plotted in the Figure 3.7. In the program, design of a binomial array of 10 elements with spacing  $d$  between the elements is done. First step is to find the excitation coefficients and then form the array factor.

In this program, finding the excitation coefficients is done with the pascal triangle method. The values of  $m$  values of  $m$  are used to represent the number of elements of the array. With the pascal triangle as shown in the Figure 3.7, the excitation coefficients are compared and find out.

$m = 1$								1																		
$m = 2$								1		1																
$m = 3$								1		2		1														
$m = 4$								1		3		3		1												
$m = 5$								1		4		6		4		1										
$m = 6$								1		5		10		10		5		1								
$m = 7$								1		6		15		20		15		6		1						
$m = 8$								1		7		21		35		35		21		7		1				
$m = 9$								1		8		28		56		70		56		28		8		1		
$m = 10$								1		9		36		84		126		126		84		36		9		1

**Figure 3.7** Pascal Triangle used for finding the amplitude coefficients for the array

The amplitude coefficients for the array are then found out by comparison method with the help of pascal triangle.

1. For Six Elements ( $m=6$ ): As six is an even number, so the comparison is done with the Equation (3.9). The following steps are carried out for finding the excitation coefficients of the array. First the row corresponding to the  $m=6$  is selected as shown in the Figure 3.8.

$m = 6$	1	5	10	10	5	1
---------	---	---	----	----	---	---

**Figure 3.8** Pascal Triangle row corresponding to  $m=6$

Then as  $m=6$  is done for even number of elements. So, the first coefficient  $a_1$  is the value next to the middle of the row as shown below.

$$a_1 = 10$$

$$a_2 = 5$$

$$a_3 = 1$$

2. For Seven Elements ( $m=7$ ): The following steps are carried out for finding the excitation coefficients of the array. First the row corresponding to the  $m=7$  is selected as shown in the Figure 3.9.

$m = 7$	1	6	15	20	15	6	1
---------	---	---	----	----	----	---	---

**Figure 3.9** Pascal Triangle row corresponding to  $m=7$

Then as  $m=7$  is done for odd number of elements. So, the first coefficient  $a_1$  is compared with the middle value of the row as shown below. The amplitude excitation of the center element is  $2a_1$ .

$$2a_1 = 20 \Rightarrow a_1 = 10$$

$$a_2 = 15$$

$$a_3 = 6$$

$$a_4 = 1$$

$$a_5 = a_2 = 15$$

$$a_6 = a_3 = 6$$

$$a_7 = a_4 = 1$$

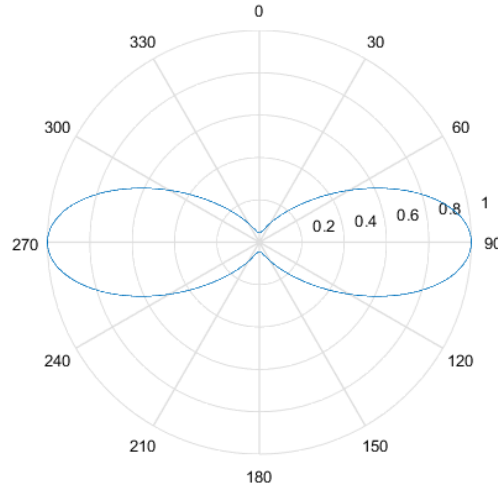
The array factor for the binomial array is represented by Equations (3.9) and (3.10) and the array factor plot is carried out once the excitation coefficients are calculated. Pascal triangle is generated using the following code as shown in the Figure 3.10.

```
for n = 0:m % Generation of pascal triangle for N elements
    C = sym([]);
    for k = 0:n
        C = horzcat(C, nchoosek(n, k));
    end
    disp(C);
end
```

**Figure 3.10** Code for generation of Pascal triangle in MATLAB

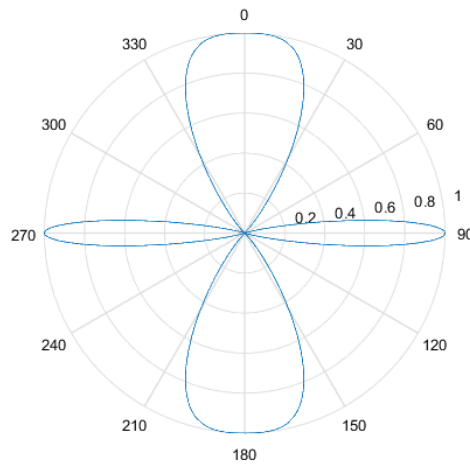


The array factor pattern plot is carried out once the excitation coefficients are calculated using the pascal triangle method. The array factor pattern for number of elements =  $N = 10$  and distance between the individual elements =  $d = \lambda/4$  is shown in the Figure 3.11.



**Figure 3.11** Array Factor pattern of a 10-element binomial array for  $d = \lambda/4$ ,  $N = 10$

The array factor pattern for number of elements =  $N = 10$  and distance between the individual elements =  $d = \lambda$  is shown in the Figure 3.12.



**Figure 3.12** Array Factor pattern of a 10-element binomial array for  $d = \lambda$ ,  $N = 10$

### 3.8.5 Dolph-Chebyshev Array:

MATLAB program to compute Array Factor pattern of N-element Linear Array which has non-Uniform Amplitude but uniform spacing. In this program computation of the Array Factor pattern of Dolph-Tschebyscheff Arrays is done and the array factor pattern is plotted in the Figure 3.7.

In the program, design of a Dolph-Tschebyscheff array of 10 elements with spacing  $d$  between the elements is done. First step is to find the excitation coefficients and then form the array factor.

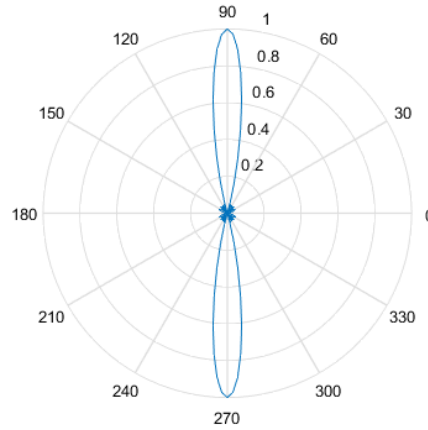
Its excitation coefficients are related to Tschebyscheff polynomials. A Dolph-Tschebyscheff array with no side lobes (or side lobes of  $-\infty$  dB) reduces to the binomial design. The MATLAB codes of the array factor pattern for Dolph-Tschebyscheff array is given in the Appendix section.

The design of a Dolph-Tschebyscheff array of 10 elements with spacing  $d$  between the elements and with a major-to-minor lobe ratio of 26 dB is studied and done. The array factor for the binomial array is represented by Equations (3.9) and (3.10) and the array factor plot is carried out once the excitation coefficients are calculated.

The excitation coefficients of a Dolph-Tschebyscheff array can be derived or calculated using calculations given by Barbiere. The coefficients using this method can be obtained using Equations (3.8a) and (3.8b).

The array factor pattern for number of elements =  $N = 10$  and distance between the individual elements =  $d = \lambda/4$  is shown in the Figure 3.13.

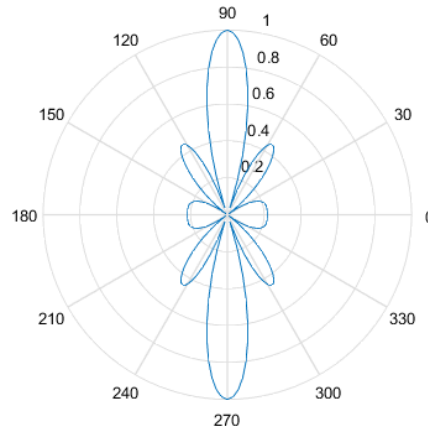
Array factor power pattern of a N = 10 element broadside Dolph-Tschebyscheff ar



**Figure 3.13** Array Factor pattern of a 10-element Dolph-Tschebyscheff array for  $d = \lambda/4$ ,  $N=10$  using Barbieri calculations

The excitation coefficients of a Dolph-Tschebyscheff array can be derived or are related to Tschebyscheff polynomials. In MATLAB, the excitation coefficients are generated using Chebwin Function. The array factor pattern of a Dolph-Tschebyscheff array of 10 elements with spacing  $d = \lambda/4$  between the elements and with a major-to-minor lobe ratio of 26 dB is shown in the Figure 3.14.

Array factor power pattern of a N = 10 element broadside Dolph-Tschebyscheff



**Figure 3.14** Array Factor pattern of a 10-element Dolph-Tschebyscheff array for  $d = \lambda/4$ ,  $N=10$  using Chebwin function

### 3.9 Conclusion:

The MATLAB software is used to run and generate the codes for the array factor pattern of the different types of linear arrays. The Array Factor pattern of the Broadside, End-Fire, Phased Array, Binomial Arrays and Dolph-Tschebyscheff Array are plotted, and the patterns are given in this chapter.

### References

- [1] C. A. Balanis, “*Antenna Theory Analysis and Design*”, John Wiley and Sons, 2016.
- [2] J. S. Stone, United States Patents No. 1,643,323 and No. 1,715,433.
- [3] C. L. Dolph, “A Current Distribution for Broadside Arrays Which Optimizes the Relationship Between Beamwidth and Side-Lobe Level,” *Proc. IRE and Waves and Electrons*, June 1946.
- [4] D. Barbieri, “A Method for Calculating the Current Distribution of Tschebyscheff Arrays,” *Proc. IRE*, January 1952, pp. 78–82

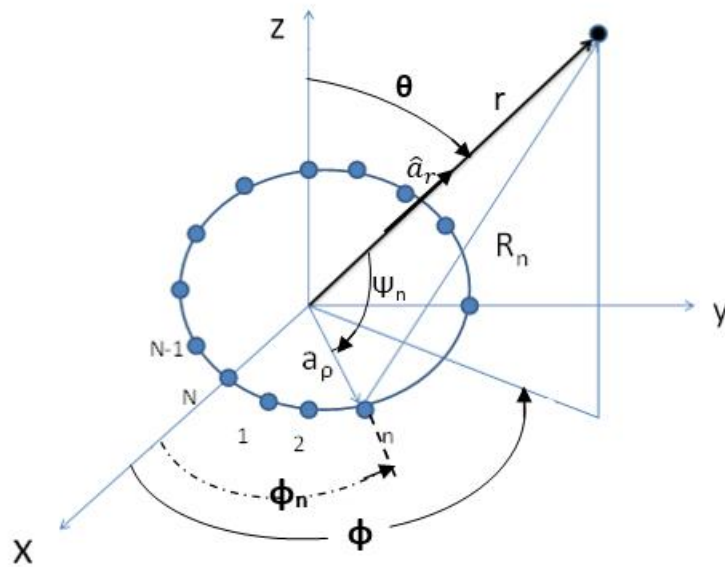
# Chapter 4

## Circular Array

### 4.1 Introduction

A circular Array is an array in which the elements are placed together in a circular ring fashion or in a circular ring geometry as shown in the Figure 4.1. Consider a single circular array of radius  $a$  and the circular array is in the  $xy$ -plane. There are  $N$  isotropic elements on the circumference of the circle. The free space far field pattern of such an array is obtained by summing the contribution from each element at some distant point.

In this chapter, the horizontal (azimuthal) pattern and vertical pattern (beam maximum pointing towards the array normal) is analysed.



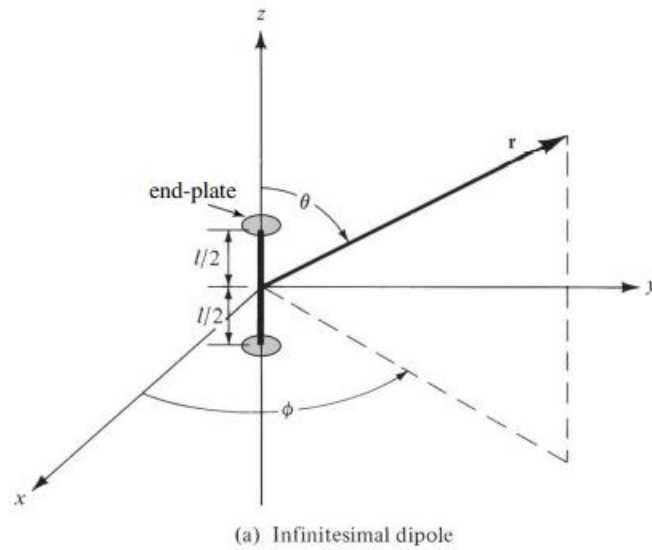
**Figure 4.1** Geometry of an N-Element Circular Array

## 4.2 Array Factor Calculation

An infinitesimal linear wire ( $l \ll \lambda$ ) is positioned symmetrically at the origin of the coordinate system and oriented along the  $z$  axis as shown in the Figure 4.2. The spatial variation of the current is assumed to be constant and given by

$$I(z') = \hat{a}_z I_0 \quad (4.1)$$

where,  $I_0 = \text{constant}$



**Figure 4.2** A in infinitesimal linear dipole positioned along the  $z$ -axis

The electric field due to the infinitesimal linear dipole at a far field region ( $kr \gg 1$ ) is given by

$$E_\theta = j\eta \frac{k I_0 l e^{-jkr}}{4\pi r} \sin\theta \quad (4.2)$$

where,  $\eta$  = intrinsic impedance

Let us assume that N isotropic elements are equally spaced on the x-y plane along a circular ring of the radius a as shown in the figure 4.1. The normalized field of the array can be written as

$$E_n(r, \theta, \phi) = \sum_{n=1}^N a_n \frac{e^{-jkR_n}}{R_n} \quad (4.3)$$

where,  $R_n$  is the distance from the nth element to the observation point.

Also,

$$R_n = (r^2 + a^2 - 2ar \cos \psi)^{1/2} \quad (4.4)$$

For  $r \gg a$ ,  $R_n$  reduces to

$$R_n = r - a \cos \psi_n = r - a(\hat{a}_\rho \cdot \hat{a}_r) = r - a \sin \theta \cos(\phi - \phi_n)$$

Thus

$$E_n(r, \theta, \phi) = \frac{e^{-jkr}}{r} \sum_{n=1}^N a_n e^{+jka \sin \theta \cos(\phi - \phi_n)} \quad (4.5)$$

where

$a_n$  = excitation coefficients (amplitude and phase) of nth element

$\phi_n = 2\pi(\frac{n}{N})$  = angular position of nth element on x-y plane

In general, the excitation coefficient of the nth element can be written as

$$a_n = I_n e^{j\alpha_n} \quad (4.6)$$

where

$I_n$  = amplitude excitation of the nth element

$\alpha_n$  = phase excitation (relative to the array centre) of the nth element

So

$$\begin{aligned}
 AF(\theta, \phi) &= \sum_{n=1}^N a_n e^{+jka \sin \theta \cos(\phi - \phi_n)} \\
 &= \sum_{n=1}^N I_n e^{+j[ka \sin \theta \cos(\phi - \phi_n) + \alpha_n]}
 \end{aligned} \tag{4.7}$$

The above equation represents the array factor of a circular array of  $N$  equally spaced elements. To direct the peak of the main beam in the  $(\theta_0, \phi_0)$  direction, the phase excitation of the  $n$ th element can be chosen to be

$$\alpha_n = -ka \sin \theta_0 \cos(\phi_0 - \phi_n) \tag{4.8}$$

The array factor can be written as

$$AF(\theta, \phi) = \sum_{n=1}^N I_n e^{+j[ka \sin \theta \cos(\phi - \phi_n) - ka \sin \theta_0 \cos(\phi_0 - \phi_n)]} \tag{4.9}$$

In terms of Bessel function, array factor of the circular array can be written as

$$AF(\theta, \phi) = \sum_{m=-\infty}^{m=\infty} J_m(k\rho_0) e^{jmN(\frac{\pi}{2} - \varepsilon)} \tag{4.10}$$

where

$$\begin{aligned}
 \rho_0 &= a[(\sin \theta \cos \phi - \sin \theta_0 \cos \phi_0)^2 \\
 &\quad + (\sin \theta \sin \phi - \sin \theta_0 \sin \phi_0)^2]^{1/2}
 \end{aligned} \tag{4.11}$$

$$\varepsilon = \tan^{-1} \left[ \frac{(\sin \theta \cos \phi - \sin \theta_0 \cos \phi_0)}{(\sin \theta \sin \phi - \sin \theta_0 \sin \phi_0)} \right] \tag{4.12}$$

where  $J(x)$  is the Bessel function of the first kind. The part of the array factor associated with the zero order Bessel function  $J_0(k\rho_0)$  is called the principal term and the remaining terms are noted as the residuals. For a circular array with a large



number of elements, the term  $J_0(k\rho_0)$  alone can be used to approximate the two-dimensional principal-plane patterns. The remaining terms contribute negligibly because Bessel functions of larger orders are very small.

### 4.3 Array Factor Plot

The horizontal (azimuthal) pattern lies in the array plane,  $\theta=\theta_0=\pi/2$  (refer to Figure 4.1). So, for horizontal cophasal pattern of the array if the beam maximum is designed to point, say, in the x-direction,  $\phi_0=0$ . Then, after putting the values of  $\theta$ ,  $\theta_0$  and  $\phi_0$  in the Equations (4.8), (4.11) and (4.12)

$$\alpha_n = -kac \cos \phi_n \quad (4.13)$$

$$\rho_0 = 2a \sin \frac{\phi}{2} \quad (4.14)$$

$$\cos \varepsilon = -\sin \frac{\phi}{2} \quad (4.15)$$

When considering the vertical pattern with the beam maximum pointing towards the z direction ( $\theta_0=0$ ) (refer to Figure 4.1), then

$$\alpha_n = 0 \quad (4.16)$$

$$\rho_0 = a \sin \frac{\theta}{2} \quad (4.17)$$

$$\cos \varepsilon = \cos \phi \quad (4.18)$$

$$\text{or} \quad \varepsilon = \phi \quad (4.19)$$

Using the Equations (4.12-4.14),

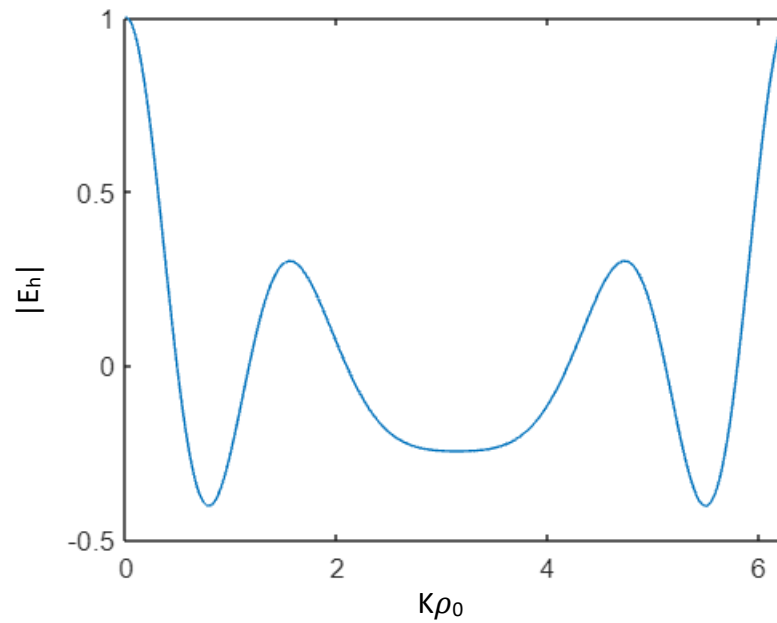
$$E = E_h = \sum_{m=-\infty}^{m=\infty} J_{mN} (2ka \sin \frac{\phi}{2}) e^{(\frac{-jmN\phi}{2})} \quad (4.20)$$

$$E = E_v = \sum_{m=-\infty}^{m=\infty} J_{mN} (ka \sin \theta) e^{(jmN(\frac{\pi}{2}-\phi))} \quad (4.21)$$

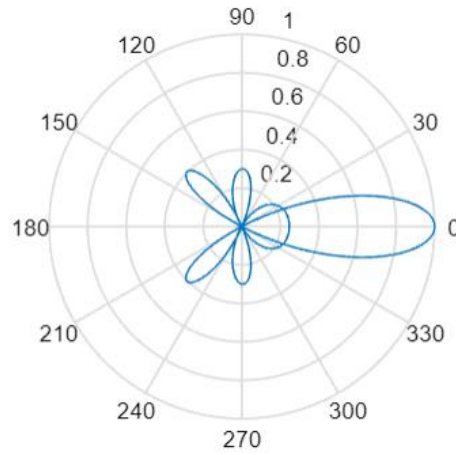
The MATLAB codes for the Array Factor pattern are given in the Appendix section. The array factor plot for the eight element circular array with uniform amplitude and phase excitation is plotted in the MATLAB software.

The array factor plot for a uniform circular array with same phase excitation is shown in the Figure 4.3. The plot shows the horizontal cophasal pattern of the array if the beam maximum is towards  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$  or x-z plane for number of elements equal to eight or  $N=8$ .

The array factor associated with the zero order Bessel function  $J_0(k\rho_0)$  or the principal term of the array factor. The polar plot of the radiation pattern is shown in the Figure 4.4 for  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$ .

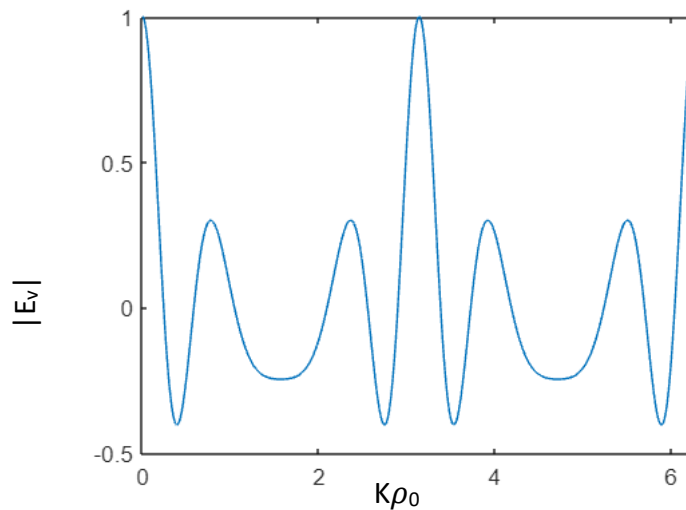


**Figure 4.3** Radiation patterns for a uniform circular array for  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$  for  $N=8$  for zero order Bessel function

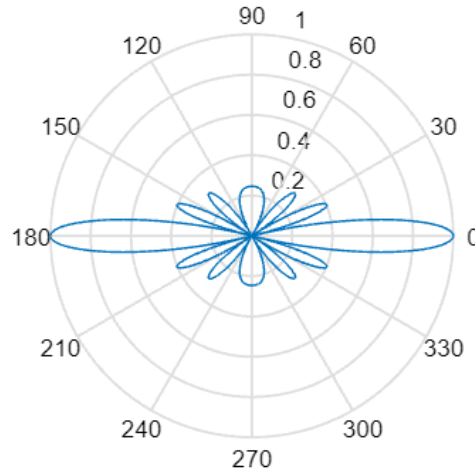


**Figure 4.4** Radiation patterns for a uniform circular array for  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$  for  $N=8$  for zero order Bessel function

The plot of the vertical cophasal pattern of the array if the beam maximum is towards  $\theta_0=0$ ,  $\phi_0=0$  or x-z plane for number of elements equal to eight or  $N=8$  is shown in the Figure 4.5. This array factor associated with the zero order Bessel function  $J_0(k\rho_0)$  or the principal term of the array factor. The polar plot of the radiation pattern is shown in the Figure 4.6 for  $\theta_0=0$ ,  $\phi_0=0$ .

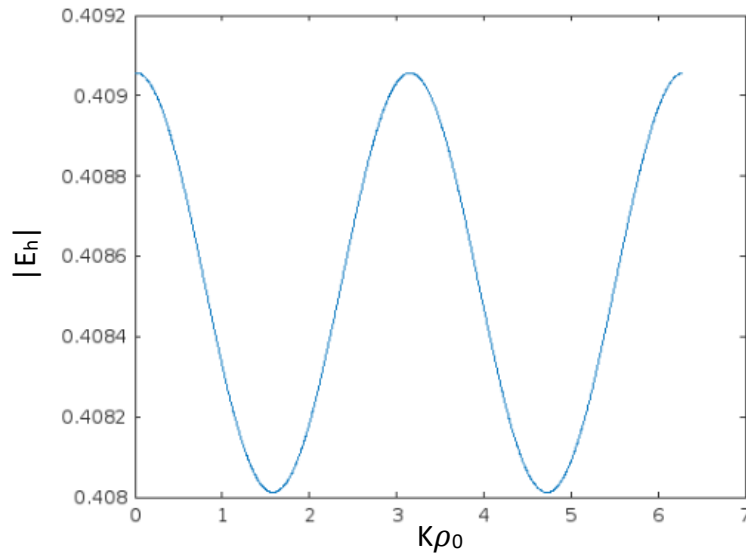


**Figure 4.5** Radiation patterns for a uniform circular array for  $\theta_0=0$ ,  $\phi_0=0$  for  $N=8$  for zero order Bessel function



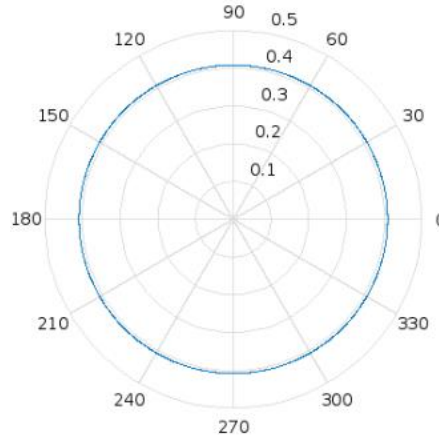
**Figure 4.6** Radiation pattern for a uniform circular array for  $\theta_0=0$ ,  $\phi_0=0$  for  $N=8$  for zero order Bessel function

The array factor associated with the Equation (4.9) or the array factor plot for a uniform circular array with same phase excitation is shown in the Figure 4.7. The plot shows the horizontal cophasal pattern of the array if the beam maximum is towards  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$  or x-z plane for number of elements equal to eight or  $N=8$ .



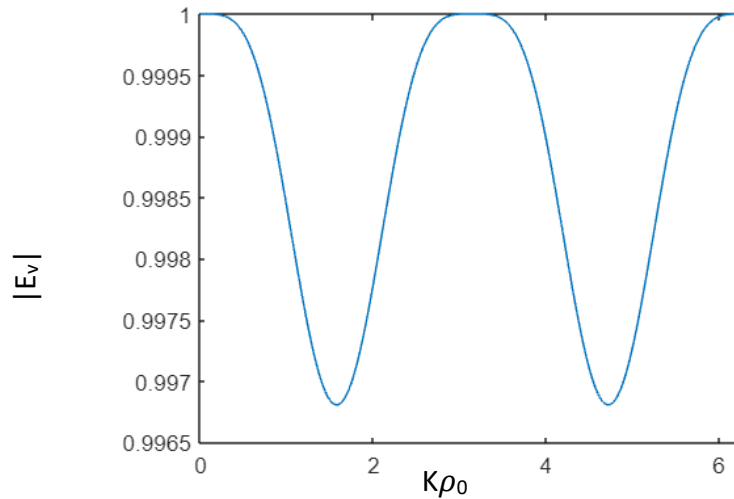
**Figure 4.7** Radiation pattern for a uniform circular array for  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$  or x-z plane for  $N=8$  for Equation (4.9)

The polar plot of the radiation pattern is shown in the Figure 4.8 for  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$ .



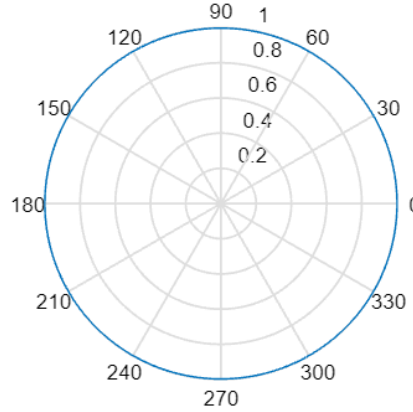
**Figure 4.8** Radiation pattern for a uniform circular array for  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$  or x-z plane for  $N=8$  for Equation (4.9)

Also, the plot of the vertical cophasal pattern of the array if the beam maximum is towards  $\theta_0=0$ ,  $\phi_0=0$  or x-z plane for number of elements equal to eight or  $N=8$  is shown in the Figure 4.9 for the array factor associated with the Equation (4.9).



**Figure 4.9** Radiation patterns for a uniform circular array for  $\theta_0=0$ ,  $\phi_0=0$  for  $N=8$  for Equation 4.9

The polar plot of the radiation pattern is shown in the Figure 4.10 for  $\theta_0=0$ ,  $\phi_0=0$ .



**Figure 4.10** Radiation patterns for a uniform circular array for  $\theta_0=0$ ,  $\phi_0=0$  for  $N=8$  for Equation 4.9

#### 4.4 Conclusion

In this chapter, the array factor plot for the circular array with uniform amplitude and phase excitation is plotted. The plot is made with the programs run in MATLAB software. The MATLAB codes for the Array Factor pattern are given in the Appendix section. The plot shows the horizontal cophasal pattern of the array if the beam maximum is towards  $\theta=\theta_0=\pi/2$ ,  $\phi_0=0$  or x-z plane for N number of elements. Here, in this chapter case study is done on  $N=8$ , where N is the number of elements in the ring array and circular array. Also, the plot of the vertical cophasal pattern of the array if the beam maximum is towards  $\theta_0=0$ ,  $\phi_0=0$  or x-z plane for number of elements equal to eight or  $N=8$  is also plotted and shown in this chapter.

#### REFERENCE

- [1] [https://en.wikipedia.org/wiki/Direction\\_finding](https://en.wikipedia.org/wiki/Direction_finding).

- [2] Sathish Chandran, “*Advances in Direction-of-Arrival Estimation*”, ARTECH HOUSE, 2006.
- [3] Frater, M. R., and M. Ryan, “*Electronic Warfare for the Digitized Battlefield, Norwood, MA*”: Artech House, 2001.
- [4] <https://ece.illinois.edu/about/history/wullenweber>.
- [5] ‘*Introduction into Theory of Direction Finding*,’ <http://www.rohde-schwarz.com>.
- [6] GETHING, P. J. D.: “High-frequency direction finding”, *Proc. IEEE*, vol. 113, pp. 49-61, Jan. 1966.
- [7] DAVIES, D. E. N., and RIZK, M. S. A. S.: “A broadband experimental null-steering antenna system for mobile communications”, *Radio Electron Engr.*, 48, Oct. 1978, pp. 511-517.





# Chapter 5

## Directive Antennas in Uniform Circular Arrays

---

### 5.1: INTRODUCTION

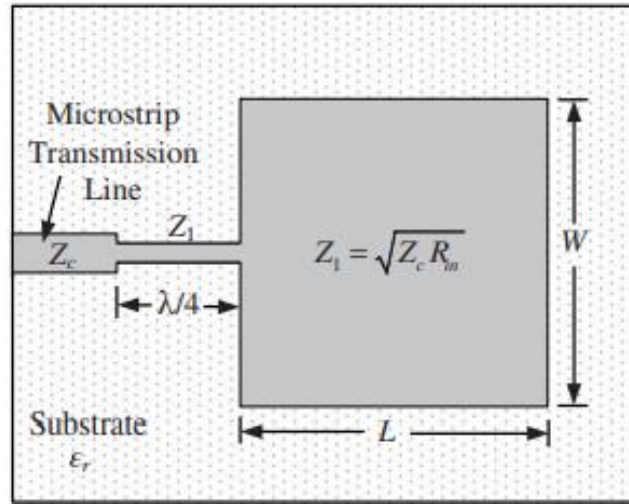
Microstrip antennas are widely used in modern wireless communication systems due to their compact size and low cost. These patch antennas have planar structure and are low-profile and thus are very easy to fabricate and so are very popular. In high-performance aircraft, spacecraft, satellite, and missile applications, where size, weight, cost, performance, ease of installation, and aerodynamic profile are constraints, these low-profile antennas may be used. Also, these patch antennas are widely popular in mobile radio and wireless communications.

The effect of directional antenna elements in uniform circular arrays (UCAs) is studied in this chapter. The directional antennas are used as the elements of a smart antenna array for DOA estimation [1]. The vast majority of the literature on smart antennas for DOA estimation and beamforming assume either isotropic antenna elements or, equivalently, omnidirectional elements with the analysis limited to the plane where the antenna gain is equal in all directions (e.g., a dipole oriented on the  $z$ -axis with  $\theta=90^\circ$ ) [2-5]. In this chapter, the use of circular arrays using directional elements or patch antennas are studied.

The microstrip patch is designed so its pattern maximum is normal to the patch (broadside radiator). There are numerous substrates that can be used for the design of microstrip antennas, and their dielectric constants are usually in the range of  $2.2 \leq \epsilon_r \leq 12$ . The radiating patch may be square, rectangular, circular, elliptical, triangular, or any other configuration.

## 5.2: DESIGN SPECIFICATIONS FOR A SINGLE ELEMENT PATCH ANTENNA

The technique used to match the patch antenna with 50 ohm microstrip line is  $\lambda/4$  impedance transformer [6] as shown in the Figure 5.1.



**Figure 5.1** Matching Technique:  $\lambda/4$  impedance transformer

The structure of the rectangular patch element is shown in the figure 5.2. The patch element is designed with Arlon AD-430 substrate with a relative permittivity of 4.3. The substrate height is 0.762 mm. The single patch element is mounted on the top of the substrate. The ground plane is mounted below the substrate. The dimensions of the substrate and ground are equal. The material for the patch element is taken as copper. The thickness of copper is 0.035 mm. The dimensions of the single element patch are shown in the Table I. The width of the microstrip feed line used to feed the patch is calculated using standard design equations. The length and width of the rectangular patch is calculated using the following equations given by [7].

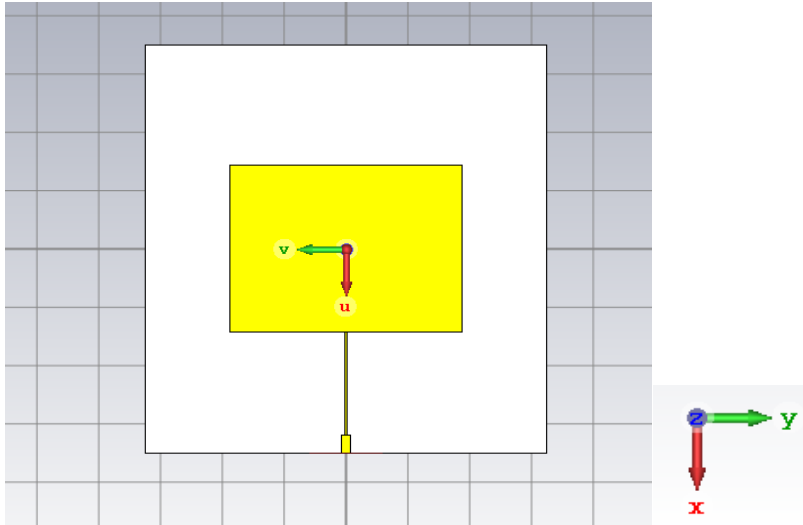
$$W_p = \frac{c}{2f_0} \sqrt{\frac{2}{\epsilon_r + 1}} \quad (5.1)$$

$$L_p = \frac{c}{2f_0 \sqrt{\epsilon_{eff}}} - 2\Delta L \quad (5.2)$$

The fields at the edges of the patch undergo fringing which makes the microstrip line look wider electrically compared to its physical dimensions. Since some of the waves travel in the substrate and some in air, an effective dielectric constant  $\epsilon_{eff}$  is introduced to account for fringing and the wave propagation in the line, which is given by [8]

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + \frac{h}{W} * 12 \right]^{-1/2} \quad (3.3)$$

$$\frac{\Delta L}{h} = 0.412 \frac{(\epsilon_{eff} + 0.3) \left( \frac{W}{h} + 0.264 \right)}{(\epsilon_{eff} - 0.258) \left( \frac{W}{h} + 0.8 \right)} \quad (3.4)$$

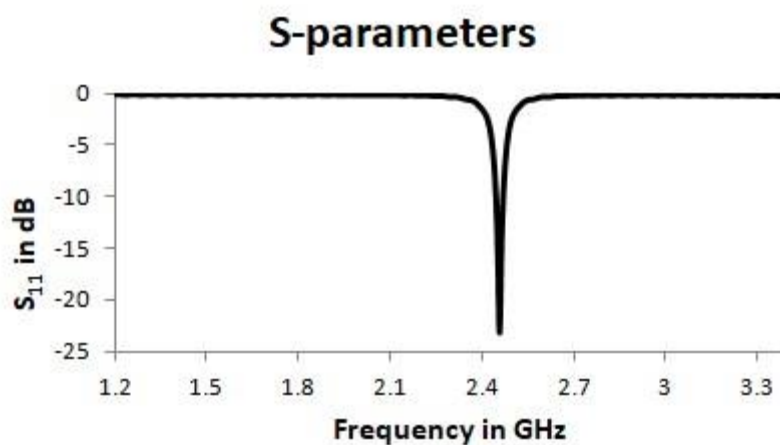


**Figure 5.2** Top View of a single patch element

**TABLE 5.1.** Design specifications for single patch element at 2.45 GHz

Antenna Dimensions	Value
Substrate dielectric constant	4.3
Substrate Height (mm)	0.762
Loss Tangent	0.002
Patch Length (L) (mm)	28.6
Patch Width (W) (mm)	37.6
Substrate Length (mm)	46
Substrate Width (mm)	56
Feed Line Width (mm)	1.48

The Return Loss plot of the single element microstrip patch antenna is shown in the Figure 5.3.



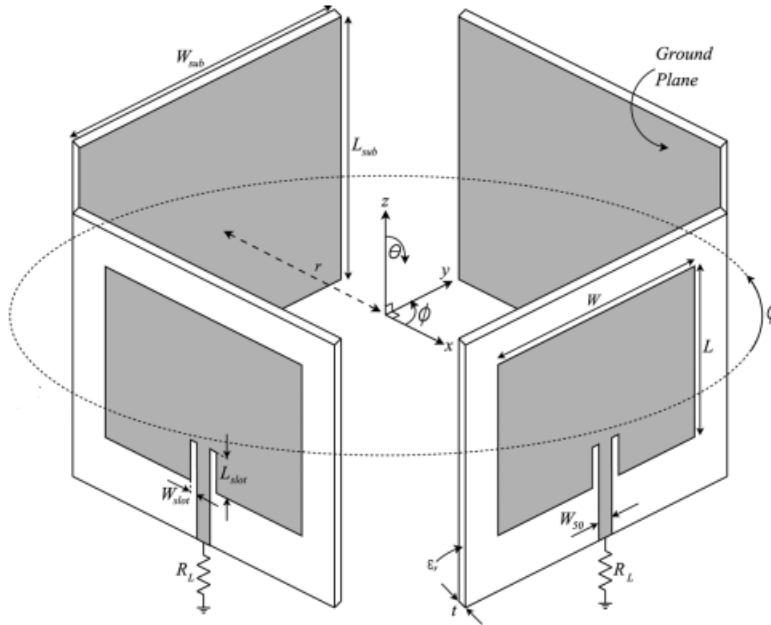
**Figure 5.3** Return Loss Plot of single element patch antenna at 2.45 GHz

Return Loss of the single element patch antenna is 23.66 dB at 2.45 GHz (refer to Figure 5.3). The patch antenna is designed in the CST Microwave Studio in the xy-plane. Next, the study is done in CST Microwave Studio with patch antenna as directive elements to form a circular array of four and eight elements and the corresponding circular array is studied on the studied based on the radiation pattern obtained. Also, the single feed is designed for the circular array in the next sections.

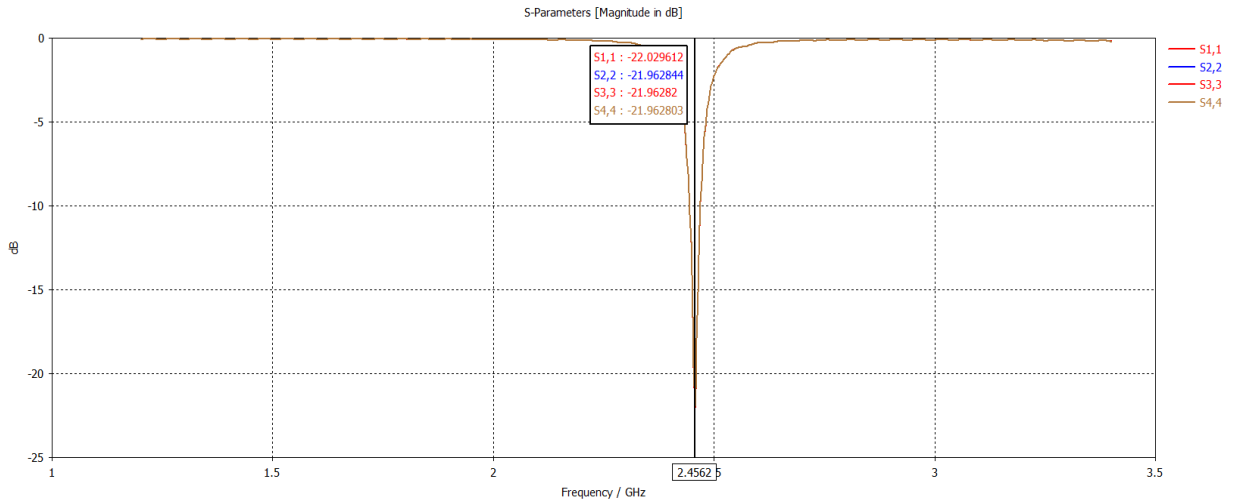
## 5.3 UNIFORM CIRCULAR ARRAY DESIGN

### 5.3.1 Four Element Uniform Circular Array

The four element uniform circular array (UCA) is shown in the Figure 5.4. Next, the effect of directional antenna elements in uniform circular arrays (UCAs) is studied in this section. The return loss plot of the four element UCA is shown in the Figure 5.5.



**Figure 5.4** Four-Element microstrip UCA [1]



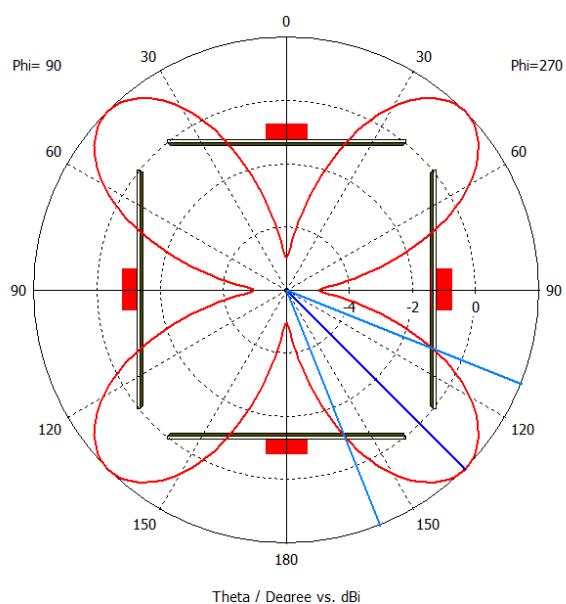
**Figure 5.5** Return Loss Plot for the Four Element Circular Array

Now, a parametric study has been done for the four element uniform circular array with patch antennas by varying the radius of the array.

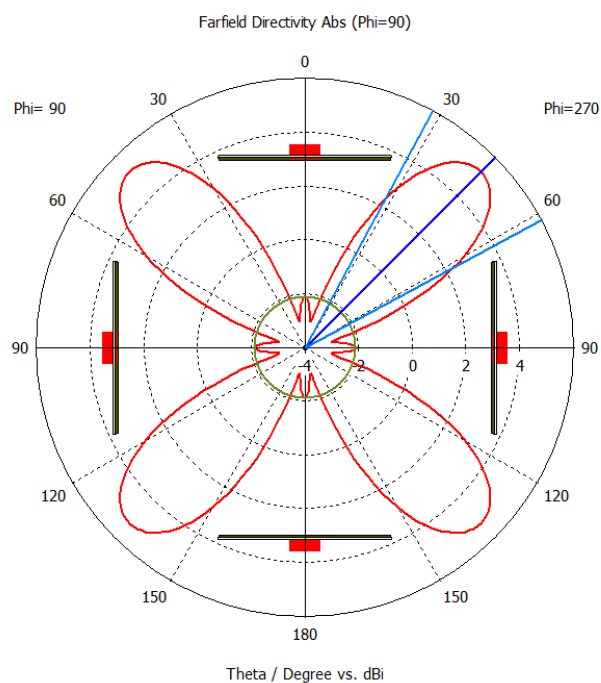
### 5.3.2 Parametric Study on Four Element Uniform Circular Array

The Radiation Pattern of Four Element Circular Array with radius as a parameter is studied using CST Microwave Studio 2019. The patterns of four element uniform circular array are shown in the with individual elements excited as per the study done taking radius as a parameter. The combined radiation pattern of the four element uniform circular array is shown in the Figure 5.6 for radius 50 mm.

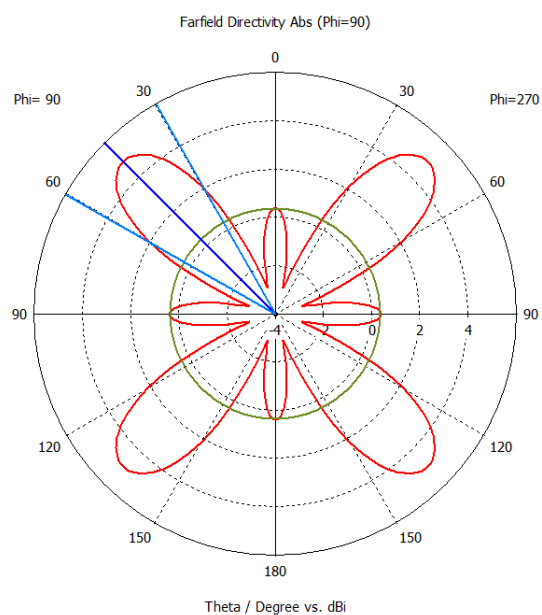
The parametric studies are done for different values of radius of four element UCA and the results are shown as in the Figure 5.7 and 5.8. The study done shows the side lobes formation in the four element UCA for radius 100 mm and 120 mm.



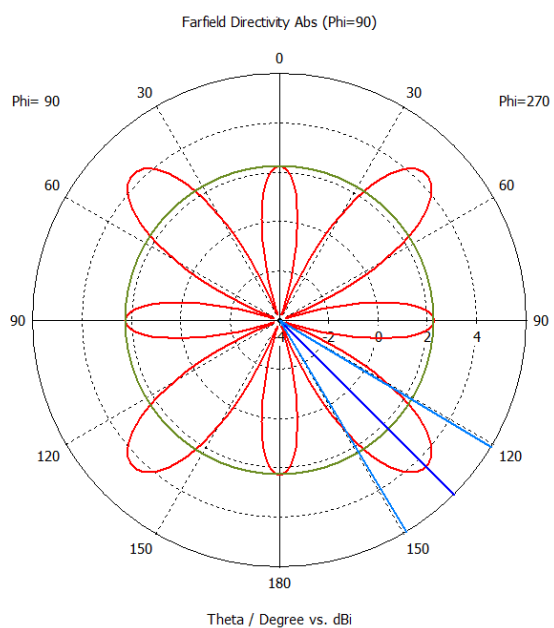
**Figure 5.6** Radiation Pattern for the combined four patch elements for  $R=50$  mm.



(a)

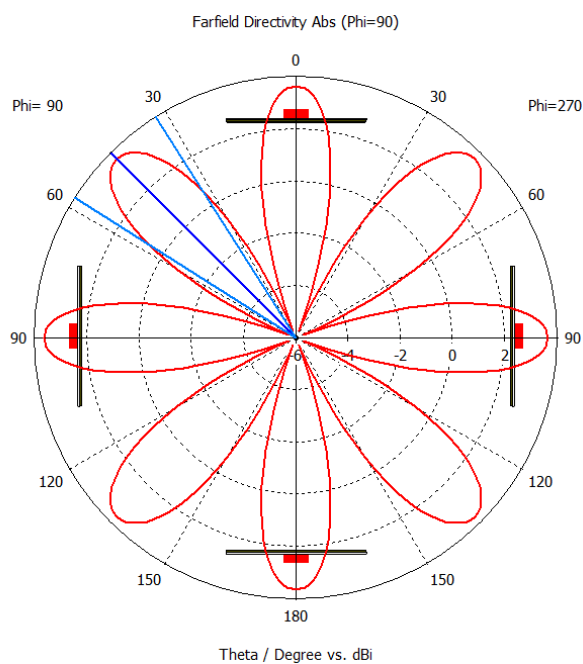


(b)



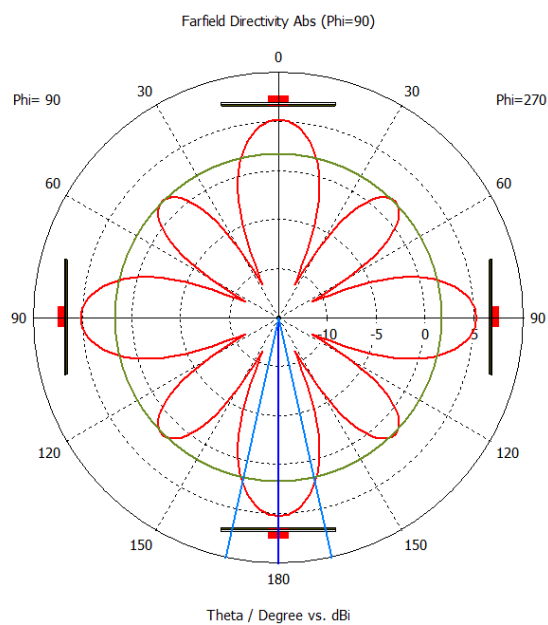
(c)



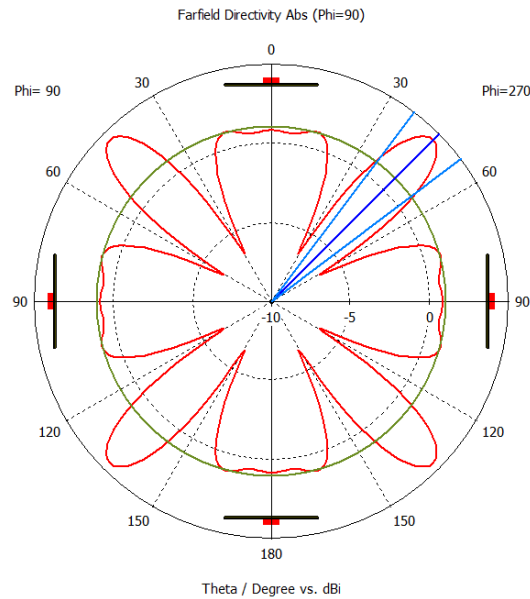


**(d)**

**Figure 5.7** Radiation Pattern of the four element circular array for radius (a) R=70 mm (b) R=80 mm (c) R=90 mm (d) R=100 mm



**(a)**

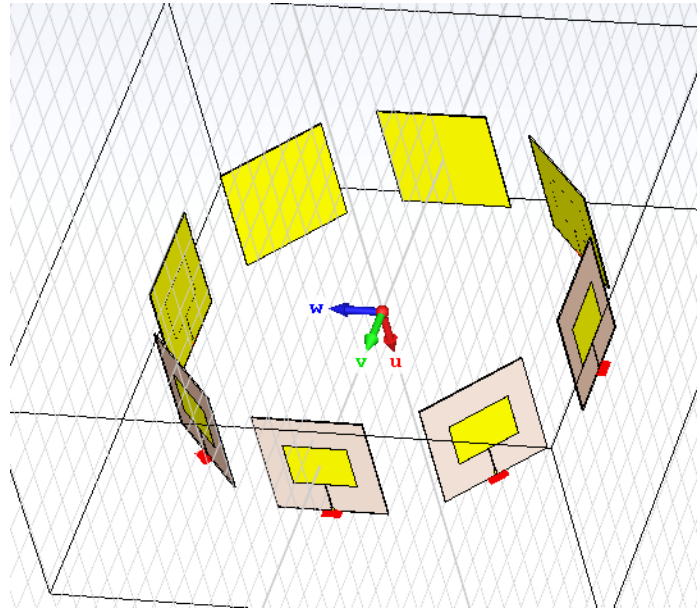


(b)

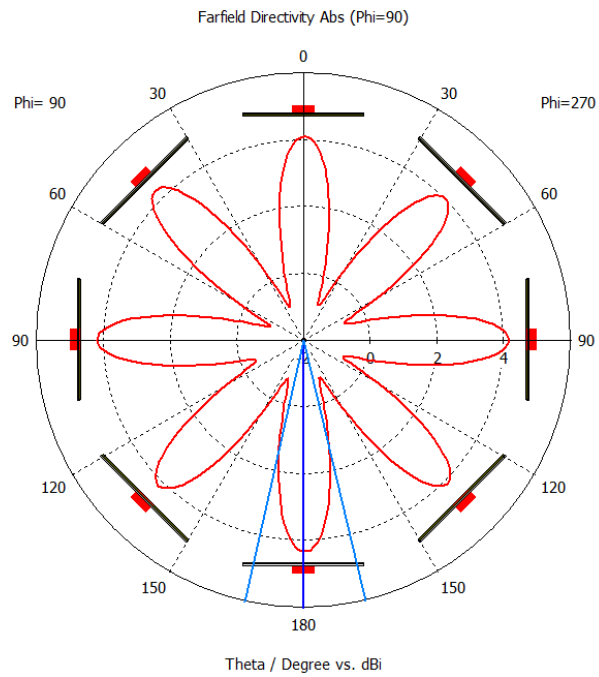
**Figure 5.8** Radiation Pattern of the four element circular array for radius (a)  $R=120$  mm (b)  $R=150$  mm

### 5.3.3 Eight Element Uniform Circular Array

The four element uniform circular array (UCA) is shown in the Figure 5.9. The Radiation Pattern of Eight Element Circular Array with radius as a parameter is studied in this section using CST Microwave Studio 2019. The patterns of eight element uniform circular array are shown in the with individual elements excited as per the study done taking radius as a parameter. The combined radiation pattern of the eight element uniform circular array is shown in the Figure 5.10 for radius 120 mm. The study done shows no side lobes formation in the eight element UCA for radius 120 mm.



**Figure 5.9** Eight-Element UCA

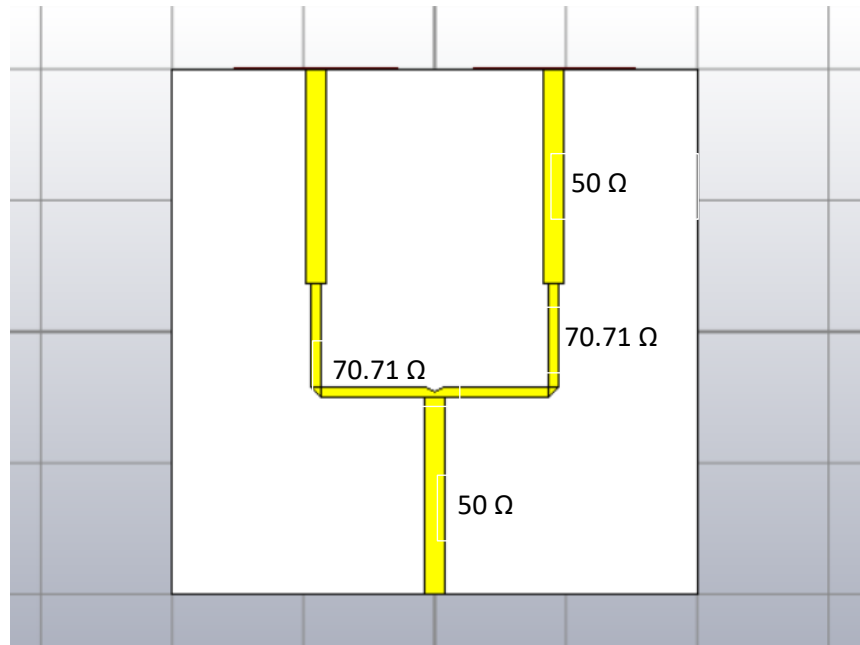


**Figure 5.10** Radiation Pattern of the four element circular array for radius 120 mm

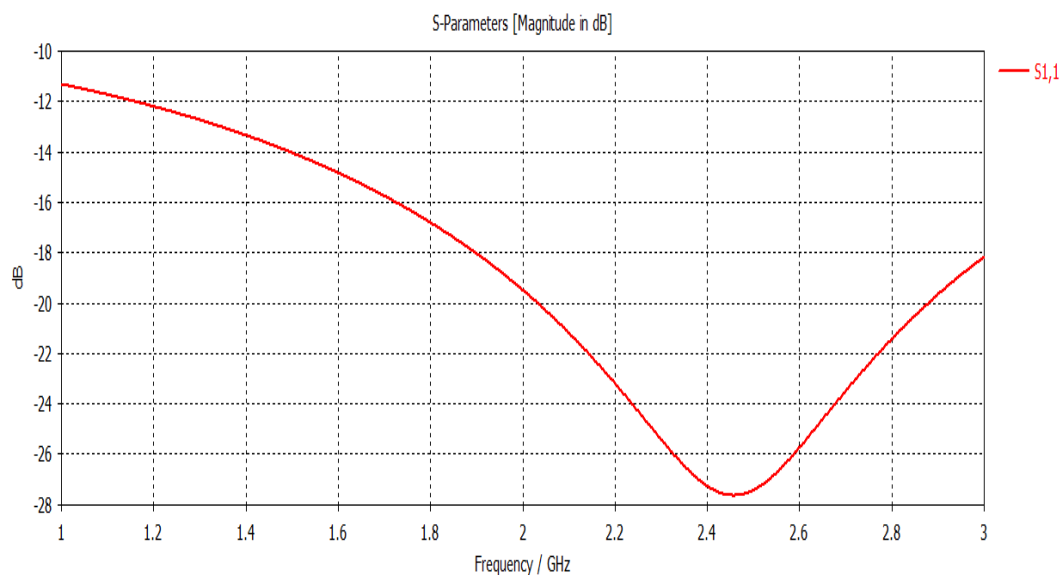
## 5.4 FEED NETWORK DESIGN

### 5.4.1 Feed Network Design- 2:1 Power Divider

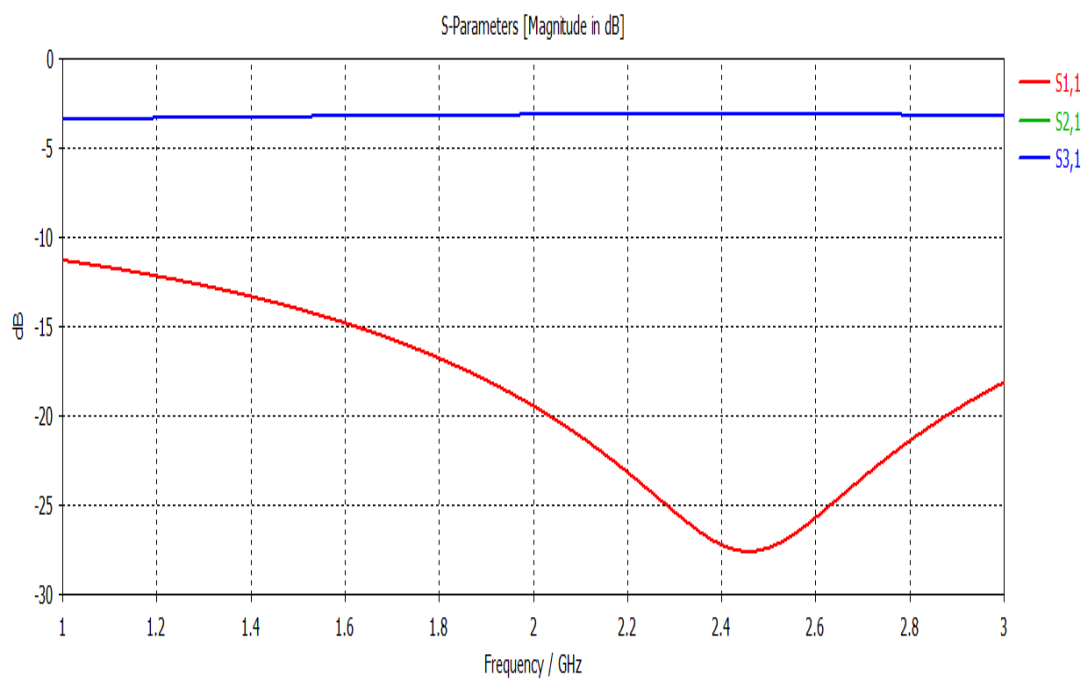
The Wilkinson power divider is implemented for the power division which is often made in microstrip line form [4]. An equal-split Wilkinson power divider in microstrip line form is designed and simulated in CST Microwave Studio as shown in the Figure 5.11. The return loss plot and insertion loss plot are shown in the Figures 5.12 and 5.13 respectively.



**Figure 5.11** An equal-split Wilkinson power divider in microstrip Line form



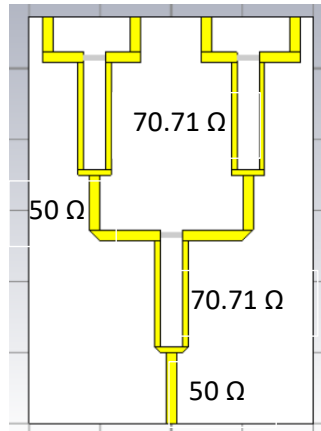
**Figure 5.12** Return Loss plot for the 2:1 power divider



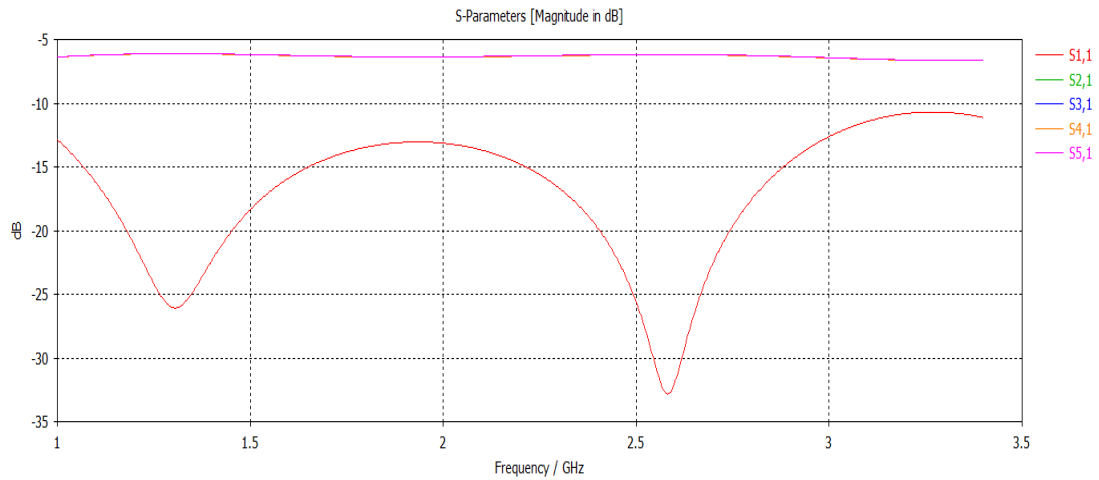
**Figure 5.13** Insertion Loss plot for the 2:1 power divider

### 5.4.2 Feed Network Design- 4:1 Power Divider

A four way Wilkinson power divider in microstrip Line form is made from 2:1 or two way power divider as shown in the Figure 5.14. The return loss plot and insertion loss plot are shown in the Figure 5.15.



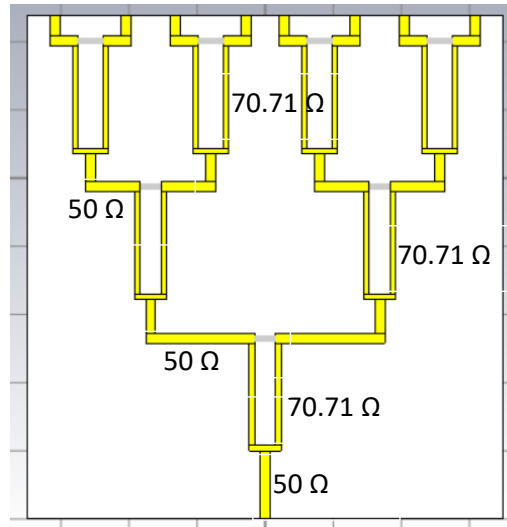
**Figure 5.14** A four way Wilkinson power divider in microstrip Line form



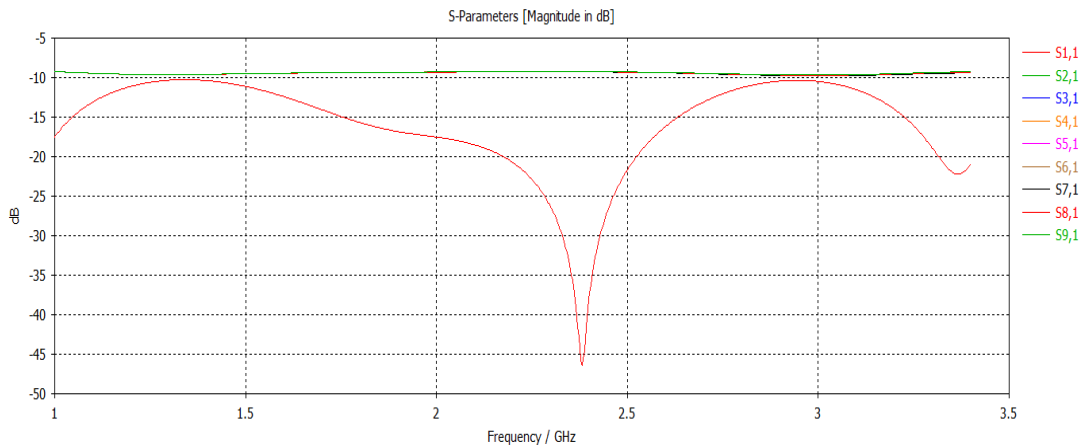
**Figure 5.15** Return Loss and Insertion Loss plot for the 4:1 power divider

### 5.4.3 Feed Network Design- 8:1 Power Divider

An eight way Wilkinson power divider in microstrip Line form is made from 4:1 or four way power divider as shown in the Figure 5.16. The return loss plot and insertion loss plot are shown in the Figure 5.17.

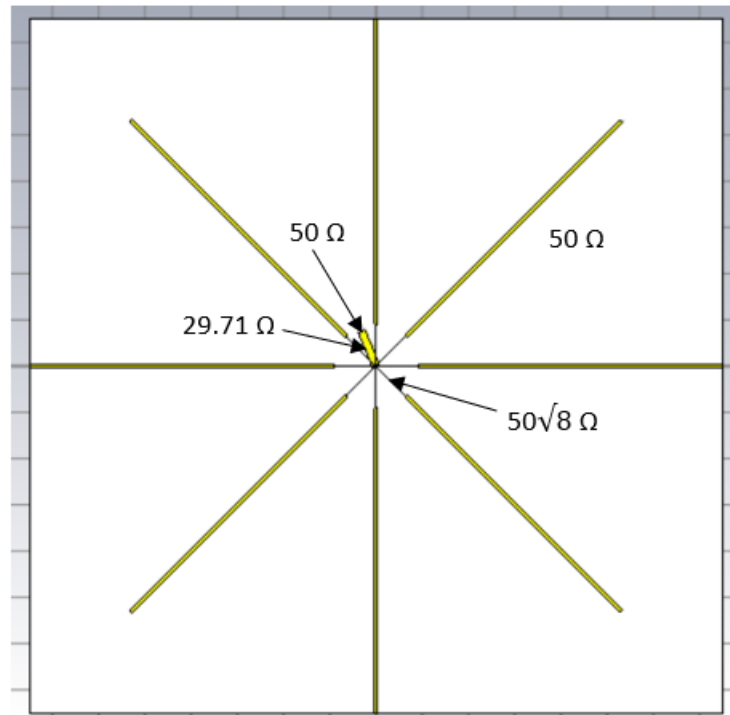


**Figure 5.16** An eight way Wilkinson power divider in microstrip Line form



**Figure 5.17** Return Loss and Insertion Loss plot for the 8:1 power divider

A Symmetrical Feed network design for eight way power divider microstrip line form is also designed as shown in the Figure 5.18. The return loss plot and insertion loss plot are shown in the Figure 5.19.



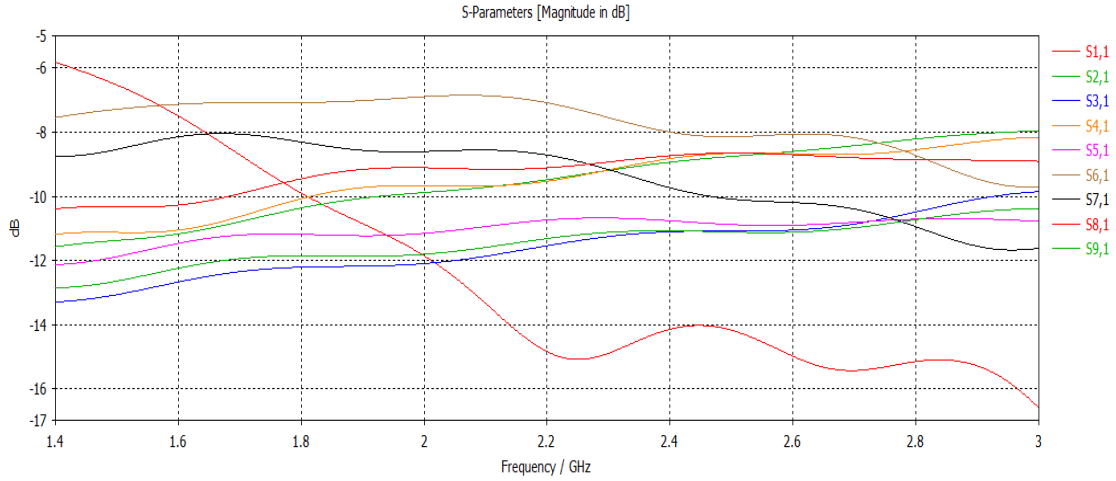
**Figure 5.18** A Symmetrical Feed network design for 8 Element circular array

The width of the microstrip lines is calculated using the microstrip line calculator and the values are tabulated in the Table II.

**TABLE 5.2. MICROSTRIP LINE IMPEDANCE DIMENSION**

Impedance	Width (mm)
$50 \Omega$	1.48
$70.71 \Omega$	0.788
$29.71 \Omega$	3.21

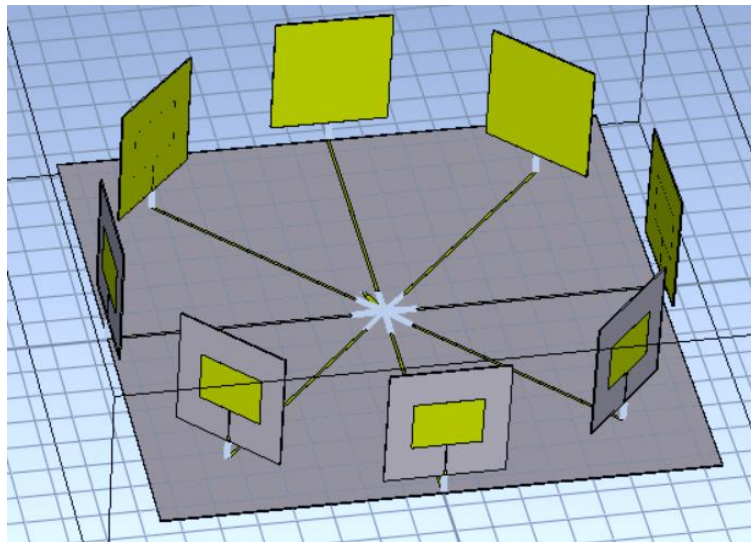




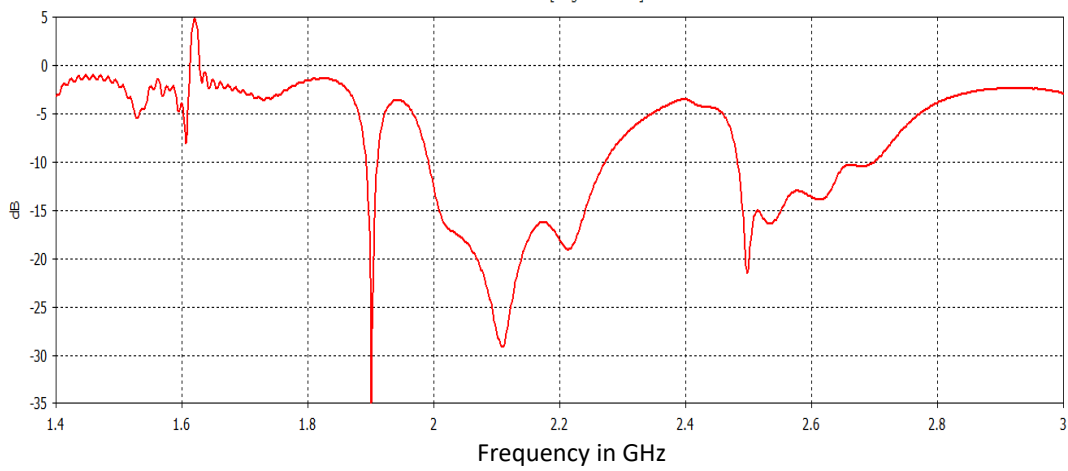
**Figure 5.19** Return Loss and Insertion Loss plot for the Symmetrical Feed network design of an eight power divider

### 5.5 Circular Array with Feed Network

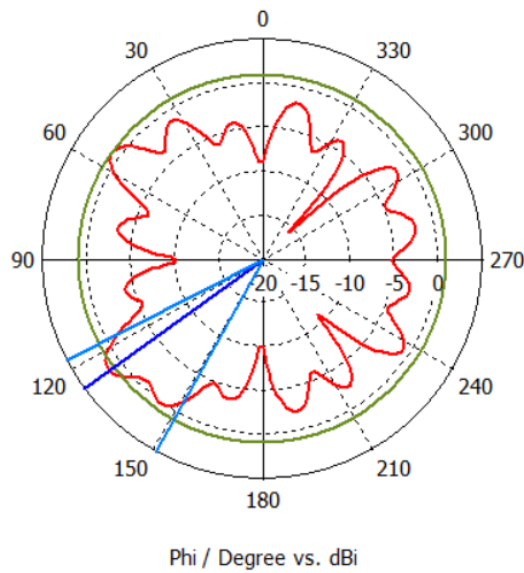
In the last section, a symmetrical feed network is designed for an eight-way power divider (refer to Figure 5.18). Now, a combined simulation is carried out in CST Microwave Studio for the eight-way circular array with symmetrical feed network as shown in the figure 5.20. The return loss plot and the radiation plot at  $\theta=90^\circ$  plane is shown in the Figures 5.21 and 5.22 respectively.



**Figure 5.20** Eight-Element Circular array with feed network design



**Figure 5.21** Return Loss plot for the eight element circular array with symmetrical feed network design



**Figure 5.22** Radiation pattern for eight element circular array with symmetrical feed network design at  $\theta=90^\circ$  plane at 2.5 GHz

As observed from the radiation pattern of the combined pattern of circular array with feed network and the radiation pattern of the circular array with individual feeds (refer to Figure 5.10), significant tuning and adjustment in the combined design is required in its design.

## 5.5 Conclusion

The effect of directional antenna elements in uniform circular arrays (UCAs) is studied in this chapter. The radiation Pattern of Four Element and Eight Circular Array with radius as a parameter is studied with CST Microwave Studio 2019. The microstrip antenna is designed with microstrip line feed. A matching network between the feed line and the patch such as quarter wavelength transformer. The Quarter wavelength transformer compensates the impedance differences between the patch and the 50 feed line. The circular array of four microstrip antenna is first designed and the results show that the circular array provides  $360^0$  scanning but has significant side lobes. This is found with the parametric study of circular array by varying the radius of the circular array. The radiation pattern of the eight element uniform circular array shows  $360^0$  coverage with almost no side lobes for the radius (equal to  $= \lambda_0$ ). The feed network for the eight elements circular array is then designed in microstrip line form. So, a symmetrical feed network design for eight way power divider microstrip line form is also designed. The return loss plot and insertion loss plot are showing the division of power in 8:1 form. Then, a combined simulation is carried out in CST Microwave Studio for the eight element circular array with symmetrical feed network. significant tuning and adjustment in the combined design is required in its design. The radiation pattern of the combined pattern of circular array with feed network shows significant tuning and adjustment in the combined design is required in its design.

## References

- [1] B. R. Jackson, S. Rajan, B. J. Liao and S. Wang, "Direction of Arrival Estimation Using Directive Antennas in Uniform Circular Arrays," in *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 2, pp. 736-747, Feb. 2015.

- [2] J. Liu, I. Li, and W. Huazhi, "Investigation of different types of array structures for smart antennas," in *Proc. Int. Conf. Microw. Millimeter Wave Technol. (ICMMT)*, 2008, pp. 1160–1163.
- [3] M. R. Islam and I. A. H. Adam, "Performance study of direction of arrival (DOA) estimation algorithms for linear array antenna," in *Proc. Int. Conf. Signal Process. Syst.*, 2009, pp. 268–271.
- [4] B. Wu, "Realization and simulation of DOA estimation using MUSIC algorithm with uniform circular arrays," in *Proc. 4th Asia-Pac. Conf. Environ. Electromagn.*, 2006, pp. 908–912.
- [5] L. Rui, S. Xiaowei, C. Lei, L. Ping, and X. Le, "The noncircular MUSIC method for uniform rectangular arrays," in *Proc. Int. Conf. Microw. Millimeter Wave Technol. (ICMMT)*, 2010, pp. 1390–1393.
- [6] C. A. Balanis, "*Antenna Theory Analysis and Design*", John Wiley and Sons, 2016.
- [7] N. Ab Wahab, Z. Bin Maslan, W. N. W. Muhamad and N. Hamzah, "Microstrip Rectangular 4x1 Patch Array Antenna at 2.5GHz for WiMax Application," *2010 2nd International Conference on Computational Intelligence, Communication Systems and Networks*, 2010, pp. 164-168.
- [8] C. A. Balanis, *Advanced Engineering Electromagnetics*, Second Edition, John Wiley & Sons, New York, 2012.
- [9] D. M. Pozar, "*Microwave Engineering*", 3rd edition, John Wiley & Sons, Inc., New York, 2004.

# Chapter 6

## Conclusion and Future Work

---

### 6.1 Conclusion

This thesis presented focused on designing circular array of microstrip antenna with feed network operating at 2.45 GHz. This thesis presented a design of symmetrical feed network of eight way power divider for the eight element circular array. The circular array is designed and simulated with microstrip antenna in CST Microwave Studio 2019.

The microstrip antenna is designed with microstrip line feed. In microstrip feed, the patch is fed by a microstrip line that is located on the same plane as the patch. Microstrip feeding is simple to model, easy to match and easy to fabricate. It is also a good choice for use in antenna-array feeding networks. A matching network between the feed line and the patch such as quarter wavelength transformer. The Quarter wavelength transformer compensates the impedance differences between the patch and the 50 feed line.

The circular array of four microstrip antenna is first designed and the results show that the circular array provides  $360^{\circ}$  scanning but has significant side lobes. This is found with the parametric study of circular array by varying the radius of the circular array. The circular array of eight element array is designed with microstrip antennas with radius equal to one unit of wavelength ( $=\lambda_0$ ). The radiation pattern of the eight element uniform circular array shows  $360^{\circ}$  coverage with almost no side lobes for the radius (equal to  $=\lambda_0$ ). The feed network for the eight element circular array is then designed in microstrip line form. First, the two way Wilkinson

power divider is designed and the design is extended to form eight way power divider. Also, a symmetrical feed network design for eight way power divider microstrip line form is also designed. The return loss plot and insertion loss plot are showing the division of power in 8:1 form. After this, a combined simulation is carried out in CST Microwave Studio for the eight-element circular array with symmetrical feed network. The radiation pattern of the combined pattern of circular array with feed network shows significant tuning and adjustment in the combined design is required in its design.

## **6.2 Future Work**

The objective of this thesis is to present a new single fed uniform circular array microstrip antenna. A uniform circular array has a unique advantage because of its symmetry property. It can scan a beam azimuthally through  $360^{\circ}$ . But designing a circular array antenna is challenging and it requires combination of design steps. The circular array is designed using patch antennas. The thesis presented shows a design of symmetrical feed network of eight-way power divider for the eight-element circular array using patch antennas.

This work can be extended by designing and simulating the symmetrical feed network and the eight element circular array together in the simulation software. And then the structure of the feed elements and the eight element circular array of microstrip antenna is to be fabricated and tested. Another issue is to derive the solution for the reduction of the side lobes in the radiation patterns of the circular array. Also, the work must be done in the design of combined simulation of eight-element circular array with symmetrical feed network for achieving the desired matching and radiation pattern.

# Appendix

---

## MATLAB Codes

Matlab program to compute Array Factor pattern of Linear  
Broadside Arrays ( $N = 10$ ,  $d = \lambda/4$ )

```
% In this program we will compute the Array Factor pattern of  
Linear  
% Broadside Arrays.  
% For Broadside Arrays ,we know that the maximum radiation is  
directed  
% towards normal to the axis of the array,i.e Theta = 90 degrees .  
% For end-Fire array ,maximum radiation is directed along the axis  
of the  
% array,i.e theta = 0 degrees or 180 degrees.  
% Here,we have choosen distance in terms of lambda,i.e distance =  
lamda/2  
% or distance =lamda/4 as per necessity.  
N=10 ; % Number of Elements  
d=0.5; % distance between the elements  
lambda=1;  
maxAngle=90; %For Broadside array  
maxRad=maxAngle*pi/180;  
k=2*pi/lambda;  
phi=linspace(0,2*pi,1000);  
beta = -k*d*cos(maxRad);%beta is the progressive phase differnece  
between the N elements  
phsi= beta + k*d*cos(phi);  
%Normalized Array Factor Calculation  
AF=sin(N*phsi/2) ./ (N*sin(phsi/2));  
AF1=abs(AF);  
polar(phi,AF1);  
title(' Array Factor pattern of a 10-element uniform amplitude  
Broadside array');  
disp(' N = 10, d =  $\lambda/4$  ');
```

### Matlab program to compute Array Factor pattern of Linear Broadside Arrays ( $N = 10$ , $d = \lambda$ )

```
% In this program we will compute the Array Factor pattern of
Linear
% Broadside Arrays.
% For Broadside Arrays ,we know that the maximum radiation is
directed
% towards normal to the axis of the array,i.e Theta = 90 degrees .
% For end-Fire array ,maximum radiation is directed along the axis
of the
% array,i.e theta = 0 degrees or 180 degrees.
% Here,we have choosen distance in terms of lambda,i.e distance =
lamda/2
% or distance =lamda as per necessity.
N=10 ; % Number of Elements
d=1; % distance between the elements
lambda=1;
maxAngle=90; %For Broadside array
maxRad=maxAngle*pi/180;
k=2*pi/lambda;
phi=linspace(0,2*pi,1000);
beta = -k*d*cos(maxRad);%beta is the progressive phase differnece
between the N elements
phsi= beta + k*d*cos(phi);
%Normalized Array Factor Calculation
AF=sin(N*phsi/2) ./ (N*sin(phsi/2));
AF1=abs(AF);
polar(phi,AF1);
title(' Array Factor pattern of a 10-element uniform amplitude
Broadside array');
disp(' N = 10, d =  $\lambda$  ');
```



Matlab program to compute Array Factor pattern of Linear End-Fire  
Arrays ( $N = 10$ ,  $d = \lambda/4$ )

```
% In this program we will compute the Array Factor pattern of
Linear
% End-Fire Arrays.
% For Broadside Arrays ,we know that the maximum radiation is
directed
% towards normal to the axis of the array,i.e Theta = 90 degrees .
% For end-Fire array ,maximum radiation is directed along the axis
of the
% array,i.e theta = 0 degrees or 180 degrees.
% Here,we have choosen distance in terms of lambda,i.e distance =
lamda/2
% or distance =lamda/4 as per necessity.
N=10 ; % Number of Elements
d=0.25; % distance between the elements
lambda=1;
maxAngle=0; %For End-Fire array ,theta = 0 degree or 180 degrees
maxRad=maxAngle*pi/180;
k=2*pi/lambda;
phi=linspace(0,2*pi,1000);
beta = -k*d*cos(maxRad);%beta is the progressive phase differnece
between the N elements
phsi= beta + k*d*cos(phi);
%Normalized Array Factor Calculation
AF=sin(N*phsi/2) ./ (N*sin(phsi/2));
AF1=abs(AF);
polar(phi,AF1);
title('N = 10, d =  $\lambda /4$ ');
```

Matlab program to compute Array Factor pattern of Phased  
(scanning) Array ( $N = 10$ ,  $d = \lambda/4$ )

```
% In this program we will compute the Array Factor pattern of
Linear
% Phased Scanning arrays
% For Broadside Arrays ,we know that the maximum radiation is
directed
% towards normal to the axis of the array,i.e Theta = 90 degrees .
% For end-Fire array ,maximum radiation is directed along the axis
of the
% array,i.e theta = 0 degrees or 180 degrees.
% For phased Scanning Arrays, the maximum radiation of the array is
required to be oriented at an angle
% theta = ( $0 \leq \theta \leq 180$ )
% Here,we have choosen distance in terms of lambda,i.e distance =
lamda/2
% or distance =lamda/4 as per necessity.
N=10 ; % Number of Elements
d=0.25; % distance between the elements
lambda=1;
maxAngle=60; %For phased-Scanning array,let us caculate towards
theta = 60 degrees
maxRad=maxAngle*pi/180;
k=2*pi/lambda;
phi=linspace(0,2*pi,1000);
beta = -k*d*cos(maxRad);%beta is the progressive phase differnece
between the N elements
phsi= beta + k*d*cos(phi);
%Normalized Array Factor Calculation
AF=sin(N*phsi/2) ./ (N*sin(phsi/2));
AF1=abs(AF);
polar(phi,AF1);
view(90,-90);
title(' Array Factor pattern of a 10-element uniform amplitude
phased Scanning array');
disp(' N = 10, d =  $\lambda /4$  ');
```

### Matlab program to compute Array Factor pattern of Binomial Array

( $N = 10$ ,  $d = \lambda/4$ )

```
% In this program, we will Design a binomial array of 10 elements
with spacing d between the elements
% and with a major-to-minor lobe ratio of 26 dB.
% we will Find the excitation coefficients and form the array
factor.
% In this program,we will find the the excitation coefficients with
the
% pascal triangle method
N=10;% number of elements
d=0.25;%distance between the elements
lambda=1;
theta = 0:pi/1000:2*pi;
m = N-1;
u=pi*d*cos(theta)/lambda;
for n = 0:m % Generation of pascal triangle for N elements
    C = sym([]);
    for k = 0:n
        C = horzcat(C, nchoosek(n, k));
    end
    disp(C);
end
for i = 0:N/2-1 % Calculation of excitation coefficients for even
elements
    for j= 0:i
        b(i+1)=C(N/2-j);
    end
end
disp(b);
disp('Excitation coefficients are-->')
%Calculation of array Factor
AF=0;
for n = 1:(N/2);
    AF = AF + b(n)*cos((2*n-1)*u);
end
AF1=abs(AF);
AFN=AF1/max(AF1);
%%Plotting of Array Factor Of 10 Element Binomial Array
polar(theta,AFN);
```

```

view(90,-90);
AFdB=20*log10(AFN);
plot(theta,(AFdB+50)/50);
axis([0 3 0 1]);
DodB=1.77*N^(0.5);
x= fprintf('Directivity = %d dB',DodB);
HPBW=1.06/(N-1)^2 ;
y = fprintf('Half Power Bandwidth = %d radians',HPBW);

```

### Matlab program to compute Array Factor pattern of Dolph-Tschebyscheff Array ( $N = 10$ , $d = \lambda/4$ ) by Barbieri

```

% In this program, we will Design a broadside Dolph-Tschebyscheff
array of 10 elements with spacing d between the elements
% we will Find the excitation coefficients and form the array
factor.
% In this program,we will find the the excitation coefficients with
the
% calucations given by Barbieri .
N=10; % Number of elemnts=10 (even)
d=0.5;
lambda=1;
RodB=26; %Major-to-minor lobe ratio
P=N-1;
Ro=10^(RodB/20);
Zo=cosh(1/P*acosh(Ro));
theta = 0:pi/100:2*pi;
u=pi*d*cos(theta)/lambda;
M=5;
for n = 1:M; % Calculations of Coefficients of Chebyshev array
    a(n)=0;
    for q=n:M;
        a(n)=a(n)+(-1)^(M-q)*Zo^(2*q-1)*factorial(q+M-2)*factorial(2*M-
1)/(factorial(q-n)*factorial(q+n-1)*factorial(M-q));
    end
end
end

```

```

disp('coefficients are -->');
disp(a/a(1));
%%Calculations of Array Factor
AF=0;
for n = 1:N/2;
    AF = AF + a(n)*cos((2*n-1)*u); % Formula for even number of
elements
end
AF1=abs(AF);
AFN=AF1/max(AF1);% normalized array Factor
AFdB=20*log10(AF1);
%%Plotting of Array Factor Of 10 Element Dolph - Chebyshev Array
polar(theta,AFN);

title ('Array factor power pattern of a N = 10 element broadside
Dolph-Tschebyscheff array');
%Cartesian plots
plot(theta,AFN);
axis([0 3 0 1]);

```

### Matlab program to compute Array Factor pattern of Dolph-Tschebyscheff Array ( $N = 10$ , $d = \lambda/4$ ) by chebwin function

```

% In this program, we will Design a broadside Dolph-Tschebyscheff
array of 10 elements with spacing d between the elements
% and with a major-to-minor lobe ratio of 26 dB.
% we will Find the excitation coefficients and form the array
factor.
% In this program,we will find the the excitation coefficients with
the
% help of Chebwin function . .
N=10; % Number of elemnts=10 (even)
lamda=1;
d=lamda/4;
theta = 0:pi/100:2*pi;
u=pi*d*cos(theta);
M=N/2;

```

```

RodB=26; %Major-to-minor lobe ratio
coeff=chebwin(N,RodB); % Calculations of Coefficients of Chebyshev
array
%%Calculations of Array Factor
AF=0;
for i =1:M
    AF=AF+coeff(i)*cos(u*(2*i-1)); % Formula for even number of
elements
end
disp('coefficients are -->');

disp(coeff);
AF1=abs(AF);
AFN=AF1/max(AF1);
AFdB=20*log10(AF1);
%%Plotting of Array Factor Of 10 Element Dolph - Chebyshev Array
plot(theta,AFN);
axis([0 3 0 1]);
plot(theta,(AFdB+70)/70); % Normalized Array Factor in dB
axis([0 3 0 1.2]);
polar(theta,AFN);
title ('Array factor power pattern of a N = 10 element broadside
Dolph-Tschebyscheff ');

```

### MATLAB program to compute Array Factor pattern of N-element Circular Array with Bessel Function

```

N=10 ; % Number of Elements
a=1;
k=10;
lambda=2*pi/k;
f=3*10.^10/lambda;
theta0=90;
phi0=0;
phi=linspace(0,2*pi,1801);
theta=90;
rho1=sin(theta)*cos(phi)-sin(theta0)*cos(phi0);
rho2=sin(theta)*sin(phi)-sin(theta0)*sin(phi0);
rho3=rho1.^2+rho2.^2;

```

```
rho0=sqrt(rho3);  
rho4=a*rho0;  
AF=besselj(0,k.*rho4);  
polar(phi,AF);  
plot(phi,AF);
```

Matlab program to compute Array Factor pattern of N-element  
Circular Array with exponential Function

```
N=10; % Number of Elements  
a=1;  
k=10;  
lambda=2*pi/k;  
f=3*10.^10/lambda;  
theta0=90;  
phi0=0;  
phi=linspace(0,pi*2,1801);  
theta=90;  
  
AF=0;  
for n=1:N  
    AF=AF+exp(1i.*k*a*(sin(theta)*cos(phi-phi(n)))-sin(theta0)*cos(phi0-  
        phi(n)));  
End  
  
AF1=abs(AF);  
AFN=AF1/N;  
plot(phi,AFN);  
polar(phi,AFN);
```