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STUDY OF GENERALIZED FUNCTIONAL IDENTITIES IN PRIME AND SEMIPRIME RINGS

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Abstract

Rings and algebras have long been studied for their own intrinsic properties. An ongoing area of interest has been to explore the implications of various special identities and, conversely, identifying sufficient conditions for a given ring to satisfy a particular identity. Ring derivation is a branch of algebra that focuses on the study of the structure of additive maps and the structure of rings by analyzing functional identities related to these additive maps. These additive maps are derivation, skew derivation, generalized derivation, generalized skew derivation, b-generalized derivation, X-generalized skew derivation, generalized (α, β) derivation etc. It is well established that there is a significant relationship between functional identities involving derivations as well as generalized derivations and the structure of rings. The key purpose of this thesis is to investigate various functional identities and generalized functional identities within the context of prime and semiprime rings. Let R be an associative ring. A basic example of a functional identity is the identity $[f(x), x] = 0$ for all $x \in R$, where $[x, y] = xy - yx$ and $f : R \to R$ is a mapping. In 1957, Posner [1] examined a specific type of functional identity in rings by considering the above function as a derivation. Later on, Brešar [2] demonstrated that if f is an additive mapping satisfying the identity investigated by Posner [1] in a prime ring R, then f must be of the form $f(x) = \lambda x + \xi(x)$, where $\lambda \in C$, $\xi : R \to C$ is an additive map and C is the extended centroid of R. Subsequently, numerous researchers have explored various functional identities and achieved remarkable results. We have also investigated some problems within this area.

This thesis is composed of eight chapters. Chapter wise brief information is given bellow:

Chapter 1 is basically devoted for introductory purpose. Some basic definitions, preliminaries and prerequisites which have been collected from other references and which are needed for the development of the subsequent chapters for this thesis.

In **Chapter 2**, we study some commutativity theorems involving generalized (α, β) derivations on left sided ideals in prime and semiprime rings. Some examples are given at the end of this chapter concluding that the hypothesis of semiprimeness or primeness in the results are not superfluous.

Recently, De Filippis [3] introduced the new map X-generalized skew derivation. This concept covers the concept of generalized skew derivation as well as b-generalized derivation.

In **Chapter 3**, we study an identity with annihilating and centralizing conditions involving X-generalized skew derivations in prime rings.

A number of authors have studied some functional identities on the evaluations of a non central valued multilinear polynomial.

In **Chapter 4**, we study an identity involving annihilating and centralizing conditions of generalized derivations acting on noncentral multilinear polynomials in prime ring.

In **Chapter 5**, we study an identity involving three generalized derivations acting on multilinear polynomial in prime rings.

In **Chapter 6**, we study a derivation which vanishes when applied over a identity involving two generalized derivations acting on noncentral multilinear polynomial. We obtain results for the inner case, and furthermore, we clarify all possible forms of the derivation and generalized derivations involved in the identity.

In **Chapter 7**, we study an identity involving two nonzero generalized skew-derivations acting on multilinear polynomials in prime rings.

An additive subgroup L of R is said to be a Lie ideal of R, if $[l, r] \in L$ for all $l \in L$ and $r \in R$. The m-th commutator of a, b is defined as $[a, b]_m = [[a, b]_{m-1}, b]$, $m = 1, 2, \ldots$. It is easy to check that $[a, b]_m = \sum_{i=0}^{m} (-1)^i \binom{m}{i} b^i a b^{m-i}$.

In **Chapter 8**, we study an m-th commutator identity involving three generalized derivations acting on elements in Lie Ideal L.

Last of all we give some references from where we get many valuable results which help us to develop the whole work.

References

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