STUDY ON FREE VIBRATION BEHAVIOUR OF ROTATING FUNCTIONALLY GRADED MICRO-DISKS

Abstract Submitted

by

Suman Pal

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Doctor of Philosophy
(Engineering)

Department of Mechanical Engineering

Faculty Council of Engineering & Technology

Jadavpur University

Kolkata, India

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Abstract

A rotating disk is considered to be a very important machine component due to its wide applications in various fields of engineering and technology. From the perspective of operating performance, durability, reliability and safety, a comprehensive knowledge on free vibration behaviour of rotating disks is essential for design and analysis of machines and instruments involving rotating disks. The coincidence of the free vibration frequency of a rotating disk with its rotational speed marks an important event in the dynamics of rotating systems. In recent times, the reality of miniaturization has shifted the focus of modern scientists and researchers towards the use of micro- and nano-size components in different conventional applications of engineering and technology. The theoretical study of micro-size structural components including rotating disks has become an emerging research area because the experimentation involving micro-size components remain a very costly and complicated affair. Moreover, the research area involving the theoretical study of micro-size components is considered challenging because the classical continuum theories are unable to capture the small-size effect. It is found that the modified couple stress theory (MCST), which is a continuum based non-classical theory, is being extensively explored by the researchers to investigate the mechanical behaviour of various micro-size structural components. Additionally, to have tailor-made material compositions for rotating micro-disks, especially in high-temperature applications, a noble family of composite materials called functionally graded (FG) materials (FGMs) can be used. In FGMs, the continuous variation of the volume fractions of the constituents enables the design engineers and scientists to achieve direction-specific properties. For rotating machineries, especially those involved in power generation in high-thermal environment, increase in the operating temperature is often desirable to increase the thermodynamic efficiency and power density of the system. In those applications, the FGMs are two-phase composites consisting of a metal phase and a ceramic phase. In such FGMs, the metal phase provides the necessary strength and toughness of the overall structure, and the ceramic phase provides the necessary thermal resistance in order to make the structure suitable for high-temperature application.

The research domain to study the free vibration behaviour of rotating micro-disks involving non-classical theories is very new, and has many research gaps. Based on these research gaps, the present thesis has been thematized to study the free vibration behaviour of rotating bidirectional FGM (BFGM) micro-disks based on the

MCST. To be specific, the following four problems have been investigated in the present thesis work: ***First problem*** - *Free vibration behaviour of a BFGM annular clamped–free micro-disk, which is rotating at constant angular speed, and operating in high-temperature environment*; ***Second problem*** - *Free vibration behaviour of a BFGM annular clamped–free micro-disk, which is rotating at constant angular speed, subjected to uniform transverse pressure, and operating in high-temperature environment*; ***Third problem*** - *Free axisymmetric bending vibration behaviour of a BFGM annular clamped–free micro-disk, which is rotating at constant angular speed, subjected to uniform transverse pressure, and operating in high-temperature environment*; ***Fourth problem*** - *Free vibration and static buckling behaviours of a BFGM annular micro-disk under different boundary conditions, which is rotating at constant angular speed, and operating in high-temperature environment*.

The present mathematical model is displacement based. It is formulated using Kirchhoff plate theory coupled with von Kármán nonlinearity for circular plates using polar coordinates. The small size-effect in the micron level is addressed within the framework of the MCST, and assuming linear elastic material behaviour. The micro-disk is assumed to be FG along the radial and thickness directions, and hence it is termed as a BFGM micro-disk. The thermal effect due to high-temperature operating environment has been incorporated into the mathematical model through thermo-elastic constitutive relations. The mechanical properties of the FGM constituents are considered temperature-dependent based on Touloukian model. The effective material properties of the BFGM material are calculated using the rule-of-mixtures (Voigt model). The problem is formulated using two steps. In the first step, the deformed configuration of the BFGM micro-disk under time-invariant centrifugal loading, uniform transverse pressure and thermal loading is determined. The governing equations for this step are derived using the principle of minimum total potential energy. In the second step, the free vibration response of the micro-disk about its deformed configuration is determined. The governing equations for this step are derived using Hamilton's principle. The governing equations for both the steps are discretized following Ritz method with the help of admissible orthogonal functions. The set of governing equations for the first step are nonlinear due to the presence von Kármán type nonlinear relations, and is solved employing Broyden's algorithm. The set of governing equations for the second step constitute an eigenvalue problem, and is solved using a standard eigen-solver.

A convergence study has been undertaken to decide on the appropriate number of functions, required for the Ritz approximation. The validity of the model has been established successfully through various comparison studies with the available results in the literature. A reasonable volume of numerical results has been generated to investigate the effects of a variety of physical parameters such as rotational speed, applied transverse pressure, thermal loading, size-dependent thickness, material gradation indices, radius ratio and FGM composition. The free vibration response of the BFGM micro-disks has been presented for both axisymmetric and asymmetric bending modes as well as for the torsional mode. The problem is mainly investigated for clamped-free BFGM rotating micro-disks. However, in order to study the effects of different boundary conditions, a separate study has also been undertaken for different boundary conditions namely, simply supported-simply supported, clamped-simply supported, simply supported-clamped and clamped-clamped. While considering different boundary conditions, the critical condition leading to static buckling of the micro-disk, when the vibration frequency becomes zero, has been identified and reported. Four different metal-ceramic FGM compositions namely, Stainless Steel/Silicon Nitride, Stainless Steel/Alumina, Stainless Steel/Zirconia and Titanium Alloy/Zirconia have been considered for presentation of results. Three-dimensional mode-shape plots along with contour plots have also been presented to visualize the axisymmetric and asymmetric bending vibration modes as well as the torsional vibration mode.

The presented formulation is quite general in nature, and can be applied for a wide variety of problems, apart from the specific problems of the thesis, by properly adjusting certain parameters. That the present model successfully solved the four research problems of the thesis clearly justifies the novelty of the model. Additionally, the fact that the model can be employed to solve a wide variety of problems related to mechanical behaviour of micro-disks/plates is a testament of the robustness of the present model. The problem-specific results presented as well as the overall summary of the results are new of its kind. These results and the corresponding findings will definitely serve as benchmark for further study in the domain of dynamics of rotating micro-disks.