# ADVANCEMENTS IN OPTIMIZATION TECHNIQUES FOR INDUSTRIAL ENGINEERING: A COMPARATIVE STUDY OF EMERGING METAHEURISTIC ALGORITHMS AND FUZZY-BASED DECISION-MAKING METHODS

BY

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(SOMNATH NANDI)

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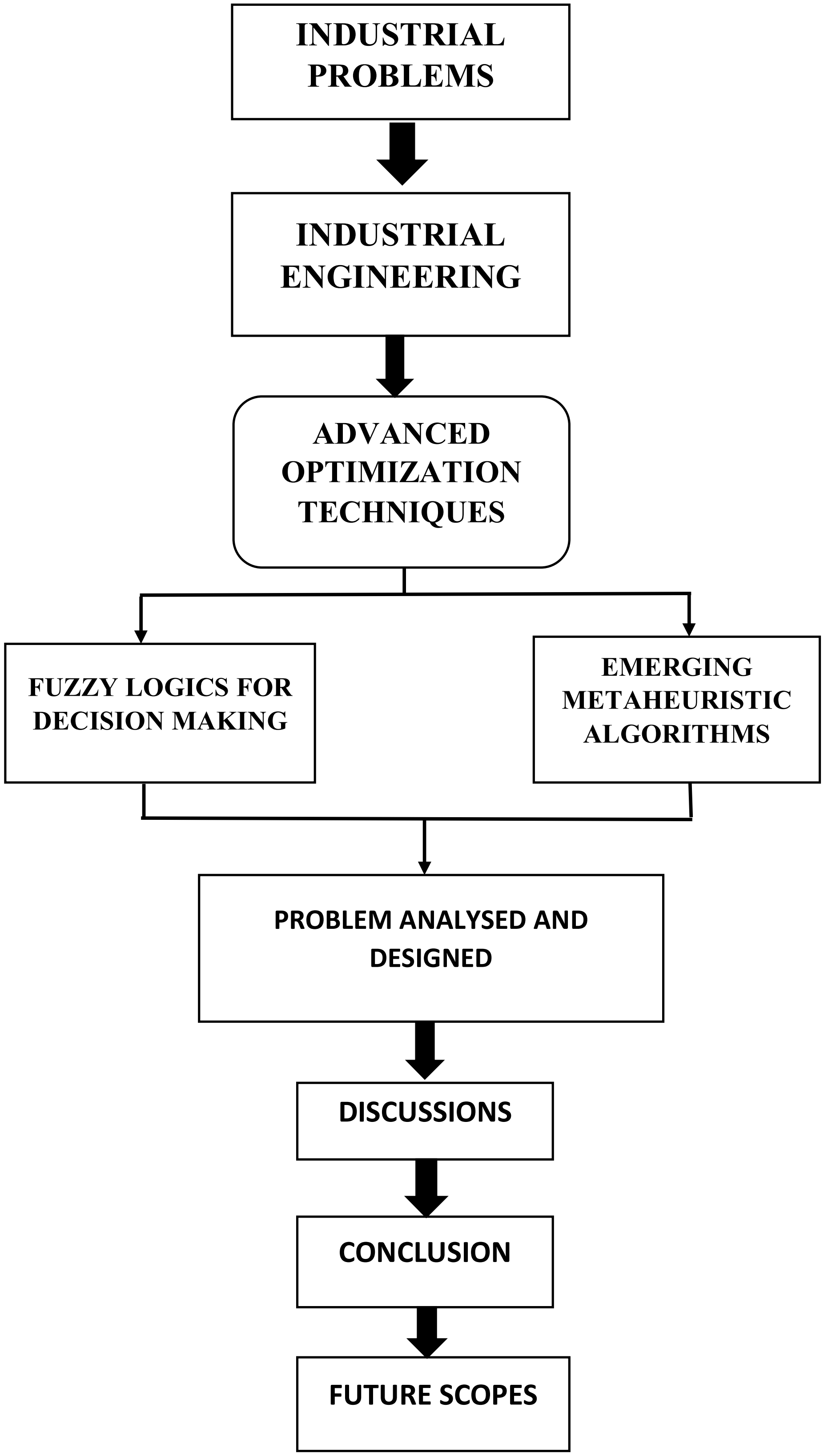
# ABSTRACT

The modern world is a competitive one. Any organization that wants to succeed must keep up with the changing demands of the global marketplace. The success factors for any organization are hence ongoing quality improvement and wise decision-making. The most effective use of time and available resources are the other key elements that greatly influence an organization's performance. Nowadays companies today face new challenges in discovering strategies and even techniques that will enable top management to identify and select from the market those assets that offer the ideal combination between the acquisition cost and economic performance. This is because businesses operate in highly competitive environments where it can be difficult to survive on the market. In manufacturing industry, Non-traditional machining (NTM) techniques are more recent production methods that emerged as a result of the fast invention of new materials that are challenging to cut and the rising demand for producing intricate part shapes. For each of these NTM processes, there are different adjustable parameters. The combination of the several input parameters of these NTM methods determines how effective they are. But because there are so many control parameters available and there are so many contradicting responses, it can be challenging to select the ideal set of input parameters for a given NTM process. Thus, in this research work, we have taken two optimization problems from the areas of manufacturing and financial supply chain management and applied two different optimization techniques to solve them. In first part we used five foraging behavior-based metaheuristic multi-objective optimization techniques for parametric optimization of the NTM processes and applied a statistical non-parametric test (Friedman test) to find the best technique. In the second part of this research proposes a fuzzy logic managerial decision tool for asset acquisition. Here, with the help of four decision-makers we applied three fuzzy logics along with the fuzzy-TOPSIS approach to choose the optimum investment strategy while taking into account their tacit knowledge. The proposed fuzzy logic managerial decision tools produced excellent results coupled with strong economic performance, in order to assist investment choices made by an investment firm on global markets. This is done from the point of view of industrial engineering in order to help society and, mostly the mankind as a whole.

**A comprehensive flowchart for this thesis:**



# ABBREVIATION

| | | |
|---|---|---|
| RSM | : | Response Surface Methodology |
| CCD | : | Central Composite Design |
| BBD | : | Box-Behnken Design |
| FFD | : | Full Factorial Design |
| HTS | : | Heat Transfer Search |
| GFRP | : | Glass Fibre-Reinforced Polymers |
| FFRP | : | Flax Fibre-Reinforced Polymer Laminates |
| Ti6Al4V | : | Titanium alloys |
| Al6063/B4C/ZrSiO4 | : | Aluminium hybrid/ Boron Carbide/ Zirconium Silicate |
| MFL | : | Metal Fibre Laminate |
| KEC | : | Kevlar Epoxy Composite |
| $Pb[Zr_xTi_{1-x}]O_3$ | : | Lead Zirconate Titanate Ceramic |
| Ti/CFRP/Ti | : | Titanium-carbon fibre-reinforced plastics-titanium hybrid laminate |
| RHA | : | Rolled Homogeneous Armor Steel |
| SOD | : | Stand of Distance (mm) |
| WP | : | Water Pressure (MPa) |
| TR | : | Transverse Rate (mm/min) |
| AMFR | : | Abrasive Mass Flow Rate (g/min) |
| DLL + | : | Delamination |
| Ra | : | Surface Roughness (μm) |
| TS | : | Transverse Speed (mm/min) |
| AFR | : | Abrasive Flow Rate (g/min) |

| | | |
|---|---|---|
| AMS | : | Abrasive Mass Size (#) |
| DOC | : | Depth of Cut (mm) |
| MRR | : | Material Removal Rate (mm3/min) |
| KR | : | Kerf Ratio (mm/mm) |
| KTA | : | Kerf Taper Angle (deg) |
| KW | : | Kerf Width |
| IA | : | Inclination Angle |
| JFS | : | Jet Feed Speed |
| SS | : | Surface Speed |
| NTA | : | Nozzle Tilted Angle |
| GA | : | Genetic Algorithm |
| SA | : | Simulated Annealing |
| TLBO | : | Teaching Learning Based Optimization |
| GWO | : | Grey Wolf Optimization |
| TGRA | : | Taguchi-Grey Relation Analysis (TGRA) |
| FR | : | Feed Rate (mm/min) |

# CHAPTER 1

# INTRODUCTION

## 1.1 An Introduction to Optimization

The term "optimization" describes the act of determining the optimum value or solution within a predetermined set of restrictions or goals. It entails methodically analysing and modifying variables or parameters in order to maximise or minimise a particular objective function. The goal of optimization is to increase effectiveness, efficiency, productivity, resource allocation, cost-effectiveness, and overall performance in a variety of professions and businesses. It makes use of computational methods, mathematical models, and algorithms to identify the best solutions in intricate systems or processes.

Optimization is a field of study that deals with finding the best solution for a given problem. It involves maximizing or minimizing a specific objective function while considering a set of constraints. Optimization problems can be found in various domains such as mathematics, engineering, economics, computer science, and many others. The goal of optimization is to find the values of decision variables that optimize the objective function, subject to certain constraints. The decision variables are the parameters or variables that can be adjusted or controlled in order to achieve the best outcome. The objective function represents the quantity that needs to be maximized or minimized, such as profit, cost, time, efficiency, or any other measurable metric. From a mathematical perspective, optimization, or mathematical programming as it is sometimes called, rests on several pillars: applied mathematics fields like numerical analysis and computer science build the bridge to the algorithmic side of the subject, while theoretical mathematics fields like analysis, topology, algebra, discrete mathematics, etc., lay the foundation of the theory. On the other hand, optimization allows us to address issues in a wide range of fields, including the technical, natural, biological, and engineering sciences, as well as economics. Additionally, optimization has a very lengthy history. Most frequently, the geometrical or mechanical issues that Archimedes, Euclid, Heron, and other ancient masters of geometry formulated and also resolved are optimization issues. We provide the issue of maximising the volume of a closed three-dimensional shape (such a sphere or cylinder) made from a two-dimensional metal sheet with a specific area as an illustration. At the same time, the idea of optimality and, in particular, how to define an optimal solution, started to be developed. In fact, a key component of any introductory optimization course is characterising distinct types of optimal solutions.

Operations research is a branch of science that focuses on decision-making in relation to controlling complicated systems and events. The phrase was first used in the 1940s, at the height of World War II (WW2), when the US and British military commands employed scientists from a variety of fields to try to find solutions to challenging issues like the best way to build convoys to avoid or protect the cargo ships from enemy (read: German) submarines, how to best cover the British Isles with radar equipment given the limited availability of radar systems, and so forth. The multidisciplinary nature of these issues and their shared focus on maximising or minimising some objective subject to restrictions might be considered as the turning point in the development of science. Decision science, which more accurately captures the range of issues that can and are addressed using optimization techniques, is a better name than operations research.

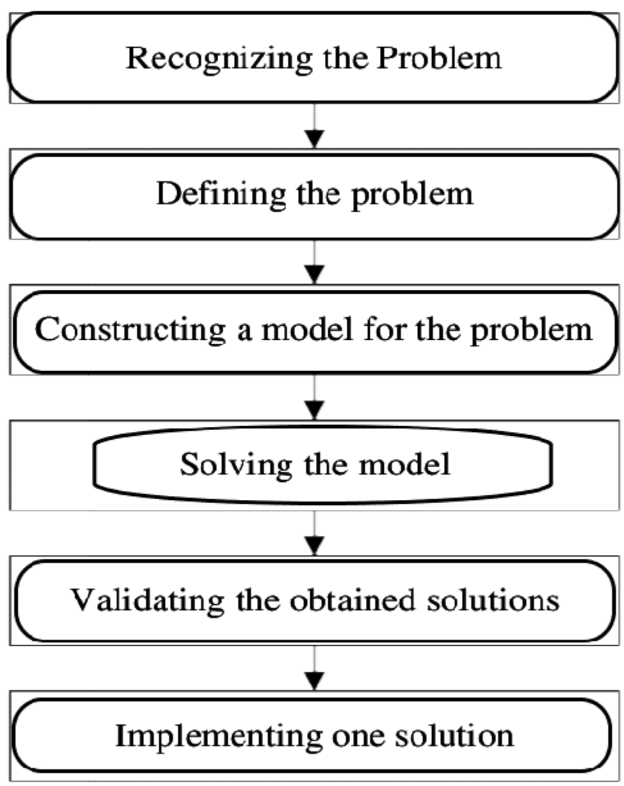



Figure 1.1 Flowchart of the optimization method

## 1.2 Type of Optimizations

Optimization techniques can be classified into several types based on the characteristics of the problem being solved and the approach used to find the optimal solution. I am describing some types of optimizations:

a) **Linear Optimization**: Linear optimization, also known as linear programming (LP), deals with optimizing linear objective functions subject to linear constraints. The decision variables and constraints are represented as linear equations or inequalities. Linear optimization is widely used in various domains, such as operations research, supply chain management, and production planning, where the relationships between variables are linear.

b) **Nonlinear Optimization**: Nonlinear optimization involves optimizing nonlinear objective functions subject to nonlinear constraints. The decision variables and constraints can involve nonlinear equations or inequalities. Nonlinear optimization is more complex than linear optimization and requires specialized techniques such as gradient-based methods (e.g., gradient descent), evolutionary algorithms (e.g., genetic algorithms), or interior point methods to find the optimal solution.

c) **Integer Optimization:** Integer optimization, also known as integer programming (IP), deals with optimizing objective functions subject to constraints, where some or all of the decision variables must take integer values. Integer optimization problems are more challenging than linear or nonlinear optimization problems because they involve combinatorial aspects and discrete decisions. Integer optimization is useful in situations where decisions need to be made among discrete options, such as selecting routes, scheduling tasks, or allocating resources.

d) **Mixed-Integer Optimization:** Mixed-integer optimization combines both continuous and integer decision variables in the optimization problem. This type of optimization involves finding the optimal values for both continuous and discrete variables simultaneously. Mixed-integer optimization problems are commonly encountered in supply chain optimization, facility location problems, and production planning, where decisions involve a combination of discrete and continuous choices.

e) **Convex Optimization:** Convex optimization deals with optimizing convex objective functions subject to convex constraints. Convex optimization problems are attractive because they have a single globally optimal solution and efficient algorithms for solving them. Convex optimization is used in various fields, including machine learning, signal processing, and control systems, where the problem structure exhibits convexity.

f) **Stochastic Optimization:** Stochastic optimization considers optimization problems with uncertain or probabilistic parameters. These problems involve optimizing objective functions that are affected by random variables or uncertain inputs. Stochastic optimization techniques, such as stochastic programming and robust optimization, aim to find solutions that are optimal under different scenarios or provide robustness against uncertain parameters.

g) **Multi-Objective Optimization:** Multi-objective optimization involves optimizing multiple conflicting objective functions simultaneously, aiming to find a set of solutions that represent the trade-offs between these objectives. Unlike single-objective optimization, which seeks a single optimal solution, multi-objective optimization generates a set of Pareto optimal solutions representing different trade-off possibilities. Multi-objective optimization is useful in decision-making processes where multiple objectives need to be considered simultaneously.

h) **Dynamic Optimization:** Dynamic optimization deals with optimization problems where decisions are made over a sequence of time periods or stages. These problems involve optimizing over time, taking into account the interdependencies and dynamics of the system. Dynamic optimization is commonly used in areas such as production planning, resource allocation, and inventory management, where decisions need to be made in a dynamic and evolving environment.

These are the most common categories of optimization techniques used in different fields. Each type has unique traits, difficulties, and specialized algorithms to identify the best answers based on the objectives and problem structure. The particular situation at hand and the desired results must be taken into consideration while selecting the best style of optimization.

## 1.3 Application of Optimization in Industrial Engineering

Optimization techniques find various applications in industrial engineering, contributing to improved productivity, efficiency, cost reduction, and decision-making. Among all the decision sciences, optimization theory is one of the most significant and rapidly expanding fields. It is described as the art and science of locating the best available solution(s) in accordance with some predefined measure(s) inside a workable domain or space delineated by a predetermined set of restrictions. The issues that optimization theory attempts to solve are essentially single-objective or multi-objective, stochastic or deterministic, discrete or continuous, offline or online. Optimization is of paramount importance in the industrial engineering domain and its applications due to several key reasons are describe below:

a) **Maximising Productivity and Efficiency:** Utilising resources like manpower, machinery, and materials can be done in the most efficient and effective methods possible with the aid of optimization strategies. Industrial engineers can increase productivity and throughput by optimising procedures, timetables for production, and resource allocation. This results in less waste, greater effectiveness, and higher productivity without wasting resources.
b) **Resource optimization and cost reduction:** Cost-cutting and resource-use efficiency both greatly benefit from optimization. Industrial engineers can find affordable fixes by optimising production schedules, inventory levels, supply chain operations, and maintenance plans. Organizations can save a lot of money by using optimization to reduce labour expenses, equipment downtime, transportation costs, and inventory holding costs.
c) **Improving Quality and Reliability:** It is possible to improve product quality, lower faults, and streamline production processes by using optimization techniques. Industrial engineers may raise the quality and consistency of their products by choosing the best process parameters, inspection schedules, and quality control techniques. In order to boost equipment reliability and decrease failure rates, optimization also helps to optimise maintenance plans and techniques.
d) **Effective Decision-Making:** Optimization provides a quantitative and data-driven approach to decision-making. By formulating problems as optimization models, industrial engineers can evaluate various scenarios, compare alternatives, and make informed decisions based on objective criteria. Optimization models consider multiple factors and constraints simultaneously, allowing decision-makers to consider complex trade-offs and find the best possible solutions.
e) **Sustainable Operations:** Industrial engineers can include sustainability factors into decision-making processes by using optimization techniques. Industrial engineers are able to create and manage sustainable systems by optimising resource use, energy use, waste production, and emissions. Finding ecologically sustainable solutions while juggling social and economic goals is made easier with optimization.

f) **Solving Complex Problems:** Industrial engineering frequently deals with intricate systems and issues including several variables, restrictions, and goals. By mathematically expressing the issues and identifying the best answers, optimization offers a methodical framework to handle such complexity. It makes it simpler to analyse and efficiently solve difficult problems by dividing them into smaller, more manageable pieces.

g) **Competitive Advantage:** Organizations functioning in industrial sectors may benefit from optimization as a competitive advantage. Companies can achieve greater efficiency, quicker delivery times, cheaper prices, and better product quality by optimising their manufacturing processes, supply networks, and resource allocation. Organizations may become more competitive, responsive, and agile thanks to optimization.

h) **Production Planning and Scheduling:** To create the best production schedules and plans that match customer demand, maximise throughput, and minimise costs, optimization models are used. These models take into account elements including labour requirements, inventory levels, equipment allocation, and production capacities. Industrial engineers can increase production schedules and overall efficiency by streamlining changeover times, balancing workloads, and optimising the order of processes.

i) **Inventory Management:** Applying optimization, which takes into account variables including demand variability, lead times, and storage costs, can help establish the best inventory levels, reorder points, and order numbers. Industrial engineers can lower the cost of maintaining inventory while assuring sufficient stock availability and minimising stock-outs by optimising inventory management. As a result, there is increased productivity, less waste, and better customer service.

j) **Supply Chain Optimization:** Supply chain activities, including critical choices like supplier selection, production site, and transportation route, are optimised by industrial engineers using optimization techniques. To create effective supply chain networks, optimization models take into account variables such as transportation costs, lead times, demand patterns, and service level requirements. Optimization improves the overall performance of the supply chain by maximising resource allocation, lowering transportation costs, and strengthening partner collaboration.

k) **Equipment Maintenance Optimization:** Optimization is used to improve maintenance schedules and methods. Industrial engineers can choose the best preventive maintenance plans, spare parts inventory levels, and condition-based maintenance tactics by analysing variables such equipment failure rates, maintenance costs, production needs, and criticality. This strategy reduces maintenance expenses, maximises equipment reliability, and minimises downtime.

l) **For Quality Control and Process Optimization:** Techniques for optimization help to enhance the quality of products and streamline production. Statistical optimization techniques and Design of Experiments (DOE) techniques aid in determining the ideal process parameters that maximise product quality while minimising variability. To reduce faults and inspection costs, industrial engineers employ optimization to establish the best inspection plans, sampling tactics, and resource allocation for quality control.

In supply chain finance, optimization is essential since it assists businesses in making decisions that will increase productivity, lower expenses, and boost profits. Here are some particular examples of how supply chain finance benefits from optimization:

a) **Cash Flow Optimization:** Optimization models can aid in optimizing cash flow by optimizing the timing and amount of payments to suppliers and the collection of receivables from customers. By effectively managing cash flow, companies can minimize working capital requirements and improve their financial stability.

b) **Financial Decision Making:** Optimization models can assist in making financial decisions related to capital investment, pricing, product mix, and capacity planning. By considering financial objectives, cost structures, and revenue generation opportunities, companies can make informed decisions that optimize their financial performance.

c) **Logistics and transportation optimization**: Optimizing the movement of goods across the supply chain can lead to significant cost savings. Optimization models help determine the optimal routes, transportation modes, and shipment consolidation strategies. By considering variables like transportation costs, delivery time windows, vehicle capacities, and customer locations, organizations can minimize transportation expenses, reduce lead times, and improve delivery performance.

d) **Supplier selection and negotiation**: Optimization models can assist in selecting the most suitable suppliers based on various criteria such as cost, quality, lead time, and reliability. By considering multiple factors simultaneously, organizations can make informed decisions about supplier selection and negotiate favourable terms and pricing. This helps in achieving cost savings and ensuring a reliable supply of goods and services.

e) **Financial risk management**: Optimization models can help in managing financial risks associated with supply chain activities. By considering variables like currency exchange rates, interest rates, and market fluctuations, organizations can optimize their financial hedging strategies. This helps mitigate the impact of volatile markets and currency fluctuations, reducing financial risks and ensuring stability in supply chain finance.

A non-traditional machining process is one that doesn't use conventional techniques for material removal or shaping, therefore optimization is crucial in these situations as well. In non-traditional machining, optimization can be useful in the following ways:

a) **Process Parameter Optimization:** Non-traditional machining processes often involve a wide range of parameters, such as power settings, feed rates, pulse durations, abrasive concentration, etc. Optimization techniques can be used to identify the optimal combination of process parameters that maximize material removal rate, minimize tool wear, improve surface finish, and achieve desired machining outcomes.

b) **Tool Path Optimization:** In non-traditional machining, the tool path can have a significant impact on the quality and efficiency of the machining process. Optimization models can be utilized to determine the optimal tool path considering factors like material properties, geometry, and desired machining objectives. By optimizing the tool path, manufacturers can reduce machining time, minimize heat-affected zones, and improve dimensional accuracy.

c) **Material Selection:** Optimization can assist in selecting the most suitable materials for non-traditional machining processes. By considering factors like material properties, cost, and desired machining outcomes, manufacturers can optimize material selection to achieve efficient and effective machining.

d) **Energy Efficiency:** Optimization models can be employed to minimize energy consumption in non-traditional machining processes. By identifying the optimal combination of process parameters, tool path, and cutting strategies, manufacturers can reduce energy usage and improve sustainability in machining operations.

e) **Cost Optimization**: Optimization techniques can help in minimizing the overall cost of non-traditional machining processes. By considering factors such as material costs, tooling costs, energy consumption, and process efficiency, manufacturers can optimize the process parameters and strategies to achieve cost-effective machining operations.

f) **Multi-objective Optimization:** Non-traditional machining often involves multiple conflicting objectives, such as maximizing material removal rate while minimizing tool wear or maximizing precision while minimizing machining time. Multi-objective optimization can help in finding the trade-off solutions that balance these objectives and achieve the best overall performance.

g) **Process Robustness and Reliability**: Optimization techniques can aid in optimizing non-traditional machining processes for robustness and reliability. By considering factors like tool life, process stability, and variability, manufacturers can optimize the process parameters and strategies to ensure consistent and reliable machining outcomes.

In summary, optimization is crucial in both supply chain management and non-traditional machining to improve efficiency, reduce costs, and enhance overall performance. By leveraging optimization techniques, companies can achieve better coordination in supply chain operations, optimize the allocation of resources, improve machining processes, and make informed decisions that drive profitability and competitiveness. These examples demonstrate how optimization is crucial to industrial engineering, allowing decision-makers to increase efficiency, cost-effectiveness, and overall performance in industrial settings by optimising processes, resources, and systems.

# CHAPTER 2

## OPTIMIZATION OF MACHINING PROCESSES

### 2.1 An Introduction to Machining Processes

In manufacturing operations, advanced machining techniques are frequently used to solve a variety of problems, including the machining of high-strength materials, making of complex-shaped profiles, the improvement of surface features, high levels of precision, miniaturisation, the reduction of waste, secondary operations, and shorter production times [1]. The use of non-traditional manufacturing processes has significantly increased over the past few years. in particular, when working with tough elements like titanium, stainless steel, composite materials, etc. Conventional techniques are no longer effective for producing precise, highly complex products with better surface finishes and dimensional accuracy. Generally, NTM can classify on the basis of what type of energy is used in the machining process or the nature of energy which are Mechanical, Chemical or Electrochemical, Thermal or electro-thermal. When we need to perform a machining operation by using the NTM process we consider many aspects these are work material, process parameters, type of shape, process capabilities, economic considerations, etc.

For the pieces produced by forming, casting, and other shaping processes, additional operations are frequently needed to achieve an exact and precise assembly. Because interchangeable features are essential for the parts to have in order for them to perform as expected over the course of their service life, the control of dimensional tolerances and surface polish for the manufactured components is of the utmost importance. In order to obtain the necessary geometry with the desired level of surface quality and accuracy, the workpiece surface must be removed from the material using a mechanism known as machining.

The earliest machining tools were constructed of sticks, stones, or animal bones. As technology advanced, hand tools were made using basic metals like iron and bronze. The tools used for machining activities up until the 17th century were either manual tools or mechanically propelled. These machines helped build battleships, wagons, furniture, and everyday utensils. Later, the introduction of steam, water, and electricity led to the development of power-driven tools. Therefore, for a wide variety of applications, power-driven tools took the place of mechanically driven tools. There are two different machining techniques: Conventional or traditional machining process, Non-traditional machining process.

## 2.2 Traditional Machining Processes

Another name for traditional machining is conventional machining. TM techniques use tools that are harder than the material being machined to remove material from the workpiece's surface. Traditional machining procedures are those that use mechanical energy to remove material from the workpiece in order to create the desired product. A tool should be able to reach a particular depth throughout the material removal process. In response to relative motion between the workpiece and the tool, the required form is produced. If any of the aforementioned components is absent, the procedure becomes conventional. Some of the various types of conventional machining processes are Turning, Milling, Drilling, Grinding, Broaching, Plaining, etc. Traditional machining techniques differ in their structural characteristics. the basic elements of conventional machines: Work holding device, Tool holding device, Work motion mechanism, Tool motion mechanism, and Support structure. Here are a few examples of typical conventional machining techniques:

a) **Turning**: Turning is a type of machining where a workpiece turns while being cut by a cutting tool. On a lathe machine, this procedure is frequently employed for cylindrical items. Turning can produce a variety of characteristics, including threads, grooves, chamfers, and straight and tapered cylindrical shapes.

**b) Milling:** The versatile machining technique of milling removes material from the workpiece using a revolving multi-point cutting tool. Flat surfaces, slots, gears, and intricate shapes can all be created using it. End mills, face mills, and ball nose cutters are just a few of the many cutting tools that can be used on milling machines, which can be vertical or horizontal in layout.

**c) Drilling:** Drilling is the process of creating holes in a workpiece using a rotating cutting tool called a drill bit. It is commonly performed on drill presses or machining centres. Drilling can produce simple holes, counterbores, countersinks, and tapped holes, among other features.

**d) Grinding**: Grinding is a precision machining process that employs an abrasive wheel to remove small amounts of material from the workpiece. It helps to increase part precision, achieve tight tolerances, and smooth surface finishes. Among other things, grinding can be used for tool and cutter grinding, surface grinding, and cylindrical grinding.

e) **Shaping**: Shaping is a machining process that involves removing material by using a single-point cutting tool to create flat surfaces or contoured profiles. It is typically performed on shaping machines and can be used for producing flat surfaces, slots, keyways, or irregular shapes.

Traditional machining techniques have many benefits, including as widespread accessibility, long-standing industry norms, and adaptability for a wide variety of materials and applications. They might, however, be restricted in their ability to machine delicate details, intricate curves, or hard or brittle materials. Non-traditional machining techniques may offer superior options in some circumstances. While conventional machining processes have been widely used and proven effective in many manufacturing applications, they do have certain limitations and disadvantages. Here are some disadvantages of traditional machining processes:

a) Certain materials, especially those that are very hard or brittle, may be challenging for conventional machining procedures to operate with. For instance, dealing with hardened steels, ceramics, or composites might provide difficulties for machining processes like turning or milling, resulting in increased tool wear, slowed cutting speeds, or even material damage. In order to get beyond these restrictions, specialized tooling or methods would be needed.

b) Traditional machining techniques frequently need a long setup and changeover period. The cutting tools, work-holding fixtures, and machine settings may need to be adjusted and aligned for each new workpiece or part arrangement. Particularly for small batch sizes or frequent part changes, this preparation time can build up, lowering overall productivity and raising production costs.

c) Conventional machining processes have limitations when it comes to machining complex or intricate shapes. While they can create a wide range of features, they may struggle with intricate internal features, sharp corners, or complex contours that require multiple axes of motion simultaneously. Achieving such complex geometries may require multiple machining operations or specialized equipment, resulting in increased process complexity and cost.

d) Significant material waste occurs during traditional machining processes in the form of chips. Cutting tools remove material by chopping away extra material, which leaves a sizable quantity of material wasted. Increased energy use, greater material costs, and more waste management requirements could result from this.

e) The friction between the cutting tool and the workpiece during traditional machining techniques produces heat. The machining part may have dimensional errors or residual strains as a result of this heat's potential to produce thermal expansion. Additionally, excessive heat might impair the function of the cutting tool, resulting in a shorter tool life and higher maintenance expenses.

### 2.3 NTM Processes

In manufacturing operations, advanced machining techniques are frequently used to solve a variety of problems, including the machining of high-strength materials, making of complex-shaped profiles, the improvement of surface features, high levels of precision, miniaturisation, the reduction of waste, secondary operations, and shorter production times [1]. Non-traditional manufacturing processes have significantly increased over the past few years. in particular, when working with tough elements like titanium, stainless steel, composite materials, etc. Conventional techniques are no longer effective for producing precise, highly complex products with better surface finishes and dimensional accuracy. Generally, NTM can classify on the basis of what type of energy is used in the machining process or the nature of energy which are Mechanical, Chemical or Electrochemical, Thermal or Electro-thermal. When we need to perform a machining operation by using the NTM process we consider many aspects these are work material, process parameters, type of shape, process capabilities, economic considerations, etc.

Several of these NTM processes-controlled factors primarily affect the responses to process performance measurements. You can select the appropriate process parameters and their levels if you are aware of how these parameters affect the responses. Therefore, choosing the best parametric combination for any NTM process is crucial for increasing output rate, surface finish, and accuracy while decreasing machining time. Finding the ideal process parameters for NTM is also challenging because the machining zone contains complicated stochastic problems such as thermal and physicochemical phenomena. The operator's technological expertise and knowledge are required for the proper selection of NTM process parameters due to their previous experiences. The suggested parametric parameter for a certain workpiece material and shape attribute combination cannot be found in the manufacturer's handbook every time. Sometimes, the selected process parameters are conservative and far from ideal, which limits how effectively the NTM process capabilities may be used. As a result, it might be difficult to guarantee that the chosen process parameters lead to NTM processes' optimal or nearly ideal machining performance, which supports the use of a variety of optimization tools in this field.

For high-quality advanced machined components, many optimization techniques have been developed; some of these are Taguchi's optimization Techniques, Genetic Algorithms (GA), Simulated Annealing (SA), Fuzzy Logic, etc. These methods have already demonstrated their value in other manufacturing process parameter optimizations. Some of these strategies have been implemented by industries for the improvement of their manufacturing processes. According to the degree of nonlinearity in the objective function and prior assumptions about the starting point, most conventional optimization approaches, which are based on gradient search methods, may become stuck at the local optimum. As a result, these traditional optimization approaches have limited uses because they cannot guarantee the global optimum. Particle swarm optimization (PSO), Bat optimization (BO), grey wolf optimization (GWO) and other unconventional search and optimization techniques based on natural phenomena (evolutionary computation) have been developed to tackle this issue.

Complicated engineering problems with chaotic disturbances, randomness, and complicated nonlinear dynamics can be solved using non-traditional optimization techniques, which standard algorithms have proven unable to do. Therefore, the goal of a non-conventional optimization strategy is to avoid the poor answers in the search space, learn from the good solutions, and ultimately get very near to the optimal option. Non-conventional algorithms are reliable and simple to modify for the specific issue at hand. The algorithm can almost always be modified and adjusted.

Several NTM processes with various machining potentials and characteristics have already entered the market in order to suit the needs of manufacturing sectors today. These NTM procedures each have advantages and disadvantages of their own. The following four NTM processes are highlighted in this study where various nature-inspired optimization methods have been successfully used to identify the best parametric combinations.

**Advantages of NTM processes:**

Non-traditional machining processes, often referred to as unconventional or advanced machining processes, are a class of production methods that don't use standard cutting tools to shape or remove material from a workpiece, like drills or milling cutters. Instead, they use a variety of techniques to get the intended result. Here are a few benefits of unconventional machining techniques:

a) **No physical contact, minimal force**: Non-traditional machining techniques, in contrast to conventional machining processes, often avoid direct contact between the tool and the workpiece. By minimising the pressures applied to the material, this absence of physical touch lowers the possibility of deformation, residual strains, or damage to fragile or brittle materials.

b) **Versatility**: The versatility of non-traditional machining techniques allows them to be used on a variety of materials, including metals, composites, ceramics, and even non-conductive materials like glass or plastic. This flexibility enables the machining of precise details and complex shapes that could be difficult to create with traditional techniques.

c) **High accuracy and precision**: Non-traditional machining techniques can produce parts with incredibly high precision and accuracy. Some processes, including laser or electron beam machining, can create features with a size of a micron or nanoscale. This accuracy is especially helpful in fields like aircraft, electronics, or medical device manufacture where precise tolerances are necessary.

d) **Complex and intricate shapes**: Complex and complicated shapes can be produced utilising non-traditional machining techniques that are sometimes difficult or impossible to manufacture using traditional techniques. Without the use of specialised gear, processes like electrical discharge machining (EDM) or water jet cutting can cut delicate curves, sharp corners, small gaps, or elaborate interior details.

e) **Hardness and heat resistance of the material**: High melting point or extremely hard materials can be machined successfully using non-traditional methods. For instance, EDM can easily machine heat-resistant alloys, carbides, and hardened steels, whereas standard machining techniques may struggle or demand specialised tools.

f) **Minimal tool wear**: In non-traditional machining, the tool and workpiece typically do not make direct physical contact, which results in little tool wear. This benefit results in longer tool life, which lessens the demand for frequent tool changes and lowers production costs.

g) **Environment-related factors**: Some non-conventional machining techniques, including water jet cutting and laser machining, don't use cutting fluids or produce harmful chips or trash. Because of this quality, they are less harmful to the environment and don't require sophisticated coolant systems or pricey waste disposal.

The applicability of a particular technique relies on the material, application, and desired result. It is essential to remember that each non-traditional machining process has unique benefits and drawbacks. To choose the best machining technique, it is essential to carefully assess the needs of a particular production operation. Following are some of the NTM techniques regarding this research project's operating principles:

### 2.3.1 Electrical Discharge Machining (EDM)

EDM is a non-traditional machining process that utilizes electrical discharges to shape and remove material from a workpiece. Also known as spark machining or spark erosion, EDM is particularly suitable for machining electrically conductive materials and is widely used in industries such as aerospace, automotive, and tooling.

**Principle of Operation:**

EDM works on the principle of controlled electrical discharges between a tool, often referred to as the electrode or tool electrode, and the workpiece, which is frequently a metal or alloy. A dielectric fluid, often deionized water, is used to submerge the tool and the workpiece while cooling the process and acting as a medium for electrical conductivity. A high-frequency electrical discharge between the tool and the workpiece occurs during EDM. The substance melts and vaporises locally as a result of the tremendous heat produced by the discharge. The molten material and debris are removed from the machining zone by the dielectric fluid. Because the tool and the workpiece are not physically in contact, the workpiece is only subjected to light mechanical forces. It is one of the most widely used NTM techniques, as shown in Figure 2.1, with the mould, tool, and die production industries using it extensively. It is primarily used to produce high-quality surfaces with great dimensional accuracy from hard conductive materials that are challenging to work with using traditional machining techniques.

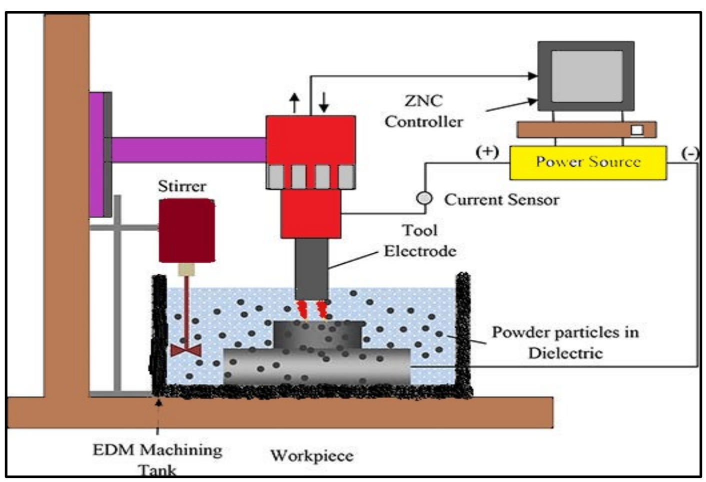



Figure 2.1 Schematic diagram of EDM setup

There are two primary types of EDM, wire EDM and sinker EDM. These are described below. Wire EDM (WEDM): In wire EDM, a thin metallic wire acts as the tool electrode. It is continuously fed through the workpiece, cutting it into the desired shape. Wire EDM is commonly used for precision cutting, creating complex shapes, and making narrow slots or contours. It is especially useful for parts with intricate designs or delicate features.

Sinker EDM (SEDM): In sinker EDM, a shaped electrode is used to create a cavity or a feature in the workpiece. The electrode, usually made of graphite or copper, is brought close to the workpiece, and electrical discharges occur between them, eroding the material. Sinker EDM is commonly employed for producing moulds, dies, and tooling, where high accuracy and intricate details are required.

**Advantages of EDM:**

a) The capacity to machine elaborates and complicated shapes, including internal features or minute details that are difficult or impossible to accomplish using traditional machining.
b) Ability to work with extremely hard materials, such as hardened tool steels or carbides, without significant difficulties.
c) In the absence of direct physical contact, there are minimum forces and a lower danger of part deformation or residual stresses.
d) High precision and accuracy, with the capacity to produce exquisite finishes and close tolerances.
e) Versatility in the machining of a variety of conductive materials, including ceramics that transmit electricity, heat-resistant materials, and rare metals.

**Disadvantages of EDM:**

a) Surface smoothness could not be as high as that attained by other techniques, such as grinding or honing.
b) Slower material removal rate compared to traditional machining processes.
c) Limited applicability to non-conductive materials.

Overall, EDM is a useful machining technique for creating delicate, exact, and complex components, especially when working with hard or challenging-to-machine materials. Due to its special abilities, it is a necessary technology in many industries where conventional machining techniques are neither practical nor effective.

### 2.3.2 Wire Electrical Discharge Machining (WEDM)

In this procedure, as shown in Figure 2.2, a thin strand of wire, commonly composed of brass, is continually passed through the workpiece while it is completely submerged in a dielectric fluid. Two wire guides are held at the top and bottom of the workpiece to guide the wire and maintain stress on it as it is continuously fed from a wire spool. A computer numerical control (CNC) system controls how these guides move. Due to the erosion-causing sparking that occurs from the sidewalls of the wire during this procedure, the kerf width (KW) obtained is typically greater than the wire diameter. Using a high-pressure dielectric fluid fed by a nozzle in the WEDM setup, the eroded materials are flushed out of the machining zone.

Figure 2.2 Schematic diagram of WEDM setup

Despite the fact that it has many benefits, this approach has some drawbacks as well. The advantages and disadvantages of wire EDM are as follows: Wire EDM is renowned for its exceptional precision and accuracy. It can achieve intricate and complex shapes with very tight tolerances, often in the range of micrometres. Wire EDM is highly versatile and can work with a wide range of conductive materials, including hard metals such as titanium, stainless steel, and hardened tool steels. It can also handle delicate and fragile materials. Unlike traditional machining methods that employ cutting tools, wire EDM does not involve any direct physical contact between the tool and the workpiece. One of the primary disadvantages of wire EDM is its relatively slow material removal rate compared to other machining processes like milling or turning. Wire EDM is generally effective for materials with limited thickness.

### 2.3.3 Ultrasonic Machining (USM)

USM is a non-traditional machining process that utilizes ultrasonic vibrations to remove material from a workpiece. It is particularly effective for machining hard and brittle materials such as ceramics, glass, and advanced composites. The metallurgical, chemical, and physical properties of the material used to make the workpiece remain unchanged by this non-thermal, non-chemical, and non-electrical process. Low material removal rate (MRR) and virtually minimal surface damage to the work material being machined are the defining characteristics of this method. It is capable of producing complex shapes with good accuracy and acceptable surface finish out of both electrically conductive and non-conductive materials, ideally those with low ductility and high hardness.

**Principle of Operation**:

Electrical energy is transformed into high-frequency mechanical vibrations during ultrasonic machining. A tool called a horn or sonotrode that is in contact with an abrasive slurry receives these vibrations and transmits them to the abrasive slurry. The material chips away as a result of the tool's ultrasonic vibrations and the abrasive particles in the slurry creating a micro-hammering impact on the workpiece surface. During USM, the sonotrode is forced on the workpiece surface while the workpiece is held in a fixture. The machining zone is continuously supplied with the abrasive slurry, which is typically a combination of abrasive particles and a liquid medium. The abrasive particles strike the workpiece when the sonotrode vibrates at high frequency (usually 20 kHz to 40 kHz), leading to localised brittle fracture and material removal.

**Advantages of USM:**

a) Materials like ceramics, glass, and composites that are challenging to machine using traditional techniques are excellent candidates for USM. It permits controlled and precise material removal without resulting in considerable damage or thermal stress.
b) High degrees of precision and accuracy are achievable using ultrasonic machining. Small chips are produced by the ultrasonic vibrations, producing exquisite surface finishes and precise tolerances. Because of this, USM is appropriate for situations where surface quality and dimensional precision are crucial.
c) The amount of heat produced during machining is small because USM is a non-thermal technique. This lessens the possibility of thermally induced workpiece damage, such as cracks, residual strains, or modifications to the material's characteristics.

**Limitations of USM**:

a) USM is primarily suitable for machining electrically conductive or brittle materials. It may not be effective for machining soft or ductile materials such as aluminium or steel.
b) USM typically has a lower rate of material removal than conventional machining techniques. It might not be appropriate for applications that call for quick material removal and great output.
c) Compared to traditional machining methods, USM typically removes less material from the workpiece. It might not be suitable for applications that require significant production and rapid material removal.

Despite these drawbacks, ultrasonic machining presents special benefits for certain applications, particularly in sectors like precision manufacturing, electronics, and aerospace.

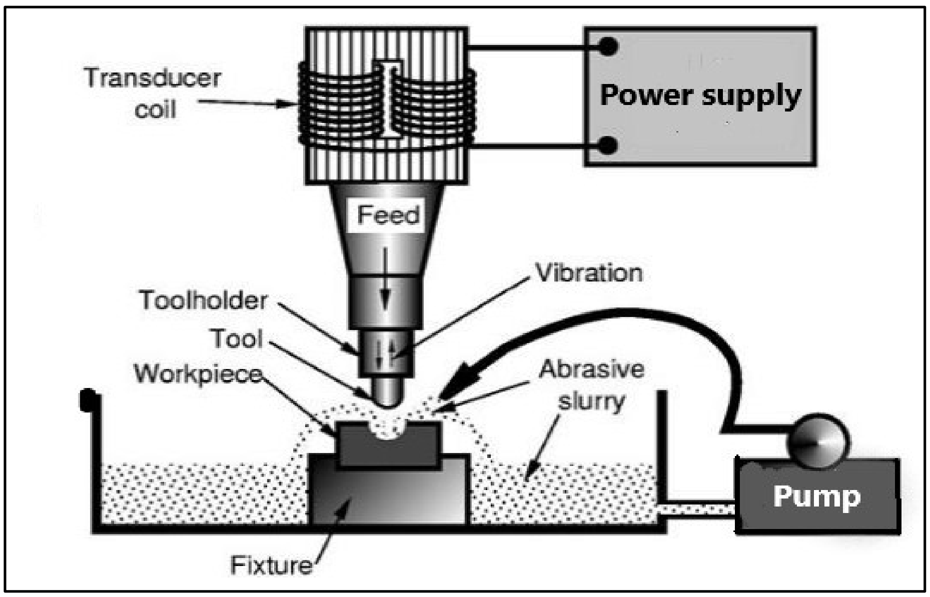



Figure 2.3 Schematic diagram of USM

### 2.3.4 Laser Beam Machining (LBM)

It is a non-traditional machining process that utilizes a high-intensity laser beam to remove material from a workpiece. It is commonly used for precision cutting, drilling, welding, and surface modification of a wide range of materials**.** In order to meet modern demands for high flexibility and productivity, noncontact machining, adaptability to automation, decreased processing cost, improved product quality, increased material utilization, processing of materials regardless of electrical conductivity, minimum heat affected zone (HAZ), and green manufacturing, the LBM, as shown in Figure 2.4**,**a thermal energy-based machining technique, is now widely used. Additionally, this procedure does not include mechanical cutting force or tool wear. The LBM method can be used to perform a variety of material processing tasks, including laser micro-drilling, cutting, micro grooving, micro-turning, marking, and scribing [3].

**Principle of Operation:**

A laser source produces a highly concentrated and coherent beam of light that is focused onto the workpiece in laser beam machining. Mirrors and lenses are used to direct and control the laser beam in order to perform the desired machining process. The material is removed as a result of the laser beam's extreme heat melting or vaporizing the material at the site of engagement. In LBM, the material is removed by a variety of methods, such as vaporization, melting, ablation, and thermal cracking. You can adjust the laser's power, wavelength, pulse duration, and beam profile to suit your particular machining and material needs.

**Advantages of LBM:**

**a)** High levels of precision and accuracy are possible using LBM. With fewer heat-affected zones, the focused laser beam may create complicated designs, sharp edges, and fine details. This qualifies it for applications that call for accurate geometry and close tolerances.

**b)** Metals, ceramics, polymers, composites, and even non-conductive materials like glass can all be machined using laser beam technology. Its adaptability across several industries is increased by the efficient processing of both reflective and non-reflective materials.

**c)** Since LBM is a non-contact technique, there is no actual physical touch between the workpiece and the laser. By removing tool wear, this lowers the possibility of damaging delicate or brittle materials. Additionally, by minimizing pressures applied to the workpiece, deformation and residual stresses are less likely to occur.

**d)** Complex forms and curves can be produced with laser beam machining because of its great precision and versatility. It can easily create complex patterns, tiny holes, thin-walled structures, and three-dimensional geometries thanks to its ability to follow computer-aided design (CAD) data.

**Limitations of LBM:**

a) Equipment for laser beam machining can be expensive, especially for powerful lasers used in industrial settings. Small-scale or low-volume production may be hindered by the initial investment and maintenance costs.

b) The more material thickness, the less efficient and effective LBM may be. Thicker materials might need more passes or a stronger laser, which could reduce productivity and lengthen processing time.
c) High-power lasers are used in laser beam machining, which can be dangerous if not carefully managed. To ensure safe operation, adequate safety precautions, such as safety goggles, ventilation, and operator training, are required.
d) The efficiency of LBM can be diminished by certain highly reflective materials like copper or aluminum, which can reflect a large proportion of laser energy. To get over this restriction, other methods might be needed, including adding a coating or using an aid gas.

Precision, adaptability, and little heat effect are all special benefits of laser beam machining. It is appropriate for a variety of industries, including the production of medical devices, electronics, automobiles, and spacecraft, thanks to its capacity to deal with a wide variety of materials and create complex geometries.

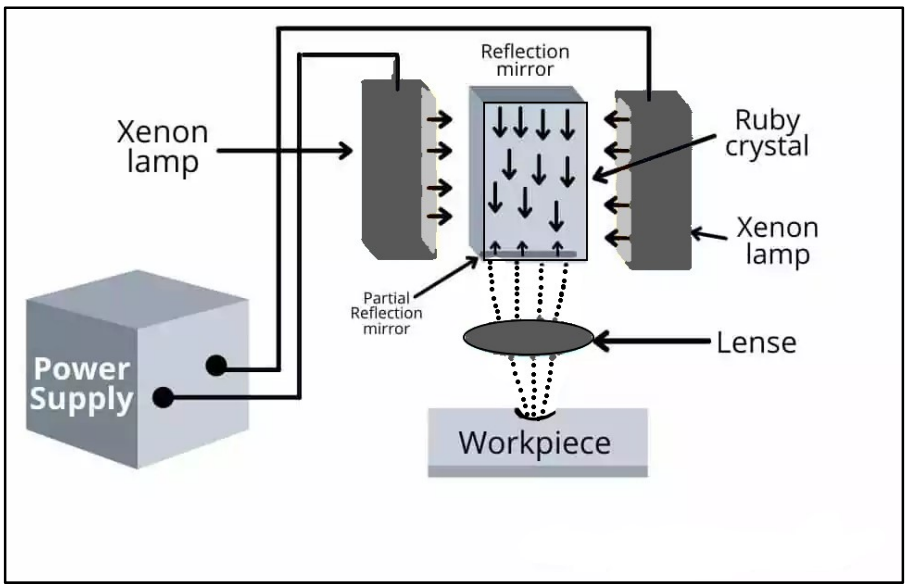



Figure 2.4 Schematic diagram of LBM

### 2.3.5 Abrasive Water-Jet Machining (AWJM)

AWJM is a non-traditional machining process, which uses a high-velocity jet of a water and abrasive particle mixture as a tool to remove material from the surface of the workpiece. It is particularly effective for cutting hard and brittle materials and is widely used in industries such as aerospace, automotive, and manufacturing. Here's a short note on abrasive water-jet machining.

**Principle of Operation**:

Abrasives are thrust by a high-velocity water jet into a mixing chamber. The mixture is then pushed via a nozzle, which enhances the jet's velocity and directs it to a precise area where it will strike the workpiece surface. As a result, the material is removed from the workpiece by the erosive action of the abrasives as the abrasive water jet strikes it at a very high velocity. The hammering effect of the abrasives on the workpiece causes a brittle fracture, and the water jet then clears the wear particles from the machining zone. Each of these systems has required careful consideration of the mechanical properties of the target material as well as the abrasive hitting angle. Three zones have been created on the AWJ surface. The expansion of the jet prior to impingement and the variation in jet energy with radial distance provide the initial damage region at the top kerf wall cut surface. The abrasive impingement angle is far greater here than it has to be to maintain the remaining cutting depth. Waviness profiles help to detect the cutting regions recognised for their smoothness and roughness, which coexist beneath the initial damage region. The machined surface imposes the abrasive particle size in the area of the smooth cutting areas. In contrast, surface characteristics of the rough cutting region are controlled by cutting parameters that affect jet kinetic energy [4].

**Advantages of AWJM**:

a) A variety of materials, including metals, composites, ceramics, glass, stone, and polymers, can be efficiently machined by AWJM. It allows for the machining of materials with varied hardness and brittleness because it is appropriate for both soft and hard materials.
b) The amount of heat produced during machining is small because AWJM is a non-thermal technique. This avoids the possibility of the workpiece experiencing heat damage, such as melting, warping, or modifications to the material's characteristics. It is especially useful for applications requiring less heat effect, such as those involving heat-sensitive materials.
c) The cutting tool and the workpiece do not come into contact with each other directly during the AWJM process. As a result, there is no longer any mechanical stress that is transferred to the material, which lessens the possibility of distortion, residual strains, or micro-cracks in the machined parts.
d) High levels of precision and accuracy can be achieved with abrasive water-jet machining. It has superb dimensional control and can create complex objects with sharp corners and minute details. This qualifies it for uses that call for precise tolerances and complex geometries.

**Limitations of AWJM:**

a) The average material removal rate for AWJM is often lower than for some conventional machining techniques. For applications that call for quick material removal and great productivity, it might not be as suitable.
b) The breadth of the material removed during cutting is referred to as the kerf width and is produced by the AWJM cutting process. Compared to techniques like laser cutting or electrical discharge machining, AWJM has a wider kerf width. In particular, for small-scale or fine-detail work, this might have an impact on the accuracy and complexity of the machined parts.
c) For industrial-scale applications, the equipment and setup expenses for abrasive water-jet machining might be quite costly.

Despite these drawbacks, abrasive water-jet machining provides exceptional benefits in terms of adaptability, non-thermal operation, and precise cutting. Its capacity to cut a variety of materials with little heat effect makes.

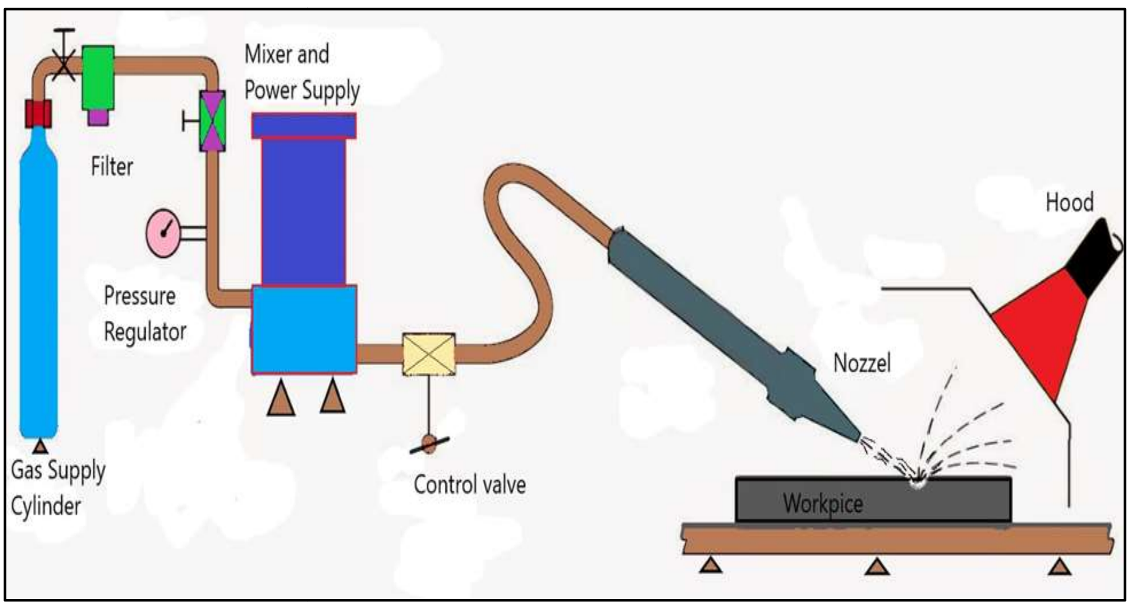



Figure 2.5 Schematic diagram of AWJM setup

## 2.4 Objectives and Scope of the Proposed Project

The NTM methods have already demonstrated their viability as effective machining technologies for producing complex shape geometries on a variety of challenging advanced engineering materials. These materials can't be processed using standard material removal procedures because of some special mechanical properties. It has been found that the input (controllable) parameters of NTM processes have a significant impact on their responses. Therefore, the NTM processes must be executed at the optimal levels of their input parameters in order to achieve the desired properties of the machined parts/components. The target response values may not always be achieved even with near-optimal settings of the NTM process parameters. Numerous mathematical strategies have already been used in this respect to identify the best parametric mixtures of various NTM processes in order to maximise their machining potential. The goals of the current research project are as follows, taking into consideration the previous requirements:

a) To identified various non-traditional machining processes for parametric analysis and optimization of multiple responses.
b) To apply a single and multi-objective optimization to determine the optimal parametric combinations of AWJM and WEDM processes.
c) To carryout detailed literature survey and implement metaheuristics (foraging behaviour) algorithms to find out the best appropriate parametric settings of WEDM and AWJM processes.
d) To make comparative analysis of the algorithms and plotting conventional and box plots for finding the optimal parametric values of the responses.
e) To validate the result of all metaheuristics algorithms by using Friedman's rank test we proven the superiority of particular of the algorithms.
f) To integrate and optimize all responses by using metaheuristics algorithms as an effective multi-objective optimization tool in order to find out the single best parametric combinations of AWJM and WEDM processes.

In the present research work, an attempt is put forward to apply five single and multi-objective mathematical techniques to search out the best parametric intermixes of different NTM processes (AWJM and WEDM). In most of the NTM processes under consideration, equal weights are usually assigned to the considered responses to make the computational steps easier.

# CHAPTER 3

## INTRODUCTION OF METAHEURISTICS OPTIMIZATION

The definition of heuristic is "to find" or "to discover solutions by trial and error." Heuristic techniques seek workable solutions to significant issues. Metaheuristic algorithms refer to the process of creating heuristic algorithms. Meta is short for beyond or higher level. In many cases, metaheuristics outperform straightforward heuristics. Since metaheuristic algorithms search more thoroughly, they have quickly taken over as the preferred way of coming up with answers to difficult real-world issues that precise methods are unable to address. Intensification and diversification, often known as exploitation and exploration, are the two main parts of any metaheuristic algorithm. While intensification implies to concentrate on the search in a limited area by taking advantage of the knowledge that a current good solution has been located in this location, diversification refers to the generation of different solutions in order to explore the search space on a global scale. This works in conjunction with choosing the best options. The best solutions are chosen, ensuring that the solutions will converge to optimality, while the diversification of the solutions through randomization prevents the solutions from becoming stuck at local optima while also broadening their diversity. Typically, a successful combination of these two key elements will ensure that the pursuit of global optimality is possible. Meta-heuristics typically behave iteratively. Throughout the optimization, the same pattern is repeated until a starting requirement is satisfied. The meta-heuristics are straightforward in that they do not require calculating the function's gradient. Users of metaheuristics need quick and efficient procedures, but they also need these approaches to be easy to use.

### 3.1 An Overview of Optimization Techniques

Optimization techniques are methods or approaches used to find the best possible solution for a given problem. These techniques are widely employed in various fields, including mathematics, engineering, computer science, economics, and more. Here is an overview of some commonly used optimization techniques [5]:

a) **Greedy Algorithms**: Greedy algorithms make locally optimal choices at each step with the hope of finding a global optimum. They are easy to implement and computationally efficient but may not always yield the best solution.
b) **Random Search**: In this technique, solutions are randomly generated and evaluated. While it is simple to implement, random search can be inefficient as it relies on chance to find an optimal solution.
c) **Gradient Descent**: Gradient descent is an iterative optimization algorithm commonly used in machine learning and mathematical optimization. It works by iteratively adjusting the parameters of a function in the direction of steepest descent to minimize a given objective function.
d) **Genetic Algorithms:** Inspired by the process of natural selection, genetic algorithms involve generating a population of potential solutions and iteratively evolving them through processes like selection, crossover, and mutation. Genetic algorithms are useful for solving complex optimization problems with large search spaces.
e) **Simulated Annealing**: Simulated annealing is a probabilistic optimization technique that mimics the annealing process in metallurgy. It starts with an initial solution and allows occasional uphill moves to escape local optima. Over time, the algorithm decreases the probability of accepting worse solutions, resulting in convergence to a global optimum.
f) **Particle Swarm Optimization**: Particle swarm optimization is a population-based stochastic optimization algorithm. It stimulates the behaviour of a swarm of particles, each representing a potential solution. These particles move through the search space, updating their positions based on their own best position and the best position found by the entire swarm.
g) **Linear Programming**: Linear programming is a mathematical technique used to optimize a linear objective function subject to linear equality and inequality constraints. It is widely applied in operations research, resource allocation, production planning, and other fields.
h) **Integer Programming**: Integer programming is an extension of linear programming where some or all variables are required to be integers. It is useful for solving optimization problems with discrete decision variables.
i) **Convex Optimization**: Convex optimization deals with optimization problems where the objective function and constraints are convex. Convex problems have desirable mathematical properties, allowing for efficient and globally optimal solutions.

Choosing the appropriate optimization technique depends on the nature of the problem, the available resources, and the desired trade-offs between solution quality and computational efficiency.

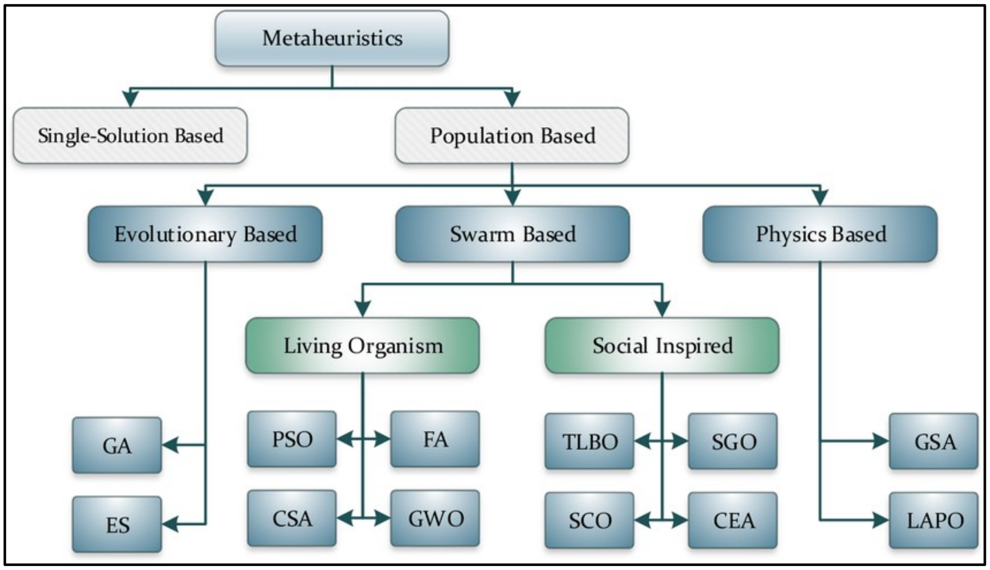



Figure 3.1 Type of metaheuristic algorithms

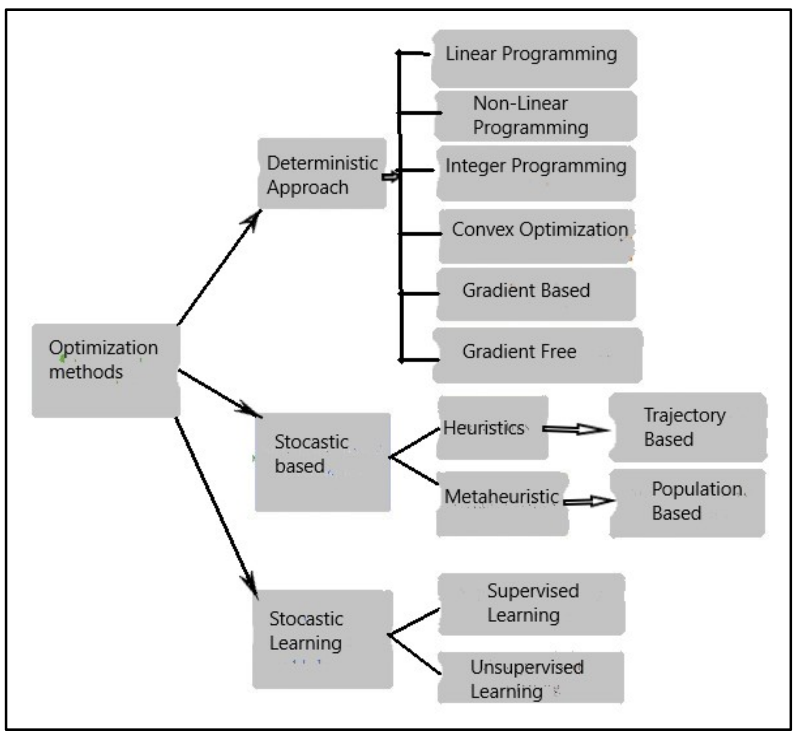



Figure 3.2 Type of the optimization methods

Almost all areas of science, engineering, and commerce regularly use optimization, which is a task that is not specific to any one profession. According to the Chambers lexicon, optimization is the "act of creating the most or best of things." Theoretically, solving a problem involves conducting an optimization task to determine which is the optimal solution. Studies on mathematical optimization put a lot of work into attempting to characterise the characteristics of such an ideal solution. On the other hand, engineering or practical optimization studies aim to find solutions that have comparable characteristics. Despite the desire for the ultimate optimal solution, time and processing resource constraints frequently persuade practitioners to settle for an approximation. The majority of conventional or traditional algorithms are deterministic. Examples of deterministic algorithms include the simplex approach used in linear programming. They are known as gradient-based algorithms. Some deterministic optimization techniques use gradient information. It does not, however, perform well if the goal function is somewhat discontinuous. A non-gradient method is favoured in this situation. Algorithms that are not gradient-based or gradient-free rely just on the values of the function; no derivative is used. Examples of gradient-free algorithms include Nelder-Mead downhill simplex and the Hooke-Jeeves pattern search.

Common difficulties with traditional optimization methods are as follows:

a) According to the chosen initial solution, it convergence to an optimal solution.
b) Most algorithms are prone to get confined to a sub-optimal solution.
c) A problem solved by one algorithm may not be efficient to solve the other problem.
d) Algorithms are not efficient to solve problems with nonlinear objectives in discrete variables and many constraints.
e) Algorithms cannot be efficiently used on a parallel computer.

Heuristic and metaheuristic algorithms are the two main categories of stochastic algorithms, but there is little distinction between the two. Heuristic broadly translates as "to find" or "to discover by trial and error." Even though it is possible to find good answers to challenging optimization problems in a reasonable time, there is no assurance that the solutions chosen will be the best ones. It hopes that these algorithms will function the majority of the time but not always. This is advantageous when we don't always seek the most ideal solutions, but rather acceptable ones. Metaheuristic algorithms are an improvement on heuristic algorithms [6].

## 3.2 Multi-Objective Optimization (MOO)

MOO, also known as multi-criteria optimization or Pareto optimization, deals with the optimization of multiple conflicting objective functions simultaneously. Unlike single-objective optimization, which seeks a single optimal solution, MOO aims to find a set of solutions that represent the trade-offs between different objectives. In MOO, the goal is not to find a single solution that optimizes all objectives simultaneously, as this is often not possible due to conflicting objectives. Instead, the objective is to find a set of solutions that represent a range of trade-offs between the objectives. These solutions are known as Pareto optimal solutions, which are non-dominated, meaning that no other solution in the search space is better for all objectives. The concept of Pareto optimality is based on the Pareto dominance principle. In MOO, one solution is said to dominate another if it is at least as good in all objectives and strictly better in at least one objective. Pareto optimal solutions represent the best compromise solutions that cannot be improved in any objective without worsening at least one other objective. The process of solving MOO problems involves exploring the trade-off surface by generating and evaluating a diverse set of candidate solutions. Various algorithms are used for MOO, including evolutionary algorithms (such as genetic algorithms and particle swarm optimization), non-dominated sorting algorithms (such as NSGA-II and SPEA2), and mathematical programming approaches. The solutions obtained from MOO provide decision-makers with a range of options to choose from, allowing them to make informed decisions based on their preferences and priorities. The final selection among the Pareto optimal solutions often involves considering additional decision criteria or applying decision-making methods such as the analytic hierarchy process (AHP) or the weighted sum method.MOO finds applications in diverse fields, including engineering design, portfolio optimization, resource allocation, logistics planning, and many others. It allows decision-makers to consider multiple conflicting objectives simultaneously, thereby supporting more comprehensive and informed decision-making processes.

Finding the optimal values for multiple desired goals is referred to as MOO. The MOO is chosen because it simplifies problems by avoiding complex equations, which is one of the reasons for its application in optimization [7]. MOO has been utilised in many scientific domains, including engineering, where decisions must be made when there may be trade-offs between two or more objectives that may be in conflict with one another. A compromise on some incompatible issues is possible due to the decision-making dilemma in MOOs. Vilfredo Pareto first established MOO. An MOO has a vector representing the objective function.

Every vector of the objective function is a function of the vector representing the solution. There are multiple options available in MOO rather than a single ideal one for every situation. The objective function vector and the decision variable space for the solution vector are both multidimensional spaces in the MOO. There is a point on the objective function space in every x-solution in the decision variable space.

The MOO problem's equations can be expressed mathematically as follows:
Max/min $f1(x), f2(x), f3(x)\ldots\ldots f_n(x)$
Subject to: $x \in X$
where $f_n(x)$ is the nth objective function, X is the feasible set, x is the solution, n is the number of objective functions, and max/min are combined object operations.

### 3.3 Meta-Heuristics Optimization Algorithms

Metaheuristic optimization algorithms have a relatively recent history, emerging in the latter half of the 20th century. The development and evolution of metaheuristics have been influenced by advancements in computational power, the need to solve complex optimization problems, and inspiration from natural phenomena and intelligent behaviour. In 1960-1970, the early foundations of metaheuristics were laid during this period. Researchers began exploring stochastic methods for optimization, such as Monte Carlo simulation and random search algorithms. These methods provided the initial inspiration for the development of more sophisticated metaheuristic approaches.The 1980s saw the emergence of several influential metaheuristic algorithms. The Genetic Algorithm (GA), proposed by John Holland and further developed by David Goldberg, gained significant attention. GAs was inspired by the principles of natural evolution and genetic inheritance, employing mechanisms like crossover and mutation to explore and exploit solution spaces. Simulated Annealing (SA), introduced by Scott Kirkpatrick, C. Daniel Gelatt Jr., and Mario P. Vecchi, drew inspiration from the annealing process in metallurgy and used temperature-based acceptance criteria to escape local optima. Throughout the history of metaheuristics, there has been a continuous focus on improving algorithm efficiency, scalability, and adaptability. Researchers have explored various variants, parameter-tuning strategies, and problem-specific adaptations to enhance the performance of metaheuristic algorithms. Additionally, the incorporation of parallel and distributed computing techniques has enabled the application of metaheuristics to tackle even more complex and large-scale optimization problems.

The definition of heuristic is "to find" or "to discover solutions by trial and error." Heuristic techniques seek workable solutions to significant issues. Metaheuristic algorithms refer to the process of creating heuristic algorithms. Meta is short for beyond or higher level. In many cases, metaheuristics outperform straightforward heuristics. Since metaheuristic algorithms search more thoroughly, they have quickly taken over as the preferred way of coming up with answers to difficult real-world issues that precise methods are unable to address.

Intensification and diversification, often known as exploitation and exploration, are the two main parts of any metaheuristic algorithm. While intensification implies to concentrate on the search in a limited area by taking advantage of the knowledge that a current good solution has been located in this location, diversification refers to the generation of different solutions in order to explore the search space on a global scale. This works in conjunction with choosing the best options. The best solutions are chosen, ensuring that the solutions will converge to optimality, while the diversification of the solutions through randomization prevents the solutions from becoming stuck at local optima while also broadening their diversity. Typically, a successful combination of these two key elements will ensure that the pursuit of global optimality is possible. Meta-heuristics typically behave iteratively. Throughout the optimization, the same pattern is repeated until a starting requirement is satisfied. The meta-heuristics are straightforward in that they do not require calculating the function's gradient. Users of metaheuristics need quick and efficient procedures, but they also need these approaches to be easy to use.

### 3.3.1 Dragonfly Optimization Agorithm (DOA)

The DOA, created by Mirjalili [8] in 2015, is one of the current heuristic optimization techniques. It’s a nature-inspired swarm intelligence optimization technique. The static and dynamic behaviours of dragonflies are hunting (exploration) and migration (exploitation) respectively. Similar to other metaheuristic algorithms, the dragonfly algorithm also carries out the target search process. They create sub-swarm groups fly back and forth over small different areas for food and attract flying prey like butterflies and mosquitoes in the exploration phase. The main characteristics of a static swarm are Local movements and abrupt changes in the flying path. In the exploitation phase, a bigger number of dragonflies migrate in one direction over long distances to distract from outward enemies. The main objective of any swarm is survival, so all individuals should be attracted towards food sources and distract outer enemies. Separation, Alignment, Cohesion, Attraction, and

Distraction are the operators for exploration and exploitation. The behaviours of each swarm are mathematically modelled as follows [8].

The separation is modelled as follows:

$$SNi = -\sum_{j=1}^{N} Y - Yj \tag{1}$$

where, $SNi$ indicates the separation of the i$^{th}$ individual, Y is the position of the current individual, Yj shows the position of the j$^{th}$ neighbouring individual and N is the number of neighbouring individuals.

Alignment is represented as follows:

$$ALi = \frac{\sum_{j=1}^{N} Uj}{N} \tag{2}$$

here, $AL_i$ is the alignment of i$^{th}$ individual, Uj is the velocity of the j$^{th}$ neighbouring individual.

The cohesion is represented as follows:

$$CHj = \frac{\sum_{j=1}^{N} Yj}{N} - Y \tag{3}$$

where $CH_i$ is the cohesion of the i$^{th}$ individual.

Attraction towards a food source is represented as follows:

$$FSi = Y^{+} - Y \tag{4}$$

where $FS_i$ is the food source of the i$^{th}$ individual, $Y^{+}$ is the position of the food source.

Distraction outwards towards an enemy is represented as follows:

$$EPi = Y^{-} + Y \tag{5}$$

where $EP_i$ is the position of the enemy of the i$^{th}$ individual, $Y^{-}$ is the position of the enemy.

Positions of artificial dragonflies are updated inside their search space and simulate their movements. Step vectors ($\Delta Y$) and position vectors ($Y$) are considered for the position and reproduction respectively. The step vector which is defined the movement of the dragonflies represented as follows:

$$\Delta\mathrm{Yt} + 1 = (\mathrm{s\,SN}_i + \mathrm{a}\,AL_i + \mathrm{c}\,CH_i + \mathrm{f}\,FS_i + \mathrm{e}\,EP_i) + \mathrm{w}\,\Delta\mathrm{Yt} \tag{6}$$

where $\Delta Yt$ is the step vector of the i$^{th}$ iteration, $s$ is the separation weight, a represents alignment weight, $c$ denotes cohesion weight, $f$ is the food factor, $e$ defined the enemy factor and $w$ is inertia weight, $t$ is the iteration counter.

The position vectors are calculated as follows:

$$Y_{t+1} = Yt + \Delta Y_{t+1} \tag{7}$$

where $Yt+1$ is the position vector for the $(t+1)^{th}$ iteration, $\Delta Yt+1$ is the step vector for the$(t+1)^{th}$ iteration, when there aren't any nearby solutions, random walk (Levy. flight) occurs. So, the $i^{th}$ dragonfly's position is modified as follows at iteration $(t + 1)$:

$$Y_{t+1} = Yt + Levy(d) \times Yt \tag{8}$$

where d stands for the position vectors' dimension. Levy flight is estimated using eqn. (9) as follows:

$$Levy(y) = 0.01 \times \frac{r_1 \times \mu}{|r_2|^{\frac{1}{\beta}}} \tag{9}$$

where β is a constant, and $r1$, $r2$ are two random values(vector) in the range of [0, 1], $\mu$ is determined as follows:

$$\mu = \left(\frac{\Gamma\ (1+\beta)\times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\ (\frac{1+\beta}{2})\times\beta\times 2^{\beta-1}/_{2}}\right)^{1/\beta} \tag{10}$$

Where, $\Gamma(y) = (y-1)\,!$

The pseudocode of the dragonfly algorithm is as follows:

| **Pseudocode of DOA** |
|---|
| IT: Max. iterations<br>DF: Dragonfly<br>Objective: The position and fitness value of Dragonfly (DF)<br>Generate initial DF population $Yi(i = 1, 2, ..., n)$<br>Generate initial step vectors of DF population $\Delta Yi\ (i = 1, 2, ..., n)$<br>while ($t \leq IT$) do<br>    calculate the fitness value of each Dragonfly (DF)<br>    update food source and enemy position<br>    Update *w*, *s*, *a*, *c*, *f* and *e*<br>    using eqn. (*1 - 5*) calculate *SN, AL, CH, FS* and *EP*<br>    update neighbouring radius<br>if a Dragonfly has at least one neighbouring Dragonfly (DF)<br>    using *eqn. (6)* update the velocity vector<br>    using *eqn. (7)* update the position vector<br>else<br>    using *eqn. (8)* update the position vector |

end if

check and correct the new positions based on the boundaries of variables.

*IT=IT+1*

End while

### 3.3.2 African Vulture Optimization Algorithm (AVOA)

One of the most recently developed heuristic optimization algorithms is the AVOA was developed by SeyedaliMirjaliliet.al.[9] in 2021. The African vulture's foraging behaviour and movement patterns are simulated by the AVO algorithm. Based on the distinct, despotic characteristic of the vulture's behaviour, the algorithm divides a large number of vultures into two groups. In order to categorise the vultures, the algorithm first determines the fitness function of each solution from starting population. The first and best vulture is used to represent the best solution, while the second is used to represent the second-best vulture. One of the top two vultures in each display is moved or replaced by members of the population. The main natural purpose of vultures, which can be described as group living to obtain food, justifies the separation of groups in this algorithm. Each group of vultures has a unique difficulty to locate food and consume it. Vultures have the characteristic to consume large quantities of food, and their hours-long search for food is what makes them escape the hungry trap. At the formulation stage of our anti-hunger compromises, the vultures attempt to stay away from the worst and come up with the best solution. This is because they believe that the population's worst option is the one that is weakest and hungriest. The other vultures in the AVOA strive to be as close to the best as possible, and the two strongest and best solutions are regarded as the strongest and best vultures.

The proposed algorithm was further developed in 4 distinct levels based on the fundamental ideas of vultures; these are:

**First phase**: Calculating the best vulture in any group

After the initial population is formed, the best solution is chosen as the best vulture of the first group, and the second-best solution is chosen as the best vulture of the second group. All other solutions are then directed towards the best solutions for the first and second groups using Equation (11).

$$K\mathrm{i} = \begin{cases} BestVulture1 \; if \; pi \; = \; L1 \\ BestVulture2 \; if \; pi \; = \; L2 \end{cases} \tag{11}$$

where L1 and L2 Prior to the search procedure with values between 0 and 1 must be measured. From each group choose the best solution and calculate the probability of finding the best solution using the Roulette wheel by equation (12).

$$Pi = \frac{Fi}{\sum_{i=1}^{n} Fi} \tag{12}$$

**Second phase:** Vulture starvation rate

Vultures are often looking for food and when satiated, vultures have high energy levels that enable them to search for food over greater distances. but if they are hungry, they do not have enough energy to fly long and look for food next to the stronger vulture and become aggressive while hungry. Equation (14) has been applied to the mathematical modelling of this phenomenon.

$$t = h \times \left(\sin\left(\frac{\pi}{2} \times \frac{it_i}{IT}\right) + \cos\left(\frac{\pi}{2} \times \frac{it_i}{IT}\right) - 1\right) \tag{13}$$

$$F_R = (2 \times rand1 + 1) \times r \times \left(1 - \frac{it_i}{IT}\right) + t \tag{14}$$

Here, $F_R$ indicates that the vultures are full, *iteration* indicates the number of iterations that have been completed, *IT* indicates the total number of iterations, and r is a random number between -1 and 1. h is chosen at random between -2 to 2. rand1 has a random value Between 0 and 1. When r < 0, the vulture is starved, and if 0, it means the vulture is satiated.

**Third Phase**: Exploration

Examining the AVOA's exploration phase. Vultures have excellent vision, a great ability to locate food, and a strong ability to spot weak, dying animals in the wild. Each vulture conducts a random search of its surroundings for food. Eq. (15) illustrates this process.

$$P(i+1) = \begin{cases} if P1 \geq randP1, then\ Eqn. (6) \\ if P1 < randP1, then\ Eqn. (8) \end{cases} \tag{15}$$

$$P(i+1) = K(i) - D(i) \times F_R \tag{16}$$

$$D(i) = |X \times K(i) - P(i)| \tag{17}$$

Where *P(i+1)* is the vulture position vector, *P(i)* is the current vector position of the vulture, and Vultures will randomly move to *X* to defend prey from other vultures.

$$P(i+1) = K(i) - F_R + rand2 \times ((ub - lb) \times rand3 + lb) \tag{18}$$

Here*, S(i)* is one of the best vultures selected by the use of Eq. (11). lb and ub are the variables' upper bound and lower bound. *rand2* provides a random value between 0 and 1.*rand3* is used to increase the coefficient of random nature.

**Fourth Phase Four:** Exploitation

The AVOA enters the exploitation phase ($|F_R| < 1$). The degree of choosing each strategy in each internal phase is determined by two parameters of *P2* and *P3*, they are valued between 0 and 1. Vultures frequently make a rotational flight used to model Spiral Motion. Two separate rotating flight and siege-fight techniques are used in the initial phase. This process is illustrated in equation (19).

$$P(i+1) = \begin{cases} if P2 \geq randP2 \text{ , } then\ Eqn. (20) \\ if P2 < randP2 \text{ , } then\ Eqn. (23) \end{cases} \tag{19}$$

Serious fights over food can arise when a large group of vultures assemble around a single food source. Physically strong vultures avoid sharing food with weaker vultures during these periods. The weaker vultures try to gather around the stronger vultures and start la little conflict in an effort to exhaust them and get food from them. Equations (20) and (21) are used to form the model.

$$P(i+1) = D(i) \times (F_R + rand4) - d(t) \tag{20}$$

$$d(t) = K(i) - P(i)$$

(21)

The rotational flight is expressed using Eqs. (22) and (23).

$$S1 = K(i) \times \left(\frac{rand5 \times P(i)}{2\pi}\right) \times \cos(P(i)) \tag{22}$$

$$S2 = K(i) \times \left(\frac{rand6 \times P(i)}{2\pi}\right) \times \cos(P(i)) \tag{23}$$

$$P(i+1) = K(i) - (S1 + S2) \tag{24}$$

Here*, rand5* and *rand6* are random numbers between 0 and 1. Finally, by the use of Eq. (24), the location of the vultures is updated.

The siege and violent struggle for food are carried out during the second stage of exploitation when the activities of the two vultures have gathered several different vulture species around the food source. In Eq. (25), this process is illustrated.

$$P(i+1) = \begin{cases} if P3 \geq randP3 \text{ , } then\ Eqn. (26) \\ if P3 < randP3 \text{ , } then\ Eqn. (27) \end{cases} \tag{25}$$

The accumulation of several types of vultures over the food source has been used to formulate this movement of vultures by Eqns. (26) and (27).

$$A1 = BestVulture1(i) - \frac{BestVulture1(i) \times P(i)}{BestVulture1(i) - P(i)^2} \times F_R \tag{26}$$

$$A2 = BestVulture2(i) - \frac{BestVulture2(i) \times P(i)}{BestVulture2(i) - P(i)^2} \times F_R \tag{27}$$

Where *BestVulture*1(i) and *BestVulture2(i)* are the best vulture of the first group and the best vulture of the second group in the current iteration respectively.

the aggregation of all vultures is carried out by using Eq. (28).

$$P(i+1) \;=\; \frac{A1 + A2}{2} \tag{28}$$

Aggressive Competition for Food is used to model by Eq. (29).

$$P(i+1) = \; K(i) - \; |d(t)| \; \times \; F_R \times \; Levy(d) \tag{29}$$

where the vulture's distance from one of the two groups' top vultures is shown by *d(t)*. Levy flight (LF) [10] patterns have been used to increase the effectiveness of the AVOA.

| **Pseudocode of AVOA** |
|---|
| Objective: The position and fitness value of African vulture.<br>The population size (*N*) and maximum number of iterations *(IT)*.<br>First best location: $P_{\text{Bestafricanvulture1}}$ is the position of African vulture.<br>Second best location: $P_{\text{Bestafricanvulture2}}$ is the position of African vulture.<br>Generate initial African vulture population *Pi* (i = 1, 2, ..., n) randomly<br>while stopping criteria not met do<br>for each africanvulture($A_V$) in population do<br>Calculate fitness for ($A_V$)<br>end for<br>find the best ($A_V$)<br>for (each Vulture (*P*i)) do<br>using Eqn. (11) calculate *K(i)*<br>using Eqn. (14) Update the $F_R$<br>if ($\|F_R\| \geq 1$) and ($P1 \geq rand(P1)$) then<br>using Eqn. (16) Update the position of ($A_V$)<br>else<br>using Eq. (18) Update the position of ($A_V$)<br>end for<br>end while<br>if ($0.5 \leq \|F_R\| < 1$) then<br>if ($P2 \geq rand(P2)$) then<br>using Eqn. (20) Update the position of ($A_V$)<br>else |

| |
|---|
| using Eqn. (23) Update the position of ($A_V$)<br>else<br>if ($P3 \geq rand(P3)$) then<br>using Eqn. (28) Update the position of ($A_V$)<br>else<br>using Eqn. (29) Update the position of ($A_V$)<br>Return $P_{Bestafricanvulture1}$ |

### 3.3.3 The Grasshopper Optimization Algorithm (GOA)

The GOA, was created by Saremia. et al. [11] in 2017, It is a new, innovative, and proposed swarm intelligence heuristic optimization technique. It uses a swarm intelligence optimization method that was inspired by nature. The grasshopper algorithm performs the target search procedure just like other metaheuristic algorithms by hunting (exploration) and migration (exploitation) and these are the static and dynamic behaviour of the grasshopper. The distinctive feature of grasshopper swarms is that they exhibit swarming behaviour both as nymphs and as adults. The grasshoppers in the swarm move slowly and take little steps when they are in the larval stage. The grasshoppers in the swarm travel slowly and take modest steps when they are in the larval stage, but when they are adults, they move abruptly and over long distances. Another important aspect of the grasshopper swarm is the search for food sources. The mathematical model employed to simulate the swarming behaviour of grasshoppers is presented as follows.

$$Xi = SI_i + GF_i + A_{W_i} \tag{30}$$

Here, Xi is the position of the $i^{th}$ grasshopper, *SIi* is the social interaction, *GFi* is the gravity force on the $i^{th}$ grasshopper, and $A_{W_i}$is the wind advection.

For random behaviour, the equation will be

$$Xi = r_1 SI_i + r_2 GF_i + 3\, A_{W_i} \tag{31}$$

Where $r_1$,$r_2$and 3are random numbers between 0 and 1.

$$SI_i = \sum_{\substack{j=1 \\ j \neq 1}}^{N} s\left(d_{ij}\right)\widehat{d_{ij}} \tag{32}$$

where $d_{ij}$is the distance between the $i^{th}$ and the $j^{th}$ grasshopper = $|x_i - x_j|$, s is a function to define the strength of social forces and $\widehat{d_{ij}}$ is a unit vector from the $i$ $^{th}$ grasshopper to the $j$ th grasshopper. $N$ is the number of grasshoppers.

$$\widehat{d_{ij}} = \frac{|xi - xj|}{d_{ij}} \tag{33}$$

$$s = f\, e^{\frac{-r}{l}} - e^{-r} \tag{34}$$

where *f* and l indicate the intensity of attraction and attractive length scale respectively.

By substituting *SI, GF*, and $A_{W_i}$ value in Eq. (31), we get

$$Xi = \sum_{\substack{j=1 \\ j\neq 1}}^{N} s\,(|\mathrm{xj} - \mathrm{xi}|)\frac{\mathrm{xj} - \mathrm{xi}}{d_{ij}} - g\widehat{e_g} + u\widehat{e_w} \tag{35}$$

$$s(r) = f\, e^{\frac{-r}{l}} - e^{-r} \tag{36}$$

where *g* is the gravitational constant and *u* is a constant drift. $\widehat{e_g}$ and $\widehat{e_w}$ show a unity vector towards the centre of the earth and in the direction of wind respectively.

However, due to the grasshoppers' rapid emergence into their comfort zone and the swarm's failure to converge to a predetermined point. To tackle optimization difficulties, the following modified version of this equation is proposed:

$$X_i^d = c\left(\sum_{\substack{j=1 \\ j\neq 1}}^{N} c\,\frac{ub-lb}{2}\, s\,(|\mathrm{xj} - \mathrm{xi}|)\frac{\mathrm{xj} - \mathrm{xi}}{d_{ij}}\right) + \widehat{T_d} \tag{37}$$

Here, $\widehat{T_d}$ simulates their tendency to move towards the source of food.

The first parameter *c* balances exploration and exploitation of the entire swarm around the target and the second *c* decreases the attraction zone, comfort zone, and repulsion zone between grasshoppers. The comfort zone is decreased by the coefficient c in proportion to the number of iterations, and its calculation is as follows:

$$c = c_{max} - t \times \frac{c_{max} - c_{min}}{IT} \tag{38}$$

where $c_{max}$ ,$c_{min}$, *t, IT* is the maximum value, minimum value, the current iteration, and maximum number of iterations respectively.

The pseudocode of the dragonfly algorithm is as follows:

| **Pseudocode of GOA** |
|---|
| Objective: Current position<br>Generate initial population $X_i$ (i = 1, 2, …, n)<br>Generate initial $c_{max}$. and $c_{min}$. value<br>Initialised the number of iterations (IT)<br>Calculate the fitness value of all grasshoppers (search agents)<br>Best search agent = T<br>While (t < IT)<br>Using eqn. (38) update the value of c<br>for each grasshopper |

using eqn. (30-34) normalised the distances between search agents
using eqn. (37) update the position of the current grasshopper
if the current grasshopper goes outside the boundaries, then bring it back
end for
update best search agent if there is a better solution
t = t+1
end while
return T

### 3.3.4 The Fruit-fly Optimization Olgorithm (FOA)

The FOA, was created by Wen-Tsao Pan [12] in 2012, It is a new, innovative, and proposed swarm intelligence optimization technique. It uses techniques for optimizing by using swarm intelligence and it was motivated by fruit flies' foraging habits. The fruit fly has greater sensory abilities than other species, especially in eyesight and smell. Fruit flies have osphresis organs that can detect a wide range of odours in the atmosphere. It can also use its keen vision once it is close to the food source to locate food and the company's crowding area and fly in that direction. The steps for finding food are as follows.

Initial fruit fly swarm location*: X-axis, Y-axis.*

Specify the random direction and distance for food while employing osphresis

$$X_{i=}\ X_{axis} + Rand(m)$$

$$Y_{i=}\ Y_{axis} + Rand(m) \tag{39}$$

Where $Rand(m)$ is a random value.

Calculate the value of judgment scent concentration (fitness function) and distance ($Di$) by these equations

$$Di = \sqrt{Xi^2 + Yi^2} \tag{40}$$

$$Si = \frac{1}{Di} \tag{41}$$

Determine the smell concentration value

$$Smell_i = fitness\ (Si) \tag{42}$$

Find the fruit fly that has the highest concentration of smell (fitness function value)

*[bestsmell, bestindex] = max.(smell)* (43)

Using their vision, the fruit fly swarm moves in that direction by these equations

*Smellbest = bestsmell, X-axis* = $X_{bestindex}$, *Y-axis* = $Y_{bestindex}$ (44)

Repeat these steps until it reaches the best or maximum conditions. According to the specifications of our optimization task, we identify the fruit fly with the maximum or minimum scent value. This is known as the "best smell." Save the Fruit fly's location index.
A pseudocode was provided that showed the processes in the fruit fly method.

| **Pseudocode of FOA** |
|---|
| Objective: Maximum smell concentration and location |
| Generate initial swarm location range (Lr) and fly range (Fr) |
| Generate the maximum number of iterations (T) |
| X-axis = rand (Lr), Y-axis = rand (Lr) |
| X'=X-axis + rand (Fr) |
| Y'= Y-axis + rand (Fr) |
| Calculate swarm fitness smell by using eqns. (2-4) |
| [bestsmell, bestindex] = min.(smell) |
| Smellbest = bestsmell, X-axis = X(bestindex), Y-axis = Y(bestindex) |
| for I = 1: T, all T fruitfly |
|     X'=X-axis + rand (Fr), Y'= Y-axis + rand (Fr) |
|     using eqns. (39-41) calculate swarm fitness |
|     [bestsmell, bestindex] = min.(smell) |
|     If bestsmell<smellbest |
|         Smellbest = bestsmell, X-axis = X(bestindex), Y-axis = Y(bestindex) |
|     end if |
| end for |
| return bestsmell, $X_{bestindex}$, $Y_{bestindex}$ |

### 3.3.5 Bird Swarm Algorithm (BSA)

A new swarm intelligent and global optimization algorithm called the BSA was created by Meng. et al. [13] in 2015 is based on the social behaviours and social interactions of birds in nature. Foraging, flight, and vigilance are the three primary social behaviours of birds and based on them the author constructed the algorithm. This algorithm is made up using some rules that describe all of the behaviours of birds as a whole:

| RULES | |
|---|---|
| 1 | Each bird can be switched to one of two statuses vigilance or foraging. |

| | |
|---|---|
| 2 | Each bird can keep track of and update both its own and the swarm's prior best experiences about the food patch while foraging. It also helps them to find a path for food and their movement |
| 3 | Each bird would make an effort to travel towards the centre while remaining vigilant. In comparison, those with bigger reserves would be more likely to lie near to the swarm's centre than others so the chances of other predators attacking birds in the centre are lower |
| 4 | Birds are constantly migrating from one location to another and alternate between producing and scrounging. A producer would be the bird with the most reserves, and a scrounger would be the bird with the least reserves. Some birds, however, are arbitrarily presumed to be either producers or scroungers |
| 5 | Producing birds take the lead in the search for food, while Scroungers follow them at random |

These rules can be used to create a mathematical model shown below:

a) According to its past experiences and the collective experiences of the flock, each bird goes in search of food by using eqn. (45).

$$y_{i,j}^{t+1} = y_{i,j}^{t} + (bp_{i,j} - x_{i,j}^{t}) \times C \times rand(0,1) + (sp_j - y_{i,j}^{t}) \times S \times rand(0,1) \qquad (45)$$

The value of element *j* of bird number *i* is $y_{i,j}^{t}$ and the generation is *t.rand (0, 1)* is uniformly distributed random numbers in between 0 and 1. $bp_{i,j}$ and $sp_j$ represent the i$^{th}$ bird's best prior position, and the swarm's best prior position respectively. The two positive constant values C and S are referred to as the cognitive coefficients and social accelerated coefficients. This is foraging behaviour and it's implemented by rule-1.

b) The vigilance behaviour is described and the motion equation can be determined as follows:

$$y_{i,j}^{t+1} = y_{i,j}^{t} + (mean_j - y_{i,j}^{t}) \times A_1 \times rand(0,1) + (bp_{k,j} - y_{i,j}^{t}) \times A_2 \times rand(-1,1) \qquad (46)$$

$$A1 = a1 \times exp\left(-\frac{bpFti}{sumFt+\epsilon} \times N\right) \qquad (47)$$

$$A2 = a2 \times \exp\left(\left(\frac{bpFti-bpFtk}{|bpFtk-bpFti|+\epsilon} \times N\right)\frac{bpFiti}{sumFt+\epsilon} \times N\right) \qquad (48)$$

Where, *k, bpFti, sumFt*, *a1* and *a2* are positive integers, the best fitness value of bird *i*, the sum of all birds' best fitness values, and positive constants (in [0, 2]) respectively. $\epsilon$ is a small constant used to avoid zero-division. The $j^{th}$ element is indicated by $mean_j$ and it is the average swarm position.

c) After producing and scrounging Rule 46 is modelled to describe flight behaviour. The following mathematical formulas can be used to characterise the actions of producers and scroungers:

$$y_{i,j}^{t+1} = y_{i,j}^{t} + rand_n(0,1) \times y_{i,j}^{t} \tag{49}$$

$$y_{i,j}^{t+1} = y_{i,j}^{t} + \left(y_{k,j}^{t} - y_{i,j}^{t}\right) \times F_L \times rand_n(0,1) \tag{50}$$

Where, $rand_n$ is a randomly chosen number with a mean of 0 and a standard deviation of 1 taken from the Gaussian distribution. $k \in [1,2\ldots, N]$, $k \neq i$, $F_L \in [0, 2]$. We assume that for every $F_Q$ unit interval, each bird goes to a different location.

The pseudocode of the Bird Swarm Algorithm is as follows:

| **Pseudo code of BSA** |
|---|
| Objective: The best solution and fitness value of birds |
| Initialised the population, $t = 0$ |
| Generate max. number of birds in the population *(N)* |
| Generate the maximum number of iterations *(T)* |
| Frequency of flight behaviour of a bird = *Fr* |
| Probability of foraging food = *P* |
| The constant parameters are *C, s, a1, a2, f* |
| While $(t < T)$ |
| If $(t \% Fr \neq 0)$ |
| For **i** in range *(1, N)* |
| If rand $(0,1) < P$ |
| Birds go on searches for food [Using eqn. (45)] |
| Else |
| Birds remain alert [Using eqn. (46)] |
| End if |
| End for |
| Else |
| Separate the swarm into producers and scroungers |
| For *i* in range (*1, N*) |
| If *i* is a producer |
| it will be Producing [Using eqn (49)] |
| Else |

it will be scrounging [Using eqn. (50)]
End if
End for
End if
Find the new current position of the search agents
If the new position is better than before then update
$t = t + 1$
End while

### 3.4 Metaheuristics-based Optimization Processes

Essential process parameters in metaheuristic algorithms are crucial as they directly influence the search behaviour, convergence speed, and solution quality. Carefully setting these parameters allows balancing exploration and exploitation, controlling the diversity of the population, and adapting to the problem characteristics. The right parameter values can significantly impact the algorithm's performance, ensuring effective exploration of the search space, avoidance of premature convergence, and the ability to find high-quality solutions. Adjusting these parameters is vital for achieving optimal results and improving the efficiency and effectiveness of metaheuristic algorithms.

To execute these optimization algorithms, we use Python code and run it on Spyder (Anacoda-3). The Python codes are run in Windows-10 Pro, Intel® Core™ i3-6006U CPU @ 2.00 GHz processor, and 4.00 GB RAM platform. All algorithms run 30 times independently and iterations are taken as 100. So, the total number of trials taken = (30×100) = 3000 to execute these analyses. Table 3.1provides a list of the essential parameter values for all the algorithms.

Table 3.1 Essential parameters

| Algorithms | Essential parameters and there values | Algorithms | Essential parameters and there values | Algorithms | Essential parameters and there values |
|---|---|---|---|---|---|
| DOA | Epoch (ep) = 1000<br>Pop size (ps) = 50 | FFA | ep = 1000<br>ps= 50<br>p= 0.5<br>b= 1 | BSA | ep = 1000<br>ps = 50<br>ff = 10<br>pff = 0.8<br>c1 = 1.5<br>c2 = 1.5<br>a1 = 1.0<br>a2 = 1.0<br>fl = 0.5 |
| AVOA | ep = 1000<br>ps = 50<br>p1 = 0.6<br>p2 = 0.4<br>p3 = 0.6<br>α = 0.8<br>γ = 2.5 | GOA | ep = 1000<br>ps = 50<br>c(min) = 0.00004<br>c(max) = 1.0 | | |

# CHAPTER 4

## PARAMETRIC OPTIMIZATION OF NTM PROCESSES

Numerous manufacturing industries have adopted and made use of the NTM techniques. These NTM procedures each have a number of significant adjustable parameters. For example, peak current, gap voltage, duty factor, and flushing pressure in the case of an EDM process; abrasive flow rate, water jet pressure, stand-off distance, and traverse speed in the case of an AWJM process, etc. Surface roughness (SR) and other reactions are significantly influenced by the settings of these process parameters at various operating levels. When there are several objectives (responses) to be satisfied, choosing the most appropriate combination of input parameters for a particular NTM process can be difficult. Identification of the ideal parametric mixes is highly desirable in order to take advantage of the full machining potential of these NTM processes, as even a small change in the setting of a single input parameter may negatively impact the machining process in a variety of ways. When choosing the appropriate parametric combination of an NTM process for a particular work material and shape feature combination, operators frequently use manufacturer handbooks or their understanding of the process. However, it has frequently been shown that the given parametric combination results in a solution that is either close to optimal or suboptimal. The manufacturer's handbook may not contain the most effective parametric combination for a given NTM process, which would prevent it from performing to its fullest potential. Selection of the optimal parametric settings for different NTM processes highly demands applications of several novel multi-objective mathematical approaches by the concerned process engineers where a set of conflicting objectives (responses) needs to be simultaneously satisfied. Thus, there is an urgent need to explore the applications of different multi-objective optimization techniques for obtaining the optimal parametric combinations for the considered NTM processes. The available process parameters and varied responses for various NTM processes are listed in Table 4.1. The specific optimization techniques can vary depending on the NTM process under consideration, but in this research, here we providing with a general approach to parametric optimization.

**Define the objective**: Clearly define the objective of the optimization, such as maximizing material removal rate or minimizing surface roughness. This objective will guide the optimization process.

**Identify process parameters**: Identify the key process parameters that significantly affect the performance of the NTM process. For example, in electrical discharge machining (EDM), parameters like pulse current, pulse duration, electrode material, and dielectric fluid properties are crucial.

**Determine the parameter ranges**: Determine the feasible range of values for each process parameter. Consider both practical limitations and the desired range of variation for effective optimization.

**Design of experiments:** Use statistical experimental design techniques, such as factorial design or response surface methodology, to plan a set of experiments that cover the parameter space adequately. The goal is to obtain data on the process performance for different parameter combinations.

**Conduct experiments:** Perform the planned experiments and record the process outputs or responses of interest. Ensure that the experiments are conducted under controlled conditions and repeated multiple times to account for variability.

**Make a model:** Use the experimental data to develop a mathematical or empirical model that relates the process parameters to the process performance. This model can be a simple regression equation or a more complex model based on machine learning algorithms.

**Perform optimization:** Utilize optimization algorithms to find the optimal parameter combination that satisfies the defined objective. Common optimization algorithms include genetic algorithms, particle swarm optimization, or gradient-based methods.

**Validate and refine:** Validate the optimized parameter combination by performing additional experiments or simulations. Refine the model, if necessary, based on the new data and repeat the optimization process if needed.

**Implement and monitor:** Implement the optimized parameter combination in the actual NTM process and monitor the performance. Continuously evaluate the process outputs and make adjustments if required.

Table 4.1 NTM process parameters and responses

| NTM process | Process parameters | Responses |
|---|---|---|
| USM | Tool material, abrasive type, grit size, power rating | MRR, SR, TWR |
| EDM | Peak current, gap voltage, pulse-on time, duty factor and flushing pressure | MRR, SR, TWR, surface crack density, white layer thickness, micro hardness |
| WEDM | Gap voltage, wire tension, capacitance, feed rate, wire type, pulse-on time, pulse-off time | Cutting speed, MRR, KW, SR, depth deviation |
| LBM | Average power, pulse frequency, rotational speed, air pressure, feed rate | SR, depth deviation |
| ECM | Applied voltage, tool feed rate, electrolyte flow rate and electrolyte concentration | MRR, SR, KW |
| PCM | concentration of the etchant, time of etching and temperature of the etchant | SR, undercut, MRR, etch factor |
| ECDM | Voltage, electrolyte concentration, inter-electrode gap and duty factor | MRR, overcut, taper |
| AWJM | Abrasive flow rate, water jet pressure, stand-off distance and traverse speed | MRR, SR, KW, kerf angle, circularity error, perpendicularity error, parallelism error |

## 4.1 Literature Reviews of NTM Processes

It's important to note that the specific details of the optimization process will depend on the NTM process being considered. Different NTM processes have unique characteristics and may require tailored optimization approaches. Consulting specialized literature and experts in the specific NTM process can provide valuable insights for optimization. I am providing two NTM processes example of parametric optimization. On a problem of such constrained design, numerous metaheuristics have been applied. The earliest MH techniques that were perhaps the most widely utilised were genetic algorithms (GA) and simulated annealing. Numerous researchers have proposed various optimization techniques over the past few decades, the particle swarm optimization (PSO), the artificial bee colony (ABC), ant colony optimization (ACO), etc. These optimization methods are also known as metaheuristic methods or non-traditional optimization methods. These optimizers have been employed in a variety of industries.

**Reddy et al. [14]** investigated the performance of an EDM process while machining PH17-4 stainless steel material using graphite powder-mixed and surfactant-mixed dielectric fluids. Peak current, surfactant concentration, and graphite powder concentration were chosen as the three crucial process parameters, while MRR, Ra, and TWR were used as the responses, in a multi-response optimization technique based on integrated Taguchi-data envelopment analysis. Taking into account the input parameters wire drum speed, pulse current, pulse on time, and pulse off time. They found that the performance of the EDM process was most significantly impacted by peak current, followed by surfactant and PC.At the best parametric setting, a confirmation experiment was done. The measured values of responses were MRR= 61.2608 mm3 /min, SR= 5.3 mm, and TWR=3.461 mm3 /min.

**Rao et al. [15]** designed and optimized the input parameters (applied voltage, electrolyte concentration, electrode feed rate, and percentage of reinforcement) of an ECM process while integrating the Taguchi method with the utility concept. They stated that the essential process parameters including applied voltage, electrolyte concentration, electrode feed rate, and percentage of reinforcement significantly impacted the responses including MRR, Ra, and radial overcut. They used ANOVA and found that feed rate, electrolyte concentration, and applied voltage had the most influence on multi-response optimization. The output of this study was significantly improved by the confirmation experiment.

**Chaki et al. [16]** developed a model by using ANN multi-objective optimization (PSO) for the input parameters of Cutting speed, Pulse energy, Pulse width of an LBM process (Nd: YA Glaser cutting). They found that during the machining of AA1200 aluminium alloy sheet reduction of kerf width, kerf deviation, and surface roughness values by 36.75%, 26.25%, and 14.94%, respectively, and an increase in MRR by 24.67%.The ANN design with the lowest Mean Square Error (MSE) and accurately predicted laser cutting process parameters within a low mean absolute error of 1.74% has demonstrated the highest prediction capabilities. Cutting speed has the greatest impact on the features of output quality, according to an ANOVA.

**Shrivastava et al. [17]** developed a model by using multi-objective optimization (GA) for the input parameters of Assist gas pressure; Standoff distance; Cutting speed; Laser power of an LBM process (Nd: YA Glaser cutting). They found that during the machining of Inconel-718,the output responses improved, with improvements in kerf deviation, kerf breadth, and kerf taper amounting to 88%, 10.63%, and 42.15%, respectively and multiple quality parameters have seen an overall improvement of 46.92%.According to the ANOVA results, the two process variables that have most influence on kerf deviation are cutting speed and laser power. Standoff distance and laser power have been determined to be the most important process variables for the KW. Gas pressure and standoff distance are the most essential parameters for the kerf taper.

**Pandey et al. [18]** designed and optimized the input parameters (Assist gas pressure, Pulse width/Pulse duration, Pulse frequency, cutting speed) of an LBM process while integrating the Fuzzy logic multi-objective optimization method. During the machining of Duralumin, they found the optimum parameter values for minimization of the kerf-width and kerf deviation at the top and bottom sides. They used a hybrid strategy to calculate the fuzzy multi-response performance index, integrating robust parameter design methodology and fuzzy logic theory. Multi-objective optimization also makes use of this performance index. By running the confirmation tests, the projected optimum results have been confirmed. The results of the study show that when laser cutting highly reflective and thermally conductive materials like Duralumin, oxygen gas pressure is the most important factor, followed by pulse frequency (which is directly proportional to pulse energy).

**Satpathy et al.[19]** designed and optimized the input parameters (Peak current, pulse-on-time ($T_{on}$), duty cycle (DC) and gap voltage) of an EDM process while Combining PCA and TOPSIS method. During the machining of AlSiC (20%SiC+ Copper), they found that high Ip, low Ton, high Duty Cycle, and moderate Gap Voltage all contributed to the achievement of maximum MRR, whereas MRR was found to be closer to its minimal value for low TWR. After using the PCA-TOPSIS method, the ideal set of input variables was found to be input current (Ip) = 3A, pulse on time (Tonne) = 75 s, duty cycle (DC) = 80%, and gap voltage (Vg) = 40V.

**Khullar et al. [20]** designed and optimized the input parameters (Ton, Toff, peak current) of an EDM process while Combining NSGA-II, RSM method. Instead of analysing one problem at a time, they study all the control factors on responses collectively. During the machining of AISI 5160 alloy steel, they found that with the help of NSGA-II, engineers can adopt the algorithm's optimal solution in order to meet their demands for improved productivity or quality. The algorithm's anticipated findings are discovered to be extremely close to the experimental results. The MRR and SR values in the confirmation experiment-1 are 1.167 g/min and 1.28 m and 1.149 g/min and 1.327 m, respectively, between the anticipated and experimental values.

**Goswami and Chakraborty [21]** employed the gravitational search algorithm (GSA) and the fireworks algorithm (FWA) for the parametric optimization of USM processes, two almost unexplored nonconventional optimization strategies. These two population-based optimization algorithms' performances is compared to that of other well-known algorithms, and the impacts of their method parameters on the obtained optimal solutions and computing speed have been looked into. They found that, although on average, it requires a little bit more computational time, the FWA technique's optimization performance is more effective to that of GSA and other widely used population-based methods.

**Radovanovic [22]** studied multi-objective optimization of abrasive water jet cutting of carbon steel S235.The three factors of TS, AMFR, and SOD were combined with the three constraints of perpendicularity tolerance, surface roughness limit, and TS for separation cut to optimise two objectives simultaneously: productivity and operating cost. Multi-objective genetic algorithm (MOGA) was used to tackle the optimization challenge.

### 4.2 Optimization of AWJM Processes

In manufacturing operations, advanced machining techniques are frequently used to solve a variety of problems, including the machining of high-strength materials, making of complex-shaped profiles, the improvement of surface features, high levels of precision, miniaturisation, the reduction of waste, secondary operations, and shorter production times [23]. The use of non-traditional manufacturing processes has significantly increased over the past few years. in particular, when working with tough elements like titanium, stainless steel, composite materials, etc. Conventional techniques are no longer effective for producing precise, highly complex products with better surface finishes and dimensional accuracy. Generally, NTM can be classified on the basis of what type of energy is used in the machining process or the nature of energy which are Mechanical, Chemical or Electrochemical, Thermal or electro-thermal. When we need to perform a machining operation by using the NTM process we consider many aspects these are work material, process parameters, type of shape, process capabilities, economic considerations, etc.

In 1983, the unconventional AWJM process based on high-pressure abrasive water jet was commercialized. AWJM is a mechanical machining NTM technique based on the principles of water erosion that uses a high-velocity jet of water and abrasive particles hits the surface and removes material from the surface of the workpiece. Abrasives are shot by a high-velocity water jet into a mixing chamber. The mixture is then fed via a nozzle, and directed towards a small area where it will strike the workpiece surface. As a result, the material is removed from the workpiece by the erosive action of the. The hammering effect of the abrasives on the machining area of the workpiece causes a brittle fracture, and the water jet then removes the wear particles from the machining zone. Based on the different methods of combining abrasive and water, the AWJ process is divided into two categories: abrasive water injection jet and abrasive slurry jet. To process soft materials, pure water jets are used. However, AWJM is a mixture of abrasive particles and water used in machining to cut hard materials. The different machining parameters of an AWJM process, such as the Jet pressure (JP), Traverse Speed (TS), abrasive flow rate (AFR), the diameter of the abrasive water-jet nozzle (D), stand-off distance (SOD), abrasive particle size (AMS), etc., affect the process performance. Each of these machining parameters also affects the responses, namely the material removal rate (MRR), surface roughness (SR), and kerf angle (KA).

Performance optimization is an important challenge because of the many different machining requirements and various settings involved in the AWJ technique. There are numerous factors (constraints) to be taken into account when performing this job. Many scholars researched in this AWJM field and used many tools based on mathematical and analytical to identify the relationship between process parameters and various responses.

**Sundararaj et al. [24]** considered mathematical models for optimization of three process parameters in AJM of material silicon applying Taguchi design methodology using gray-fuzzy (GF), and identified the causes of variation in MRR, surface roughness, flatness, inclination angle with respect to different process parameters (i.e., water pressure, stand of distance, abrasive mass flow rate).

**Akhai et al. [25]** considered mathematical models for optimization of three process parameters in AWJM of material Al-6061 applying Taguchi design methodology using TGRA, and identified the causes of variation in MRR, surface roughness (Ra), and kerf angle (KA) with respect to different process parameters (stand of distance (SOD), abrasive mass flow rate (AMFR), transverse speed (TS). They said that SOD and AMFR are highly significant to affect MRR but the effect of TS is very less compared to them.

**Dahiya et al**. **[26]** investigated mathematical models for the optimization of four process parameters in the Central composite design (CCD) technique for the material GFRP utilising Single performance optimization, and he found the parameters that affected the variance in DLL+ with regard to the various process parameters. They found that while WP has a relatively small impact compared to TR and AMFR, they are both highly significant in affecting DLL+.

**Rammohan et al. [27]** investigated mathematical models for the optimization of three process parameters in the Full factorial design (FFD) technique for the material Rolled homogeneous armour steel (RHA) and he found Higher MRR was obtained with a higher value of WP and a higher surface finish was achieved with a higher value of SOD, and a lower value of TS. SOD significantly impacts on the response KA.

**Rajesh et al. [29]** investigated mathematical models for the optimization of four process parameters in the CCD technique for the material Flax fibre-reinforced polymer laminates (FFRP) utilising RSM, and he found the factors that most affect the response Ra, and WP and FR with regard to the various process parameters (WP, SOD, FR, AMFR). They found that while TS has a relatively small impact compared to SOD and AMFR, they are both highly significant in affecting MRR. Ra was influenced by AFR and WP as well, but they are not particularly important.

**Fuse et al**. **[30]** investigated mathematical models for the optimization of three process parameters in the Box-Behnken design(BBD) technique for the material Ti6Al4V utilising Heat Transfer Search (HTS), and he found by using both single- and multi-objective optimization, a multi-objective optimization method (HTS) provides the efficient outcome of the constructed models of MRR, SR, and TA.

**Thamizhvalavan et al**. **[31]** investigated an optimization model for the optimization of four process parameters (WP, TR, AFR, AMS) in the Box-Behnken design(BBD) technique for the material Al6063/B4C/ZrSiO4 and he found that in this composite material, AMS influences the low value of Ra and the greater Depth of cut (DOC) and MRR.

Some other literature reviews on AWJM are described in the table 4.2:

Table 4.2 Literature reviews

| SL. NO. | AUTHOR NAME &YEAR | MATERIAL USED | PARAMETERS | RESPONCES | DESIGN TECHNIQUE | OPTIMIZATION METHOD | CONCLUSIONS |
|---|---|---|---|---|---|---|---|
| 1 | Kumar et al.(20 | KEC | WP,SOD,TS,AM | KTA | CCD | RSM | The most essential parameters affecting the KTA are the WP and TS |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20) [32] | | FR | | | | and if we decrease TS and increase WP then also KTA value decreased. |
| 2 | Selvam (2020) [33] | Carbon+S-Glassfibers + SiC nano particles | WP,SOD,TS,AMFR | KTA, Ra | | RSM | WP and AFR are essential parameters to get a better value of Ra.<br>For machining this material KTA is reduced by adding SIC particles and by decreasing SOD. |
| 3 | Dhanawade et al. (2020) [34] | $Pb[Zr_x Ti_{1-x}] O_3$ and carbon fiber | WP,SOD,TS | Ra | CCD | RSM | In this paper Ra decreased with an increase in WP and with a decrease in SOD. Here WP and TR are the essential parameters in machining CFRP composite. |
| 4 | Kumar et al. (2019) [35] | Ti /CFRP/ Ti | WP,SOD,TS | MRR, Ra,TR | CCD | RSM | Here TS and WP are the maximum influencing parameters on MRR, Damage factor, and Ra but SOD has the most influence on TR. |
| 5 | Tiwari et al. (2018) [36] | Alumina ceramic | WP,AFR,TS | MRR, RA,TA | BBD | RSM | In this paper MRR and Ra increased with an increase in WP but by increasing AFR but a decrease in TS, AFR and increased MRR. An increase in TA was obtained with a decrease in WP and AFR but an increase in TS. |
| 6 | Chakraborty et al. (2018) [37] | Alumina Ceramic | WP,JFS,AMFRa,SS,NTA | MRR, Ra | | GA,SA, TLBO, GWO | In this paper a new unexplored optimization algorithm gives quite satisfactory results in both single and multi-objective optimization when it compared with other algorithms, such as SA,GA, and TLBO |

### 4.2.1 Problem Statement

In order to determine the effects of different abrasive water-jet machining (AWJM) process parameters on responses, Balamurugan et al. [38] performed an experimental analysis on Predicting correlations in abrasive water jet cutting parameters of Lanthanum phosphate/ Yttria composite ($LaPO_4/Y_2O_3$). The $LaPO_4/Y_2O_3$ as a workpiece material (composite) is cut with an AWJM of Model DIP 6D-2230. The orifice and tungsten carbide nozzle in AWJM are 0.25 mm and 0.67 mm in diameter, respectively. Abrasive particles are made of silicon carbide with an 80-mesh size. There were 20 experiments conducted in total, and the MRR (g/s), KA (degree), and Ra (µm) were collected. Jet Pressure (JP), Stand-off-distance (SOD), and Traverse Speed (TS) are the process parameters and Material removal rate (MRR), kerf angle (KA), and surface roughness (Ra) are the responses. For this machining operation, the various process parameters and their set levels are shown in Table 4.3 based on data from experiments a Response Surface Regression was used to build the empirical equations for the three output responses using single objective optimization.

$$MRR = 0.099 - 0.00087 \times \mathrm{JP} - 0.02502 \times \mathrm{SOD} - 0.000875 \times \mathrm{TS} + 0.000003 \times JP^2 - 0.000343 \times \mathrm{SOD}^2 + 0.000002 \times \mathrm{TS}^2 + 0.000123 \times \mathrm{JP} \times \mathrm{SOD} + 0.000001 \times \mathrm{JP} \times \mathrm{TS} + 0.000084 \times \mathrm{SOD} \times \mathrm{TS} \quad (51)$$

$$KA = 2.58 - 0.0200 \times \mathrm{JP} - 0.519 \times \mathrm{SOD} - 0.0012 \times \mathrm{TS} + 0.000039 \times JP^2 + 0.0398 \times SOD^2 - 0.000127 \times TS^2 + 0.001850 \times \mathrm{JP} \times \mathrm{SOD} + 0.000073 \times \mathrm{JP} \times \mathrm{TS} - 0.001050 \times \mathrm{SOD} \times \mathrm{TS} \quad (52)$$

$$Ra = -2.04 + 0.0190 \times \mathrm{JP} - 0.369 \times \mathrm{SOD} + 0.0292 \times \mathrm{TS} - 0.000027 \times JP^2 + 0.0513 \times SOD^2 - 0.000172 \times TS^2 + 0.000963 \times \mathrm{JP} \times \mathrm{SOD} - 0.000023 \times \mathrm{JP} \times \mathrm{TS} + 0.00110 \times \mathrm{SOD} \times \mathrm{TS} \quad (53)$$

Table 4.3 Process parameters and their levels

| SL. No | Process parameters | Symbol | Variation levels | | |
|---|---|---|---|---|---|
| | | | -1 | 0 | 1 |
| 1 | Jet Pressure (bar) | JP | 220 | 240 | 260 |
| 2 | Stand-off-distance (mm) | SOD | 1 | 2 | 3 |
| 3 | Traverse Speed (mm/min) | TS | 20 | 30 | 40 |

For this experimental investigation, we used a multi-objective optimization technique by giving different relative importance (weights). An optimal Pareto front was then created to produce non-dominated solutions set, and by using this set we find the best solution for this particular problem [39].The model of this optimization problem will be as follows:

$$\text{Objective}_1 = \text{Maximize ( MRR)} \tag{54}$$

$$\text{Objective}_2 = \text{Minimize (KA)} \tag{55}$$

$$\text{Objective}_3 = \text{Minimize (Ra)} \tag{56}$$

Total independent variables (IV) = JP, SOD, TS

By converting these three single objective functions to a single multi-objective function, we make an equation and the equation is below:

$$Objectives : (\text{Objective}_1, \text{Objective}_2, and\ \text{Objective}_3) = f(IV) \tag{57}$$

For multi-objective optimization, we are using The Weighted Sum Method (WSM) to optimize this problem by combining all three responses and the model is as below:

$$Z = \left[(-)\frac{w_1 \times MRR}{MRR_{min}} + \frac{w_2 \times KA}{KA_{min}} + \frac{w_3 \times Ra}{Ra_{min}}\right] \tag{58}$$

$$w_1 + w_2 + w_3 = 1 \tag{59}$$

Where, Z is a multi-objective function. $w_1$, $w_2$, and $w_3$ are the assigned weights for MRR, KA and Ra respectively. In the present problem we have taken the weight value for MRR, KA and Ra are the same, ach (1/3). An objective's weight is chosen based on its relative importance. For the purpose of solving this optimization problem, self-developed Python language programs have been used. The result or outcome of objective function from these algorithms depends on these essential parameters of every metaheuristic algorithm. In order to get the proper results, the value of these parameters must be properly chosen.

### 4.2.2 Results and Discussion

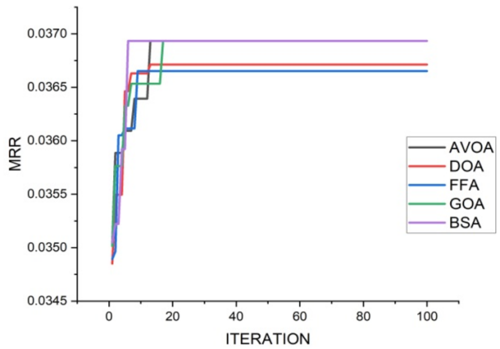
The entire function evaluation is maintained constant to make it easier to compare these five metaheuristic algorithms' performances. To take into consideration of the stochastic character of each method, it was independently executed 30 times. Each run had a max of 30 search agents, but the maximum number of iterations was set at 100, resulting in 3000 function evaluations.

Table 4.4 Comparison of current results with solutions with statistical results of MRR, KA, and Ra optimization

| ALGORITHMS | RESPONCES | PARAMETERS | | | Mean | SD | OPTIMUM | *IMPROVEMENT ( %)* |
|---|---|---|---|---|---|---|---|---|
| | | JP (bar) | SOD (mm) | TS (mm/min) | | | | |
| **Balamurugan et.al.** | MRR | 220 | 1 | 40 | | | 0.0215 | 0 |
| | KA | 220 | 1.68 | 20 | | | 0.359 | |
| | Ra | 220 | 1.32 | 20 | | | 1.28 | |
| ***AVOA*** | *MRR* | 260 | 3 | 20 | 0.0368 | 0.0003 | 0.0369 | 71.47 |
| | *KA* | 220 | 1.317 | 20 | 0.203 | 0.0022 | 0.202 | 43.73 |
| | *Ra* | 220 | 1.67 | 20 | 1.15 | 0.0059 | 1.152 | 10.00 |
| ***DOA*** | MRR | 260 | 3 | 28.31 | 0.0366 | 0.0003 | 0.0367 | 70.54 |
| | KA | 234.6 | 1.293 | 20.16 | 0.2037 | 0.0022 | 0.2028 | 43.51 |
| | Ra | 226.2 | 1.314 | 22.05 | 1.159 | 0.0065 | 1.158 | 9.53 |
| ***FFA*** | *MRR* | 260 | 2 | 25 | 0.0365 | 0.0003 | 0.0366 | 70.07 |
| | *KA* | 220 | 1.678 | 20 | 0.2087 | 0.0039 | 0.2072 | 42.28 |
| | *Ra* | 220 | 1.319 | 20 | 1.166 | 0.0136 | 1.162 | 9.22 |
| ***GOA*** | MRR | 260 | 3 | 20 | 0.0366 | 0.0003 | 0.0369 | 71.47 |
| | KA | 220 | 1.67 | 20 | 0.2032 | 0.0018 | 0.202 | 43.73 |
| | Ra | 220 | 1.317 | 20 | 1.161 | 0.0082 | 1.161 | 9.30 |
| ***BSA*** | *MRR* | 260 | 3 | 20 | 0.0366 | 0.0003 | 0.0369 | 71.47 |
| | *KA* | 220 | 1.67 | 20 | 0.2036 | 0.0023 | 0.202 | 43.73 |
| | *Ra* | 220 | 1.318 | 20 | 1.16 | 0.0079 | 1.1583 | 9.51 |

**For MRR:**

To compare the five metaheuristic algorithms, we employed convergence diagrams of 30 trials with MRR optimization; these plots are shown in Figure 4.1. Except for DOA and FFA, all the other algorithms in single objective optimization showed a noticeable increase in the best solution after the first few iterations, as can be seen from the plot. The convergence patterns of the AVOA, GOA, and BSA were found to be same for MRR optimization. BSA, AVOA, and GOA all shown continuous progress in the best solution search for the first few iterations, but after 20 to 30 iterations, there was no improvement. Table 4.4 displays the statistical metrics used to evaluate the algorithms' performance over the course of 30 independent trials.



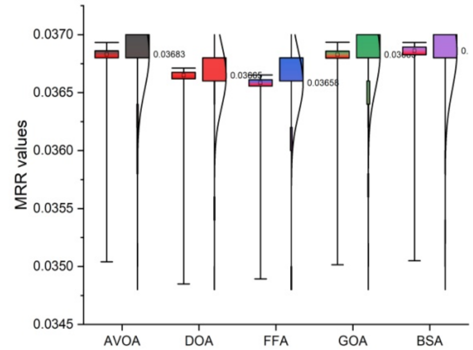




Figure 4.1 Convergence diagram between optimum MRR and iterations

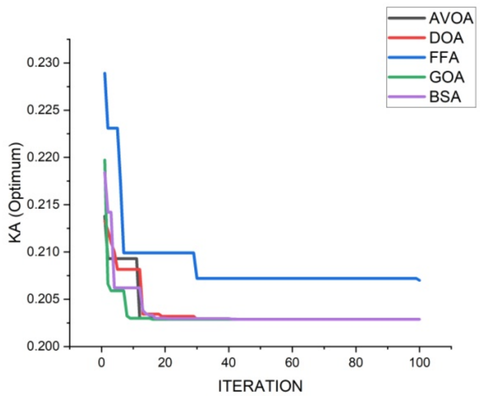
Figure 4.2 Box plots showing the range of all function analyses during MRR optimization

It was found that the best MRR values provided by the other three algorithms was 0.0368, while DOA and FFA provided 0.0367 and 0.0366, respectively. It therefore proves that these three algorithms (AVOA, GOA, and BSA) were successful in predicting the best-known value in each of the 30 trials but FFA cannot reach the optimal value after 100 iterations. As a result, when compared to the perform used by Balamurugan et al. [38], the success rate—i.e., the proportion of times the method used by AVOA, DOA, FFA, GOA, and BSA reached the best or optimum value—was seen to be 71.46%, 70.53%, 70.07%, and 71.46%, 71.46%, respectively. We presented the 100 iterations value for each algorithm as box plots (as box plots are powerful tools to find the repeatability of these algorithms) in Figure 4.2 to provide a more detailed study of the performance of these algorithms.

It is crucial to examine this plot in conjunction with Table 4.4 in order to come to a clear conclusion and prevent misinterpretation. From Table4.4 we can show that for MRR highest variation for BSA and AVOA but for FFA there is minimal variation between the optimum and mean values. In comparison to the other algorithms, AVOA and BSA are observed to require fewer function evaluations to get to the optimal value, which indicates that they reached the optimal zone more quickly.

**For KA:**

The convergence of the algorithms while minimising the KA is shown in Figure 4.3. It was noticed that all other algorithms, with the exception of FFA, show a similar trend of convergence. Table 4.4 also contains a summary of the statistical results and optimization results of the 30 different KA optimization trials. The change in KA values over iterations is seen in Figure 4.3.



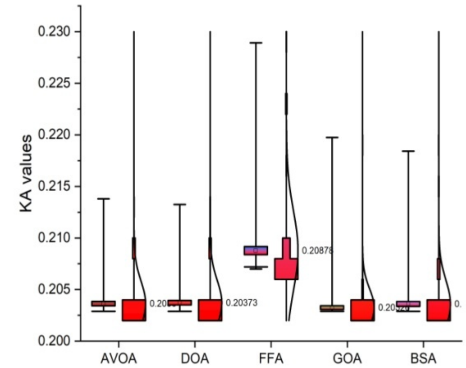




Figure 4.3 Convergence diagram between optimum KA and iterations

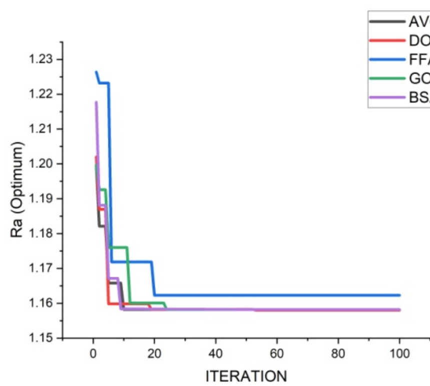
Figure 4.4 Box plots showing the range of all function analyses during KA optimization

The optimum optimization values, similar to MRR optimization, are offered by AVOA, GOA, and BSA, with the exception of DOA and FFA in KA optimization. Table 4.4 also displays the statistical characteristics that were used to evaluate the algorithms' efficacy across 30 independent trials. It was found that whereas the best value reported by the other three algorithms was 0.202, the optimum KA values provided by DOA and FFA were 0.2342 and 0.2072, respectively. Therefore, it shows that these three algorithms (AVOA, GOA, and BSA) were successful in forecasting the best optimum value in each of the 30 trials. Table 4.4 presents the optimal process parameters and the KA as given by the various metaheuristic algorithms and improved the solutions of Balamurugan et al. [38] were improved by 34.7% and 42.2% for DOA and FFA, respectively. Each of the other algorithms was reported to have performed better than the current answers in the literature by 71.4%. Figures 4.2 and Figure 4.4 have fairly similar overall patterns for the spread and distribution of the function evaluations. This suggests that whether the optimization problem is of the minimization or maximisation kind has no effect on the processes.

According to Table 4.4, the difference between the optimum and mean values of FFA for KA and GOA is considerably lower than that for KA. As a significant portion of GOA is in the optimum zone, it can be seen that GOA reaches the zone faster than the other algorithms. Therefore, it took less function evaluations to get to at the optimal value.

**For Ra:**

The convergence of the algorithms while minimising the Ra is shown in Figure 4.5**.** All other algorithms, with the exception of FFA, were found to exhibit a similar trend of convergence. Table 4.4also contains the results of the 30 distinct Ra optimization trials and statistical outcome summaries. In Ra optimization, AVOA, DOA, and BSA provide us the best optimum optimization values, with the exception of GOA and FFA. Table 4.4 displays the statistical parameters used to evaluate the algorithms' performance throughout 30 independent trials also. Figure 4.5 describes the variation of Ra values with respect to iterations.



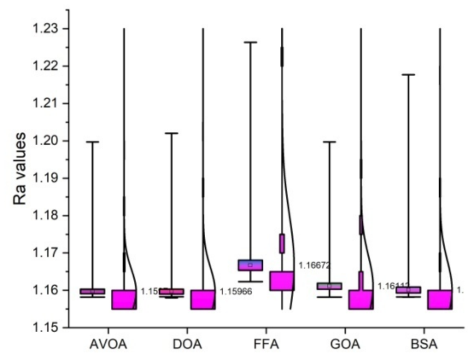




Figure 4.5 Convergence diagram between optimum Ra and iterations

Figure 4.6 Box plots showing the range of all function analyses during Ra optimization

It was found that the best Ra values given by the other three algorithms (AVOA, DOA, and BSA) were, 1.152, 1.158, and 1.1583, respectively, compared to the optimum Ra values obtained from GOA and FFA, which were 1.161 and 1.162.It therefore proves that these three algorithms (AVOA, DOA, and BSA) were successful in predicting the best optimum value in each of the 30 trials but FFA cannot reached the optimum value. However, AVOA, DOA, and BSA were reported to have done better than the present solutions in the literature (Balamurugan et al. [38]) by 10%, 9.53%, and 9.50% respectively but for GOA and FFA, the solutions were improved by 9.29% and 9.21%, respectively. The distribution of the total function evaluations in a typical trial while minimising Ra is depicted in Figure 4.6.

**Final result:**

As seen in Table 4.4, there is little deviation for AVOA between the optimal and mean values but there is a significant deviation for FFA for Ra. AVOA was shown to enter the optimal zone more quickly than the other algorithms as it shows a significant portion of the AVOA is in the optimal zone. Hence it required fewer function evaluations to obtain the optimal value.

It is significant to note that all five algorithms provided a range of optimal values for the process parameters. If we compare the overall optimization process (except time) then we can say that we can get the best optimum value for MRR and KA optimization by using AVOA, GOA, and BSA and for Ra optimization AVOA gives us the best value. When we take execution time as another variable to determine all algorithm's efficiency then we can say AVOA is the best algorithm to solve this particular single objective optimization problem (Figure 4.7). Hence, we can conclude that AVOA is the best algorithm to find the best optimal value for this problem. Optimization algorithms rank based on each optimization are given below:

---

MRR (Response 1) = **AVOA**= GOA= BSA > DOA> FFA

KA (Response 2) = **AVOA**= GOA= BSA > FFA> DOA

Ra (Response 3) = **AVOA**> DOA> BSA> GOA> FFA

Computational time= **AVOA**> FFA > BSA > DOA > GOA

---

This suggests that the search space for the goal function was multimodal. It is important to note that Balamurugan et al. [38] employed an RSM CCD-based model to determine the optimal values. The optimum values of responses obtained by Balamurugan et al. [38] were MRR = 0.0215g/s, KA = 0.359degrees and Ra = 1.28µm. By using these five metaheuristic algorithms we found the best-known optimal parameters of these three responses were MRR = 0.0369g/s, KA = 0.202 degrees, and SR = 1.15µm.

Each algorithm evaluated the same number of functions (30 ×100 = 3000), but the time it took for each algorithm to complete the optimization process varied. In order to do this, the CPU time for each algorithm was measured and averaged over 10 separate trials for each response. Figure 4.7 shows that, in terms of execution time, AVOA, FFA, and BSA were the three most expensive algorithms, whereas DOA and GOA were the least expensive.

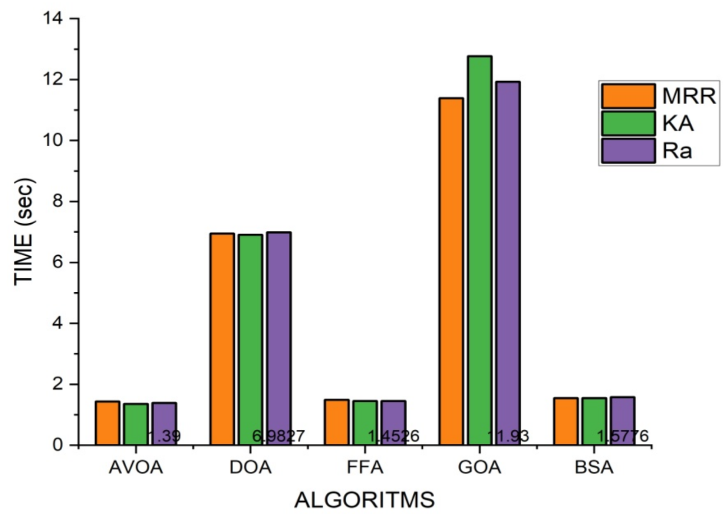



Figure 4.7 Bar chart of algorithms and their computational time

The performance of all five recently developed algorithms outperformed the available solutions by Balamurugan et al. [38]. Balamurugan et al. [38] although typically fails to find the global optima in this problem. For MRR, KA, and Ra results, the current metaheuristics analysed in this research produced at least 70%, 34%, and 9% better solutions than Balamurugan et al. [38]. This enhancement may be brought about by the fact that metaheuristics start with a random population and iteratively enhance it over generations by modifying the solutions. According to their execution time, we can say AVOA, FFA, and BSA are approximately nine times faster than GOA and approximately five times faster than DOA.

**Friedman test:**

To evaluate whether or not there was a statistically significant difference between the AVOA, DOA, FFA, GOA, and BSA derived objective function values for the 30 runs, Friedman's non-parametric test was performed. The test is run using the instructions provided in this test by using Python 3.0. The table 4.5 below displays the obtained results. Where Q means chi-square value, DF means degree of freedom.

Since the p-value is less than 0.01, in both examples, indicates that the null hypothesis, $H_0$ is rejected at 1% level of significance for all three responses in both examples which means that objective function is the same for all five metaheuristic algorithms can be rejected. This is appropriate confirmation to conclude that the results obtained by AVOA, DOA, FFA, GOA, and BSA are statistically significant.

General Guidelines for rejection of null hypothesis: p-value ≤ 0.01 then rejection of null hypothesis ($H_0$) at 1% level of significance. p-value ≤ 0.05 then rejection of the null hypothesis ($H_0$) at 5% level of significance. p-value ≤ 0.1 then rejection of the null hypothesis ($H_0$) at 10% level of significance. p-value > 0.1 then unable to reject the null hypothesis ($H_0$) because never consider more than 10% error or randomness in our test theory results. Additionally, the p-value indicated the existence of notable variations among the compared algorithms [40].

Table 4.5 Friedman's test results for AWJM

| Source | Q | DF | p-value |
|---|---|---|---|
| **MRR** | 252.2075 | 4 | 2.18E-53 |
| **KA** | 275.7947 | 4 | 1.80E-58 |
| **Ra** | 268.8526 | 4 | 5.64E-57 |

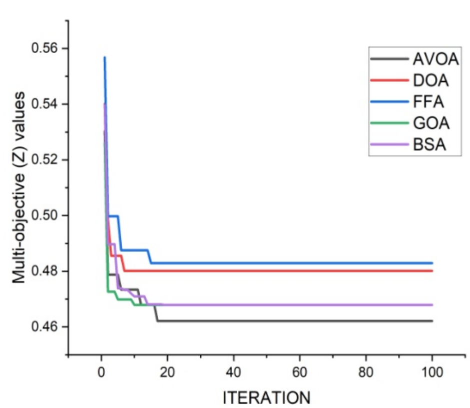
| **Algorithms** | | **AVOA** | **DOA** | **FFA** | **GOA** | **BSA** |
|---|---|---|---|---|---|---|
| Average | MRR | 0.03683 | 0.036647 | 0.0365841 | 0.036827 | 0.036801 |
| Rank-Sum | | 213 | 377 | 464 | 220 | 226 |
| Avg. Rank | | 2.13 | 3.77 | 4.64 | 2.2 | 2.26 |
| **Rank** | | **1** | **4** | **5** | **2** | **3** |
| Average | KA | 0.203644 | 0.203726 | 0.2087755 | 0.203256 | 0.20361 |
| Rank-Sum | | 179.5 | 213 | 500 | 248.5 | 359 |
| Avg. Rank | | 1.795 | 2.13 | 5 | 2.485 | 3.59 |
| **Rank** | | **1** | **2** | **5** | **3** | **4** |
| Average | Ra | 1.159714 | 1.159664 | 1.1667184 | 1.16112 | 1.160107 |
| Rank-Sum | | 165.5 | 203 | 494 | 286.5 | 351 |
| Avg. Rank | | 1.655 | 2.03 | 4.94 | 2.865 | 3.51 |
| **Rank** | | **1** | **2** | **5** | **3** | **4** |

**Multi-objective result:**

The results of the multi-objective optimization are presented in Table 4.6, and it demonstrate that all five algorithms are capable of generating better response values than those achieved by Balamurugan et al. [38]. For a fair comparison of all the algorithms, the search agents and iterations were limited to 30 and 100, respectively. The total number of function evaluations for each algorithm was limited to 3000 in order to provide a neutral comparison. Except for DOA, AVOA and FFA offer the best optimum optimization values for MRR in multi-objective optimization at 0.0362 and 0.0354, respectively. This demonstrates that the optimal optimum value was correctly predicted by the two algorithms (AVOA and FFA) in each of the 30 trials. The solutions for DOA, GOA, and BSA were improved by 57.21%, 62.79%, and 62.79%, respectively, even with AVOA and FFA were stated to have performed better than the current answers in the literature (Balamurugan et al.'s [38]) by 64.65% and 68.37%, respectively. If we compare between five metaheuristic algorithms, all but DOA generate more substantial optimum results in KA optimization but if we compare all with current literature paper (Balamurugan et al.'s [38]**)** then it can be seen that all algorithms performed better than it by 43.45%, 38.72%, 42.34%, 42.90%, and 42.90% for AVOA, DOA, FFA, GOA, and BSA respectively. In comparison to the best Ra values obtained using FFA, which was 1.185, it was discovered that the best Ra values provided by the other three algorithms (AVOA, DOA, GOA, and BSA) were, respectively, 1.16, 1.163, 1.158, and 1.159. As a result, it demonstrates that the three algorithms (AVOA, DOA, GOA, and BSA) were successful in determining the best optimum value in each the trial. However, AVOA, DOA and BSA were reported to have performed 9.38%, 9.14%, and7.42% better than the existing solutions in the literature (Balamurugan et al. [38]), respectively, while GOA and FFA saw enhancements of 9.53% and 9.45%, respectively. The variation of Z values for this multi-objective Problem with regard to iterations is shown in Figure 4.8. The best optimum value for MRR and KA optimization can be obtained by employing AVOA, according to a comparison of the single objective optimization process as a whole. The best algorithms to address this specific problem are AVOA and FFA, according to multi-objective optimization, which is one method of evaluating all algorithms' efficiency. As computational time is another variable to determine all algorithm’s efficiency then by determining and analysing Figure 4.9**,** we can say AVOA takes minimum time to execute this code. Hence from this point of view we can say AVOA is the best algorithm to solve this particular single objective as well as a multi-objective optimization problem. Optimization algorithms rank based on multi-objective optimization are given: AVOA> FFA > BSA > DOA > GOA. An attempt is made to determine the best-performing algorithm based on some performance measures among the metaheuristics under consideration. The AVOA method excels over the others for all responses in terms of having the highest MRR, the lowest KA and Ra, the lowest convergence time at the fastest processing speed, and the most accurate solutions with the least variability.

Table 4.6 Comparison of current results for multi-objective (Z) optimization

| **ALGORITHMS** | **RESPONCES** | **PARAMETERS** | | | **OPTIMUM (Z)** | ***IMPROVEMENT (%)*** | **min. Z** |
|---|---|---|---|---|---|---|---|
| | | JP (bar) | SOD (mm) | TS (mm/min) | | | |
| **Balamurugan et al. [38]** | *MRR* | 220 | 1 | 40 | 0.0215 | | |
| | *KA* | 220 | 1.68 | 20 | 0.359 | - | |
| | *Ra* | 220 | 1.32 | 20 | 1.28 | | |
| ***AVOA*** | *MRR* | | | | 0.0354 | 64.65 | |
| | *KA* | 220 | 1.4217 | 20 | 0.203 | 43.45 | 0.46 |
| | *Ra* | | | | 1.16 | 9.38 | |
| ***DOA*** | MRR | | | | 0.0338 | 57.21 | |
| | KA | 220 | 1.49 | 20.1 | 0.22 | 38.72 | 0.48 |
| | Ra | | | | 1.163 | 9.14 | |
| ***FFA*** | *MRR* | | | | 0.0362 | 68.37 | |
| | *KA* | 220 | 2 | 20 | 0.207 | 42.34 | 0.48 |
| | *Ra* | | | | 1.185 | 7.42 | |
| ***GOA*** | MRR | | | | 0.035 | 62.79 | |
| | KA | 220 | 1.418 | 20 | 0.205 | 42.90 | 0.47 |
| | Ra | | | | 1.158 | 9.53 | |
| ***BSA*** | *MRR* | | | | 0.035 | 62.79 | |
| | *KA* | 220 | 1.418 | 20 | 0.205 | 42.90 | 0.47 |
| | *Ra* | | | | 1.159 | 9.45 | |



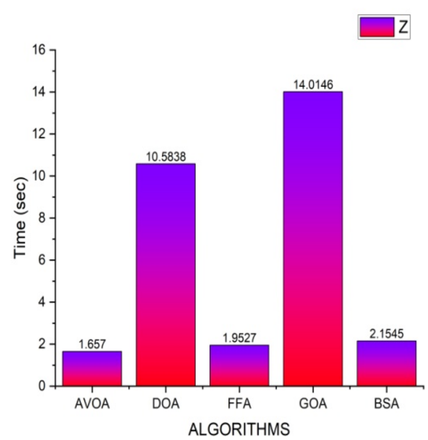




Figure 4.8 Convergence diagram between optimum Z and iterations

Figure 4.9 Bar chart between computation time and algorithms

### 4.3 Optimization of a WEDM Process

A special type of conventional electrical discharge machining (EDM) method is called WEDM. WEDM, an electro-thermal metal removal method, is frequently used in the automotive, aerospace, and nuclear sectors to machine complicated designs with irregular forms out of a variety of difficult-to-machine electrically conductive materials. In WEDM where the electrode is a moving wire constantly in motion. To achieve the required shape, size, and accuracy of the workpiece, the movement of the electrode is controlled dynamically. Generally, the electrode is made of thin brass, copper, or tungsten of diameter 0.05 - 0.3 mm. A mechanical device keeps the wire under tension to prevent incorrect forms. Material is removed during the WEDM process by erosion brought on by frequent, quick, and discrete spark discharges between the wire and the workpiece while it is submerged in a dielectric fluid. The dielectric fluid used is mainly de-ionized water or kerosene. A little amount of the workpiece is melted and vaporised by the spark and then removed by dielectric fluid [41]. The workpiece is mounted on the machine bed and is concurrently moved along the X and Y axes. Since the workpiece and the wire are not in direct contact, contact stresses are reduced. Tools and dies, as well as aeronautical, medical, and military equipment, as well as electrical equipment, are made by WEDM. Some of the necessary inputs are wire electrodes, workpieces, electrical, and non-electrical characteristics. It is crucial to select the appropriate input settings for improved performance. The workpiece can be finished with few or no additional procedures.

There are numerous methods for carrying out the experiments to optimize process parameters, including the following: Taguchi method, fuzzy, grey relation analysis, heuristics and metaheuristics. The most effective design tool for manipulative quality systems is the Taguchi Method [34]. This offers a practical, straightforward technique for raising the concert's quality and budget. Using the S/N and raw data, the ANOVA was utilised to identify significant and non-significant components. The ANOVA table and response curves were examined to find the ideal values of significant process parameters in terms of mean response characteristics [42].

In the WEDM process, the input process parameters used are the servo feed (SF), water pressure (WP), wire material, wire offset (Woff), wire tension (WT), wire feed rate (WFR), peak current (Ip), servo voltage (SV), pulse-off time (Toff), and pulse-on time (Ton), while the process responses used are the surface integrity aspects, wire wear rate, KW, MRR, and

SR. Since the turn of the millennium, the author community has made a number of attempts to increase material removal rate, cutting speed, minimise surface roughness, kerf, dimensional deviation, residual stress, recast layer thickness, wire wear rate, and improve surface integrity aspects by using various methods. These efforts have been made because WEDM can benefit greatly from these factors, which can also help to increase cost-effective payback.

**Jangra et al. [43]** used the Taguchi approach in conjunction with grey relational analysis to optimise the performance characteristic in wire electrical discharge machining. During the rough-cutting process, performance attributes such as dimensional lag, surface roughness, and cutting speed were examined. Using a mixed L18 orthogonal array, process parameters (WT, Ton, wire speed, Ip, and $T_{off}$) were examined.

**Shandilya et al. [44]** described the RSM and BPNN based mathematical modeling for average cutting speed of SiCp/6061 Al metal matrix composite (MMC) during WEDM. WFR, Toff, servo voltage (Vs), and Ton were the four WEDM parameters selected as the machining process parameters. RSM mathematical models of average cutting speed were used to compare the performance of the created ANN models.

**Garg et al. [45]** presented an experimental investigation of process parameters during the machining of a newly developed Al/10% ZrO2(p) metal matrix composite (MMC) such as WT, WFR, time between pulses, Vs, short pulse time, and pulse width. They discovered that the Vs, short pulse period, pulse width, and time between pulses were the most crucial variables determining cutting velocity.

**Yadav et al. [46]** examined the effects of process variables such as WFR, Ip, Ton, Vs, and Toff on cutting rate in the WEDM machining of AISI D2 steel Using the L27 Taguchi's orthogonal array and assessed the experimental data using ANOVA. It was discovered that the cutting rate is significantly influenced by the Vs, Toff, and Ton.

**Hewidy et al. [47]** created mathematical models based on the RSM, for relating the effects of Ip, duty factor, WT, and water pressure during the WEDM machining of Inconel-601 material on the rate of metal removal. According to their research, the MRR often rose when the Ip and WP values increased. This tendency held true up until the creation of arcing; nevertheless, after reaching a certain value, an increase in Ip results in a drop in MRR.

**Ghodsiyeh et al. [48]** used the design of experiment (DOE) approach to explore the behaviour of three control parameters when we apply WEDM on titanium alloy (Ti6Al4V). A brass wire electrode device with a 0.25 mm diameter was used to cut the sample. The MRR parameters were identified using the ANOVA method. They discovered that the MRR was strongly impacted by the Ip.

**Eltaweel et al. [49]** examined the optimum conditions for WEDM on CK45 steel. They consider the primary variables determining WEDM performance requirements were feeding speed, duty factor, water pressure, WT, and wire speed. The MRR was assessed in terms of process performance. They demonstrated that the MRR was most significantly influenced by the feeding speed and duty factor.

**Mohapatra et al. [50]** worked with the micro-structural analysis, and optimization of high-quality gears. WF, wire tension WT, Ton, and Toff are input parameters for gears. The goal was to explore the micro-structural characterization of stainless-steel fine pitch spur gears and optimise MRR. The gear has a pressure angle of 20°, a base diameter of 28.19mm, a face width of 5.23 mm, an addendum diameter of 36.66mm, and a pitch circle diameter of 30mm. Brass was used to make the 0.25mm-diameter wire. They discovered that low pulse on time and high pulse off time, wire feed rate, and wire tension all contributed to the highest MRR.

**Mohapatra et al. [51]** used suitable methods, such as Taguchi quality loss design technique and desirability with grey Taguchi technique, to optimise the process parameters of WFR, Ton, Vs, WT, Toff in order to achieve the highest MRR during the production of a fine pitch spur gear made of copper. They found that the loss function lowers as WFR, Ton, and WT rise. This was caused by the fact that larger sparks force the wire to move more quickly while carrying a greater load, which causes the loss function to decrease. A faster cutting speed due to an increase in Ton produces differing MRR values.

**Chakraborty and bose [52]** used entropy based grey relation analysis to identify the optimal cutting parameters: servo feed setting, Ton, Ip, corner angle, Vg, and Toff for MRR during WEDM process of Inconel-718 by Taguchi L27 OA design of experiments. They discovered from the ANOVA table that Ip, Tonne, Tofff, and Corner Angle have lower p-values (less than 0.05), making them the most efficient cutting parameters that influence performance parameters.

**Ramakrishnan et al. [53]** developed artificial neural network (ANN) models and multi-response optimization techniques to predict and choose the most excellent process parameters for WEDM process on Inconel-718 material to conduct experiments and brass wire of 0.25mm diameter as tool electrode. They conducted trials with various process parameters, including delay time, ignition current, Ton, and WFR, using the Taguchi L9 OA design of experiment approach. They discovered that wire feed rate SR is less significant than the importance of the Ton, delay time, and ignition current.

**Routara et al. [54]** presented the multi-response optimization of MRR and Ra under regulated process parameters, such as duty factor, gap voltage, wire feed rate, and gap current, using a planned experimental design based on a Taguchi L9 orthogonal array. In order to quantify performance that deviates from actual value, multi-response optimization approaches were used.

**Rajyalakshmi et al. [55]** have investigated the influence of machining parameters of wire-EDM during the machining of Inconel-825. The analysis of surface characteristics like surface roughness of Inconel-825 was carried out, and an excellent multiple linear regression model had been developed relating the process parameters, and machining performance indicating the suitability of the proposed model in predicting surface roughness.

**Kumar et al. [56]** created mathematical models based on the Taguchi method, for relating the effects of $T_{on}$, $T_{off}$, and WFR during the WEDM machining of SKD61 alloy on the MRR and Surface finish (SF). They found that while kerf breadth grows with flow rate and Vs, it reduces directly with increasing WT and wire running speed. With an increase in WT and running speed, the average wire amplitude drops. However, as the dielectric flow rate increases, the average wire amplitude rises. On average wire amplitude, servo voltage has little bearing.

**Mohammad et al. [57]** have investigated the influence of machining parameters of wire-EDM during the machining of EN34 steel. They created mathematical model based on the Taguchi method, for relating the effects of $T_{on}$, $T_{off}$, and WFR during the WEDM machining of SKD61 alloy on the MRR and S/N ratio. They found that the results of the experiment indicate that rotor speed was the parameter that significantly affects the various performance aspects. Factor rotor diameter, factor rotor opener speed, and factor navel type are in order of importance, with factor yarn linear density coming in second.

**Chaudhari et al. [58]** developed a model and optimized based on Heat-transfer search algorithm (HTS) and found that the most important factors impacting SR and MH were determined to be $T_{off}$ and current, although $T_{off}$ and current also had a substantial impact on

MRR. To determine the importance of input variables on the various output responses, contour plot analyses were performed. For four separate case studies that were taken into consideration, the HTS algorithm was proven to be effective at predicting and optimising the input values. Validation tests were used to validate the same. Between projected and actual values, there was a strong correlation.

**Magabe et al. [59]** developed NSGA-II models and optimization techniques to predict and choose the most excellent process parameters for WEDM process on Shape memory alloy (Ni 55.8 Ti) material to conduct experiments. They conducted trials with various process parameters, using the Taguchi design of experiment approach. They discovered that in terms of process parameters that correlate to various MRR and Ra, NSGA-II successfully predicts the solution. Some other literature reviews are also describing in table 4.7:

Table 4.7 Literature reviews

| SL. NO. | AUTHOR NAME& YEAR | MATERIAL USED | PARAMETERS | RESPONCES | OPTIMIZATION METHOD | CONCLUSIONS |
|---|---|---|---|---|---|---|
| 1 | Kumar et al. [60] | Composites of aluminium Al-SiC-B4C | $T_{on}$, $T_{off}$, V, I, WFR, B4C | Kerf width, cutting speed | RSM | Here the cutting speed was moderately impacted by the wire feed rate. The most heat energy was produced when the Ip was larger, but longer pulse durations allow the heat energy to reach the cross-sectional areas of the composites. |
| 2 | Patel et al. [61] | EN31 alloys steel | $T_{on}$, $T_{off}$, V, I, WFR, dielectric flushing pressure | Kerf width, MRR, SF | AHP and MOORA | Here the ranking of all 27 alternatives using the AHP/MOORA approach described and based on the weighted assessment value. They understand that from the executed experimental design, out of the 27 trials, experiment 19 or alternative number 19 provides the finest multi-performance features of the WEDM method. |
| 3 | Conde | Nitinol | | | SA | They proposed using an |

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|---|---|---|---|---|---|---|
| | etal. [62] | | | | | Elman-based Layer Recurrent Neural Network (LRNN) to forecast how accurate WEMD components will be. The findings show that there was a very low 6m average difference between network predictions and real components, indicating very strong network performance. Wire pathways with varying radius can be created by combining the generated LRNN's predictions with the SA optimization process, thereby minimising radial deviations caused by wire deformations. |
| 4 | Rao etal. [63] | Inconel -690 alloy | Ton, Toff, Ip, and Vs | MRR, SR | Modified Flower Pollination Algorithm (FPA), GA. PSO | They proposed a modified Flower Pollination algorithm (FPA) and developed a mathematical model to predict responses. ANOVA has been used to evaluate the percentage contributions of each process parameter on different responses. Ton, Ip, and their interaction greatly affect MRR and SR values. |
| 5 | Choudhuri etal. [64] | AISI H21 grade steel | $T_{on}$ | Wire Consumption( WC), SR | Fuzzy-logic and PSO | They evaluated the modelling, sensitivity analysis, and optimization capabilities of two methods, RSM and ANN, and predicted that ANN model is superior to RSM, demonstrating the advantage of ANN in mapping the nonlinear behaviour of the system. In order to optimise the WEDM process's process parameters, the PSO technique is combined with |

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| | | | | | | the ANN fitness function. As a result, they got Ton is a significant parameter as with its increase leads to an increase in SR and decrease in WC respectively. |
| 6 | Kulkarni et al. [65] | HCHCr, Brass wire | $T_{on}$, $T_{off}$, WFR, WT, LF, and UF | MRR | GWO | They used the well-known Taguchi approach for the optimization of a single answer using Taguchi's L25 Orthogonal Array. In order to optimise the WEDM process parameters. Ton has the most influence on MRR, whereas WF has the least effect. The following other factors that affect MRR in that order: Toff, upper flush (UF), lower flush (LF), and WT. |
| 7 | Nayak et al. [66] | Inconel 718 deep cryo-treated | Part Thickness, Taper angle (TA), Ton, Ip, WS, and WT | Angular error, Surface roughness, and Cutting speed | Maximum deviation theory, BPNN, and Levenberg Marquardt Algorithm (LMA) | They analysed the modelling by Taguchi design of experiment, taking six input characteristics into account: part thickness, TA, pulse length, discharge current, WS, and WT. They proposed an ANN model, which was then optimised to find the optimal parametric combination using a new meta-heuristic technique known as the bat method to determine the performance quality. |
| 8 | Tonday et al. [67] | Inconel 825 | SV, WT, Flushing Pressure (FP), and Ton | MRR, SR | LRT | They used the Taguchi method and regression analysis used to optimise the machining variables. They also constructed an equation to connect the input and output variables and then investigated that the output parameters MRR and Ra are |

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|---|---|---|---|---|---|---|
| | | | | | | both highly affected by the spark voltage. |
| 9 | Nain et al. [68] | Udimet-L605 | $T_{on}$, $T_{off}$, Ip, WFR, SV | MRR, SR | Support Vector Machine algorithm, Non-linear and multi-linear regression, GRA | They employed three models, including SVM methods based on PUK kernel, non-linear regression, and multi-linear regression, and they favoured the best model based on its assessment parameters examination of performance and graphs to analyse the variation between experimental and anticipated results. The rate of MRR and Ra are significantly impacted by the Ton. |
| 10 | Sen et al. [69] | Maraging steel 300 | $T_{on}$, $T_{off}$, Ip, WT, and SV | CS, SR, and WC | BPNN, FL, and Teaching Learning Based Optimization Algorithm (TLBO) | For efficient modelling, neural network models are developed to forecast cutting speed, surface roughness, and wire consumption. The multi-objective issue was reduced to a single objective using fuzzy logic, and parametric optimizations were carried out using both the GA technique and teaching-learning-based optimization. they stated that cutting speed and Ra significantly rise with increasing Ton and drastically decrease with increasing Toff, whilst wire consumption lowers. |
| 11 | Raj and Senthilvelan [70] | Titanium alloy (Ti6Al 4V) | $T_{on}$, $T_{off}$, WFR | MRR, SR | BBD, Desirability Function | The objective of their research is to improve the Wire-EDM cutting conditions for greater Ra and MRR. They used. BBD approach to design and optimized the problem. They found that the Ton and Toff are the crucial variables that affect Ra, while |

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|---|---|---|---|---|---|---|
| | | | | | | the Toff significantly affects MRR. |

### 4.3.1 Problem Statement

In order to determine the effects of different WEDM process parameters on responses, Gupta et al. [71] performed an experimental analysis on Predicting correlations in wire electrical discharge machining parameters of Pure and Heat-Treated Titanium Alloy (Ti-6Al-4V). According to their extensive range of uses in the industrial and commercial sectors, the machining properties of titanium alloy were researched. Unfortunately, there hasn't been much research on how hardening affects WEDM machining performance, and very few researchers provided a comparison of studies on how to optimise WEDM process parameters for the various heat-treated titanium alloys. In order to better understand how to machine hardened titanium alloys, systematic research was done, as well as a comparison of heat-treated titanium alloys. The Ti-6Al-4V as a workpiece material (Alloy) is cut with a WEDM of EUROCUT MARK-2, Electronica Machine Tools which is a five-axis numerical computer-controlled wire-cut EDM machine. The processed samples were rectangular 15 mm × 10 mm × 10 mm bars. The Cu wire electrode was attached to the negative terminal, and the work material was attached to the positive terminal. The machine's highest travel range was 350(x) × 400(y) × 250(z) mm. Additionally, Cu wire was utilised as the WEDM's electrode tools because copper is a good electrical conductor. The wire had a diameter of 0.25 mm. The WEDM process's machining factors, such as the servo reference voltage, wire feed, and wire tension, significantly affect the quality of the final product. There were 20 experiments conducted in total, and the cutting speeds of four heat-treated materials were collected. The servo reference voltage (Vs), wire feed (Fw), and wire tension (Tw) are the process parameters and the cutting speeds (Vcp, Vca, Vcq, Vch) are the response of all four materials (Pure Sample, Annealed, Oil Quenched, Hardened). For this machining operation, the various process parameters and their set levels are shown in Table 4.8. Based on data from experiments a Response Surface Regression was used to build the empirical equations for the four output responses using single objective optimization.

$$\mathbf{Vcp} = 2.13215 - 0.031765 \times V + 0.013180 \times F - 0.036167 \times T \quad (60)$$

$$\mathbf{Vca} = +2.96740 - 0.045313 \times V - 0.11445 \times F + 0.62678 \times T + 0.0025 \times V \times F - 0.008333 \times V \times T - 0.020833 \times F \times T - 0.00069765 \times V^2 \quad (61)$$

$\mathbf{Vcq} = 4.11016 - 0.12227 \times \mathrm{V} - 0.15806 \times \mathrm{F} + 0.43326 \times \mathrm{T} + 0.0065 \times \mathrm{V} \times \mathrm{F} - 0.0016667 \times \mathrm{V} \times \mathrm{T} - 0.045833 \times \mathrm{F} \times \mathrm{T} + 0.000560470 \times V^2$ (62)

$\mathbf{Vch} = 5.97863 - 0.20669 \times \mathrm{V} - 0.36113 \times \mathrm{F} - 0.51005 \times \mathrm{T} + 0.015000 \times \mathrm{V} \times \mathrm{F} - 0.016667 \times \mathrm{V} \times \mathrm{T} + 0.083333 \times \mathrm{F} \times \mathrm{T} + 0.00114253 \times V^2$ (63)

Table 4.8Variation levels of all parameters

| Sl. No | Process parameters | Symbol | Variation levels High | Low |
|---|---|---|---|---|
| 1 | Servo reference voltage (V) | Vs | 18 | 27 |
| 2 | wire feed (m/min) | Fw | 8 | 10 |
| 3 | wire tension (N) | Tw | 0.3 | 1.4 |

We employed a multi-objective optimization for our investigation by assigning some relative weights (importance). The model of this optimization problem will be as follows:

$$\mathrm{Objective}_1 = \mathrm{Maximize}\ (V_{CP}) \quad (64)$$

$$\mathrm{Objective}_2 = \mathrm{Maximize}\ (V_{\mathrm{CA}}) \quad (65)$$

$$\mathrm{Objective}_3 = \mathrm{Maximize}\ (V_{COQ}) \quad (66)$$

$$\mathrm{Objective}_1 = \mathrm{Maximize}\ (V_{CH}) \quad (67)$$

Total independent variables (IV) = Vs, Fw, Tw

By converting these three single objective functions to a single multi-objective function we make an equation and the equation is below:

$$O\mathrm{bjectives} : (\mathrm{Objective}_1, \mathrm{Objective}_2, \mathrm{Objective}_3\ and\ \mathrm{Objective}_4) = \mathrm{f(IV)}$$

For multi-objective optimization, we are using The Weighted Sum Method (WSM) to optimize this problem by combining all three responses and the model is as below:

$$Z = \left[(-)\frac{w_1 \times \mathrm{Vcp}}{\mathrm{Vcp}_{max}} + (-)\frac{w_2 \times \mathrm{Vca}}{\mathrm{Vca}_{max}} + (-)\frac{w_3 \times \mathrm{Vca}}{\mathrm{Vcoq}_{max}} + (-)\frac{w_4 \times \mathrm{Vch}}{\mathrm{Vch}_{max}}\right] \quad (68)$$

$$w_1 + w_2 + w_3 + w_4 = 1$$

Where, Z is a multi-objective function. $w_1$, $w_2$,$w_3$ and $w_4$ are the assigned weights for Vcp, Vca,Vca, and Vchrespectively. In the present problem we have taken the weight value for Vcp, Vca, Vca, and Vchare the same, each (1/4). An objective's weight is chosen based on its relative importance. For the purpose of resolving this optimization problem, self-developed python language programs have been used. The result or outcome of objective function from these algorithms depends on these essential parameters of every metaheuristic algorithm. In order to get the proper results, the value of these parameters must be properly chosen.

### 4.3.2 Results and Discussion

The entire function evaluation is maintained constant to make it easier to compare these five metaheuristic algorithms' performances. To take into consideration of the stochastic character of each method, it was independently executed 30 times. Each run had a max of 30 search agents, but the maximum number of iterations was set at 100, resulting in 3000 function evaluations. The five metaheuristics' convergence plots for an example trial with Vc optimization are shown in below figures.

Table 4.9 Single objective optimization values of all responses

| | | PARAMETERS | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **ALGORITHMS** | **RESPONCES** | Vs (V) | Fw (m/min) | T (N) | **Mean** | **SD** | **OPTIMUM** | ***IMPROVEMENT (%)*** |
| **Gupta et.al.** | PURE | 18.2 | 9 | 0.9 | | | 1.65 | 0 |
| | ANNELED | 18.2 | 9 | 0.9 | | | 1.75 | |
| | QUENCHED | 18.2 | 9 | 0.9 | | | 1.7 | |
| | HARDEND | 18.2 | 9 | 0.9 | | | 1.65 | |
| ***AVOA*** | PURE | 18 | 10 | 0.3 | 1.68 | 0.00 | 1.68 | **1.90** |
| | ANNELED | 18 | 8 | 1.4 | 1.80 | 0.01 | 1.80 | **3.10** |
| | QUENCHED | 18 | 8 | 1.4 | 1.81 | 0.02 | 1.81 | **6.71** |
| | HARDEND | 18 | 8 | 0.3 | 1.85 | 0.03 | 1.86 | **12.51** |
| ***DOA*** | PURE | 18.05 | 9.71 | 0.65 | 1.67 | 0.01 | 1.68 | 1.57 |
| | ANNELED | 18.27 | 9.24 | 1.23 | 1.79 | 0.01 | 1.79 | 2.44 |
| | QUENCHED | 18.09 | 8.04 | 1.26 | 1.79 | 0.01 | 1.79 | 5.52 |
| | HARDEND | 18.06 | 8.31 | 0.48 | 1.83 | 0.01 | 1.83 | 10.91 |
| ***FFA*** | PURE | 18.08 | 10.00 | 0.46 | 1.67 | 0.01 | 1.68 | 1.56 |
| | ANNELED | 18.00 | 8.00 | 1.40 | 1.80 | 0.01 | 1.80 | 3.09 |
| | QUENCHED | 18.00 | 8.00 | 1.40 | 1.81 | 0.02 | 1.81 | 6.70 |
| | HARDEND | 18.00 | 8.00 | 0.66 | 1.85 | 0.02 | 1.86 | 12.49 |
| ***GOA*** | PURE | 18 | 10 | 0.3 | 1.68 | 0.00 | 1.68 | 1.84 |
| | ANNELED | 18 | 10 | 1.4 | 1.80 | 0.01 | 1.80 | 3.08 |
| | QUENCHED | 18 | 10 | 1.4 | 1.80 | 0.02 | 1.81 | 6.69 |
| | HARDEND | 18 | 8 | 0.3 | 1.85 | 0.03 | 1.86 | 12.50 |
| **BSA** | PURE | 18 | 10 | 0.3 | 1.68 | 0.00 | 1.68 | 1.87 |
| | ANNELED | 18 | 8 | 1.4 | 1.80 | 0.01 | 1.80 | 3.05 |
| | QUENCHED | 18 | 8 | 1.4 | 1.81 | 0.01 | 1.81 | 6.69 |
| | HARDEND | 18 | 8 | 0.3 | 1.85 | 0.01 | 1.86 | 12.50 |

**For Vcp:**

In single objective optimization we found that Except for DOA and FFA, all the other algorithms exhibited a noticeable increase in the best solution after the first few iterations, as can be seen from the plot. For Vcp optimization, it was observed that the GOA, and BSA convergence trends were the same but AVOA gives slightly higher than others. For the first few iterations, BSA, AVOA, and GOA showed continuous improvement in the best solution search, but there was no improvement in them after 20 to 30 iterations (Figure 4.10).The algorithms' effectiveness as measured by 30 independent trials' statistical parameters is shown in Table 4.9. It was found that the optimum Vcp values provided by DOA and FFA were 1.675856 and 1.6757402, respectively, whereas the best value reported by the other three algorithms was 1.6813299 (AVOA), 1.68039 (GOA), and 1.680893299 (BSA). Therefore, it demonstrates that in all 30 trials, AVOA was successful in predicting the best-known value. As a result, the success rate—i.e., the proportion of times the method used by AVOA, DOA, FFA, GOA, and BSA reached the best or optimum value—was seen to be 1.90%, 1.57%, 1.56%, and 1.84%, 1.87%, respectively, compared to that used by Gupta et al. [71]. We presented the 100 iterations value for each algorithm as box plots (as box plots are powerful tools to find the repeatability of these algorithms) in Figure 4.11 to provide a more detailed study of the performance of these algorithms. It is crucial to examine this plot in conjunction with Table 4.9 in order to come to a clear conclusion and prevent misinterpretation. We can show that for Vcp, the highest variation for BSA and AVOA but for FFA there is minimal variation between the optimum and mean values. In comparison to the other algorithms, AVOA and BSA are observed to require fewer function evaluations to get to the optimal value, which indicates that they reached the optimal zone more quickly.

**For Vca:**

The algorithms' convergence while maximising the Vca is shown in Figure 4.12. All other algorithms, with the exception of DOA, were found to follow a similar trend of convergence. The fluctuation of Vca values in relation to iterations is shown in Figure 4.12. Except for DOA in Vca optimization, AVOA, GOA, FFA, and BSA can reach the best-optimization values like Vcp optimization. The algorithms' effectiveness as measured by 30 independent trials' statistical parameters is shown in Table 4.9 also. It was found that the optimum Vca values provided by DOA and BSA were 1.79269564and 1.80329, respectively, whereas the best value reported by the other three algorithms was 1.8042 (AVOA), 1.8041 (FFA), and

1.8039 (GOA). Therefore, these three algorithms (AVOA,GOA, FFA) were successful in predicting the best optimum value. Table 4.9 presents the optimal process parameters and the Vca as given by the various metaheuristic's algorithms and improved solutions of Gupta et.al. [71] by 2.44% and 3.05% for DOA and BSA, respectively. Each of the other algorithms (AVOA, FFA, GOA) were reported to have performed better than the current answers in the literature by 3.10%, 3.09%, and 3.08% respectively. Figures 4.11 and 4.13 have fairly similar overall patterns for the spread and distribution of the function evaluations. This suggests that whether the optimization problem is of the minimization or maximisation kind has no effect on the processes. Table 4.9 demonstrates that FFA, GOA, and BSA has a large variation for Vca, whereas there is little variation for DOA and AVOA between the optimal and mean values. DOA was shown to enter the optimal zone more quickly than the other algorithms as it shows a significant portion of the DOA is in the optimal zone. So, it required fewer function evaluations to obtain the optimal value.

**For Vcq:**

The convergence of the algorithms while maximizing the Vcq is shown in Figure 4.14. All other algorithms, with the exception of DOA, were found to exhibit a similar trend of convergence. AVOA provide us the best optimization value, with the exception of GOA, BSA and FFA.The statistical measures used for evaluating the algorithms' performance over 30 separate trials are also shown in Table 4.9. The best Vcq values given by the other four algorithms (AVOA, FFA, GOA and BSA) were, 1.81402, 1.81393, 1.8136, and 1.81364, respectively, compared to the optimum Vcq values obtained from DOA, which were 1.79382. It therefore proves that these four algorithms (AVOA, FFA, GOA, and BSA) were successful in predicting the best optimum value. However, AVOA, and FFA reported to have done better than the present answers in the literature (Gupta et.al. [71]) by 6.71%, and 6.70% respectively but for DOA, GOA and FFA, the solutions were improved by 5.52%, 6.69% and 6.69%, respectively. The distribution of the total function evaluations in a typical trial while maximizing Vcq is depicted in Figure 4.14 and 4.15.According to Table 4.9, there is not much variation for the variables AVOA, FFA, and GOA between the ideal and mean values, but there is a sizable variation for the variables DOA and BSA for Vcq. As a major amount of AVOA, FFA, and GOA are in the optimal zone, it can be seen that these algorithms enter it more quickly than the other algorithms. Consequently, it took less function evaluations to arrive at the optimal value.

**Vch result:**

The algorithms' convergence while maximising the Vch is shown in Figure 4.16. All other algorithms, with the exception of DOA, were found to follow a similar trend of convergence. The variation of Vca values in relation to iterations is shown in Figure 4.17. The best optimization values, such Vca and Vcq optimization, are provided by AVOA, GOA, FFA, and BSA, with the exception of DOA in Vca optimization. The best value produced by the other three methods was 1.856432 (AVOA), 1.85603 (FFA), 1.8563321 (BSA), and 1.85633 (GOA), however the optimal Vch values provided by DOA is 1.8299. This proves that all 30 trials were successful in predicting the best optimum value by these three algorithms (AVOA, GOA, FFA, BSA). Table 4.9 presents the optimal process parameters and the Vch values as given by the various metaheuristics algorithms and improved the solutions of Gupta et.al. [71] by 10.91% for DOA. Each of the other algorithms (AVOA, FFA, GOA, BSA)were reported to have performed better than the current answers in the literature by 12.51%, 12.49%, 12.50%and 12.50% respectively. This suggests that whether the optimization problem is of the minimization or maximisation kind has no effect on the processes. Table 4.9 demonstrates that FFA and GOA have a large variation for Vch, whereas there is little variation for DOA, BSA and AVOA between the optimal and mean values. DOA and BSA were shown to enter the optimal zone more quickly than the other algorithms as it shows a significant portion in the optimal zone. So, it required fewer function evaluations to obtain the optimal value.

**Overall result:**

It is significant to note that all five algorithms provided a range of optimal values for the process parameters. If we compare the overall optimization process (except time) then we can say that we can get the best optimum value for Vcp optimization by using AVOA and BSA and for Vca and Vcq optimization AVOA, GOA and FFA give us the best value. When we take execution time as another variable to determine all algorithms' efficiency then we can say AVOA is the best algorithm to solve this particular single objective optimization problem (Figure 4.18). Hence, we can conclude that AVOA is the best algorithm to find the best optimal value for this problem.

Optimization algorithms rank based on each optimization are given below:

Vcp (Response 1) = **AVOA**>BSA > GOA > DOA > FFA

Vca (Response 2) = **AVOA**> FFA > GOA >BSA >DOA

Vcq (Response 3) = **AVOA**> FFA > GOA = BSA > DOA

Vch (Response 4) = **AVOA**> GOA = BSA > FFA > DOA

Computational time= **AVOA**> FFA > BSA > DOA > GOA

This suggests that the search space for the goal function was multimodal. It is important to note that Gupta et al. [71] employed an RSM BBD-based model to determine the optimal values. The optimum values of responses obtained by Gupta et al. [71] were Vc = 1.65, 1.75, 1.70, 1.65 for pure, annealed, quenched, and hardened titanium alloys respectively. By using these five metaheuristic algorithms we found the best-known optimal parameters of these three responses were Vc = 1.68, 1.80, 1.81, 1.86 for pure, annealed, quenched, and hardened titanium alloys respectively. Each algorithm evaluated the same number of functions (30 ×100 = 3000), but the time it took for each algorithm to complete the optimization process varied. In order to do this, the CPU time for each algorithm was measured and averaged over 10 separate trials for each response. Figure 4.18 shows that, in terms of execution time, AVOA, FFA, and BSA were the three most expensive algorithms, whereas DOA was the least expensive.

**Friedman test:**

To evaluate whether or not there was a statistically significant difference between the AVOA, DOA, FFA, GOA, and BSA derived objective function values for the 30 runs, Friedman's non-parametric test was performed. The test is run using the instructions provided in this test by using Python 3.0. The table 4.10 displays the obtained results. Where Q means chi-square value, DF means degree of freedom. We can reject the null hypothesis (the comparing algorithms perform similarly) that the mean of the objective function is the same for each of the five algorithms because the p-value is less than 0.05.

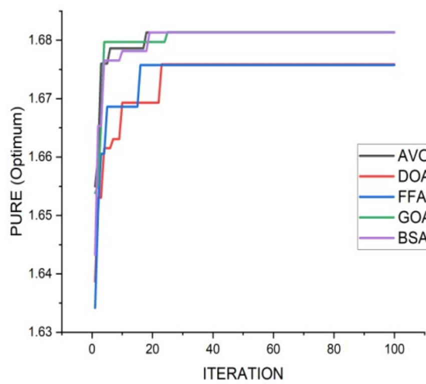
Since the p-value is less than 0.01, in both examples, indicates that the null hypothesis, $H_0$ is rejected at 1% level of significance for all four responses in both examples which means that objective function is the same for all five metaheuristic algorithms can be rejected. This is appropriate confirmation to conclude that the results obtained by AVOA, DOA, FFA, GOA, and BSA are statistically significant.

General Guidelines for rejection of null hypothesis: p-value $\leq 0.01$ then rejection of null hypothesis ($H_0$) at 1% level of significance. p-value $\leq 0.05$ then rejection of the null hypothesis ($H_0$) at 5% level of significance. p-value $\leq 0.1$ then rejection of the null hypothesis ($H_0$) at 10% level of significance. p-value $> 0.1$ then unable to reject the null hypothesis ($H_0$) because never consider more than 10% error or randomness in our test theory results. Additionally, the p-value indicated the existence of notable variations among the compared algorithms [40].

Table 4.10 Friedman test results

| Source | Q | DF | p-value |
|---|---|---|---|
| **MRR** | 252.2075 | 4 | 2.18E-53 |
| **KA** | 275.7947 | 4 | 1.80E-58 |
| **Ra** | 268.8526 | 4 | 5.64E-57 |

| **Algorithms** | | **AVOA** | **DOA** | **FFA** | **GOA** | **BSA** |
|---|---|---|---|---|---|---|
| Average | PURE | 1.680366 | 1.673364 | 1.673984 | 1.679598 | 1.680049 |
| Rank-Sum | | 157.5 | 413 | 487 | 269 | 173.5 |
| Avg. Rank | | 1.575 | 4.13 | 4.87 | 2.69 | 1.735 |
| **Rank** | | **1** | **4** | **5** | **3** | **2** |
| Average | ANNELED | 1.798851 | 1.79083 | 1.799672 | 1.799445 | 1.800791 |
| Rank-Sum | | 194.5 | 456 | 295 | 206.5 | 359 |
| Avg. Rank | | 1.945 | 4.56 | 2.95 | 2.065 | 4.47 |
| **Rank** | | **1** | **5** | **3** | **2** | **4** |
| Average | QUENCHED | 1.810204 | 1.791917 | 1.809696 | 1.800609 | 1.809658 |
| Rank-Sum | | 128 | 484 | 247.5 | 410.5 | 528 |
| Avg. Rank | | 1.28 | 4.84 | 2.475 | 4.105 | 5.28 |
| **Rank** | | **1** | **4** | **2** | **3** | **5** |
| Average | HARDENED | 1.853336 | 1.828793 | 1.854392 | 1.853583 | 1.853204 |
| Rank-Sum | | 134 | 495 | 371 | 246 | 254 |
| Avg. Rank | | 1.34 | 4.95 | 3.71 | 2.46 | 2.54 |
| **Rank** | | **1** | **5** | **4** | **2** | **3** |



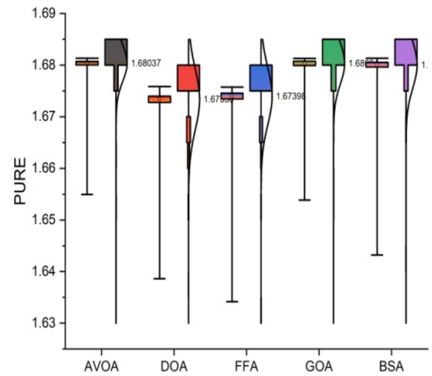
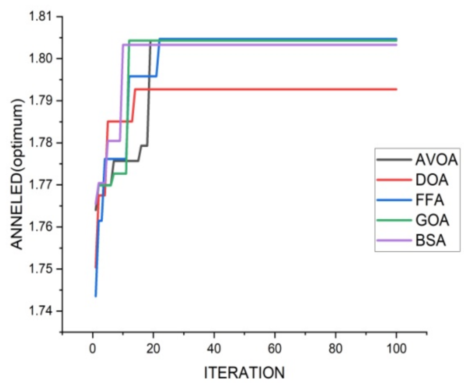




Figure 4.10 Convergence diagram between optimum Vcp and iterations

Figure 4.11 Box plots showing the range of all function analyses during Vcp optimization



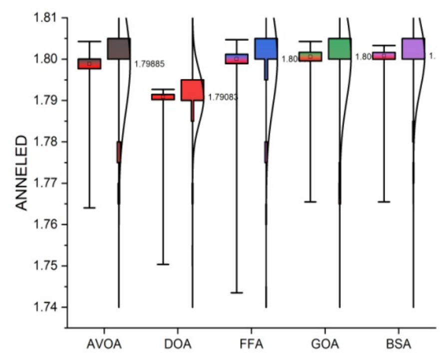
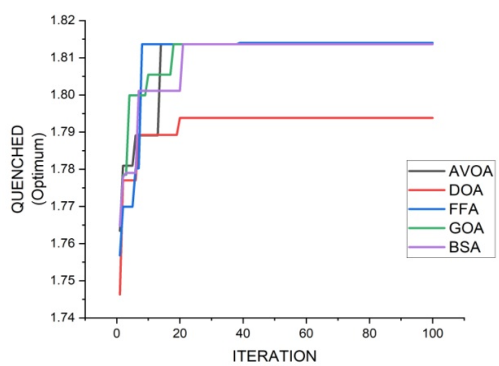




Figure 4.12 Convergence diagram between optimum Vca and iterations

Figure 4.13 Box plots showing the range of all function analyses during Vca optimization



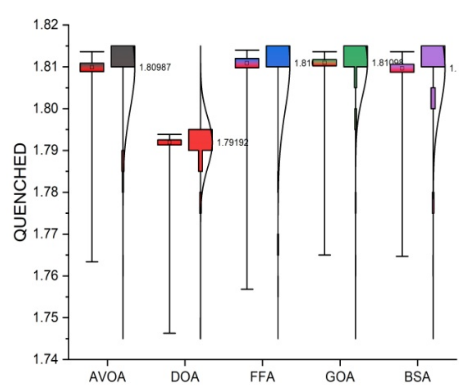
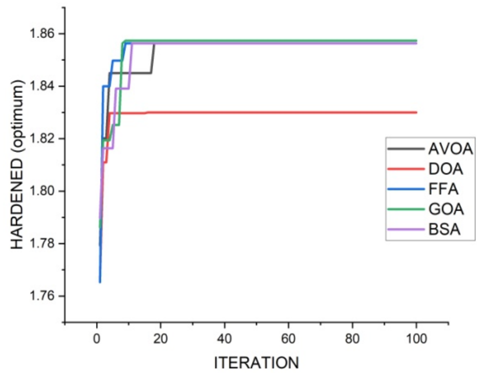




Figure 4.14 Convergence diagram between optimum Vcq and iterations

Figure 4.15 Box plots showing the range of all function analyses during Vcq optimization



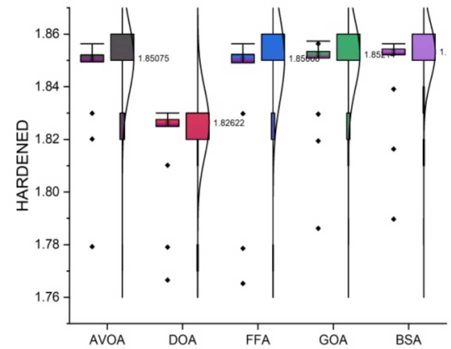




Figure 4.16 Convergence diagram between optimum Vch and iterations

Figure 4.17 Box plots showing the range of all function analyses during Vch optimization

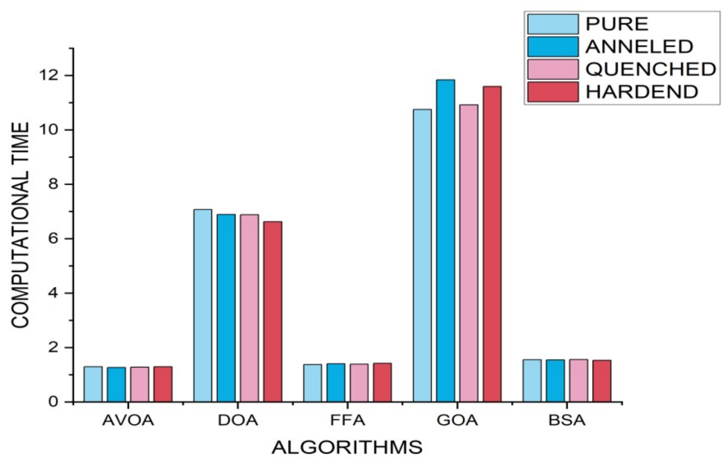



Figure 4.18 Bar chart between algorithms and computational time

**Multi-objective result:**

The results of the multi-objective optimization are presented in Table 4.11**,** and it demonstrate that all five algorithms are capable of generating better response values than those achieved by Gupta et al. [71]. For a fair comparison of all the algorithms, the search agents and iterations were limited to 30 and 100, respectively. The total number of function evaluations for each algorithm was limited to 3000 in order to provide a neutral comparison. Except for GOA and FFA, AVOA, DOA and BSA offer the best optimum optimization values for Vcp in multiobjective optimization (1.66, 1.659 and 1.659, respectively). This demonstrates that the optimal optimum value was correctly predicted by the algorithms AVOA in each of the 30 trials. The solutions for DOA, FFA, GOA and BSA were improved by 0.55%, 0.48%, 0.48% and 0.55%, respectively; even with AVOA was stated to have performed better than the current answers in the literature (Gupta et al. [71]) by 0.61%. If we compare between five metaheuristic algorithms, all but AVOA and DOA generate more substantial optimum results in Vca optimization but if we compare all with the current literature paper (Gupta et al. [71]) then it can be seen that all algorithms performed better than it by 1.71%, 1.71%, 1.14%, 1.14%, and 1.14% for AVOA, DOA, FFA, GOA, and BSA respectively. In comparison to the best Vca values obtained using AVOA and DOA, which were 1.78 each, it was discovered that the best Vcq values provided by the other four algorithms (FFA, DOA, GOA, and BSA) were, respectively, 1.78, 1.79, 1.78, and 1.79. As a result, it demonstrates that the four algorithms (FFA, DOA, GOA, and BSA) were successful in determining the best optimum value in each the trial. However, AVOA was reported to have performed 1.18% better than the existing solutions in the literature (Gupta et al. [71]), respectively, while DOA and BSA saw enhancements of 5.29%, each. If we compare between five metaheuristic algorithms with the current literature paper (Gupta et al. [71]), AVOA generates more substantial optimum results in Vch optimization of 3.03% and it can be seen that all algorithms performed better than it by 1.82%, 2.42%, 2.42%, and 1.82% for DOA, FFA, GOA, and BSA respectively. In comparison to the best Vch values obtained using AVOA, which was 1.7. Optimization algorithms rank based on each multi-objective optimization are given below:

Vcp (Response 1) = **AVOA**> BSA = DOA > GOA = FFA

Vca (Response 2) = **AVOA**= DOA > GOA = BSA = FFA

Vcq (Response 3) = **BSA = DOA**> FFA = GOA > AVOA

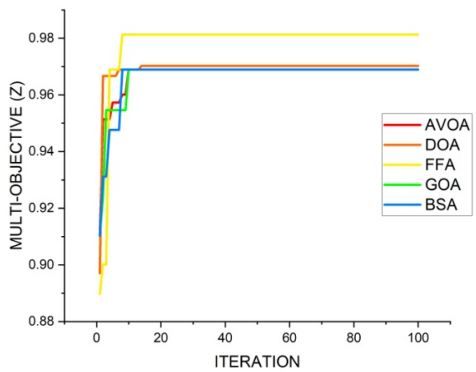
Vch (Response 4) = **AVOA**> FFA = GOA > DOA = BSA

Computational time = **AVOA**> FFA > BSA > DOA > GOA

Table 4.11 Comparison of current results for multi-objective (Z) optimization

| ALGORITHMS | RESPONSES | PARAMETERS Vs (V) | Fw (m/min) | T (N) | OPTIMUM (Z) | *IMPROVEMENT (%)* | Max. Z |
|---|---|---|---|---|---|---|---|
| **Gupta et al. [71]** | PURE | 18.2 | 9 | 0.9 | 1.65 | - | - |
| | ANNEALED | 18.2 | 9 | 0.9 | 1.75 | | |
| | QUENCHED | 18.2 | 9 | 0.9 | 1.7 | | |
| | HARDENED | 18.2 | 9 | 0.9 | 1.65 | | |
| ***AVOA*** | PURE | 18.1 | 8.43 | 1.38 | 1.66 | 0.61 | 0.968 |
| | ANNEALED | | | | 1.78 | 1.71 | |
| | QUENCHED | | | | 1.72 | 1.18 | |
| | HARDENED | | | | 1.7 | 3.03 | |
| ***DOA*** | PURE | 18.263 | 8 | 1.4 | 1.659 | 0.55 | 0.97 |
| | ANNEALED | | | | 1.78 | 1.71 | |
| | QUENCHED | | | | 1.79 | 5.29 | |
| | HARDEND | | | | 1.68 | 1.82 | |
| ***FFA*** | PURE | 18 | 8.3 | 1.4 | 1.658 | 0.48 | 0.981 |
| | ANNEALED | | | | 1.77 | 1.14 | |
| | QUENCHED | | | | 1.78 | 4.71 | |
| | HARDEND | | | | 1.69 | 2.42 | |
| ***GOA*** | PURE | 18 | 8.4 | 1.39 | 1.658 | 0.48 | 0.968 |
| | ANNELED | | | | 1.77 | 1.14 | |
| | QUENCHED | | | | 1.78 | 4.71 | |
| | HARDENED | | | | 1.69 | 2.42 | |
| ***BSA*** | PURE | 18 | 8.43 | 1.38 | 1.659 | 0.55 | 0.968 |
| | ANNEALED | | | | 1.77 | 1.14 | |
| | QUENCHED | | | | 1.79 | 5.29 | |
| | HARDENED | | | | 1.68 | 1.82 | |

The variation of Z values for this multi-objective Problem with regard to iterations is shown in Figure 4.19. The best optimum value for Vcp, Vch and Vca optimization can be obtained by employing AVOA but for Vcq we got best value for GOA and BSA, according to a comparison of the single objective optimization process as a whole. The best algorithms to address this specific problem according to multi-objective optimization's efficiency is AVOA.As computational time is another variable to determine all algorithm's efficiency then by comparing and analysing with Figure 4.20**,** we can say AVOA takes minimum time to execute this code. An attempt is made to determine the best-performing algorithm based on some performance measures among the metaheuristics under consideration. The AVOA method excels over the others for all responses in terms of having the highest Vcp, Vca, Vcq, and Vch, the lowest convergence time at the fastest processing speed, and the most accurate solutions with the least variability.



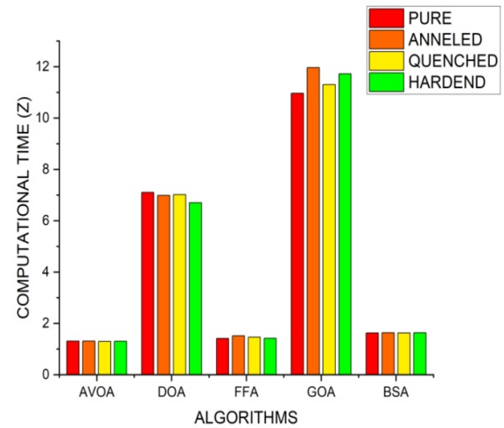




Figure 4.19 Convergence diagram between optimum Z and iterations

Figure 4.20 Bar chart between algorithms and computational time of multi-objective optimization

# CHAPTER 5

## OPTIMIZATION OF A FINANCIAL INVESTMENT METHOD

### 5.1 An Introduction of Decision Making

**"The age of Decision is as old as human civilization."**

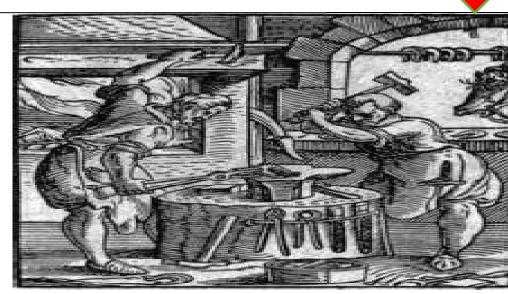
| | | |
|---|---|---|
|  | |  |
| **Stone age** | | **Pyramid age** |
|  | |  |
| **Victorian age** | | **Medieval age** |
|  | |  |
| **Lime age** | | **Lean age** |


Figure 5.1 Industrial revolution from Stone age to Industry 4.0 (De-Ra Metallica written by L.G Agricola in 1556).

The transition from the Stone Age to Industry 4.0 represents a vast evolutionary journey in human civilization marked by significant technological advancements and societal transformations. Here's a short note on this transition:

**The Stone Age**: The Stone Age was characterized by primitive tools made from stone, wood, and bone. Humans were hunter-gatherers, relying on their immediate environment for survival. This era lasted for thousands of years, witnessing slow progress and minimal technological development.

**Pyramid Age**: The Pyramid Age, also known as the Ancient Egyptian civilization, refers to a time when Egypt thrived and built impressive structures like the pyramids. This era, dating back to around 2600 BCE, showcased advanced architectural techniques, sophisticated hieroglyphic writing, and a complex belief system centred around pharaohs and the afterlife.

**Medieval Age:** The Medieval Age, also called the Middle Ages, spanned roughly from the 5th to the 15th century. It was characterized by feudalism, where societies were structured around nobility, serfs, and the Catholic Church. This period witnessed significant political, social, and economic changes, including the rise of powerful monarchies and the development of Gothic architecture.

**Victorian & Lime Age:** Stone was first used as a building material, which led to the beginning of lime manufacture. In Mesopotamia, the earliest organised production began about 10,000 years ago. The Victorian Age refers to the period during Queen Victoria's reign in the United Kingdom from 1837 to 1901. It was marked by industrialization, urbanization, and social reforms. The Victorian era saw tremendous advancements in technology, such as the steam engine and railways, as well as cultural shifts, including increased social awareness and the expansion of the British Empire.

**Lean Manufacturing Age**: Lean manufacturing, also known as lean production or simply "Lean," refers to a management philosophy and approach to manufacturing that focuses on minimizing waste, optimizing processes, and improving efficiency. The core principle of lean manufacturing is to eliminate activities and processes that do not add value to the final product or service. Lean manufacturing aims to create a culture of continuous improvement, where waste is identified and eliminated systematically [72]. The concept of Lean originated from the Toyota Production System (TPS), developed by the Japanese automotive manufacturer Toyota. TPS became renowned for its ability to achieve high levels of productivity, quality, and customer satisfaction while minimizing costs and lead times. Lean manufacturing techniques include various tools and practices, such as value stream mapping, 5S methodology, Just-in-Time (JIT) production, Kanban systems, and Kaizen events. These methodologies aim to create efficient and flexible production systems that respond to customer demand while eliminating waste in all forms, such as overproduction, waiting times, excess inventory, unnecessary transportation, and defects.

**Industry 4.0:** It represents the current phase of the industrial revolution, integrating digital technologies and automation into manufacturing processes. It encompasses advancements such as the Internet of Things (IoT), Artificial intelligence (AI), robotics, and big data analytics. Industry 4.0 aims to create interconnected, efficient, and smart factories that optimize production, reduce costs, and enable customization and flexibility in manufacturing. It is characterized by the fusion of physical and digital systems, paving the way for increased productivity, data-driven decision-making, and improved efficiency in various industries.

Since the dawn of civilization, mental processes have been extensively researched. However, until the relatively recent advent of cognitive and information sciences, these studies were primarily limited to language and logic because thought and language and logic are closely related [73]. The Greeks were enthusiastic students and proponents of reasoned thought. They valued logic in arguments and frequently utilized it to support their positions in court. They also began instructing their young people to think logically. Although Aristotle explored non-logical mechanisms of the mind, this portion of his theories has been lost, and the educational system of Western Civilization was built on logical schemas and modes of thinking.

### 5.1.1 Iceberg Decision Model:

> "Knowledge is Elixir for making holistic, eclectic and exotic decisions in order to survive comfortably and conveniently in highly volatile demand environment without jeopardizing the stability of the motherland"

The term "Iceberg" in the context of a cognitive mind of a decision analyst likely refers to the concept of the "Iceberg Model" or the "Iceberg Theory." This model is commonly used in decision analysis and psychology to describe the different levels of information and factors that influence human decision-making. The Iceberg Model suggests that decision-making processes involve both conscious and unconscious factors (Figure 5.2). It compares the decision-making process to an iceberg, where only a small portion is visible above the waterline, while the majority remains hidden beneath the surface [74].

In the context of a decision analyst's cognitive mind, the Iceberg Model implies that there are conscious aspects of decision-making that are easily observable, such as rational thinking, logical analysis, and explicit knowledge. However, there are also deeper, unconscious aspects that influence decision-making, including personal biases, emotions, values, and intuition. By acknowledging the hidden factors beneath the surface, decision analysts can strive for a more comprehensive understanding of the decision-making process. They can consider the influence of both explicit and implicit factors on their judgments, leading to more informed and effective decision-making. Tacit knowledge refers to the knowledge and expertise that individuals possess but may find challenging to articulate or transfer to others explicitly. It includes skills, insights, intuitions, and experiences that are deeply ingrained in an individual's subconscious. In the context of decision-making, tacit knowledge plays a crucial role and can be highly valuable for decision-makers. Here's why:

- **Experience-based insights**: Decision makers accumulate tacit knowledge through years of experience and exposure to various situations. This experiential knowledge helps decision-makers develop a deep understanding of patterns, nuances, and interconnections within their domain. It enables them to make informed judgments, recognize subtle cues, and anticipate potential outcomes based on their previous encounters.
- **Intuition and gut feelings**: Tacit knowledge often manifest as intuition or gut feelings. Decision makers may have a "hunch" or an instinctive sense about the right

course of action, even without being able to explicitly articulate the reasons behind it. These intuitive insights are often informed by tacit knowledge and can be valuable in situations where time is limited, and extensive analysis is not feasible.

- **Complex decision contexts**: Many decision contexts involve complexity, ambiguity, and uncertainty. Tacit knowledge can help decision-makers navigate these challenging situations by providing them with a deeper understanding and a broader perspective. It allows decision makers to draw on implicit information and make judgments based on a holistic view rather than relying solely on explicit, readily available data.
- **Informing judgment and risk assessment**: Tacit knowledge can influence the judgment and risk assessment abilities of decision makers. It helps them recognize subtle cues, assess the significance of factors beyond the surface level, and make more nuanced evaluations. Decision makers with tacit knowledge are often better equipped to handle novel or unique situations and can make well-informed decisions even in the absence of complete information.
- **Facilitating learning and adaptation:** Tacit knowledge contributes to continuous learning and adaptation. Decision makers can reflect on their past experiences, successes, and failures, and use their tacit knowledge to refine their decision-making approaches. Tacit knowledge allows decision makers to recognize patterns, identify potential pitfalls, and adjust their strategies accordingly.

In summary, tacit knowledge brings valuable insights, intuition, and expertise to the decision-making process. It complements explicit knowledge and helps decision makers navigate complex situations, make informed judgments, and adapt their approaches. Harnessing and leveraging tacit knowledge can enhance decision-making capabilities and contribute to better outcomes.

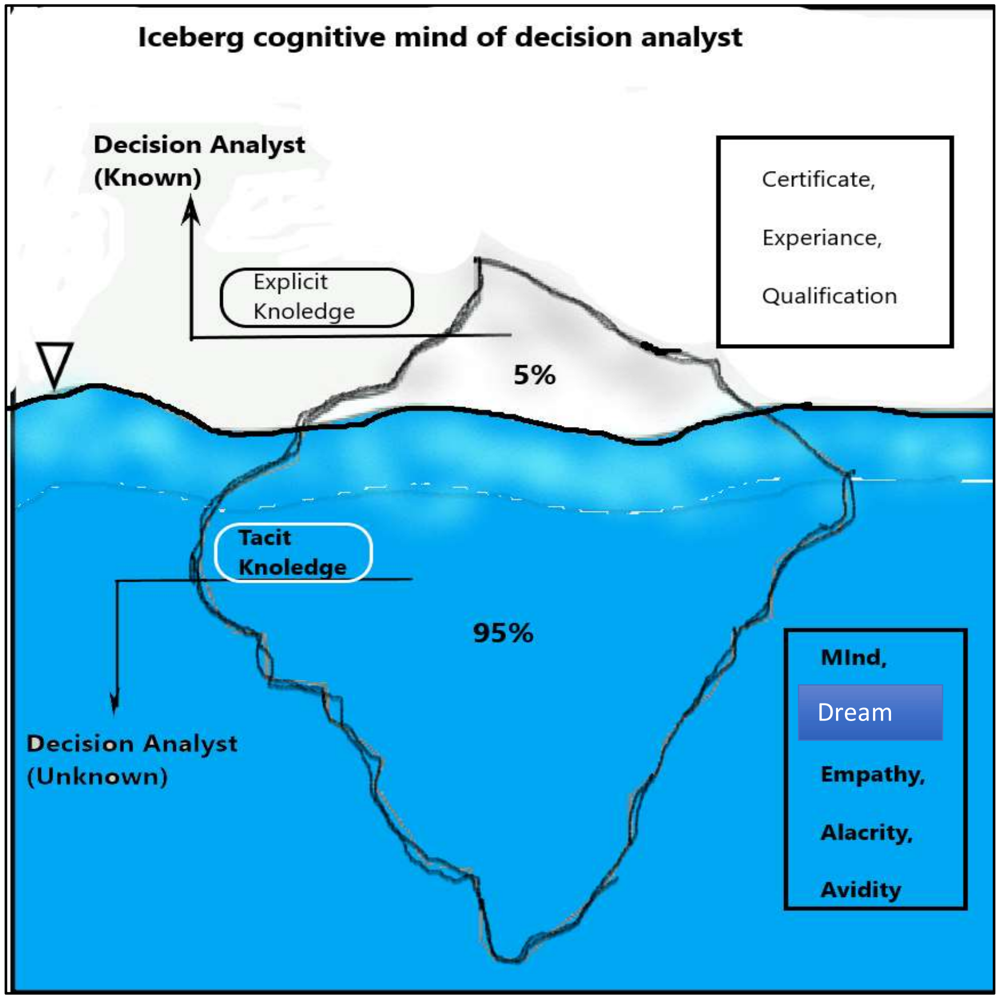



Figure 5.2 Iceberg cognitive mind of decision analyst

1. **Explicit knowledge** refers to knowledge that can be easily articulated, codified, and transferred through formal language and communication. It is the type of knowledge that can be documented, stored in databases, textbooks, or manuals. Explicit knowledge is typically structured, systematic, and easily shared among individuals or organizations. Examples of explicit knowledge include scientific principles, formulas, procedures, and factual information.
2. **Tacit knowledge**, on the other hand, is more subjective and personal. It refers to knowledge that is difficult to express or communicate in a formal and explicit manner. Tacit knowledge is deeply rooted in personal experiences, insights, intuitions, and skills that are gained through practice and observation. This type of knowledge is often context-specific and resides within an individual's mind. It is difficult to transfer tacit knowledge directly from one person to another, as it requires shared experiences and interaction.

Tacit knowledge is often crucial in areas such as expertise, creativity, and problem-solving. It encompasses skills, judgment, and know-how that individuals have developed through their unique experiences. Examples of tacit knowledge include riding a bicycle, playing a musical instrument, or recognizing patterns in complex data. Some examples of tacit knowledge are:

a) **Hedonic treadmill**: The hedonic treadmill refers to the tendency of people to quickly adapt to changes in their lives, whether positive or negative and return to their baseline level of happiness. It suggests that material possessions or external circumstances provide only temporary boosts to happiness, and individuals constantly seek new sources of happiness to maintain their well-being.

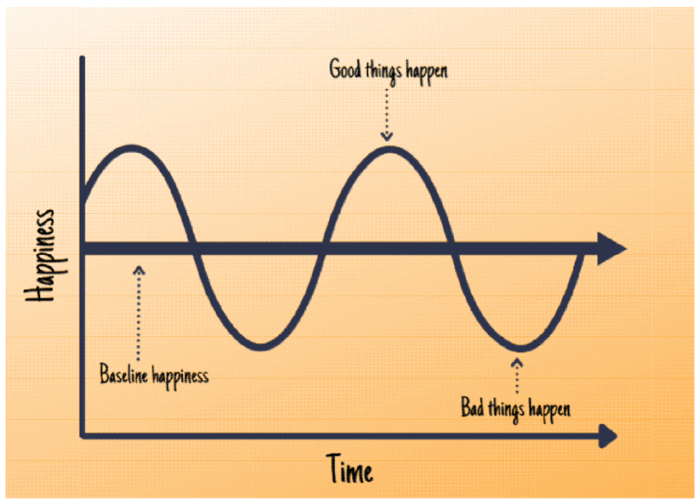



Figure 5.3 The hedonic treadmill [75]

b) **Pavlov experiment**: The Pavlov experiment, conducted by Ivan Pavlov in the late 19th century, focused on classical conditioning. Pavlov conditioned dogs to associate a neutral stimulus, such as the sound of a bell, with the presentation of food. Over time, the dogs began to salivate in response to the bell alone, even without the presence of food. This demonstrated that organisms can learn to associate a neutral stimulus with a reflex response, leading to a conditioned response. Pavlov's experiment laid the foundation for understanding how learning occurs through the association of stimuli in the environment. It had a significant impact on psychology, shaping the field of behaviourism and influencing our understanding of conditioning and learning processes.

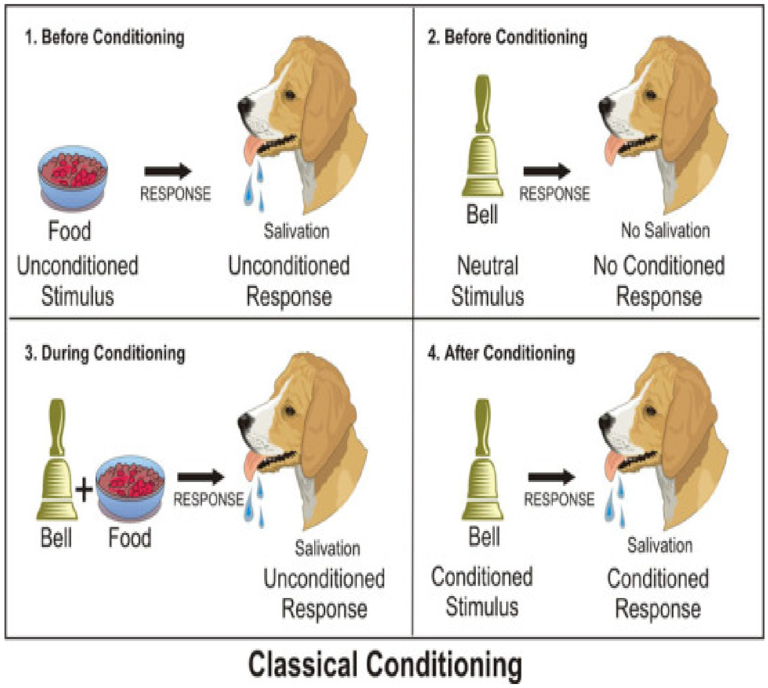



Figure 5.4 Pavlov's experiment [76]

c) **Neuroplasticity**: It can be influenced by various factors, including learning experiences, physical exercise, environmental enrichment, and even psychological factors like motivation and attention. By understanding and harnessing neuroplasticity, researchers and clinicians can develop interventions and rehabilitation strategies to promote brain health, enhance cognitive abilities, and aid in the recovery from neurological conditions. Neuroplasticity refers to the brain's ability to change and reorganize itself in response to learning and experience. In the context of decision making, neuroplasticity allows the brain to modify its decision-making processes based on feedback and new information. It enables us to learn from our experiences, adapt our decision-making strategies to changing circumstances, and form new connections in the brain to optimize our decision making. At the same time, neuroplasticity can also contribute to the development and reinforcement of cognitive biases that influence our decision making. Overall, neuroplasticity plays a crucial role in shaping how we make decisions and how we can improve our decision-making abilities through learning and adaptation.

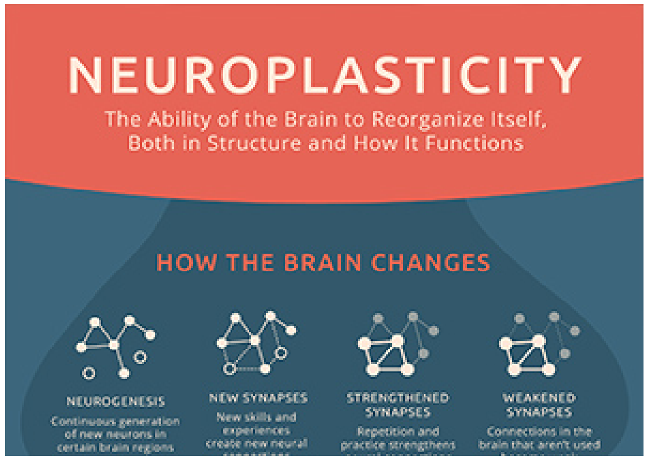



Figure 5.5 Neuroplasticity [77]

d) **Logo therapy**: Logotherapy, developed by psychiatrist Viktor Frankl, is a form of existential therapy that focuses on finding meaning and purpose in life. It posits that the search for meaning is a fundamental human motivation and that individuals can endure and overcome suffering if they find meaning in their experiences. Logo therapy emphasizes the importance of taking responsibility, making choices, and finding meaning in one's actions, relationships, and challenges. It encourages individuals to shift their perspective and develop a positive attitude towards life's difficulties. The therapy utilizes techniques such as Socratic dialogue, paradoxical intention, and dereflection to help clients gain insight, discover their values, and find meaning in their existence.

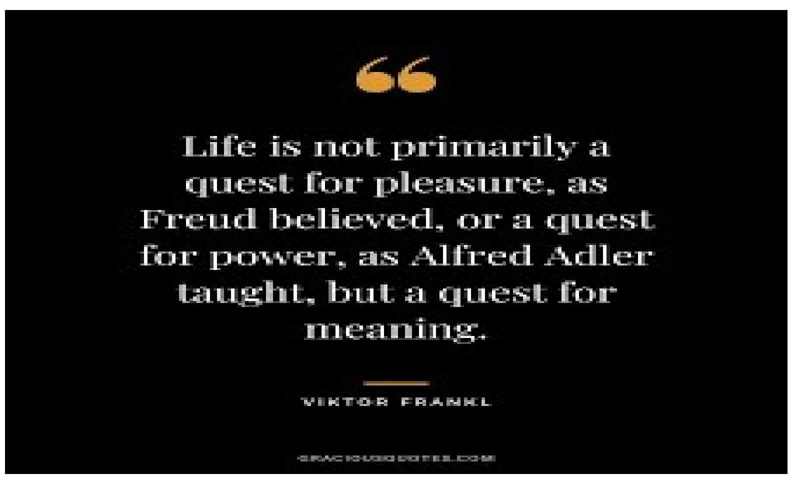



Figure 5.6 Logotherapy [78]

e) **Flow-diagram:** Mihaly Csikszentmihalyi is a psychologist known for his work on the concept of "flow," which describes a state of optimal experience characterized by complete absorption and focused engagement in an activity. Csikszentmihalyi proposed a flow diagram to illustrate the conditions that lead to the experience of flow. In decision making, a flow diagram helps break down the decision-making process into smaller, manageable steps. Each step is represented by a specific shape or symbol in the diagram, such as rectangles for actions or decisions, diamonds for decision points, and arrows to show the flow of the process. The flow diagram begins with an initial step or starting point and proceeds through various decision points, where choices or alternatives need to be considered. At each decision point, the flow diagram branches out based on different outcomes or paths that can be taken. This visual representation helps individuals analyse the consequences of each choice and consider the possible outcomes.

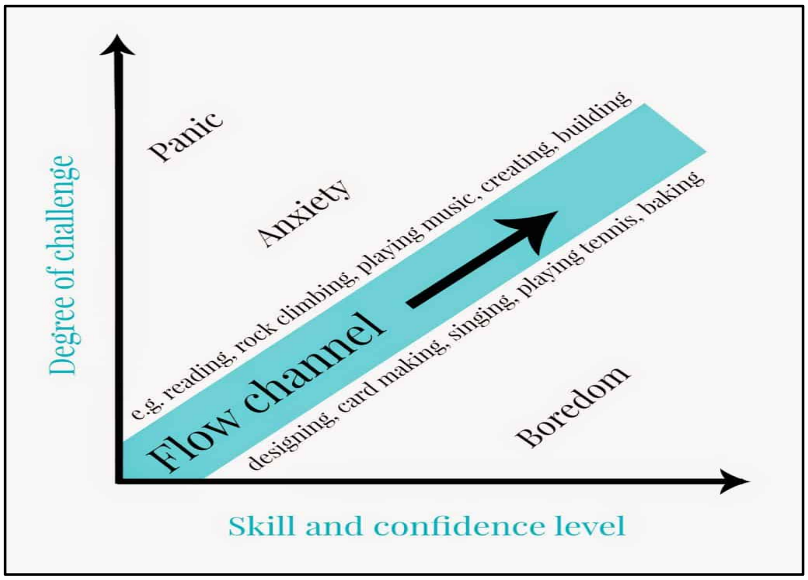



Figure 5.7 Flow diagram

Overall, neuroplasticity highlights the dynamic nature of the brain and its potential for change and adaptation throughout life. It emphasizes the importance of creating stimulating environments, engaging in lifelong learning, and adopting healthy habits to support brain plasticity and overall cognitive well-being. The flow diagram by Csikszentmihalyi provides a visual representation of the conditions necessary for experiencing flow and highlights the importance of the balance between challenge and skill, concentration, a sense of control, intrinsic motivation, and goal clarity. Achieving flow can lead to enhanced performance, increased creativity, and a greater sense of well-being in various domains of life.

Both explicit and tacit knowledge are important in various domains. Explicit knowledge forms the foundation for learning and education, while tacit knowledge contributes to practical skills and expertise. Effective knowledge management involves a balance between capturing and sharing explicit knowledge and creating opportunities for individuals to exchange and develop tacit knowledge through collaboration, experience sharing, and on-the-job learning.

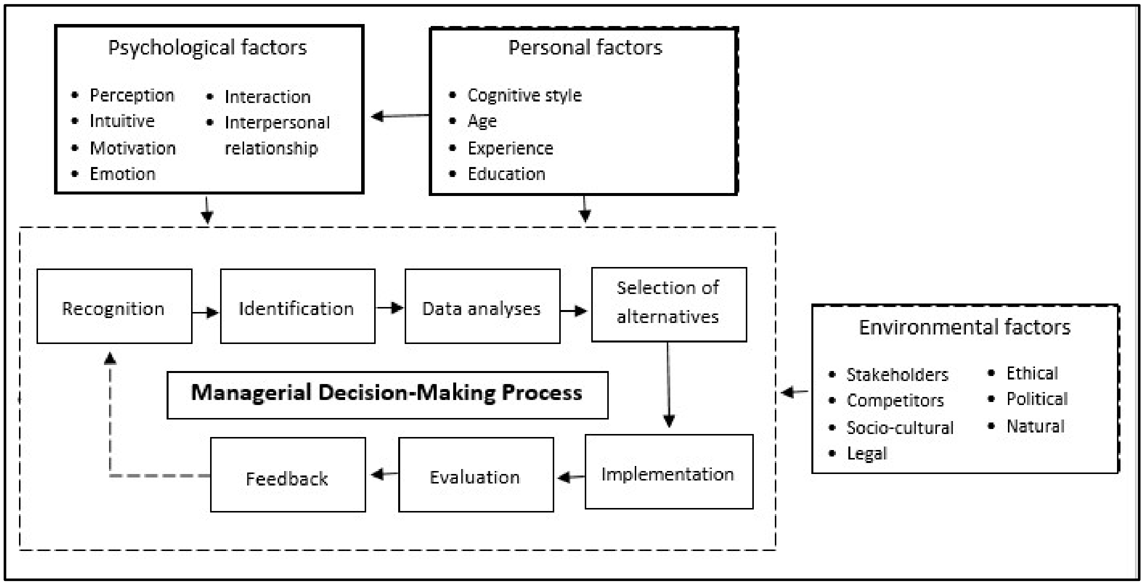



Figure 5.8 The factors and processes in managerial decision-making

### 5.1.2 **Importance of DM in Fuzzy Algorithms**:

According to present scenario, when a manager handles an information system needing to make a decision, the inputs necessary to make a decision come from two sources. One is what we call human judgement. It is the experience, the expertise, the feelings and the knowledge that constitute judgement. The second is what we call information - a processed set of data that a person needs to reach a decision. A decision can be made or by employing a certain amount of information and a particular technique. In these cases, the decision-maker has been replaced by a computer. At the same time, it is a well-known fact that any '(DM)'s ability to make precise but significantly certain statements about complex external world decreases as their complexity increases. Precision and certainty seem to be incompatible. This principle explains the considerable intellectual investment required to approximate reasoning. The key idea is the representation of the fact as an evaluation and of a rule as a transformation of evaluation. Such an evaluation is a function, as is the fuzzy set proposed by Zadeh [79]**,** As classical set theory is governed by a logic that permits a proposition to possess one of only two values: true or false. This logic does not accord well with the need to represent vague concepts. We see things in shades of gray: not only in black and white. The key idea of fuzzy set theory is that an element has a degree of membership in a fuzzy set. Thus, a proposition need not be simply true or false, but may be partially true to any degree. We usually assume that this degree is a real number in the interval [0,1]. Optimization is viewed as a movement in an order structure. If optimization is described by a set and a function, there are two possible ways to consider it. If the set is seen as an object and the function as a transformation, then exploring sets means ranking elements in time and a two valued logic is sufficient for observation. Multi-criteria optimization, however, requires a framework in which the pair set-function is an object. Substituting the world of sets with the world of their evaluations leads to a partial membership and to a continuous logic by which to perceive that membership. In a multi-criteria fuzzy optimization problem, the criteria are formulated as fuzzy sets. Any fuzzy optimization problem is a movement in the structure of predications towards a synthesis. Finding "the best" solution means finding a particular fuzzy set in this structure.

In various real life decision situations, particularly to financial decision systems fuzzy programming has a great role as both structured and unstructured decision-making are taken care of. Fuzzy multi-objective programming at one end has a multi-criterion modelling role and at the same time can incorporate the knowledge-base of the DM in judgement stage. Instead of being described as a "compromise programming" methodology, it is such a flexible programming approach in contrast to conventional multi-objective programming. As knowledge is a key component, the emphasis is therefore more on expert systems than just decision-making systems. It can be targeted as a "intelligent decision system" that takes into account plenty, competing, unrelated, and even inaccurate criteria.

### 5.1.3 The Behavioral Pattern of Every DM to get a Consilient and Confluent Result:

The behavioural pattern of every decision maker aims to achieve a consilient and confluent result by considering multiple perspectives, seeking diverse input, and fostering collaboration. Decision makers should demonstrate open-mindedness, empathy, and effective communication to encourage the sharing of ideas and knowledge. They should actively listen, critically analyse information, and manage biases to make well-informed decisions. By embracing a holistic and integrative approach, decision makers can synthesize various viewpoints, bridge gaps, and facilitate consensus, leading to a harmonious and cohesive outcome that reflects the collective wisdom of the group (Figure 5.9).

The behavioral patterns of a decision-maker refer to the tendencies, biases, and cognitive processes that influence how they approach and make decisions. It is important to note that these behavioral patterns are not exhaustive and can vary among individuals. Being aware of these patterns can help decision-makers recognize and mitigate their impact, enabling them to make more objective and rational decisions. Additionally, incorporating diverse perspectives, seeking feedback, and employing decision-making frameworks can help counteract these biases and improve decision outcomes. We put behavioral pattern in table 5.1.

$$B = f(C, O)$$

Where, B = Behavior of decision analyst, f = explicit function, C = covert, O = overt.

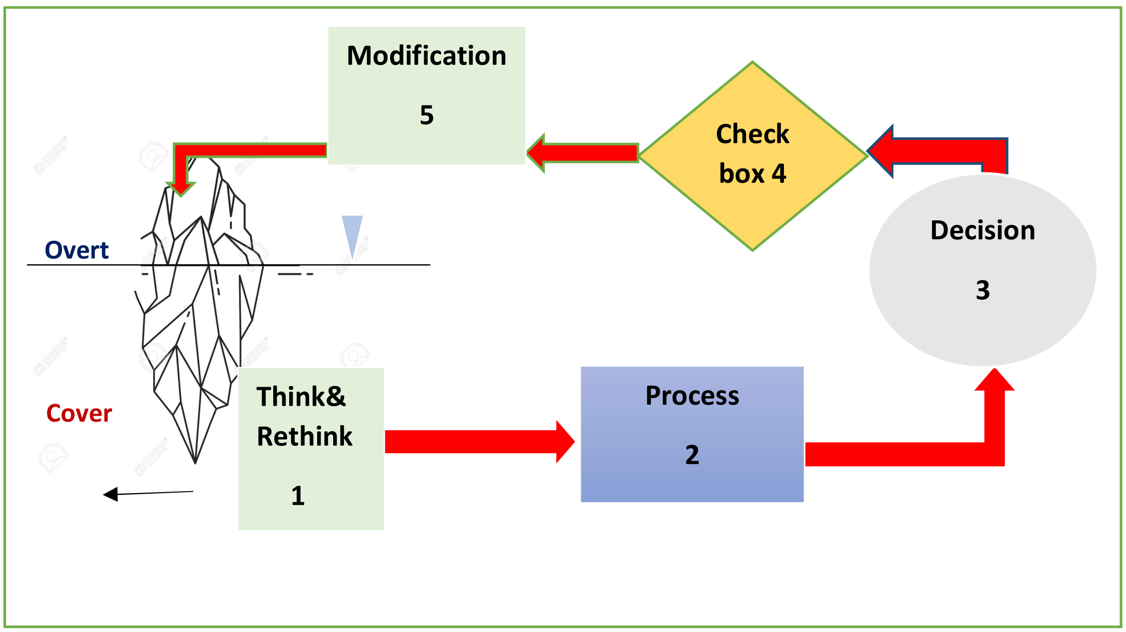



Figure 5.9 Behavioral pattern of every decision maker

**"Every product is produced thrice"**: Visualization, Design, Manufacturing (Socrates, 469-399BC).

$$\boldsymbol{Action = f\ (Thought)}$$

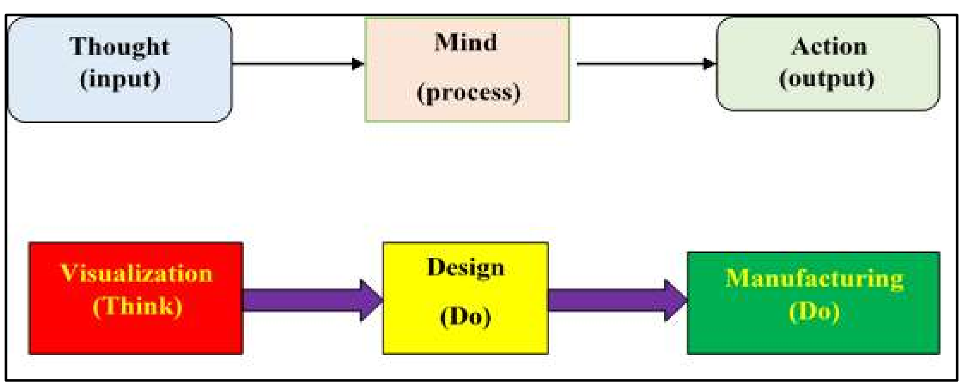
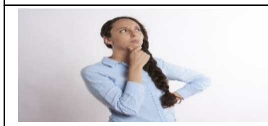
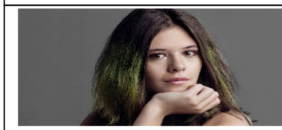
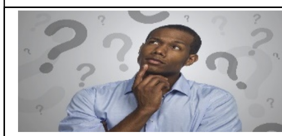



**Figure 5.10** Input-output relation based on the cognitive science of mind.

Table-5.1 Behavioural patterns are described

| Decision maker | Gender | Educational Qualification | Experience | Temperament |
|---|---|---|---|---|
|  | Male | M. E | 10 | High |
|  | Female | Undergraduate | 15 | Low |
|  | Transgender | MBA | 17 | Medium |
|  | Male | PhD | 11 | Excellent |

Research on the behavioural patterns of decision-makers based on gender, education, and experience has provided valuable insights into understanding how these factors influence decision-making processes and outcomes.

**Gender Differences:** Studies have found that men tend to exhibit more confident and risk-taking decision-making styles compared to women, who are often more cautious and risk-averse. Women generally perceive risks to be higher than men and are more likely to consider potential losses and negative outcomes in their decision-making. Women often show a greater tendency towards collaboration and consensus-building in decision-making, while men may display more assertiveness and competitiveness.

**Education Level:** Individuals with higher education levels, such as advanced degrees, tend to possess stronger analytical skills, enabling them to engage in more rational and systematic decision-making processes. Highly educated decision-makers may have better information processing capabilities, allowing them to gather and evaluate information more effectively. Despite their education levels, decision-makers may still be susceptible to cognitive biases, such as confirmation bias or overconfidence, which can influence their judgment.

**Experience:** Decision-makers with more experience in a particular domain often exhibit higher levels of expertise, leading to better decision-making performance. Experienced decision-makers may have a more nuanced understanding of risks, leading to better risk assessment and potentially more balanced risk-taking behaviours. Decision-makers with extensive experience may rely on heuristics or rules of thumb, developed over time, which can positively or negatively impact their decision outcomes.

Interventions such as training programs and diversity initiatives can play a crucial role in mitigating biases and enhancing decision-making effectiveness across diverse groups. Training programs can provide decision makers with skills to recognize and address cognitive biases, improve communication, and foster inclusive decision-making environments. Diversity initiatives aim to create a culture that values diverse perspectives, experiences, and backgrounds, promoting innovation and better decision outcomes.

### 5.2 Fuzzy Logic For Optimization

Fuzzy optimization is a branch of optimization theory that incorporates fuzzy logic to handle problems with uncertain or imprecise information. It combines optimization techniques with fuzzy set theory to address real-world problems that involve vague or ambiguous constraints and objectives. In traditional optimization problems, precise mathematical models are used to represent the variables, constraints, and objectives. However, in many real-life situations, the available information is often incomplete or uncertain. Fuzzy optimization provides a framework to deal with such situations by allowing for fuzzy constraints and objectives. In fuzzy optimization, fuzzy sets are used to represent imprecise or uncertain data. Fuzzy sets generalize classical crisp sets by allowing elements to have degrees of membership ranging between 0 and 1, representing the degree of belongingness of an element to a set. Fuzzy optimization models incorporate fuzzy sets in the definition of variables, constraints, and objectives, enabling the optimization process to handle uncertainty and imprecision. Fuzzy optimization techniques can be applied to various fields, including engineering, finance, decision-making, logistics, and resource allocation. Some common applications of fuzzy optimization include portfolio optimization, production planning, supply chain management, and scheduling problems. To solve fuzzy optimization problems, various algorithms and approaches have been developed, including fuzzy linear programming, fuzzy nonlinear programming, fuzzy dynamic programming, genetic algorithms, particle swarm optimization, and evolutionary algorithms. These methods aim to find optimal or near-optimal solutions that satisfy the fuzzy constraints and objectives. Overall, fuzzy optimization provides a valuable framework for dealing with optimization problems in situations where uncertainty and imprecision are present, allowing decision-makers to handle real-world complexities effectively [80]. As an application-oriented fuzzy technology, that is, as a tool for modelling, problem-solving, and data mining that has been proven superior to existing methods in many cases and an attractive add-on to classical approaches in the case. Lofti Zadeh's fuzzy set theory, which forms the basis of conventional fuzzy sets, has been widely adopted and applied in various fields. However, it is important to acknowledge some of the drawbacks associated with this theory:

1. **Lack of Representation for Ambiguity:** Conventional fuzzy sets primarily focus on representing uncertainty through membership degrees, but they do not explicitly address ambiguity or the lack of knowledge regarding an element's membership. Ambiguity arises when there is hesitancy or vagueness in assigning an element to a set. Conventional fuzzy sets do not provide a direct mechanism to represent ambiguity or the lack of confidence in membership assignments.
2. **Difficulty in Handling Granularity**: Conventional fuzzy sets are limited in their ability to capture fine-grained distinctions or granularity in membership degrees. The representation of membership degrees between 0 and 1 may not be sufficient in situations that require more precise assessments. For example, when dealing with subjective opinions or preferences, conventional fuzzy sets may not accurately capture the subtle differences in degrees of membership.
3. **Inability to Represent Non-Membership:** Conventional fuzzy sets primarily focus on membership degrees, but they do not explicitly represent non-membership or the degree to which an element does not belong to a set. This limitation can be problematic in scenarios where non-membership information is crucial for decision-making or where the absence of an element from a set carries significance.
4. **Limited Mathematical Framework:** The mathematical foundations of conventional fuzzy sets are based on set theory and predicate logic. While these foundations provide a solid basis for reasoning and analysis, they may not be flexible enough to handle more complex and advanced mathematical operations. This limitation can restrict the application of conventional fuzzy sets in certain areas where more sophisticated mathematical frameworks are required.
5. **Subjectivity in Membership Assignments:** Membership degrees in conventional fuzzy sets are typically assigned subjectively by experts or decision-makers based on their intuition or knowledge. The subjectivity in membership assignments can introduce bias or inconsistency, leading to potential challenges in interpretation and application.

   It is important to note that these drawbacks should not undermine the contributions and practical applications of conventional fuzzy sets. They highlight areas where more advanced fuzzy set theories, such as PF, IF, or NF sets, have emerged to address these limitations and provide more comprehensive and refined representations of uncertainty, ambiguity, and non-membership.

**Relation between fuzzy logic with iceberg theory:** The Fuzzy Decision-Making process and the Iceberg Theory are not directly related concepts. However, we can explore how they both involve hidden or underlying factors that influence decision-making. The Fuzzy Decision-Making process is a methodology used when decisions involve uncertain or imprecise information. It allows decision-makers to deal with ambiguity and vagueness by using fuzzy logic, which allows for degrees of membership or uncertainty. The process considers multiple criteria and factors, assigning degrees of importance and evaluating options based on their overall desirability. In relation to the Iceberg Theory, both concepts recognize that decision-making involves more than what is immediately visible or apparent on the surface. The Iceberg Theory suggests that there are hidden or underlying factors at play, while Fuzzy Decision-Making acknowledges that decisions often rely on uncertain or imprecise information. Both concepts highlight the need to delve deeper, beyond surface-level information, to gain a more comprehensive understanding of the situation. When making decisions using the Fuzzy Decision-Making process, decision-makers consider a range of factors and criteria, including those that may not be readily apparent or quantifiable. Just as the Iceberg Theory encourages the exploration of hidden factors, the Fuzzy Decision-Making process involves assessing various criteria, often incorporating subjective judgments and considering multiple perspectives.

In this sense, the Iceberg Theory and the Fuzzy Decision-Making process share a common thread of emphasizing the importance of going beyond superficial information. They both encourage decision-makers to explore hidden or uncertain factors, delve into underlying complexities, and consider a broader range of criteria and perspectives to make more informed and comprehensive decisions.

### 5.2.1 **Types of Fuzzy Optimization:**

Fuzzy algorithms are computational methods based on fuzzy logic that deal with uncertainty and imprecision. They have gained significant attention in various domains due to their ability to handle complex and ambiguous data. This literature review aims to provide a comprehensive overview of various fuzzy algorithms, highlighting their applications, advantages, limitations, and recent developments. By analyzing a wide range of scholarly articles and research papers, this review synthesizes the existing knowledge on fuzzy algorithms and identifies potential future research directions.

**Fuzzy Clustering Algorithms**:

- Fuzzy C-Means (FCM) algorithm and its variants
- Possibilistic clustering algorithms
- Fuzzy clustering algorithms for image segmentation

**Fuzzy decision trees and rule-based classifiers**:

- Fuzzy support vector machines
- Fuzzy neural networks

**Fuzzy Optimization Algorithms:**

- Fuzzy particle swarm optimization
- Fuzzy genetic algorithms
- Hybrid fuzzy optimization algorithms

**Fuzzy Time Series Forecasting Algorithms**:

- Fuzzy logic-based forecasting models
- Fuzzy clustering-based time series forecasting
- Fuzzy neural networks for time series prediction

**Applications of Fuzzy Algorithms:**

- Fuzzy algorithms in pattern recognition and image processing
- Fuzzy algorithms in data mining and knowledge discovery
- Fuzzy algorithms in control systems and robotics
- Fuzzy algorithms in decision support systems

Fuzzy sets are a fundamental concept in fuzzy logic, which allows for the representation and handling of uncertainty and imprecision in data. There are several types of fuzzy sets that are commonly used. Here are some of the main types:

1. **Intuitionistic Fuzzy Set (IFS)**: IFS is an extension of fuzzy set theory that was introduced by Krassimir Atanassov in the 1980s. While traditional fuzzy sets allow for degrees of membership and non-membership, intuitionistic fuzzy sets introduce an additional parameter called the degree of hesitancy or indeterminacy. This degree represents the hesitation or uncertainty about the membership status of an element in a set [81].

An intuitionistic fuzzy set is defined by three membership functions: membership function ($\mu$), non-membership function ($\nu$), and hesitation function ($\lambda$). These functions assign degrees to each element, indicating the degree of membership, non-membership, and hesitancy, respectively. Mathematically, for a given set X, an intuitionistic fuzzy set A is represented as:

$A = \{(x, \mu A(x), \nu A(x), \lambda A(x)) \mid x \in X\}$

where $\mu A(x)$ represents the degree of membership of x in A, $\nu A(x)$ represents the degree of non-membership of x in A, and $\lambda A(x)$ represents the degree of hesitancy about the membership status of x in A.

The values of $\mu A(x)$, $\nu A(x)$, and $\lambda A(x)$ range between 0 and 1, where 0 indicates no membership, no non-membership, or no hesitancy, and 1 indicates complete membership, complete non-membership, or complete hesitancy, respectively.

Intuitionistic fuzzy sets provide a more flexible representation of uncertainty compared to traditional fuzzy sets. They can handle situations where the degree of hesitation or uncertainty about the membership of an element is significant. This makes them useful in decision-making, pattern recognition, expert systems, and other applications where imprecise or uncertain information needs to be modeled. Operations such as union, intersection, and complement can be defined for intuitionistic fuzzy sets based on their membership, non-membership, and hesitation functions. Various aggregation and de-fuzzification methods have also been proposed to process intuitionistic fuzzy information and make decisions based on it. Intuitionistic fuzzy sets have been further extended to intuitionistic fuzzy logic, intuitionistic fuzzy clustering, and other related areas. They provide a rich framework for handling uncertainty and hesitancy in decision-making and information processing.

**Advantages of IFS:**

a) Handling Uncertainty: IFS provides a more comprehensive representation of uncertainty compared to traditional fuzzy sets. It incorporates an additional parameter called the degree of hesitancy, which allows for modeling and managing uncertainty more effectively.

b) Capturing Indeterminacy: IFS can capture the concept of indeterminacy or lack of knowledge about the membership status of an element. This makes it suitable for situations where decision-makers have limited or incomplete information.
c) Expressing Ambiguity: IFS offers a framework for expressing ambiguity or vagueness in a more explicit manner. It provides a means to represent situations where multiple interpretations or possibilities exist.
d) Flexibility in Decision-making: IFS allows decision-makers to express their subjective judgments and preferences in a more nuanced way. It provides a richer representation of their uncertainties, hesitations, and degrees of belief.
e) Enhanced Decision Support: IFS can facilitate better decision support systems by enabling more realistic and flexible modeling of uncertainty. It allows for capturing and incorporating experts' opinions and subjective assessments in a more accurate manner.

**Disadvantages of IFS:**

a) Increased Complexity: The inclusion of an additional parameter, the degree of hesitancy, in IFS increases the complexity of computations and decision-making processes. Handling and processing IFS may require more computational resources compared to traditional fuzzy sets.
b) Lack of Standardization: There is no standardized methodology for determining or assigning the degree of hesitancy. This can lead to inconsistencies or subjectivity in the interpretation and representation of uncertainty.
c) Limited Applicability: The use of IFS may not be suitable for all types of problems or decision-making scenarios. Some situations may require more precise or well-defined measures of uncertainty, and IFS may not provide the desired level of granularity.
d) Interpretation Challenges: Interpreting the results or outputs of an IFS-based system can be challenging. The additional parameter of hesitancy adds complexity to the interpretation process and may require additional effort to understand and communicate the results effectively.
e) Lack of Extensive Applications: Although IFS has been proposed as an extension to fuzzy set theory, its application is not as widespread or well-established as traditional fuzzy sets. This can limit the availability of resources and expertise in implementing and utilizing IFS in practical scenarios.

It's important to note that the advantages and disadvantages of IFS can vary depending on the specific problem domain and context of application. Consideration should be given to the particular requirements and characteristics of the problem before deciding to adopt IFS as a modeling tool.

2. **Neutrosophic Fuzzy Set (NFS)**: NFS is a hybrid framework that combines neutrosophy and fuzzy set theory. It was introduced by Florentin Smarandache in 1998 as an extension of traditional fuzzy sets. NFS allows for the representation of indeterminacy, ambiguity, and inconsistency in a more comprehensive way by introducing three membership functions: truth-membership function, indeterminacy-membership function, and falsity-membership function. The membership values in a neutrosophic fuzzy set are not limited to the traditional range of 0 to 1. Instead, they can take values in the interval [0,1], allowing for a more expressive representation of uncertainty. The truth-membership function represents the degree of truth or acceptance of an element in the set, the indeterminacy-membership function represents the degree of indeterminacy or hesitancy, and the falsity-membership function represents the degree of falsity or rejection. Mathematically, for a given set X, a neutrosophic fuzzy set A is represented as:

   $A = \{(x, \mu T(x), \mu I(x), \mu F(x)) \mid x \in X\}$

   where $\mu T(x)$, $\mu I(x)$, and $\mu F(x)$ represent the truth-membership, indeterminacy-membership, and falsity-membership degrees of element x, respectively [82]:

   Neutrosophic fuzzy sets allow for a more nuanced representation of uncertainty and imprecision compared to traditional fuzzy sets. They are particularly useful in situations where decision-makers need to handle indeterminacy and inconsistency simultaneously. Neutrosophic fuzzy sets have been applied in various fields such as decision-making, expert systems, pattern recognition, image processing, and control systems. Similar to fuzzy sets, neutrosophic fuzzy sets can be combined using various operations such as union, intersection, and complement. Different aggregation and defuzzification techniques have also been developed to process neutrosophic fuzzy information and make decisions based on it. Overall, neutrosophic fuzzy sets provide a versatile framework for handling uncertainty, indeterminacy, and inconsistency, offering a more comprehensive representation of complex information.

**Advantages of NFS:**

a) Handling Uncertainty, Ambiguity, and Contradiction: NFS provides a framework for representing and managing uncertainty, ambiguity, and contradiction simultaneously. It allows for the explicit modelling of these three aspects, which can be valuable in decision-making processes.
b) Comprehensive Representation: NFS offers a more comprehensive representation of complex information compared to traditional fuzzy sets. It enables decision-makers to express their uncertainty, ambiguity, and contradictory opinions in a more nuanced way.
c) Flexibility in Decision-making: NFS allows decision-makers to express their subjective judgments and preferences in a more flexible manner. It accommodates different degrees of uncertainty, ambiguity, and contradiction, enabling a more realistic representation of their beliefs.
d) Enhanced Decision Support: NFS can enhance decision support systems by capturing and incorporating experts' opinions and subjective assessments, even when they are contradictory or uncertain. It provides a structured framework for managing and processing such information.
e) Extensibility and Integration: NFS can be integrated with other decision-making techniques and frameworks to enhance their capabilities. It can be combined with traditional fuzzy sets, probability theory, and other approaches to address complex decision-making problems.

**Disadvantages of NFS:**

a) Complexity: The inclusion of three parameters—truth-membership, indeterminacy-membership, and falsity-membership—increases the complexity of computations and decision-making processes. Working with NFS may require more computational resources and expertise compared to traditional fuzzy sets.
b) Interpretation Challenges: Interpreting the results or outputs of an NFS-based system can be challenging due to the presence of contradictory information. Understanding and communicating the implications of conflicting membership degrees can be difficult.
c) Lack of Standardization: There is no standardized methodology for determining or assigning membership degrees in NFS. This can lead to inconsistencies or subjectivity in the interpretation and representation of uncertainty, ambiguity, and contradiction.

d) Limited Applicability: The use of NFS may not be suitable for all types of problems or decision-making scenarios. Some situations may require more precise or well-defined measures of uncertainty, while NFS provides a more qualitative or subjective representation.
e) Limited Availability: Compared to traditional fuzzy sets, the availability of resources, tools, and expertise in implementing and utilizing NFS may be limited. This can pose challenges in terms of practical application and accessibility.

As with any modeling technique, it is important to consider the specific requirements, characteristics, and limitations of NFS when deciding to adopt it in a particular problem domain. The advantages and disadvantages may vary depending on the context of the application and the nature of the problem at hand.

3. **Pythagorean Fuzzy Sets (PFS)**: NFS are an extension of fuzzy sets that were introduced by Ye-Jun Liu in 2007. PFS aim to provide a more precise representation of uncertainty and ambiguity by incorporating a membership degree and a non-membership degree, along with an additional parameter called the indeterminacy degree. In a Pythagorean fuzzy set, the membership degree represents the degree to which an element belongs to the set, similar to traditional fuzzy sets. The non-membership degree represents the degree to which an element does not belong to the set. The indeterminacy degree quantifies the degree of ambiguity or uncertainty associated with the element's membership status [83].

   Mathematically, for a given set X, a Pythagorean fuzzy set A is represented as:

   $A = \{(x, \mu A(x), \nu A(x), \pi A(x)) \mid x \in X\}$

   where $\mu A(x)$ represents the membership degree of x in A, $\nu A(x)$ represents the non-membership degree of x in A, and $\pi A(x)$ represents the indeterminacy degree of x in A.

   The values of $\mu A(x)$, $\nu A(x)$, and $\pi A(x)$ range between 0 and 1, where 0 indicates no membership, no non-membership, or no indeterminacy, and 1 indicates complete membership, complete non-membership, or complete indeterminacy, respectively.

   Pythagorean fuzzy sets provide a more comprehensive representation of uncertainty compared to traditional fuzzy sets by explicitly incorporating the indeterminacy degree. This allows for a more precise characterization of ambiguous or uncertain information. They have been applied in decision-making, pattern recognition, expert systems, and other domains where uncertainty and ambiguity need to be taken into account. Similar to traditional fuzzy sets, Pythagorean fuzzy sets can be combined using operations such as union, intersection, and complement. Various aggregation and de-fuzzification techniques have also been proposed to process Pythagorean fuzzy information and make decisions based on it. Pythagorean fuzzy sets provide a flexible framework for representing and managing uncertainty and ambiguity, offering a richer and more nuanced approach compared to traditional fuzzy sets in certain situations.

**Advantages of PFS:**

a) Enhanced Representation of Uncertainty: Pythagorean fuzzy sets provide a more precise representation of uncertainty compared to traditional fuzzy sets. By incorporating membership and non-membership degrees, along with an indeterminacy degree, PFS offer a more comprehensive and detailed depiction of uncertain information.
b) Flexible Modelling of Ambiguity: PFS allow for capturing and expressing ambiguity or vagueness more effectively. The indeterminacy degree enables the modelling of situations where multiple interpretations or possibilities exist, providing a richer representation of ambiguous information.
c) Granular Degree of Membership: PFS allow for a granular degree of membership by considering both membership and non-membership degrees. This added granularity enables a more accurate representation of the degree to which an element belongs or does not belong to a set.
d) Improved Decision-Making: The enhanced representation of uncertainty and ambiguity provided by PFS can lead to improved decision-making. Decision-makers can make more informed choices by considering the precise membership, non-membership, and indeterminacy degrees associated with each element.
e) Compatibility with Existing Fuzzy Set Operations: PFS maintain compatibility with traditional fuzzy set operations such as union, intersection, and complement. This allows for leveraging existing fuzzy set theory tools and techniques when working with PFS.

**Disadvantages of PFS:**

a) Increased Complexity: The incorporation of membership, non-membership, and indeterminacy degrees in PFS increases the complexity of calculations and decision-making processes. Handling and processing PFS may require additional computational resources and expertise compared to traditional fuzzy sets.

b) Interpretation Challenges: Interpreting the results or outputs of PFS-based systems can be challenging due to the presence of multiple degrees of membership and non-membership. The complexity of interpretation may make it more difficult to understand and communicate the results effectively.

c) Lack of Standardization: There is no standardized methodology for determining or assigning the membership, non-membership, and indeterminacy degrees in PFS. This can lead to inconsistencies or subjectivity in their interpretation and representation.

d) Limited Availability of Tools and Resources: Compared to traditional fuzzy sets, the availability of tools, libraries, and resources specific to working with PFS may be limited. This can pose challenges in terms of practical application and accessibility.

### 5.2.2 Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) Method

Multi-Criteria Decision Making, or MCDM, is an area of research that investigates decision-making issues that contained numerous conflicting criteria. The purpose of MCDM methods is to offer organised ways to assess and rank various solutions in accordance with these standards. These approaches are frequently employed in a variety of fields where complex decision-making and trade-offs between distinct goals are present. One of the MCDM techniques frequently used to address decision-making issues is TOPSIS. It operates on the principle of selecting an ideal solution and evaluating alternatives according to how closely they resemble this ideal solution. The TOPSIS method is a decision-making technique used to evaluate and rank alternative options based on multiple criteria. It is widely employed in various domains, including business, engineering, and social sciences. The TOPSIS method involves a systematic process starting with the identification and selection of relevant criteria. The criteria are then weighted to reflect their relative importance in the decision-making process. A decision matrix is constructed, with each row representing an alternative and each column representing a criterion. The matrix is normalized to standardize the values and eliminate any scale differences. Next, the positive ideal solution (PIS) and negative ideal solution (NIS) are determined. The PIS represents the best possible values for each criterion, while the NIS represents the worst. The relative closeness of each alternative to the PIS and

NIS is calculated using the Euclidean distance. Alternatives with shorter distances to the PIS and longer distances to the NIS are considered more favourable. Finally, the alternatives are ranked based on their relative closeness, with the one closest to the PIS and farthest from the NIS being the most preferred.

The TOPSIS method provides a practical and structured approach for decision makers to evaluate alternatives and make informed choices considering multiple criteria simultaneously. TOPSIS method is used to determine the best alternative from the concepts of the compromise solution. The best compromise solution should have the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the negative ideal solution. The procedures of TOPSIS can be described as follows. Let $A = \{A_1, A_2, \ldots, A_m\}$be the set of alternatives, $CC = \{C_1, C_2, \ldots, C_n\}$ be the set of criteria and $D = \{d_{ij}\}, i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, be the performance ratings with the criteria weight vector $W = \{w_j | j = 1, 2, \ldots, n\}$.

TOPSIS method is presented with these following steps.

**Step 1**. Normalization the decision matrix

The normalized value $d_{ij}^N$ is calculated as follows:

➢ For benefit criteria (higher the better), $d_{ij}^N = (d_{ij} - d_j^-)/(d_j^+ - d_j^-)$, where $d_j^+ = max_i(d_{ij})$ and $d_j^- = min_i(d_{ij})$ or setting $d_j^+$is the aspired or desired level and $d_j^-$is the worst level.

➢ For c cost criteria (lower the better), $d_{ij}^N = (d_j^- - d_{ij})/(d_j^- - d_j^+)$.

**Step 2.** Calculation of weighted normalized decision matrix

In the weighted normalized decision matrix, the modified ratings are calculated as the following way:

$$v_{ij} = w_j \times d_{ij}^N \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$

Where,$w_j$ is the weight of the $j^{th}$ criteria such that $w_j \geq 0$ for $j = 1, 2, \ldots, n$ and

$\sum_{j=1}^{n} w_j = 1$.

**Step 3.** Determination of the positive and the negative ideal solutions

The positive ideal solution (PIS) and the negative ideal solution (NIS) are derived as follows

$$PIS = A^+ = \{v_1^+, v_2^+, \ldots, v_n^+\} = \left\{\left(\max_j v_{ij} | j \in J_1\right), \left(\min_j v_{ij} | j \in J_2\right) | j = 1, 2, \ldots, n\right\}$$

And

$$NIS = A^{-} = \{v_1^-, v_2^-, \dots, v_n^-\} = \left\{\left(\min_j v_{ij} | j \in J_1\right), \left(\max_j v_{ij} | j \in J_2\right) | j = 1, 2, \dots, n\right\}$$

where, $J_1$ and $J_2$are the benefit and cost-type criteria, respectively.

**Step 4**. Calculation of the separation measures for each alternative from the PIS and the NIS

The separation values for the PIS can be measured by using the n-dimensional Euclidean distance, which is given as:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, \qquad i = 1, 2, \dots, m$$

Similarly, separation values for the NIS are

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, \qquad i = 1, 2, \dots, m$$

**Step 5**. Calculation of the relative closeness coefficient to the positive ideal solution

The relative closeness coefficient for the alternative $A_i$ with respect to $A^+$ is

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \text{for} i = 1, 2, \dots, m$$

**Step 6**. Ranking the alternatives

According to the relative closeness coefficient to the ideal alternative, a larger value of $C_i$ indicates the better alternative $A_i$.

TOPSIS provides a ranking of alternatives based on their relative closeness, helping decision makers identify the most preferred option. It is a part of the broader MCDM field as it contributes to the set of methods and techniques used to support decision-making processes that involve multiple criteria. MCDM encompasses a wide range of methods, and TOPSIS is one of the popular and effective approaches within this framework. A new method termed fuzzy-TOPSIS is suggested because the traditional methods for investment selection are unable to handle the imprecise or vague nature of linguistic assessment. The purpose of this study is to compare and contrast the Pythagorean and intuitionistic fuzzy TOPSIS approaches for the financial investment problem. An investment firm employs the suggested techniques.

The outcomes from both strategies are provided after identifying the factors that influence investment decision-making.

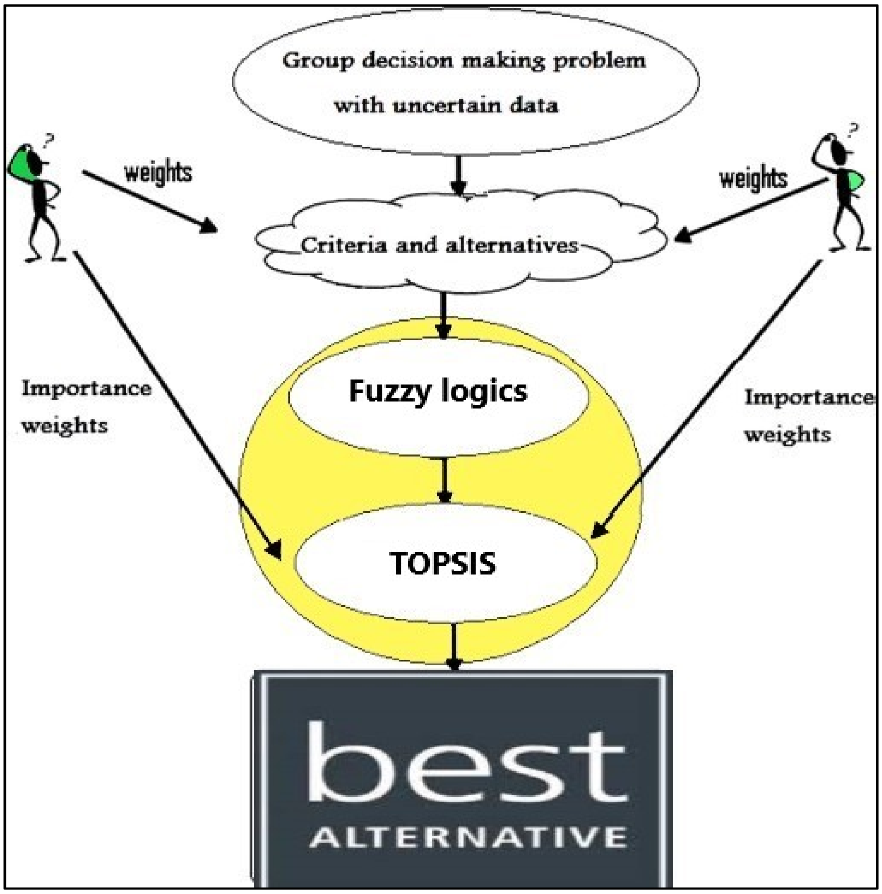



Figure 5.11 Fuzzy-TOPSIS techniques for group decision making problem

## 5.3 A Financial Decision System

The financial investment system in industrial engineering involves allocating capital to optimize productivity and profitability in manufacturing processes. It encompasses analysing project feasibility, estimating financial returns, managing costs, and evaluating risks. This system aims to maximize efficiency, reduce waste, and enhance operational performance by strategically investing in technologies, equipment, and human resources. Through diligent financial planning and decision-making, industrial engineers ensure the efficient allocation of resources to drive sustainable growth and achieve long-term financial objectives.

A financial decision system is a tool or framework used by individuals, organizations, or financial professionals to analyse and make informed decisions regarding financial matters. It typically involves a combination of financial analysis, data evaluation, and decision-making techniques to optimize financial outcomes. The financial investment system in supply chain management (SCM) focuses on allocating resources to optimize the flow of goods, information, and finances across the supply chain network. It involves evaluating investment opportunities in areas such as logistics infrastructure, technology implementation, inventory management, and supplier relationships. This system aims to enhance operational efficiency, reduce costs, mitigate risks, and improve customer satisfaction. By strategically investing in supply chain capabilities, organizations can achieve competitive advantages, increase profitability, and foster resilience in a dynamic business environment. Every organisation, irrespective of its size, and mission, can be viewed as a financial entity. In financial decision system, the objective of the firm is to maximise the value of firm to its equity shareholders. This means that the objective of the firm is to maximise the market value of its equity shares. This generally appears to provide a rational guide for business-decision-making and promote efficient allocation of resources in the economic system. Hence, when a firm maximises the market value of its equity, it ensures that the decisions are consistent with the risk-return preference of investors. This suggests that it allocates resources optimally. According to Fairchild [84], inadequate financial supply chain integration can cause disruptions in the physical supply chain. The supply chain's financial data monitors every transaction connected to the actual flow of items in conjunction with that movement. The fact that information transfers at a different rate from the product or service is crucial because companies that have tried to improve the efficiency of their physical supply chains but have only achieved limited success are still dissatisfied by the gaps that exist between the activities that take place in the supply chain and the financial data that is related to them.

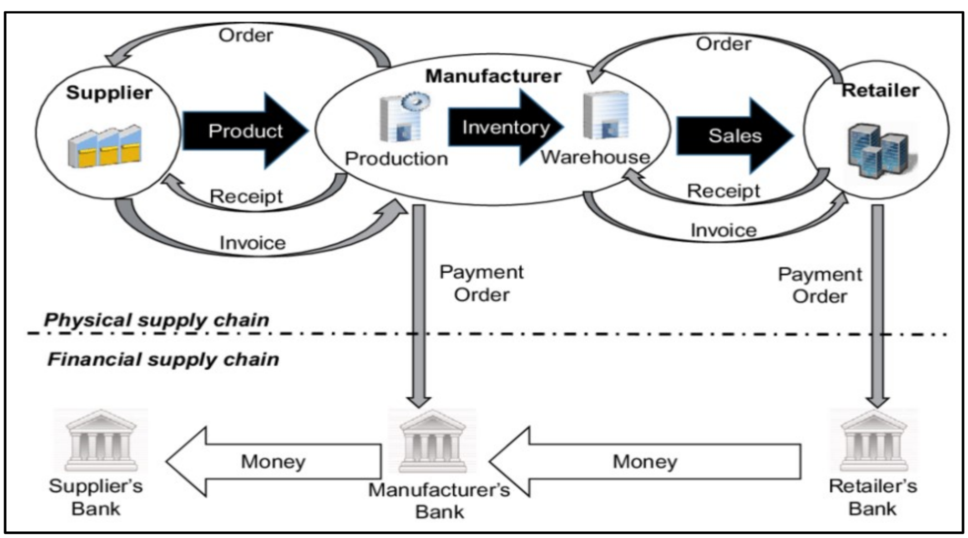



Figure 5.12 Physical and financial SCM network

## 5.4 Objective of the Work

When one talks about the financial decision system, it involves both structured and unstructured decision-making. Because, one has to depend on a set of data, procedures, rules etc. and at the same time some knowledge. Hence, both decision system and knowledge system are the two important components of any financial decision system. The present research is based on best investment aspect of financial management in supply chain management and the applicability of intelligent decision system by considering both explicit and tacit part of DM. As explained in earlier study, when employing fuzzy approaches, fuzzy rules develop and defined through human judgement and may involve a degree of subjectivity. However, defining criteria weights and establishing fuzzy rules can take some time. In order to make the process of establishing fuzzy rules easier, we concentrate on the cognitive and behavioural aspects of DM that are currently being pushed. We can summarise it in some points below:

1. Fuzzy logic's significant accomplishment in decision making has paved the way for its utilization in various domains, such as finance. Nevertheless, there is a lack of an up-to-date and comprehensive assessment of fuzzy logic's applications in the financial sector. Therefore, this research aims to thoroughly assess the viability and practicality of employing fuzzy logic as an efficient approach in financial studies, specifically in investment decision-making.
2. We develop a methodology to identify the optimal investment area among four possibilities and by considering four criteria in order to address a financial decision-making issue in an organisation.
3. We study how our subconscious minds (tacit knowledge) influence decision-making for managerial purposes using the iceberg theory.
4. To make a final resilient decision, we employed three fuzzy logic-based optimization techniques (Pythagorean. Intutuinistic, Neutrosophic) and fuzzy- TOPSIS method, comparing the outcomes and determining the most suitable option for investment.

## 5.5 Literature Review

**“A Research literature review is a systematic, explicit, and reproducible method for identifying, evaluating, and synthesizing the existing body of completed and recorded work produced by researchers, scholars, and practitioners" (Fink 2003).**

A comprehensive literature review clears the way for the research and encourages the researcher to consider the work of other researchers, which inspires the researcher to carry out important research. Studying the benefits, constraints, and scope of the research field is also helpful (Boote & Belle 2005). Scholars are interested in the finance investment problem because it is a multi-attribute decision-making problem that involves numerous attributes. To efficiently address financial performance evaluation,

**Dong et al. [85]** created a cosine similarity-based Qualitative Flexible multiple criteria technique (QUALIFLEX) methodology for MCDM using HFLTSs. In addition to having higher flexibility and reduced computing complexity, the new fuzzy envelope of HFLTS is also very simple for DM to implement. The defined modified cosine similarity metric takes the HFLTS's degree of reluctance into account, making it more fair for decision-making.

**Geoffrion et al [86]** used a method based on the Frank-Wolfe method, a specific mathematical programming technique. Assuming the DM is able to provide an overall value function as an aggregate of multi-objectives, the procedure incorporates a large step gradient method for solving a vector-maximum problem. The procedure, however, simply needs the local data necessary to carry out the computation and does not require that this function be explicitly specified. Both linear and non-linear problems can be solved using this approach. Its computation is straightforward, and its convergence happens quickly. The explicit trade-off information provided by DM, however, is challenging to articulate.

**Tabaraee et al. [87]** used three factors and seven primary criteria to present the fuzzy Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE) method to evaluate the investment projects. To show how the values of final production costs and capacity factor affect the ranking of alternatives, a thorough sensitivity analysis was conducted. They looked into how investment costs impacted how renewable power projects were ranked. Iranian fossil resources are so inexpensive that they can cut investment expenses by up to 50%.

**Zeng et al**. **[88]** was created a TOPSIS approach for intuitionistic fuzzy MCDM and applied to investment selection. In the same formulation, it can take into account both the decision maker's attitude and the variables' subjective relevance. The key benefit of this approach is that it may convey the relative weights of the subjective information of an attribute and the decision-makers' attitude. Additionally, it offers a more accurate depiction of the decision-making process because the decision-maker can take into account a wide range of scenarios depending on his interests by interacting with the many intuitionistic fuzzy ordered weighted averaging weighted averaging (IFOWAWAD) operator parameters.

**Zhan et al**. **[89]** suggested an enhanced multi-attribute decision-making method based on mental accounting and prospect theory, for investment project schemes. He developed a fuzzy information fusion strategy for investment selection that aims to demonstrate its usefulness by keeping the original opinions of experts and risk preference data based on expert reliability. In order to determine in advance whether the decision maker chosen is trustworthy and appropriate for scheme selection, a final investment selection example is applied.

**Zhou et al. [90]** developed a portfolio selection strategy based on hesitant fuzzy information. They look into the potential for investment and the boundaries of efficiency for these proposed qualitative portfolio models. Additionally, precise portfolio selection procedures are offered. Finally, a sample of choosing the best risk investment portfolio is shown. We can draw the conclusion that the suggested qualitative portfolio models employed for the three types of risk investors are successful based on the aforementioned study and example. This technique for choosing a portfolio can be utilised to invest in qualitative risks in real-world situations.

**Walczak et al. [91]** developed a model that combines randomness and fuzziness, such as stochastic returns with fuzzy information, for portfolio optimization. To express the stochastic return on individual securities with unclear information, they used random fuzzy variables. They create random fuzzy simulations and simulation-based genetic algorithms to solve the suggested models in order to discover the best portfolio. They used a numerical example using made-up data that was then provided to show the method's viability.

**Qin [92]** established a model that pioneered the multi-criteria decision-making (MCDM) challenges, by addressing the cognitive constraints and the variables that can cause rational decisions to deviate. The main aim of this research is to develop a methodology to make the right decision for the investment firm supply chain.

**Eskandarpou et al. [93]** created a model with multiple goals to improve the post-sales network's sustainability and decrease overall costs, completion times, and environmental pollution. They create random fuzzy simulation and simulation-based genetic algorithms to solve the suggested models in order to discover the best portfolio. A numerical example using made-up data was then provided to show the method's viability.

**Darabi et al. [94]** proposed an interval-valued hesitant fuzzy entropy system MCDM that describes the supplier selection problem, and MCDM techniques are useful in this context. The analysis of the literature reveals that MCDM methods are a key component of many supplier evaluation and selection procedures.

**Darabi et al. [95]** proposed an interval-valued hesitant fuzzy entropy system as a grading system for evaluating green suppliers. They ranked three suppliers after evaluating their quality, cost, technological prowess, distribution, and environmental competency to test the effectiveness of their strategy.

**Mohammed et al. [96]** proposed combining fuzzy AHP and fuzzy TOPSIS. To evaluate and rank providers based on three different sets of criteria (i.e., cost, green, and social), In a case study, they chose suppliers and worked out the best order quantities using an improved multi-objective optimization model. Some researchers are developing decision-making processes in extensive fuzzy settings. For the purpose of choosing environmentally friendly suppliers for a supply chain.

**Mousavi et al. [97]** introduced an interval type-2 trapezoidal fuzzy-decision model based on probabilistic statistical ideas. They demonstrate a precise process that prevents data loss for the GDM in ambiguous circumstances. Decisions were made by decision-makers in this study based on IT2TrFNs. In this proposed method, both economic and environmental considerations have been taken into account.

**Mina et al**. **[98]** integrated AHP and TOPSIS with a fuzzy inference system to evaluate and rank circular suppliers in a petrochemical company. The expertise of six industry experts was utilised, and the six providers were assessed and graded using the suggested methodology. Experts approved the findings, which showed the success of the suggested strategy. Additionally, by contrasting the findings of the proposed methodology with those of two other approaches, the accuracy of its performance was demonstrated.

**Roger et al. [99]** proposed a combination of AI techniques, including fuzzy logic, neural networks and evolutionary programming and it outperforms traditional techniques. They used the fuzzy logic and neural networks as a hybrid approach and it tends to be the most common technique and demonstrates the best performance. They identified the efficiency of this method. They came to the conclusion that fuzzy logic has demonstrated to be particularly effective when tackling uncertainty and ambiguity, two of the most prevalent features connected to financial analysis.

**Simon [100]** enunciated his theory of bounded rationality because of his concern that utility theory presumed 'a well organized and stable system of preferences and a skill in computation' that was unrealistic in many decision contexts. He said that due to the cost or the practical impossibility of searching among all possible acts for the optimal, the decision maker simply looked for the first 'satisfactory' alternative that met some pre-specified target. This approach is known as 'satisficing approach'. 'Satisficing' means 'satisfying' and 'sufficing'.

**Ramot et al. [101]** introduced the notion of complex fuzzy set which was able to deal with the phase variable with the explanation of the use of phase in tracking the cycle of solar activity. Further, they explained that the degree of membership can accordingly be increased by using the phase element. Now, the degree of membership depends on both the amplitude and phase variables. The limitation of using CFS is that it only deals with the maximum value of membership of sunspots whereas the nearby values are sometimes neglected which can also play an important role in the tracking of the solar activity.

**Smarandache [102]** contributed the unique concept of indeterminacy to the above-mentioned theories, which plays a vital role in obtaining solutions to various uncertain situations. This novel concept is known as the neutrosophic set, this concept of the neutrosophic set not only increase the clarity but also increase the basic information related to neutrality.

**Kifayat et al. [103]** presented the geometrical aspects and features of these generalizations - "fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets and picture fuzzy sets". In order to observe the constrained nature of their structures, this article provided a detailed background on IFSs, Pythagorean fuzzy sets (PFS), and picture fuzzy sets. With the aid of diagrams and numerical examples, it is discussed how the flaws in the current structures can be fixed by utilising the spherical fuzzy set and T-spherical fuzzy set frameworks. The limits of these similarity measurements between IFSs and PFSs were then explored. Some new similarity metrics were created to broaden the scope of these similarity measurements.

**Zopounidis and doumpos [104]** give a thorough overview of MCDA's contributions to the subject of finance, emphasising the techniques employed and how they are applied in practise. They said that because MCDA approaches provide a very methodical and practical framework for decision problems, they appear to have a bright future in the field of financial management and also the application of MCDA approaches to support real-time financial decision-making necessitates the creation of multi-criteria decision support systems that are integrated, user-friendly, and tailored to solve financial issues.

**Nath and Sarkar [105]** proposed a fuzzy-TOPSIS model to chose the best advanced manufacturing technology. In this research they described fuzzy logic, or more specifically, the fuzzy sets theory, is one of the fundamental components of soft computing that must be applied to the models in order to imitate (approximately) human decision making. Finally, they employed sensitivity analysis to determine the best course of action in the event that the attitude co-efficient varied in value.

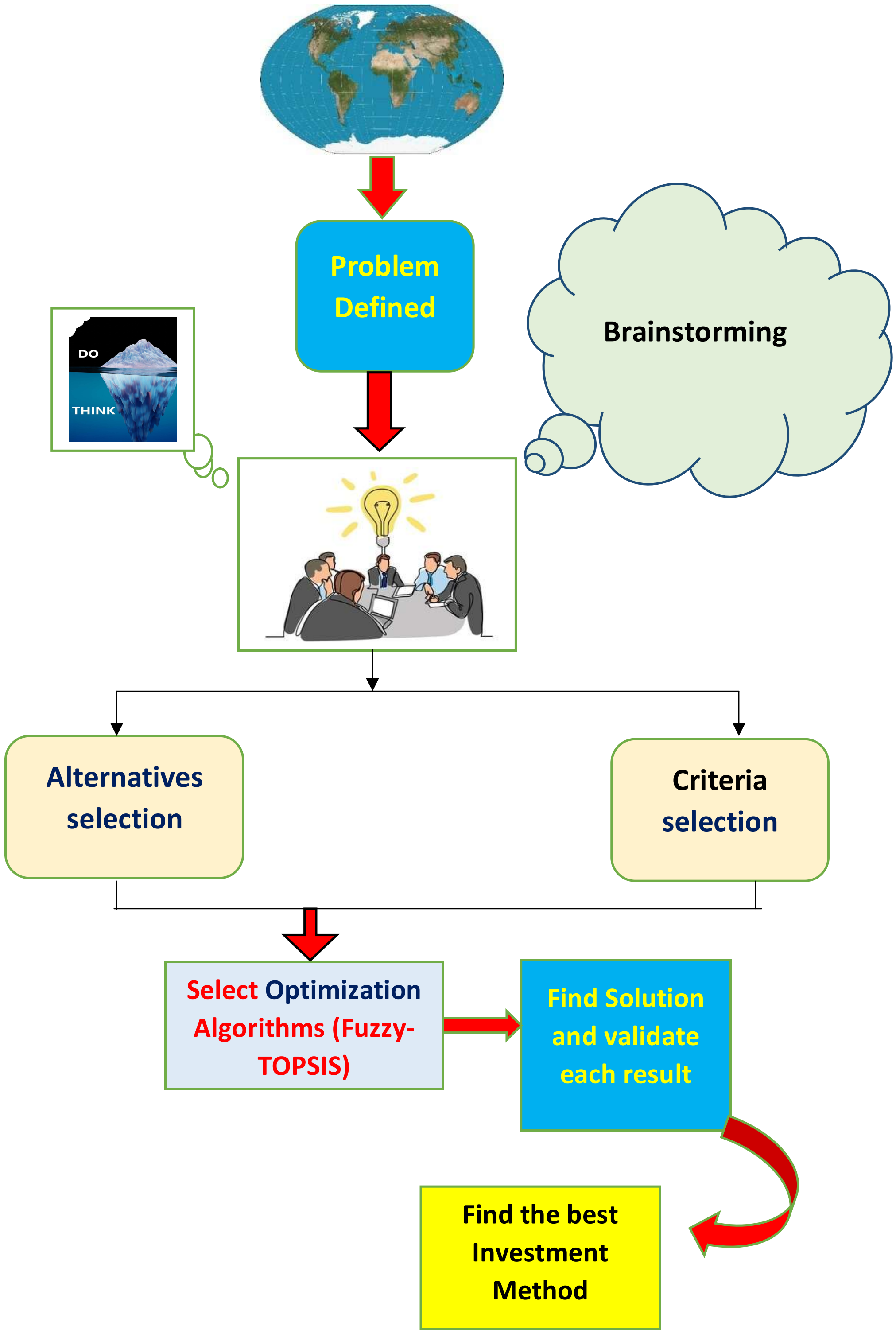



Figure 5.12 Steps for solving a problem by fuzzy algorithms

# CHAPTER 6

# A PRACTICAL EXAMPLE TO EVALUATE PERFORMANCE OF INVESTMENT ALTERNATIVES

In this section, a sample evaluation of an investment firm's performance is provided. An investment firm seeks to place a certain amount of capital in the best option. Four businesses, A1, A2, A3, and A4, are viable options for investment. A biscuit firm is A1, a food company is A2, a bike company is A3, and a mobile company is A4. Four criteria will be used to evaluate each potential company: The benefit criteria are C1, C2, and the cost criteria are C3, C4. C1 stands for economic benefit, C2 for social benefit, C3 for environmental pollution, and C4 for political issues. The problem is addressed using the fuzzy TOPSIS decision-making model that has been proposed, and the following is a step-by-step description of the computing process:

## 6.1 Numerical Analysis:

In fuzzy logic, criteria weights represent the relative importance or significance assigned to different criteria used in decision-making processes. These weights indicate how much each criterion contributes to the overall evaluation or assessment of a system. The criteria weights are typically defined by experts or decision-makers based on their domain knowledge and preferences. When considering economic benefit, social benefit, environmental pollution, and political issues, the criteria weights in fuzzy logic can be determined using various methods, such as pair-wise comparison or expert judgment. Here is a general description of how each criterion's weight can be assigned:

I. **Economic Benefit (C1)**: The weight assigned to economic benefit reflects the importance placed on financial considerations. It takes into account factors like profitability, cost-effectiveness, revenue generation, and economic growth. Decision-makers may consider factors such as potential monetary gains, return on investment, or long-term financial sustainability when assigning weights to this criterion.

II. **Social Benefit (C2)**: The weight assigned to social benefit captures the significance given to the impacts on individuals, communities, or society as a whole. It considers factors like social welfare, quality of life, human well-being, and equity. The decision-makers may consider aspects such as social justice, public health, education, employment opportunities, and societal cohesion when assigning weights to this criterion.

III. **Environmental Pollution (C3)**: The weight assigned to environmental pollution represents the importance placed on environmental considerations and sustainability. It takes into account factors such as pollution levels, resource depletion, climate change, and ecological impact. Decision-makers may consider the significance of reducing pollution, conserving natural resources, promoting clean technologies, and mitigating environmental risks when assigning weights to this criterion.

IV. **Political Issues (C4)**: The weight assigned to political issues reflects the significance given to factors related to governance, policy, and public opinion. It considers aspects like political stability, legal frameworks, regulatory compliance, and public acceptance. Decision-makers may consider factors such as political feasibility, public trust, stakeholder engagement, and adherence to laws and regulations when assigning weights to this criterion.

So, we are solving this problem by using three different fuzzy logic and find the outcomes from all techniques:

### 6.1.1 Pythagorean Fuzzy Set (PF)

PF set is used to express the evaluations for four potential companies based on four criteria. Criteria weights play a crucial role in a decision maker's point of view in fuzzy sets as they determine the relative importance or significance assigned to different criteria or attributes. The criteria weights help quantify the degree of importance or preference for each criterion in the decision-making process. So, from expert opinion, we take criteria weights of each fuzzy sets.

For all fuzzy sets: **(0.15, 0.25, 0.35, 0.25)**

The investing firm constructs the Pythagorean fuzzy decision matrix Dp, which may be stated as follows:

**Table 6.1** Pythagorean fuzzy linguistic ratings and Pythagorean fuzzy numbers

| **Linguistic term** | **Corresponding Pythagorean fuzzy number** |
|---|---|
| Very poor (VP) | (0.15, 0.85) |
| Poor (P) | (0.25, 0.75) |
| Medium poor (MP) | (0.35, 0.65) |
| Medium (M) | (0.50, 0.45) |
| Medium good (MG) | (0.65, 0.35) |
| Good (G) | (0.75, 0.25) |
| Very good (VG) | (0.85, 0.15) |

a) So, Pythagorean design matrix will be: (We get this matrix from expert's point of view or DM)

**Dp**=

| **Alternatives** | **(0.15) C1** | **(0.25) C2** | **(0.35) C3** | **(0.25) C4** |
|---|---|---|---|---|
| **A1** | P (0.9,0.3) | P (0.7,0.6) | P (0.5,0.8) | P (0.6,0.3) |
| **A2** | P (0.4,0.7) | P (0.9,0.2) | P (0.8,0.1) | P (0.5,0.3) |
| **A3** | P (0.8,0.4) | P (0.7,0.5) | P (0.6,0.2) | P (0.7,0.4) |
| **A4** | P (0.7,0.2) | P (0.8,0.2) | P (0.8,0.4) | P (0.6,0.6) |
| **PIS** | P (0.9,0.5) | P (0.9,0.2) | P (0.8,0.1) | P (0.7,0.4) |
| **NIS** | P (0.4,0.7) | P (0.7,0.6) | P (0.5,0.8) | P (0.5,0.3) |

R.K YAGA Proposed (Pythagorean weighted) aggregation operator to solve MAGDM problem with Pythagorean fuzzy information.

b) $\mathrm{C(Ai)} = P\left(\sum_{i=1}^{4} Wj\; U_{1j}, \sum_{j=1}^{4} Wi\; V_{ij}\right)$

Where Wj and Wi are the weights of these three criteria. U and V are values of the criteria. So, by calculating these values we get:

c) $C(A1) = P\{(0.15 * 0.9 + 0.25 * 0.7 + 0.35 * 0.5 + 0.25 * 0.6),\ (0.15 * 0.3 + 0.25 * 0.6 + 0.35 * 0.8 + 0.25 * 0.3)\} = P(0.635, 0.550)$

d) $C(A2) = P\{(0.15 * 0.4 + 0.25 * 0.9 + 0.35 * 0.8 + 0.25 * 0.5),\ (0.15 * 0.7 + 0.25 * 0.2 + 0.35 * 018 + 0.25 * 0.3)\} = P(0.690, 0.265)$

e) $C(A3) = P\{(0.15 * 0.8 + 0.25 * 0.7 + 0.35 * 0.6 + 0.25 * 0.6),\ (0.15 * 0.4 + 0.25 * 0.5 + 0.35 * 0.2 + 0.25 * 0.4)\} = P(0.680, 0.355)$

f) $C(A4) = P\{(0.15 * 0.7 + 0.25 * 0.8 + 0.35 * 0.8 + 0.25 * 0.6),\ (0.15 * 0.2 + 0.25 * 0.2 + 0.35 * 0.4 + 0.25 * 0.6)\} = P(0.735, 0.370)$

Using score function, based on comparison method we obtain:

a) $C\{S(A1)\} = (0.635^2 - 0.550^2) = 0.100725$

b) $C\{S(A3)\} = (0.680^2 - 0.355^2) = 0.3367$

c) $C\{S(A2)\} = (0.69^2 - 0.265^2) = 0.4058$

d) $C\{S(A4)\} = (0.735^2 - 0.370^2) = 0.4033$

The best choice to invest money according to Pythagorean fuzzy logic would be **A1 (Biscuits firms).**

### 6.1.2 Intuitionistic Fuzzy Set (IFS)

IFSs are used to express the evaluations for four potential companies based on these criteria. The investing firm constructs the intuitionistic fuzzy decision matrix Di, which may be stated as follows:

**Step-1:** So, IFS design matrix will be: (We get this matrix from expert's point of view or DM)**: Di** =

| **Alternatives** | **0.15**<br>**C1 (+)** | **0.25**<br>**C2 (+)** | **0.35**<br>**C3 (-)** | **0.25**<br>**C4 (-)** |
|---|---|---|---|---|
| **A1** | (0.6,0.3) | (0.5,0.35) | (0.3,0.4) | (0.4,0.5) |
| **A2** | (0.4,0.55) | (0.7,0.2) | (0.8,0.1) | (0.5,0.3) |
| **A3** | (0.6.5,0.3) | (0.6,0.3) | (0.6,0.2) | (0.5,0.35) |
| **A4** | (0.7,0.2) | (0.65,0.2) | (0.6,0.3) | (0.4,0.4) |

**Step-2:** The same weights are used in this problem also from DMs point of view. Weights are as follows: W1 = 0.15, W2 = 0.25, W3 = 0.35, W4 = 0.25.

**Step-3:** We determine criteria values after establishing the criteria weighting vector by this equation:

$$x_{ij} = (\mu_{ij}, v_{ij}) = \left(1 - (1 - \mu_{ij})^{w_{ij}}, v_{ij}^{w_{ij}}\right)$$

Weighted intuitionistic decision matrix, **Dw** =

| | **C1 (+)** | **C2 (+)** | **C3 (-)** | **C4 (-)** |
|---|---|---|---|---|
| **A1** | (0.12841,0.834) | (0.1591,0.7691) | (0.1173,0.7256) | (0.1198,0.8408) |
| **A2** | (0.0737,0.9142) | (0.2599,0.66874) | (0.43067,0.44668) | (0.159103,0.7400) |
| **A3** | (0.12841,0.8347) | (0.20472,0.74008) | (0.27436,0.56932) | (0.159,0.7691) |
| **A4** | (0.16522,0.7855) | (0.23083,0.6687) | (0.27436,0.65613) | (0.1198,0.795) |

**Step-4:** Each alternative's Intuitionistic fuzzy positive-ideal solution (IFPIS, $A^+$) and Intuitionistic fuzzy negative-ideal solution (IFNIS, $A^-$) with respect to the criterion can be calculated using Equations respectively:

$$A^+ = \{< Cj, \big((max\ \mu ij(Cj)|\ j \in G), (\min \mu ij(Cj)|\ j \in B)\big), \big((min\ vij(Cj)|\ j \in G), (\max vij(Cj)|\ j \in B)\big) > |\ i \in m\}$$

$$A^- = \{< Cj, \big((min\ \mu ij(Cj)|\ j \in G), (\max \mu ij(Cj)|\ j \in B)\big), \big((max\ vij(Cj)|\ j \in G), (\min vij(Cj)|\ j \in B)\big) > |\ i \in m\}$$

$A^+$ = (0.165, 0.785), (0.259, 0.668), (0.117, 0.725), (0.119, 0.840)

$A^-$ = (0.0737, 0.9142), (0.159, 0.769), (0.4306, 0.4466), (0.159, 0.740)

**Step-5:** Use these equations to determine the distance between alternatives and intuitionistic fuzzy ideal solutions (IFPIS and INFNIS):

$$d_{IFS}(Ai, A^-) = \sqrt{\sum_{j=1}^{n} \left[(\mu_A(Cj) - \mu_{A^-}(Cj))^2 + \left((v_A(Cj) - v_{A^-}(Cj)\right)^2 + (\pi_A(Cj) - \pi_{A^-}(Cj))^2\right]}$$

$$d_{IFS}(Ai, A^+) = \sqrt{\sum_{j=1}^{n} \left[(\mu_A(Cj) - \mu_{A^+}(Cj))^2 + \left((v_A(Cj) - v_{A^+}(Cj)\right)^2 + (\pi_A(Cj) - \pi_{A^+}(Cj))^2\right]}$$

### $d_{IFS}(Ai, A^-)$ calculations

**C1** =

| | | | SUM |
|---|---|---|---|
| 0.001338 | 0.0497 | 0.0001 | 0.051138 |
| 0.008324 | 0.1292 | 0.0014 | 0.138924 |
| 0.00133 | 0.0497 | 0.00017 | 0.0512 |
| 0 | 0.0005 | 0 | 0.0005 |

**C2** =

| | | | SUM |
|---|---|---|---|
| 0.009979 | 0.01023 | 1.6E-06 | 0.020211 |
| 0 | 0 | 2.75E-06 | 2.75E-06 |
| 0.002945 | 0.00519 | 0.00031 | 0.008445 |
| 0.0007930 | 0 | 0.00075 | 0.001543 |

**C4** =

| | | | SUM |
|---|---|---|---|
| 0 | 0 | 3.18508E-06 | 3.19E-06 |
| 0.00160829 | 0.0099834 | 0.003577 | 0.015169 |
| 0.001608298 | 0.0050182 | 0.00094 | 0.007566 |
| 0 | 0.0020007 | 0.001922 | 0.003923 |

**C3** =

| | | | SUM |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.098391805 | 0.077460023 | 0.001250207 | 0.177102 |
| 0.024762284 | 0.024234606 | 2.84153E-06 | 0.049 |
| 0.024762284 | 0.004742548 | 0.007831225 | 0.037336 |

$d_{IFS}(Ai, A^{+})$ ***calculations***

**C1** =

| | | | SUM |
|---|---|---|---|
| 0.002993896 | 0.834773 | 0.000600784 | 0.838368 |
| 0 | 0.914228 | 0 | 0.914228 |
| 0.002993896 | 0.834773 | 0.000600784 | 0.838368 |
| 0.008377264 | 0.785515 | 0.001365862 | 0.795258 |

**C2** =

| | | | SUM |
|---|---|---|---|
| 0 | 0 | 6.97763E-08 | 0 |
| 0.01018428 | 0.010052006 | 4.32307E-07 | 0.020237 |
| 0.002091166 | 0.000836204 | 0.000282646 | 0.00321 |
| 0.005160904 | 0.010052006 | 0.000807711 | 0.016021 |

**C3** =

| | | | SUM |
|---|---|---|---|
| 0.09774467 | 0.078198326 | 0.001089119 | 0.177032 |
| 0 | 0 | 0 | 0 |
| 0.024223696 | 0.015209134 | 0.001044215 | 0.040477 |
| 0.024223696 | 0.044156232 | 0.002969619 | 0.07135 |

**C4** =

| | | | SUM |
|---|---|---|---|
| 0.001529728 | 0.010180087 | 0.003817346 | 0.015527 |
| 0 | 0 | 0 | 0 |
| 0 | 0.000850339 | 0.000856391 | 0.001707 |
| 0.001529728 | 0.003054853 | 0.000261113 | 0.004846 |

**Table 6.2** Distances of every alternative

| **Alternatives** | $\boldsymbol{d_{IFS}(Ai, A^-)}$ | $\boldsymbol{d_{IFS}(Ai, A^+)}$ |
|---|---|---|
| **A1** | 0.017838 | 0.257732 |
| **A2** | 0.082799 | 0.233616 |
| **A3** | 0.029053 | 0.22094 |
| **A4** | 0.010825 | 0.221868 |

**Step-6:** Determine the relative closeness coefficient (CC) for each choice, then rank the alternatives in order of preference by using this equation:

$$CC_I = \frac{d_{IFS}(Ai, A^-)}{d_{IFS}(Ai, A^+) + d_{IFS}(Ai, A^-)}$$

Where $0 \leq CCi \leq 1$, (i=1,2,3…,m)

Table 6.3 CC values

| | $\boldsymbol{CC_I}$ |
|---|---|
| **A1** | 1.257732 |
| **A2** | 1.233616 |
| **A3** | 1.22094 |
| **A4** | 1.221868 |

**Step-7:** The distance measurement, the closeness ratio, and the ranking are calculated by this table:

Table 6.4 Rank of all alternatives

| Alternatives | $d_{IFS}(Ai, A^-)$ | $d_{IFS}(Ai, A^+)$ | $CC_I$ | **Rank** |
|---|---|---|---|---|
| **A1** | 0.017838 | 0.257732 | 1.257732 | **1** |
| **A2** | 0.082799 | 0.233616 | 1.233616 | **2** |
| **A3** | 0.029053 | 0.22094 | 1.22094 | **4** |
| **A4** | 0.010825 | 0.221868 | 1.221868 | **3** |

As a result, we can observe that the four options were rated in the following order to invest money by the firm according to intutionistic fuzzy logic**: A1- A2- A4- A3** and the best choice would be **A1 (Biscuit firms).**

### 6.1.3 Neutrosophic Fuzzy Set (NFS)

Neutrosophy is a philosophy that depicts the origin, nature, and scope of neutralities and their roles in the formation of ideas in its spectra, Smarandache F (1999). A neutrosophic set $N$ can be expressed as; $N \subset X$. where X is the universal space of points.

Linguistic terms play a vital role in neutrosophic fuzzy logic by enabling human-like representation and interpretation of uncertain and imprecise information. These terms provide a bridge between mathematical models and human cognition, allowing us to express and reason with linguistic concepts. They enhance the interpretability and transparency of neutrosophic fuzzy systems, making them suitable for decision-making in complex and uncertain domains. Linguistic terms enable effective communication between experts and facilitate the incorporation of domain knowledge into the modeling process, ultimately enhancing the practical utility and acceptance of neutrosophic fuzzy logic.

The net $N$ can be characterized by a truth membership function $T_N$(x), an indeterminacy membership function $I_N$(x) and a falsity membership function $F_N$(x). All the three neutrosophic components are independent subsets of $[^-0,1^+]$. Therefore, the relation of the three membership functions may be illustrated as,

Wang H et al. (2010): $^-0 \leq T_N(x)+ I_N(x)+ F_N(x) \leq 3^+$

Rivieccio U (2008) and Ghaderi SF et. al. (2012) explained that in a neutrosophic set, indeterminacy membership function $I_N$(x) may be include vagueness, redundancy, imprecision, contradiction etc. Smarandache F. (2005) had sub classified indeterminacy membership function into subcomponents naming contradiction, uncertainty and unknown.

**Single-valued Neutrosophic Set (SVNS)**

Inspite of being a powerful general formal framework neutrosophic set needs to be specified from a technical point of view. Wang H. (2010) defined some neutrosophic operators based on set theory is presented as SVNS. Various properties which are connected to the operations and relations over SVNS such as complement, containment, union, intersections, differences, truth favouritism and falsity favourism of SVNS, are illustrated. The operators related to this paper are derived in article 4.4.

**Single valued neutrosophic set integrated TOPSIS method**

A multi-criteria group decision-making (MCGDM) with *m* alternatives and *n* attributes Let us assume that A = {$A_1$, $A_2$, ……, $A_m$} be a discrete set of alternatives, and C = {$C_1$, $C_2$, ……, $C_n$} be the set of attributes. The rating provided by the decision maker describes the performance of alternative $A_i$ against attribute $C_j$. Also, assuming that $W$ = {$W_1$, $W_2$, ……, $W_n$} be the weight vector assigned for the attributes $C_1$, $C_2$; ...; $C_n$ by the decision-makers.

$$D = \langle d_{ij} \rangle_{m \times n} = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \cdots \\ A_m \end{array} & \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{pmatrix} \end{array}$$

**Step 1:** *Determination of the most important attribute*

Generally, there are multiple responses or attributes in decision making problems, where some of them are significant and others may not be so significant. Therefore, it is important to select the attributes priority for the decision-making situation. The most significant attributes may be chosen with the help of decision-makers' opinions.

**Step 2:** It is assumed that the rating of each alternative with respect to each attribute is expressed as SVNS for MADM problem. The neutrosophic values associated with the alternatives for MADM problems can be represented in the following decision matrix:

$$
\begin{array}{ccc}
 & & \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\
D_{\bar{N}} = \langle d_{ij}{}^{s} \rangle_{m\times n} & \begin{array}{c} A_1 \\ A_2 \\ \dots \\ A_m \end{array} & \begin{pmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \dots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \dots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \dots & \dots & \dots & \dots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{2m} \rangle & \dots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{pmatrix} \\
= \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n} = & &
\end{array}
$$

$D_{\bar{N}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$, $T_{ij}, I_{ij}$ and $F_{ij}$ denote the degree of truth-membership value, indeterminacy membership value and falsity-membership value of alternative $A_i$ with respect to attribute $C_i$ satisfying the following properties under the single-valued neutrosophic environment:

1. $0 \le T_{ij} \le 1; 0 \le I_{ij} \le 1;\ 0 \le F_{ij} \le 1$
2. $0 \le T_{ij} + I_{ij} + F_{ij} \le 3, \quad$ for $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$

The ratings of each alternative over the attributes are best illustrated by the neutrosophic cube proposed by Dezert in 2002. The vertices of neutrosophic cube are (0, 0, 0); (1, 0, 0); (1, 0, 1); (0, 0, 1); (0, 1, 0); (1, 1, 0); (1, 1, 1) and (0, 1, 1). The area of ratings in the neutrosophic cube is classified in three categories, namely 1. Highly acceptable neutrosophic ratings, 2. tolerable neutrosophic rating, and 3. unacceptable neutrosophic ratings.

**Definition 1:**

$$U = \langle T_{ij}, I_{ij}, F_{ij} \rangle \text{where } 0.5 < T_{ij} < 1, 0 < I_{ij} < 0.5 \text{ and } 0 < F_{ij} < 0.5$$
$$\text{for } i = 1,2, \dots, m \text{ and } j = 1,2, \dots, n$$

**Definition 2:**

$$V = \langle T_{ij}, I_{ij}, F_{ij} \rangle \text{where, } T_{ij} = 0, 0 < I_{ij} \le 1 \text{ and } 0 < F_{ij} \le 1$$
$$\text{for } i = 1,2, \dots, m \text{ and } j = 1,2, \dots, n$$
$$Z = \langle T_{ij}, I_{ij}, F_{ij} \rangle \text{where,} \qquad 0 < T_{ij} < 0.5, 0.5 < I_{ij} < 1 \text{ and } 0.5 < F_{ij} < 1$$
$$\text{for } i = 1,2, \dots, m \text{ and } j = 1,2, \dots, n$$

Fuzzification of SVNS $\bar{N} = \{(x|\langle T_{\bar{N}}(x), I_{\bar{N}}(x), F_{\bar{N}}(x)\rangle)|x \in X\}$ can be defined as a process of mapping $\bar{N}$ into fuzzy set $\tilde{F} = \{(x|\mu_{\tilde{F}}(x))|x \in X\}$ i.e. $f: \bar{N} \to \bar{F}$

$$\mu_{\tilde{F}}(x) = \begin{cases} 1 - \sqrt{\left\{\left(1 - T_{\bar{N}}(x)\right)^2 + I_{\bar{N}}(x)^2 + F_{\bar{N}}(x)^2\right\}/3} & \text{for } \forall x \in U \cup Z \\ 0 & \text{for } \forall x \in V \end{cases}$$

**Step 3:**

$$\boldsymbol{\psi_k = \frac{1 - \sqrt{\{(1 - T_k(x))^2 + (I_k(x))^2 + (F_k(x))^2\}/3}}{\sum_{k=1}^{p}\left(1 - \sqrt{\{(1 - T_k(x))^2 + (I_k(x))^2 + (F_k(x))^2\}/3}\right)}}$$

$$\boldsymbol{and \sum_{k=1}^{p} \psi_k = 1}$$

**Step 4:**

$D = \left(d_{ij}\right)_{m\times n} where$

$d_{ij} = SVNSWA_{\psi}\left(d_{ij}{}^{(1)}, d_{ij}{}^{(2)}, \dots, d_{ij}{}^{(p)}\right)$

$= \psi_1 d_{ij}{}^{(1)} \oplus \psi_2 d_{ij}{}^{(2)} \oplus \dots \oplus \psi_p d_{ij}{}^{(p)}$

$$= \langle 1 - \prod_{k=1}^{p}\left(1 - T_{ij}{}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(I_{ij}{}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(F_{ij}{}^{(p)}\right)^{\psi_k} \rangle$$

$$D = \langle d_{ij}\rangle_{m\times n} = \langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n} = \begin{array}{c|cccc} & C_1 & C_2 & \dots & C_n \\ \hline A_1 & \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \dots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ A_2 & \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \dots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \dots & \dots & \dots & \dots & \dots \\ A_m & \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{2m}\rangle & \dots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{array}$$

**Step 5:**

Let $w_k{}^j = \left(w_j{}^{(1)}, w_j{}^{(2)}, \dots, w_j{}^{(p)}\right)$be the neutrosophic number (NN) assigned to the attribute $C_i$ by the kth decision maker. Then the combined weight $W = \{w_1, w_2, \dots, w_n\}$of the attribute can be determined by using SVNWA aggregation operator

$w_j = SVNSWA_{\psi}\left(w_j{}^{(1)}, w_j{}^{(2)}, \dots, w_j{}^{(p)}\right)$

$= \psi_1 w_j{}^{(1)} \oplus \psi_2 w_j{}^{(2)} \oplus \dots \oplus \psi_p w_j{}^{(p)}$

$$= \langle 1 - \prod_{k=1}^{p}\left(1 - T_j{}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(I_j{}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(F_j{}^{(p)}\right)^{\psi_k} \rangle$$

$W = \{w_1, w_2, \dots, w_n\}$

where$w_j = \langle T_j, I_j, F_j\rangle$for $j = 1, 2, \dots, n$

**Step 6:**

The aggregated weighted neutrosophic decision matrix can be obtained by using the multiplication formula of two neutrosophic sets as:

$D \otimes W = D^w = \langle d_{ij}^{w_j} \rangle_{m\times n} = \langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle_{m\times n} =$

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ \\ \\ \\ \end{array} & \begin{pmatrix} \langle T_{11}{}^{w_1}, I_{11}{}^{w_1}, F_{11}{}^{w_1} \rangle & \langle T_{12}{}^{w_1}, I_{12}{}^{w_1}, F_{12}{}^{w_1} \rangle & \dots & \langle T_{1n}{}^{w_n}, I_{1n}{}^{w_n}, F_{1n}{}^{w_n} \rangle \\ \langle T_{21}{}^{w_2}, I_{21}{}^{w_2}, F_{21}{}^{w_2} \rangle & \langle T_{22}{}^{w_2}, I_{22}{}^{w_2}, F_{22}{}^{w_2} \rangle & \dots & \langle T_{2n}{}^{w_2}, I_{2n}{}^{w_2}, F_{2n}{}^{w_2} \rangle \\ \dots & \dots & \dots & \dots \\ \langle T_{m1}{}^{w_m}, I_{m1}{}^{w_m}, F_{m1}{}^{w_m} \rangle & \langle T_{m2}{}^{w_m}, I_{m2}{}^{w_m}, F_{m2}{}^{w_m} \rangle & \dots & \langle T_{mn}{}^{w_m}, I_{mn}{}^{w_m}, F_{mn}{}^{w_m} \rangle \end{pmatrix} \end{array}$$

Here, $d_{ij}^{w_j}$

$= \langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ is an element of the aggregated weighted neutrosophic decision matrix $D^w$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

**Step 7:** Relative Neutrosophic Positive Ideal Solution (RNPIS),

$$Q_N^+ = [d_1^{w+}, d_2^{w+}, \dots, d_n^{w+}] \text{ where, } d_j^{w+} = \langle T_j^{w+}, I_j^{w+}, F_j^{w+} \rangle \text{ for } j = 1, 2, \dots, n$$

$$T_j^{w+} = \left\{ \left( \max_i \{T_{ij}^{w+}\} \mid j \in J_1 \right), \left( \min_i \{T_{ij}^{w+}\} \mid j \in J_2 \right) \right\}$$

$$I_j^{w+} = \left\{ \left( \min_i \{I_{ij}^{w+}\} \mid j \in J_1 \right), \left( \max_i \{I_{ij}^{w+}\} \mid j \in J_2 \right) \right\}$$

$$F_j^{w+} = \left\{ \left( \min_i \{F_{ij}^{w+}\} \mid j \in J_1 \right), \left( \max_i \{F_{ij}^{w+}\} \mid j \in J_2 \right) \right\}$$

Relative Neutrosophic Negative Ideal Solution (RNNIS),

$$Q_N^- = [d_1^{w-}, d_2^{w-}, \dots, d_n^{w-}] \text{ where, } d_j^{w-} = \langle T_j^{w-}, I_j^{w-}, F_j^{w-} \rangle \text{ for } j = 1, 2, \dots, n$$

$$T_j^{w-} = \left\{ \left( \min_i \{T_{ij}^{w+}\} \mid j \in J_1 \right), \left( \max_i \{T_{ij}^{w+}\} \mid j \in J_2 \right) \right\}$$

$$I_j^{w+} = \left\{ \left( \max_i \{I_{ij}^{w+}\} \mid j \in J_1 \right), \left( \min_i \{I_{ij}^{w+}\} \mid j \in J_2 \right) \right\}$$

$$F_j^{w+} = \left\{ \left( \max_i \{F_{ij}^{w+}\} \mid j \in J_1 \right), \left( \min_i \{F_{ij}^{w+}\} \mid j \in J_2 \right) \right\}$$

where $J_1$ and $J_2$ be benefit type attribute and cost type attribute respectively

**Step 8:** Normalized Euclidean Distance measure of each alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ from the RNPIS $\langle T_{ij}^{w+}, I_{ij}^{w+}, F_{ij}^{w+} \rangle$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ can calculated as:

$$D_{Eu}^{i+}\left(d_{ij}^{w_j}, d_j^{w+}\right)$$

$$= \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left\{ \left( T_{ij}^{w_j}(x_j) - T_j^{w+}(x_j) \right)^2 + \left( I_{ij}^{w_j}(x_j) - I_j^{w+}(x_j) \right)^2 + \left( F_{ij}^{w_j}(x_j) - F_j^{w+}(x_j) \right)^2 \right\}}$$

Similarly, the Normalized Euclidean Distance measure of each alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ from the RNNIS $\langle T_{ij}^{w-}, I_{ij}^{w-}, F_{ij}^{w-} \rangle$ can calculated as:

$$D_{Eu}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right)$$
$$= \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\left(T_{ij}^{w_j}(x_j) - T_j^{w-}(x_j)\right)^2 + \left(I_{ij}^{w_j}(x_j) - I_j^{w+}(x_j)\right)^2 + \left(F_{ij}^{w_j}(x_j) - F_j^{w+}(x_j)\right)^2\right\}}$$

**Step 9:** Calculate the relative closeness with the following equations and rank the alternatives according the descending order the relative closeness.

As per C.L. Hwang, K. Yoon (1981), Relative Closeness (RC) = $\frac{D_{Eu}^{i-}(d_{ij}^{w_j}, d_j^{w-})}{D_{Eu}^{i-}(d_{ij}^{w_j}, d_j^{w-}) + D_{Eu}^{i+}(d_{ij}^{w_j}, d_j^{w+})}$

**Solution of this Problem**:

Table 6.5 Linguistic terms for rating of attributes and decision makers:

| **Linguistic terms** | **SVNNs** |
|---|---|
| Very good/very important (VG/VI) | 0.90, 0.10, 0.10 |
| Good/important (G/I) | 0.80, 0.20, 0.15 |
| Fair/medium (F/M) | 0.50, 0.40, 0.45 |
| Bad/unimportant (B/UI) | 0.35, 0.60, 0.70 |
| Very bad/very unimportant (VB/VUI) | 0.10, 0.80, 0.90 |

Table 6.6 Importance of decision makers expressed with SVNNs

| | **DM-1** | **DM-2** | **DM-3** | **DM-4** |
|---|---|---|---|---|
| **LT** | **VI** | **I** | **M** | **I** |
| $\overline{W}$ | **0.90, 0.10, 0.10** | **0.80, 0.20, 0.15** | **0.50, 0.40, 0.45** | **0.80, 0.20, 0.15** |

Determination of the weights of decision makers:

the weight of the $k$th decision maker can be written as:

$$\psi_k = \frac{1 - \sqrt{\{(1 - T_k(x))^2 + (I_k(x))^2 + (F_k(x))^2\}/3}}{\sum_{k=1}^{p}\left(1 - \sqrt{\{(1 - T_k(x))^2 + (I_k(x))^2 + (F_k(x))^2\}/3}\right)} \quad (16)$$

and $\sum_{k=1}^{p} \psi_k = 1$

$E_k = (T_K, I_K, F_K)$ = Be the neutrosophic number

P = No of DM

| | **NF linguistics** | **T** | **I** | **F** | Crisp Importance |
|---|---|---|---|---|---|
| **DM-1** | VI | 0.9 | 0.1 | 0.10 | 0.292 |
| **DM-2** | I | 0.8 | 0.2 | 0.15 | 0.265 |
| **DM-3** | M | 0.5 | 0.4 | 0.45 | 0.178 |
| **DM-4** | I | 0.8 | 0.2 | 0.15 | 0.265 |

Table 6.7 Linguistic terms for rating the candidates with SVNNs

| | **Linguistic terms** | **SVNNs** |
|---|---|---|
| **EG** | Extremely good/high (EG/EH) | 1.00, 0.00, 0.00 |
| **VG** | Very good/high (VG/VH) | 0.90, 0.10, 0.05 |
| **G** | Good/high (G/H) | 0.80, 0.20, 0.15 |
| **MG** | Medium good/high (MG/MH) | 0.65, 0.35, 0.30 |
| **M** | Medium/fair (M/F) | 0.50, 0.50, 0.45 |
| **MB** | Medium bad/medium law (MB/ML) | 0.35, 0.65, 0.60 |
| **B** | Bad/law (B/L) | 0.20, 0.75, 0.80 |
| **VB** | Very bad/low (VB/VL) | 0.10, 0.85, 0.90 |
| **VVB** | Very very bad/low (VVB/VVL) | 0.05, 0.90, 0.95 |

Table 6.8 Assessments of Alternatives and Attribute Weights given by Four Decision Makers

| **Alternatives (Ai)** | **DM** | **C1** | **C2** | **C3** | **C4** |
|---|---|---|---|---|---|
| **A1** | DM-1 | G | VVB | MG | M |
| | DM-2 | G | VB | MG | M |
| | DM-3 | G | VVB | MG | MG |
| | DM-4 | G | VVB | MG | M |
| **A2** | DM-1 | VB | B | G | G |
| | DM-2 | VB | B | G | G |
| | DM-3 | VB | MB | G | VG |
| | DM-4 | VB | VB | G | MG |
| **A3** | DM-1 | EG | M | EG | EG |
| | DM-2 | EG | M | EG | VG |
| | DM-3 | EG | MG | EG | EG |
| | DM-4 | EG | MG | EG | EG |
| **A4** | DM-1 | MG | B | B | VVB |
| | DM-2 | M | B | B | VB |

| | | | | | |
|---|---|---|---|---|---|
| | DM-3 | M | MB | B | VB |
| | DM-4 | M | VB | B | VVB |
| **Weights** | DM-1 | VI | VI | I | M |
| | DM-2 | I | VI | I | I |
| | DM-3 | I | I | M | M |
| | DM-4 | I | VI | M | VI |

Table 6.9

| *(Ai)* | DM | *C1* | *C2* | *C3* | *C4* |
|---|---|---|---|---|---|
| ***A1*** | **DM-1** | 0.80, 0.20, 0.15 | 0.05, 0.90, 0.95 | 0.65, 0.35, 0.30 | 0.50, 0.50, 0.45 |
| | **DM-2** | 0.80, 0.20, 0.15 | 0.10, 0.85, 0.90 | 0.65, 0.35, 0.30 | 0.50, 0.50, 0.45 |
| | **DM-3** | 0.80, 0.20, 0.15 | 0.05, 0.90, 0.95 | 0.65, 0.35, 0.30 | 0.65, 0.35, 0.30 |
| | **DM-4** | 0.80, 0.20, 0.15 | 0.05, 0.90, 0.95 | 0.65, 0.35, 0.30 | 0.50, 0.50, 0.45 |
| ***A2*** | **DM-1** | 0.10, 0.85, 0.90 | 0.20, 0.75, 0.80 | 0.80, 0.20, 0.15 | 0.80, 0.20, 0.15 |
| | **DM-2** | 0.10, 0.85, 0.90 | 0.20, 0.75, 0.80 | 0.80, 0.20, 0.15 | 0.80, 0.20, 0.15 |
| | **DM-3** | 0.10, 0.85, 0.90 | 0.35, 0.65, 0.60 | 0.80, 0.20, 0.15 | 0.90, 0.10, 0.05 |
| | **DM-4** | 0.10, 0.85, 0.90 | 0.10, 0.85, 0.90 | 0.80, 0.20, 0.15 | 0.65, 0.35, 0.30 |
| ***A3*** | **DM-1** | 1.00, 0.00, 0.00 | 0.50, 0.50, 0.45 | 1.00, 0.00, 0.00 | 1.00, 0.00, 0.00 |
| | **DM-2** | 1.00, 0.00, 0.00 | 0.50, 0.50, 0.45 | 1.00, 0.00, 0.00 | 0.90, 0.10, 0.05 |
| | **DM-3** | 1.00, 0.00, 0.00 | 0.65, 0.35, 0.30 | 1.00, 0.00, 0.00 | 1.00, 0.00, 0.00 |
| | **DM-4** | 1.00, 0.00, 0.00 | 0.65, 0.35, 0.30 | 1.00, 0.00, 0.00 | 1.00, 0.00, 0.00 |
| ***A4*** | **DM-1** | 0.65, 0.35, 0.30 | 0.20, 0.75, 0.80 | 0.20, 0.75, 0.80 | 0.05, 0.90, 0.95 |
| | **DM-2** | 0.50, 0.50, 0.45 | 0.20, 0.75, 0.80 | 0.20, 0.75, 0.80 | 0.10, 0.85, 0.90 |
| | **DM-3** | 0.50, 0.50, 0.45 | 0.35, 0.65, 0.60 | 0.20, 0.75, 0.80 | 0.10, 0.85, 0.90 |
| | **DM-4** | 0.50, 0.50, 0.45 | 0.10, 0.85, 0.90 | 0.20, 0.75, 0.80 | 0.05, 0.90, 0.95 |
| **Weights** | **DM-1** | 0.90, 0.10, 0.10 | 0.90, 0.10, 0.10 | 0.80, 0.20, 0.15 | 0.50, 0.40, 0.45 |
| | **DM-2** | 0.80, 0.20, 0.15 | 0.90, 0.10, 0.10 | 0.80, 0.20, 0.15 | 0.80, 0.20, 0.15 |

| | | | | | |
|---|---|---|---|---|---|
| | **DM-3** | 0.50, 0.40, 0.45 | 0.80, 0.20, 0.15 | 0.50, 0.40, 0.45 | 0.50, 0.40, 0.45 |
| | **DM-4** | 0.50, 0.40, 0.45 | 0.90, 0.10, 0.10 | 0.50, 0.40, 0.45 | 0.80, 0.20, 0.15 |

Table 6.10

| *($A_i$)* | *DM* | $C_1$ | | | $C_2$ | | | $C_3$ | | | $C_4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ |
| | **DM-1** | 0.80 | 0.20 | 0.15 | 0.05 | 0.90 | 0.95 | 0.65 | 0.35 | 0.30 | 0.50 | 0.50 | 0.45 |
| $A_1$ | **DM-2** | 0.80 | 0.20 | 0.15 | 0.10 | 0.85 | 0.90 | 0.65 | 0.35 | 0.30 | 0.50 | 0.50 | 0.45 |
| | **DM-3** | 0.80 | 0.20 | 0.15 | 0.05 | 0.90 | 0.95 | 0.65 | 0.35 | 0.30 | 0.65 | 0.35 | 0.30 |
| | **DM-4** | 0.80 | 0.20 | 0.15 | 0.05 | 0.90 | 0.95 | 0.65 | 0.35 | 0.30 | 0.50 | 0.50 | 0.45 |
| | **DM-1** | 0.10 | 0.85 | 0.90 | 0.20 | 0.75 | 0.80 | 0.80 | 0.20 | 0.15 | 0.80 | 0.20 | 0.15 |
| $A_2$ | **DM-2** | 0.10 | 0.85 | 0.90 | 0.20 | 0.75 | 0.80 | 0.80 | 0.20 | 0.15 | 0.80 | 0.20 | 0.15 |
| | **DM-3** | 0.10 | 0.85 | 0.90 | 0.35 | 0.65 | 0.60 | 0.80 | 0.20 | 0.15 | 0.90 | 0.10 | 0.05 |
| | **DM-4** | 0.10 | 0.85 | 0.90 | 0.10 | 0.85 | 0.90 | 0.80 | 0.20 | 0.15 | 0.65 | 0.35 | 0.30 |
| | **DM-1** | 1.00 | 0.00 | 0.00 | 0.50 | 0.50 | 0.45 | 1.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 |
| $A_3$ | **DM-2** | 1.00 | 0.00 | 0.00 | 0.50 | 0.50 | 0.45 | 1.00 | 0.00 | 0.00 | 0.90 | 0.10 | 0.05 |
| | **DM-3** | 1.00 | 0.00 | 0.00 | 0.65 | 0.35 | 0.30 | 1.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 |
| | **DM-4** | 1.00 | 0.00 | 0.00 | 0.65 | 0.35 | 0.30 | 1.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 |
| | **DM-1** | 0.65 | 0.35 | 0.30 | 0.20 | 0.75 | 0.80 | 0.20 | 0.75 | 0.80 | 0.05 | 0.90 | 0.95 |
| $A_4$ | **DM-2** | 0.50 | 0.50 | 0.45 | 0.20 | 0.75 | 0.80 | 0.20 | 0.75 | 0.80 | 0.10 | 0.85 | 0.90 |
| | **DM-3** | 0.50 | 0.50 | 0.45 | 0.35 | 0.65 | 0.60 | 0.20 | 0.75 | 0.80 | 0.10 | 0.85 | 0.90 |
| **W** | **DM-1** | 0.90 | 0.10 | 0.10 | 0.90 | 0.10 | 0.10 | 0.80 | 0.20 | 0.15 | 0.50 | 0.40 | 0.45 |
| | **DM-2** | 0.80 | 0.20 | 0.15 | 0.90 | 0.10 | 0.10 | 0.80 | 0.20 | 0.15 | 0.80 | 0.20 | 0.15 |
| | **DM-3** | 0.50 | 0.40 | 0.45 | 0.80 | 0.20 | 0.15 | 0.50 | 0.40 | 0.45 | 0.50 | 0.40 | 0.45 |
| | **DM-4** | 0.50 | 0.40 | 0.45 | 0.90 | 0.10 | 0.10 | 0.50 | 0.40 | 0.45 | 0.80 | 0.20 | 0.15 |

Table 6.11 Weights of Decision Makers

| | | |
|---|---|---|
| **DM-1** | **$Y_1$** | 0.292 |
| **DM-2** | **$Y_2$** | 0.265 |
| **DM-3** | **$Y_3$** | 0.178 |
| **DM-4** | **$Y_4$** | 0.265 |

Table 6.12

| *($A_i$)* | *DM* | *$C_1$* | | | *$C_2$* | | | *$C_3$* | | | *$C_4$* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ |
| | **DM-1** | 0.625 | 0.625 | 0.574 | 0.985 | 0.970 | 0.985 | 0.736 | 0.736 | 0.703 | 0.817 | 0.817 | 0.792 |
| $A_1$ | **DM-2** | 0.653 | 0.653 | 0.605 | 0.972 | 0.958 | 0.972 | 0.757 | 0.757 | 0.727 | 0.832 | 0.832 | 0.809 |
| | **DM-3** | 0.751 | 0.751 | 0.713 | 0.991 | 0.981 | 0.991 | 0.830 | 0.830 | 0.807 | 0.830 | 0.830 | 0.807 |
| | **DM-4** | 0.653 | 0.653 | 0.605 | 0.987 | 0.972 | 0.987 | 0.757 | 0.757 | 0.727 | 0.832 | 0.832 | 0.809 |
| | **DM-1** | 0.970 | 0.954 | 0.970 | 0.937 | 0.919 | 0.937 | 0.625 | 0.625 | 0.574 | 0.625 | 0.625 | 0.574 |
| $A_2$ | **DM-2** | 0.972 | 0.958 | 0.972 | 0.943 | 0.927 | 0.943 | 0.653 | 0.653 | 0.605 | 0.653 | 0.653 | 0.605 |
| | **DM-3** | 0.981 | 0.971 | 0.981 | 0.926 | 0.926 | 0.913 | 0.751 | 0.751 | 0.713 | 0.664 | 0.664 | 0.587 |
| | **DM-4** | 0.972 | 0.958 | 0.972 | 0.972 | 0.958 | 0.972 | 0.653 | 0.653 | 0.605 | 0.757 | 0.757 | 0.727 |
| | **DM-1** | 0.000 | 0.000 | 0.000 | 0.817 | 0.817 | 0.792 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $A_3$ | **DM-2** | 0.000 | 0.000 | 0.000 | 0.832 | 0.832 | 0.809 | 0.000 | 0.000 | 0.000 | 0.544 | 0.544 | 0.452 |
| | **DM-3** | 0.000 | 0.000 | 0.000 | 0.830 | 0.830 | 0.807 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | **DM-4** | 0.000 | 0.000 | 0.000 | 0.757 | 0.757 | 0.727 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | **DM-1** | 0.736 | 0.736 | 0.703 | 0.937 | 0.919 | 0.937 | 0.937 | 0.919 | 0.937 | 0.985 | 0.970 | 0.985 |
| $A_4$ | **DM-2** | 0.832 | 0.832 | 0.809 | 0.943 | 0.927 | 0.943 | 0.943 | 0.927 | 0.943 | 0.972 | 0.958 | 0.972 |
| | **DM-3** | 0.884 | 0.884 | 0.867 | 0.926 | 0.926 | 0.913 | 0.961 | 0.950 | 0.961 | 0.981 | 0.971 | 0.981 |
| **W** | **DM-1** | 0.510 | 0.510 | 0.510 | 0.510 | 0.510 | 0.510 | 0.625 | 0.625 | 0.574 | 0.817 | 0.765 | 0.792 |
| | **DM-2** | 0.653 | 0.653 | 0.605 | 0.544 | 0.544 | 0.544 | 0.653 | 0.653 | 0.605 | 0.653 | 0.653 | 0.605 |
| | **DM-3** | 0.884 | 0.849 | 0.867 | 0.751 | 0.751 | 0.713 | 0.884 | 0.849 | 0.867 | 0.884 | 0.849 | 0.867 |
| | **DM-4** | 0.832 | 0.785 | 0.809 | 0.544 | 0.544 | 0.544 | 0.832 | 0.785 | 0.809 | 0.653 | 0.653 | 0.605 |

Table 6.13 Construction of the aggregated neutrosophic decision matrix based on the assessments of decision makers

| *(A<sub>i</sub>)* | *C1* | | | *C2* | | | *C3* | | | *C4* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ |
| **$A_1$** | 0.800 | 0.200 | 0.150 | 0.064 | 0.886 | 0.936 | 0.650 | 0.350 | 0.300 | 0.531 | 0.469 | 0.419 |
| **$A_2$** | 0.100 | 0.850 | 0.900 | 0.205 | 0.756 | 0.784 | 0.800 | 0.200 | 0.150 | 0.795 | 0.205 | 0.148 |
| **$A_3$** | 1.000 | 0.000 | 0.000 | 0.573 | 0.427 | 0.376 | 1.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 |
| **$A_4$** | 0.550 | 0.450 | 0.400 | 0.205 | 0.756 | 0.784 | 0.200 | 0.750 | 0.800 | 0.072 | 0.878 | 0.928 |
| **W** | 0.755 | 0.222 | 0.217 | 0.887 | 0.113 | 0.107 | 0.700 | 0.272 | 0.244 | 0.692 | 0.277 | 0.251 |

Table 6.14 Aggregated neutrosophic decision matrix

| | **C1** | **C2** | **C3** | **C4** |
|---|---|---|---|---|
| **$A_1$** | (0.8,0.2,0.15) | (0.064,0.886,0.936) | (0.65,0.35,0.3) | (0.531,0.469,0.419) |
| **$A_2$** | (0.1,0.85,0.9) | (0.205,0.756,0.784) | (0.8,0.2,0.15) | (0.795,0.205,0.148) |
| **$A_3$** | (1,0,0) | (0.573,0.427,0.376) | (1,0,0) | (1,0,0) |
| **$A_4$** | (0.55,0.45,0.4) | (0.205,0.756,0.784) | (0.2,0.75,0.8) | (0.072,0.878,0.928) |
| **W** | (0.755,0.222,0.217) | (0.887,0.113,0.107) | (0.7,0.272,0.244) | (0.692,0.277,0.251) |

Table 6.15 Aggregated weighted neutrosophic decision matrix

| *($A_i$)* | *C1* | | | *C2* | | | *C3* | | | *C4* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ | $T_k(x)$ | $I_k(x)$ | $F_k(x)$ |
| **$A_1$** | 0.604 | 0.378 | 0.334 | 0.056 | 0.899 | 0.943 | 0.455 | 0.527 | 0.471 | 0.367 | 0.616 | 0.565 |
| **$A_2$** | 0.075 | 0.883 | 0.922 | 0.181 | 0.783 | 0.807 | 0.560 | 0.417 | 0.357 | 0.550 | 0.425 | 0.362 |
| **$A_3$** | 0.755 | 0.222 | 0.217 | 0.508 | 0.492 | 0.443 | 0.700 | 0.272 | 0.244 | 0.692 | 0.277 | 0.251 |
| **$A_4$** | 0.415 | 0.572 | 0.530 | 0.181 | 0.783 | 0.807 | 0.140 | 0.818 | 0.849 | 0.050 | 0.911 | 0.946 |
| **PIS** | 0.755 | 0.883 | 0.922 | 0.508 | 0.899 | 0.943 | 0.700 | 0.818 | 0.849 | 0.692 | 0.911 | 0.946 |
| **NIS** | 0.075 | 0.222 | 0.217 | 0.056 | 0.492 | 0.443 | 0.140 | 0.272 | 0.244 | 0.050 | 0.277 | 0.251 |

Table 6.16 Determination of the distance measure of each alternative from the RNPIS and the RNNIS and relative closeness coefficient

| *($A_i$)* | ***C1*** | | | ***C2*** | | | ***C3*** | | | ***C4*** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| For **PIS** | *$(T_k(x) - PIS)^2$* | *$(I_k(x) - PIS)^2$* | *$(F_k(x) - PIS)^2$* | *$(T_k(x) - PIS)^2$* | *$(I_k(x) - PIS)^2$* | *$(F_k(x) - PIS)^2$* | *$(T_k(x) - PIS)^2$* | *$(I_k(x) - PIS)^2$* | *$(F_k(x) - PIS)^2$* | *$(T_k(x) - PIS)^2$* | *$(I_k(x) - PIS)^2$* | *$(F_k(x) - PIS)^2$* |
| **$A_1$** | 0.02280 | 0.25574 | 0.34511 | 0.20422 | 0.00000 | 0.00000 | 0.06001 | 0.08483 | 0.14288 | 0.10551 | 0.08711 | 0.14508 |
| **$A_2$** | 0.46166 | 0.00000 | 0.00000 | 0.10677 | 0.01344 | 0.01849 | 0.01959 | 0.16038 | 0.24148 | 0.02014 | 0.23633 | 0.34027 |
| **$A_3$** | 0.00000 | 0.43733 | 0.49695 | 0.00000 | 0.16609 | 0.25022 | 0.00000 | 0.29823 | 0.36579 | 0.00000 | 0.40240 | 0.48199 |
| **$A_4$** | 0.11567 | 0.09661 | 0.15357 | 0.10677 | 0.01344 | 0.01849 | 0.31351 | 0.00000 | 0.00000 | 0.41224 | 0.00000 | 0.00000 |
| **PIS** | 0.755 | 0.883 | 0.922 | 0.508 | 0.899 | 0.943 | 0.700 | 0.818 | 0.849 | 0.692 | 0.911 | 0.946 |
| **NIS** | 0.075 | 0.222 | 0.217 | 0.056 | 0.492 | 0.443 | 0.140 | 0.272 | 0.244 | 0.050 | 0.277 | 0.251 |

| *($A_i$)* | ***C1*** | | | ***C2*** | | | ***C3*** | | | ***C4*** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| For **NIS** | *$(T_k(x) - NIS)^2$* | *$(I_k(x) - NIS)^2$* | *$(F_k(x) - NIS)^2$* | *$(T_k(x) - NIS)^2$* | *$(I_k(x) - NIS)^2$* | *$(F_k(x) - NIS)^2$* | *$(T_k(x) - NIS)^2$* | *$(I_k(x) - NIS)^2$* | *$(F_k(x) - NIS)^2$* | *$(T_k(x) - NIS)^2$* | *$(I_k(x) - NIS)^2$* | *$(F_k(x) - NIS)^2$* |
| **$A_1$** | 0.27928 | 0.02421 | 0.01380 | 0.00000 | 0.16609 | 0.25022 | 0.09920 | 0.06495 | 0.05144 | 0.10064 | 0.11506 | 0.09820 |
| **$A_2$** | 0.00000 | 0.43733 | 0.49695 | 0.01566 | 0.08504 | 0.13266 | 0.17635 | 0.02121 | 0.01286 | 0.25014 | 0.02196 | 0.01230 |
| **$A_3$** | 0.46166 | 0.00000 | 0.00000 | 0.20422 | 0.00000 | 0.00000 | 0.31351 | 0.00000 | 0.00000 | 0.41224 | 0.00000 | 0.00000 |
| **$A_4$** | 0.11516 | 0.12284 | 0.09802 | 0.01566 | 0.08504 | 0.13266 | 0.00000 | 0.29823 | 0.36579 | 0.00000 | 0.40240 | 0.48199 |

Table 6.17 Distance measure and relative closeness coefficient of each alternative

| ***Alternatives ($A_i$)*** | ***d ($A_i$, PIS)*** | ***d ($A_i$, NIS)*** | **RC** | **Rank** |
|---|---|---|---|---|
| **$A_1$** | 0.26130 | 0.32443 | 0.55 | **2** |
| **$A_2$** | 0.38409 | 0.37221 | 0.49 | **3** |
| **$A_3$** | 0.49457 | 0.34054 | 0.41 | **4** |
| **$A_4$** | 0.00000 | 0.42010 | 1.00 | **1** |

The best choice to invest money according to neutrosophic fuzzy logic would be **A4 (Mobile company).**

## 6.2 Results and Discussion

According to the available literature, fuzzy logic has already been used in a variety of financial applications. However, compared to other sectors where it is used more frequently, such as control systems, engineering, and environmental sciences, it is still far from attaining its full potential. Fuzzy logic has demonstrated to be very effective when tackling uncertainty and ambiguity, which are two of the most prevalent qualities associated to financial analysis. This is one of our primary conclusions after assessing the results reported in research in which fuzzy logic has been utilized. By comparing the developed models of Intuitionistic fuzzy, Neutrosophic fuzzy and Pythagorean fuzzy logics under various levels of uncertainty with various attributes and criteria in fuzzy sets, it can be stated that the optimum investment strategy for the firm's money has been identified. We found that Intuitionistic and Pythagorean fuzzy logics give us the same result for investment (A1-Biscuits firm) but for Neutrosophic fuzzy logic, we found that Mobile company (A4) is the best investment alternative. This difference may be due to the inconsistency in the thought process of the DM. We have analyzed it based on cognitive theory and highlighted the importance of the subconscious part of every DM to make a fair judgment. In earlier research, fuzzy rules are developed and defined through human judgment by DM when implementing fuzzy methods, and this may include a degree of subjectivity. From the decision-makers' point of view, the alternatives' relative importance varies, which has an impact on the decision-making problem's outcome. In order to make a managerial decision, we can conclude that tacit knowledge and behavioral factors are also crucial.

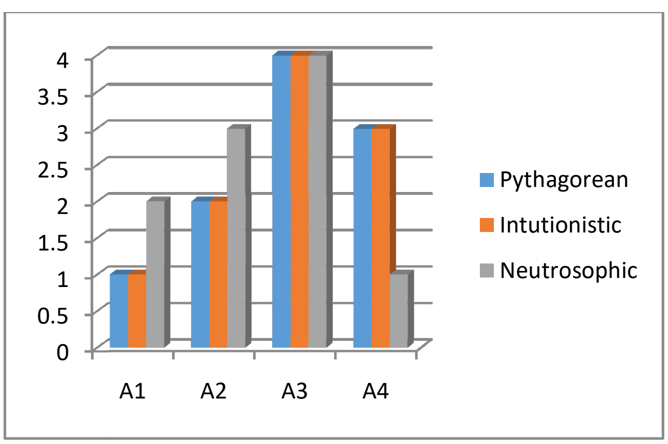



Figure 5.13 Comparison results of all three methods

# 7.0 CONCLUSIONS

In the present thesis, we have studied and proposed some new optimization techniques (foraging behaviour metaheuristic algorithm and extension of fuzzy sets) with various results and applications. Five metaheuristic techniques—AVOA, DOA, FFA, GOA, and BSA—are used in this research to find the best parametric combinations of several NTM processes using single and multi-objective optimization in first part. In the second section, after evaluating the outcomes of studies using fuzzy logic, we draw several key conclusions; the model developed and helped to understand how the Iceberg theory impacts on making decisions (DM) and every individual who is managing his finances need to be aware of how following a disciplined approach could impact his financial health, which is one of the most prevalent features associated with financial analysis. The following conclusions can be drawn from the derived solutions:

1. For the AWJM process, the application of AVOA approach searches out the best parametric combination in single objective optimization as JP =220 bar each, SOD = 3, 1.37, and 1.67 mm for MRR, KA, and Ra respectively, and TS= 20 mm/min each, resulting in 71.47%, 43.73%, and 10.00% improvements in the values of MRR, KA, and Ra respectively. But for multi-objective optimization, optimum parameter values are JP =220 bar, SOD = 1.4217 mm, and TS = 20 mm/min resulting in 64.65%, 43.45%, and 9.38% improvements in the values of MRR, KA, and Ra respectively.
2. In WEDM optimization here also the AVOA determines the optimal parametric combinations in single and multi-objective optimization. The optimum value of all parameters in single objective optimization are Vs =18 volt each, Fw = 10, 8, 8, and 8 m/min for PURE, ANNELED, QUENCHED, and HARDENED respectively, and T = 0.3, 1.4, 1.4 and 0.3 N for PURE, ANNELED, QUENCHED, and HARDENED respectively, resulting in 1.90%, 3.10%, 6.71%, and 12.51% improvements in the values of PURE, ANNELED, QUENCHED, and HARDENED respectively. But for multi-objective they achieve 0.61%, 1.71%, 3.03% and 5.29% improvements.

3. The Friedman statistical rank test showed that AVOA was ranked first for both problems. Moreover, the p-value confirmed the existence of significant differences among the compared algorithms.
4. Finally, a real time optimization is conducted using an AWJM and WEDM process to validate the optimization performance of the five single and multi-objective foraging behavior metaheuristic techniques. By taking computational time also a comparison parameter and we observed that among these all five algorithms, AVOA method would provide the best values of the considered responses. Thus, for this AWJM and WEDM process, all parameters would provide satisfactory values of the considered responses.
5. The literature has seldom explored the uncertainty related to machining and manufacturing processes, possibly due to the significant computational time involved. However, the current approach can prove advantageous in addressing such situations as well.
6. We introduce specialized and new approaches to fuzzy logic and compare between three logics in order to enhance decision-making processes in finantial investment areas. Traditional decision support methods in different domains rely on precise and clear numerical evaluations. These methods provide a practical option for using an evaluation technique that is capable of assessing verbal descriptions and expressions, enabling decision-makers to express themselves independently based on their own evaluations.
7. From cognitive perspective we found that tacit knowledge is invaluable for decision-makers as it provides contextual understanding, enhances problem-solving abilities, and contributes to sound judgment and intuition. It fosters innovation, enables adaptive decision-making, and facilitates organizational learning. With its experiential and intuitive nature, tacit knowledge complements explicit knowledge by offering unique insights and perspectives. Decision-makers who tap into their tacit knowledge can make informed judgments, navigate complex situations, and generate creative solutions. Sharing and leveraging tacit knowledge within an organization enhances decision-making capabilities and contributes to overall success.
8. This work further emphasizes that, based on the results that have been shown in both the industrial and financial investment fields, a combination of fuzzy logic and evolutionary programming surpasses conventional methodologies.

# 8.0 FUTURE SCOPE

It is necessary to point out the weaknesses and limitations of our research. Other metaheuristics algorithms such as hunting based, physics-based metaheuristics algorithms and modern techniqueslike MCDM, combinative distance-based assessment (CODAS), multi-attributive border approximation area comparison (MABAC)with fuzzy logic may be combined as an important future path of this research work to create a vast pool of various approaches from which the most appropriate may be successfully found. Equal weights tend to be given to the considered responses in the vast majority of the NTM processes under examination in order to simplify the computational procedures. Future research may focus on the effects of varying the weights given to the replies on the optimal parametric settings obtained by the five multi-objective mathematical tools.Furthermore, the proposed methods for multi-objective optimization can be utilized to identify the best combinations of parameters for various machining processes, including both conventional and non-conventional techniques. These methods have the potential for broader applications in complex problems of resolving tactical and operational decision-making challenges including social, political, environmental. To streamline the process and reduce human errors in tackling complex decision-making problems. In addition to keeping an eye on the performance of such a modified model, it would be appropriate to assess the various degrees of uncertainty in the input parameters themselves. The validity and accuracy of a variety of fuzzy sets that can enhance the model should be checked. Additionally, examining how interventions, such as training programs or diversity initiatives, can mitigate potential biases and enhance decision-making effectiveness across diverse groups, would be valuable for organizations and policymakers. Overall, it would be beneficial to concentrate the paper on a bigger dataset utilising, for example stocks or stocks of continental businesses from a financial perspective as well as for other Organizations like energy, software, design, construction, etc. Last but not least, it would be desirable to update the further developed model and integrate it, for example, with neural networks.

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