“Love for Variety”, Outside Option, and Platforms

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By

Sovik Mukherjee

(Registration Number: AOOEC1100117)

Department of Economics

Jadavpur University

Kolkata - 700032, India

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Certified that the Thesis entitled

"Love for Variety", Outside Option, and Platforms submitted by me for the award of the Degree of Doctor of Philosophy in Arts at Jadavpur University is based upon my work carried out under the Supervision of Prof. Tanmoyee Banerjee (Chatterjee), Professor, Department of Economics, Jadavpur University and Prof. Vivekananda Mukherjee, Professor, Department of Economics and Finance, BITS Pilani Hyderabad Campus. And that neither this thesis nor any part of it has been submitted before for any degree or diploma anywhere / elsewhere.

Countersigned by the Supervisors :

Tanmoyee Banerjee

Prof. Tanmoyee Banerjee (Chatterjee)
Professor, Department of Economics
Jadavpur University
Kolkata
Dated: 20.11.2024

PROFESSOR
DEPARTMENT OF ECONOMICS
JADAVPUR UNIVERSITY
KOLKATA - 700 032

Sovik Mukherjee

Candidate : Sovik Mukherjee
Dated : 20.11.2024

Vivekananda Mukherjee

Prof. Vivekananda Mukherjee
Professor, Department of Economics and Finance
BITS Pilani Hyderabad Campus
Hyderabad
Dated: 20.11.2024

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No one who achieves success does so without acknowledging the help of others. The wise and confident acknowledge this help with gratitude. - Alfred North Whitehead

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Kolkata *Sovik Mukherjee*

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Preface

Platforms are referred to as “two-sided” markets when they bring together two user categories, namely, buyers and sellers, to enable a transaction from which both sides benefit. These days, it is hard to imagine our lives without platforms. We use them in virtually all areas of our daily lives: for socializing, buying books and clothing apparel, listening to music and watching movies, selling old items, renting rides, housing opportunities, and much more. Social media platforms like Facebook, entertainment platforms like Netflix and Disney Hotstar, e-commerce platforms like Amazon, Flipkart, and eBay, ride-hailing platforms like Ola and Uber, and food delivery apps like Zomato and Swiggy, among others, are common names. How are the platforms different from the firms we study in the conventional Microeconomics? How do they interact with the conventional firms? Do we need to regulate them? These are the questions that intrigued me at the start of the current thesis.

The thesis has been divided into five chapters. The first chapter gives a detailed outline of the entire thesis: background, a survey of existing literature, research objectives, methodology, and contributions.

It has three core chapters: Chapters 2, 3, and 4. Chapter 2 analyses the effect of a ‘love for variety’ preference pattern, a non-zero outside option on the buyer side, and uniform-price Bertrand-price competition among the sellers using a delegation common agency framework in a differentiated product oligopoly market. Chapters 3 and 4 extend the model developed in Chapter 2 to understand the choices of a platform.

Chapter 3 introduces a monopoly platform and focuses on its role in increasing the number of varieties in a differentiated product market where buyers have a ‘love for variety’ preferences and Bertrand-price competition exists among the participating sellers. The importance of different platform-efficiency-related, seller-side and buyer-side parameters in characterizing the equilibrium has been highlighted.

Using a framework similar to Chapter 3's, in a spatial competition setting, Chapter 4 incorporates the location choice of a platform (imagined as a shopping mall) in a differentiated product market and studies its interaction with an outside option (being a homogeneous goods market) located on one end of a linear city.

Chapter 5 concludes the thesis and discusses the policy implications of the results derived in the core chapters. It also outlines the future research agenda. I am hopeful that the contents of this thesis, described in each of the core chapters, will benefit researchers, business practitioners, policymakers, antitrust regulators, and urban planners interested in platform economics. All opinions, omissions, and errors remain my own.

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Chapter 1

Introduction

1.1 Background

Platforms refer to a market situation where distinct groups interact with each other through an intermediary. The number of members on the opposite side determines the value of joining the platform. This is called cross-side externality benefits. A platform internalizes this cross-sided externality in its profit-maximizing behavior. As a third party, the platform creates a place or space where two consumer groups, the buyers and the sellers, can get together to carry out the transaction. The platform's structure is pertinent and holds good if and only if the consumers on both the buyers' side and the sellers' side cannot come to an "efficient agreement"[1] outside the platform. In 2024, five of the top ten biggest companies by market cap are structured around platforms.[2] Some examples of sector-specific online or digital platforms include social media platforms like Facebook, e-commerce platforms like Amazon and eBay, credit card platforms like VISA and Master Card, Google web portal as an advertising platform, software platforms like Sony PlayStation video games, entertainment platforms like Netflix and Disney Hotstar, hosted platforms like Airbnb and OYO, ride-hailing platforms like Ola and Uber, food delivery apps like Zomato and Swiggy, and the list goes on. In reality, it is not only about competing platforms but also monopoly platforms. In the actual world, there are several instances of monopoly platforms. As Mukherjee and Mukherjee (2020) point out, the monopoly platform paradigm seems suitable when there is only one magazine or newspaper or a monopoly shopping mall in some area or the Yellow Pages directory of an incumbent telephone company, to name a few. Another example of a monopoly platform is Bharat Matrimony. In the Indian

[1] Rochet & Tirole (2004) define efficiency as the maximum joint utility of the consumers on both sides. The platforms charge some fees, which can be either a fixed membership fee, a per-unit usage fee, or a combination of both, to make such an "efficient agreement" possible.

[2] These five companies are - Apple, Microsoft, Amazon, Alphabet (Google), and Meta Platforms. Check the complete list of "Top 10 biggest companies in the world by market cap in 2024" at https://www.forbesindia.com/article/explainers/top-10-largest-companies-world-market-cap/86341/1

context, when the matchmaking industry began in 1997, it was the sole online website. Online platforms have certain unquestionable benefits that have helped them gain a larger client base over time and achieve faster growth rates. What makes them so popular is the simplicity of conducting business seamlessly with anyone living anywhere in the world by eliminating physical restrictions.

One of the issues that intrigued me is that people usually think of platforms as online or digital and often do not pay more attention to offline ones like shopping malls. The motivation for using shopping malls comes from the rapid rise in the number of malls worldwide[3] and their contribution to determining the quality of urban life. Whether the mall should locate itself in or away from the city has essential implications for forming city structures (Malykhin & Ushchev, 2018), thus making the location choice important. Shopping malls have become attractive because they allow consumers access to different goods and varieties of a particular good in one place without spending much time and money commuting between shops. Moreover, the mall offers a wide range of stimuli: coffee shops, food court establishments, and movie theaters, making it a desirable place to spend time (Farrag et al., 2010). Also, it is common for malls not to charge buyers any membership/entry fee for visiting the malls, thereby adding to their attractiveness. Though COVID-19 changed how people shop, with buyers preferring a more "touch-free"[4] experience, the footfall in malls across the globe did take a hit. India was no exception.[5] However, since 2023, companies including Tata Trent, PVR, Aditya Birla Fashion, and Reliance Retail have greatly expanded the number of their stores.[6] A report in The

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[3] The size of the shopping mall market is expected to rise from USD 5.90 trillion in 2022 to USD 9.41 trillion by 2031. See https://www.grandviewresearch.com/industry-analysis/shopping-centers-market-report

[4] Reported at https://www.across-magazine.com/the-mall-as-a-platform/

[5] https://www.trade.gov/country-commercial-guides/india-online-marketplace-and-e-commerce

[6] Reported at https://www.mytotalretail.com/article/how-malls-are-making-a-comeback-in-2024/

Economic Times (January 4, 2024) states, "In 2023, as many as 11 shopping malls became operational, covering 59,48,395 square feet of space across the top eight cities. In the previous year, nine malls came into eight markets, totaling 34,49,222 square feet area. Hyderabad witnessed the completion of three shopping malls, while Pune and Chennai had two each. One shopping mall each came up in the Mumbai Metropolitan Region, Delhi-NCR, Bengaluru, and Ahmedabad."[7] So, malls as a platform have again started flourishing in large cities in India, despite e-commerce recently becoming the next big thing in the retail sector and business houses concentrating on internet platforms. The thesis will focus on online platforms like e-commerce and offline ones like shopping malls to develop the theoretical models.

Besides these advantages, the platforms, be it online or offline as discussed, are known for offering many varieties of products for sale, produced by different sellers, in one place, attracting buyers. This aspect is one of the key focus areas of the thesis. The buyers get access to a variety of products in one place, reducing their search costs. For instance, large e-commerce platforms in India like Amazon and Flipkart, among others, give buyers the option to choose from a wide range of goods and services provided by vendors featured on their websites. The same is true for the malls. These days, operating system platforms like Google's Android and Windows are the foundation of many consumer electronics devices, including smartphones, smart televisions, and even car navigation systems. These platforms enable users to download and utilize hundreds of applications created by third-party developers[8] compatible with their respective platforms. Also, About 500 million Stock Keeping Units (SKUs) were available on Amazon in 2016; that figure has risen to an incredible 4.5 billion for USA alone in 2023.[9] Thus,

[7] Read at: https://economictimes.indiatimes.com/industry/services/property-/-cstruction/why-bigger-and-more-shopping-malls-are-coming-up-in-top-cities/articleshow/106550628.cms?from=mdr

[8] For instance, we use the food delivery app Swiggy on our Android phones.

[9] Reported at https://amzscout.net/blog/amazon-statistics/

variety has become the "bread and butter" of such marketplaces.[10] The more apps, films, content, etc., a user can choose from online, the better it is. Similarly, there are more shops in a mall to choose from, so buyers will be attracted.

In modern-day economies, one question that has been doing the rounds is the issue of the coexistence of 'brick-and-mortar' shops vis-à-vis shopping malls/e-commerce platforms selling varieties of a differentiated product. Typically, a 'brick-and-mortar' store stocks fewer product varieties than a shopping mall/e-commerce platform. For instance, Amazon lists 57 times as many book titles as a typical physical bookstore, according to Brynjolfsson et al. (2003). In 2020, Walmart's online store had 75 million SKUs[11], whereas an offline Supercenter in the US stocked only 120,000 different items.[12] Then, how does the existence of the 'brick-and-mortar' store affect the shopping malls/e-commerce platforms and *vice versa*? This makes the theme of 'love for variety' and the availability of an outside option very much applicable in the present context regarding two-sided platforms.

There are different ways of modeling 'love for variety', namely, 'ideal variety' (Spence, 1976; Salop, 1979; Perloff & Salop, 1985) and 'love for variety' (Dixit & Stiglitz, 1977). In the models developed in the thesis, the focus is on the Dixit-Stiglitz type of preferences. To match this idea of 'love for variety' preferences where the buyer consumes some amount of every variety of the product available at the market, we started working with a Dixit-Stiglitz type of (Dixit & Stiglitz, 1977) utility function. As we started working on the modeling section, first, we noticed that in the existing literature on Bertrand-type price-competition models in differentiated product oligopoly markets, on their demand side specification, usually assumes

[10] Reported at https://platformpapers.substack.com/p/five-ways-to-boost-platform-growth

[11] https://multichannelmerchant.com/ecommerce/walmart-gives-shopifys-1-million-sellers-access-marketplace/

[12] See Brynjolfsson et al. (2022)

the presence of a representative buyer[13] with continuous choice in a commodity space (Singh & Vives, 1984; Häckner, 2000). To capture the presence of multiple varieties in a differentiated oligopoly market, Anderson et al. (1995) introduce consumer heterogeneity, but it comes at the cost of the presence of multiple price equilibria, leading to problems of existence. Second, in many of the initial works in this area of differentiated products, a monopolistically competitive set-up was used, contrary to a differentiated oligopoly market, where an *a priori* assumption of "a large number of firms", each selling a small amount[14] implies firms' actions having a negligible impact on other firms' profits were used. To put it simply, strategic price interactions among firms were absent. Also, the existing literature on platforms does not determine the number of varieties sold in the differentiated product market and assumes it to be exogenously given. In the context of two-sided platforms, it is, thus, evident that the requirement for these dimensions is of paramount importance. Besides, the endogenous location choice configuration is missing from studies in the existing literature for an offline platform like a shopping mall.

In this context, the existing literature fails to provide a unifying framework for studying these dimensions: 'love for variety' with heterogeneous buyer preferences, strategic price interactions among the sellers of the varieties of the differentiated product, an outside option in a differentiated oligopolistic framework and the location choice of a platform (in case it is an offline platform). Hence, the motivation behind choosing such a theme for doctoral research. The thesis attempts to integrate these dimensions into the theoretical modeling frameworks combined with real-life examples to motivate the same.

The survey of literature relevant to the thesis follows in the next section.

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[13] Kirman (1992) commented on the debatability of the representative agent set-up and stated that it "deserves a decent burial, as an approach to economic analysis that is not only primitive, but fundamentally erroneous."

[14] See Yang and Heijdra (1993)

1.2 Survey of Literature

Focusing on 'love for variety' and a non-zero outside option, the thesis aims to extend the existing literature in three specific dimensions: first, in the context of differentiated oligopoly in a delegation common agency framework; next, for a two-sided monopoly platform; and finally, in the context of spatial competition with the location choice of a platform. This section looks into relevant studies in the existing literature pertaining to the discussion in the upcoming chapters.

To begin with, in the traditional one-dimensional product differentiation literature, two models prevail, namely horizontal (Hotelling, 1929) and vertical (Mussa & Rosen, 1978; Gabszewicz & Thisse, 1979; Shaked & Sutton, 1982) differentiation. Following Lancaster (1979), two products are said to be horizontally differentiated when both products have a positive demand whenever they are offered at the same price. Neither product dominates the other in terms of characteristics, and heterogeneity in preferences over characteristics explains why both products are in the market. On the other hand, these are considered vertically differentiated if one product captures the whole demand when both are supplied at the same price. In other words, one product dominates the other in terms of 'quality,' 'brand name,' etc., and such perceived differences in product characteristics give the consumer a preference for ordering based on her willingness to pay. Given the complexity of product characteristics, most products embody both types of differentiation (Neven & Thisse, 1990; Caplin & Nalebuff, 1991; Anderson et al., 1992; Gabszewicz & Wauthy, 2012). However, this strand of literature was silent as to how one should design a differentiation model when a consumer consumes a little bit of every available good rather than her most preferred product. Contrary to the standard vertical and horizontal differentiation models, the pioneering work of Dixit and Stiglitz (1977) introduced us to the idea of a 'love for variety' preference where the consumer consumes some amount of each of the varieties of the differentiated product.

Monopolistic competition has been instrumental in expanding theoretical and empirical research boundaries in several economic domains in the last few decades, ranging from international trade, growth economics, and industrial organization to economic geography, among others. Dixit and Stiglitz's (1977) "household preference aggregator" (see Stiglitz, 2017) has been used to illustrate monopolistic competition in almost all of these studies. However, one of the major drawbacks is the absence of strategic price interactions among the firms (Brakman & Heijdra, 2002), which assumes that the number of firms is infinitely large.[15]

Moving away from assuming the absence of strategic price interactions among the varieties, Chapter 2 starts with a differentiated oligopoly market where the sellers compete in prices in delegation common agency games (see Martimort & Stole, 2009). As argued, having an alternative consumption program in our context is important. Their approach, however, does not accommodate buyers' non-zero outside options or 'love for variety', a choice criterion frequently cited in the literature on differentiated product markets. The impact of such a framework on the price-setting game's supermodular structure[16] has also not been explored in the context of a strong extensive margin effect discussed in Chapter 2. Papers like Anderson et al. (1992) and Cosandier et al. (2018) discuss the violation of supermodularity through the network effect due to differences in tastes and incomplete product awareness, respectively. Additionally, the coexistence of a 'love for variety' preference pattern and non-zero outside option makes our framework suitable for discussing the interaction between

[15] The CES utility function is used in the Dixit-Stiglitz-Krugman (DSK) (Dixit & Stiglitz, 1977; Krugman, 1981) model of monopolistic competition to capture the love of variety aspect. It assumes that a sufficiently large number of varieties exist, which makes the elasticity of the demand curve faced by the firms equal the "CES between varieties" (Cox & Ruffin, 2010), meaning that the impact of other firms' prices is minimal. Therefore, a free entry equilibrium solves for the number of varieties with fixed markups.

[16] As the current thesis does not discuss how supermodularity operates in oligopoly theory in detail, one can refer to Amir (2005) for the details.

‘brick-and-mortar’ stores and shopping malls/e-commerce platforms. There are two strands of literature. Industry-specific papers like Spanke (2020), Belussi and Rakic (2019), and Bhatnagar and Yadav (2022) argue that ‘brick-and-mortar’ shops and shopping malls/e-commerce platforms will grow simultaneously, contrary to the dominant view in the literature on country-specific studies, which predicts that it will not, more so after COVID-19 (see Salem & Nor, 2020; Keels, 2021; Afridi et al., 2021; Li et al., 2022 among others). Chapter 2 adds to this literature.

Coming to two-sided platforms, since the seminal works of Katz and Shapiro (1985) and others, markets with network externalities have been extensively studied. Two-sided platforms are essentially markets with a special kind of network externality. As described in Rochet and Tirole (2003), the higher the number of sellers, the value of the buyer from joining the platform increases, and the same thing is true from the buyer side, exemplifying a cross-side externality advantage. In other words, the prices charged by the platform from agents on one side of the market affect the participation of the other side in the market. In the context of the relevant literature on platforms, platform pricing in monopoly setting has a rich literature pioneered by Rochet and Tirole (2004, 2006). Rochet and Tirole (2004) incorporate both the per-unit usage and fixed membership charges on both sides of the market. The working definition we use in our context follows from Rochet and Tirole (2004) and has been refined by Roson (2005) and stated as - a platform to be called “two-sided” if

> “……… any change in the price structure (or distribution) on one side would affect participation levels not only on that side but also on the opposite side and the number of interactions on the platform. Thus, it would become important to consider who pays what, in order to get both sides on board.”

In Chapter 3, we use only the platform's fixed membership fee levied on both the buyers and the sellers. Following Rochet and Tirole (2006), a combination of fixed and variable fees can also be introduced in our model without any loss of generality. Papers by Weyl (2010), Armstrong (2006), Caillaud and Jullien (2003), and Rochet and Tirole (2003) have pointed out that it is common in reality where one side of the market is charged while the other side is subsidized. This phenomenon is common in newspapers, social sites, credit card markets, and shopping malls, to name a few. We use the same concept in Chapter 4, where we assume that the mall, being a two-sided platform, does not charge the buyer side.[17] The differentiated products on the platform are discussed by papers like Galeotti and Moraga-González (2009) and Hagiu (2009), which are closer to the scope of the thesis. The buyers' non-zero outside option is equally important because it theoretically captures the trade-off between the 'brick-and-mortar' shops and the platform (as discussed in Belleflamme & Peitz, 2021). This chapter introduces a non-zero outside option in a monopoly platform framework, unlike Galeotti and Moraga-González (2009) and Hagiu (2009). The key finding in Hagiu (2009) relevant to our context involves the impact of the exogenously defined "intensity" of consumers' preferences for variety, in other words, a fall in the substitutability among varieties, on the platform's pricing structures. The paper finds that the monopoly platform earns more profits on the seller side as the higher the consumer's preference for variety, the lesser will be the extent of rivalry among producers and, consequently, more rent extraction power in the hands of the producers. Thus, increasing the fee on the seller side. It also finds opposite results under competing platforms. However, Hagiu (2009) reaches this conclusion for a monopoly platform by means of comparing the share of profits on both sides defined based on an exogenously fixed number

[17] Apart from sometimes charging a very negligible parking fee. For instance, the rulings of the Kerala High Court in 2022 and Gujarat High Court in 2019 can be references that do not allow malls to charge parking fees. In Pune, the local bodies have made parking free at the malls.

of varieties. The overall impact on the profit of the platform has not been captured in his paper. Chapter 3 of the thesis fills addresses this research gap. For a monopoly platform, Galeotti and Moraga-González (2009) find that with an increase in the exogenously given number of varieties, competition among the sellers rises, and the monopoly platform lowers the fee on the seller side relative to the buyer side. However, a fall in the substitutability among the varieties of the differentiated product weakens competition among sellers. It raises the platform's profit; hence, a higher fee is charged from both the buyer and seller sides. We add to this by exploring the possibility where the platform earns a higher profit even by charging a smaller fee on the seller side. Contrary to a monopoly platform setup, Economides and Katsamakas's (2006) model in the context of competing platforms shows that a strong preference for a variety of operating system platforms may lead to higher total profits in a proprietary platform (e.g., Windows) as compared to an open-source platform (e.g., Linux). A similar result was discussed in Manten and Saha (2012). The more recent literature on platforms (Bergemann & Bonatti, 2024; Bergemann et al., 2022; Bounie et al., 2021; Kirpalani & Philippon, 2020) focuses on the exploitation of sellers' and buyers' information acquired by the platform in the process of the transaction. Also, the other area where the issue of variety available on platforms has been addressed is how a firm's distribution channel structure impacts its optimal product variety decisions (Rajagopalan & Xia, 2012; Guo & Heese, 2017; Sweeney et al., 2022). However, in the context of a monopoly platform, it does not look at the issue of 'love for variety' from the information issue perspective or the distribution channel structure. In the context of platforms, the existing literature does not address the platform's role in increasing the number of varieties. However, in a different setting than ours, Nyarko and Pellegrina (2022), in the African context, compare the intertemporal coexistence of a centralized agricultural commodity exchange (similar to our platform) vis-à-vis decentralized bilateral trade between farmer and trader. They emphasize the role of the centralized exchange and show that more farmers get a chance to enter the market in the presence of the centralized commodity exchange, unlike in the case of bilateral trading. Their conclusion shows that more farmers enter in the presence of a centralized exchange. This chapter of the thesis, in contrast to its predecessors, not only endogenously determines the number of varieties sold on the platform and outside it but also highlights the importance of different platform-efficiency-related, seller-side, and buyer-side parameters in characterizing the equilibrium.

Moving on, another dimension of the literature on 'love for variety' that the thesis addresses is the literature on spatial competition and location choice of a platform. There is a literature that looks at the choice of location of firms in spatial markets under three different cases – i) where firms are engaged in monopolistic competition (Ago, 2008; Ago et al., 2017), ii) Cournot competition in a linear city (Hamilton et al., 1989; Anderson & Neven, 1991), and iii) Cournot competition in a circular city (Gupta et al., 1997; Matsumura & Shimizu, 2005; Chen & Lai, 2008). However, the choice of location of a platform, for instance, a shopping mall, has not been sufficiently addressed in the literature. In this context, one of the pioneering works is by Smith and Hay (2005), where they study the interactions between three alternative modes of retail organization: the street marketplace, malls, and supermarkets. However, the location choice of such marketplaces is fixed exogenously, and each supplies the same goods. It is also assumed that the volumes and prices of the products purchased are independent of firms' strategies, consumers' locations, and the location of each of the marketplaces. However, the cross-sided network effects, which are common to platforms, have not been discussed. Brando et al. (2014) consider a setting where a shopping mall and a supermarket compete by supplying the same range of goods, and locations are fixed on two extremes of the linear city. Ushchev et al. (2015) extend this model by introducing i) a downtown retail market and a monopoly shopping mall on both ends of the linear city and ii) buyers have a 'love for variety' preference. Some varieties are sold in the retail market, and some are sold in the mall. The downtown retail market is characterized by free entry, while the monopoly shopping mall allows sellers by charging a per-slot fee. Though the location of the marketplaces is fixed, the size of the marketplaces gets endogenously determined. In the literature on social media platforms, a recent paper by Mishra and Sarkar (2023) studied a platform duopoly where the platforms are located on the extremes of a linear city, similar to Uschev et al. (2015). The difference between Mishra and Sarkar (2023) and Ushchev et al. (2015) is that the former assumes a duopoly involving two-sided platforms, while the latter assumes one platform and one marketplace characterized by monopolistic competition. Consumers, depending on their location, can either single-home or multi-home. Thus, one of the significant research gaps in the existing literature in this regard is not endogenizing the location choice of a two-sided platform, like a shopping mall. The role of the mall in increasing the number of varieties is something that the literature has not addressed in this context. Furthermore, the other dimension in the literature that needs to be examined is the use of Bertrand-price competition in this type of spatial structure. While the prices derived in Brando et al. (2014) depend on the exogenous number of varieties, Ushchev et al. (2015) derive the Dixit-Stiglitz prices (Dixit & Stiglitz, 1977). However, this result is empirically not supported (see Lach, 2002; Kaplan et al., 2019). Chapter 4 tries to fill in these research gaps. The impact of an increase in the substitutability of the varieties on the number of varieties sold both on the platform and outside it and also on the platform's profit is a common theme in the literature (Belleflamme & Peitz, 2019). To the best of our knowledge, other parameters like the platform's efficiency in servicing its clients on both sides, the fixed costs faced by the sellers, improvements in the outside option for Chapter 3, and, in addition to these, travel costs in Chapter 4 have not been adequately discussed in the existing literature.

The following section outlines the research questions, and the methodology used for the theoretical modeling in the thesis.

1.3 Research Objectives and Methodology Used

Building on the research gaps discussed in the last section, the three key chapters address the following objectives.

Chapter 2 analyzes how introducing a 'love for variety' preference and a non-zero outside option for the buyers impact the oligopoly prices in a differentiated product oligopoly market. Also, the chapter tries to give insights into how the extensive margin of demand affects the supermodular structure of such a price-setting game. Next up, the model characterizes the equilibrium to endogenously determine the number of varieties sold and the number of buyers in the market. The chapter highlights the role of the extensive margin of demand effect and, in this context, finds out how the prices of the varieties and the number of sellers in the market get affected by the marginal cost incurred by the sellers, substitutability among the varieties of the differentiated product, and an improvement in the outside option.

Chapter 3 discusses how the presence of a platform increases the number of varieties in a market and the factors that determine the number of varieties sold on the platform. The platform internalizes the externalities either side of the market creates for the other side and decides the optimum membership fee for both sides. The comparative static exercises focus on the impact of the platform's costs of servicing its clients on both sides and the fixed costs faced by the sellers in their operation on the number of varieties sold both on the platform and outside it and also on the platform's profit. Moreover, we explore how the presence of an outside option for buyers (like the presence of 'brick-and-mortar' shops) affects a platform and whether greater differentiation of varieties is good for a platform.

Chapter 4 models the platform's location choice using a framework similar to Chapter 3. In this setting, the chapter assesses the impact of such location choice on the price of the variety of the differentiated product. It analyzes how buyer-side parameters like 'love for variety' preferences and travel costs impact consumer behavior. Like Chapter 3, it emphasizes the role played by the shopping mall in increasing the number of varieties in the differentiated product market. In this light, the chapter finds out how the mall's membership fee on the seller side and its profit are affected by the cost of servicing buyers and sellers, the fixed cost of the sellers, substitutability among the varieties of the differentiated product, and travel costs. In line with the theme of the thesis, we study how the presence of an outside option, like a homogeneous goods market, affects the mall.

A short description of the methodology follows before briefly outlining the main contributions of the thesis. The thesis uses conventional theoretical modeling frameworks used in industrial organization to accomplish the given objectives. The sequential games we develop in each of the three core chapters involve the choices made by the agents in the different stages, and accordingly, the payoffs are realized. The models have been solved using the backward induction method.

The following section discusses the results and compiles the contributions of the three core chapters to the existing literature.

1.4 Results and Contribution to the Literature

As already discussed, Chapter 2 of the thesis introduces 'love for variety' and a non-zero outside option for the buyers in a delegation common agency framework under a differentiated oligopoly market, which is new to the literature. The central theme of the chapter is the presence of a strong extensive margin of demand effect. In the presence of such an effect, the chapter finds that the supermodular structure of a standard Bertrand specification may get changed. The price of a variety of the differentiated product may fall because of a rise in the marginal cost of production contrary to standard textbook results. Even the price and the number of varieties of the differentiated product may rise with an increase in the substitutability among the varieties of the differentiated product or with the availability of a stronger outside option. To validate this claim, the chapter provides empirical evidence to justify that the revenue of both the e-commerce platform and the 'brick-and-mortar' shops rises simultaneously in the presence of a stronger outside option.

Chapter 3 contributes to the existing literature on the economics of platforms in multiple ways: First, the model incorporates buyers' 'love for variety' preferences; Second, it allows the number of varieties of a differentiated product in the market and on the platform to be determined endogenously, and shows that platform benefits the buyers by increasing the number of varieties available at the market; Third, it characterizes the platform's pricing on both the side of the market for changes in costs to servicing to the clients, changes in fixed costs on the sellers' side, improved outside option of the buyers, and the lower substitutability of the varieties at the market. Besides, the comparative static exercises highlight the role of these factors in influencing the number of varieties sold both on the platform and outside it and the platform's profit. We find that a platform has more incentive to serve its sellers efficiently than its buyers. It dislikes developments that lower the sellers' fixed costs and create alternative options for buyers, such as buying from 'brick-and-mortar' stores. Additionally, the platform encourages sellers to sell varieties that are not close substitutes for one another. As commonly discussed in the existing literature that a fall in substitutability among varieties implies that stronger preferences for variety but less competition. Sellers are expected to gain more out of it. In equilibrium, thus, the platform charges both the seller side and the buyer side a higher membership fee and its profit rises. Given the 'love for variety' preferences and the endogenous extensive margin of demand, we add to the literature by exploring the possibility where the platform earns a higher profit even by charging a lower fee on the seller side. It is motivated to encourage the expansion of diversity in the variety of products offered in the market.

The contribution Chapter 4 makes can be summed up as follows. First, it combines in a single framework: spatial competition, 'love for variety' preferences on the buyer side, and Bertrand-price competition on the seller side to study the interaction between a shopping mall selling differentiated products and a homogeneous goods market in a linear city; Second, the mall chooses both its location in the linear city and the membership fees it charges from the participating sellers. Interestingly, since the mall's location affects the demand for the differentiated product, the prices of the varieties are influenced by the mall's location, which the thesis proves in this chapter. Third, the representative agent framework used in standard monopolistic competition models, assuming away price interactions, is replaced by introducing heterogeneity on the buyer side with varying preferences for purchasing the differentiated product and on the seller side with their varying transaction costs. Fourth, it enables endogenous determination of the number of the differentiated product produced both inside and outside the mall, demonstrating that the mall helps consumers by expanding the number of varieties available in the market. Fifth, it characterizes the mall's choice of location on the linear city and pricing on the seller side of the market for changes in costs to servicing to the clients, changes in fixed costs incurred by the sellers, improved outside option of the buyers, lower substitutability of the varieties at the market, and the transport costs incurred by the buyers for traveling to the mall. We highlight the impact of the mall's ability to choose its location when analyzing the impact of these parametric changes on the mall's profit. In contrast to Chapter 3, it has been noted in this chapter that under specific conditions on the shopping mall's revenue and cost sides, the shopping mall's profit increases as it becomes more efficient in serving both the buyers and the sellers. The mall might support developments that lower the seller's fixed costs, depending on its location and the impact on its profit. The comparative static exercise pertaining to the sellers' fixed costs in Chapter 3 did not present this case. It is also interesting to note that depending on the mall's revenue and cost considerations, changes in the travel costs on the buyer side, improvement in the outside option, and substitutability among varieties may increase the mall's profit and the number of varieties sold through it.

The following section discusses the outline of the chapters.

1.5 Outline of the Chapters

The first chapter outlines the background of the thesis. It has three main chapters, Chapters 2,3 and 4, each briefly described in this section. Chapter 5 concludes the thesis.

The current chapter establishes the main idea of the thesis and gives a background to what motivated this work. It surveys the existing literature and highlights the research objectives addressed in each core chapter. The theoretical methodology followed has been clearly defined. The significant contributions of the thesis to the existing theory are thoroughly illustrated in this chapter, followed by an outline of how the thesis has been organized.

Chapter 2, entitled "*Love for Variety*", *Outside Option, and Extensive Margin of Demand*, incorporates 'love for variety' and non-zero outside option for the buyers in the delegation common agency framework with uniform pricing. The chapter points out that it is not only the intensive margin of demand (the quantity demanded of a variety the buyer continues to purchase) that matters, the role of an extensive margin (participation of buyers in the market) is also important. It allows for a case where the monopoly price is lower than the oligopoly price even when the different varieties of the differentiated product are substitutes of each other. The chapter contributes to the standard 'love for variety' literature, as it introduces the strategic price competition aspect, which is usually ignored on the basis of *a priori* assumption of a large number of sellers. A buyer holding the Dixit-Stiglitz kind of utility function representing such preference, if decides to buy the differentiated product, owing to her 'love for variety' consumes some amount of every variety of the product available at the market. The chapter has argued that this creates a complementarity on the extensive margin of the delegation common agency game. So, even in the presence of substitutability among the varieties of the differentiated product, if the price of a particular variety rises, the demand for all other varieties falls in the presence of a strong extensive margin effect. This effect violates the supermodular structure of the price-setting game. The number of sellers and buyers is determined endogenously in the present framework. Then, the comparative static results of the model derive the effect of change in the degree of substitutability among the varieties and the effect of change in the outside option to the buyers both on the extensive and intensive margin of demand and on the equilibrium number of varieties produced in the differentiated product market. The role of a strong extensive margin effect in explaining the results is specifically highlighted. The co-existence of the ‘love for variety’ preference pattern and non-zero outside option makes the model discussed in the chapter suitable for discussing the interaction between ‘brick-and-mortar’ stores and shopping malls/e-commerce platforms.

Chapter 3 on “*Love for Variety*” *and Monopoly Platform* studies the role of a monopoly platform in increasing the number of varieties in a differentiated product market where the buyers have a ‘love for variety’ preference, and Bertrand-price competition exists among the participating sellers. In the model presented in this chapter, a buyer has a Dixit-Stiglitz type of ‘love for variety’ utility function, which implies that a higher number of varieties consumed increases the buyer's utility. The buyers are identical, except that their preference for the differentiated product differs from each other. Depending on her preference, the buyer also has the option of purchasing a homogeneous product. Each seller sells a specific variety of the differentiated product. Owing to her ‘love for variety’ utility function, she purchases some amount of every variety of the differentiated product, whether the seller is located at the platform or outside it. The sellers differ from each other in terms of their transaction costs. However, all of them, irrespective of their location inside or outside the platform, involve themselves in Bertrand-price competition with each other. The platform charges a membership fee for both buyers and sellers. The platform also incurs a cost for servicing both the buyers and the sellers. Only the registered agents can carry out transactions on the platform. We assume that buyers and sellers single-home on the platform if registered with it. The unregistered sellers and buyers do not pay any fee. The number of varieties sold on the platform, outside the platform and the number of buyers in the differentiated product market gets endogenously determined in the model. The platform internalizes the externalities either side of the market creates for the other side and decides the membership fee to charge to both sides. The chapter then discusses the comparative static exercises. It highlights the role of factors like the platform's costs of servicing the buyers and the sellers, the fixed costs faced by the seller side of the market in their operation, outside options of the buyers, and the substitutability of the varieties of the product, on the membership charged by the platform on either side of the market, the number of varieties sold both on the platform, and outside it, and the platform's profit.

Following the introduction of a platform in Chapter 3, Chapter 4, titled "*Love for Variety*" *and Location Choice*, incorporates the location choice of the platform in a differentiated product market and studies its interaction with an outside option in the form of a homogeneous goods market. This chapter presents a theoretical model of a monopoly shopping mall, being the platform, selling differentiated products and a homogeneous goods market in a linear city framework over a [0,1] continuum. The location of the homogeneous goods market is fixed on one end of the linear city and has not been explicitly modeled. At the same time, the shopping mall chooses its location at any point in $[0,1]$ and a membership fee to be charged on the seller side. The sellers of the differentiated product observe the membership fee and then decide whether to join the mall or operate outside of it at the same location. Bertrand-price competition among sellers to determine the prices of their products follows. Interestingly, the prices of the varieties are influenced by the choice of the location of the mall. The number of varieties sold on and off the mall, the location of the marginal buyer, and the number of buyers are endogenously determined. The theoretical model presented in this chapter combines several dimensions − spatial competition, 'love for variety' preferences on the buyer side, intensive and extensive margins of demand, travel costs, and Bertrand-price competition among the sellers participating in the differentiated product market. The choice of location of the mall is of great significance in the context of the results derived in the chapter. The chapter also discusses the comparative static exercises that follow the impact of buyer-side parameters like - the transport costs incurred by the buyers for traveling to the mall, outside options of the buyers, and the substitutability of the varieties of the product, seller-side parameters like - the fixed cost of the sellers and factors related to the platform's cost of servicing the buyers and the sellers on the location choice of the shopping mall, the membership fee charged to the sellers, the number of varieties sold in the mall and the mall's profit.

The thesis concludes in Chapter 5 by summarizing the critical findings obtained in Chapters 2, 3, and 4. It also highlights the policy implications and lays out future research directions. A list of bibliographic references follows.

Chapter 2

"Love for Variety", Outside Option, and Extensive Margin of Demand*

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2.1 Introduction

In delegation common agency games, where the sellers compete in terms of non-linear prices in a differentiated product oligopoly market, Martimort and Stole (2009) show that in the presence of demand substitutability between the products, the oligopoly market creates less distortion compared to a monopoly market both at the extensive margin (as it allows participation of greater number of buyers in the market), and at the intensive margin (as it sells higher quantity to them). However, their framework neither allows for 'love for variety', a preference specification widely used in the literature on differentiated product market, nor allows for non-zero outside option for the buyers. An analysis of 'love for variety' is important as it has been the cornerstone of many strands of economics literature like international trade theory (Krugman, 1979, 1980, 1995; Montagna, 2001; Ottaviano *et al*., 2002; Cox & Ruffin, 2010; Oyamada, 2020; to name a few), growth theory (Romer, 1990; Jones, 1995; Smulders & van de Klundert, 2003) and location theory (Krugman,1997; Brakman *et al.,* 2001; Martin & Ottaviano, 2001; Ottaviano & Thisse, 2002; 2004; Behrens & Thisse, 2006; Schiff, 2015). The non-zero outside option of the buyer is also important because it theoretically captures the competition between the 'brick-and-mortar' shops and the shopping malls, as given in Belleflamme and Peitz (2015). The present chapter introduces these attributes in the delegation common agency framework[1] with uniform pricing and shows that it allows for a case where the monopoly price is lower than the oligopoly price even when the different varieties of the differentiated product are substitutes of each other. The comparative static results of the model derive the effect of change in the degree of substitutability among the varieties

[1] Martimort and Stole (2009) defines such framework as the one where multiple sellers attempt to sell their products to a set of buyers who can refuse to buy from any one of them. Unlike the present chapter, they discuss the case where the sellers compete with each other in terms of non-linear prices and the buyers have zero outside option. They also do not discuss the 'love-for-variety' preference pattern.

and the effect of change in the outside option to the buyers both on the extensive and intensive margin of demand and on the equilibrium number of varieties produced in the differentiated product market. Counterintuitively, it shows that under certain conditions, an increase in substitutability among the varieties may raise the oligopoly price, and an improvement in outside option may raise the number of varieties in the market. We highlight the role of a strong extensive margin effect in explaining the results.

The chapter contributes to the existing literature by using the properties of the 'love for variety' preference pattern in the delegation common agency framework. A buyer holding the Dixit-Stiglitz kind of utility function representing such preference[2], if decides to buy the differentiated product, owing to her 'love for variety' consumes some amount of every variety of the product available at the market. The chapter argues that this creates a complementarity on the extensive margin of the delegation common agency game. If the price of a particular variety rises, the marginal buyer's indirect utility of the differentiated product falls, and she stops buying all other varieties of the product available in the market. Therefore, the behavior of one seller creates a negative externality for the other sellers in the market through the network effect. Consequently, even in the presence of substitutability among the varieties of the differentiated product, if the price of a particular variety rises, the demand for all other varieties falls in the presence of a strong extensive margin effect. This effect violates the supermodular structure of the price-setting game as described in Vives (1990). In supermodular games, there is strategic complementarity between the prices charged by the sellers of the differentiated product. We show in this chapter that in the presence of a strong extensive margin effect, it is possible to have strategic substitutability in price-setting differentiated product games, which derives

[2] See Dixit and Stiglitz (1977).

counterintuitive results. The violation of supermodularity through the network effect has been discussed earlier through other channels by papers like Anderson et al. (1992) and Cosandier et al. (2018). Our chapter adds to this literature. In the delegation common agency game literature apart from the 'love for variety' preference pattern, the chapter adds the effect of non-zero outside option. The chapter also contributes to the standard 'love for variety' literature, as it introduces the strategic price competition aspect in it, which is usually ignored based on *a priori* assumption of a large number of sellers (Brakman & Heijdra, 2002). The number of sellers is determined endogenously in the present framework, as in Madden and Pezzino (2011). However, in contrast to the other models in this area, we also endogenously determine the number of buyers in our framework.

The coexistence of the 'love for variety' preference pattern and non-zero outside option makes our framework suitable for discussing the interaction between 'brick-and-mortar' stores and shopping malls/e-commerce platforms. Typically, a 'brick-and-mortar' store stocks a lesser number of varieties of a product than a shopping mall/e-commerce platform. Then, how does the existence of the 'brick-and-mortar' store affect the shopping malls/e-commerce platforms and *vice versa*? The results of our model suggest that in the presence of a strong extensive margin of demand effect, they may be complements of each other instead of being in conflict as conventionally apprehended, and a price-cartel among the sellers in shopping malls/e-commerce platforms may not be a bad idea. The intuition has important implications for regulatory interventions.

The chapter is organized as follows: Section 2 presents the model and derives the results. Section 3 presents the comparative static exercises. The following section concludes.

2.2 The Model

Consider a market of a differentiated product with N_S number of sellers, each producing a unique variety of the product by using an increasing returns technology with marginal cost of production $c > 0$ and fixed cost $F > 0$ independent of the scale of production. There is a free entry of firms in the market, and N_S gets endogenously determined. Due to free entry, each firm earns zero profit at the equilibrium.[3]

2.2.1 The Demand Side of the Market

The buyers have the same level of income $I > 0$, but are heterogeneous in their preference. Although each of them derives their utility from the consumption of the basket of N_S varieties of the differentiated product and the other commodities, they differ from each other in their valuation of the basket of the differentiated product. The parameter $\mu > 0$ captures the valuation a buyer puts on the basket of differentiated product. We assume, μ is distributed uniformly over $[0, 1]$. A buyer of the type μ consumes $q_j > 0$ units of the j^{th} variety ($\forall j = 1, \ldots \ldots N_S$) of the differentiated product and spends q_0 on the consumption of all other commodities as a composite good. Each of the buyers also has an option of participating in an alternative consumption program from where she derives a utility of $\bar{v}$.

The utility function representing the preference of a buyer of the type μ, is given by,

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[3] One can think of the sequence of events in our chapter as follows:

Period 1: The number of firms and therefore, the number of varieties of the differentiated product is determined.

Period 2: The Bertrand competition between the firms determine the price of each of the varieties.

Period 3: Given the number of varieties and the price of each of them, a consumer determines whether to participate at the market and if she participates, determines her demand for a variety.

The model is solved using the backward induction method.

$$u(\mu) = \max\left\{\left(q_0 + \mu\left[\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta}\right), \bar{v}\right\} \tag{1}$$

where $\rho \in (0, 1)$ denotes the substitutability among the varieties of the differentiated product in the buyer's preference irrespective of her type, and $\mu\left[\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta}$ is the utility obtained from the consumption of the basket of differentiated product. The extent of substitutability falls with lower values of ρ. The preference parameter $\beta > 0$ represents the substitutability between the differentiated product and the expenditure on other commodities. We assume $\beta < \rho$ to ensure that the utility function is concave in q_j.

Suppose $p_j > 0$ is the price of the differentiated product *j*, ($\forall j = 1, \ldots \ldots N_S$). A buyer of the type μ maximizes her utility by choosing $q_j \geq 0 (\forall\, j = 1, \ldots \ldots N_S)$ subject to her budget constraint, given by

$$q_0 + \sum_{j=1}^{N_S} p_j q_j \leq I \;\forall\, j = 1, \ldots \ldots N_S. \tag{2}$$

At the equilibrium choice, the budget constraint binds. Substituting for q_0 from equation (2) in equation (1), the maximization problem of the μ^{th} type buyer becomes

$$\max \emptyset_\mu = I - \sum_{j=1}^{N_S} p_j q_j + \mu\left[\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta} \quad w.r.t\ 'q_j' \quad \forall\ j = 1, \ldots \ldots N_S.$$

The choice of $q_j \geq 0$ for all $j = 1, \ldots \ldots N_S$ satisfies,

$$\frac{\partial \emptyset_\mu}{\partial q_j} \leq 0 \Rightarrow -p_j + \mu\beta\left[\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta-1}\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)^{\frac{1-\rho}{\rho}} {q_j}^{\rho-1} \leq 0. \tag{3}$$

From equation (3) for all $j = 1, \dots \dots N_S$ the μ type consumer's demand for the j^{th} variety is derived as $q_j\left(p_1, \dots p_j \dots, p_{N_S}, \mu, \rho, \beta\right)$. For $q_j > 0$ for all $j = 1, \dots \dots N_S$, $\frac{\partial \emptyset_\mu}{\partial q_j} = 0$ is the first order condition for maximization, when the satisfaction of the second order condition requires $\frac{\partial^2 \emptyset_\mu}{\partial {q_j}^2} < 0$. Given $\rho \in (0, 1)$ and $\beta < \rho$, the model ensures that the second-order condition holds.

For simplicity, let us define, $\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)^{\frac{1}{\rho}} = k$. If $q_j \neq 0$ for all $j = 1, \dots \dots N_S$, $k > 0$.

Using the definition of k and substituting the value of q_j that solves equation (3), in $\emptyset_\mu$, the indirect utility function of a buyer of type μ is written as,

$$v(\mu) = I - \sum_{j=1}^{N_S} p_j q_j + \mu k^{\beta}. \tag{4}$$

$v(\mu)$ is nothing but the value of $\emptyset_\mu$ at $q_j\left(p_1, \dots p_j \dots, p_{N_S}, \mu, \rho, \beta\right)$. Since $\frac{\partial v}{\partial \mu} = -\sum_{j=1}^{N_S} \left[\frac{\partial \emptyset_\mu}{\partial q_j}\right] \frac{\partial q_j}{\partial \mu} + k^{\beta}$ and $\frac{\partial \emptyset_\mu}{\partial q_j} = 0$ from equation (3), $\frac{\partial v}{\partial \mu} = k^{\beta} > 0$ for all values of μ in $[0, 1]$. Therefore, $v(\mu)$ is a continuous and monotonically increasing function of μ in $[0, 1]$. Among the buyers, a buyer who puts a higher weight on the differentiated product enjoys a higher level of indirect utility.

We also know from equation (3) that as $\mu \to 0, q_j \to 0$. Then it follows from equation (4) that since $k \to 0, v(\mu) \to I$. Since $v(\mu)$ is monotonically increasing function of μ, $v(1) > I$.

Assumption 1: $\bar{v} \in [I, v(1)]$.

Assumption 1, along with the fact that $v(\mu)$ is a monotonically increasing continuous function of μ in $[0,1]$, implies that there exists a value of $\mu = \bar{\mu}(p_1, \dots \dots, p_{N_S}, \rho, \bar{v}, I, \beta)$ in $(0, 1)$ such that $v(\bar{\mu}) = \bar{v}$. It follows from equation (1) that a buyer of type μ participates in the differentiated product market if and only if $v(\mu) \geq \bar{v}$. In other words, the types of buyers participating in the differentiated product market are given by $[\bar{\mu}, 1]$.

Notice that Assumption 1 can be violated in two different ways. If $\bar{v} < I$, the buyers, irrespective of their types, purchase the differentiated product. If $\bar{v} > v(1)$, none of the buyers purchase the differentiated product. Both these cases are uninteresting from the perspective of the present analysis. Therefore, they are not discussed any further.

Since μ is distributed uniformly over $[0, 1]$, the number of buyers participating in the differentiated product market is given by:

$$N_B = \Pr[\mu \geq \bar{\mu}(\cdot)]. \tag{5}$$

The purchase decision of type μ buyer is guided by equation (3). Therefore, the aggregate demand for the j^{th} variety of the differentiated product (Q_j) is derived as

$$Q_j = \int_{\bar{\mu}(\cdot)}^{1} q_j(\cdot) d\mu \quad \forall \ j = 1, \dots \dots N_S, \tag{6}$$

where $q_j(\cdot)$ satisfies equation (3). Notice that Q_j derived in equation (6) is a function of $(p_1, \dots \dots, p_{N_S}, \rho, \bar{v}, I, \beta)$.

Lemma 1: If $q_j > 0, \frac{\partial q_j}{\partial p_j} < 0$ for all $\mu \in [0,1]$ and for all $j = 1, \dots \dots N_S$.

Proof: See Appendix A2. □

Lemma 1 states that for every buyer, irrespective of their type, with a 'love for variety' preference, each variety of the differentiated product is like a normal good, having a negatively sloped demand function.

Lemma 2: If $q_j > 0, \frac{\partial \bar{\mu}(\cdot)}{\partial p_j} > 0$ for all $j = 1, \dots \dots N_S$.

Proof: See Appendix A2. □

Ceteris paribus an increase in the price of the j^{th} variety of the differentiated product, which the buyer purchases at the initial equilibrium, reduces the indirect utility level of the marginal buyer of the type $\bar{\mu}$ below $\bar{v}$. Therefore, she stops buying the differentiated product and avails the outside option. Only the buyers with a higher valuation of the differentiated product continue to purchase it. Therefore, $\bar{\mu}$ rises at the new equilibrium.

Lemma 3: $\frac{\partial Q_j}{\partial p_j} < 0$ for all $j = 1, \dots \dots N_S$.

Proof: See Appendix A2. □

Lemma 3 states that the aggregate demand of each variety of the differentiated product is negatively sloped like normal commodities. As the price of a variety rises, its aggregate demand (demand from all types of buyers taken together) falls. As in Lemma 1, all other things remaining the same, as the price of a variety rises, not only does its demand from every type of buyer who continues to purchase the variety falls (i.e., the intensive margin of demand shrinks), but also as suggested by Lemma 2 some types of the buyers may stop buying the product as well (the extensive margin of demand shrinks).

Lemma 4:

$\frac{\partial Q_j}{\partial p_i} \lesseqgtr 0$ if and only if $\left[\int_{\bar{\mu}(\cdot)}^{1} \frac{\partial q_j(\cdot)}{\partial p_i} d\mu\right] - \left[Q_j \frac{\partial \bar{\mu}(\cdot)}{\partial p_i}\right] \lesseqgtr 0$ for all i, $j = 1, \dots \dots N_S; i \neq j$.

Proof: See Appendix A2.

□

Lemma 4 states that *ceteris paribus*, a rise in the price of the i^{th} variety of the differentiated product has an uncertain effect on the aggregate demand of the j^{th} variety of the product. There are two different effects that come into play as the price of the i^{th} variety rises. The first is the usual *substitution effect* (SE) that raises the demand for the j^{th} variety as the more expensive i^{th} variety is substituted by the buyers through consumption of the substitute varieties. The second is the *extensive margin effect on the buyers' side* (EME), by which because of the rise in the price of any one of the N_S varieties of the differentiated product, the identity of the marginal buyer, who is indifferent between buying and not buying the differentiated product, changes. As the basket of the differentiated product becomes costlier, some buyers prefer not to buy the differentiated product at all. Therefore, all the sellers face a negative demand externality because of the rise in price of any one of the varieties. The EME has rarely been discussed in the existing literature on differentiated product oligopoly models with continuous demand function[4]. An earlier paper by Cosandier et al. (2018) discusses a similar effect in a context different from this chapter. They discuss it in the context of product awareness. If the EME is ignored as in the conventional literature, our model implies $\frac{\partial Q_j}{\partial p_i} > 0$ i.e., the i^{th} and j^{th} variety are gross substitutes of each other. But, if the EME is considered, and if it is strong enough, it can outweigh the positive substitution effect to have $\frac{\partial Q_j}{\partial p_i} < 0$. In such a case, the i^{th} and j^{th} variety are gross complements of each other. This observation has important implications for the outcome of the price-setting game played between the sellers on the supply side of the market, which is discussed below.

[4]Anderson et al. (1992) discusses a network effect in models of discrete demand, which has similar implications as in our model. In Martimort and Stole (2009), this effect is referred as complementarity at extensive margin. But the delegated common agency game in their framework cannot have complementarity at the extensive margin. It can occur either in the case of demand complementarity in the delegated games or in the intrinsic common agency games.

2.2.2 The Supply Side of the Market

The Bertrand-price setting game

Each seller, in its pricing decision, takes the aggregate demand function derived in (6), which is negatively sloped, as given. The j^{th} seller's profit is calculated as,

$$\pi_j = [p_j - c]Q_j(p_1, \dots, p_j, \dots, p_{N_S}, \rho, \bar{v}, I, \beta) - F \,. \tag{7}$$

The j^{th} seller ($\forall j = 1, \dots \dots N_S$) autonomously chooses p_j to maximize its profit π_j. The prices are chosen simultaneously.

Let us define, $\varphi_j(p_1, \dots, p_{j-1}, p_j, p_{j+1}, \dots, p_{N_S}, \rho, \bar{v}, I, \beta) = c - \frac{Q_j(p_1, \dots, p_j, \dots, p_{N_S}, \cdot)}{\frac{\partial Q_j}{\partial p_j}(p_1, \dots, p_j, \dots, p_{N_S}, \cdot)}$.

Assuming an interior solution to the seller's problem, from the first order condition of profit maximization, the optimum price of the j^{th} seller $p_j = R_j(p_1, \dots, p_{j-1}, p_{j+1}, \dots, p_{N_S}, \rho, \bar{v}, I, \beta)$ must satisfy the following equation:

$$p_j = \varphi_j(p_1, \dots, p_{j-1}, p_j, p_{j+1}, \dots, p_{N_S}, \rho, \bar{v}, I, \beta). \tag{8}$$

The sufficient condition that satisfies the second-order condition of each firm's profit maximization problem is: $\frac{\partial^2 Q_j}{\partial p_j^2} < 0$. R_j defines the reaction function of the j^{th} seller with respect to the prices charged by the seller of the other varieties. The Nash equilibrium $(p_1^*, p_2^*, \dots., p_j^*, \dots., p_{N_S}^*)$ of the price-setting game played by the sellers in the differentiated product market lies at the intersection of N_S different reaction functions of N_S number of sellers, each given by equation (8) for $j = 1, \dots \dots N_S$.

Lemma 5: $\frac{\partial \varphi_j}{\partial p_j} < 0$ for all $j = 1, \dots \dots N_S$.

Proof: See Appendix A2. □

A Nash equilibrium of the Bertrand-price competition game would exist if given $(p_1^*, p_2^*, \dots, p_{j-1}^*, p_{j+1}^* \dots, p_{N_S}^*)$, equation (8) holds, such that firm j (for all $j = 1, \dots \dots N_S$) chooses p_j^* to maximize its own profit. Satisfying this equation, p_j^* becomes the best response strategy to $(p_1^*, p_2^*, \dots, p_{j-1}^*, p_{j+1}^* \dots, p_{N_S}^*)$. The same argument holds for all $j = 1, \dots \dots N_S$. Since $\frac{\partial \varphi_j}{\partial p_j}$ exists, φ_j is a continuous function over $p_j \geq 0$. Lemma 5 demonstrates that φ_j is a monotonically declining function over $p_j \geq 0$. Therefore, the conditions $\varphi_j(p_j = 0, \cdot) > 0$ and $\frac{\partial^2 \varphi_j}{\partial p_j^2} \leq 0$ ensure that a unique interior Nash equilibrium exists. We assume these sufficient conditions to hold in our further discussion.

As explained in Lemma 4, the conventional literature on Bertrand game in differentiated product oligopoly market assumes $\frac{\partial Q_j}{\partial p_i} > 0$. This helps to satisfy $\frac{\partial^2 \pi_j}{\partial p_i \partial p_j} = \left[\frac{\partial Q_j}{\partial p_i} + (p_j - c)\frac{\partial^2 Q_j}{\partial p_i \partial p_j}\right] > 0$ that makes the game strictly supermodular.[5] Consequently, the prices become strategic complements with each other with positively sloped reaction function for each of the firms competing in the market. However, if the EME dominates the SE in the present model and EME is strong enough, it is possible that $\frac{\partial Q_j}{\partial p_i} < 0$. If $\frac{\partial^2 Q_j}{\partial p_i \partial p_j} \approx 0$, with a strong EME, the condition for supermodularity gets violated and in the Bertrand game prices become strategic substitutes of each other. In such a situation, each firm has a negatively sloped reaction function with respect to the prices charged by any other firm.

[5] See Vives (1990).

Assumption 2: $\frac{\partial^2 Q_j}{\partial p_i \partial p_j} \approx 0$.

Assumption 2 helps us to focus on the role played by the EME in the Bertrand game. However, notice that even when Assumption 2 is violated, $\frac{\partial^2 \pi_j}{\partial p_i \partial p_j} = \left[\frac{\partial Q_j}{\partial p_i} + (p_j - c)\frac{\partial^2 Q_j}{\partial p_i \partial p_j}\right]$ can still be negative under the necessary condition $\left[\frac{\partial Q_j}{\partial p_i} + (p_j - c)\frac{\partial^2 Q_j}{\partial p_i \partial p_j}\right] < 0$. If $\frac{\partial^2 Q_j}{\partial p_i \partial p_j} > 0$, the necessary condition holds if and only if $\frac{\partial Q_j}{\partial p_i}$ is strongly negative, i.e., the extensive margin effect overwhelmingly dominates the substitution effect. The strength of these effects depends on the parameters of the Dixit-Stiglitz 'love for variety' utility function specified in equation (1), which has been made explicit in Lemma 4 above.

Lemma 6: $\frac{\partial \varphi_j}{\partial p_i} \lesseqgtr 0$ if and only if $\frac{\partial Q_j}{\partial p_i} \lesseqgtr 0$ for all i, $j = 1, \dots \dots N_S; i \neq j$.

Proof: See Appendix A2. □

Lemma 6 implies that, as suggested in Lemma 4, if the *i^th^* seller raises its price and the EME dominates the SE, $\varphi_j$which is negatively sloped with respect to p_j(Lemma 5), as shown in Figure 2.1 below, shifts in the downward direction. Earlier p_j^* used to solve equation (8), as shown in Figure 2.1. But now it solves for $p_j^{*\prime\prime} < p_j^*$. Therefore, the prices of any two sellers in the price-setting game act as strategic substitutes for each other. The reaction function of the j^{th} seller for all i, $j = 1, \dots \dots N_S$ slopes negatively. On the other hand, if the SE dominates the EME, as the i^{th} seller raises its price, in Figure 2.1 φ_j shifts in the upward direction. Therefore, equation (8) now solves for $p_j^{*\prime} > p_j^*$ i.e., the prices of any two sellers in the price-setting game act as strategic complements of each other. The reaction function of the j^{th} seller for all i, $j = 1, \dots N_S$ slopes positively.

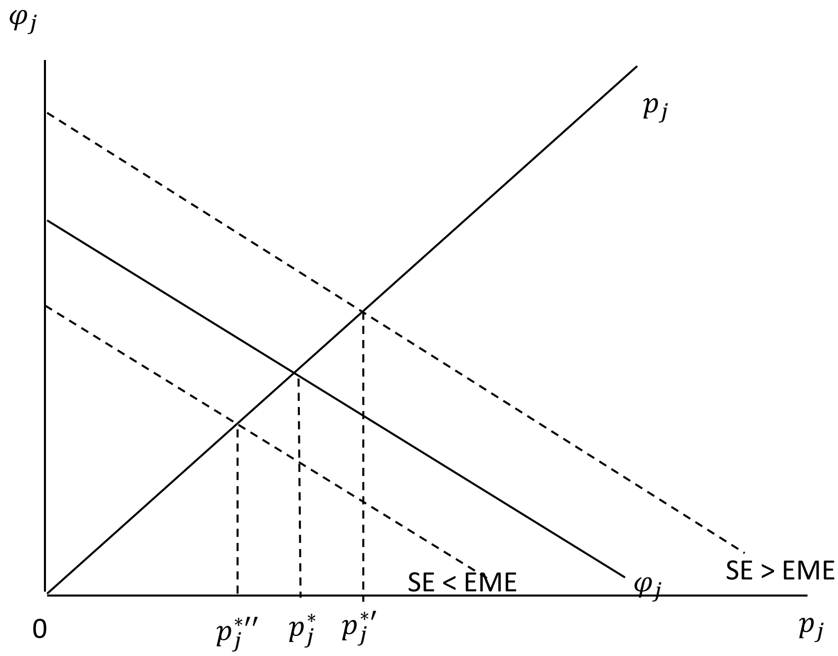

Figure 2.1: Optimum reaction of the j^{th} seller to change in p_i

In our model, every variety of the differentiated product enters symmetrically in the utility function specified in equation (1), and therefore, every seller faces the same demand function. They also have the same cost function. Therefore, the sellers are symmetric. Since all sellers face the same set of parameters $(\rho, \bar{v}, I, \beta, c)$ while pricing their own variety, it must be the case that the Nash equilibrium in prices is symmetric i.e. $p_1^* = \cdots p_j^* = \cdots = p_{N_S}^* = p^*(\rho, \bar{v}, I, \beta, c)$ i.e. all the sellers charge the same price to the buyers irrespective of the variety they produce. Notice that since $\frac{\partial Q_j}{\partial p_j} < 0$ from Lemma 3, equation (7) implies, $p^* > c$ i.e., the seller of each variety prices its product over the unit cost of production and enjoys a monopoly margin of $(p^* - c) > 0$.

Let us denote the joint profit-maximizing monopoly price in the differentiated product market as p^M that solves,

$\max \pi = [p - c]Q(p, \rho, \bar{v}, I, \beta) - F.$

Proposition 1 notes how p^* compares with p^M and c, which is also the competitive price of the varieties in the differentiated product market.

Proposition 1: *If* $\frac{\partial^2 Q_j}{\partial p_i \partial p_j} \approx 0$,

(i) $p^* = p^M > c$ *if EME = SE.*

(ii) $p^* > p^M > c$ *if EME > SE.*

(iii) $p^M > p^* > c$ *if* $EME < SE$.

Proof: This follows from the discussion above. □

If the EME dominates the SE, each seller raising its price creates a negative externality for the sellers of the other varieties of the differentiated product, which it does not internalize. Therefore, each charges a higher price than the coordinated equilibrium price, p^M. On the other hand, if the SE dominates the EME, each seller raising its price creates a positive externality for the sellers of the other varieties of the differentiated product, which it does not internalize. Therefore, each charges a lower price than the coordinated equilibrium price, p^M. If the two effects balance each other, we obtain, $p^* = p^M$. The conventional literature with a representative buyer does not anticipate the possibility of finding $p^* \geq p^M$ that arises in our model. In the literature with a heterogeneous buyer (e.g., Martimort and Stole, 2009), $p^* > p^M$ can never occur in a delegation common agency game with demand substitutability.

Since each variety is sold at the same price $p^*(\rho, \bar{v}, I, \beta, c)$ at the market, and each variety enters symmetrically into the utility function of the μ type consumer given in equation (1), from equation (3) it must be the case that an identical amount is demanded of each variety, given by

$$q^* = \left(\frac{\mu\beta}{p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}. \tag{9}$$

Since $\beta < \rho$, notice that as the number of varieties rises, each buyer purchases a smaller amount of each variety of the product, similar to the standard 'love for variety' models.

It can be argued that under certain conditions (derived in Appendix B2), an extensive margin $\bar{\mu}(N_S, p^*(\rho, \bar{v}, I, \beta, c), \bar{v})$ exists at the differentiated product market. All buyers having $\mu \geq \bar{\mu}$ ($N_S, p^*(\rho, \bar{v}, I, \beta, c), \bar{v}$) participate in the market, and all buyers having $\mu < \bar{\mu}(N_S, p^*(\rho, \bar{v}, I, \beta, c), \bar{v})$ do not participate in the market[6].

Using equations (6) and (9), the aggregate demand of a specific variety is written as,

$$Q^* = \left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}} \left(\frac{1-\beta}{2-\beta}\right)\left(1\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right). \tag{10}$$

Free Entry

With free entry, each seller participating in the market must earn zero profit. After substitution of p^* in equation (7), the zero profit condition yields,

$$Q^* = \frac{F}{p^*-c}. \tag{11}$$

On substitution of Q^* from equation (10) in equation (11), the equilibrium number of sellers N_S^* of the differentiated product is solved from the following equation[7]:

$$\left(\frac{F}{p^*-c}\right)\left(\frac{p^*}{\beta}\right)^{\frac{1}{1-\beta}} = N_S^{\frac{\beta-\rho}{\rho(1-\beta)}} \left(\frac{1-\beta}{2-\beta}\right)\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right). \tag{12}$$

---

[6] See Appendix B2 for the proof.

[7] The uniqueness of N_S^*can be proved in the following way. Equation (12) can be rewritten as:

$$\left(\frac{F}{p^*-c}\right) = \left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}} \left(\frac{1-\beta}{2-\beta}\right)\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right).$$

Notice that the LHS of the equation is positive and independent of N_S.

The RHS of the equation is, however, a function of N_S. Let us call it $X(N_S)$. Notice that $X'(N_S) = \left(\frac{-\beta^2}{\rho^2 p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}-1} \left(\frac{\beta-\rho}{2-\beta}\right)\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right)\left(\frac{1}{\rho(1-\beta)}\right) > 0$ for all $N_S \geq 0$. Therefore, $X(N_S)$ is a monotonically rising continuous function of N_S for all $N_S \geq 0$. Moreover, as $N_S \to 0$, since $X(N_S)$ approaches a $(\frac{0}{0})$ form, the application of L'Hospital rule implies: $X(N_S) \to 0$.

Therefore, it must be the case that there exists a unique value of $N_S > 0$ at which equation (12) holds.

2.3 Comparative Static Results

Let us define L as Lerner index of monopoly power of the sellers of the differentiated product.

Assumption 3: $L > 1 - \beta$.

Since we show the importance of extensive margin of demand in the chapter, Assumption 3 simplifies the subsequent analysis without a major change in the results in the presence of a sufficiently large extensive margin effect on the demand side. It is also reasonable in our context in the sense that it assigns a monopoly power to the sellers of different varieties bounded away from zero.

Proposition 2: (Change in ρ)

(i) *Given* $N_S, \frac{\partial p^*}{\partial \rho} < 0$;

(ii) $\frac{\partial N_S}{\partial \rho} >=< 0$ *if and only if* $\left[\left(\frac{\partial p^*}{\partial \rho}\right)\left[\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2} + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S{}^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \rho}\right)\right] <=$

> 0;

(iii) *If* $\frac{\partial N_S}{\partial \rho} \leq 0, \frac{\partial p^*}{\partial \rho} < 0$; *if* $\frac{\partial N_S}{\partial \rho} > 0,\ \frac{\partial p^*}{\partial \rho} >=< 0$ *if and only if* $\left(-\frac{\frac{\partial Q^*}{\partial \rho}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \rho}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) +\right.$

$\left. Q^*\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) >=< 0.$

Proof: See Appendix A2. □

As ρ rises, the varieties of the differentiated product become closer substitutes of each other in the buyers' preference. As a result, the indirect utility received from the consumption of the differentiated product falls and the marginal buyer, who was indifferent between buying and not buying the differentiated product, no longer buys the differentiated product. Given N_S, the extensive margin of demand for the differentiated product shrinks as $\bar{\mu}$ rises. Since each buyer who participates in the differentiated product market buys an equal amount of each of the available varieties, the exit of some buyers adversely affects the demand enjoyed by each of the existing sellers. They react to the situation by reducing their price that boosts up the demand (i.e.,$\frac{\partial p^*}{\partial \rho} < 0$). As the price falls, the indirect utility of the buyers buying the differentiated product rises, and the identity of the marginal buyers changes again as some buyers come back to the differentiated product market. If this effect is strong enough to outweigh the initial negative demand effect arising from the shrinkage in the extensive margin, $\left[\left(\frac{\partial p^*}{\partial \rho}\right)\left[\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2} + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \rho}\right)\right] < 0$ and the number of sellers rises in the market. The opposite is likely to happen if the strategic price behavior of the sellers fails to outweigh the negative demand shock. If the adjustment in the extensive margin works in the positive direction and the number of sellers rises in the differentiated market, the rise in number of varieties of the differentiated product reinforces the positive effect on the level of indirect utility and the extensive margin of demand. Consequently, the price of the differentiated product may rise as well. The third part of the proposition mentions the condition under which the price of the differentiated product may rise. Notice that the extensive margin effect on the demand side of the market plays an important role in derivation of the results in Proposition 2. It creates a possibility that the increased substitutability among the varieties of the differentiated product increases both the price and the number of sellers in the differentiated product market.

Proposition 3: (Change in c)

(i) *Given* N_S, $\frac{\partial p^*}{\partial c} > 0$;

(ii) $\frac{\partial N_S}{\partial c} < 0$;

(iii) $\frac{\partial p^*}{\partial c} >=< 0$ *if and only if* $\left(1 - \frac{\frac{\partial Q^*}{\partial c}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial c}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)\right) >=< 0.$

Proof: See Appendix A2. □

As the marginal cost of producing each differentiated variety rises, given the marginal revenue at the equilibrium, the profit-maximizing seller of a differentiated variety raises the price of her product to maintain the monopoly margin. The buyers of the differentiated product adjust their demand with respect to the price rise in the downward direction, but no one leaves the market. Therefore, the extensive margin of demand remains unchanged. As the sellers were already operating under the zero-profit condition, the falling profit consequent on the rise in marginal cost triggers the exit of the sellers from the market. As the number of sellers falls, the number of available varieties of the differentiated product falls. Therefore, the indirect utility of the buyers falls and the marginal buyer who was indifferent between buying and not buying the differentiated product leaves the market. Since every buyer buys an equal amount from all the sellers, as the marginal buyers exit, given price, demand for every seller falls. They react by charging a lower price. The third part of Proposition 3 states that the price of the differentiated product rises due to a rise in the marginal cost of production if and only if the price falls due to the shrinking extensive margin of demand is strictly dominated by the initial price rise due to the rise in the marginal cost of production. Otherwise, the price of the differentiated product may either remain unchanged or may fall due to the rise in the marginal cost of production.

Proposition 4: (Change in $\bar{v}$)

(i) *Given* $N_S, \frac{\partial p^*}{\partial \bar{v}} < 0$;

(ii) $\frac{\partial N_S}{\partial \bar{v}} >=< 0$ *if and only if* $\left[\left(\frac{\partial p^*}{\partial \bar{v}}\right)\left[\left(\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}\right) + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \bar{v}}\right)\right] <=$

> 0;

(iii) *If* $\frac{\partial N_S}{\partial \bar{v}} \leq 0, \frac{\partial p^*}{\partial \bar{v}} < 0$; *If* $\frac{\partial N_S}{\partial \bar{v}} > 0, \frac{\partial p^*}{\partial \bar{v}} >=< 0$ *if and only if* $\left(-\frac{\frac{\partial Q^*}{\partial \bar{v}}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \bar{v}}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)\right)$

$>=< 0$.

Proof: See Appendix A2. □

At the initial equilibrium, all other things remaining the same, an increase in the utility from the outside option induces the marginal buyer, who was indifferent between buying and not buying the differentiated product, not to buy the differentiated product anymore. As a result, the number of buyers falls in the differentiated product market. Since each buyer in the differentiated product market buys an equal amount from every seller, given price, the demand for every variety falls in the extensive margin. Therefore, the sellers adjust their prices in the downward direction. As the price falls, the indirect utility of the buyers buying the differentiated product rises, and the identity of the marginal buyers changes again as some buyers come back to the differentiated product market. If this effect is strong enough to outweigh the initial negative demand effect arising from the shrinkage in the extensive margin, $\left[\left(\frac{\partial p^*}{\partial \bar{v}}\right)\left[\left(\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}\right) + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \bar{v}}\right)\right] < 0$. Therefore, as mentioned in part (ii) of the proposition the number of sellers rises in the market. The opposite is likely to happen if the strategic price behavior of the sellers fails to outweigh the negative demand shock. If the adjustment in the extensive margin works in the positive direction and the number of sellers rises in the differentiated market, the rise in the number of varieties of the differentiated product reinforces the positive effect on the level of indirect utility and the extensive margin of demand. Consequently, the price of the differentiated product may rise as well. The third part of the proposition mentions the condition satisfying which the price of the differentiated product may rise.

Notice that the extensive margin effect on the demand side of the market plays an important role in the derivation of the results in Proposition 4. It creates the possibility that an improved outside option of the differentiated product market, counterintuitively, increases both the price and the number of sellers in the differentiated product market.

Proposition 4 also highlights the equilibrium features of the markets where the ‘brick-and-mortar’ shops and the shopping malls/e-commerce platforms coexist. In our model, the ‘brick-and-mortar’ shops act as competitors to the malls and platforms by their existence. The buyers would like to visit the malls/platforms if and only if their utility from going there exceeds the utility they receive from visiting the ‘brick-and-mortar’ shops. If the ‘brick-and-mortar’ shops act as a greater threat to the malls/platforms by siphoning off the buyers, the sellers of the differentiated product at the malls/e-commerce platforms are expected to react strategically to bring them back to the market by lowering their price. If the lowering of price triggers a strong participation response from the buyers at the malls/e-commerce platforms, it is possible that eventually, the price, the number of varieties, and the number of sellers all rise at the malls/platforms.

Amazon has the highest share in the US e-commerce market, which had the second-largest turnover in the World in the year 2021[8]. One notable feature of the US retail market is that Amazon, since 2015, also started operating in the 'brick-and-mortar' segment with Amazon Books. But the revolution came after Amazon bought Whole Foods in 2017[9]. Since 2018, the count of the number of physical outlets of Amazon under the different heads of Amazon Fresh & Amazon Go Grocery, Amazon Books, Amazon Go, Amazon 4-Star, and Whole Foods has steadily been on the rise, as seen in Figure 2.2 below[10]. Forty-four other e-commerce companies, including eBay and Myntra, have also followed through[11]. In terms of our model, the implication is to strengthen the outside option for potential buyers purchasing from the e-commerce platform. Proposition 4 suggests that with the stronger outside option, if Assumption 2 holds, more is the number of varieties of the differentiated product produced in the market and the higher is the price of the product. Therefore, with a stronger outside option, one would expect to see the revenue of both the e-commerce platform (excluding their 'brick-and-mortar' counterpart) and the 'brick-and-mortar' shops rise simultaneously, as shown in the data presented in Figure 2.2 below.

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[8] Retrieved from https://www.census.gov/retail/index.html on 20.06.2022 and U.S. leading e-retailers by market share 2021 | Statista accessed on 27.07.2022.

[9] https://www.cnbc.com/2018/06/15/a-year-after-amazon-announced-whole-foods-deal-heres-where-we-stand.html accessed on 20.06.2022.

[10] Amazon also shut down small number of offline retail stores on account of technological obsolescence. We have kept them out of the count.

[11] https://www.insider-trends.com/top-45-online-retailers-who-went-offline/ accessed on 27.07.2022.

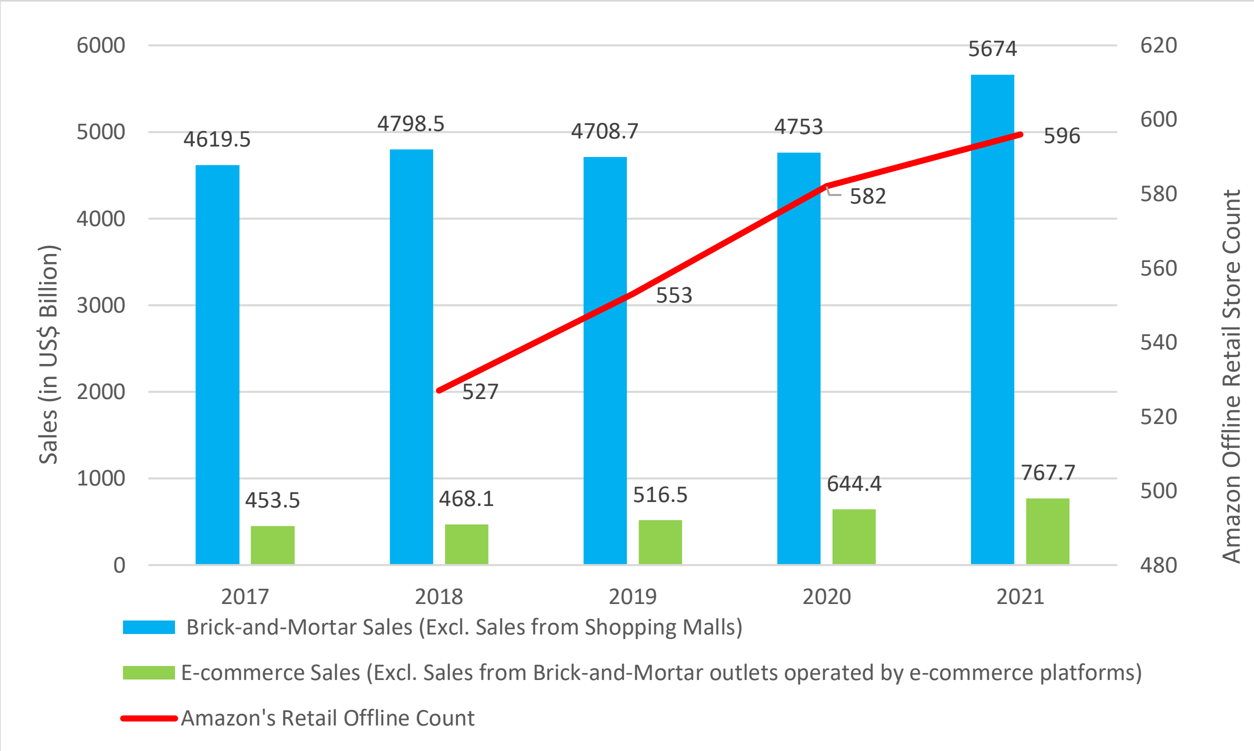

Figure 2.2: E-commerce sales (excluding the 'brick-and-mortar' shops operated by e-commerce companies), 'brick-and-mortar' sales (excluding sales from shopping malls), and the number of Amazon retail shops over 2017-2021 in the US

Source: Data for the e-commerce sales and adjusted 'brick-and-mortar' sales compiled from https://www.census.gov/retail/index.html on 20.07.22 and https://www.forbes.com/sites/jasongoldberg/2022/02/16/brick-and-mortar-sales-grew-faster-than-e-commerce-in-2021/?sh=128cd806dde5.
https://www.ibisworld.com/industry-statistics/market-size/shopping-mall-management-united-states/
Amazon's retail offline data have been compiled from https://www.statista.com/statistics/1155873/amazon-store-openings-number/ https://edition.cnn.com/2021/03/17/business/amazon-fresh-grocery-stores/index.html

Figure 2.2 shows that in the US, both e-commerce sales and 'brick-and-mortar' sales have grown within a short span of time (presumably with unchanged demand) at the compounded annual growth rate of 11.1% and 4.2%, respectively, along with the number of offline stores operated by e-commerce companies like Amazon. This supports modeling the interaction between the two presented in the chapter.

2.4 Conclusions

The chapter models a Bertrand-price competition in a differentiated product oligopoly market with an innovation on the demand side. It allows the buyers to differ from each other in their valuation of the differentiated product with a common outside option available to all. The buyers' preference shows 'love for variety', which is modeled using the 'Dixit-Stiglitz' utility function. The 'love for variety' specification introduces complementarity in the extensive margin of buyers' participation in the model. Therefore, we show that even in the presence of substitutability between the varieties of the differentiated product, the oligopoly price in such markets can be greater than the monopoly price. The model allows us to study the responsiveness of extensive margin of demand to parameters like the degree of substitutability between varieties of the differentiated product and the attractiveness of the outside option. The number of buyers and sellers in the differentiated product market also gets endogenously determined in the model. Since each seller produces and sells a specific variety of the product, the number of varieties of the product gets determined as well. We show that the presence of a strong extensive margin effect may change many of the conventional results. First, it may alter the supermodular structure of the standard Bertrand specification. In such a situation, the oligopoly price in the differentiated product market can be greater than the coordinated monopoly price. Second, the increase either in the substitutability between the varieties of the differentiated product or in the availability of a stronger outside option may increase both the price of the product and the number of varieties produced in the market. Third, a rise in the marginal cost of production of the varieties of the differentiated product may lead to a fall in the price of the product at the market.

The model relates to real-life market situations, where the 'love for variety' matters and the buyers choose between a 'brick-and-mortar' shop, selling a particular variety vis-à-vis a shopping mall/e-commerce platform selling a number of different varieties of a product. The results suggest that in the presence of a strong extensive margin of demand effect, the 'brick-and-mortar' shops, the shopping malls, and the e-commerce platforms may complement each other instead of being in conflict, as conventionally thought, and a price-cartel among the sellers of the differentiated product may not be a bad idea.

The present chapter can be extended further. It can be used in spatial economics models to study the retail agglomeration. It can also be used to study the functioning of e-commerce platforms and the relationship between platforms and their subscribers. The next chapter, considering both intensive and extensive margins of demand, introduces a theoretical model of a monopoly platform with a 'love for variety' preference of the buyers and price competition among sellers participating in the differentiated product market.

Appendix A2

Proof of Lemma 1.

If $q_j > 0$, equation (3) is satisfied with equality. Taking the total differential from equation (3) and collecting the terms:

$$\frac{\partial q_j}{\partial p_j} = -\frac{\frac{\partial^2 \varphi_\mu}{\partial p_j \partial q_j}}{\frac{\partial^2 \phi_\mu}{\partial {q_j}^2}}. \tag{A.1}$$

On the RHS of (A.1), $\frac{\partial^2 \varphi_\mu}{\partial p_j \partial q_j} = -\left(1 - \left[\mu\beta\left(\frac{\beta-\rho}{\rho}\right)\left[\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)\right]^{\frac{\beta-1}{\rho}-1} {q_j}^{\rho-1} \sum_{\mathrm{i}\neq\mathrm{j}} \rho {q_i}^{\rho-1} \frac{\partial q_i}{\partial p_j}\right]\right) < 0$

since $\frac{\partial q_i}{\partial p_j} > 0$ because of the demand-substitutability between any two varieties and $\beta < \rho$. Since $\frac{\partial^2 \phi_\mu}{\partial {q_j}^2} < 0$ by the second order condition, the statement of the lemma follows. □

Proof of Lemma 2.

At $v(\bar{\mu}) = \bar{v}$, equation (4) is written as:

$$\bar{v} = I - \sum_{j=1}^{N_S} p_j q_j + \bar{\mu} k^{\beta}. \tag{A.2}$$

By application of equation (3) and by use of the definition of k, from equation (A.2):

$$\frac{\partial \bar{\mu}}{\partial p_j} = \frac{q_j}{k^{\beta}}.$$

The statement of the lemma follows from the fact that $q_j > 0$ and $k > 0$. □

Proof of Lemma 3.

By application of Leibniz rule, from equation (6):

$$\frac{\partial Q_j}{\partial p_j} = -\left[Q_j \frac{\partial \bar{\mu}(\cdot)}{\partial p_j}\right] + \left[\int_{\bar{\mu}(\cdot)}^{1} \frac{\partial q_j(\cdot)}{\partial p_j} d\mu\right]. \tag{A.3}$$

Since $Q_j > 0$, the statement of the lemma follows from equation (A.3) by the application of Lemmas 1 and 2. □

Proof of Lemma 4.

Follows from (A.3). □

Proof of Lemma 5.

From definition,

$$\varphi_j(p_1, \dots, p_{j-1}, p_j, p_{j+1}, \dots, p_{N_S}, \rho, \bar{v}, I, \beta) = c - \frac{Q_j(p_1, \dots, p_j, \dots, p_{N_S}, \cdot)}{\frac{\partial Q_j}{\partial p_j}(p_1, \dots, p_j, \dots, p_{N_S}, \cdot)}.$$

Therefore, $\frac{\partial \varphi_j}{\partial p_j} = -1 + \frac{Q_j \frac{\partial^2 Q_j}{\partial p_j^2}}{(\frac{\partial Q_j}{\partial p_j})^2}$. (A.4)

Since $\frac{\partial^2 Q_j}{\partial p_j^2} < 0$, from (A.4) the statement follows.□

Proof of Lemma 6.

From definition of $\varphi_j(p_1, \dots, p_{j-1}, p_j, p_{j+1}, \dots, p_{N_S}, \rho, \bar{v}, I, \beta)$,

$$\frac{\partial \varphi_j}{\partial p_i} = -\frac{\frac{\partial Q_j}{\partial p_i}}{\frac{\partial Q_j}{\partial p_j}} + \frac{Q_j \frac{\partial^2 Q_j}{\partial p_i \partial p_j}}{(\frac{\partial Q_j}{\partial p_j})^2}. \tag{A.5}$$

The statement follows by the application of Assumption 2, Lemmas 3, and 4.□

Proof of Proposition 2.

(i) From equation (8),

$$\frac{\partial p^*}{\partial \rho}\left[1 + N_S\left\{1 - \frac{Q^*}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.\frac{\partial^2 Q^*}{\partial p^2}\right\}\right] = -\frac{\frac{\partial Q^*}{\partial \rho}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \rho}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) + Q^* \frac{\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}. \tag{A.6}$$

Since $\left[1+N_S\left\{1-\frac{Q^*}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.\frac{\partial^2 Q^*}{\partial p^2}\right\}\right]>0,$

$$\text{Sign}\left(\frac{\partial p^*}{\partial \rho}\right)=\text{Sign}\left(-\frac{\frac{\partial Q^*}{\partial \rho}}{\frac{\partial Q^*}{\partial p}}+\frac{\partial N_S}{\partial \rho}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)+Q^*\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right). \tag{A.7}$$

Given $N_S, \frac{\partial N_S}{\partial \rho}=0$. Therefore, from (A.6),

$$\text{Sign}\left(\frac{\partial p^*}{\partial \rho}\right)=\text{Sign}\left(-\frac{\frac{\partial Q^*}{\partial \rho}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right).$$

From Lemma 3, $\frac{\partial Q^*}{\partial p}<0$, which is also confirmed by equation (10). Since $\frac{\partial \bar{\mu}}{\partial p}>0$ from Lemma 2, equation (10) implies $\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)<0$. It also follows from equation (10) that,

$$\frac{\partial Q^*}{\partial \rho}=\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{2-\beta}\right)N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left[\log N_S\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right)\frac{\beta(\beta-1)}{(\rho-\rho\beta)^2}-\left(\frac{2-\beta}{1-\beta}\right)\bar{\mu}^{\left(\frac{2-\beta}{1-\beta}\right)-1}\frac{\partial \bar{\mu}}{\partial \rho}\right]; \tag{A.8}$$

To determine the sign of $\frac{\partial Q^*}{\partial \rho}$, we need to know the sign of $\frac{\partial \bar{\mu}}{\partial \rho}$. At $v(\bar{\mu})=\bar{v}$ equation (4) is written as: $\bar{v}=I-p^*N_Sq^*+\bar{\mu}k^\beta$. Then, by application of equation (3),

$$\frac{\partial \bar{\mu}}{\partial \rho}=\frac{\partial N_S}{\partial \rho}\left(\frac{q^*p^*}{k^\beta}\right)+\left(\frac{N_Sp^*}{k^\beta}\right)\left(\frac{q^*logN_S}{\rho^2}-\frac{q^*(\beta-\rho)}{\rho\beta}\frac{1}{N_S}\frac{\partial N_S}{\partial \rho}\right)+\left(\frac{\bar{\mu}\beta}{k}\right)q^*N_S^{\frac{1}{\rho}}\left[\left(\frac{1}{\rho^2}\right)log\,N_S+\frac{logN_S}{\rho^2}+\frac{(\beta-\rho)}{\rho\beta}\frac{1}{N_S}\frac{\partial N_S}{\partial \rho}-\left(\frac{1}{N_S\rho}\right)\frac{\partial N_S}{\partial \rho}\right]. \tag{A.9}$$

Given $N_S, \frac{\partial N_S}{\partial \rho}=0$. Therefore, from (A.9), $\frac{\partial \bar{\mu}}{\partial \rho}>0$. Given $\beta<1$, from (A.8) it follows that, $\frac{\partial Q^*}{\partial \rho}<0$.

Therefore, sign $\left(-\frac{\frac{\partial Q^*}{\partial \rho}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)<0$. The statement of the first part of the proposition follows.

(ii) From equation (12):

$$\frac{\partial N_S}{\partial \rho} = -\frac{\left[\left(\frac{\partial p^*}{\partial \rho}\right)\left[\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2} + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \rho}\right)\right]}{\left[\frac{\beta}{\rho^2(1-\beta)}\right]N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\log N_S\left(\frac{1-\beta}{2-\beta}\right)\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right)}.$$

Since the denominator of the term on the RHS of the above equation is positive,

$$\text{Sign}\left(\frac{\partial N_S}{\partial \rho}\right) = -\text{Sign}\left[\left(\frac{\partial p^*}{\partial \rho}\right)\left[\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2} + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \rho}\right)\right]. \quad \text{(A.10)}$$

Since $\frac{\partial \bar{\mu}}{\partial p} > 0$ from Lemma 2 and $\frac{\partial \bar{\mu}}{\partial \rho} > 0$, $\frac{\partial p^*}{\partial \rho} < 0$ as proved in the first part of the proposition, from (A.10) by use of Assumption 3 the statement of the second part of the proposition follows.

(iii) If $\frac{\partial N_S}{\partial \rho} \neq 0$, as suggested by (A.6) the sign of $\left(\frac{\partial p^*}{\partial \rho}\right)$ is determined by the sign of $\left(-\frac{\frac{\partial Q^*}{\partial \rho}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \rho}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) + Q^*\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)$.

Since $\frac{\partial Q^*}{\partial p} < 0$ and by following the assumptions of the model, from equation (10),

$\frac{\partial Q^*}{\partial N_S} = \left(\frac{-\beta^2}{\rho^2 p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}-1}\left(\frac{\beta-\rho}{2-\beta}\right)\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right)\left(\frac{1}{\rho(1-\beta)}\right) > 0;$

$\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right) = N_S^{\frac{\beta-\rho}{\rho(1-\beta)}-1}\left(\frac{\beta}{2-\beta}\right)\left[\left\{\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right)\beta^{\frac{1}{1-\beta}}\left(\frac{1}{1-\beta}p^{*\frac{-1}{1-\beta}-1}\right)\right\} + \left\{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}\bar{\mu}^{\left(\frac{2-\beta}{1-\beta}\right)-1}\frac{\partial \bar{\mu}}{\partial p^*}\left(\frac{2-\beta}{1-\beta}\right)\right\}\right]\left(\frac{1}{\rho^2}\right) > 0,$ $\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) > 0.$

Therefore, if $\frac{\partial N_S}{\partial \rho} \leq 0$ then it follows that $\frac{\partial p^*}{\partial \rho} < 0$; $\frac{\partial N_S}{\partial \rho} > 0$ makes the sign of $\left(-\frac{\frac{\partial Q^*}{\partial \rho}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \rho}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) + Q^*\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)$ uncertain. The statement of the third part of the proposition follows. □

Proof of Proposition 3.

(i) From equation (8),

$$\frac{\partial p^*}{\partial c}\left[1+N_S\left\{1-\frac{Q^*}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.\frac{\partial^2 Q^*}{\partial p^2}\right\}\right]=1-\frac{\frac{\partial Q^*}{\partial c}}{\frac{\partial Q^*}{\partial p}}+\frac{\partial N_S}{\partial c}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)+Q^*\frac{\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.$$

Since $\left[1+N_S\left\{1-\frac{Q^*}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.\frac{\partial^2 Q^*}{\partial p^2}\right\}\right]>0$,

$$\text{Sign}\left(\frac{\partial p^*}{\partial c}\right)=\text{Sign}\left(1-\frac{\frac{\partial Q^*}{\partial c}}{\frac{\partial Q^*}{\partial p}}+\frac{\partial N_S}{\partial c}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)+Q^*\frac{\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right). \tag{A.11}$$

Given $N_S, \frac{\partial N_S}{\partial \rho}=0$. Therefore, from (A.11),

$$\text{Sign}\left(\frac{\partial p^*}{\partial c}\right)=\text{Sign}\left(1-\frac{\frac{\partial Q^*}{\partial c}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right).$$

From Lemma 3, $\frac{\partial Q^*}{\partial p}<0$, which is also confirmed by equation (10). Equation (10) also implies $\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)=0$. From equation (10), we have

$$\frac{\partial Q^*}{\partial c}=-\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left[\bar{\mu}^{\left(\frac{2-\beta}{1-\beta}\right)-1}\frac{\partial\bar{\mu}}{\partial c}\right]. \tag{A.12}$$

To determine the sign of $\frac{\partial Q^*}{\partial c}$, we need to know the sign of $\frac{\partial\bar{\mu}}{\partial c}$. At $v(\bar{\mu})=\bar{v}$ equation (4) is written as: $\bar{v}=I-p^*N_Sq^*+\bar{\mu}k^\beta$. Then, by application of equation (3),

$$\frac{\partial\bar{\mu}}{\partial c}=\frac{\partial N_S}{\partial c}\left[\left(\frac{q^*p^*}{k^\beta}\right)-\left(\frac{\bar{\mu}\beta}{k}\right)N_S^{\frac{1}{\rho}}\left[\frac{1}{\rho}\left(\frac{q^*}{N_S}\right)\right]\right]. \tag{A.13}$$

Given $N_S, \frac{\partial N_S}{\partial c}=0$. Therefore, from (A.13) $\frac{\partial\bar{\mu}}{\partial c}=0$. So from (A.12), $\frac{\partial Q^*}{\partial c}=0$.

Since $\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)=0$ and $\frac{\partial Q^*}{\partial c}=0$, the sign of $\left(1-\frac{\frac{\partial Q^*}{\partial c}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)>0$. The statement of the first part of the proposition follows.

(ii) From equation (12):

$$\frac{\partial N_S}{\partial c}=-\frac{\left\{\left(\frac{\partial p^*}{\partial c}\right)\left[\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}+N_S{}^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial\bar{\mu}}{\partial p}\right]+\frac{F}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}\right\}}{\left[\frac{\rho-\beta}{\rho(1-\beta)}\right]N_S^{\frac{\beta-\rho}{\rho(1-\beta)}-1}\left(\frac{1-\beta}{2-\beta}\right)\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right)}.$$

Since the denominator of the term on the RHS of the above equation is positive,

$$\text{Sign}\left(\frac{\partial N_S}{\partial c}\right)=-\text{Sign}\left[\left(\frac{\partial p^*}{\partial c}\right)\left[\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}+N_S{}^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial\bar{\mu}}{\partial p}\right]+\frac{F}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}\right]. \tag{A.14}$$

Since $\frac{\partial\bar{\mu}}{\partial p}>0$ from Lemma 2 and $\frac{\partial p^*}{\partial c}>0$ as proved in the first part of the proposition, from (A.14) by use of Assumption 3 the statement of the second part of the proposition follows.

(iii) Since $\frac{\partial N_S}{\partial c}<0$, as suggested by (A.11) the sign of $\left(\frac{\partial p^*}{\partial c}\right)$ is determined by the sign of $\left(1-\frac{\frac{\partial Q^*}{\partial c}}{\frac{\partial Q^*}{\partial p}}+\frac{\partial N_S}{\partial c}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)+Q^*\frac{\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)$.

Since $\frac{\partial}{\partial c}\left(\frac{\partial Q^*}{\partial p}\right)=0$, the sign of $\left(\frac{\partial p^*}{\partial c}\right)$ is determined by the sign of $\left(1-\frac{\frac{\partial Q^*}{\partial c}}{\frac{\partial Q^*}{\partial p}}+\frac{\partial N_S}{\partial c}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)\right)$. From (A.12), by using (A.13),

$$\frac{\partial Q^*}{\partial c}=-\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\left(\frac{2-\beta}{1-\beta}\right)-1}\frac{\partial N_S}{\partial c}\left[\left(\frac{q^*p^*}{k\beta}\right)-\left(\frac{\bar{\mu}\beta}{k}\right)N_S^{\frac{1}{\rho}}\left[\frac{1}{\rho}\left(\frac{q^*}{N_S}\right)\right]\right]. \tag{A.15}$$

Since $\frac{\partial N_S}{\partial c}<0$ from the second part of the proposition and $\left[\left(\frac{q^*p^*}{k\beta}\right)-\left(\frac{\bar{\mu}\beta}{k}\right)N_S^{\frac{1}{\rho}}\left[\frac{1}{\rho}\left(\frac{q^*}{N_S}\right)\right]\right]<0$, from (A.15), $\frac{\partial Q^*}{\partial c}<0$.

Since $\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) > 0$ and $\frac{\partial Q^*}{\partial p} < 0$, the statement of the third part of the proposition follows. □

Proof of Proposition 4.

(i) From equation (8),

$$\frac{\partial p^*}{\partial \bar{v}}\left[1 + N_S\left\{1 - \frac{Q^*}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.\frac{\partial^2 Q^*}{\partial p^2}\right\}\right] = -\frac{\frac{\partial Q^*}{\partial \bar{v}}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \bar{v}}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) + Q^* \frac{\frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.$$

Since $\left[1 + N_S\left\{1 - \frac{Q^*}{\left(\frac{\partial Q^*}{\partial p}\right)^2}.\frac{\partial^2 Q^*}{\partial p^2}\right\}\right] > 0$,

$$\text{Sign}\left(\frac{\partial p^*}{\partial \bar{v}}\right) = \text{Sign}\left(-\frac{\frac{\partial Q^*}{\partial \bar{v}}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \bar{v}}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) + Q^* \frac{\frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right). \tag{A.16}$$

Given $N_S, \frac{\partial N_S}{\partial \rho} = 0$. Therefore, from (A.16),

$$\text{Sign}\left(\frac{\partial p^*}{\partial \bar{v}}\right) = \text{Sign}\left(-\frac{\frac{\partial Q^*}{\partial \bar{v}}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right).$$

From Lemma 3, $\frac{\partial Q^*}{\partial p} < 0$, which is also confirmed by equation (10). Equation (10) also implies $\frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right) = 0$ and,

$$\frac{\partial Q^*}{\partial \bar{v}} = -\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}} N^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\left(\frac{2-\beta}{1-\beta}\right)-1}\right)\frac{\partial \bar{\mu}}{\partial \bar{v}}. \tag{A.17}$$

To determine the sign of $\frac{\partial Q^*}{\partial \bar{v}}$, we need to know the sign of $\frac{\partial \bar{\mu}}{\partial \bar{v}}$. At $v(\bar{\mu}) = \bar{v}$ equation (4) is written as: $\bar{v} = I - p^* N_S q^* + \bar{\mu} k^\beta$. Then, by application of equation (3),

$$\frac{\partial \bar{\mu}}{\partial \bar{v}} = \frac{1}{k^\beta} + \frac{\partial N_S}{\partial \bar{v}}\left[\left(\frac{q^* p^*}{k^\beta}\right) - \left(\frac{\bar{\mu}\beta}{k}\right) N_S^{\frac{1}{\rho}}\left[\frac{1}{\rho}\left(\frac{q^*}{N_S}\right)\right]\right]. \tag{A.18}$$

Given $N_S, \frac{\partial N_S}{\partial \bar{v}} = 0$. Therefore, from (A.18) $\frac{\partial \bar{\mu}}{\partial \bar{v}} > 0$. So, from (A.17), $\frac{\partial Q^*}{\partial \bar{v}} < 0$.

Since $\frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right) = 0$ and $\frac{\partial Q^*}{\partial \bar{v}} < 0$, the sign of $\left(-\frac{\frac{\partial Q^*}{\partial \bar{v}}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) < 0$. The statement of the first part of the proposition follows.

(ii) From equation (12):

$$\frac{\partial N_S}{\partial \bar{v}} = -\frac{\left[\left(\frac{\partial p^*}{\partial \bar{v}}\right)\left[\left(\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}\right) + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \bar{v}}\right)\right]}{\left[\frac{\rho-\beta}{\rho(1-\beta)}\right]N_S^{\frac{\beta-\rho}{\rho(1-\beta)}-1}\left(\frac{1-\beta}{2-\beta}\right)\left(1-\bar{\mu}^{\frac{2-\beta}{1-\beta}}\right)}.$$

Since the denominator of the term on the RHS of the above equation is positive,

$$\text{Sign}\left(\frac{\partial N_S}{\partial \bar{v}}\right) = -\text{Sign}\left[\left[\left(\frac{\partial p^*}{\partial \bar{v}}\right)\left[\left(\frac{F\left[\frac{1}{1-\beta}[L-(1-\beta)]\right]}{\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}}(p^*-c)^2}\right) + N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\frac{1}{1-\beta}}\right)\frac{\partial \bar{\mu}}{\partial p}\right] + \left(N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}\bar{\mu}^{\frac{1}{1-\beta}}\frac{\partial \bar{\mu}}{\partial \bar{v}}\right)\right]\right]. \quad \text{(A.19)}$$

Since $\frac{\partial \bar{\mu}}{\partial p} > 0$ from Lemma 2 and $\frac{\partial \bar{\mu}}{\partial \bar{v}} > 0$, $\frac{\partial p^*}{\partial \rho} < 0$ as proved in the first part of the proposition, from (A.19) by use of Assumption 3 the statement of the second part of the proposition follows.

(iii) If $\frac{\partial N_S}{\partial \bar{v}} \neq 0$, as suggested by (A.16) the sign of $\left(\frac{\partial p^*}{\partial \bar{v}}\right)$ is determined by the sign of $\left(-\frac{\frac{\partial Q^*}{\partial \bar{v}}}{\frac{\partial Q^*}{\partial p}} + \frac{\partial N_S}{\partial \bar{v}}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) + Q^* \frac{\frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)$.

Notice that $\frac{\partial Q^*}{\partial p} < 0, \frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q^*}{\partial p}\right) = 0$ from equation (10) and $\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}} + Q^* \frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right) > 0$ as argued above. From equation (A.17) and (A.18),

$$\frac{\partial Q^*}{\partial \bar{v}} = -\left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}} N^{\frac{\beta-\rho}{\rho(1-\beta)}}\left(\bar{\mu}^{\left(\frac{2-\beta}{1-\beta}\right)-1}\right)\left[\frac{1}{k^\beta} + \frac{\partial N_S}{\partial \bar{v}}\left[\left(\frac{q^* p^*}{k^\beta}\right) - \left(\frac{\bar{\mu}\beta}{k}\right) N_S^{\frac{1}{\rho}}\left[\frac{1}{\rho}\left(\frac{q^*}{N_S}\right)\right]\right]\right]. \quad \text{(A.20)}$$

On the RHS of (A.20), $\left[\left(\frac{q^*p^*}{k^\beta}\right)-\left(\frac{\overline{\mu}\beta}{k}\right)N_S^{\frac{1}{\rho}}\left[\frac{1}{\rho}\left(\frac{q^*}{N_S}\right)\right]\right]<0$. Therefore if $\frac{\partial N_S}{\partial \bar{v}} \leq 0$, the RHS of (A.20) is negative, which implies $\frac{\partial Q^*}{\partial \bar{v}}<0$. Since $\left(-\frac{\frac{\partial Q^*}{\partial \bar{v}}}{\frac{\partial Q^*}{\partial p}}+\frac{\partial N_S}{\partial \bar{v}}\left(-\frac{\frac{\partial Q^*}{\partial N_S}}{\frac{\partial Q^*}{\partial p}}+Q^*\frac{\frac{\partial}{\partial N_S}\left(\frac{\partial Q^*}{\partial p}\right)}{\left(\frac{\partial Q^*}{\partial p}\right)^2}\right)\right)<0$ in this case, $\frac{\partial p^*}{\partial \bar{v}}<0$. If $\frac{\partial N_S}{\partial \bar{v}}>0$, from (A.20) $\frac{\partial Q^*}{\partial \bar{v}}>=<0$ if and only if $\left[\frac{1}{k^\beta}+\frac{\partial N_S}{\partial \bar{v}}\left[\left(\frac{q^*p^*}{k^\beta}\right)-\left(\frac{\overline{\mu}\beta}{k}\right)N_S^{\frac{1}{\rho}}\left[\frac{1}{\rho}\left(\frac{q^*}{N_S}\right)\right]\right]\right]<=>0$. The statement of the third part of the proposition follows. □

Appendix B2

Evaluating $v(\bar{\mu})$ at $q^*, \overline{\mu}(N_S, p^*, \bar{v})$ is derived as a solution to the following equation:

$$Y(\bar{\mu}, N_S, p^*) = \bar{v} - I + X(\bar{\mu}, N_S, p^*)$$

where, $X(\mu, N_S, p^*) = p^* N_S q^* = \mu^{\frac{1}{1-\beta}} N_S^{\frac{\beta(1-\rho)}{\rho(1-\beta)}} (\frac{\beta}{p*\beta})^{\frac{1}{1-\beta}}$ is the contribution of expenditure on indirect utility defined as 'budget effect' and $Y(\mu, N_S, p^*) = \mu k^{\beta} = \mu^{\frac{1}{1-\beta}} N_S^{\frac{\beta(1-\rho)}{\rho(1-\beta)}} (\frac{\beta}{p*})^{\frac{\beta}{1-\beta}}$ is the contribution of preference on indirect utility defined as 'preference effect'. Since, $\bar{v} > I$, $Y(0, N_S, p^*) = X(0, N_S, p^*) = 0$ and $\frac{\partial Y}{\partial \mu} > 0$, $\frac{\partial X}{\partial \mu} > 0$, the existence of $\bar{\mu}$ requires $\frac{\partial Y}{\partial \mu} > \frac{\partial X}{\partial \mu}$ for all values of μ in $[0,1]$.

Let us assume, $\frac{\partial Y}{\partial \mu} > \frac{\partial X}{\partial \mu}$ for all values of μ in $[0,1]$.

The assumption ensures, an extensive margin $\bar{\mu}(N_S,\ p^*(\rho, \bar{v}, I, \beta, c), \bar{v})$ exists at the differentiated product market such that all buyers having $\mu \geq \bar{\mu}(N_S, p^*(\rho, \bar{v}, I, \beta, c), \bar{v})$ participate in the market and all buyers having $\mu < \bar{\mu}(N_S, p^*(\rho, \bar{v}, I, \beta, c), \bar{v})$ do not participate in the market.

Chapter 3

“Love for Variety” and Monopoly Platform

3.1 Introduction

Having explored the co-existence of e-commerce platforms and 'brick-and-mortar' stores in Chapter 1, we explicitly introduce a two-sided platform vis-à-vis an outside option in the form of 'brick-and-mortar' stores in Chapter 2. Two-sided platforms refer to a market situation where two distinct groups of economic agents, namely, buyers and sellers, interact with each other and gain access to the members on the opposite side using a common platform. The platform reduces the transaction costs of the interaction and achieves efficiency. Post-COVID-19, the e-commerce sector has the most prominent two-sided platforms. Names like Amazon, Flipkart, Myntra, eBay, Snapdeal, etc., are now familiar in every household. The World market for e-commerce platforms has seen rapid expansion and is expected to increase to US$6.5 trillion in the next five years (Market Insights, Statista, 2024). India is not an exception. Statista's Market Insights for India (2024) highlights that the value of India's e-commerce market is projected to rise to US$0.1 trillion in the same period. Hence, the motivation.

The platforms are known for offering a large number of varieties of products for sale, produced by different sellers, in one place, which attracts buyers to it. The buyers get access to a variety of products in one place, reducing their search costs. However, the theoretical literature on platforms developed so far, which we briefly review below, has not paid sufficient attention to buyers' 'love for variety'. Specifically, the questions we address are: Does the presence of a platform increase the number of varieties in a market? What determines the number of varieties sold on a platform? Next, whether the cost of servicing buyers and sellers and fixed costs on the sellers' side affect the platform's membership fee and profit. We also ask how the presence of outside option of the buyers (like the presence of 'brick-and-mortar' shops) affects a platform and whether greater differentiation of varieties is good for a platform. To answer these questions, this chapter presents a theoretical model of a platform with a 'love for variety' preference of the buyers. The model endogenously determines the number of varieties sold in a platform. It shows that the presence of a platform increases the number of varieties produced of a differentiated product compared to a situation of 'no-platform' equilibrium. It also focuses on the platform's incentive to increase its efficiency in servicing the sellers vis-à-vis the buyers and the platform's reaction in terms of pricing to both sides as a response to a reduction in the sellers' fixed cost of production.

In the model presented in this chapter, a buyer has a Dixit-Stiglitz type of 'love for variety' utility function (Dixit & Stiglitz, 1977), which implies that a higher number of varieties consumed increases the utility of the buyer. The buyers are identical, except that their preference for the differentiated product differs from each other. Depending on her preference, the buyer also has the option of purchasing a homogeneous product. Each seller sells a specific variety of the differentiated product. Owing to her 'love for variety' utility function, she purchases some amount of every variety of the differentiated product, whether the seller is located at the platform or outside it. The platform charges a membership fee for both buyers and sellers. The sellers differ from each other in terms of their transaction costs. However, all of them, irrespective of their location inside or outside the platform, involve themselves in price competition with each other. The comparative static exercises highlight the role of factors like the platform's costs of servicing the buyers and the sellers, the fixed costs faced by the seller side of the market in their operation, outside options of the buyers, and the substitutability of the varieties of the product, on the membership charged by the platform on either side of the market, the number of varieties sold both on the platform, and outside it, and the platform's profit.

The platform internalizes the externalities that either side of the market creates for the other side and decides the membership fee to charge to both sides. The results show that if the costs of servicing the clients on one particular side of the market fall, the platform lowers the membership fee charged to that side by raising the membership fee on the other side. For example, if the costs of servicing the buyers fall, the buyers' membership fee is cross-subsidized by raising the membership fee to the sellers. The number of varieties on the platform rises accordingly. The platform exploits the higher intensive and extensive margin of demand enjoyed by the sellers to extract higher rent from them. The opposite happens if the costs of servicing the sellers fall. Interestingly, the results suggest that, unlike the standard monopoly case, its efficiency in servicing the clients does not necessarily translate into a higher profit for the platform. The platform's profit rises if it is more efficient in servicing the sellers but falls if it is more efficient in servicing the buyers. Therefore, a platform has more incentive to become efficient in servicing the sellers rather than buyers. If the sellers have a lower fixed cost, although the buyers enjoy a lower membership fee, the sellers may face a higher membership fee if the platform expects them to gain due to the lowering of the fixed cost. The number of varieties sold on the platform depends on the membership fee charged to the sellers. It falls if a higher fee is charged. It rises for a lower membership fee. However, since the platform's profit falls, it may not like developments that lower the seller's fixed costs. If the buyers have a better outside option of buying out of the differentiated product market, like buying from a 'brick-and-mortar' shop. In that case, the platform reacts by lowering the membership fee for the buyers. However, the seller's fee and the number of varieties sold on the platform may rise or fall depending on the platform's calculation of the seller's gain and loss. The results suggest that even if the differentiated product market loses its buyers to the 'brick-and-mortar' shops, the platform may raise the membership fee for the sellers. Then again, there are conditions when an increase in the utility from participating in a 'brick-and-mortar' shop and a rise in the number of sellers on the platform coexists[*]. The platform, however, loses profit if 'brick-and-mortar' shops turn out to be an attractive option for

[*] In a recent The Times of India report (August 23, 2024), it was pointed out that Indian consumers, in towns or metro cities, exhibit 'love for variety' and it has always been "and" and not "either-or" when it comes to shopping from Amazon and physical retail shops.
Read at: https://timesofindia.indiatimes.com/blogs/toi-edit-page/consumer-indias-amma-jaans/

shopping. If the varieties of differentiated products are close substitutes, sellers' monopoly power falls, and the buyers leave the differentiated product market. In such a situation, the platform lowers the membership fee to the buyers to bring them back into the market. Since the sellers may gain or lose out of the process, they are charged accordingly. In our framework, if the platform expects them to gain, it charges a higher membership fee. The opposite happens otherwise. Since the platform loses if the varieties the sellers sell in the differentiated product market are close substitutes, it may incentivize them to take initiatives to increase their product diversity.

Platform pricing in monopoly setting has a rich literature pioneered by Rochet and Tirole (2003, 2006). The papers like Caillaud and Jullien (2001, 2003), Armstrong (2006), Weyl (2010), Hagiu and Halaburda (2014), Economides and Hermalin (2015), Belleflamme and Peitz (2015), Hovenkamp (2020), Jeon et al. (2022) explored various aspects of it further. The differentiated products on the platform are discussed by papers like Hagiu (2009), Galeotti, and Moraga-González (2009) and closer to the scope of the present chapter. We incorporate the non-zero outside option of the buyer, which is equally important because it theoretically captures the trade-off between the 'brick-and-mortar' shops and the platform (as discussed in Belleflamme & Peitz, 2021). Contrary to a monopoly platform setup, Economides and Katsamakas (2006) model in the context of competing platforms shows that a strong preference for a variety of operating system platforms may lead to higher total profits in a proprietary platform (e.g., Windows) as compared to an open-source platform (e.g., Linux). A similar result was discussed in Manten and Saha (2012). In this context of competing platforms, few studies have introduced Bertrand (Tan & Zhou, 2021; Teh et al., 2023) and Cournot competition (Correia-da Silva et al., 2019) among the platforms in terms of either fixed membership fees or per-unit usage fees but with an ad-hoc formulation of both the buyer and the seller sides to focus on the oligopolistic interactions among the platforms. Coming to the more recent literature on platforms dealing with differentiated multi-product sellers, (Zhu & Liu, 2018; Etro, 2021; Hagiu et al., 2022; Choi et al., 2020;

Acemoglu et al., 2022; Bergemann et al., 2022; Bergemann & Bonatti, 2024; Kirpalani & Philippon, 2020; Bounie et al., 2021) the focus is on the exploitation of sellers' and buyers' information acquired by the platform in the process of the transaction. Also, the other area where the issue of variety available on platforms has been addressed is how a firm's distribution channel structure impacts its optimal product variety decisions (Rajagopalan & Xia, 2012; Guo & Heese, 2017; Sweeney et al., 2022). However, this chapter does not look at the issue of 'love for variety' from the information issue perspective or the distribution channel structure.

Our framework introduces a 'love for variety' preference pattern for buyers in a framework similar to Hagiu (2009), Galeotti and Moraga-González (2009) and endogenizes the intensive and extensive margin of demand, similar to Chapter 2. Unlike Hagiu (2009), Galeotti and Moraga-González (2009), the number of varieties of the differentiated product gets endogenously determined in the model. It shows that the presence of a platform increases the number of varieties of the differentiated product produced in the market. It also adds insight into the platform's incentive to increase its efficiency in servicing the buyers' and sellers' sides of the market, which is not mentioned in the literature. It shows that the platform has an incentive to be more efficient in servicing the sellers than the buyers. It argues that the platform may not like developments that lower the seller's fixed costs. It shows that an increase in the number of sellers on the platform coexists with an increase in the utility of participating in a 'brick-and-mortar' store. Even with a rise in the substitutability among the varieties, the platform can raise the membership fee on the seller side. These results are new to the literature. We also get a result similar to Belleflamme and Peitz (2019), Chawla and Mondal (2024) regarding a fall in the platform's profit with a rise in substitutability between the varieties of the differentiated product. Unlike ours, strategic price interaction among sellers is absent in their model.

The next section of the chapter presents the model and derives the results. Section 3 presents the comparative static exercises. Section 4 contains the conclusion.

3.2 The Model

3.2.1 A Baseline Model

The differentiated product market we talk about comprises three types of agents: a monopoly platform, buyers (B), and sellers (S). Each seller produces a unique variety of the product by using an increasing return-to-scale technology with a fixed cost of production $F > 0$ and a unit cost of production $c > 0$. The j^{th} seller also incurs a fixed transaction cost T_j for carrying out business activities. This transaction cost can be thought of as a fixed cost of advertising or publicity apart from the fixed costs of production. We assume T_j is uniformly distributed among sellers in the interval $[0, 1]$. The platform caters to a two-sided market by hosting buyers and sellers who want to transact through it. The platform charges a one-time membership fee F_B to each buyer and F_S to each seller for their participation at the platform irrespective of the number of units transacted through it[†]. By joining the platform, the j^{th} seller avoids the transaction cost T_j.

In our model, the measure of the number of buyers is 1, N_B of them enter the market for the differentiated product. Each of the buyers has a 'love for variety' utility function, and therefore, those who enter the differentiated product market buy from every seller present in the market independent of whether they are members of the platform or not. So, the buyers who enter the differentiated product market in our model also join the platform if some sellers of the differentiated product operate on the platform.

[†] Participating in popular marketplaces like Amazon, Flipkart, and eBay, among others, comes with regular fees for listing, selling, and promoting products along with a per transaction fee. For simplicity, we have kept only the membership fee. However, the pricing scheme involving only membership fees is reasonable when monitoring transactions is difficult (see Gal-Or & Shi, 2022). This happens in the case of exhibitions, yellow pages, magazines, etc. The benefit of registration/membership is that the members do not have to fill in their identity details every time they log on to the platform. So, it saves the members' transaction costs. For buyers, it is like paying a monthly/annual subscription, say for Amazon Prime, irrespective of the number of units they buy.

We also assume that the measure of the number of potential sellers in the differentiated product market approaches 1, N_S of them entering the market in the absence of the platform. We show that in the presence of the platform, every seller enters the market, and $\widetilde{N}_S$ of them operates as a member of the platform. The values of N_B, N_S and $\widetilde{N}_S$ gets endogenously determined in the model. Since the platform increases the number of varieties produced in the market for the differentiated product, the buyers with a 'love for variety' utility function gain from the existence of the platform.

The platform incurs a cost, C_S, for servicing each seller, and a cost, C_B, for servicing each buyer. Only the registered agents can carry out transactions on the platform. We assume that both buyers and sellers single-home on the platform if they are registered with it. The unregistered sellers and buyers do not pay any fee. The profit of the platform, therefore, is written as:

$$\pi = (F_S - C_S)\widetilde{N}_S + (F_B - C_B)N_B. \tag{1}$$

The sequence of decisions in the model is as follows.

Period 1: the platform decides about (F_B, F_S) by maximizing π;

Period 2: the sellers observe F_S, and then decide whether to join the platform;

Period 3: the Bertrand competition among sellers determines the prices of their products;

Period 4: given F_B, the number of varieties available in the differentiated product market and on the platform, and their prices, a consumer decides whether to join the differentiated product market and the platform or to stay out of it.

Then the payoffs are realized.

The model is solved using the backward induction method.

We first solve the buyer's problem in Period 4.

3.2.2 The Buyers

In our model, the utility function of the μ-type buyer is represented by:

$$u(\mu) = \max\left\{\left(q_0 + \mu\left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta}\right), \bar{v}\right\} \tag{2}$$

where μ is the valuation attached by the buyer on the basket of the N_S varieties of the differentiated product. We assume μ is distributed uniformly over $[0, 1]$. She also derives utility from a composite commodity on which she spends q_0. However, a buyer may choose to stay out of the differentiated product market and participate in an alternative consumption program like transacting in 'brick-and-mortar' stores in which a single product is available with no brand variation. She derives a utility of $\bar{v}$ from the alternative program. $\rho \in (0, 1)$ denotes the substitutability among the varieties of the differentiated product in the buyer's preference irrespective of her type, and $\mu\left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta}$ is the utility obtained by the μ type buyer from the consumption of the basket of varieties of the differentiated product. The extent of substitutability falls with lower values of ρ. The preference parameter $\beta > 0$ represents the substitutability between the differentiated product and the composite commodity. We assume $\beta < \rho$ to ensure that the utility function in (2) is concave in q_j.

Given $p_j > 0$ is the price of the differentiated product *j*, ($\forall j = 1, \dots \widetilde{N}_S \dots N_S$), a buyer of type μ maximizes her utility by choosing $q_j \geq 0$ ($\forall j = 1, \dots \widetilde{N}_S \dots N_S$) subject to her budget constraint

$$q_0 + \sum_{j=1}^{N_S} p_j\, q_j \leq I - F_B \qquad \forall\, j = 1, \dots \widetilde{N}_S \dots N_S. \tag{3}$$

where I is the income of the buyer. Given the budget constraint binds, substituting for q_0 from equation (3) in equation (2), the maximization problem of the μ^{th} type buyer becomes

$$\max \emptyset_{\mu} = (I - F_B) - \sum_{j=1}^{N_S} p_j q_j + \mu\left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta} \quad w.r.t\ 'q_j' \ \ \forall\ j = 1, \dots \widetilde{N}_S \dots N_S.$$

The equilibrium choice of $q_j \geq 0$ for all $j = 1, \dots \widetilde{N}_S \dots N_S$ satisfies,

$$\frac{\partial \emptyset_\mu}{\partial q_j} \leq 0 \Rightarrow -p_j + \mu\beta\left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta-1}\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1-\rho}{\rho}} q_j{}^{\rho-1} \leq 0. \tag{4}$$

The μ type consumer's demand for the j^{th} variety is derived from equation (4) for all $j = 1, \dots \widetilde{N}_S \dots N_S$ as $q_j(p_1, \dots p_j \dots \dots, p_{N_S}, \mu, \rho)$. For $q_j > 0$ for all $j = 1, \dots \widetilde{N}_S \dots N_S$. $\frac{\partial \emptyset_\mu}{\partial q_j} = 0$ is the first-order condition for maximization. Given $\rho \in (0, 1)$ and $\beta < \rho$, the model ensures that the second-order condition, $\frac{\partial^2 \emptyset_\mu}{\partial q_j{}^2} < 0$, holds.

For simplicity, let us define, $\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}} = k$. If $q_j > 0$ for all $j = 1, \dots \widetilde{N}_S \dots N_S$, $k > 0$.

Using the definition of k and substituting the value of q_j that solves equation (4), in $\emptyset_\mu$, the indirect utility function of a buyer of type μ is written as,

$$v(\mu) = (I - F_B) - \sum_{j=1}^{N_S} p_j\, q_j + \mu k^{\beta}. \tag{5}$$

The indirect utility $v(\mu)$ is nothing but the value of $\emptyset_\mu$ at $q_j(p_1, \dots p_j \dots, p_{N_S}, \mu, \rho)$.

Since $\frac{\partial v}{\partial \mu} = -\sum_{j=1}^{N_S}\left[\frac{\partial \emptyset_\mu}{\partial q_j}\right]\frac{\partial q_j}{\partial \mu} + k^{\beta}$ and $\frac{\partial \emptyset_\mu}{\partial q_j} = 0$ from equation (4), $\frac{\partial v}{\partial \mu} = k^{\beta} > 0$ for all values of μ in $[0, 1]$. Therefore, $v(\mu)$ is a continuous and monotonically increasing function of μ in $[0, 1]$. Among the buyers, a buyer who has a higher valuation of the differentiated product enjoys a higher level of indirect utility. We also know from equation (4) that as $\mu \to 0, q_j \to 0$. Then it follows from equation (5) that since $k \to 0, v(\mu) \to (I - F_B)$. Also, notice that since $v(\mu)$ is monotonically increasing function of μ, it must be $v(1) > (I - F_B)$.

Assumption 1: $\bar{v} \in ((I - F_B), v(1))$[‡].

Assumption 1, along with the fact that $v(\mu)$ is a monotonically increasing continuous function of μ in [0,1], implies that there exists a value of $\mu = \bar{\mu}(p_1, \dots \dots, p_{N_S}, \rho, \bar{v}, F_B)$ in $(0, 1)$ such

[‡] Notice that Assumption 1 can be violated in two different ways, both of which are not interesting for our purpose. If $\bar{v} < (I - F_B)$, the buyers irrespective of their types purchase the differentiated product. If $\bar{v} > v(1)$, none of the buyers purchase the differentiated product.

that $v(\bar{\mu}) = \bar{v}$. It follows from equation (2) that a buyer of type μ participates in the differentiated product market if and only if $v(\mu) \geq \bar{v}$. The buyer of type $\bar{\mu}$ is the marginal buyer who is indifferent between purchasing the differentiated product and availing of the outside option. We assume she purchases the differentiated product. Therefore, the types of buyers participating in the differentiated product market are given by $[\bar{\mu}, 1]$.

Since μ is distributed uniformly over $[0, 1]$, the number of buyers participating in the differentiated product market is given by:

$$N_B = \Pr[\mu \geq \bar{\mu}(\cdot)] = 1 - \bar{\mu}(\cdot). \tag{6}$$

The aggregate demand for the j^{th} variety of differentiated product (Q_j) is

$$Q_j = \int_{\bar{\mu}(\cdot)}^{1} q_j(\cdot) d\mu \quad \forall\ j = 1, \dots \widetilde{N}_S \dots N_S, \tag{7}$$

where $q_j(\cdot)$ satisfies equation (4). Notice that Q_j derived in equation (7) is a function of $(p_1, \dots \dots, p_{N_S}, \rho, \bar{v}, F_B)$.

Lemma 1: (similar to Lemma 1 of Chapter 2) If $q_j > 0$,

a. $\frac{\partial q_j}{\partial p_j} < 0,$

b. $\frac{\partial q_j}{\partial \rho} < 0$

for all $\mu \in [0,1]$ and for all $j = 1, \dots \dots N_S$.

For every buyer irrespective of their type, Lemma 1 states that with 'love for variety' preference, each variety of the differentiated product is like a normal good having a negatively sloped demand function. With a rise in ρ, the amount purchased of each variety of the product falls as varieties are now closer substitutes of each other in the buyers' preference. From equation (5), after substituting $v(\bar{\mu}) = \bar{v}$, we obtain:

$$\frac{\partial \bar{\mu}}{\partial p_j} = \frac{q_j}{k^{\beta}}. \tag{8}$$

Lemma 2: If $q_j > 0$

$a. \frac{\partial \bar{\mu}(\cdot)}{\partial p_j} > 0,$

$b. \frac{\partial N_B}{\partial p_j} < 0$

for all $j = 1, \dots \dots N_S$.

Proof: The proof of $\frac{\partial \bar{\mu}(\cdot)}{\partial p_j} > 0$ follows from Lemma 2 of Chapter 2, and for $\frac{\partial N_B}{\partial p_j} < 0$ see the Appendix A3. □

Ceteris paribus an increase in the price of the j^{th} variety of the differentiated product, which is purchased by the marginal buyer $\bar{\mu}$ at the initial equilibrium, reduces her indirect utility level below $\bar{v}$. Therefore, she stops buying the differentiated product and avails herself of the outside option. Only the buyers with a higher valuation of the differentiated product than the marginal buyer continue to purchase it. Therefore, $\bar{\mu}$ rises at the new equilibrium, resulting in the number of buyers in the differentiated product market falling.

From equation (7):

$$\frac{\partial Q_j}{\partial p_j} = -\left[Q_j \frac{\partial \bar{\mu}(\cdot)}{\partial p_j}\right] + \left[\int_{\bar{\mu}(\cdot)}^{1} \frac{\partial q_j(\cdot)}{\partial p_j} d\mu\right]. \tag{9}$$

Lemma 3: (similar to Lemma 3 of Chapter 2) $\frac{\partial Q_j}{\partial p_j} < 0$ for all $j = 1, \dots \dots N_S$.

Lemma 3 states that the aggregate demand of each variety of the differentiated product is negatively sloped like normal commodities: as the price of a variety rises, its aggregate demand (demand from all types of buyers taken together) falls. Intuitively, this happens from two different sources. First, as Lemma 1a suggests, all other things remain the same, as the price of a variety rises, its demand from every type of buyer, who continues to purchase the variety, falls, i.e., the demand shrinks in intensive margin. Second, as Lemma 2a suggests, as the price of a variety rises, some buyers stop buying the product, i.e., the demand shrinks in extensive margin, too.

Lemma 4: a. If $q_j > 0, \frac{\partial \bar{\mu}(\cdot)}{\partial F_B} > 0, \frac{\partial \bar{\mu}(\cdot)}{\partial \bar{v}} > 0, \frac{\partial \bar{\mu}(\cdot)}{\partial \rho} > 0$ for all $j = 1, \dots \dots N_S$;

b. $\frac{\partial N_B}{\partial F_B} < 0, \frac{\partial N_B}{\partial \bar{v}} < 0, \frac{\partial N_B}{\partial \rho} < 0$

Proof: See the Appendix A3. □

Ceteris paribus an increase either in F_B, or in $\bar{v}$ or in ρ, reduces the indirect utility level of the marginal buyer of the type $\bar{\mu}$. Therefore, she stops buying the differentiated product and avails herself of the outside option. Only the buyers with a higher valuation of the differentiated product continue to purchase it. Therefore, $\bar{\mu}$ rises at the new equilibrium and N_B falls.

Assumption 2: $\frac{\partial^2 N_B}{\partial \rho \partial F_B} \approx 0$.

The assumption that the effect of change in the demand side parameter, ρ, on $\frac{\partial N_B}{\partial F_B}$ is negligible, keeps the comparative static exercises of the model tractable.

Lemma 5: $\frac{\partial Q_j}{\partial F_B} < 0, \frac{\partial Q_j}{\partial \bar{v}} < 0, \frac{\partial Q_j}{\partial \rho} < 0$, for all $j = 1, \dots \dots N_S$.

Proof: See the Appendix A3. □

Since the membership fee F_B and outside option $\bar{v}$ does not have an impact on the intensive margin of demand, its entire effect on the aggregate demand of a variety works through the extensive margin. As suggested in Lemma 4, buyers with relatively lower valuations of the differentiated product stop buying the product with a rise in F_B. Since the marginal buyer exits, the aggregate demand for the product falls. A similar explanation follows for a rise in $\bar{v}$.

The different varieties of the differentiated product become closer substitutes of one another in the buyers' preference as ρ rises. This results in a fall in the indirect utility of using the differentiated product. The extensive margin of demand for each variety of the differentiated product shrinks. The intensive margin of demand also shrinks, as suggested by Lemma 1(b). Therefore, the aggregate demand for each variety falls as the varieties become closer substitutes for each other.

Assumption 3: $\frac{\partial}{\partial F_B}\left(\frac{\partial Q_j}{\partial p_j}\right) \approx 0, \frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q_j}{\partial p_j}\right) \approx 0, \frac{\partial}{\partial \rho}\left(\frac{\partial Q_j}{\partial p_j}\right) \approx 0.$

Assumption 2 implies that the slope of the aggregate demand function faced by the j^{th} seller at the market for the differentiated product is not very responsive to the change in $F_B, \bar{v}$, and ρ. The assumption simplifies the model and makes it tractable. On violation of Assumption 2, if $\frac{\partial}{\partial F_B}\left(\frac{\partial Q_j}{\partial p_j}\right) < 0, \frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q_j}{\partial p_j}\right) < 0, \frac{\partial}{\partial \rho}\left(\frac{\partial Q_j}{\partial p_j}\right) \leq 0$, all the results of the model go through. If they are weakly positive, then again all the results of the model go through.

3.2.3 The Sellers

Having solved the buyers' problem in period 4, we now move backward to period 3 to solve the sellers' price determination problem.

Each seller in its pricing decision takes the aggregate demand function derived in (7), which is negatively sloped (from Lemma 3), as given.

The j^{th} seller's profit is:

$\pi_j^{PL} = [p_j^{PL} - c]Q_j(.) - F_S - F$

if she operates at the platform, and,

$\pi_j^{O} = [p_j^{O} - c]Q_j(.) - T_j - F$

if she operates outside it, where p_j^{PL} is the price charged in the platform, and p_j^{O} is the price charged outside the platform. p_j^{PL} and p_j^{O} are determined through maximization of π_j^{PL} and π_j^{O} respectively. Irrespective of where she operates, given the buyers' 'love for variety' preferences, it can be seen that a rise in the number of buyers will lead to a rise in the aggregate demand of the seller of the j^{th} variety thereby leading to a rise in her profit. The first-order condition of profit maximization by choice of $p_j^{\omega} > 0$, assuming an interior solution, must satisfy, $\frac{\partial \pi_j^{\omega}}{\partial p_j^{\omega}} = 0$ for $\omega = PL, 0$, which implies:

$$p_j^{\omega} = c - \frac{Q_j(p_1,\ldots,p_j,\ldots,p_{N_S},\cdot)}{\frac{\partial Q_j}{\partial p_j}(p_1,\ldots,p_j,\ldots,p_{N_S},\cdot)}. \tag{10}$$

Since (F_S, F, T_j) do not affect p_j^{ω} for the sellers inside or outside the platform, $p_j^{\omega} = p_j$ for $\omega = PL, 0$. The sufficient condition that satisfies the second-order condition of each seller's profit maximization problem $\frac{\partial^2 \pi_j^{\omega}}{\partial p_j^2} < 0$ is: $\frac{\partial^2 Q_j}{\partial p_j^2} < 0$.

In our model, every variety of the differentiated product enters symmetrically in the utility function specified in equation (2), and therefore every seller faces the same demand function. They also have the same variable cost function. Although the nature of the fixed costs incurred by a seller varies depending on whether it is on the platform or off it, the fixed costs do not play a role in the Bertrand price-setting game that the sellers play among themselves. Since all sellers face the same set of parameters $(\rho, \bar{v}, F_B)$ while pricing their variety irrespective of whether they are on the platform or off it, it must be the case that the Nash equilibrium in prices is symmetric i.e.$p_1^* = \cdots p_j^* = \cdots = p_{N_S}^* = p^*(\rho, \bar{v}, F_B)$ i.e., all the sellers charge the same price to the buyers, regardless of the variety they produce.

Lemma 6: $\frac{\partial p^*}{\partial F_B} < 0, \frac{\partial p^*}{\partial \bar{v}} < 0, \frac{\partial p^*}{\partial \rho} < 0$.

Proof: See the Appendix A3. □

As F_B rises, from Lemma 5, for each seller, the demand for the variety of the differentiated product it produces shrinks. In response, the sellers lower their prices to boost the demand. Since with a rise in $\bar{v}$ and ρ, we observe a similar effect on Q_j, concerning their effect on p^*, a similar explanation follows.

Notice that regarding the effect of the rise in ρ i.e., the increased substitutability between the varieties of the differentiated product, the result reported in Lemma 6 is similar to Dixit and Stiglitz (1977), Armstrong (2006), Hagiu (2009), and Belleflamme and Peitz (2018a) among others.

Since each variety is sold at the same price $p^*(\rho, \bar{v}, F_B)$ at the market and each variety enters symmetrically into the utility function of the μ type buyer given in equation (2), from equation (4) it must be the case that an identical amount $q = \left(\frac{\mu\beta}{p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}$ is demanded of each variety by the μ-type buyer. It can be argued that under certain conditions $\bar{\mu}(N_S,\ p^*(\rho, \bar{v}, F_B), \bar{v})$ exists[§]. All buyers having $\mu \geq \bar{\mu}(N_S, p^*(\rho, \bar{v}, F_B), \bar{v},\ \rho, F_B)$ participate at the market, and all buyers having $\mu < \bar{\mu}(N_S, p^*(\rho, \bar{v}, F_B), \bar{v},\ \rho, F_B)$ do not participate in the market.

So, from equation (7) we derive the aggregate demand for each variety as

$$Q(.) = \left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}} \left(\frac{1-\beta}{2-\beta}\right)\left(1 - \bar{\mu}^{\frac{2-\beta}{1-\beta}}\right) \text{ for all } j = 1, \dots \dots N_S. \quad (11)$$

Since $p_j^{PL} = p_j^O = p^*(\rho, \bar{v}, F_B)$, the profit functions of the seller at the platform and the outside, are written as:

$$\pi_j^{PL} = [p^*(\rho, \bar{v}, F_B) - c]Q(.) - F_S - F, \quad (12)$$

and,

$$\pi_j^O = [p^*(\rho, \bar{v}, F_B) - c]Q(.) - T_j - F. \quad (13)$$

Notice from equations (12) and (13) that while π_j^O is monotonically declining in T_j, π_j^{PL} is independent of it.

The j^{th} seller operates in the differentiated product market if and only if $\pi_j^O \geq 0$ i.e.

$$T_j \leq [p^*(\rho, \bar{v}, F_B) - c]Q(.) - F.$$

Let us define,

$$T_0 = [p^*(\rho, \bar{v}, F_B) - c]Q(.) - F. \quad (14)$$

Therefore, the j^{th} seller operates in the market if and only if $T_j \leq T_0$.

[§] See Appendix B2 of Chapter 2 for the proof.

The j^{th} seller operates on the platform if and only if $\pi_j^{PL} \geq \pi_j^O$. After the substitution of π_j^{PL} and π_j^O from equation (12) and (13), the condition for the j^{th} seller's participation in the platform turns out as $T_j \geq F_S$.

Assumption 4: $F_S \leq T_0 < 1$.

Assumption 4 implies that the sellers with $T_j \in [F_S, 1]$ operate on the platform, and the sellers with $T_j < F_S$ operates outside of it. The sellers with $T_j \in (T_0, 1]$ if operates outside the platform makes a loss and exits the differentiated product market. In violation of Assumption 2, if $F_S > T_0$ no seller enters the platform, which is not a case of our interest.

The figure below represents the π_j^O and π_j^{PL} as function of T_j and explains the participation of sellers on the platform.

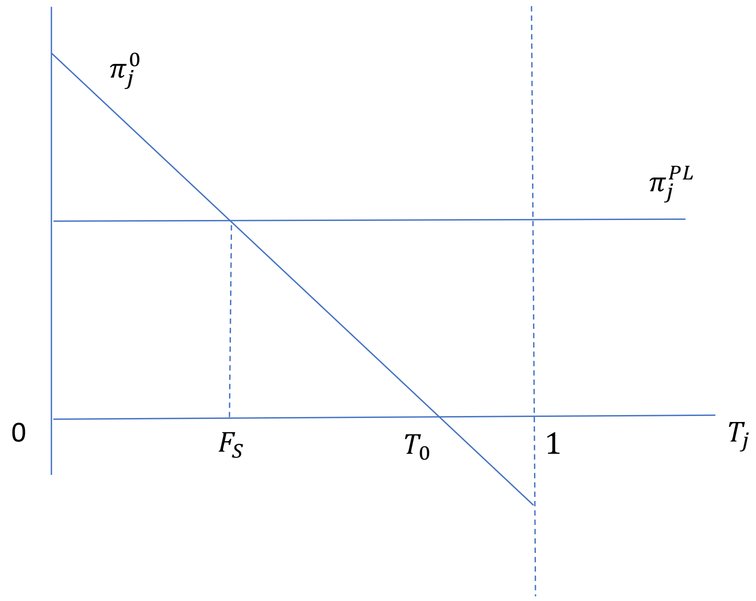

Figure 3.1: Participation of the sellers on the platform

Figure 3.1 shows clearly, as explained above, since the j^{th} seller with its $T_j \in [F_S, 1]$ has $\pi_j^{PL} \geq \pi_j^O$, participates on the platform. A seller with its T_j in $[0, F_S)$ having $\pi_j^{PL} < \pi_j^O$ stays outside the platform. A subset of the sellers that participate on the platform with their $T_j \in (T_0, 1]$, would not enter the differentiated product market in the absence of the platform. If $F_S > T_0$, all the sellers independent of their T_j earns negative profit by participating on the platform. Therefore, no seller participates on the platform.

Clearly, $N_S = \Pr[T_j \leq T_0]$

and,

$\tilde{N}_S = \Pr[T_j > F_S] = 1 - \Pr\,[T_j \leq F_S]$.

Since T_j is distributed uniformly over $[0, \bar{T}]$

$$(N_S = T_0, \tilde{N}_S = 1 - F_S). \tag{15}$$

Since $\tilde{N}_S = 1 - F_S$ in the presence of the platform, the number of sellers that operate outside the platform is F_S. The number of potential entrants who enter and produce in the differentiated market in the presence of the platform is 1, which is higher than T_0, the number of varieties produced in the absence of the platform. Therefore, the platform benefits the buyers with a 'love for variety' preference pattern by increasing the number of varieties produced in the market.

Proposition 1: *The existence of a platform increases the number of varieties produced in the market for the differentiated product and benefits the buyers.*

Proof: This follows from the discussion above. □

One of the salient features of the present model is the endogenous determination of the number of varieties in the differentiated product market in the platform's presence, which is not addressed in the existing literature on platforms. The chapters like Hagiu (2009), Galeotti and Moraga-González (2009), although they deal with differentiated products on the platforms, do not determine the number of varieties in the model. Proposition 1, in contrast, proves that in the presence of platforms, the number of varieties produced in the differentiated product market rises. We even found a piece of empirical evidence for the same in the existing literature.

In their study, Brynjolfsson et al. (2022) analyzed weekly transaction data between 2015 and 2019 regarding different categories of books sold in China's largest e-commerce platform, Taobao. Interestingly, they show that the number of sold varieties went up by 98 percent during this period, and what was even more surprising was that 70 percent of the overall book sales in the Chinese book market** was attributed to this e-commerce platform.

From Figure 3.1, it must be clear that a lower value of T_0 creates a wider scope of the sellers for participating on the platform. But it also severely restricts the choice of F_S by the platform. The platform knows that no seller participates on the platform unless its choice of membership fee for the sellers satisfies $F_S \leq T_0$.

Lemma 7 notes the effect of changing demand side parameters like F_B, $\bar{v}$, ρ, and F on T_0 and $\frac{\partial T_0}{\partial F_B}$.

Lemma 7: $\frac{\partial T_0}{\partial F_B} < 0$, $\frac{\partial T_0}{\partial \bar{v}} < 0$, $\frac{\partial T_0}{\partial \rho} < 0$, $\frac{\partial T_0}{\partial F} < 0$.

Proof: See the Appendix A3. □

With a rise in F_B, since the buyers' disposable income falls, the indirect utility received from the consumption of the differentiated product falls, and the marginal buyer, who was indifferent between buying and not buying the differentiated product, no longer buys the product. Given N_S, therefore the extensive margin of demand for the differentiated product shrinks as $\bar{\mu}$ rises, as explained in Lemma 4. Since each buyer who participates in the differentiated product market buys an equal amount of each of the available varieties, irrespective of whether the variety is available on the platform or off it, the exit of some buyers adversely affects the demand enjoyed

** The 2019 Annual Report of the Chinese Book Market published in 2020 provides details about the Chinese book market in 2019.

by each of the existing sellers. They react to the situation by reducing their price to boost the demand for their variety (Lemma 6). As the price falls, the indirect utility of the buyers buying the differentiated product rises, and the identity of the marginal buyers changes again as some buyers come back to the differentiated product market, generating a positive effect on T_0. However, in our model, this effect is not strong enough to outweigh the initial negative demand effect arising from the shrinkage in the extensive margin, and hence, T_0 falls. The negative effect of a rise in $\bar{v}$ and ρ on T_0 has a similar explanation since Lemma 4 suggests a rise in $\bar{\mu}$ and a fall in N_B as a consequence of a rise in $\bar{v}$ and ρ. A rise in F has a direct equiproportional negative impact on T_0.

Assumption 5: $\frac{\partial^2 T_0}{\partial F_B^2} \approx 0, \frac{\partial^2 T_0}{\partial \bar{v} \partial F_B} \approx 0, \frac{\partial^2 T_0}{\partial \rho \partial F_B} \approx 0.$

The assumption that the effect of change in demand side parameters like F_B, $\bar{v}$, and ρ, on $\frac{\partial T_0}{\partial F_B}$ is negligible, keeps the model tractable.

3.2.4 The Platform's Choice

The profit of the platform is:

$$\pi = (F_S - C_S)\widetilde{N}_S + (F_B - C_B)N_B \tag{16}$$

where $\widetilde{N}_S = 1 - F_S$ from equation (15), and $N_B = N_B(p^*(.), N_S(.), \rho, \bar{v}, F_B)$.

The first term on the right-hand side of equation (16) characterizes the net revenue earned from the seller side of the platform. Similarly, the second term represents the net revenue earned from the buyer side of the platform. The platform maximizes π by choosing $\{F_S, F_B\}$ subject to the constraint $F_S \leq T_0$.

Assumption 6: $T_0 < \frac{(1+C_S)}{2}$

In the case of an unconstrained optimization problem, the value that F_S takes is $\frac{(1+C_S)}{2}$. If assumption 6 holds it implies that the platform cannot charge a fee which gives it an unconstrained optimum. Therefore, $F_S = T_0$ becomes binding.

The Lagrange function of the optimization problem is:

$Z = (F_S - C_S)(1 - F_S) + (F_B - C_B)N_B + \lambda[T_0 - F_S]$.

Assuming the existence of an interior solution $\{F_S > 0, F_B > 0,\ \lambda > 0\}$, the first-order conditions of maximization satisfy the following equations:

$$\frac{\partial Z}{\partial \lambda} = 0 \Rightarrow\ T_0 - F_S = 0; \tag{17}$$

$$\frac{\partial Z}{\partial F_S} = 0\ \Rightarrow (1 + C_S - 2F_S) - \lambda = 0; \tag{18}$$

$$\frac{\partial Z}{\partial F_B} = 0\ \Rightarrow (F_B - C_B)\frac{\partial N_B(.)}{\partial F_B} + N_B(.) + \lambda\frac{\partial T_0}{\partial F_B} = 0. \tag{19}$$

Combining equations (18) and (19) we have,

$$1 + C_S - 2F_S = -\frac{(F_B - C_B)\frac{\partial N_B(.)}{\partial F_B} + N_B(.)}{\frac{\partial T_0}{\partial F_B}}. \tag{20}$$

The LHS of equation (20) is the net marginal revenue of the platform from the sellers' side and the RHS is the net marginal revenue of the platform from the buyers' side. Equation (20) is similar to the first-order condition of a multi-market monopoly's profit maximization problem where the net marginal revenues of the multiple markets it serves are equated and also appear in chapters like Rochet and Tirole (2006) in a different form[††].

Let us define the net marginal revenue of the platform from the sellers' side as

$$NMR_S = 1 + C_S - 2F_S, \tag{21}$$

[††] If Assumption 3 is violated with a strongly positive $\frac{\partial}{\partial F_B}\left(\frac{\partial Q_j}{\partial p_j}\right)$ that results in $\frac{\partial p^*}{\partial F_B} > 0$ and $\frac{\partial T_0}{\partial F_B} > 0$ in Lemma 6 and Lemma 7 respectively, it is always $1 + C_S - 2F_S > -\frac{(F_B - C_B)\frac{\partial N_B(.)}{\partial F_B} + N_B(.)}{\frac{\partial T_0}{\partial F_B}}$. Then the platform does not exist since it stops servicing the buyer side of the market.

and the net marginal revenue of the platform from the buyers' side as

$$NMR_B = -\frac{(F_B - C_B)\frac{\partial N_B(.)}{\partial F_B} + N_B(.)}{\frac{\partial T_0}{\partial F_B}}. \tag{22}$$

Substituting for T_0 from equation (14), equation (17) is written as

$$F_S = [p^*(\rho, \bar{v}, F_B) - c]Q(.) - F. \tag{23}$$

After substitution of F_S from equation (23), equation (20) solves for $F_B(C_S, C_B, F, \bar{T}, \rho, \bar{v})$. Then substituting $F_B(C_S, C_B, F, \bar{T}, \rho, \bar{v})$ in equation (23) we obtain the equilibrium value of F_S as:

$$F_S = [p^*(\rho, \bar{v}, c, F_B(C_S, C_B, F, \bar{T}, \rho, \bar{v}) - c]Q(.) - F. \tag{24}$$

Notice that in the equilibrium the platform charges membership fees to the seller-side in such a way that the sellers are pushed to their participation constraint. Each of the sellers on the platform earns $\pi_j^{PL} = 0$. The sellers with their T_j lying in the range $[T_{0,}1]$ enter the platform. However, the sellers with their T_j lying in the range $[0, T_0)$ operate outside the platform and earn positive profit. The second-order condition for the platform's profit maximization problem, derived from equations (17), (18) and (19) requires that,

$$\frac{\partial T_0}{\partial F_B}\left[\frac{\partial (NMR_B)}{\partial F_B} + \frac{\partial (NMR_S)}{\partial F_B} - \frac{\partial T_0}{\partial F_B}\frac{\partial (NMR_S)}{\partial F_S}\right] + NMR_B\frac{\partial^2 T_0}{\partial F_B^2} \geq 0.$$

Lemma 8. a. $\frac{\partial (NMR_S)}{\partial F_B} > 0.$

b. $\frac{\partial (NMR_S)}{\partial C_S} > 0, \frac{\partial (NMR_S)}{\partial C_B} = 0, \frac{\partial (NMR_S)}{\partial F} > 0, \frac{\partial (NMR_S)}{\partial \bar{v}} > 0, \frac{\partial (NMR_S)}{\partial \rho} > 0.$

Proof: See the Appendix A3. □

As F_B rises, T_0 falls (Lemma 7) as the demand shrinks, and the sellers have a lower ability to charge a high price. Since the platform always chooses $F_S = T_0, F_S$ falls. More sellers participate in the platform. Therefore, with a rise in F_B, NMR_S rises. The same logic applies to the rise in F, $\bar{v}$ and ρ, because in each of these cases as Lemma 7 suggests T_0 falls. Therefore, F_S falls and

NMR_S rises. As C_S rises margin of profit earned from each seller i.e., $(F_S - C_S)$ falls. However, this has a positive effect on NMR_S since this lowers the revenue loss of the platform as sellers leave the platform because of a charge of a higher F_S. A change in C_B does not have any effect on NMR_S.

Lemma 9. a. $\frac{\partial(NMR_B)}{\partial F_B} < 0$;

b. $\frac{\partial(NMR_B)}{\partial C_S} = 0$, $\frac{\partial(NMR_B)}{\partial C_B} > 0$, $\frac{\partial(NMR_S)}{\partial F} = 0$, $\frac{\partial(NMR_B)}{\partial \bar{v}} < 0$, $\frac{\partial(NMR_B)}{\partial \rho} < 0$.

Proof: See the Appendix A3. □

A change in F_B affects NMR_B in two different ways. The first is a direct effect: as F_B rises, the number of buyers, i.e., N_B falls. Therefore, NMR_B falls. The second is an indirect effect which comes through a change in $\frac{\partial T_0}{\partial F_B}$ and $\frac{\partial N_B}{\partial F_B}$, which we have assumed zero in Assumptions 2 and 5 for tractability. Therefore, Lemma 9 states that as F_B rises, NMR_B falls. Similar are the explanations of the negative effects of the change in $\bar{v}$ and ρ on NMR_B. As C_B rises margin of profit earned from each buyer, i.e., $(F_B - C_B)$ falls. However, this has a positive effect on NMR_B as it charges a higher F_B. It reduces the revenue loss of the platform when the buyers leave the platform as a result of the rise in F_B. A change in C_S and F do not have any effect on NMR_B.

The figure below plots the NMR_S and NMR_B as positive and negative functions of F_B respectively on the assumption that they intersect such that an interior solution of F_B exists from the platform's profit maximization problem.

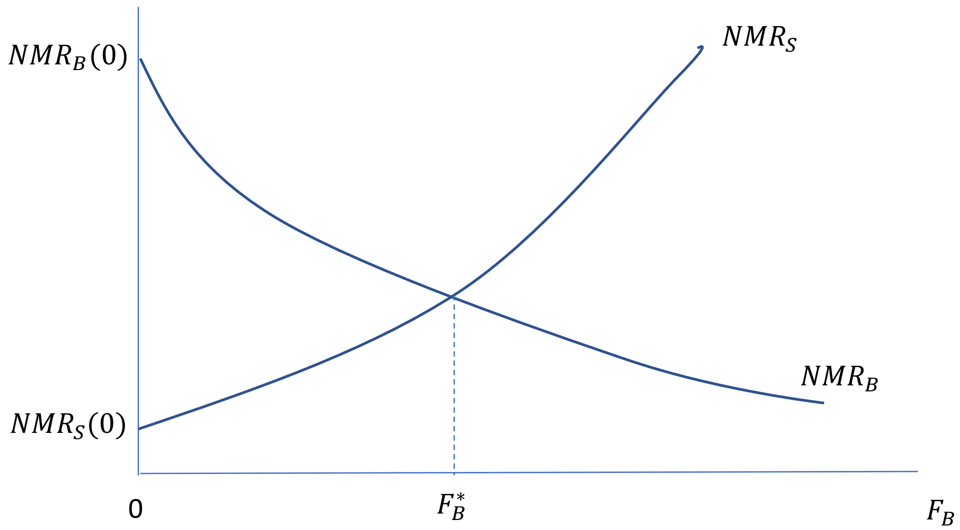

Figure 3.2: Determination of the optimum membership fee on the buyers' side

Lemma 8(a) and Lemma 9(a) explain the slopes of NMR_S and NMR_B in Figure 3.2. The condition $NMR_B(0) > NMR_S(0)$ ensures that a unique intersection of the two curves exists at a positive value of F_B that represents the unique solution to equation (20). On violation of this condition $F_B^* \leq 0$. A change in the parameters like C_B, C_S, F, $\bar{v}$ and ρ will change the position of the curves as predicted in Lemma 8(b) and 9(b). Consequently, the platform's equilibrium choice of F_B will change. The next section presents the comparative static results.

3.3 Comparative Static Results

Proposition 2: *As the platform's cost of servicing a seller C_S falls (rises),*

(i) the platform charges a higher (lower) membership fee to the buyers and a lower (higher) membership fee to the sellers;

(ii) the number of varieties in the platform rises (falls), both the number of varieties sold at the differentiated product market outside the platform and the number of buyers at the differentiated product market falls (rises);

(iii) the profit of the platform rises (falls).

Proof: See the Appendix A3. □

Proposition 2 characterizes the behavior of a platform that, other things remaining the same, becomes more efficient in servicing its seller side. A more efficient platform has a lower C_S. A lower C_S implies a lower value of NMR_S (from Lemma 8(b)) at the existing F_B^* resulting in $NMR_B > NMR_S$. In Figure 3.2, it shows up in the downward shift of the NMR_S curve. Therefore, at the new equilibrium, the profit-maximizing platform adjusts F_B at the upward direction. As F_B rises, the indirect utility of the marginal buyer falls below $\bar{v}$ and she leaves the differentiated product market. Since the number of buyers in the market falls, and the price charged for each variety of the differentiated product falls, T_0 falls and the platform's ability to charge a high price to the sellers participating on the platform falls. It lowers F_S to retain them on the platform. Therefore, the number of varieties on the platform rises as some sellers who are unable to survive outside the platform due to lower demand move into the platform. The platform by adjusting its pricing behavior ensures that it earns a higher profit. Interestingly, unlike the standard monopoly case, the efficiency of the platform does not translate into a lower price charged by the platform to the buyers. The buyers are charged a higher price to cross-subsidize the sellers. As the platform becomes more efficient in its dealing with the sellers a lower number of buyers can purchase a greater variety of a product on the platform.

Proposition 3: *As the platform's cost of servicing a buyer C_B falls (rises),*

(i) the platform charges a lower (higher) membership fee to the buyers and a higher (lower) membership fee to the sellers;

(ii) the number of varieties in the platform falls (rises), both the number of varieties sold at the differentiated product market outside the platform and the number of buyers at the differentiated product market rises (falls);

(iii) the profit of the platform falls (rises).

Proof: See the Appendix A3. □

Proposition 3 characterizes the behavior of a platform that, other things remaining the same, becomes more efficient in servicing its buyer side. A more efficient platform has a lower C_B. A lower C_B implies a lower value of NMR_B (from Lemma 9(b)) at the existing F_B^* resulting in $NMR_S > NMR_B$. In Figure 3.2, it shows up in the downward shift of the NMR_B curve. Therefore, at the new equilibrium, the profit-maximizing platform adjusts F_B at the downward direction. As F_B falls, the indirect utility of the marginal buyer rises above $\bar{v}$ and the buyers with a lower valuation of the differentiated product now enter the market. Since the number of buyers in the market rises, each seller charges a higher price for the variety it produces, T_0 rises, and the platform's constraint to charge a higher price to the sellers' side relaxes. It charges a higher F_S to the sellers. Consequently, the number of varieties sold on the platform falls. More sellers operate outside the platform and earn positive profits. The platform's profit falls. The sellers are charged a higher price to cross-subsidize the buyers.

Interestingly, here, unlike the standard monopoly case, the platform's efficiency does not translate into a higher profit for the platform. The Proposition shows that a monopoly platform has no incentive to service the buyers efficiently. However, becoming more efficient in servicing the sellers is always rewarding, as demonstrated in Proposition 2.

Earlier chapters in the literature like Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), Armstrong (2006), and Weyl (2010), do not discuss the impact of change in the cost of servicing to buyers and sellers on the platform on the membership fee charged to them, and the number of varieties sold on the platform.

Proposition 4: *As the fixed costs F of the sellers participating in the differentiated product market fall (rise)*

(i) *the platform charges a higher (lower) membership fee to the buyers;*

(ii) *if* $\left[Q(.)\frac{\partial p^*}{\partial F_B} + (p^* - c)\frac{\partial Q}{\partial F_B}\right]\frac{\partial F_B}{\partial F} < (>)1,$ *the platform charges a higher (lower) membership fee to the sellers, the number of varieties in the platform falls (rises), the number of varieties sold at the differentiated product market outside the platform rises (falls),*

(iii) *the number of buyers in the differentiated product market falls (rises);*

(iv) *the profit of the platform rises (falls).*

Proof: See the Appendix A3. □

Proposition 4 deduces the platform's behavior if the fixed costs of the seller change. Without loss of generality, suppose the fixed costs (F) fall because of some infrastructural development that equally benefits all the suppliers. As F falls at the initial equilibrium, each seller starts earning positive profit. Therefore, the platform reacts by increasing F_S. As F_S rises NMR_S falls below NMR_B at the initial equilibrium and F_B adjusts in the upward direction. As F_B rises, the buyers' indirect utility falls, and the marginal buyer exits the differentiated product market. Since the demand falls for every seller in the differentiated product market, each one starts lowering their price to retain buyers. As the price drops, their revenue falls. If $\left[Q(.)\frac{\partial p^*}{\partial F_B} +\right.$

$(p^* - c)\frac{\partial Q}{\partial F_B}\Big]\frac{\partial F_B}{\partial F} < 1$, the fall in revenue is lower than the fall in the fixed cost, and the positive direct effect of a fall in F dominates the negative indirect effect of a fall in the revenue, increasing the seller's profit. Consequently, the platform's initial decision of raising F_S gets reinforced since by its choice of F_S it always keeps the sellers under the participation constraint. Since the membership fee at the platform for both the buyers and sellers rises, the number of sellers on the platform and the number of buyers on the differentiated product market fall. However, if $\Big[Q(.)\frac{\partial p^*}{\partial F_B} + (p^* - c)\frac{\partial Q}{\partial F_B}\Big]\frac{\partial F_B}{\partial F} > 1$, the fall in revenue is higher than the fall in the fixed cost, and each seller's profit falls. Therefore, the platform ends up lowering F_S to keep the existing sellers participating on the platform. Since the membership fee is lower, new sellers also enter the platform. In both cases, the platform earns a higher profit.

Proposition 5: *If the outside option of the buyers improves (worsens) due to a rise (fall) in* $\bar{v}$

(i) *the platform charges a lower (higher) membership fee to the buyers;*

(ii) *if* $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \bar{v}}Q(.) > (<) - [\frac{\partial p^*}{\partial \bar{v}}Q(.) + (p^* - c)\frac{\partial Q}{\partial \bar{v}}]$

the platform charges a higher (lower) membership fee to the sellers, and the number of varieties in the platform falls (rises), the number of varieties sold at the differentiated product market outside the platform rises (falls),

(iii) *the number of buyers in the differentiated product market falls (increases);*

(iv) *the profit of the platform falls.*

Proof: See the Appendix A3. □

As $\bar{v}$ rises, the number of buyers in the differentiated market falls, and due to the reduced demand, each seller charges a lower price. Therefore, the platform lowers F_S. Consequently NMR_B falls, and NMR_S rises. In Figure 3.2, the NMR_B curve shifts in the downward direction and the NMR_S curve shifts in the upward direction, resulting in $NMR_S > NMR_B$ at the initial equilibrium. Therefore, the platform adjusts F_B in the downward direction. As F_B falls in the new equilibrium, the indirect utility of the marginal buyer purchasing from the differentiated product market rises. Therefore, some buyers return to the differentiated product market, and the sellers increase the price to gain revenue. If $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \bar{v}}Q(.) > -[\frac{\partial p^*}{\partial \bar{v}}Q(.) + (p^* - c)\frac{\partial Q}{\partial \bar{v}}]$ the revenue gain from the indirect effect outweighs the revenue loss from the direct impact; therefore, the platform charges a higher F_S to keep the sellers on their participation constraint. If $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \bar{v}}Q(.) < -[\frac{\partial p^*}{\partial \bar{v}}Q(.) + (p^* - c)\frac{\partial Q}{\partial \bar{v}}]$ the opposite happens. In either case, the number of buyers in the differentiated product market and the platform's profit fall. The counterintuitive aspect of Proposition 5 is that even if the differentiated product market loses its buyers to the 'brick-and-mortar' shops, the platform may raise the membership fee for the sellers participating on the platform. Also, there are conditions when a rise in the utility from participating in a 'brick-and-mortar' shop and growth in the number of sellers on the platform co-occur.

Proposition 6: *As the substitutability among the varieties of the differentiated product rises (falls) as* ρ *rises (falls)*

(i) *the platform charges a lower (higher) membership fee to the buyers;*

(ii) *if* $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \rho}Q(.) > (<) - [\frac{\partial p^*}{\partial \rho}Q(.) + (p^* - c)\frac{\partial Q}{\partial \rho}]$

the platform charges a higher (lower) membership fee to the sellers, and the number of varieties in the platform falls (rises), the number of varieties sold at the differentiated product market outside the platform rises (falls),

(iii) *the number of buyers in the differentiated product market falls (increases);*

(iv) *the profit of the platform falls.*

Proof: See the Appendix A3. □

As ρ rises, the substitutability of the varieties of the differentiated product falls. Therefore, the marginal buyer's indirect utility in purchasing from the differentiated product falls, and she exits the market. The number of buyers in the differentiated market falls, and due to the reduced demand, each seller charges a lower price. Therefore, the platform lowers F_S. Consequently NMR_B falls, and NMR_S rises. In Figure 3.2, similar to the case of a rise in $\bar{v}$, the NMR_B curve shifts in the downward direction and the NMR_S curve shifts in the upward direction, resulting in $NMR_S > NMR_B$ at the initial equilibrium. Therefore, the platform adjusts F_B in the downward direction. As F_B falls in the new equilibrium, the indirect utility of the marginal buyer purchasing from the differentiated product market rises. Therefore, some buyers return to the differentiated product market, and the sellers increase the price to gain revenue. If $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \rho}Q(.) > (<) -[\frac{\partial p^*}{\partial \rho}Q(.) + (p^* - c)\frac{\partial Q}{\partial \rho}]$ the revenue gain from the indirect effect outweighs the revenue loss from the direct impact; therefore, the platform charges a higher F_S to keep the sellers on their participation constraint. If $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \rho}Q(.) > (<) -[\frac{\partial p^*}{\partial \rho}Q(.) + (p^* - c)\frac{\partial Q}{\partial \rho}]$ the opposite happens. In either case, the number of buyers in the differentiated product market and the platform's profit fall. With regards to the fall in the platform's profit, papers like Belleflamme and Peitz (2019) and Chawla and Mondal (2024) get similar results in a monopolistic competition framework where price interaction among sellers is absent.

In the existing literature, Hagiu (2009) discusses a similar result by comparing the share of profits on both the buyer side and the seller side but does not discuss the impact on the overall profits of the platform. Galeotti and Moraga-González (2009), in a different setting, derive similar results regarding the membership fee charged to the buyers and the profit earned by the platform like ours. However, in the context of 'love for variety' preferences and both endogenous intensive and extensive margin of demand, we show that the platform may charge the sellers a higher or lower membership fee depending on whether the seller loses or gains out of it. It is interesting to note that the earlier papers have not addressed the possibility of platform earning higher profits with a higher membership fee on the buyer side but charging a lower fee on the seller side. If the membership fee falls, more varieties sell on the platform. If the membership fee rises, the number of varieties sold on the platform falls. Since the platform loses if the varieties the sellers sell in the differentiated product market are close substitutes, it may incentivize the sellers to increase their product diversity.

3.4 Conclusions

The model presented in the chapter involves a two-sided monopoly platform like the shopping malls and e-commerce platforms that serves the buyers and sellers in a differentiated product market where the buyers have Dixit-Stiglitz type 'love for variety' preference. The sellers compete with each other in terms of prices. The platform internalizes the externalities either side of the market creates for the other side and decides the membership fee to charge to both sides. The model endogenously determines both the number of varieties produced in the differentiated product market and the number of varieties sold on the platform. The model shows that the existence of the platform benefits the buyers by delivering more varieties of the differentiated product in the market. However, the membership fee charged to the buyers and sellers and the number of varieties sold on the platform depends on factors like the costs of servicing the buyers and sellers, the fixed costs faced by the sellers in their operation, outside options of the buyers, and the substitutability among the varieties of the differentiated product.

The results show that if the costs of servicing the clients on one particular side of the market fall, the platform lowers the membership fee charged to that side by raising the membership fee on the other side. For example, if the costs of servicing the buyers fall, the buyers' membership fee is cross-subsidized by raising the membership fee to the sellers. The number of varieties on the platform rises. The opposite happens if the costs of servicing the sellers fall. Interestingly, the results suggest that, unlike the standard monopoly case, its efficiency in servicing the clients does not necessarily translate into a higher profit for the platform. The platform's profit rises if it is more efficient in servicing the sellers but falls if it is more efficient in servicing the buyers. Therefore, a platform has more incentive to become efficient in servicing the sellers rather than buyers. If the sellers have a lower fixed cost, although the buyers enjoy a lower membership fee, the sellers may face a higher membership fee if the platform expects them to gain due to the lowering of the fixed cost. The number of varieties sold on the platform depends on the membership fee charged to the sellers. It falls if a higher fee is charged. It rises for a lower membership fee. However, since the platform's profit falls, it may not like developments that lower the seller's fixed costs. Suppose the buyers have a better outside option of buying out of the differentiated product market, like buying from a 'brick-and-mortar' shop. In that case, the platform reacts by lowering the membership fee for the buyers. However, the seller's fee and the number of varieties sold on the platform may rise or fall depending on the platform's calculation of the seller's gain and loss. The results suggest that even if the differentiated product market loses its buyers to the 'brick-and-mortar' shops, the platform may raise the membership fee for the sellers. However, there are instances in which an increase in the number of sellers on the platform goes along with an increase in the utility from participating in a "brick-and-mortar" store. The platform, however, loses profit if 'brick-and-mortar' shops turn out to be an attractive option for shopping. If the varieties of differentiated products are close substitutes, sellers' monopoly power falls, and the buyers leave the differentiated product market. In such a situation, the platform lowers the membership fee to the buyers to bring them back into the market. Since the sellers may gain or lose out of the process, they are charged accordingly. If the platform expects them to gain, it charges a higher membership fee. The opposite happens otherwise. Since the platform loses if the varieties the sellers sell in the differentiated product market are close substitutes, it may incentivize them to take initiatives to increase their product diversity.

The chapter contributes to the existing literature on the economics of platforms in multiple ways: First, it accommodates the 'love for variety' preference pattern of buyers; Second, it allows endogenous determination of the number of varieties of a differentiated product in the market and on the platform, and shows that platform benefits the buyers by increasing the number of varieties available at the market; Third, it characterizes the platform's pricing on both the side of the market for changes in costs to servicing to the clients, changes in fixed costs on the sellers' side, improved outside option of the buyers, and the lower substitutability of the varieties at the market. As we analyze the impact of these parametric changes on the platform's profit, we find that a platform has more incentive to be efficient in servicing its sellers than buyers; it does not like lowering the fixed costs of the sellers and developing outside options for the buyers like purchase from 'brick-and-mortar' shops. The platform also likes sellers to sell varieties that are not close substitutes for each other. It has an incentive to promote the development of the diversity of varieties sold in the market.

The model has some limitations. First, it does not explicitly explore the matching of buyers and sellers. Alternatively, it focuses on the benefits created by a platform by lowering transaction costs for the sellers and increasing the product variety for the buyers. Second, it focuses only on the platform's membership fee decision and does not consider its per-unit user fee decisions or its choice of location. Third, in our model, the buyers differ only in their preference for purchasing the differentiated product and the sellers in their transaction costs. The simplifying assumptions have kept the model tractable. To generalize the results, we plan to relax some of these assumptions in future extensions. The analysis would also be more comprehensive with a paradigm that considers buyer-side congestion effects that highlight the same-sided negative externalities on the buyer side. We also plan to introduce competition among the platforms in the setup of the present model in our future work and compare the welfare aspects of these two models, namely, the current model with a monopoly platform and the extension with platform competition.

Extending the present work, in a setting of spatial competition and 'love for variety' preferences, the next chapter incorporates the platform's location choice and studies its interaction with an outside option in the form of a homogeneous goods market located on one end of a linear city.

Appendix A3

Proof of Lemma 2.

From equation (6), $\frac{\partial N_B}{\partial p_j} = -\frac{\partial \bar{\mu}}{\partial p_j}$. (A.1)

Since $\frac{\partial \bar{\mu}}{\partial p_j} > 0$ from the first part of the Lemma, the RHS of (A.1) is negative. The statement of the Lemma follows. □

Proof of Lemma 4.

(a) At $v(\bar{\mu}) = \bar{v}$, from equation (5):

$\frac{\partial \bar{\mu}}{\partial \bar{v}} = \frac{\partial \bar{\mu}}{\partial F_B} = \frac{1}{k^{\beta}}$.

Since $k^{\beta} > 0$, the RHS of the above equations is positive. Therefore, the LHS is also positive.

Since $k = \left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}$, from equation (5) again we derive:

$$\frac{\partial \bar{\mu}}{\partial \rho} = \left(\frac{N_S p_j}{k^{\beta}}\right)\left(\frac{q_j log N_S}{\rho^2}\right) + \left(\frac{\bar{\mu}\beta}{k}\right) q_j N_S^{\frac{1}{\rho}} \left[\left(\frac{1}{\rho^2}\right) log\ N_S + \frac{log N_S}{\rho^2}\right] > 0\,.$$

The statement follows.

(b) From equation (6):

$\frac{\partial N_B}{\partial F_B} = -\frac{\partial \bar{\mu}}{\partial F_B}$. (A.2)

$\frac{\partial N_B}{\partial \bar{v}} = -\frac{\partial \bar{\mu}}{\partial \bar{v}}$; (A.3)

$\frac{\partial N_B}{\partial \rho} = -\frac{\partial \bar{\mu}}{\partial \rho}$. (A.4)

Since $\frac{\partial \bar{\mu}}{\partial F_B} > 0$, $\frac{\partial \bar{\mu}}{\partial \bar{v}} > 0$, and $\frac{\partial \bar{\mu}}{\partial \rho} > 0$ from the first part of the Lemma, the RHS of (A.2), (A.3) and (A.4) is negative. The statement of the Lemma follows. □

Proof of Lemma 5.

By application of Leibniz rule, from equation (7):

$$\frac{\partial Q_j}{\partial F_B} = -\left[Q_j \frac{\partial \bar{\mu}(\cdot)}{\partial F_B}\right] + \left[\int_{\bar{\mu}(\cdot)}^{1} \frac{\partial q_j(\cdot)}{\partial F_B} d\mu\right]. \tag{A.5}$$

$$\frac{\partial Q_j}{\partial \bar{v}} = -\left[Q_j \frac{\partial \bar{\mu}(\cdot)}{\partial \bar{v}}\right] + \left[\int_{\bar{\mu}(\cdot)}^{1} \frac{\partial q_j(\cdot)}{\partial \bar{v}} d\mu\right]. \tag{A.6}$$

$$\frac{\partial Q_j}{\partial \rho} = -\left[Q_j \frac{\partial \bar{\mu}(\cdot)}{\partial \rho}\right] + \left[\int_{\bar{\mu}(\cdot)}^{1} \frac{\partial q_j(\cdot)}{\partial \rho} d\mu\right]. \tag{A.7}$$

Since $Q_j > 0$, $\frac{\partial \bar{\mu}(\cdot)}{\partial F_B}, \frac{\partial \bar{\mu}}{\partial \bar{v}}, \frac{\partial \bar{\mu}}{\partial \rho}$ is positive from Lemma 4(a), and $\frac{\partial q_j}{\partial F_B} = 0, \frac{\partial q_j}{\partial \bar{v}} = 0, \frac{\partial q_j}{\partial \rho} < 0$ (from Lemma 1(b)) the RHS of (A.5), (A.6), and (A.7) is negative.

Therefore, the statement of the Lemma follows. □

Proof of Lemma 6.

Evaluating equation (9) at $p^*(\rho, \bar{v}, F_B)$, for each seller we have $\frac{\partial \pi_j^{\omega}}{\partial p}(p^*, \rho, \bar{v}, F_B) = 0$. Taking total differential and collecting the terms:

$$\frac{dp^*}{dF_B} = -\frac{\frac{\partial}{\partial F_B}\left(\frac{\partial \pi_j^{\omega}}{\partial p}\right)}{\frac{\partial^2 \pi_j^{\omega}}{\partial p^2}}; \tag{A.8}$$

$$\frac{dp^*}{d\bar{v}} = -\frac{\frac{\partial}{\partial \bar{v}}\left(\frac{\partial \pi_j^{\omega}}{\partial p}\right)}{\frac{\partial^2 \pi_j^{\omega}}{\partial p^2}} \tag{A.9}$$

and,

$$\frac{dp^*}{d\rho} = -\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial \pi_j^{\omega}}{\partial p}\right)}{\frac{\partial^2 \pi_j^{\omega}}{\partial p^2}}. \tag{A.10}$$

Since $\frac{\partial^2 \pi_j^{\omega}}{\partial p^2} < 0$ from the second order condition of the sellers' profit maximization, from (A.8), (A.9) and (A.10) it follows that the $sign\left[\frac{\partial p^*}{\partial F_B}\right] = sign\left[\frac{\partial}{\partial F_B}\left(\frac{\partial \pi_j^{\omega}}{\partial p}\right)\right]$, the $sign\left[\frac{\partial p^*}{\partial \bar{v}}\right] = sign\left[\frac{\partial}{\partial \bar{v}}\left(\frac{\partial \pi_j^{\omega}}{\partial p}\right)\right]$ and the $sign\left[\frac{\partial p^*}{\partial \rho}\right] = sign\left[\frac{\partial}{\partial \rho}\left(\frac{\partial \pi_j^{\omega}}{\partial p}\right)\right]$ respectively.

Notice that,

$$\frac{\partial \pi_j^{\omega}}{\partial p} = (p^* - c)\frac{\partial Q_j}{\partial p_j} + Q_j. \tag{A.11}$$

Using Assumption 2, from (A.11) we obtain: the $sign\left[\frac{\partial p^*}{\partial F_B}\right] = sign\left[\frac{\partial Q_j}{\partial F_B}\right]$, the $sign\left[\frac{\partial p^*}{\partial \bar{v}}\right] = sign\left[\frac{\partial Q_j}{\partial \bar{v}}\right]$ and the $sign\left[\frac{\partial p^*}{\partial \rho}\right] = sign\left[\frac{\partial Q_j}{\partial \rho}\right]$ respectively. Now applying Lemma 5 the statement follows. □

Proof of Lemma 7. From equation (14) we have

$$\frac{\partial T_0}{\partial F_B} = \left[Q(.)\frac{\partial p^*(.)}{\partial F_B}\right] + \left[(p^*(\rho, \bar{v}, F_B) - c)\frac{\partial Q(.)}{\partial F_B}\right], \tag{A.12}$$

$$\frac{\partial T_0}{\partial \bar{v}} = \left[Q(.)\frac{\partial p^*(.)}{\partial \bar{v}}\right] + \left[(p^*(\rho, \bar{v}, F_B) - c)\frac{\partial Q(.)}{\partial \bar{v}}\right], \tag{A.13}$$

and,

$$\frac{\partial T_0}{\partial \rho} = \left[Q(.)\frac{\partial p^*(.)}{\partial \rho}\right] + \left[(p^*(\rho, \bar{v}, F_B) - c)\frac{\partial Q(.)}{\partial \rho}\right]. \tag{A.14}$$

Given $\frac{\partial Q_j}{\partial p_j} < 0$, $(p^*(\rho, \bar{v}, F_B) - c) > 0$ from equation (9). Since $\frac{\partial p^*(.)}{\partial F_B} < 0, \frac{\partial p^*(.)}{\partial \bar{v}} < 0$, and $\frac{\partial p^*(.)}{\partial \rho} < 0$ from Lemma 6 and $\frac{\partial Q(.)}{\partial F_B} < 0$, $\frac{\partial Q(.)}{\partial \bar{v}} < 0$, and $\frac{\partial Q(.)}{\partial \rho} < 0$ from Lemma 5, from equations (A.12), (A.13) and (A.14) it follows that $\frac{\partial T_0}{\partial F_B} < 0, \frac{\partial T_0}{\partial \bar{v}} < 0$, and $\frac{\partial T_0}{\partial \rho} < 0$.

Proof of Lemma 8.

Substituting $F_S = T_0$ from equation (17) in equation (21) and differentiating with respect to F_B we obtain:

$$\frac{\partial(NMR_S)}{\partial F_B} = -2\frac{\partial T_0}{\partial F_B}.$$

Since $\frac{\partial T_0}{\partial F_B} < 0$ from Lemma 7, $\frac{\partial(NMR_S)}{\partial F_B} > 0$.

The sign of $\frac{\partial(NMR_S)}{\partial F_s}$ directly follows from equation (21).

b. From equation (21):

$$\frac{\partial(NMR_S)}{\partial C_S} = 1 > 0,\ \frac{\partial(NMR_S)}{\partial C_B} = 0, \frac{\partial(NMR_S)}{\partial \bar{v}} = -2\frac{\partial T_0}{\partial \bar{v}}, \text{ and } \frac{\partial(NMR_S)}{\partial \rho} = -2\frac{\partial T_0}{\partial \rho}.$$

The statement follows from application of Lemma 7. □

Proof of Lemma 9.

a. From equation (22):

$$\frac{\partial(NMR_B)}{\partial F_B} = -\frac{2\frac{\partial N_B(.)}{\partial F_B}}{\frac{\partial T_0}{\partial F_B}} + \frac{(F_B - C_B)\frac{\partial N_B(.)}{\partial F_B} + N_B(.)}{\left(\frac{\partial T_0}{\partial F_B}\right)^2}\frac{\partial^2 T_0}{\partial F_B^2} - \frac{(F_B - C_B)\frac{\partial^2 N_B}{\partial F_B^2}}{\frac{\partial T_0}{\partial F_B}}. \tag{A.15}$$

Since $\bar{\mu}$ is a linear function of F_B which follows from equation (5) at $v(\bar{\mu}) = \bar{v}$, from equation (6), N_B is also a linear function of F_B. Therefore, $\frac{\partial^2 N_B}{\partial F_B^2} \approx 0$. From Assumption 5: $\frac{\partial^2 T_0}{\partial F_B^2} \approx 0$. Since $\frac{\partial N_B(.)}{\partial F_B} < 0$ from Lemma 4(b) and $\frac{\partial T_0}{\partial F_B} < 0$ from Lemma 7, the RHS of (A.15) is negative. Therefore, the statement follows.

b. From equation (22):

$$\frac{\partial(NMR_B)}{\partial C_S} = 0, \frac{\partial(NMR_B)}{\partial F} = 0;$$

$\frac{\partial(NMR_B)}{\partial C_B} = \frac{\frac{\partial N_B}{\partial F_B}}{\frac{\partial T_0}{\partial F_B}} > 0$ since $\frac{\partial N_B(.)}{\partial F_B} < 0$ from Lemma 4.b and $\frac{\partial T_0}{\partial F_B} < 0$ from Lemma 7;

$\frac{\partial(NMR_B)}{\partial \bar{v}} = -\frac{\frac{\partial N_B(.)}{\partial \bar{v}}}{\frac{\partial T_0}{\partial F_B}} < 0$, and $\frac{\partial(NMR_B)}{\partial \rho} = -\frac{\frac{\partial N_B(.)}{\partial \rho}}{\frac{\partial T_0}{\partial F_B}} < 0$, which follows from Assumption 2, Assumption 5, Lemma 4(b) and Lemma 7. □

Proof of Proposition 2:

(i) From equation (20):

$$\frac{\partial NMR_S}{\partial C_S} + \frac{\partial NMR_S}{\partial F_B}\frac{\partial F_B}{\partial C_S} = \frac{\partial NMR_B}{\partial F_B}\frac{\partial F_B}{\partial C_S}.$$

It follows from the above equation that

$$\frac{\partial F_B}{\partial C_S} = \frac{-\frac{\partial NMR_S}{\partial C_S}}{\frac{\partial NMR_S}{\partial F_B} - \frac{\partial NMR_B}{\partial F_B}}. \tag{A.16}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial F_B} > 0$ and $\frac{\partial NMR_B}{\partial F_B} < 0$ respectively. Therefore, the denominator of the term on the RHS of (A.16) is positive. From Lemma 8(b) $\frac{\partial NMR_S}{\partial C_S} > 0$. Therefore, the term on the RHS of (A.16) is negative and it follows that $\frac{\partial F_B}{\partial C_S} < 0$.

Again, from equation (23):

$$\frac{\partial F_S}{\partial C_S} = \left\{Q(.)\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial C_S} + (p^* - c)\frac{\partial Q(.)}{\partial F_B}\frac{\partial F_B}{\partial C_S}\right\}. \tag{A.17}$$

Since $\frac{\partial p^*}{\partial F_B} < 0$ from Lemma 6, $\frac{\partial Q_j}{\partial F_B} < 0$ Lemma 5,and $\frac{\partial F_B}{\partial C_S} < 0$ as proved above, the RHS of (A.17) is positive. Therefore, $\frac{\partial F_S}{\partial C_S} > 0$.

(ii) From equation (15):

$$\frac{\partial \tilde{N}_S}{\partial C_S} = -\left(\frac{\partial F_S}{\partial C_S}\right). \tag{A.18}$$

Since $\frac{\partial F_S}{\partial C_S} > 0$ from the first part of the Proposition, the RHS of (A.18) is negative. Therefore, $\frac{\partial \tilde{N}_S}{\partial C_S} < 0$. Again, from equation (15):

$$\frac{\partial N_S}{\partial C_S} = \left(\frac{\partial T_0}{\partial F_B}\frac{\partial F_B}{\partial C_S}\right). \tag{A.19}$$

Since $\frac{\partial T_0}{\partial F_B} < 0$ from Lemma 7 and $\frac{\partial F_B}{\partial C_S} < 0$ from the first part of the Proposition. Therefore, the RHS of (A.19) is positive and the LHS is also positive. Now, from equation (5):

$$\frac{\partial N_B}{\partial C_S} = -\frac{\partial \bar{\mu}}{\partial F_B}\frac{\partial F_B}{\partial C_S}. \tag{A.20}$$

Since from Lemma 5(a), $\frac{\partial \bar{\mu}}{\partial F_B} > 0$ and from the first part of the Proposition $\frac{\partial F_B}{\partial C_S} < 0$, the RHS of (A.20) is positive. Therefore, $\frac{\partial N_B}{\partial C_S} > 0$. □

(iii) From equation (16) at the equilibrium:

$$\frac{\partial \pi}{\partial C_S} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial F_B}\frac{\partial F_B}{\partial C_S} + \frac{\partial[(F_B - C_B)N_B]}{\partial F_B}\frac{\partial F_B}{\partial C_S},$$

using the definition of NMR_S and NMR_B from equations (21) and (22), which can be written as:

$$\frac{\partial \pi}{\partial C_S} = \frac{\partial F_B}{\partial C_S}[NMR_S - NMR_B . \frac{\partial T_0}{\partial F_B}]. \tag{A.21}$$

Since $NMR_S = NMR_B$ from equation (20), (A.21) is written as:

$$\frac{\partial \pi}{\partial C_S} = \frac{\partial F_B}{\partial C_S} NMR_S[1 - \frac{\partial T_0}{\partial F_B}]. \tag{A.22}$$

Since $\frac{\partial F_B}{\partial C_S} < 0$, $NMR_S > 0$, and $\frac{\partial T_0}{\partial F_B} < 0$, the RHS of (A.22) is negative.

Therefore, $\frac{\partial \pi}{\partial C_S}$ is negative. □

Proof of Proposition 3:

(i) From equation (20):

$$\frac{\partial NMR_S}{\partial F_B}\frac{\partial F_B}{\partial C_B} = \frac{\partial NMR_B}{\partial F_B}\frac{\partial F_B}{\partial C_B} + \frac{\partial NMR_B}{\partial C_B}.$$

which implies,

$$\frac{\partial F_B}{\partial C_B} = \frac{\frac{\partial NMR_B}{\partial C_B}}{\frac{\partial NMR_S}{\partial F_B} - \frac{\partial NMR_B}{\partial F_B}}. \tag{A.23}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial F_B} > 0$ and $\frac{\partial NMR_B}{\partial F_B} < 0$ respectively. Therefore, the denominator of the term on the RHS of (A.23) is positive. From Lemma 9(b), $\frac{\partial NMR_B}{\partial C_B} > 0$. Therefore, the term on the numerator of RHS of (A.23) is positive and it follows that $\frac{\partial F_B}{\partial C_B} > 0$.

Again, from equation (23):

$$\frac{\partial F_S}{\partial C_B} = \left\{Q(.)\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial C_B} + (p^* - c)\frac{\partial Q(.)}{\partial F_B}\frac{\partial F_B}{\partial C_B}\right\}. \tag{A.24}$$

Since $\frac{\partial p^*}{\partial F_B} < 0$ from Lemma 6, $\frac{\partial Q_j}{\partial F_B} < 0$ Lemma 5 and $\frac{\partial F_B}{\partial C_B} > 0$ as proved above, the RHS of (A.24) is negative. Therefore, $\frac{\partial F_S}{\partial C_B} < 0$.

(ii) From equation (15):

$$\frac{\partial \tilde{N}_S}{\partial C_B} = -\left(\frac{\partial F_S}{\partial C_B}\right). \tag{A.25}$$

Since $\frac{\partial F_S}{\partial C_B} < 0$ from the first part of the Proposition, the RHS of (A.25) is positive. Therefore, $\frac{\partial \tilde{N}_S}{\partial C_B} > 0$.

Again, from equation (15):

$$\frac{\partial N_S}{\partial C_B} = \left(\frac{\partial T_0}{\partial F_B}\frac{\partial F_B}{\partial C_B}\right). \tag{A.26}$$

Since $\frac{\partial T_0}{\partial F_B} < 0$ from Lemma 8 and $\frac{\partial F_B}{\partial C_B} > 0$ from the first part of the Proposition. Therefore, the RHS of (A.26) is negative and the LHS is also negative.

Now, from equation (5):

$$\frac{\partial N_B}{\partial C_B} = -\frac{\partial \bar{\mu}}{\partial F_B}\frac{\partial F_B}{\partial C_B}. \tag{A.27}$$

Since from Lemma 5(a) $\frac{\partial \bar{\mu}}{\partial F_B} > 0$ and from the first part of the Proposition $\frac{\partial F_B}{\partial C_B} > 0$, the RHS of (A.27) is negative. Therefore, $\frac{\partial N_B}{\partial C_B} < 0$. □

(iii) From equation (16) at the equilibrium:

$$\frac{\partial \pi}{\partial C_B} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial F_B}\frac{\partial F_B}{\partial C_B} + \frac{\partial[(F_B - C_B)N_B]}{\partial F_B}\frac{\partial F_B}{\partial C_B},$$

using the definition of NMR_S and NMR_B from equations (21) and (22), which can be written as:

$$\frac{\partial \pi}{\partial C_B} = \frac{\partial F_B}{\partial C_B}[NMR_S - NMR_B . \frac{\partial T_0}{\partial F_B}]. \tag{A.28}$$

Since $NMR_S = NMR_B$ from equation (20), (A.28) is written as:

$$\frac{\partial \pi}{\partial C_B} = \frac{\partial F_B}{\partial C_B} NMR_S[1 - \frac{\partial T_0}{\partial F_B}]. \tag{A.29}$$

Since $\frac{\partial F_B}{\partial C_B} > 0$, $NMR_S > 0$, and $\frac{\partial T_0}{\partial F_B} < 0$, the RHS of (A.29) is positive.

Therefore, $\frac{\partial \pi}{\partial C_B}$ is positive. □

Proof of Proposition 4:

(i) From equation (20) since

$$\frac{\partial NMR_S}{\partial F} + \frac{\partial NMR_S}{\partial F_B}\frac{\partial F_B}{\partial F} = \frac{\partial NMR_B}{\partial F_B}\frac{\partial F_B}{\partial F},$$

it follows that

$$\frac{\partial F_B}{\partial F} = \frac{-\frac{\partial NMR_S}{\partial F}}{\frac{\partial NMR_S}{\partial F_B} - \frac{\partial NMR_B}{\partial F_B}}. \tag{A.30}$$

From Lemma 8, $\frac{\partial NMR_S}{\partial F_B} > 0$ and $\frac{\partial NMR_S}{\partial F} > 0$. From Lemma 9(a) $\frac{\partial NMR_B}{\partial F_B} < 0$. Therefore, the denominator of the term on the RHS of (A.30) is positive. The sign of $\frac{\partial F_B}{\partial F}$ is therefore negative.

(ii) From equation (23):

$$\frac{\partial F_S}{\partial F} = [Q(.)\frac{\partial p^*}{\partial F_B} + (p^* - c)\frac{\partial Q(.)}{\partial F_B}]\frac{\partial F_B}{\partial F} - 1 \tag{A.31}$$

Since $\frac{\partial p^*}{\partial F_B} < 0$ from Lemma 6, $\frac{\partial Q}{\partial F_B} < 0$ Lemma 5, and $\frac{\partial F_B}{\partial F} < 0$, as proved above, the RHS of (A.31) has an uncertain sign. $\frac{\partial F_S}{\partial F} >=< 0$ if and only if $\left[Q(.)\frac{\partial p^*}{\partial F_B} + (p^* - c)\frac{\partial Q(.)}{\partial F_B}\right]\frac{\partial F_B}{\partial F} >=< 1$.

From equation (21):

$$\frac{\partial(NMR_S)}{\partial F} = -2\frac{\partial F_S}{\partial F}.$$

Therefore, $\frac{\partial(NMR_S)}{\partial F} <=> 0$ if and only if $\frac{\partial F_S}{\partial F} >=< 0$.

From equation (15):

$\frac{\partial \tilde{N}_S}{\partial F} = -\left(\frac{\partial F_S}{\partial F}\right)$ and $\frac{\partial N_S}{\partial F} = \left(\frac{\partial F_S}{\partial F}\right)$.

The sign of $\frac{\partial \tilde{N}_S}{\partial F}$ and $\frac{\partial N_S}{\partial F}$ follows the sign of $\frac{\partial F_S}{\partial F}$ derived in the second part of the Proposition.

Now, from equation (5):

$$\frac{\partial N_B}{\partial F} = -\frac{\partial \bar{\mu}}{\partial F_B}\frac{\partial F_B}{\partial F}.$$

Since $\frac{\partial \bar{\mu}}{\partial F_B} > 0$ from Lemma 5(a), and $\frac{\partial F_B}{\partial F} < 0$ from the first part of the Proposition, $\frac{\partial N_B}{\partial F} > 0$.

(iii) From equation (14) at the equilibrium:

$$\frac{\partial \pi}{\partial F} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial F_B}\frac{\partial F_B}{\partial F} + \frac{\partial[(F_B - C_B)N_B]}{\partial F_B}\frac{\partial F_B}{\partial F},$$

using the definition of NMR_S and NMR_B from equations (21) and (22), which we write as:

$$\frac{\partial \pi}{\partial F} = \frac{\partial F_B}{\partial F}[NMR_S - NMR_B.\frac{\partial T_0}{\partial F_B}].$$

Since $NMR_S = NMR_B$ from equation (20),

$$\frac{\partial \pi}{\partial F} = \frac{\partial F_B}{\partial F}NMR_S[1 - \frac{\partial T_0}{\partial F_B}]. \quad \text{(A.32)}$$

On the RHS of (A.32), $NMR_S > 0$, and $\frac{\partial T_0}{\partial F_B} < 0$ (from Lemma 7) and $\frac{\partial F_B}{\partial F} < 0$ from the first part of the Proposition. Therefore, $\frac{\partial \pi}{\partial F} < 0$. The statement follows. □

Proof of Proposition 5:

(i) From equation (20):

$$\frac{\partial NMR_S}{\partial F_B}\frac{\partial F_B}{\partial \bar{v}} + \frac{\partial NMR_S}{\partial \bar{v}} = \frac{\partial NMR_B}{\partial F_B}\frac{\partial F_B}{\partial \bar{v}} + \frac{\partial NMR_B}{\partial \bar{v}}.$$

It follows from the above equation that,

$$\frac{\partial F_B}{\partial \bar{v}} = \frac{\frac{\partial NMR_B}{\partial \bar{v}} - \frac{\partial NMR_S}{\partial \bar{v}}}{\frac{\partial NMR_S}{\partial F_B} - \frac{\partial NMR_B}{\partial F_B}}. \tag{A.33}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial F_B} > 0$ and $\frac{\partial NMR_B}{\partial F_B} < 0$ respectively. Therefore, the term's denominator is positive on the RHS of (A.33). From Lemma 8(b) and 9(b) $\frac{\partial NMR_S}{\partial \bar{v}} > 0$ and $\frac{\partial NMR_B}{\partial \bar{v}} < 0$. Therefore, the numerator of the term on the RHS of (A.33) is negative. Therefore, $\frac{\partial F_B}{\partial \bar{v}} < 0$.

(ii) From equation (23):

$$\frac{\partial F_S}{\partial \bar{v}} = Q(.)\left[\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \bar{v}} + \frac{\partial p^*}{\partial \bar{v}}\right] + (p^* - c)\frac{\partial Q}{\partial \bar{v}}. \tag{A.34}$$

Since $\frac{\partial p^*}{\partial F_B} < 0$ and $\frac{\partial p^*}{\partial \bar{v}} < 0$ from Lemma 6, $\frac{\partial Q}{\partial \bar{v}} < 0$ Lemma 5, and $\frac{\partial F_B}{\partial \bar{v}} < 0$, as proved above, the RHS of (A.31) has an uncertain sign.$\frac{\partial F_S}{\partial \bar{v}} >=< 0$ if and only if $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \bar{v}}Q(.) > (<) - [\frac{\partial p^*}{\partial \bar{v}}Q(.) + (p^* - c)\frac{\partial Q}{\partial \bar{v}}]$. From equation (21):

$$\frac{\partial (NMR_S)}{\partial \bar{v}} = -2\frac{\partial F_S}{\partial \bar{v}}.$$

Therefore, $\frac{\partial(NMR_S)}{\partial\bar{v}}$ <=> 0 if and only if $\frac{\partial F_S}{\partial\bar{v}}$ >=< 0.

From equation (15):

$\frac{\partial\tilde{N}_S}{\partial\bar{v}} = -\left(\frac{\partial F_S}{\partial\bar{v}}\right)$ and $\frac{\partial N_S}{\partial\bar{v}} = \left(\frac{\partial F_S}{\partial\bar{v}}\right)$.

The sign of $\frac{\partial\tilde{N}_S}{\partial\bar{v}}$ and $\frac{\partial N_S}{\partial\bar{v}}$ follows the sign of $\frac{\partial F_S}{\partial\bar{v}}$ derived in the second part of the Proposition.

Now, from Lemma 4(b):

$\frac{\partial N_B}{\partial\bar{v}} = -\frac{\partial\bar{\mu}}{\partial\bar{v}} < 0$.

(iii) From equation (14) at the equilibrium:

$\frac{\partial\pi}{\partial\bar{v}} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial F_B}\frac{\partial F_B}{\partial\bar{v}} + \frac{\partial[(F_B - C_B)N_B]}{\partial F_B}\frac{\partial F_B}{\partial\bar{v}}$,

using the definition of NMR_S and NMR_B from equations (21) and (22), which we write as:

$\frac{\partial\pi}{\partial\bar{v}} = \frac{\partial F_B}{\partial\bar{v}}[NMR_S - NMR_B.\frac{\partial T_0}{\partial F_B}]$.

Since $NMR_S = NMR_B$ from equation (20),

$$\frac{\partial\pi}{\partial\bar{v}} = \frac{\partial F_B}{\partial\bar{v}} NMR_S[1 - \frac{\partial T_0}{\partial F_B}]. \quad (A.35)$$

On the RHS of (A.35), $NMR_S > 0$, and $\frac{\partial T_0}{\partial F_B} < 0$ (from Lemma 7) and $\frac{\partial F_B}{\partial\bar{v}} < 0$ from the first part of the Proposition. Therefore, $\frac{\partial\pi}{\partial\bar{v}} < 0$. The statement follows. □

Proof of Proposition 6:

(i) From equation (20):

$\frac{\partial NMR_S}{\partial F_B}\frac{\partial F_B}{\partial\rho} + \frac{\partial NMR_S}{\partial\rho} = \frac{\partial NMR_B}{\partial F_B}\frac{\partial F_B}{\partial\rho} + \frac{\partial NMR_B}{\partial\rho}$.

It follows from the above equation that,

$$\frac{\partial F_B}{\partial \rho} = \frac{\frac{\partial NMR_B}{\partial \rho} - \frac{\partial NMR_S}{\partial \rho}}{\frac{\partial NMR_S}{\partial F_B} - \frac{\partial NMR_B}{\partial F_B}}. \tag{A.36}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial F_B} > 0$ and $\frac{\partial NMR_B}{\partial F_B} < 0$ respectively. Therefore, the term's denominator is positive on the RHS of (A.36). From Lemma 8(b) and 9(b) $\frac{\partial NMR_S}{\partial \rho} > 0$ and $\frac{\partial NMR_B}{\partial \rho} < 0$. Therefore, the numerator of the term on the RHS of (A.36) is negative. Therefore, $\frac{\partial F_B}{\partial \rho} < 0$.

(ii) From equation (23):

$$\frac{\partial F_S}{\partial \rho} = Q(.)\left[\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \rho} + \frac{\partial p^*}{\partial \rho}\right] + (p^* - c)\frac{\partial Q}{\partial \rho}. \tag{A.37}$$

Since $\frac{\partial p^*}{\partial F_B} < 0$ and $\frac{\partial p^*}{\partial \rho} < 0$ from Lemma 6, $\frac{\partial Q}{\partial \rho} < 0$ Lemma 5, and $\frac{\partial F_B}{\partial \bar{v}} < 0$, as proved above, the RHS of (A.37) has an uncertain sign.$\frac{\partial F_S}{\partial \rho} >=< 0$ if and only if $\frac{\partial p^*}{\partial F_B}\frac{\partial F_B}{\partial \rho}Q(.) > (<) - [\frac{\partial p^*}{\partial \rho}Q(.) + (p^* - c)\frac{\partial Q}{\partial \rho}]$. From equation (21):

$$\frac{\partial (NMR_S)}{\partial \rho} = -2\frac{\partial F_S}{\partial \rho}.$$

Therefore, $\frac{\partial (NMR_S)}{\partial \rho} <=> 0$ if and only if $\frac{\partial F_S}{\partial \rho} >=< 0$.

From equation (15):

$$\frac{\partial \tilde{N}_S}{\partial \rho} = -\left(\frac{\partial F_S}{\partial \rho}\right) \text{ and } \frac{\partial N_S}{\partial \rho} = \left(\frac{\partial F_S}{\partial \rho}\right).$$

The sign of $\frac{\partial \tilde{N}_S}{\partial \rho}$ and $\frac{\partial N_S}{\partial \rho}$ follows the sign of $\frac{\partial F_S}{\partial \rho}$ derived in the second part of the Proposition.

Now, from Lemma 4(b):

$$\frac{\partial N_B}{\partial \rho} = -\frac{\partial \bar{\mu}}{\partial \rho} < 0 .$$

(iii) From equation (14) at the equilibrium:

$$\frac{\partial \pi}{\partial \rho} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial F_B}\frac{\partial F_B}{\partial \rho} + \frac{\partial[(F_B - C_B)N_B]}{\partial F_B}\frac{\partial F_B}{\partial \rho},$$

using the definition of NMR_S and NMR_B from equations (21) and (22), which we write as:

$$\frac{\partial \pi}{\partial \rho} = \frac{\partial F_B}{\partial \rho}[NMR_S - NMR_B.\frac{\partial T_0}{\partial F_B}].$$

Since $NMR_S = NMR_B$ from equation (20),

$$\frac{\partial \pi}{\partial \rho} = \frac{\partial F_B}{\partial \rho} NMR_S[1 - \frac{\partial T_0}{\partial F_B}]. \tag{A.38}$$

On the RHS of (A.38), $NMR_S > 0$, and $\frac{\partial T_0}{\partial F_B} < 0$ (from Lemma 7) and $\frac{\partial F_B}{\partial \rho} < 0$ from the first part of the Proposition. Therefore, $\frac{\partial \pi}{\partial \rho} < 0$. The statement follows. □

Chapter 4

"Love for Variety" and Location Choice

4.1 Introduction

The last chapter discussed the choice of membership fees by a monopoly platform for buyers and sellers and the participation of buyers and sellers in the platform when buyers have a ‘love for variety’ preference. This chapter uses a similar framework to discuss the platform’s location choice. The platform in this chapter is imagined as a shopping mall, which decides where to locate in a linear city, given that there are fringe sellers of the differentiated product who will locate themselves in the precisely same location just outside the mall, and there are sellers of homogenous products located at one end of the city. The consumers are uniformly distributed across all the city locations. The platform decides its location in the city and the membership fee to charge to the sellers.

In the modern era, shopping malls have become a part of urban life worldwide. India is not an exception. Statista (2024) reported that the number of shopping malls in India went up from 188 in 2012 to 271 in 2022 – roughly a rise of 45 percent. Malls have become attractive because they allow consumers access to different goods and varieties of a particular good in one place without spending much time and money commuting between shops. It is common for malls across the globe not to charge buyers any membership/entry fee[*] for visiting the malls (Hasker & Inci, 2014). In this context, ‘love for variety’ and traveling costs are two major factors influencing consumers’ decision to visit a mall. It also affects the quality of urban life and the formation of city structures (Schiff, 2015). The model presented in this chapter incorporates the features of the ‘love for variety’ of the buyers, the transport costs of accessing the mall, and zero entry fees charged by the mall.

[*] See Mishra and Sarkar (2023) for the cases where buyers are charged in a two-sided platform framework.

The model presented in the chapter assumes that the sellers of the differentiated product who do not opt to locate inside the mall by paying the membership fee charged by the mall sell their products just outside the mall precisely in the exact location. This is a common feature of malls in developing parts of the World (Rajagopal, 2009)[†]. Also, this is a standard feature in other parts of the world. There is a shopping street in the city with shops outside the mall. Buyers either buy the differentiated product, purchasing all the varieties from both the mall and the sellers outside the mall, or the homogeneous product from the sellers located at one end of the linear city.

The chapter presents a theoretical model of a monopoly shopping mall selling differentiated products and a homogeneous goods market in a linear city. The location of the homogeneous goods market is fixed on one end of the linear city and has not been explicitly modeled, while the shopping mall chooses its location and a membership fee to be charged on the seller side. The sellers of the differentiated product observe the membership fee and then decide whether to join the mall or operate outside of it at the same location. Bertrand-price competition among sellers to determine the prices of their products follows. Interestingly, since the mall's location affects the demand for differentiated products, the prices of the varieties are influenced by the mall's location. Thus, the theoretical model presented in the chapter combines several dimensions - spatial competition, 'love for variety' preferences on the buyer side, intensive and extensive margins of demand, travel costs, and Bertrand-price competition among the sellers participating in the differentiated product market. The choice of location of the mall is of great significance in the context of the results derived in the chapter. The number of

[†] As an example, in the New Market-Lindsay Street area in Kolkata, the Simpark Mall selling varieties of clothing apparel and outside the mall, there is a buzzing informal sector comprising small sellers selling similar products off Lindsay Street (Ghosh, 2022). Similarly, one can consider the Gariahat area – the shopping hub in South Kolkata (Sekhani et al., 2019). The sellers locate just outside the Bazaar Kolkata Mall.

varieties sold on and off the mall, the location of the marginal buyer, and the number of buyers are endogenously determined. In the presence of a mall, the number of varieties produced of a differentiated product increases compared to a 'no-mall' situation. As the number of varieties sold in the mall rises, the number of varieties sold outside the mall falls. It also focuses on the mall's incentive to increase its efficiency in servicing both buyers and sellers, the mall's reaction in terms of pricing the seller side as a response to a reduction in the sellers' fixed cost of production. Additionally, the location of the shopping mall, the membership fee charged to the sellers, and the number of varieties sold in the mall also depend on factors like the transport costs incurred by the buyers for traveling to the mall, outside options of the buyers, and the substitutability of the varieties of the product.

The results of the comparative static exercises show that if the costs of servicing the sellers fall, the mall charges a lower membership fee to the sellers. The model highlights the role of the location choice of the mall in explaining these results. As the costs of servicing the sellers fall, at the new equilibrium, the profit-maximizing mall adjusts its location away from the location of the homogeneous product market given its revenue-cost considerations and creates a negative cross-sided externality on the buyer side. As a result, some of the buyers close to the homogeneous goods market drop out of the differentiated product market as their travel cost rises for accessing the differentiated product market situated around the mall, the price charged for each variety of the differentiated product falls and the mall's ability to charge a high fee to the sellers joining the mall falls. It thus lowers the membership fee to retain the sellers in the mall. Therefore, the number of varieties in the mall rises as some sellers move in.

In contrast, a fall in the cost of servicing buyers leads to a rise in the membership fee charged on the seller side. Interestingly, under certain conditions on both the revenue and the cost sides of the shopping mall, the mall has an incentive to become efficient in servicing both the buyers and the sellers. This result is similar to a standard monopoly case. If the sellers have a lower fixed cost, they face a higher membership fee as the shopping mall expects them to gain due to the lowering of the fixed cost. Thus, at the initial equilibrium, the net marginal revenue of the mall from the seller side falls below the marginal cost on the buyer side, and the mall chooses its location away from the differentiated product market. Some buyers close to the homogeneous goods market leave the differentiated product market, given the rise in their transport costs to travel to the mall. Since the demand falls for every seller in the differentiated product market, each one starts lowering their price to retain buyers. There are two opposing forces on every seller's profit, namely, a positive effect through a fall in the fixed costs and a negative effect through a fall in revenue due to a fall in the prices of the varieties. Given this, the mall accordingly adjusts the membership fee on the seller side, and as a result, the number of varieties sold in the mall adjusts. Thus, the mall may prefer developments that lower the seller's fixed costs depending on the impact on the mall's profit. Suppose the buyers have a better outside option as their valuation of the product sold at the homogeneous goods market rises. In such a case, the number of buyers falls in the differentiated product market. The mall chooses its location closer to the homogeneous goods market. With this, some buyers who have been buying homogeneous products earlier start buying differentiated products now as their travel costs fall and they have a 'love for variety' preference. However, the seller's fee and the number of varieties sold in the mall may rise or fall depending on the mall's calculation of the seller's gain and loss. The results suggest that even if the differentiated product market loses its buyers to the homogeneous goods market, the mall may raise the membership fee for the sellers contrary to standard spatial competition models where location is exogenously assumed. Also, there are conditions under which the homogeneous goods market and the number of varieties in the mall grow together. The results concerning the number of varieties sold in the mall and the mall's profit are similar to the changes in the substitutability among the varieties and transport costs incurred by the buyers. The chapter emphasizes the role of the mall's location choice in explaining the results.

In the existing literature, Fujita and Thisse (2013) have singled out two dominant approaches to endogenizing urban structure and the choice of firms involved in spatial competition. In the first approach, used in papers like Fujita and Ogawa (1982) and Lucas and Rossi-Hansberg (2002), firms are involved in perfect competition and choose to locate in the center of the city. The second approach, used by De Palma et al. (1985), Fujita (1988), Abdel-Rahman (1988), Berliant and Konishi (2000) to more recent ones which study different forms of marketplaces and locations of firms therein, like Brandão et al. (2014), Ushchev et al. (2015) rests on assuming monopolistic competition, traveling costs, and product differentiation as the key factors shaping the urban landscape. In contrast to the dominant theme of using monopolistic competition for studying the equilibrium spatial distributions of firms (Ago, 2008; Ago et al., 2017), few papers like Hamilton et al. (1989) and Anderson and Neven (1991) study Cournot competition among firms in such spatial markets using a linear city framework while Gupta et al. (1997), Matsushima (2001), Matsumura and Shimizu (2005), and Chen and Lai (2008) do it for a circular city.

Closely related to the theme of this chapter, one of the pioneering works is by Smith and Hay (2005), where they study the interactions between three alternative modes of retail organization: the street marketplace, malls, and supermarkets. However, the location of marketplaces is exogenously fixed in their model, and each supplies the same goods. It is also assumed that the volumes and prices of the products purchased are independent of firms' strategies, consumers' locations, and the location of each marketplace. Brando et al. (2014) consider a setting where a shopping mall and a supermarket compete by supplying the same range of goods, and locations are fixed on two extremes of the linear city. Ushchev et al. (2015) extend this model by introducing i) a downtown retail market and a monopoly shopping mall on both ends of the linear city and ii) buyers have a 'love for variety' preference. Some varieties are sold in the retail market, and some are sold in malls. The downtown retail market is characterized by free entry, while the monopoly shopping mall allows sellers by charging a per-slot fee. Though the location of the marketplaces is fixed, the size of the marketplaces gets endogenously determined. In the literature on social media platforms, a recent paper by Mishra and Sarkar (2023) studied a platform duopoly where the platforms are located on the extremes of a linear city, similar to Uschev et al. (2015). A similar structure in a different context has been used in Bounie et al. (2024). The difference between Mishra and Sarkar (2023) and Ushchev et al. (2015) is that the former assumes a duopoly involving two-sided platforms, while the latter assumes one platform and one marketplace characterized by monopolistic competition. Consumers, depending on their location, can either single-home or multi-home. Thus, one of the major research gaps in the existing literature is not endogenizing the location choice of a two-sided platform, like a shopping mall. The chapter contributes to this literature.

Unlike Smith and Hay (2005), Brando et al. (2014), and Ushchev et al. (2015), we endogenize the location choice of the shopping mall, keeping the location of the homogeneous goods market fixed. Also, contrary to Smith and Hay (2005) and Brando et al. (2014), the model formulated in the chapter does not have any overlapping cases of buyers going to both marketplaces, differentiated and homogeneous, as the goods sold are entirely different. All the buyers in our model have a 'love for variety' preference and purchase all the varieties irrespective of whether they are available in the mall or outside it, unlike Ushchev et al. (2015), where only a few purchase all the varieties. Following Malykhin and Ushchev (2018), our model introduces quadratic transportation costs for keeping the model tractable, contrary to the linear transportation costs introduced by Brando et al. (2014) and Ushchev et al. (2015). Unlike Bronnenberg (2015), we do not consider travel costs to rise with an increase in variety.

Until now, no other paper in the literature uses Bertrand-price competition in such a setting. The choice of location of a mall, among other factors, is of great significance in deriving the prices of varieties in our model, unlike Brando et al. (2014) and Ushchev et al. (2015). While the prices derived by Brando et al. (2014) depend on the exogenous number of varieties, Ushchev et al. (2015) derive the Dixit-Stiglitz prices (Dixit & Stiglitz, 1977). Our model overcomes this major drawback in the firm's pricing behavior, which is independent of the firm's location – a counterfactual attribute given the empirical evidence of spatial price dispersion (Lach, 2002; Kaplan et al., 2019). In Brando et al. (2014), the number of varieties is exogenously fixed, while Ushchev et al. (2015) use the free-entry condition in monopolistic competition models to derive the number of varieties unlike our model. In the differentiated product market, as the number of varieties sold in the mall rises, the number of varieties sold outside the mall falls. We find empirical evidence for this result in the Indian context, which has a large informal sector. Roy (2014) shows it for Kolkata, Kalhan (2007) for Mumbai, Naidu and Naidu (2016) in Chennai and Coimbatore, and Mathews (2018) for Kollam. Ushchev et al. (2015) show that with a rise in travel costs, the mall's profit rises or falls under different conditions on the exogenously defined size of the mall. We came up with a similar result depending on the mall's revenue and cost side conditions, but the size of the mall is endogenous in our case.

Moreover, given the endogenous location choice of the mall, this chapter comes up with some results that are new to the literature. The location choice of the mall influences the consumers' choice of how much of each variety they want to consume, given the transportation costs of traveling to the mall. The results in the chapter add insights into the incentive the mall has by increasing its efficiency in servicing both buyers and sellers. It argues that the mall may like developments that lower the seller's fixed costs. Under certain conditions, the homogeneous goods market and the number of varieties in the mall grow together. This result can have important implications in the context of regulatory intervention for limiting the size of malls, like in Italy (Schivardi & Viviano, 2011) and the UK (Vialard et al., 2017), for supporting the 'brick-and-mortar' stores. The profit of the mall and the number of varieties sold in the mall may rise with a rise in the transport costs, improvement in the outside option, and substitutability among the varieties of the differentiated product.

The remainder of the chapter is organized as follows: Section 2 describes the model, followed by the comparative static exercises in Section 3. Section 4 concludes.

4.2 The Model

4.2.1 A Baseline Model

Consider a linear city of unit length having its population uniformly distributed over $[0,1]$. The city is supplied with a differentiated product with many varieties, each sold by different sellers. A shopping mall at location $a \in [0,1]$ of the city hosts some sellers. The sellers, who are not members of the shopping mall, sell their products outside the mall at the same location. The population of the city has unit mass. They buy either the differentiated product at a location x or a homogeneous product from the sellers located at 1. We assume while the location of the sellers of the homogeneous product is fixed at 1, the mall can choose its location a at any point in $[0,1]$. For each buyer, there is a transport cost tx^2 for traveling a distance x in the city, where $t > 0$. Travel costs and transport costs have been synonymously used in the chapter. Each seller of the differentiated product uses an increasing return-to-scale technology for production with a fixed cost $F > 0$ and unit cost $c > 0$. The j^{th} seller (for $j = 1,2, \dots, N_S$) also incurs a transaction cost T_j for carrying out business activities. We assume T_j is uniformly distributed among sellers in the interval $[0, 1]$‡.

The mall acts as a two-sided platform by hosting buyers and sellers of the differentiated product who want to transact through it. It brings down the transaction cost of a seller in exchange for a membership fee F_S. If F_S is high, a seller having a lower T_j may not join the mall. We assume, for simplification, that the mall does not charge any membership fee on the buyers' side. In our model, each buyer has a 'love for variety' utility function. N_B of the population of buyers of measure 1 enter the market for the differentiated product and buy from every seller present at a, be it in the mall or outside of it. We assume the measure of the number

‡ The model can be generalized by assuming a generic continuous distribution of T over a support $[0, \overline{T}]$ with twice continuously differentiable cumulative distribution function with similar results.

of potential sellers in the differentiated product market is 1. N_S of them enter the market in the absence of the mall. We show that in the presence of the mall, every seller enters the market, and $\widetilde{N}_S$ of them operate in the mall. The values of N_B, N_S and $\widetilde{N}_S$ get endogenously determined in the model. Since the shopping mall increases the number of varieties sold in the market, the buyers with a 'love for variety' utility function gain from the mall's existence.

There is a servicing cost of the shopping mall is C_S on the seller's side, and C_B, on the buyer's side.

The sequence of decisions in the model is as follows.

Period 1: the shopping mall decides about (a, F_S);

Period 2: the sellers of the differentiated product observe F_S, and then decide whether to join the mall or operate outside of it at a;

Period 3: the Bertrand competition among sellers determines the prices of their products;

Period 4: given a, the number of varieties available in the shopping mall, and their prices, a consumer decides whether to go shopping at the mall or purchase the homogeneous product at 1; Then the payoffs are realized.

The model is solved using the backward induction method.

We first solve the buyer's problem in Period 4.

4.2.2 The Buyers

Independent of their different locations in the city, each buyer has an identical income $I > 0$ and has a 'love for variety' utility function represented by:

$$u = \max\left\{\left(q_0 + \left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta}\right), \bar{v}\right\}. \tag{1}$$

She derives her utility either from the consumption of the basket of N_S varieties of the differentiated product available at location a or from the consumption of a homogeneous product available at location 1. The term $\left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta}$ in (1), is the utility obtained from the consumption of the basket of differentiated products, where $\rho \in (0,1)$ denotes the substitutability among the varieties of the product in her preference. The extent of substitutability falls with lower values of ρ. The preference parameter $\beta > 0$ represents the substitutability between the differentiated product and a composite commodity, which is also available at the location a on which she spends q_0. We assume $\beta < \rho$ to ensure that the utility function is concave in q_j. The location of the buyer plays an important role in her purchase decision.

Given $p_j > 0$ is the price of the differentiated product j, ($\forall j = 1, \dots \dots N_S$), a buyer located at location x maximizes her utility by choosing $q_j \geq 0$ ($\forall j = 1, \dots \dots N_S$) subject to her budget constraint

$$q_0 + \sum_{j=1}^{N_S} p_j\, q_j \leq I - t(a-x)^2 \quad \forall\, j = 1, \dots \dots N_S. \tag{2}$$

Given the budget constraint binds, substituting for q_0 from equation (2) in equation (1), the maximization problem of the buyer at location x becomes,

$$\max \emptyset_x = I - t(a-x)^2 - \sum_{j=1}^{N_S} p_j q_j + \left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta} \quad w.r.t\ 'q_j' \ \ \forall\ j = 1, \dots \dots N_S.$$

The equilibrium choice of $q_j \geq 0$ for all $j = 1, \dots \dots N_S$ satisfies,

$$\frac{\partial \emptyset_x}{\partial q_j} \leq 0 \Rightarrow -p_j + \beta\left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta-1} \left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1-\rho}{\rho}} q_j{}^{\rho-1} \leq 0. \tag{3}$$

The x type consumer's demand for the j^{th} variety is derived from equation (3) for all $j = 1, \dots \dots N_S$ as $q_j\left(p_1, \dots p_j \dots \dots, p_{N_S}, \rho, \beta\right)$. For $q_j > 0$ for all $j = 1, \dots \dots N_S$. $\frac{\partial \emptyset_x}{\partial q_j} = 0$ is the first-order condition for maximization. Given $\rho \in (0, 1)$ and $\beta < \rho$, the model ensures that the second-order condition, $\frac{\partial^2 \emptyset_x}{\partial {q_j}^2} < 0$, holds.

For simplicity, let us define, $\left(\sum_{j=1}^{N_S} {q_j}^{\rho}\right)^{\frac{1}{\rho}} = k$. If $q_j > 0$ for all $j = 1, \dots \dots N_S$, $k > 0$.

Using the definition of k and substituting the value of q_j that solves equation (3), in $\emptyset_x$, the indirect utility function of a buyer at location x from the purchase of the differentiated product at location a is,

$$v(x) = I - t(a-x)^2 - \sum_{j=1}^{N_S} p_j\, q_j + k^{\beta}. \tag{4}$$

The indirect utility $v(x)$ is nothing but the value of $\emptyset_x$ at $q_j\left(p_1, \dots p_j \dots, p_{N_S}, \rho, \beta\right)$. From equation (4), $\frac{\partial v}{\partial x} = 2t(a-x) > 0$ & $\frac{\partial^2 v}{\partial x^2} = -2t < 0$ for all values of x in $[0, 1]$ with its maximum at $x = a$.

The indirect utility obtained from buying the homogeneous product at location 1 is $[\bar{v} - t(1-x)^2]$, which is monotonically rising in x.

The buyer at x will be indifferent between buying the differentiated product from a and buying the homogeneous product from 1 if and only if

$$v(x) = \bar{v} - t(1-x)^2. \tag{5}$$

Assumption 1: (i) $a < 1$.

(ii) $I - t(a-1)^2 - \sum_{j=1}^{N_S} p_j\, q_j + k^{\beta} < \bar{v} < I - \sum_{j=1}^{N_S} p_j\, q_j + k^{\beta} = v(a)$.

Since $v(x)$ attains its maximum at $x = a$, Assumption 1 implies that a buyer located in the differentiated product market would always prefer to buy the differentiated product than to buy the homogeneous product, which is intuitively obvious since the buyers have 'love for variety' preference. The second part of the assumption is less obvious. It also implies that a buyer located at 1 prefers to buy the homogeneous product rather than buying the differentiated product from a different location even if she has 'love for variety'. So transport cost matters in purchasing decisions.

Assumption 1 implies $v(0) = \left[I \; - ta^2 - \sum_{j=1}^{N_S} p_j \, q_j + \; k^{\beta}\right] > [\bar{v} - t]$ and a unique solution to equation (5) exists in [0,1]. Let the location of the buyer who is indifferent between buying the differentiated product at a and buying the homogeneous product at 1 be $\bar{x}$, which solves equation (5) as:

$$\bar{x} = a + \frac{1-a}{2} + \frac{I - \sum_{j=1}^{N_S} p_j \, q_j + k^{\beta} - \bar{v}}{2t(1-a)}. \tag{6}$$

Figure 4.1 below explains the determination of $\bar{x}$.

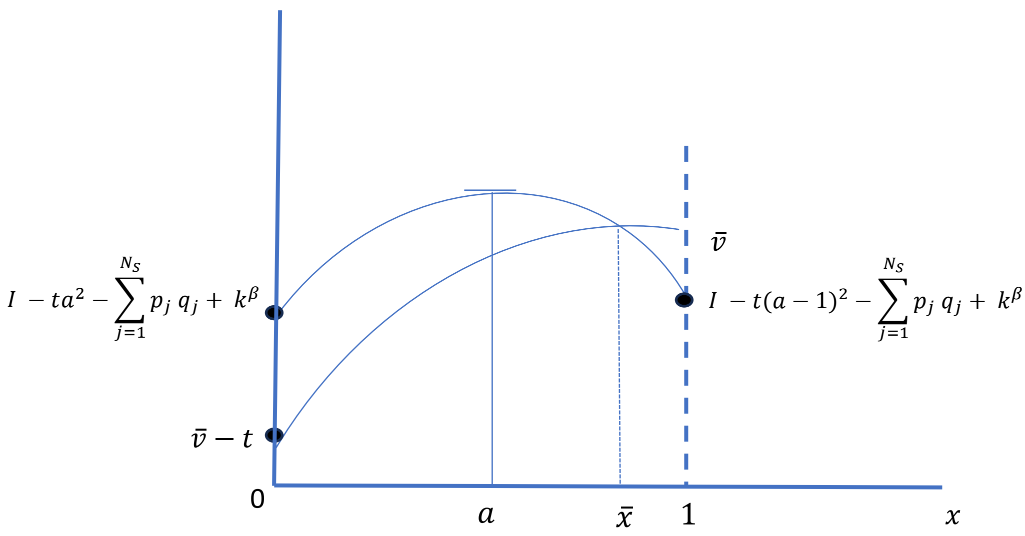

Figure 4.1: Determination of $\bar{x}$

From Figure 4.1 it is clear that all buyers having $x \leq \bar{x}(N_S, p_1, \ldots p_j \ldots, p_{N_S}, \rho, \bar{v}, t, a)$ have $v(x) \geq \bar{v} - t(1-x)^2$, therefore buys the differentiated product from a and all buyers having $x > \bar{x}(N_S, p_1, \ldots p_j \ldots, p_{N_S}, \rho, \bar{v}, t, a)$ have $v(x) < \bar{v} - t(1-x)^2$, therefore buys the homogeneous product from a.

Since the population is distributed uniformly over $[0, 1]$, the number of buyers participating in the differentiated product market is:

$$N_B = \Pr[x \leq \bar{x}(\cdot)] = \bar{x}(\cdot). \tag{7}$$

The number of buyers in the homogeneous product market is $[1 - \bar{x}(\cdot)]$.

The aggregate demand for the j^{th} variety of differentiated product (Q_j) is:

$$Q_j = \int_0^{\bar{x}(\cdot)} q_j(\cdot)dx \;\; \forall \;\; j = 1, \dots \dots N_S, \tag{8}$$

where $q_j(\cdot)$ satisfies equation (3) and $\bar{x}(\cdot)$ follows from equation (6). Notice that Q_j derived in equation (8) is a function of $(p_1, \dots \dots, p_{N_S}, \rho, \bar{v}, t, a)$.

Lemma 1: If $q_j > 0$,

$a. \frac{\partial q_j}{\partial p_j} < 0$,

$b. \frac{\partial q_j}{\partial \rho} < 0$

for all $x \in [0,1]$ and for all $j = 1, \dots \dots N_S$.

Proof: See the Appendix A4. □

For every buyer, irrespective of location, Lemma 1 states that with a 'love for variety' preference, each variety of the differentiated product, be it in the mall or outside, is like a normal good having a negatively sloped demand function. With a rise in ρ, the amount purchased of each variety of the product falls as varieties are now closer substitutes of each other in the buyers' preference.

Lemma 2: If $q_j > 0$

$a. \frac{\partial \bar{x}(\cdot)}{\partial p_j} < 0$,

$b. \frac{\partial N_B}{\partial p_j} < 0$

for all $j = 1, \dots \dots N_S$.

Proof: See the Appendix A4. □

Ceteris paribus an increase in the price of the j^{th} variety of the differentiated product, which is purchased by the marginal buyer at the initial equilibrium, reduces the indirect utility level of the marginal buyer of type $\bar{x}$ below $[\bar{v} - t(1-x)^2]$. Therefore, she stops buying the differentiated product and avails herself of the outside option at 1. Only the buyers closer to the shopping mall, thus having a lower travel cost, continue to purchase it. As a result, the number of buyers in the differentiated product market falls.

Lemma 3: $\frac{\partial Q_j}{\partial p_j} < 0$ for all $j = 1, \dots \dots N_S$.

Proof: See the Appendix A4. □

Lemma 3 states that the aggregate demand of each variety of the differentiated product is negatively sloped like normal commodities: as the price of a variety rises its aggregate demand (demand from buyers at all the locations taken together) falls. Intuitively Lemma 1a suggests, all other things remaining the same, as the price of a variety rises, the demand from a buyer for the j^{th} variety irrespective of her location, who continues to purchase the variety, falls (i.e., shrinkage in the intensive margin of demand). Second, Lemma 2a suggests that as the price of a variety rises, some buyers stop buying the product (shrinkage in the extensive margin of demand).

Lemma 4: a. If $q_j > 0, \frac{\partial \bar{x}(\cdot)}{\partial \rho} < 0, \frac{\partial \bar{x}(\cdot)}{\partial \bar{v}} < 0, \frac{\partial \bar{x}(\cdot)}{\partial t} < 0, \frac{\partial \bar{x}(\cdot)}{\partial a} > 0$ for all $j = 1, \dots N_S$;

b. $\frac{\partial N_B}{\partial \rho} < 0, \frac{\partial N_B}{\partial \bar{v}} < 0, \frac{\partial N_B}{\partial t} < 0, \frac{\partial N_B}{\partial a} > 0.$

Proof: See the Appendix A4. □

Ceteris paribus an increase either in ρ, $\bar{v}$ or in t reduces the indirect utility level of the marginal buyer of the type $\bar{x}$. Therefore, she stops buying differentiated products and avails herself of the outside option. Only the buyers closer to the shopping mall, thus having a lower travel cost, continue to purchase it. Therefore, $\bar{x}$ falls at the new equilibrium, and so does N_B. However, if the shopping mall chooses its location, a, closer to 1, some buyers who were buying the homogeneous product earlier start buying the differentiated product since their travel cost to a falls and they have a 'love for variety'. As a result both $\bar{x}(\cdot)$ and N_B rises.

Lemma 5: $\frac{\partial Q_j}{\partial \rho} < 0, \frac{\partial Q_j}{\partial \bar{v}} < 0, \frac{\partial Q_j}{\partial t} < 0, \frac{\partial Q_j}{\partial a} > 0$, for all $j = 1, \dots \dots N_S$.

Proof: See the Appendix A4. □

Since the location of the mall, a, per unit travel cost, t, and the utility from the outside option $\bar{v}$, does not have an impact on the intensive margin of demand, their effect on the aggregate demand of a variety works through the change in the extensive margin of demand as explained in Lemma 4a.

The different varieties of the differentiated product become closer substitutes of one another in the buyers' preference as ρ rises. This results in a fall in the indirect utility of using the differentiated product. Also, consequent to a rise in ρ, a shrinkage in the extensive margin of demand for the differentiated product happens through a fall in $\bar{x}$. Because every buyer in the differentiated product market buys an equal amount of each variety offered, the exit of some buyers adversely affects the demand enjoyed by each existing seller, whether in the mall or outside of it. The negative impact on the intensive margin reinforces the negative impact on the extensive margin.

4.2.3 The Sellers in the Differentiated Product Market

Having solved the buyers' problem in period 4, we now move backward to period 3 to solve the sellers' price determination problem.

In its pricing decision, each seller takes the aggregate demand function derived in (8), which is negatively sloped (from Lemma 3), as given.

The j^{th} seller's profit is:

$$\pi_j^{PL} = [p_j^{PL} - c]Q_j(.) - F_S - F$$

if she operates in the mall, and,

$$\pi_j^{O} = [p_j^{O} - c]Q_j(.) - T_j - F$$

if she operates outside of it, where p_j^{PL} is the price charged in the mall, and p_j^O is the price charged outside the mall. p_j^{PL} and p_j^O are determined through the maximization of π_j^{PL} and π_j^O respectively. Following Mukherjee and Mukherjee (2023), the first-order condition of profit maximization with respect to p_j, assuming an interior solution must satisfy,

$$p_j = c - \frac{Q_j\left(p_1, \ldots, p_j, \ldots, p_{N_S}, \cdot\right)}{\frac{\partial Q_j}{\partial p_j}\left(p_1, \ldots, p_j, \ldots, p_{N_S}, \cdot\right)}. \tag{9}$$

The sufficient condition that satisfies the second-order condition of each seller's profit maximization problem is: $\frac{\partial^2 Q_j}{\partial p_j^2} < 0$.

In our model, every variety of the differentiated product enters symmetrically in the utility function specified in equation (1), and therefore every seller faces the same demand function. They also have the same variable cost function. Although the nature of the fixed costs incurred by a seller varies depending on whether it is in the mall or outside of it, the fixed costs do not play a role in the Bertrand price-setting game that the sellers play among themselves. Since all sellers face the same set of parameters $(\rho, \bar{v}, t, a)$ while pricing their variety irrespective of whether they are in the mall or outside of it, it must be the case that the Nash equilibrium in prices is symmetric i.e.$p_1^* = \cdots p_j^* = \cdots = p_{N_S}^* = p^*(\rho, \bar{v}, t, a)$ i.e. all the sellers charge the same price to the buyers of the differentiated product irrespective of the variety they produce.

Since each variety is sold at the same price $p^*(\rho, \bar{v}, t, a)$ at the market and each variety enters symmetrically in the utility function of a buyer given in equation (1), from equation (3) it must be the case that an identical amount $q = \left(\frac{\beta}{p^*}\right)^{\frac{1}{1-\beta}} N_S^{\frac{\beta-\rho}{\rho(1-\beta)}}$ is demanded of each variety. So, from equation (8) we obtain,

$Q(.) = Q_j(p^*(.), N_S, \rho, \bar{v}, t, a)$ for all $j = 1, \ldots \ldots N_S$.

Assumption 2: $\frac{\partial}{\partial \rho}\left(\frac{\partial Q_j}{\partial p_j}\right) \approx 0, \frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q_j}{\partial p_j}\right) \approx 0, \frac{\partial}{\partial t}\left(\frac{\partial Q_j}{\partial p_j}\right) \approx 0, \frac{\partial}{\partial a}\left(\frac{\partial Q_j}{\partial p_j}\right) \approx 0.$

Assumption 2 implies that the slope of the aggregate demand function faced by the j^{th} seller at the market for the differentiated product is a little responsive to the change in ρ, $\bar{v}$, *t,* and a. The assumption simplifies the model and makes it tractable. On violation of Assumption 2, if $\frac{\partial}{\partial \rho}\left(\frac{\partial Q_j}{\partial p_j}\right) \leq 0, \frac{\partial}{\partial \bar{v}}\left(\frac{\partial Q_j}{\partial p_j}\right) < 0, \frac{\partial}{\partial t}\left(\frac{\partial Q_j}{\partial p_j}\right) < 0, \frac{\partial}{\partial a}\left(\frac{\partial Q_j}{\partial p_j}\right) > 0$ all the results of the model go through. If they are weakly positive with respect to ρ, $\bar{v}$, and *t*, then again all the results of the model go through, while if it is weakly negative with respect to *a,* then again, all the results of the model go through.

Lemma 6: $\frac{\partial p^*}{\partial \rho} < 0, \frac{\partial p^*}{\partial \bar{v}} < 0, \frac{\partial p^*}{\partial t} < 0, \frac{\partial p^*}{\partial a} > 0$.

Proof: See the Appendix A4. □

As $\bar{v}$ rises, from Lemma 5, for each seller, the demand for the variety of the differentiated product it produces shrinks. In response, the sellers lower their prices to boost the demand. With a rise in ρ and *t*, we observe a similar effect on Q_j. Concerning their effect on p^*, a similar explanation follows.

The result with regards to the impact on price on account of a change in substitutability among the varieties of the differentiated product matches with the result in the Dixit-Stiglitz model (Dixit & Stiglitz, 1977) and with similar results in recent papers like Armstrong (2006), Hagiu (2009), Belleflamme and Peitz (2018) among others.

In the existing literature on location choice theory, the negative effect of a fall in the indirect utility because of a rise in *t* has been discussed (see Tirole, 1988). Interestingly, we come up with two opposing effects as *t* rises. The first is a negative direct effect of a fall in the aggregate demand because of a fall in the indirect utility, and the second effect is a positive indirect effect on the slope of the demand curve, which is a consequence of the impact on the extensive margin. Given Assumption 2, we rule out the positive effect to emphasize the negative effect of a rise in travel costs. Similarly, for a, there is a direct positive effect of an increase in the aggregate demand following a fall in the distance cost, and the second effect is a negative indirect effect on the slope of the demand curve. We assume away the indirect negative effect through Assumption 2.

Since $p_j^{PL} = p_j^O = p^*(\rho, \bar{v}, t, a)$, the profit functions of the seller in the shopping mall and the outside can be written as:

$$\pi_j^{PL} = [p^*(\rho, \bar{v}, t, a) - c]Q(.) - F_S - F, \tag{10}$$

and,

$$\pi_j^O = [p^*(\rho, \bar{v}, t, a) - c]Q(.) - T_j - F. \tag{11}$$

Notice from equations (11) and (12) that while π_j^O is monotonically declining in T_j, π_j^{PL} is independent of it.

The j^{th} seller operates in the market if and only if $\pi_j^O \geq 0$ i.e.

$T_j \leq [p^*(\rho, \bar{v}, t, a) - c]Q(.) - F.$

Let us define,

$$T_0 = [p^*(\rho, \bar{v}, t, a) - c]Q(.) - F. \tag{12}$$

The j^{th} seller operates in the market if and only if $T_j \leq T_0$.

The j^{th} seller operates in the mall if and only if $\pi_j^{PL} \geq \pi_j^O$ i.e. $T_j \geq F_S$.

Assumption 3: $F_S \leq T_0 < 1$.

Assumption 3 implies that the sellers with $T_j \in [F_S, 1]$ operate in the mall, and the sellers with $T_j < F_S$ operate outside of it. The sellers with $T_j \in (T_0, 1]$ that operates outside the mall makes a loss and exits the differentiated product market. On the violation of Assumption 3, if $F_S > T_0$ no seller enters the mall, which is not a case of interest to us.

The figure below represents the π_j^O and π_j^{PL} as function of T_j and explains the participation of sellers in the shopping mall.

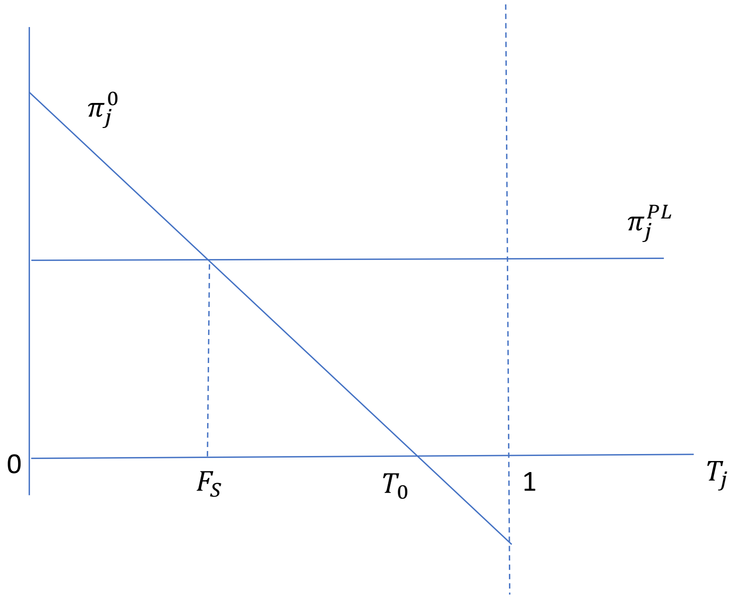

Figure 4.2: Participation of the sellers in the mall

Figure 4.2 shows clearly, as explained above, since the j^{th} seller with its $T_j \in [F_S, 1]$ has $\pi_j^{PL} \geq \pi_j^O$, operates in the mall. A seller with its T_j in $[0, F_S)$ having $\pi_j^{PL} < \pi_j^O$ stays outside the mall. A subset of the sellers that participate in the mall with their $T_j \in (T_0, 1]$, would not enter the differentiated product market in the absence of the mall. If $F_S > T_0$, all the sellers, independent of their T_j earns negative profit by entering the mall. Therefore, no seller operates in the mall.

Clearly, $N_S = \Pr[T_j \leq T_0]$

and,

$\widetilde{N}_S = \Pr[T_j > F_S] = 1 - \Pr\,[T_j \leq F_S]$.

Since, T_j is distributed uniformly over $[0,1]$,

$(N_S = T_0, \widetilde{N}_S = 1 - F_S)$. (13)

Equation (13) implies that as the mall lowers its membership fee, the number of varieties sold in the mall rises while outside the mall it falls. The opposite happens otherwise. Also, since $\widetilde{N}_S = 1 - F_S$, in the presence of the mall, the number of sellers that operate outside the mall is F_S. Since all the potential entrants in the differentiated market enter and produce, the number of varieties produced is 1 in the presence of the mall, which is higher than T_0, the number of varieties produced in the absence of the mall. Therefore, the shopping mall benefits the buyers with a 'love for variety' preference pattern by increasing the number of varieties produced in the market.

Proposition 1: *The shopping mall increases the equilibrium number of varieties produced in the differentiated product market.*

Proof: Follows from equation (13) above. □

A similar result was obtained in Chapter 3.

Lemma 7: $\frac{\partial T_0}{\partial \rho} < 0, \frac{\partial T_0}{\partial \bar{v}} < 0, \frac{\partial T_0}{\partial t} < 0, \frac{\partial T_0}{\partial a} > 0, \frac{\partial T_0}{\partial F} < 0$.

Proof: See the Appendix A4. □

With a rise in ρ, the products become close substitutes. The indirect utility received from the consumption of the differentiated product falls and the marginal buyer, who was indifferent between buying and not buying the differentiated product, no longer buys the product. Given N_S, therefore the extensive margin of demand for the differentiated product shrinks as $\bar{x}$ falls, as explained in Lemma 4. Since each buyer who participates in the differentiated product market buys an equal amount of each available variety, irrespective of whether the variety is available in the mall or outside, the exit of some buyers adversely affects the demand enjoyed by each of the existing sellers. They react to the situation by reducing their price to boost the demand for their variety (Lemma 6). As the price falls, the indirect utility of the buyers buying the differentiated product rises, and the identity of the marginal buyers changes again as some buyers come back to the differentiated product market, generating a positive effect on T_0. However, this effect is not strong enough to outweigh the initial negative demand effect arising from the shrinkage in the extensive margin, and hence, T_0 falls. The negative effect of a rise in $\bar{v}$ and t on T_0 has a similar explanation since Lemma 4 suggests a rise in $\bar{x}$ and a fall in N_B as a consequence of a rise in $\bar{v}$ and t. The opposite happens for a. For a rise in F, the profit of the sellers falls, and the marginal seller no longer finds it profitable to be in the differentiated product market, generating a negative impact on T_0.

Assumption 4: $\frac{\partial^2 T_0}{\partial \rho \partial a} \approx 0, \frac{\partial^2 T_0}{\partial \bar{v} \partial a} \approx 0, \frac{\partial^2 T_0}{\partial t \partial a} \approx 0, \frac{\partial^2 T_0}{\partial a^2} \approx 0.$

Without loss of generality, the effect of changing demand side parameters like $\bar{v}$, ρ, *t* and *a* on $\frac{\partial T_0}{\partial a}$ is assumed to be very negligible and close to zero for keeping the model tractable.

4.2.4 The Monopoly Shopping Mall's Choice

The monopoly shopping mall services all its buyer and seller members by facilitating their transactions. We assume it incurs a constant cost C_S in servicing each seller and a constant cost C_B in servicing each buyer. Its profit, therefore, is written as:

$$\tilde{\pi} = (F_S - C_S)\widetilde{N}_S - C_B N_B, \qquad (14)$$

where $\widetilde{N}_S = 1 - F_S$ from (11), and $N_B = N_B(p^*(.), N_S(.), \rho, \bar{v}, t, a)$.

The first term on the right-hand side of equation (14) characterizes the net revenue earned from the seller side of the mall. Since the mall does not charge the buyers' side, the second term represents the cost incurred for servicing the buyer side. The mall maximizes $\tilde{\pi}$ by choosing $\{F_S, a\}$ subject to the constraints $F_S \leq T_0$[§] and $a < 1$.

[§]For the constrained optimization problem to hold, $T_0 < \frac{(1+C_S)}{2}$. A similar assumption was used in the last chapter.

The Lagrange function of the optimization problem is:

$Z = (F_S - C_S)(1 - F_S) - C_B N_B + \lambda[T_0 - F_S].$

Assuming the existence of an interior solution $\{F_S > 0,\ a > 0,\ \lambda > 0\}$, the first-order conditions of maximization satisfy the following equations:

$$\frac{\partial Z}{\partial F_S} = 0 \implies (1 + C_S - 2F_S) - \lambda = 0; \tag{15}$$

$$\frac{\partial Z}{\partial a} = 0 \implies -C_B \frac{\partial N_B(.)}{\partial a} + \lambda \frac{\partial T_0}{\partial a} = 0; \tag{16}$$

$$\frac{\partial Z}{\partial \lambda} = 0 \implies F_S = T_0. \tag{17}$$

Combining equations (16) and (17) we have,

$$1 + C_S - 2F_S = \frac{C_B \frac{\partial N_B(.)}{\partial a}}{\frac{\partial T_0}{\partial a}}. \tag{18}$$

The LHS of equation (18) is the net marginal revenue of the mall from the sellers' side, and the RHS is the net marginal cost of the mall on the buyers' side.

Let us define the net marginal revenue of the mall from the sellers' side as

$$NMR_S = 1 + C_S - 2F_S, \tag{19}$$

and the marginal cost of the mall on the buyers' side, as

$$MC_B = \frac{C_B \frac{\partial N_B(.)}{\partial a}}{\frac{\partial T_0}{\partial a}}. \tag{20}$$

Since the solution must satisfy equation (18), and $T_0 = [p^*(\rho, \bar{v}, t, a) - c]Q(.) - F$,

$$F_S = [p^*(\rho, \bar{v}, t, a) - c]Q(.) - F. \tag{21}$$

After substitution of F_S from equation (21), equation (18) solves for $a(C_S, C_B, F, \rho, \bar{v}, t)$. Thus, in equilibrium, the solution of *a* can be expressed as: $a^* = \min\{a(C_S, C_B, F, \rho, \bar{v}, t), 1\}$.

Then substituting $a(C_S, C_B, F, \rho, \bar{v}, t)$ in equation (22) we obtain the equilibrium value of F_S as:

$$F_S = [p^*(\rho, \bar{v}, t, a(C_S, C_B, F, \rho, \bar{v}, t) - c]Q(.) - F \tag{22}$$

Notice that in the equilibrium, the shopping mall charges membership fees to the seller side in such a way that the sellers are pushed to their participation constraint. Each of the sellers in the mall earns $\pi_j^{PL} = 0$. The sellers, with their T_j lying in the range $[T_{0,}1]$ enter the mall. However, the sellers, with their T_j lying in the range $[0, T_0)$ operate outside the mall and earn positive profit.

Lemma 8. a. $\frac{\partial(NMR_S)}{\partial a} < 0$.

b. $\frac{\partial(NMR_S)}{\partial C_S} > 0$; $\frac{\partial(NMR_S)}{\partial C_B} = 0$.

c. $\frac{\partial(NMR_S)}{\partial \rho} > 0, \frac{\partial(NMR_S)}{\partial \bar{v}} > 0, \frac{\partial(NMR_S)}{\partial t} > 0, \frac{\partial(NMR_S)}{\partial F} > 0$.

Proof: See the Appendix A4. □

As a rises, T_0 rises (Lemma 7). As the demand increases, the sellers can charge a high price. Since the shopping mall always chooses $F_S = T_0, F_S$ rises. A smaller number of sellers enter the mall. Therefore, NMR_S falls.

As $\bar{v}$ rises, T_0 falls (Lemma 7) as the demand shrinks, and the sellers have a lower ability to charge a high price. Since the shopping mall always chooses $F_S = T_0, F_S$ falls. More number of number of sellers enter the mall. Therefore, NMR_S rises. The same logic applies to a rise ρ, *t,* and *F*. Similar to Chapter 3, as C_S rises NMR_S also rises while as C_B rises, NMR_S remains unchanged as there is no effect on p^* and $Q(.)$.

Lemma 9. a. $\frac{\partial(MC_B)}{\partial a} > 0$.

b. $\frac{\partial(MC_B)}{\partial C_S} = 0, \frac{\partial(MC_B)}{\partial F} = 0, \frac{\partial(MC_B)}{\partial C_B} > 0$.

c. $\frac{\partial(MC_B)}{\partial \bar{v}} < 0, \frac{\partial(MC_B)}{\partial \rho} < 0, \frac{\partial(MC_B)}{\partial t} < 0$.

Proof: See the Appendix A4. □

The above lemma characterizes the impact of different buyer-side and seller-side parameters on the net marginal cost of servicing the buyers' side. From the sellers' side, the parameters C_S and F do not affect MC_B. While from the buyer's side, it can be seen that the parameters

$\bar{v}, \rho$ and t do have a negative impact on MC_B. And, as expected, MC_B rises with a rise in C_B. A change in $\bar{v}$ affects MC_B in two different ways. The first effect comes from $\frac{\partial N_B}{\partial a}$ which falls as $\bar{v}$ rises, i.e., the change in the number of buyers as a result of a coming closer to 1 falls as $\bar{v}$ rises. So, N_B falls. The second effect comes from a change in $\frac{\partial T_0}{\partial a}$ as a result of a change in $\bar{v}$ which we have assumed to be zero in Assumption 4 for tractability. Therefore, Lemma 9 states that as $\bar{v}$ rises, MR_B falls. Similar is the explanations of the negative effects of the change in ρ and t on MR_B.

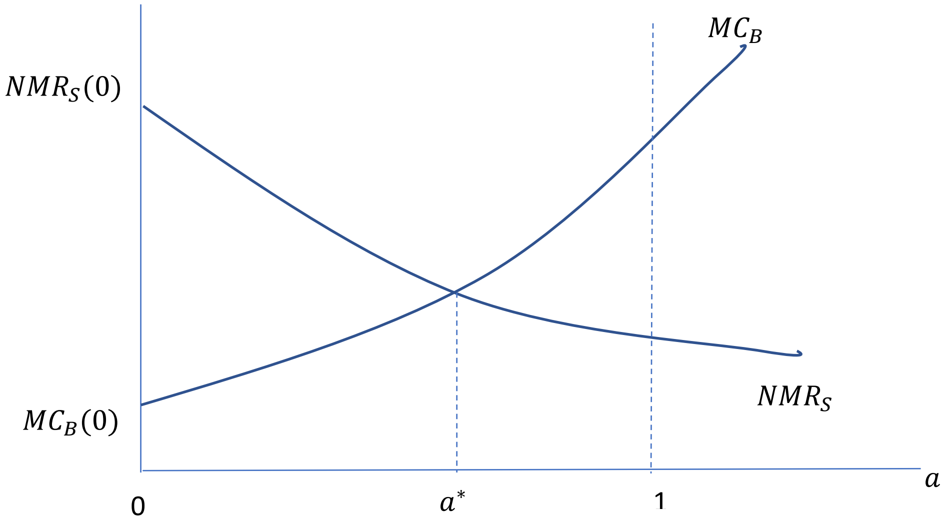

Figure 4.3: Determination of the location of the shopping mall

Figure 4.3 above shows the determination of a in our model. Since by implications of Lemma 8(a) and Lemma 9(a) NMR_S is monotonically decreasing function of a and MC_B is monotonically increasing function of a, in Figure 4.3, NMR_S and MC_B have been drawn as downward-sloping and upward-rising curves, respectively. The condition $NMR_S(0) > MC_B(0)$ ensures that a unique intersection of the two curves exists at a positive value of a that represents the unique interior solution to equation (18). In violation of this condition $a^* = 0$. If $NMR_S(0) > MC_B(0)$, we have a stable equilibrium at $a^* > 0$. The following propositions discuss the implications of the results of Lemmas 7 and 8 on equilibrium.

4.3 Comparative Static Results

Proposition 2: *As the mall's cost of servicing a seller C_S falls (rises),*

(i) the shopping mall chooses a location 'a' away from 1;

(ii) the mall charges a lower (higher) membership fee to the sellers;

(iii) the number of varieties in the shopping mall rises (falls), both the number of varieties sold at the differentiated product market outside the mall and the number of buyers at the differentiated product market falls (rises);

(iv) if $\frac{\partial T_0}{\partial a} < (>) 1$, *the profit of the mall rises (falls);*

Proof: See the Appendix A4. □

Proposition 2 characterizes the behavior of a mall that, other things remaining the same, becomes more efficient in servicing its seller side. A more efficient shopping mall has a lower C_S. As C_S falls, the margin of profit earned from each seller i.e., $(F_S - C_S)$ rises. However, this has a negative effect on NMR_S since this lowers the revenue gain of the mall as sellers enter the mall with a fall in F_S. A lower C_S, in Figure 4.3, shows up in the downward shift of the NMR_S curve. It thus, implies a lower value of NMR_S (from Lemma 8(b)) at the existing a, resulting in $MC_B > NMR_S$ at the initial equilibrium. Therefore, at the new equilibrium, the profit-maximizing mall adjusts a away from the location of the homogeneous product market thereby creating a negative cross-sided externality on the buyer side. As a falls, some of the buyers close to 1 drop out of the differentiated product market given their travel cost to a rises, the number of buyers in the market falls, and the price charged for each variety of the differentiated product falls, T_0 falls, and the mall's ability to charge a high price to the sellers joining the mall falls. It thus lowers F_S to retain them in the mall. Therefore, the number of varieties in the mall rises as some sellers unable to survive outside the mall due to lowered demand move into the mall. There are two opposing effects on the mall's profit. First, on the buyer side, the profit rises as the mall saves costs on servicing the buyers on the mall as fewer buyers participate. Second, on the seller side, the profit may fall or rise as it serves more sellers on the mall at a lower membership fee and incurs a lower servicing cost. If $\frac{\partial T_0}{\partial a} < 1,$ as it chooses the new location away from 1, the membership fee charged to the sellers $F_S = T_0$ does not fall much and hence, the revenue on the seller side may not fall much. Therefore, the fall in cost outweighs the fall in revenue on the seller side. Combining the positive effects on both the buyer and the seller sides, the profit of the mall rises. The opposite happens if $\frac{\partial T_0}{\partial a} > 1$. The profit of the mall falls. Interestingly, Proposition 2 shows that, unlike the standard monopoly case, the efficiency of the mall may not translate into higher profit for it.

Proposition 3: *As the mall's cost of servicing a buyer C_B falls (rises),*

(i) the shopping mall chooses a location 'a' close to 1;

(ii) the mall charges a higher (lower) membership fee to the sellers;

(iii) the number of varieties in the shopping mall falls (rises), both the number of varieties sold at the differentiated product market outside the mall and the number of buyers at the differentiated product market rises (falls);

(iv) if $\frac{\partial T_0}{\partial a} < (>) 1$, the profit of the mall falls (rises);

Proof: See the Appendix A4. □

Proposition 3 characterizes the behavior of a mall that, other things remaining the same, becomes more efficient in servicing its buyer side. A more efficient shopping mall has a lower C_B. A lower C_B implies a lower value of MC_B (from Lemma 9(b)) at the existing a^* resulting in $NMR_S > MC_B$. In Figure 4.3, it shows up in the downward shift of the MC_B curve. Therefore, at the new equilibrium, the profit-maximizing mall adjusts a in the upward direction closer to 1. As a rises, some of the buyers close to 1 who were buying the homogeneous product earlier start buying the differentiated product since their travel cost to a falls and they have 'love for variety', the number of buyers in the market rises, and the price charged for each variety of the differentiated product rises, T_0 rises and the mall's ability to charge a high price to the sellers joining the mall rises. It charges a higher F_S to the sellers. Consequently, the number of varieties sold in the mall falls. More sellers operate outside the shopping mall and earn positive profits. Similar to Proposition 1, there are two opposing effects on the mall's profit. First, on the buyer side, the profit of the mall may rise or fall as it services more number of buyers at a lower cost. Second, on the seller side, the profit may rise or fall as the mall serves lesser number of sellers at a higher membership fee but at the same time incurs a lower cost of servicing as fewer sellers participate. If $\frac{\partial T_0}{\partial a} < 1,$ as it chooses the new location closer to 1, the membership fee charged to the sellers $F_S = T_0$ does not rise much and hence, the revenue on the seller side may not rise much. Therefore, the negative effects on profit from both the buyer and the seller sides dominate the positive effects from both sides. The profit of the mall falls. The opposite happens when $\frac{\partial T_0}{\partial a} > 1$. In fact, with $\frac{\partial T_0}{\partial a} < 1$, the shopping mall will be more efficient in servicing the sellers. While with $\frac{\partial T_0}{\partial a} > 1$, the efficiency incentives reverses with the mall becoming more efficient in servicing the buyers.

Earlier papers in the literature like Brandão et al. (2014), Ushchev et al. (2015), do not discuss the impact of change in the cost of servicing buyers and sellers in the mall on the membership fees charged to them, and the number of varieties sold in the mall.

Proposition 4: *As the fixed costs F of the sellers for participating in the differentiated product market fall (rise)*

(i) the shopping mall chooses a location 'a' away from 1;

(ii) if $\left[Q(.)\frac{\partial p^*}{\partial a} + (p^* - c)\frac{\partial Q(.)}{\partial a}\right]\frac{\partial a}{\partial F} > (< 1)$, *the shopping mall charges a lower (higher) membership fee to the sellers, the number of varieties in the shopping mall rises (falls), the number of varieties sold at the differentiated product market outside the mall falls (rises);*

(iii) the number of buyers in the differentiated product market falls (rises);

(iv) if $\frac{\partial T_0}{\partial a} < (>)1$, *the profit of the mall rises (falls);*

Proof: See the Appendix A4. □

Proposition 4 deduces the shopping mall's behavior if the fixed costs of the seller change because of some infrastructural development that benefits all the suppliers equally. As F falls at the initial equilibrium, each seller starts earning positive profit. Therefore, the mall reacts by increasing F_S. As F_S rises NMR_S falls below MC_B at the initial equilibrium and a falls and moves away from 1. The reason for this is similar to what we have argued in Proposition 1. As a falls, some of the buyers close to 1 drop out of the differentiated product market given their travel costs to a rises, the number of buyers in the market falls. Since the demand falls for every seller in the differentiated product market, each one starts lowering their price to retain buyers. As the price drops, their revenue falls. If $\left[Q(.)\frac{\partial p^*}{\partial a} + (p^* - c)\frac{\partial Q}{\partial a}\right]\frac{\partial a}{\partial F} > 1$, the fall in revenue is more than the fall in the fixed cost, and the negative indirect effect of a fall in the revenue dominates the positive direct effect of a fall in F, reducing the seller's profit. Consequently, the mall lowers F_S to retain the sellers in the mall. Since the membership fee in the mall for the sellers falls, the number of sellers in the mall rises. However, if $\left[Q(.)\frac{\partial p^*}{\partial a} + (p^* - c)\frac{\partial Q}{\partial a}\right]\frac{\partial a}{\partial F} <$

1, the opposite happens. There are two opposing effects on the mall's profit. First, on the buyer side, the profit rises as the mall saves costs on servicing the buyers on the mall as fewer buyers participate. Second, on the seller side, the profit may fall as the mall serves a higher number of sellers at a lower membership fee but at the same time incurs a higher cost of servicing as more sellers participate. If $\frac{\partial T_0}{\partial a} < 1$, as it chooses the new location away from 1, the membership fee charged to the sellers $F_S = T_0$ does not fall much. Thus, the first effect dominates the second effect and the profit of the mall rises. The opposite happens if $\frac{\partial T_0}{\partial a} > 1$. The profit of the mall falls.

Proposition 5: *If the outside option of the buyers improves (worsens) due to a rise (fall) in* $\bar{v}$

(i) the shopping mall chooses a location 'a' close to 1;

(ii) if $\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \bar{v}}\right) > (<) - \frac{\partial T_0}{\partial \bar{v}}$

the mall charges a higher (lower) membership fee to the sellers, and the number of varieties in the shopping mall falls (rises), the number of varieties sold at the differentiated product market outside the mall rises (falls);

(iii) the number of buyers in the differentiated product market falls (rises);

(iv) if $\frac{\partial T_0}{\partial a} < (>) 1$*, the profit of the mall rises (falls);*

Proof: See the Appendix A4. □

As $\bar{v}$ rises, the shopping mall prefers to choose a location close to the homogeneous goods market. The number of buyers in the differentiated market falls, and due to the reduced demand, each seller charges a lower price. The mall does not control the participation of buyers as there is no fee charged from the buyer side. Therefore, to retain the existing sellers in the mall the shopping mall lowers the value of F_S. Consequently MC_B falls, and NMR_S rises. In Figure 4.3, the MC_B curve shifts in the downward direction, and the NMR_S curve shifts in the upward direction, resulting in $NMR_S > MC_B$ at the initial equilibrium. Therefore, the mall chooses the location of a close to 1. As a rises in the new equilibrium, some of the buyers close to 1 who were buying the homogeneous product earlier start buying the differentiated product since their travel cost to a falls and they have 'love for variety'. Therefore, some buyers return to the differentiated product market, and the sellers increase the price to gain revenue. The strength of the direct effect of an improvement of the outside option on F_S in equilibrium and the strength of the indirect effect through location choice on F_S in equilibrium matters. If $\left[\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \bar{v}}\right) > -\frac{\partial T_0}{\partial \bar{v}}\right]$, the revenue gain from the indirect effect of a rise in the number of buyers outweighs the revenue loss from the direct impact of a price rise; therefore, the mall charges a higher F_S to keep the sellers on their participation constraint. If $\left[\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \bar{v}}\right) < -\frac{\partial T_0}{\partial \bar{v}}\right]$, the opposite happens. There are two opposing effects on the mall's profit. First, on the buyer side, the profit rises as the mall saves costs on servicing the buyers on the mall as fewer buyers participate. Second, on the seller side, the profit of the mall may rise or fall as it services a lesser number of sellers by charging a higher membership fee but at the same time incurs a lower cost of servicing as fewer sellers participate. If $\frac{\partial T_0}{\partial a} < 1$, as it chooses the new location close to 1, the membership fee charged to the sellers $F_S = T_0$ does not rise much and hence, the revenue on the seller side may not rise much. Thus, the first effect dominates the second effect. The profit of the mall rises. The opposite happens if $\frac{\partial T_0}{\partial a} > 1$. The profit of the mall falls.

The counter-intuitive aspect of Proposition 5 is that even if the differentiated product market loses its buyers to the homogeneous product market, the mall may raise the membership fee for the sellers joining the mall. Also, there are conditions under which the shops selling the homogeneous product and the number of sellers in the mall grow together. A similar result was obtained in Chapter 2. Adding to it, we also derive conditions when the profit of the shopping mall rises with an improvement in the outside option. In a different setting, Ushchev et al. (2015) with two competing marketplaces show that as the size of the downtown retail market grows the profit of the shopping mall falls. A similar result was found in Shimomura and Thisse (2012). Given our setting, this result is counter-intuitive to the ones in the existing literature.

Proposition 6: *As the substitutability among the varieties of the differentiated product rises (falls) as ρ rises (falls)*

(i) the shopping mall chooses a location 'a' close to 1;

(ii) If $\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \rho}\right) > (<) - \frac{\partial T_0}{\partial \rho}$

the mall charges a higher (lower) membership fee to the sellers, and the number of varieties in the shopping mall falls (rises), the number of varieties sold at the differentiated product market outside the mall rises (falls);

(iii) the number of buyers in the differentiated product market falls (rises)

(iv) if $\frac{\partial T_0}{\partial a} < (>) 1$*, the profit of the mall rises (falls);*

Proof: See the Appendix A4. □

As ρ rises, the substitutability of the varieties of the differentiated product falls. Therefore, the marginal buyer's indirect utility in purchasing from the differentiated product falls, and she exits the market. The number of buyers in the differentiated market falls, and due to the reduced demand, each seller starts charging a lower price. The shopping mall lowers F_S. In Figure 4.3, like the case of a rise in $\bar{v}$, the MC_B curve shifts in the downward direction, and the NMR_S curve shifts in the upward direction, resulting in $NMR_S > MC_B$ at the initial equilibrium. Therefore, the mall chooses the location of a close to 1. As a rises in the new equilibrium, some of the buyers close to 1 who were buying the homogeneous product earlier start buying the differentiated product since their travel cost to a falls and they have 'love for variety'. As a result, some buyers return to the differentiated product market, and the sellers increase the price to gain revenue. If $\left[\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \rho}\right) > -\frac{\partial T_0}{\partial \rho}\right]$, the revenue gain from the indirect effect of a rise in the number of buyers outweigh the revenue loss from the direct impact of a price rise; therefore, the mall charges a higher F_S to keep the sellers on their participation constraint. If $\left[\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \rho}\right) < -\frac{\partial T_0}{\partial \rho}\right]$, the opposite happens. There are two opposing effects on the mall's profit. First, on the buyer side, the profit rises as the mall saves costs on servicing the buyers on the mall as fewer buyers participate. Second, on the seller side, the profit of the mall may rise or fall as it services lesser number of sellers by charging a higher membership fee but at the same time incurs a lower cost of servicing as fewer sellers participate. If $\frac{\partial T_0}{\partial a} < 1$, as it chooses the new location close to 1, the membership fee charged to the sellers $F_S = T_0$ does not rise much and hence, the revenue on the seller side may not rise much. Thus, the first effect dominates the second effect. The profit of the mall rises. The opposite happens if $\frac{\partial T_0}{\partial a} > 1$. The profit of the mall falls. The effect on the profit of the mall is similar to Proposition 5.

According to Ushchev et al. (2015), an increase in the substitutability among the varieties of the differentiated product, some being sold in the downtown retail and some being sold in the shopping mall, makes both marketplaces equally attractive only for those consumers buying from either of the extremes. However, in our chapter, as the marketplaces selling homogeneous goods vis-à-vis differentiated products are considered, such a comparison does not arise. Rather, we see that the number of varieties in the mall may rise as the substitutability among the differentiated product rises.

Proposition 7: *If the travel cost of the buyer rises (falls) through a rise (fall) in* t

(i) the shopping mall chooses a location 'a' closer to 1;

(ii) If $\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial t}\right) > (<) -\frac{\partial T_0}{\partial t}$

the mall charges a higher (lower) membership fee to the sellers, and the number of varieties in the shopping mall falls (rises), the number of varieties sold at the differentiated product market outside the mall rises (falls);

(iii) the number of buyers in the differentiated product market falls (rises)

(iv) if $\frac{\partial T_0}{\partial a} < (>) 1$*, the profit of the mall rises (falls);*

Proof: See the Appendix A4. □

As t rises, MC_B falls (from Lemma 9), NMR_S rises (from Lemma 8). Therefore, at the initial choice of location of the mall, $NMR_S > MC_B$. The mall changes its location closer to 1. Consequently, the number of buyers at the mall increases. As the travel cost to the malls falls, some buyers close to 1 who were buying the homogeneous product earlier start buying the differentiated product. If $\left[\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial t}\right) > -\frac{\partial T_0}{\partial t}\right]$, the revenue gain from the indirect effect of a rise in the number of buyers outweighs the revenue loss from the direct impact of a price rise; therefore, the mall charges a higher F_S to keep the sellers on their participation constraint. If $\left[\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial t}\right) < -\frac{\partial T_0}{\partial t}\right]$, the opposite happens. Similar to Propositions 5 and 6, the mall's profit is affected by this in two opposite effects. First, as the number of buyers in the mall reduces, the cost of servicing the buyers falls, increasing the mall's profit. Second, as the mall charges a higher membership fee to a lower number of sellers, its revenue from the sellers rises, but the cost of servicing the sellers falls. It earns a higher profit. If $\frac{\partial T_0}{\partial a} < 1$, the membership fee does not rise much as a rises. Therefore, the first effect dominates the second effect, and the mall's profit rises. The opposite happens if $\frac{\partial T_0}{\partial a} > 1$. The profit of the mall falls.

Ushchev et al. (2015) in their paper show that travel costs may lead to a rise or fall in the profit of the shops located in the two different marketplaces, viz. downtown retailers and shopping mall under different conditions on the exogenously defined size of the mall. We also establish that under certain conditions defined on the revenue and the cost side of the mall, as discussed in Proposition 7, with a rise in the travel costs the profit of the sellers in the mall rises or falls. Additionally, we show how the location choice and profit of the mall respond to changes in travel costs. Interestingly, the impact of $\bar{v}, \rho$ and t on $\bar{x}(\cdot)$ (Lemma 4a), Q_j (Lemma 5) and p^* (Lemma 6) and subsequently on T_0, NMR_S and MC_B is very similar and hence the results in Propositions 5, 6, and 7 are also similar.

4.4 Conclusions

The model presented in the chapter involves a monopoly shopping mall selling differentiated products and a homogeneous goods market in a linear city framework combining spatial competition and 'love for variety' preferences on the buyer side. The location of the homogeneous goods market is fixed on one end of the linear city, while the shopping mall chooses its location and a membership fee on the seller's side. The mall does not charge the buyers. The model combines several dimensions like spatial competition, 'love for variety' preferences on the buyer side, intensive and extensive margins of demand, travel costs, and Bertrand-price competition among the sellers participating in the differentiated product market in a single framework. Interestingly, the mall's location choice has a role in determining the prices of the varieties through Bertrand-price competition. The location of the shopping mall, the membership fee charged to the sellers, and the number of varieties sold in the mall also depend on factors like the transport costs incurred by the buyers for traveling to the mall, outside options of the buyers, and the substitutability of the varieties of the product.

The number of varieties sold on and off the mall is endogenously determined in the model. In the presence of a mall, the number of varieties produced of a differentiated product increases compared to a 'no-mall' situation. As already discussed, in all the comparative static exercises, the mall's choice of location in the linear city is of great significance. As the costs of servicing the sellers fall, at the initial equilibrium, the net marginal cost of the mall on the buyers' side is relatively higher than the net marginal revenue of the mall from the sellers' side. Therefore, at the new equilibrium, the mall adjusts its location away from the location of the homogeneous product market, given its revenue-cost considerations. This creates a negative cross-sided externality, common to a monopoly platform, on the buyer side. The adjustment in the location of the mall determines the membership fee on the seller side and the prices of varieties. The results show that if the costs of servicing the sellers fall, the mall charges a lower membership fee on the seller side.

In contrast, a fall in the cost of servicing buyers leads to a rise in the membership fee charged on the seller side. The number of varieties on the platform rises with a fall in the membership fee on the seller side. The opposite happens if the costs of servicing the buyers fall. Interestingly, like the standard monopoly case, under certain conditions on the shopping mall's revenue and cost sides, the mall has an incentive to become efficient in servicing both the buyers and the sellers. If the sellers have a lower fixed cost, they may face a higher membership fee if the shopping mall expects them to gain due to the lowering of the fixed cost. The number of varieties sold in the mall accordingly depends on the choice of location of the mall and the membership fee charged to the sellers. It may prefer developments that lower the seller's fixed costs depending on the choice of its location and the consequent impact on its profit. In the case of an improved outside option, the results suggest that even if the differentiated product market loses its buyers to the homogeneous goods market, the mall may raise the membership fee for the sellers contrary to standard spatial competition models where location is exogenously fixed. Also, there are certain conditions under which the homogeneous goods market and the number of varieties in the mall grow together. In the context of antitrust regulations, this result can be helpful for the policymakers who think that large shopping malls like Wal-Mart or METRO Cash & Carry will encroach on the small-scale retail space. If the varieties of differentiated products are close substitutes, the mall's choice of location may raise the number of varieties in the mall. Similar results concerning the number of varieties sold in the mall and the mall's profit can be noticed in the case of a rise in the transport costs incurred by the buyers for traveling to the mall. Interestingly, under certain conditions on the revenue and cost sides of the shopping mall, the profit of the shopping mall may rise with changes in the travel costs on the buyer side, improvement in the outside option, and substitutability among varieties.

The chapter contributes to the existing literature in multiple ways: First, it combines spatial competition and 'love for variety' preferences to study the interaction between a shopping mall selling differentiated products and a homogeneous goods market in a linear city where the sellers in the differentiated product market engage in Bertrand-price competition; Second, the mall chooses not only the membership fees it charges from the participating sellers but also its location in the linear city. Third, the representative agent framework used in standard monopolistic competition models, assuming away price interactions, is replaced by introducing heterogeneity on the buyer side with varying preferences for purchasing the differentiated product and on the seller side with their varying transaction costs. Fourth, it allows endogenous determination of the number of varieties of a differentiated product in and outside the mall and shows that the mall benefits the buyers by increasing the number of varieties available at the market. Fifth, it characterizes the mall's choice of location on the linear city and pricing on the seller side of the market for changes in costs to servicing to the clients, changes in fixed costs on the sellers' side, improved outside option of the buyers, lower substitutability of the varieties at the market, and the transport costs incurred by the buyers for traveling to the mall. We highlight the impact of the mall's ability to choose its location when analyzing the impact of these parametric changes on the mall's profit.

There are some limitations in the model. First, the model has fixed the location of the homogeneous good market and does not explicitly model it and its location choice. Second, it does not explicitly model the sellers' location outside the mall and is assumed to be at the same place as the mall. Third, it focuses only on the mall's membership fee decision and does not consider its per-unit/per-slot fee decisions. The simplifying assumptions have kept the model tractable. To generalize the results, we plan to relax some of these assumptions in future extensions. Given that the sellers outside the mall can join another shopping mall, our model can be used to study the competition between shopping malls. Another extension can be introducing a Salop-type circular city instead of a linear Hotelling-type town. Exploring the impact of population density in the linear city, alternative functional forms of transport costs, and the issue of regulating the mall also remains a part of our future research works.

Appendix A4

Proof of Lemma 1.

If $q_j > 0$, equation (3) is satisfied with equality. Taking the total differential from equation (3) and collecting the terms:

$$\frac{\partial q_j}{\partial p_j} = -\frac{\frac{\partial^2 \varphi_x}{\partial p_j \partial q_j}}{\frac{\partial^2 \emptyset_x}{\partial q_j{}^2}}. \tag{A.1}$$

On the RHS of the above expression,

$\frac{\partial^2 \varphi_\mu}{\partial p_j \partial q_j} = -\left(1 - \left[\beta\left(\frac{\beta-\rho}{\rho}\right)\left[\left(\sum_{j=1}^{N_S} q_j{}^\rho\right)\right]^{\frac{\beta-1}{\rho}-1} q_j{}^{\rho-1} \sum_{i\neq j} \rho q_i{}^{\rho-1} \frac{\partial q_i}{\partial p_j}\right]\right) < 0$ since $\frac{\partial q_i}{\partial p_j} > 0$ because of the demand-substitutability between any two varieties and $\beta < \rho$. Since $\frac{\partial^2 \emptyset_\mu}{\partial q_j{}^2} < 0$ by the second order condition, the first part of the lemma follows.

Similarly, $\frac{\partial q_j}{\partial \rho} = \frac{\beta k^\beta\left[-\frac{\log\left(\sum_{j=1}^{N_S} q_j{}^\rho\right)}{\rho^2} + q_j{}^\rho \log q_j\right]}{\frac{\partial^2 \emptyset_\mu}{\partial q_j{}^2}} < 0.$

where, the term in the numerator is > 0 and $\frac{\partial^2 \emptyset_\mu}{\partial q_j{}^2} < 0$ follows from the second order condition.

The statement of the lemma follows. □

Proof of Lemma 2.

From equation (7) and by use of the definition of k :

$$\frac{\partial \bar{x}(\cdot)}{\partial p_j} = -\frac{q_j}{2t(1-a)} \tag{A.2}$$

The statement of the first part of the lemma follows from the fact that $q_j > 0$ and $t > 0$. □

From equation (5), $\frac{\partial N_B}{\partial p_j} = \frac{\partial \bar{x}}{\partial p_j}$. (A.3)

Since $\frac{\partial \bar{x}}{\partial p_j} < 0$ from the first part of the lemma, the RHS of (A.3) is negative.

The statement of the lemma follows. □

Proof of Lemma 3.

By application of Leibniz rule, from equation (9):

$$\frac{\partial Q_j}{\partial p_j} = \left[Q_j \frac{\partial \bar{x}(\cdot)}{\partial p_j}\right] + \left[\int_0^{\bar{x}(\cdot)} \frac{\partial q_j(\cdot)}{\partial p_j} dx\right]. \tag{A.4}$$

Since $Q_j > 0$, the statement of the lemma follows from equation (A.4) by application of Lemma 1 and 2. □

Proof of Lemma 4.

(a) From equation (7) it follows:

$$\frac{\partial \bar{x}}{\partial \rho} = -\frac{1}{2(1-a)}\left[\left(\frac{N_S p_j}{t}\right)\left(\frac{q_j \beta log N_S}{\rho^2(1-\beta)}\right) + \left(\frac{\beta k^{\beta}}{tk}\right) q_j N_S^{\frac{1}{\rho}} \log N_S \left[\frac{1}{\rho^2(1-\beta)}\right]\right] < 0 \because \frac{\partial N_S}{\partial \rho} = 0 \text{ given } N_S.$$

$\frac{\partial \bar{x}}{\partial \bar{v}} = -\frac{1}{2t(1-a)}$. Since $t > 0$, the RHS of the above equations is negative.

$$\frac{\partial \bar{x}}{\partial t} = -\frac{I - \sum_{j=1}^{N_S} p_j \, q_j + k^{\beta} - \bar{v}}{2t^2(1-a)}$$

$$\frac{\partial \bar{x}}{\partial a} = \frac{1}{2} + \frac{I - \sum_{j=1}^{N_S} p_j \, q_j + k^{\beta} - \bar{v}}{2t(1-a)^2}$$

The statement follows.

(b) From equation (8):

$$\frac{\partial N_B}{\partial \rho} = \frac{\partial \bar{x}}{\partial \rho}. \tag{A.5}$$

$$\frac{\partial N_B}{\partial \bar{v}} = \frac{\partial \bar{x}}{\partial \bar{v}}. \tag{A.6}$$

$$\frac{\partial N_B}{\partial t} = \frac{\partial \bar{x}}{\partial t}. \tag{A.7}$$

$$\frac{\partial N_B}{\partial a} = \frac{\partial \bar{x}}{\partial a}. \tag{A.8}$$

Since $\frac{\partial \bar{x}}{\partial \rho} < 0, \frac{\partial \bar{x}}{\partial \bar{v}} < 0, \frac{\partial \bar{x}}{\partial t} < 0$ from the first part of the lemma, the RHS of (A.5), (A.6), (A.7) is negative while the RHS of (A.8) is positive as $\frac{\partial \bar{x}}{\partial a} > 0$. The statement of the lemma follows. □

Proof of Lemma 5.

By application of Leibniz rule, from equation (9):

$$\frac{\partial Q_j}{\partial \rho} = \left[Q_j \frac{\partial \bar{x}(\cdot)}{\partial \rho}\right] + \left[\int_0^{\bar{x}(\cdot)} \frac{\partial q_j(\cdot)}{\partial \rho} dx\right]. \tag{A.9}$$

$$\frac{\partial Q_j}{\partial \bar{v}} = \left[Q_j \frac{\partial \bar{x}(\cdot)}{\partial \bar{v}}\right] + \left[\int_0^{\bar{x}(\cdot)} \frac{\partial q_j(\cdot)}{\partial \bar{v}} dx\right]. \tag{A.10}$$

$$\frac{\partial Q_j}{\partial t} = \left[Q_j \frac{\partial \bar{x}(\cdot)}{\partial t}\right] + \left[\int_0^{\bar{x}(\cdot)} \frac{\partial q_j(\cdot)}{\partial t} dx\right]. \tag{A.11}$$

$$\frac{\partial Q_j}{\partial a} = \left[Q_j \frac{\partial \bar{x}(\cdot)}{\partial a}\right] + \left[\int_0^{\bar{x}(\cdot)} \frac{\partial q_j(\cdot)}{\partial a} dx\right]. \tag{A.12}$$

Since $Q_j > 0$, $\frac{\partial \bar{x}}{\partial \rho}, \frac{\partial \bar{x}}{\partial \bar{v}}$, and $\frac{\partial \bar{x}(\cdot)}{\partial t}$, is negative from Lemma 4(a), and $\frac{\partial q_j}{\partial \bar{v}} = 0$, $\frac{\partial q_j}{\partial t} = 0$, $\frac{\partial q_j}{\partial \rho} < 0$, (from Lemma 1) the RHS of (A.9), (A.10) and (A.11) is negative. While the RHS of (A.12) is positive as $\frac{\partial \bar{x}(\cdot)}{\partial a}$ is positive and $\frac{\partial q_j(\cdot)}{\partial a} = 0$.

Therefore, the statement of the lemma follows. □

Proof of Lemma 6.

Evaluating equation (10) at $p^*(\rho, \bar{v}, t, a, c)$, for the marginal seller we have

$$\frac{\partial \pi}{\partial p^*}(p^*, \bar{v}, .) = 0$$

$$\Longrightarrow \frac{\partial^2 \pi}{\partial p^{*2}} . \frac{\partial p^*}{\partial \bar{v}} + \frac{\partial}{\partial \bar{v}}\left(\frac{\partial \pi}{\partial p^*}\right) = 0$$

$$\Longrightarrow \frac{\partial p^*}{\partial \bar{v}} = -\frac{\frac{\partial}{\partial \bar{v}}\left(\frac{\partial \pi}{\partial p^*}\right)}{\frac{\partial^2 \pi}{\partial p^{*2}}}$$

where $\frac{\partial^2 \pi}{\partial p^{*2}} < 0$ from the second order condition of profit maximization and, $\frac{\partial}{\partial \bar{v}}\left(\frac{\partial \pi}{\partial p^*}\right) = [p^* - c]\frac{\partial}{\partial \bar{v}}\left[\frac{\partial Q(.)}{\partial p^*}\right] + \frac{\partial Q(.)}{\partial \bar{v}}$. Now, $\frac{\partial Q(.)}{\partial \bar{v}} < 0$ from Lemma 5. Now, given Assumption 2, $\frac{\partial}{\partial \bar{v}}\left(\frac{\partial \pi}{\partial p^*}\right) < 0$ as $\frac{\partial Q(.)}{\partial \bar{v}} < 0$ from Lemma 5.

Therefore, $sign\ \left[\frac{\partial p^*}{\partial \bar{v}}\right] = sign\ \left[\frac{\partial}{\partial \bar{v}}\left(\frac{\partial \pi}{\partial p^*}\right)\right]$.

Similarly,

$$\frac{\partial p^*}{\partial \rho} = -\frac{\frac{\partial}{\partial \rho}\left(\frac{\partial \pi}{\partial p^*}\right)}{\frac{\partial^2 \pi}{\partial p^{*2}}}$$

where $\frac{\partial^2 \pi}{\partial p^{*2}} < 0$ from the second order condition of profit maximization and, $\frac{\partial}{\partial \rho}\left(\frac{\partial \pi}{\partial p^*}\right) = [p^* - c]\frac{\partial}{\partial \rho}\left[\frac{\partial Q(.)}{\partial p^*}\right] + \frac{\partial Q(.)}{\partial \rho}$. Now $\frac{\partial Q(.)}{\partial \rho} < 0$ from Lemma 5 and given Assumption 2, $\frac{\partial}{\partial \rho}\left(\frac{\partial \pi}{\partial p^*}\right) < 0$.

Therefore, $sign\ \left[\frac{\partial p^*}{\partial \rho}\right] = sign\ \left[\frac{\partial}{\partial \rho}\left(\frac{\partial \pi}{\partial p^*}\right)\right]$.

And,

$$\frac{\partial p^*}{\partial t} = -\frac{\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial p^*}\right)}{\frac{\partial^2 \pi}{\partial p^{*2}}}$$

where $\frac{\partial^2 \pi}{\partial p^{*2}} < 0$ from the second order condition of profit maximization and, $\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial p^*}\right) = [p^* - c]\frac{\partial}{\partial t}\left[\frac{\partial Q(.)}{\partial p^*}\right] + \frac{\partial Q(.)}{\partial t}$. Now, $\frac{\partial Q(.)}{\partial t} < 0$ from Lemma 5 and given Assumption 2, $\frac{\partial}{\partial t}\left[\frac{\partial Q(.)}{\partial p^*}\right] < 0$.

Therefore, $sign\ \left[\frac{\partial p^*}{\partial t}\right] = sign\ \left[\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial p^*}\right)\right]$.

Similarly,

$$\frac{\partial p^*}{\partial a} = -\frac{\frac{\partial}{\partial a}\left(\frac{\partial \pi}{\partial p^*}\right)}{\frac{\partial^2 \pi}{\partial p^{*2}}}$$

where $\frac{\partial^2 \pi}{\partial p^{*2}} < 0$ from the second order condition of profit maximization and, $\frac{\partial}{\partial a}\left(\frac{\partial \pi}{\partial p^*}\right) = [p^* - c]\frac{\partial}{\partial a}\left[\frac{\partial Q(.)}{\partial p^*}\right] + \frac{\partial Q(.)}{\partial a}$. From Assumption 2 and $\frac{\partial Q(.)}{\partial a} > 0$ from Lemma 5, $\frac{\partial}{\partial a}\left(\frac{\partial \pi}{\partial p^*}\right) > 0$.

Therefore, $sign\ \left[\frac{\partial p^*}{\partial a}\right] = sign\ \left[\frac{\partial}{\partial a}\left(\frac{\partial \pi}{\partial p^*}\right)\right]$.

Therefore, the statement of the lemma follows. □

Proof of Lemma 7.

$\frac{\partial T_0}{\partial \rho} = \left[Q(.)\frac{\partial p^*(.)}{\partial \rho}\right] + \left[(p^*(\rho, \bar{v}, t, a) - c)\frac{\partial Q(.)}{\partial \rho}\right] < 0$

$\frac{\partial p^*(.)}{\partial \rho} < 0$ from Lemma 6 and $\frac{\partial Q(.)}{\partial \rho} < 0$ from Lemma 5.

Similarly, $\frac{\partial T_0}{\partial \bar{v}} = \left[Q(.)\frac{\partial p^*(.)}{\partial \bar{v}}\right] + \left[(p^*(\rho, \bar{v}, t, a) - c)\frac{\partial Q(.)}{\partial \bar{v}}\right] < 0$

which follows from $\frac{\partial p^*(.)}{\partial \bar{v}} < 0$ from Lemma 6 and $\frac{\partial Q(.)}{\partial \bar{v}} < 0$ from Lemma 5.

And,

$\frac{\partial T_0}{\partial t} = \left[Q(.)\frac{\partial p^*(.)}{\partial t}\right] + \left[(p^*(\rho, \bar{v}, t, a) - c)\frac{\partial Q(.)}{\partial t}\right] < 0$

as $\frac{\partial p^*(.)}{\partial t} < 0$ from Lemma 6 and $\frac{\partial Q(.)}{\partial t} < 0$ from Lemma 5.

$\frac{\partial T_0}{\partial a} = \left[Q(.)\frac{\partial p^*(.)}{\partial a}\right] + \left[(p^*(\rho, \bar{v}, t, a) - c)\frac{\partial Q(.)}{\partial a}\right] > 0$

$\frac{\partial p^*(.)}{\partial a} > 0$ from Lemma 6 and $\frac{\partial Q(.)}{\partial a} > 0$ from Lemma 5.

Also, $\frac{\partial T_0}{\partial F} = -1$ follows from the definition of T_0.

The statement follows. □

Proof of Lemma 8.

a. From equation (19): $\frac{\partial (NMR_S)}{\partial a} = -2\frac{\partial T_0}{\partial a}$.

Since $\frac{\partial T_0}{\partial a} > 0$ from Assumption 3, $\frac{\partial (NMR_S)}{\partial a} < 0$.

b. From equation (19):

$\frac{\partial (NMR_S)}{\partial C_S} = 1, \ \frac{\partial (NMR_S)}{\partial C_B} = 0$

c. $\frac{\partial (NMR_S)}{\partial F} = -2\frac{\partial T_0}{\partial F} = 2$ follows from Lemma 7.

Similarly, one can derive for $\frac{\partial (NMR_S)}{\partial \rho} = -2\frac{\partial T_0}{\partial \rho}$, $\frac{\partial (NMR_S)}{\partial \bar{v}} = -2\frac{\partial T_0}{\partial \bar{v}}$,

and, $\frac{\partial (NMR_S)}{\partial t} = -2\frac{\partial T_0}{\partial t}$.

The statement of the lemma follows. □

Proof of Lemma 9.

From equation (20):

a. $\frac{\partial(MC_B)}{\partial a} = \frac{\left(\frac{\partial T_0}{\partial a}\right)\left[C_B.\frac{\partial}{\partial a}\left(\frac{\partial N_B(.)}{\partial a}\right)\right]-\left(C_B\frac{\partial N_B(.)}{\partial a}.\frac{\partial^2 T_0}{\partial a^2}\right)}{\left(\frac{\partial T_0}{\partial a}\right)^2} > 0$. It follows from Assumption 4 that $\frac{\partial^2 T_0}{\partial a^2} \approx 0$

and since $\frac{\partial}{\partial a}\left(\frac{\partial N_B(.)}{\partial a}\right) = \frac{I - \sum_{j=1}^{N_S} p_j\, q_j + k^{\beta} - \bar{v}}{t(1-a)^3} > 0$ from Lemma 4(b).

b. $\frac{\partial(MC_B)}{\partial C_S} = 0, \frac{\partial(NMC_B)}{\partial F} = 0, \frac{\partial(NMC_B)}{\partial C_B} = \frac{\frac{\partial N_B(.)}{\partial a}}{\frac{\partial T_0}{\partial a}} > 0.$

c. $\frac{\partial(MC_B)}{\partial \rho} = \frac{\left(\frac{\partial T_0}{\partial a}\right)\left[C_B.\frac{\partial}{\partial \rho}\left(\frac{\partial N_B(.)}{\partial a}\right)\right]-\left(C_B\frac{\partial N_B(.)}{\partial a}.\frac{\partial^2 T_0}{\partial \rho \partial a}\right)}{\left(\frac{\partial T_0}{\partial a}\right)^2}$. It follows from Assumption 4 that $\frac{\partial^2 T_0}{\partial \rho \partial a} \approx 0$ and

since $\frac{\partial}{\partial \rho}\left(\frac{\partial N_B(.)}{\partial a}\right) = -\frac{-\sum q_j \frac{\partial p_j}{\partial \rho} + \left(\frac{\beta(1-a)}{\rho}\left[\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\beta-1}\left(\sum_{j=1}^{N_S} q_j{}^{\rho}\right)^{\frac{1-\rho}{\rho}} \sum q_j{}^{\rho} log q_j\right) + 2\left[I - \sum_{j=1}^{N_S} p_j\, q_j + k^{\beta} - \bar{v}\right]}{2t(1-a)^3} < 0$

from equation (3), the second part of Lemma 1 and Lemma 4(b).

Similarly,

$\frac{\partial(MC_B)}{\partial \bar{v}} = \frac{\left(\frac{\partial T_0}{\partial a}\right)\left[C_B.\frac{\partial}{\partial \bar{v}}\left(\frac{\partial N_B(.)}{\partial a}\right)\right]-\left(C_B\frac{\partial N_B(.)}{\partial a}.\frac{\partial^2 T_0}{\partial \bar{v} \partial a}\right)}{\left(\frac{\partial T_0}{\partial a}\right)^2} < 0$ as $\frac{\partial}{\partial \bar{v}}\left(\frac{\partial N_B(.)}{\partial a}\right) = \frac{-1}{2t(1-a)^2} < 0$ from Lemma 4(b) and

given $\frac{\partial^2 T_0}{\partial \bar{v} \partial a} \approx 0$ from Assumption 4.

$\frac{\partial(MC_B)}{\partial t} = \frac{\left(\frac{\partial T_0}{\partial a}\right)\left[C_B.\frac{\partial}{\partial t}\left(\frac{\partial N_B(.)}{\partial a}\right)\right]-\left(C_B\frac{\partial N_B(.)}{\partial a}.\frac{\partial^2 T_0}{\partial t \partial a}\right)}{\left(\frac{\partial T_0}{\partial a}\right)^2} < 0$ as $\frac{\partial}{\partial t}\left(\frac{\partial N_B(.)}{\partial a}\right) = -\frac{I - \sum_{j=1}^{N_S} p_j\, q_j + k^{\beta} - \bar{v}}{2t^2(1-a)^2} < 0$ from

Lemma 4(b) and given $\frac{\partial^2 T_0}{\partial \rho \partial a} \approx 0$ from Assumption 4.

The statement of the lemma follows. □

Proof of Proposition 2:

(i) From equation (18):

$$\frac{\partial NMR_S}{\partial C_S} + \frac{\partial NMR_S}{\partial a}\frac{\partial a}{\partial C_S} = \frac{\partial MC_B}{\partial a}\frac{\partial a}{\partial C_S}.$$

It follows from the above equation that

$$\frac{\partial a}{\partial C_S} = \frac{-\frac{\partial NMR_S}{\partial C_S}}{\frac{\partial NMR_S}{\partial a} - \frac{\partial MC_B}{\partial a}}. \tag{A.13}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial a} < 0$ and $\frac{\partial MC_B}{\partial a} > 0$ respectively. Therefore, the denominator of the term on the RHS of (A.13) is negative. From Lemma 8(b), $\frac{\partial (NMR_S)}{\partial C_S} > 0$. Therefore, the term on the RHS of (A.13) is positive and it follows that $\frac{\partial a}{\partial C_S} > 0$.

Again, from equation (22):

$$\frac{\partial F_S}{\partial C_S} = \left\{Q(.)\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial C_S} + (p^* - c)\frac{\partial Q(.)}{\partial a}\frac{\partial a}{\partial C_S}\right\}. \tag{A.14}$$

Since $\frac{\partial p^*}{\partial a} > 0$ from Lemma 6, $\frac{\partial Q_j}{\partial a} > 0$ Lemma 5, and $\frac{\partial a}{\partial C_S} > 0$ as proved above, the RHS of (A.14) is positive. Therefore, $\frac{\partial F_S}{\partial C_S} > 0$.

(ii) From equation (13):

$$\frac{\partial \tilde{N}_S}{\partial C_S} = -\left(\frac{\partial F_S}{\partial C_S}\right). \tag{A.15}$$

Since $\frac{\partial F_S}{\partial C_S} > 0$ from the first part of the proposition, the RHS of (A.15) is negative. Therefore, $\frac{\partial \tilde{N}_S}{\partial C_S} < 0$. Again, from equation (13):

$$\frac{\partial N_S}{\partial C_S} = \left(\frac{\partial T_0}{\partial a}\frac{\partial a}{\partial C_S}\right). \tag{A.16}$$

Since $\frac{\partial T_0}{\partial a} > 0$ from Lemma 7 and $\frac{\partial a}{\partial C_S} > 0$ from the first part of the proposition. Therefore, the RHS of (A.16) is positive and the LHS is also positive. Now, from equation (7):

$$\frac{\partial N_B}{\partial C_S} = \frac{\partial \bar{x}}{\partial a} \frac{\partial a}{\partial C_S}. \tag{A.17}$$

Since from Lemma 4(a), $\frac{\partial \bar{x}}{\partial a} > 0$ and from the first part of the proposition $\frac{\partial a}{\partial C_S} > 0$, the RHS of (A.17) is positive. Therefore, $\frac{\partial N_B}{\partial C_S} > 0$.

(iii) From equation (14) at the equilibrium:

$$\frac{\partial \tilde{\pi}}{\partial C_S} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial a} \frac{\partial a}{\partial C_S} + \frac{\partial[(-C_B)N_B]}{\partial a} \frac{\partial a}{\partial C_S},$$

using the definition of NMR_S and MC_B from equations (19) and (20), which can be written as:

$$\frac{\partial \tilde{\pi}}{\partial C_S} = \frac{\partial a}{\partial C_S}[NMR_S - MC_B . \frac{\partial T_0}{\partial a}]. \tag{A.18}$$

Since $NMR_S = MC_B$ from equation (18), (A.18) is written as:

$$\frac{\partial \tilde{\pi}}{\partial C_S} = \frac{\partial a}{\partial C_S} NMR_S[1 - \frac{\partial T_0}{\partial a}]. \tag{A.19}$$

Since $\frac{\partial a}{\partial C_S} > 0$, $NMR_S > 0$, and $\frac{\partial T_0}{\partial a} > 0$, the RHS of (A.19) will depend on $sign\left(1 - \frac{\partial T_0}{\partial a}\right)$.

Therefore, $sign\left(\frac{\partial \tilde{\pi}}{\partial C_S}\right) = sign\left(1 - \frac{\partial T_0}{\partial a}\right)$. □

Proof of Proposition 3:

(i) From equation (18):

$$\frac{\partial NMR_S}{\partial a} \frac{\partial a}{\partial C_B} = \frac{\partial MC_B}{\partial a} \frac{\partial a}{\partial C_B} + \frac{\partial MC_B}{\partial C_B}.$$

which implies,

$$\frac{\partial a}{\partial C_B} = \frac{\frac{\partial MC_B}{\partial C_B}}{\frac{\partial NMR_S}{\partial a} - \frac{\partial MC_B}{\partial a}}. \tag{A.20}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial a} < 0$ and $\frac{\partial MC_B}{\partial a} > 0$ respectively. Therefore, the denominator of the term on the RHS of (A.20) is negative. From Lemma 9(b), $\frac{\partial MC_B}{\partial C_B} > 0$. Therefore, the term on the RHS of (A.20) is negative and it follows that $\frac{\partial a}{\partial C_B} < 0$.

Again, from equation (22):

$$\frac{\partial F_S}{\partial C_B} = \left\{Q(.)\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial C_B} + (p^* - c)\frac{\partial Q(.)}{\partial a}\frac{\partial a}{\partial C_B}\right\}. \tag{A.21}$$

Since $\frac{\partial p^*}{\partial a} > 0$ from Lemma 6, $\frac{\partial Q_j}{\partial a} > 0$ Lemma 5 and $\frac{\partial a}{\partial C_B} < 0$ as proved above, the RHS of (A.21) is negative. Therefore, $\frac{\partial F_S}{\partial C_B} < 0$.

(ii) From equation (13):

$$\frac{\partial \tilde{N}_S}{\partial C_B} = -\left(\frac{\partial F_S}{\partial C_B}\right). \tag{A.22}$$

Since $\frac{\partial F_S}{\partial C_B} < 0$ from the first part of the proposition, the RHS of (A.22) is positive. Therefore, $\frac{\partial \tilde{N}_S}{\partial C_B} > 0$.

Again, from equation (13):

$$\frac{\partial N_S}{\partial C_B} = \left(\frac{\partial T_0}{\partial a}\frac{\partial a}{\partial C_B}\right). \tag{A.23}$$

Since $\frac{\partial T_0}{\partial a} > 0$ from Lemma 7 and $\frac{\partial a}{\partial C_B} < 0$ from the first part of the proposition. Therefore, the RHS of (A.23) is negative and the LHS is also negative.

Now, from equation (7):

$$\frac{\partial N_B}{\partial C_B} = \frac{\partial \bar{x}}{\partial a}\frac{\partial a}{\partial C_B}. \tag{A.24}$$

Since from Lemma 4(a) $\frac{\partial \bar{x}}{\partial a} > 0$ and from the first part of the proposition $\frac{\partial a}{\partial C_B} < 0$, the RHS of (A.24) is negative. Therefore, $\frac{\partial N_B}{\partial C_B} < 0$.

(iii) From equation (14) at the equilibrium:

$$\frac{\partial \tilde{\pi}}{\partial C_B} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial a}\frac{\partial a}{\partial C_B} + \frac{\partial[(-C_B)N_B]}{\partial a}\frac{\partial a}{\partial C_B},$$

using the definition of NMR_S and MC_B from equations (19) and (20), which can be written as:

$$\frac{\partial \tilde{\pi}}{\partial C_B} = \frac{\partial a}{\partial C_B}[NMR_S - MC_B.\frac{\partial T_0}{\partial a}]. \tag{A.25}$$

Since $NMR_S = MC_B$ from equation (18), (A.25) is written as:

$$\frac{\partial \tilde{\pi}}{\partial C_B} = \frac{\partial a}{\partial C_B}NMR_S[1 - \frac{\partial T_0}{\partial a}]. \tag{A.26}$$

Since $\frac{\partial a}{\partial C_B} < 0$, $NMR_S > 0$, and $\frac{\partial T_0}{\partial a} > 0$, the RHS of (A.26) will depend on $sign\left(1 - \frac{\partial T_0}{\partial a}\right)$.

Therefore, $sign\left(\frac{\partial \tilde{\pi}}{\partial C_S}\right) = -sign\left(1 - \frac{\partial T_0}{\partial a}\right)$. □

Proof of Proposition 4:

(i) From equation (18) it follows that

$$\frac{\partial NMR_S}{\partial F} + \frac{\partial NMR_S}{\partial a}\frac{\partial a}{\partial F} = \frac{\partial MC_B}{\partial a}\frac{\partial a}{\partial F},$$

which implies

$$\frac{\partial a}{\partial F} = \frac{-\frac{\partial NMR_S}{\partial F}}{\frac{\partial NMR_S}{\partial a} - \frac{\partial MC_B}{\partial a}}. \tag{A.27}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial a} < 0$ and $\frac{\partial MC_B}{\partial a} > 0$ respectively. Therefore, the denominator of the term on the RHS of (A.27) is negative. However, $\frac{\partial NMR_S}{\partial F} > 0$. The sign of $\frac{\partial a}{\partial F}$ is therefore positive.

(ii) From equation (22):

$$\frac{\partial F_S}{\partial F} = [Q(.)\frac{\partial p^*}{\partial a} + (p^* - c)\frac{\partial Q(.)}{\partial a}]\frac{\partial a}{\partial F} - 1 \tag{A.28}$$

Since $\frac{\partial p^*}{\partial a} > 0$ from Lemma 6, $\frac{\partial Q}{\partial a} > 0$ Lemma 5, and $\frac{\partial a}{\partial F} > 0$, as proved above, the RHS of (A.28) has an uncertain sign. $\frac{\partial F_S}{\partial F} >=< 0$ if and only if $\left[Q(.)\frac{\partial p^*}{\partial a} + (p^* - c)\frac{\partial Q(.)}{\partial a}\right]\frac{\partial a}{\partial F} >=< 1$.

From equation (13):

$\frac{\partial \tilde{N}_S}{\partial F} = -\left(\frac{\partial F_S}{\partial F}\right)$ and $\frac{\partial N_S}{\partial F} = \left(\frac{\partial F_S}{\partial F}\right)$.

The sign of $\frac{\partial \tilde{N}_S}{\partial F}$ and $\frac{\partial N_S}{\partial F}$ follows the sign of $\frac{\partial F_S}{\partial F}$ derived in the second part of the proposition.

Now, from equation (7):

$\frac{\partial N_B}{\partial F} = \frac{\partial \bar{x}}{\partial a}\frac{\partial a}{\partial F}$.

Since $\frac{\partial \bar{x}}{\partial a} > 0$ from Lemma 4(a), and $\frac{\partial a}{\partial F} > 0$ from the first part of the proposition, $\frac{\partial N_B}{\partial F} > 0$.

(iii) From equation (14) at the equilibrium:

$\frac{\partial \tilde{\pi}}{\partial F} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial a}\frac{\partial a}{\partial F} + \frac{\partial[(-C_B)N_B]}{\partial a}\frac{\partial a}{\partial F}$,

using the definition of NMR_S and MC_B from equations (19) and (20), which we write as:

$\frac{\partial \tilde{\pi}}{\partial F} = \frac{\partial a}{\partial F}[NMR_S - MC_B.\frac{\partial T_0}{\partial a}]$.

Since $NMR_S = MC_B$ from equation (18),

$$\frac{\partial \tilde{\pi}}{\partial F} = \frac{\partial a}{\partial F} NMR_S [1 - \frac{\partial T_0}{\partial a}]. \tag{A.29}$$

On the RHS of (A.29), $NMR_S > 0$, and $\frac{\partial T_0}{\partial a} > 0$ (from Lemma 7) and $\frac{\partial a}{\partial F} > 0$ from the first part of the proposition. Therefore, the RHS of (A.29) will depend on $sign\left(1 - \frac{\partial T_0}{\partial a}\right)$. □

Proof of Proposition 5:

(i) From equation (18):

$$\frac{\partial NMR_S}{\partial a}\frac{\partial a}{\partial \bar{v}} + \frac{\partial NMR_S}{\partial \bar{v}} = \frac{\partial MC_B}{\partial a}\frac{\partial a}{\partial \bar{v}} + \frac{\partial MC_B}{\partial \bar{v}}.$$

It follows from the above equation that,

$$\frac{\partial a}{\partial \bar{v}} = \frac{\frac{\partial MC_B}{\partial \bar{v}} - \frac{\partial NMR_S}{\partial \bar{v}}}{\frac{\partial NMR_S}{\partial a} - \frac{\partial MC_B}{\partial a}}. \tag{A.30}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial a} < 0$ and $\frac{\partial MC_B}{\partial a} > 0$ respectively. Therefore, the term's denominator is negative on the RHS of (A.30). From Lemma 8(c) and 9(c) $\frac{\partial NMR_S}{\partial \bar{v}} > 0$ and $\frac{\partial MC_B}{\partial \bar{v}} < 0$. Therefore, the numerator of the term on the RHS of (A.30) is negative. Therefore, $\frac{\partial a}{\partial \bar{v}} > 0$.

(ii) From equation (22):

$$\frac{\partial F_S}{\partial \bar{v}} = Q(.)\left[\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial \bar{v}} + \frac{\partial p^*}{\partial \bar{v}}\right] + (p^* - c)\left[\frac{\partial Q}{\partial \bar{v}} + \frac{\partial Q}{\partial a}\frac{\partial a}{\partial \bar{v}}\right]. \tag{A.31}$$

Since $\frac{\partial p^*}{\partial a} > 0$ and $\frac{\partial p^*}{\partial \bar{v}} < 0$ from Lemma 6, $\frac{\partial Q}{\partial \bar{v}} < 0$ Lemma 5, and $\frac{\partial a}{\partial \bar{v}} > 0$, as proved above, the RHS of (A.31) has an uncertain sign. The RHS of equation (A.31) can be written as

$$\frac{\partial F_S}{\partial \bar{v}} = \frac{\partial T_0}{\partial \bar{v}} + \left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \bar{v}}\right)$$

From Lemma 7 and the first part of the proposition, it follows that $\frac{\partial F_S}{\partial \bar{v}} >=< 0$ if and only if $\left(\frac{\partial T_0}{\partial a} \times \frac{\partial a}{\partial \bar{v}}\right) > (<) - \frac{\partial T_0}{\partial \bar{v}}$.

From equation (13):

$\frac{\partial \tilde{N}_S}{\partial \bar{v}} = -\left(\frac{\partial F_S}{\partial \bar{v}}\right)$ and $\frac{\partial N_S}{\partial \bar{v}} = \left(\frac{\partial F_S}{\partial \bar{v}}\right)$.

The sign of $\frac{\partial \tilde{N}_S}{\partial \bar{v}}$ and $\frac{\partial N_S}{\partial \bar{v}}$ follows the sign of $\frac{\partial F_S}{\partial \bar{v}}$ derived in the second part of the proposition.

Now, from Lemma 4b) it follows that: $\frac{\partial N_B}{\partial \bar{v}} = \frac{\partial \bar{x}}{\partial \bar{v}} < 0$.

(iii) From equation (14) at the equilibrium:

$$\frac{\partial \tilde{\pi}}{\partial \bar{v}} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial a}\frac{\partial a}{\partial \bar{v}} + \frac{\partial[(-C_B)N_B]}{\partial a}\frac{\partial a}{\partial \bar{v}},$$

using the definition of NMR_S and MC_B from equations (19) and (20), which we write as:

$$\frac{\partial \tilde{\pi}}{\partial \bar{v}} = \frac{\partial a}{\partial \bar{v}}[NMR_S - MC_B.\frac{\partial T_0}{\partial a}].$$

Since $NMR_S = MC_B$ from equation (18),

$$\frac{\partial \tilde{\pi}}{\partial \bar{v}} = \frac{\partial a}{\partial \bar{v}} NMR_S[1 - \frac{\partial T_0}{\partial a}]. \tag{A.32}$$

On the RHS of (A.35), $NMR_S > 0$, and $\frac{\partial T_0}{\partial a} > 0$ (from Lemma 7) and $\frac{\partial a}{\partial \bar{v}} > 0$ from the first part of the proposition. So, the RHS of (A.32) will depend on $sign\left(1 - \frac{\partial T_0}{\partial a}\right)$.

Therefore, $sign\left(\frac{\partial \tilde{\pi}}{\partial \bar{v}}\right) = sign\left(1 - \frac{\partial T_0}{\partial a}\right)$. The statement follows. □

Proof of Proposition 6:

(i) From equation (18):

$$\frac{\partial NMR_S}{\partial a}\frac{\partial a}{\partial \rho}+\frac{\partial NMR_S}{\partial \rho}=\frac{\partial MC_B}{\partial a}\frac{\partial a}{\partial \rho}+\frac{\partial MC_B}{\partial \rho}.$$

It follows from the above equation that,

$$\frac{\partial a}{\partial \rho}=\frac{\frac{\partial MC_B}{\partial \rho}-\frac{\partial NMR_S}{\partial \rho}}{\frac{\partial NMR_S}{\partial a}-\frac{\partial MC_B}{\partial a}}. \tag{A.33}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial a}<0$ and $\frac{\partial MC_B}{\partial a}>0$ respectively. Therefore, the term's denominator is negative on the RHS of (A.33). From Lemma 8(c) and 9(c) $\frac{\partial NMR_S}{\partial \bar{v}}>0$ and $\frac{\partial MC_B}{\partial \bar{v}}<0$. Therefore, the numerator of the term on the RHS of (A.33) is negative. Therefore, $\frac{\partial a}{\partial \rho}>0$.

(ii) From equation (22):

$$\frac{\partial F_S}{\partial \rho}=Q(.)\left[\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial \rho}+\frac{\partial p^*}{\partial \rho}\right]+(p^*-c)\left[\frac{\partial Q}{\partial \rho}+\frac{\partial Q}{\partial a}\frac{\partial a}{\partial \rho}\right]. \tag{A.34}$$

Since $\frac{\partial p^*}{\partial a}>0$ and $\frac{\partial p^*}{\partial \rho}<0$ from Lemma 6, $\frac{\partial Q}{\partial \rho}<0$ Lemma 5, and $\frac{\partial a}{\partial \rho}>0$, as proved above, the RHS of (A.34) has an uncertain sign. The RHS of equation (A.34) can be written as

$$\frac{\partial F_S}{\partial \rho}=\frac{\partial T_0}{\partial \rho}+\left(\frac{\partial T_0}{\partial a}\times\frac{\partial a}{\partial \rho}\right)$$

From Lemma 7 and the first part of the proposition, it follows that $\frac{\partial F_S}{\partial \rho}>=<0$ if and only if $\left(\frac{\partial T_0}{\partial a}\times\frac{\partial a}{\partial \rho}\right)>(<)-\frac{\partial T_0}{\partial \rho}$.

From equation (13):

$\frac{\partial \tilde{N}_S}{\partial F} = -\left(\frac{\partial F_S}{\partial \rho}\right)$ and $\frac{\partial N_S}{\partial F} = \left(\frac{\partial F_S}{\partial \rho}\right)$.

The sign of $\frac{\partial \tilde{N}_S}{\partial F}$ and $\frac{\partial N_S}{\partial F}$ follows the sign of $\frac{\partial F_S}{\partial \rho}$ derived in the second part of the proposition.

Now, from Lemma 4b) it follows that: $\frac{\partial N_B}{\partial \rho} = \frac{\partial \bar{x}}{\partial \rho} < 0$.

(iii) From equation (14) at the equilibrium:

$$\frac{\partial \tilde{\pi}}{\partial \rho} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial a}\frac{\partial a}{\partial \rho} + \frac{\partial[(-C_B)N_B]}{\partial a}\frac{\partial a}{\partial \rho},$$

using the definition of NMR_S and MC_B from equations (19) and (20), which we write as:

$$\frac{\partial \tilde{\pi}}{\partial \rho} = \frac{\partial a}{\partial \rho}[NMR_S - MC_B . \frac{\partial T_0}{\partial a}].$$

Since $NMR_S = MC_B$ from equation (18),

$$\frac{\partial \tilde{\pi}}{\partial \rho} = \frac{\partial a}{\partial \rho} NMR_S [1 - \frac{\partial T_0}{\partial a}]. \tag{A.35}$$

On the RHS of (A.35), $NMR_S > 0$, and $\frac{\partial T_0}{\partial a} > 0$ (from Lemma 7) and $\frac{\partial a}{\partial \rho} > 0$ from the first part of the proposition. So, the RHS of (A.35) will depend on $sign\left(1 - \frac{\partial T_0}{\partial a}\right)$.

Therefore, $sign\left(\frac{\partial \tilde{\pi}}{\partial \rho}\right) = sign\left(1 - \frac{\partial T_0}{\partial a}\right)$. The statement follows. □

Proof of Proposition 7:

(i) From equation (18):

$$\frac{\partial NMR_S}{\partial a} . \frac{\partial a}{\partial t} + \frac{\partial NMR_S}{\partial t} = \frac{\partial MC_B}{\partial a} . \frac{\partial a}{\partial t} + \frac{\partial MC_B}{\partial t}. \tag{A.36}$$

It follows from (A.36) that,

$$\frac{\partial a}{\partial t}=\frac{\frac{\partial MC_B}{\partial t}-\frac{\partial NMR_S}{\partial t}}{\frac{\partial NMR_S}{\partial a}-\frac{\partial MC_B}{\partial a}}. \tag{A.37}$$

From Lemma 8(a) and Lemma 9(a), $\frac{\partial NMR_S}{\partial a}<0$ and $\frac{\partial MC_B}{\partial a}>0$ respectively. Therefore, the denominator of the term on the RHS of (A.36) is negative. From Lemma 9(c) $\frac{\partial MC_B}{\partial t}<0$ and $\frac{\partial NMR_S}{\partial t}>0$. Therefore, the term on the RHS of (A.36) is positive and it follows that $\frac{\partial a}{\partial t}>0$.

(ii) From equation (22):

$$\frac{\partial F_S}{\partial t}=\left\{\left[\frac{\partial p^*}{\partial t}+\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial t}\right]Q(.)\ +(p^*-c)\left[\frac{\partial Q(.)}{\partial t}+\frac{\partial Q(.)}{\partial a}\frac{\partial a}{\partial t}\right]\right\}. \tag{A.38}$$

Since $\frac{\partial p^*}{\partial t}<0, \frac{\partial p^*}{\partial a}>0$ from Lemma 6, and $\frac{\partial Q(.)}{\partial t}<0$, $\frac{\partial Q_j}{\partial a}>0$ Lemma 5, the RHS of (A.38) can be written as

$$\frac{\partial F_S}{\partial t}=\frac{\partial T_0}{\partial t}+\left(\frac{\partial T_0}{\partial a}\times\frac{\partial a}{\partial t}\right). \tag{A.39}$$

Since from Lemma 7, $\frac{\partial T_0}{\partial t}<0, \frac{\partial T_0}{\partial a}>0$ and from the first part $\frac{\partial a}{\partial t}>0,$ then

$$sign\left(\frac{\partial F_S}{\partial t}\right)=sign\left(\frac{\partial T_0}{\partial t}+\left(\frac{\partial T_0}{\partial a}\times\frac{\partial a}{\partial t}\right)\right).$$

Therefore, the statement follows.

Now, from equation (13):

$$\frac{\partial \tilde{N}_S}{\partial t}=-\frac{\partial F_S}{\partial t}.$$

Since $\frac{\partial F_S}{\partial t}\gtreqless 0$ depending on $\left(\frac{\partial T_0}{\partial a}\times\frac{\partial a}{\partial t}\right)\gtreqless-\frac{\partial T_0}{\partial t}$ which follows from the part (ii) of the proposition. The sign of $\frac{\partial \tilde{N}_S}{\partial t}$ follows accordingly. Similarly, $\frac{\partial N_S}{\partial t}=\frac{\partial F_S}{\partial t}$.

The sign of $\frac{\partial \tilde{N}_S}{\partial t}$ and $\frac{\partial N_S}{\partial t}$ follows the sign of $\frac{\partial F_S}{\partial t}$ derived in the second part of the proposition.

(iii) Now, from Lemma 4(b):

$\frac{\partial N_B}{\partial t} = \frac{\partial \bar{x}}{\partial t}$ where, $\frac{\partial \bar{x}}{\partial t} < 0$ from Lemma 4(a).

(iv) From equation (14) at the equilibrium:

$$\frac{\partial \tilde{\pi}}{\partial t} = \frac{\partial[(F_S - C_S)\tilde{N}_S]}{\partial a} \frac{\partial a}{\partial t} + \frac{\partial[(-C_B)N_B]}{\partial a} \frac{\partial a}{\partial t},$$

using the definition of NMR_S and MC_B from equations (19) and (20), which can be written as:

$$\frac{\partial \tilde{\pi}}{\partial t} = \frac{\partial a}{\partial t}[NMR_S - MC_B . \frac{\partial T_0}{\partial a}]. \tag{A.40}$$

Since $NMR_S = MC_B$ from equation (19), (A.40) is written as:

$$\frac{\partial \tilde{\pi}}{\partial t} = \frac{\partial a}{\partial t} NMR_S[1 - \frac{\partial T_0}{\partial a}]. \tag{A.41}$$

On the RHS of (A.40), $\frac{\partial a}{\partial t} > 0$, $NMR_S > 0$, and $\frac{\partial T_0}{\partial a} > 0$. Therefore,

$$sign\left(\frac{\partial \tilde{\pi}}{\partial t}\right) = sign\left(1 - \frac{\partial T_0}{\partial a}\right).$$

The statement follows. □

Chapter 5

Conclusion

Millions of products are offered for sale on platforms, be it online or offline. Given the consumer's 'love for variety' preference, the interaction of such platforms with conventional 'brick-and-mortar' stores has become a burning issue. The thesis presents a unified framework comprising 'love for variety' with heterogeneous buyer preferences, strategic price interactions among the sellers of the varieties of the differentiated product, and an outside option common to all buyers that will be useful in this context of two-sided platforms. The framework is then extended to analyze the behavior of a platform, especially its pricing behavior and its location choice in a linear city.

5.1 Summary of Results

Chapter 2 models Bertrand-price competition in a market with differentiated product oligopoly. We introduce a demand-side innovation in the model where buyers differ from each other in their valuation of the differentiated product with a common outside option available to all. The 'Dixit-Stiglitz' utility function models the buyers' 'love for variety' preferences. In the presence of substitutability among the varieties of the differentiated product, if the price of a particular variety rises, the demand for all other varieties falls in the presence of a strong extensive margin effect. This introduces complementarity in the extensive margin of buyers' participation in the model, and it happens because of the 'love for variety' specification of preferences. Thus, we show that the oligopoly price in such markets can be greater than the monopoly price even in the presence of substitutability among the varieties of the differentiated product. Using this framework, we have also studied the responsiveness of parameters like the degree of substitutability between varieties of the differentiated product and the attractiveness of the outside option on the extensive margin of demand. The model presented in Chapter 2 endogenously determines the number of buyers and sellers in the differentiated product market and, consequently, the number of varieties. It finds that the presence of a strong extensive margin effect may change many of the conventional results. First, it may alter the supermodular structure of the standard Bertrand specification. In such a situation, the oligopoly price in the differentiated product market can be greater than the coordinated monopoly price. Second, the increase either in the substitutability between the varieties of the differentiated product or in the availability of a stronger outside option may increase both the price of the product and the number of varieties produced in the market. Third, a rise in the marginal cost of production of the varieties of the differentiated product may lead to a fall in the price of the product at the market. These findings are new to the literature.

In Chapter 3, we construct a model involving a two-sided monopoly platform, such as malls and e-commerce platforms, that caters to both sides, namely, buyers and sellers, in a market for differentiated products. The buyers have a Dixit-Stiglitz type of ‘love for variety’ preferences, while the sellers compete with one another in terms of pricing. The platform internalizes the externalities either side of the market creates for the other side and decides the membership fee to be charged on both sides. As in Chapter 2, the number of varieties produced in the differentiated product market and the number of varieties sold on the platform are both endogenously determined by the model. It characterizes the platform’s pricing on both the side of the market and the number of varieties sold on the platform with respect to changes in costs to servicing the clients, changes in fixed costs on the sellers’ side, improved outside option of the buyers, and the lower substitutability of the varieties at the market.

The comparative static exercises of Chapter 3 derive some interesting results. The findings show that if the costs of servicing the buyers fall, the buyers’ membership fee is cross-subsidized by raising the membership fee for the sellers. The opposite happens if the costs of servicing the sellers fall. In the context of a monopoly platform, these two outcomes highlight the presence of cross-sided externalities. Unlike the standard monopoly case, an efficient platform will not always have higher profits. If the platform is more efficient at serving sellers, its profit increases; if it is more efficient at serving buyers, its profit decreases. The number of varieties sold on the platform accordingly adjusts in both cases. If the platform anticipates that the sellers would benefit from the reduced fixed cost, they may be charged a higher membership fee even as the buyers benefit from a lower membership fee. Accordingly, sellers' membership fee determines how many varieties are sold on the platform. However, the platform might not prefer innovations that reduce the sellers' fixed costs because this leads to a fall in its profit. Imagine that buyers have a better alternative to purchasing from the market for differentiated products, such as a 'brick-and-mortar' store. The platform responds to that by reducing the buyers' membership fees. The platform's calculation of the seller's gain or loss is important in deciding the seller's fee and the number of varieties sold on the platform. So, the platform may raise the membership fee for the sellers even when the differentiated product market loses its buyers to the 'brick-and-mortar' shops. In the case where the membership fees charged by the platform fall, an increase in the number of sellers on the platform coexists with an increase in the utility of participating in a 'brick-and-mortar' store. However, the platform's profit falls if 'brick-and-mortar' stores prove to be a desirable place to shop. Suppose the varieties of differentiated products are close substitutes. In that case, a similar impact occurs for an improvement in the outside option. It can be observed that the platform charges a lower membership fee on the buyer side. However, it can be higher on the seller side if the sellers gain from a change in the substitutability of the varieties. Accordingly, the number of varieties on the platform falls while the number of varieties outside the platform rises. The opposite happens if the sellers lose. In both situations, the number of buyers in the differentiated product market and the platform's profit fall. Sellers of the differentiated product varieties may be encouraged to diversify their offerings since the platform suffers if the types they offer in the differentiated product market are close substitutes.

Chapter 4 discusses the platform's location choice using a framework similar to Chapter 3. The model in this chapter involves a monopoly shopping mall selling differentiated products and a homogeneous goods market in a linear city framework combining spatial competition and 'love for variety' preferences on the buyer side. While the shopping mall chooses its location and the membership fee on the seller side, the location of the homogeneous goods market is fixed on one end of the linear city. The mall does not charge the buyers. It has been assumed that the sellers of the differentiated product who do not opt to locate inside the mall by paying the membership fee charged by the mall sell their products just outside the mall precisely in the exact location. The number of varieties sold on and off the mall is endogenously determined in the model. In the presence of a mall, the number of varieties produced of a differentiated product increases compared to a 'no-mall' situation. My model conjoins several dimensions in a single framework, like spatial competition, 'love for variety' preferences on the buyer side, intensive and extensive margins of demand, travel costs, and Bertrand-price competition among the sellers participating in the differentiated product market. Interestingly, the mall's location choice has a role in determining the prices of the varieties through Bertrand-price competition. The key contribution of Chapter 4 is the characterization of the mall's choice of location on the linear city and the pricing on the seller side of the market in equilibrium. The chapter analyzes the impact of changes in the platform-related parameters of the model, like the costs of servicing the clients, on these key characterizations. Also, seller-side parameters, like changes in fixed costs, and buyer-side parameters, like improved outside option of the buyers, lower substitutability of the varieties at the market, and the transport costs incurred by the buyers for traveling to the mall, add to the insights.

Like Chapter 3, we also perform comparative static exercises in Chapter 4. In this chapter's comparative static exercises, the mall's choice of location in the linear city is of great significance. As the costs of servicing the sellers fall, at the initial equilibrium, the net marginal cost of the mall on the buyers' side is relatively higher than the net marginal revenue of the mall from the sellers' side. Therefore, at the new equilibrium, the mall adjusts its location away from the location of the homogeneous product market, given its revenue-cost considerations. This creates a negative cross-sided externality on the buyer side, common to a monopoly platform. The adjustment in the location of the mall determines the membership fee on the seller side and the prices of varieties. The results show that if the costs of servicing the sellers fall, the mall charges a lower membership fee on the seller side. The cross-subsidization happens in the opposite direction for a fall in the cost of servicing the buyers. Unlike Chapter 3, here in this chapter, it has been observed that under certain conditions on the shopping mall's revenue and cost sides, the mall's profit rises with increased efficiency in servicing both the buyers and the sellers. Depending on its location and the resulting effect on its profit, the mall may favor developments that reduce the seller's fixed costs. This situation did not arise in the comparative static exercise with respect to the seller's fixed cost in Chapter 3. In the case of an improved outside option, the impact on the revenue of the sellers of the varieties of the differentiated product matters for the choice of the membership fee in equilibrium. So even if the differentiated product market loses its buyers to the homogeneous goods market, the mall may raise the membership fee for the sellers, leading to a fall in the number of varieties available in the mall contrary to standard spatial competition models where location is exogenously fixed. Similar results regarding the number of varieties of the differentiated product sold in the mall can be observed when the substitutability among varieties and the transportation cost for buyers to go to the mall increases. To add to these, under certain conditions on the revenue and cost sides of the shopping mall, the profit of the shopping mall may rise with changes in the travel costs on the buyer side, improvement in the outside option, and substitutability among varieties.

This section summarizes the results, while the following section highlights the policy implications of the three core chapters. The last section of this chapter discusses the scope of future research to conclude the thesis.

5.2 Policy Implications

Each of the three main chapters in the thesis has its unique policy implications. The model in Chapter 2 relates to real-life market situations, where the 'love for variety' matters and the buyers choose between a 'brick-and-mortar' shop, selling a particular variety vis-à-vis a shopping mall/e-commerce platform selling a number of different varieties of a product. The findings imply that contrary to popular belief, the 'brick-and-mortar' shops, the shopping malls, and the e-commerce platforms may complement each other in the presence of a strong extensive margin of demand effect. Instead of having price competition, the formation of a price-cartel can be a possibility.

In Chapter 3, we have seen factors like the efficiency of the monopoly platform in servicing the buyers and the sellers, fixed operational costs faced by the sellers, the outside option of the buyers, and the substitutability among the varieties of the differentiated product have an impact on the platform's profit. Since online platform giants like Amazon tend to avoid taxes (Mukherjee & Mukherjee, 2020), regulating their profits is an important aspect of the platform literature. The effect of a rise in per-unit tax on both the buyer and seller sides is similar to a rise in the cost of servicing both these sides in our model. In that case, it translates to the fact that raising per-unit taxes on the buyer side can lead to higher profits for the platform, while the platform's profit falls if the regulator imposes the tax on the seller side. This result is similar to the "lucky break" concept introduced by Belleflamme and Toulemonde (2018b) in the context of competing platforms. Given the cross-sided network effects, imposing a specific tax on one of the sides of a competing platform may lead to a rise in the platform's profit. Also, as an alternative to imposing taxes on the monopoly platform's profit, the regulator can promote the growth of the 'brick-and-mortar' stores, resulting in a fall in the profits of the monopoly platform. These results can have important implications for controlling the profits of a monopoly platform.

Chapter 4 shows the possibility of the coexistence of a homogeneous goods market and a shopping mall. In the context of antitrust regulations, this result can be useful for the policymakers who think that large shopping malls will encroach on the small-scale retail space. This result can also have important implications in the context of regulatory intervention for limiting the size of malls, in support of the 'brick-and-mortar' stores. Even it can have important implications from the political economy perspective. That is, whether building shopping malls to improve the quality of urban life will add to the ruling party's vote share. The contributions of the thesis can be very pertinent in this regard.

5.3 Future Scope of Research

Even though this study has several significant strengths for analysis, there is scope to add new dimensions to the work presented in the three core chapters in the future. We discuss them separately, starting with Chapter 2.

Chapter 2 has already been extended in the context of two-sided platforms in Chapter 3 and spatial economics in Chapter 4. It can be extended further in the context of international trade, where gains from variety can have important implications for trade policy, and in Romer-type endogenous growth theory models for explaining long-run growth. This work remains to be done.

In Chapter 3 of the thesis, some simplifying assumptions have been made to keep the model tractable. As discussed, we have not explicitly explored the matching of buyers and sellers. In the model presented in Chapter 3, only the platform's membership fee decision has been considered. However, adding the per-unit usage fee decision can add to the richness of the model. Also, the buyers differ only in their preference for purchasing the differentiated product and the sellers in their transaction costs. The intention is to relax some of these assumptions and explore new aspects of two-sided platforms. Moreover, adding platform competition and buyer-side congestion to the current model's structure can be another possible extension of the model.

Some simplifying assumptions discussed in Chapter 4 include the location of the homogeneous good market fixed on one end of a linear city. The location of the sellers just outside the mall has not been explicitly modeled, and they are assumed to be located precisely at the same location as that of the mall. Future extensions will relax some of these assumptions to make the results more generic. Given that sellers outside the mall can join another shopping mall, the model can be used to study the competition between shopping malls. Another extension can be introducing a Salop-type circular city instead of a linear Hotelling-type town. Exploring the impact of population density in the linear city, other functional forms of transport costs, and the issue of regulating the mall also remains a part of future research works.

The issues discussed in this work are very relevant in the present context. It has highlighted some policy implications and suggested some extensions to research further.

References

Abdel-Rahman, H. M. (1988). Product differentiation, monopolistic competition and city size. *Regional Science and Urban Economics*, *18*(1), 69-86.

Acemoglu, D., Makhdoumi, A., Malekian, A., & Ozdaglar, A. (2022). Too much data: Prices and inefficiencies in data markets. *American Economic Journal: Microeconomics*, *14*(4), 218-256.

Afridi, F. E. A., Jan, S., Ayaz, B., & Irfan, M. (2021). The impact of Covid-19 on E-business practices and consumer buying behavior in a developing country. *Amazonia Investiga*, *10*(38), 97-112.

Ago, T. (2008). Central agglomeration of monopolistically competitive firms. *Journal of Economic Geography*, *8*(6), 811-823.

Ago, T., Hamoudi, H., & Lefouili, Y. (2017). Firm location and monopolistic competition. *Chapters in Regional Science*, *96*(1), 211-219.

Amir, R. (2005). Supermodularity and complementarity in economics: An elementary survey. *Southern Economic Journal*, *71*(3), 636-660.

Anderson, S. P., & Neven, D. J. (1991). Cournot competition yields spatial agglomeration. *International Economic Review*, 793-808.

Anderson, S. P., De Palma, A., & Nesterov, Y. (1995). Oligopolistic competition and the optimal provision of products. *Econometrica*, *63*(6), 1281-1301.

Anderson, S., De Palma, A., and Thisse, J. (1992). *Interpretations of the logit discrete choice models and the theory of product differentiation* (Working Chapter No. 1017). Université Catholique de Louvain, Center for Operations Research and Econometrics (CORE).

Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics*, 37(3), 668-691.

Behrens, K., and Thisse, J. F. (2006). Agglomeration versus product variety: Implications for regional inequalities. *Journal of Regional Science*, *46*(5), 867-880.

Belleflamme, P., & Peitz, M. (2021). *The Economics of Platforms*. Cambridge, United Kingdom: Cambridge University Press.

Belleflamme, P., & Peitz, M. (2019). Managing competition on a two-sided platform. *Journal of Economics & Management Strategy*, *28*(1), 5-22.

Belleflamme, P., & Peitz, M. (2018a). Platforms and network effects. In L. Porchon, and M. Marini (Eds.), *Handbook of Game Theory and Industrial Organization, Volume II: Applications* (286 - 317). Cheltenham : Edward Elgar Publishing.

Belleflamme, P., & Toulemonde, E. (2018b). Tax incidence on competing two-sided platforms. *Journal of Public Economic Theory*, *20*(1), 9-21.

Belleflamme, P., and Peitz, M. (2015). *Industrial Organization: Markets and Strategies*. Cambridge: Cambridge University Press.

Belussi, F., & Rakic, K. (2020). Online and Brick-and-Mortar Strategy in Fashion Industry: Co-Existence or Cannibalization?. doi: 10.32474/LTTFD.2019.03.000157

Bergemann, D., & Bonatti, A. (2024). Data, competition, and digital platforms. *American Economic Review*, *114*(8), 2553-2595.

Bergemann, D., Bonatti, A., & Gan, T. (2022). The economics of social data. *The RAND Journal of Economics*, *53*(2), 263-296.

Berliant, M., & Konishi, H. (2000). The endogenous formation of a city: Population agglomeration and marketplaces in a location-specific production economy. *Regional Science and Urban Economics*, *30*(3), 289-324.

Bhatnagar, S., & Yadav, R. (2022). Can Multiple Channels Coexist?: The Case of the Indian FMCG Sector. In *Multidisciplinary Perspectives Towards Building a Digitally Competent Society* (pp. 70-86). IGI Global, USA.

Bounie, D., Dubus, A., & Waelbroeck, P. (2024). Competition and the Two Margins of Privacy. *Available at SSRN 4815096*.

Bounie, D., Dubus, A., & Waelbroeck, P. (2021). Selling strategic information in digital competitive markets. *The RAND Journal of Economics*, *52*(2), 283-313.

Brakman, S., Garretsen, H., and Van Marrewijk, C. (2001). *An introduction to geographical economics: Trade, location and growth*. Cambridge: Cambridge University Press.

Brakman, S. and Heijdra, B. J. (eds.) (2002). *The Monopolistic Competition Revolution After Twenty-Five Years*. Cambridge: Cambridge University Press.

Brandão, A., Correia-da-Silva, J., & Pinho, J. (2014). Spatial competition between shopping centers. *Journal of Mathematical Economics*, *50*, 234-250.

Bronnenberg, B. J. (2015). The provision of convenience and variety by the market. *The RAND Journal of Economics*, *46*(3), 480-498.

Brynjolfsson, E., Chen, L., & Gao, X. (2022). *Gains from Product Variety: Evidence from a Large Digital Platform* (No. 30802). National Bureau of Economic Research, Inc.

Brynjolfsson, E., Hu, Y., & Smith, M. D. (2003). Consumer surplus in the digital economy: Estimating the value of increased product variety at online booksellers. *Management science*, *49*(11), 1580-1596.

Caillaud, B., & Jullien, B. (2003). Chicken & egg: Competition among intermediation service providers. *Rand Journal of Economics*, *34*(2), 309-309.

Caillaud, B., & Jullien, B. (2001). Competing cybermediaries. *European Economic Review*, *45*(4-6), 797-808.

Caplin, A., & Nalebuff, B. (1991). Aggregation and imperfect competition : On the existence of equilibrium. *Econometrica*, *59*(1), 25-59.

Chawla, N., & Mondal, D. (2024). Seller competition on two-sided platforms. *Journal of Economics*, *142*(3), 213-245.

Chen, C. S., & Lai, F. C. (2008). Location choice and optimal zoning under Cournot competition. *Regional Science and Urban Economics*, *38*(2), 119-126.

Choi, G., Nam, C., Kim, S., Jung, H. J., & Lee, C. H. (2020). Where does knowledge-sharing motivation come from? The case of third-party developer in mobile platforms. *Journal of Knowledge Management*, *24*(7), 1681-1704.

Correia-da-Silva, J., Jullien, B., Lefouili, Y., & Pinho, J. (2019). Horizontal mergers between multisided platforms: Insights from Cournot competition. *Journal of Economics & Management Strategy*, *28*(1), 109-124.

Cosandier, C., Garcia, F., and Knauff, M. (2018). Price competition with differentiated goods and incomplete product awareness. *Economic Theory*, *66*(3), 681-705.

Cox, W. M., and Ruffin, R. J. (2010). Variety, globalization, and social efficiency. *Southern Economic Journal*, *76*(4), 1064-1075.

De Palma, A., Ginsburgh, V., Papageorgiou, Y. Y., & Thisse, J. F. (1985). The principle of minimum differentiation holds under sufficient heterogeneity. *Econometrica: Journal of the Econometric Society*, 767-781.

Dixit, A. K., and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *The American Economic Review*, *67*(3), 297-308.

Economides, N., & Hermalin, B. E. (2015). The strategic use of download limits by a monopoly platform. *The RAND Journal of Economics*, *46*(2), 297-327.

Economides, N., & Katsamakas, E. (2006). Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry. *Management science*, *52*(7), 1057-1071.

Etro, F. (2021). Product selection in online marketplaces. *Journal of Economics & Management Strategy*, *30*(3), 614-637.

Farrag, D. A., El Sayed, I. M., & Belk, R. W. (2010). Mall shopping motives and activities: a multimethod approach. *Journal of International Consumer Marketing*, *22*(2), 95-115.

Fujita, M. (1988). A monopolistic competition model of spatial agglomeration: Differentiated product approach. *Regional Science and Urban Economics*, *18*(1), 87-124.

Fujita, M., & Ogawa, H. (1982). Multiple equilibria and structural transition of non-monocentric urban configurations. *Regional Science and Urban Economics*, *12*(2), 161-196.

Fujita, M. & Thisse, J. (2013). Economics of Agglomeration: Cities, Industrial Location, and Globalization. Cambridge University Press: Cambridge, England, UK.

Gabszewicz, J. J., & Thisse, J. F. (2013). Spatial competition and the location of firms. In *Regional and Urban Economics Parts 1 & 2* (pp. 39-109). Routledge: Oxfordshire, England, UK.

Gabszewicz, J. J., & Thisse, J. F. (1979). Price competition, quality and income disparities. *Journal of Economic Theory*, *20*(3), 340-359.

Gabszewicz, J. J., & Wauthy, X. Y. (2012). Nesting horizontal and vertical differentiation. *Regional Science and Urban Economics*, *42*(6), 998-1002.

Galeotti, A., & Moraga-González, J. L. (2009). Platform intermediation in a market for differentiated products. *European Economic Review*, *53*(4), 417-428.

Gal-Or, E., & Shi, Q. (2022). Designing entry strategies for subscription platforms. *Management Science*, *68*(10), 7597-7613.

Ghosh, S. (2022). The Economics of Informal Markets: The Case of Street Vendors in Kolkata. In M. Khemka, T. N. Sahu, & S. Ghosh (Eds.), *Indian Business@ 75: Research on Trends and Prospects* (pp 225 – 236). Allied Publishers, Noida.

Guo, S., & Heese, H. S. (2017). Product variety and distribution channel structure. *International Journal of Production Research*, *55*(12), 3392-3410.

Gupta, B., Pal, D., & Sarkar, J. (1997). Spatial Cournot competition and agglomeration in a model of location choice. *Regional Science and Urban Economics*, *27*(3), 261-282.

Häckner, J. (2000). A Note on Price and Quantity Competition in Differentiated Oligopolies. *Journal of Economic Theory*, *93*, 233-239.

Hagiu, A., Teh, T. H., & Wright, J. (2022). Should platforms be allowed to sell on their own marketplaces?. *The RAND Journal of Economics*, *53*(2), 297-327.

Hagiu, A., & Hałaburda, H. (2014). Information and two-sided platform profits. *International Journal of Industrial Organization*, *34*, 25-35.

Hagiu, A. (2009). Two-sided platforms: Product variety and pricing structures. *Journal of Economics & Management Strategy*, *18*(4), 1011-1043.

Hamilton, J. H., Thisse, J. F., & Weskamp, A. (1989). Spatial discrimination: Bertrand vs. Cournot in a model of location choice. *Regional Science and Urban Economics*, *19*(1), 87-102.

Hasker, K., & Inci, E. (2014). Free parking for all in shopping malls. *International Economic Review*, *55*(4), 1281-1304.

Hotelling, H. (1929). Stability in Competition. *The Economic Journal*, *39*(153), 41-57.

Hovenkamp, H. (2020). Antitrust and platform monopoly. *Yale LJ*, *130*(8), 1952-2050.

Jeon, D. S., Kim, B. C., & Menicucci, D. (2022). Second-degree price discrimination by a two-sided monopoly platform. *American Economic Journal: Microeconomics*, *14*(2), 322-369.

Jones, C. I. (1995). R & D-based models of economic growth. *Journal of Political Economy*, *103*(4), 759-784.

Kalhan, A. (2007). Impact of malls on small shops and hawkers. *Economic and political Weekly*, *42*(22), 2063-2066.

Kaplan, G., Menzio, G., Rudanko, L., & Trachter, N. (2019). Relative price dispersion: Evidence and theory. *American Economic Journal: Microeconomics*, *11*(3), 68-124

Katz, M. L., & Shapiro, C. (1985). Network externalities, competition, and compatibility. *The American economic review*, *75*(3), 424-440.

Keels, G. N. (2021). *Brick-and-mortar retail stores disappearing in the digital age: Marketing strategies for sustainability*. University of Maryland University College.

Kirman, A. P. (1992). Whom or what does the representative individual represent?. *Journal of Economic Perspectives*, *6*(2), 117-136.

Kirpalani, R., & Philippon, T. (2020). *Data sharing and market power with two-sided platforms* (No. w28023). National Bureau of Economic Research. Retrieved from https://www.tse-fr.eu/sites/default/files/TSE/documents/conf/2021/kirpalani.pdf

Klemperer, P. (1992). Equilibrium Product Lines: Competing Head-to-Head May Be Less Competitive. *American Economic Review*, *82*(4), 740-755.

Krugman, P. R. (1997): *Development, Geography and Economic Theory*, Cambridge, MA: MIT Press.

Krugman, P. R. (1995). Increasing returns, imperfect competition and the positive theory of international trade. *Handbook of International Economics*, *3*, 1243-1277.

Krugman, P. R. (1981). Intra industry specialization and the gains from trade. *Journal of Political Economy*, *89*(5), 959-973.

Krugman, P. R. (1980). Scale economies, product differentiation, and the pattern of trade. *The American Economic Review*, *70*(5), 950-959.

Krugman, P. R. (1979). Increasing returns, monopolistic competition, and international trade. *Journal of International Economics*, *9*(4), 469-479.

Lach, S. (2002). Existence and persistence of price dispersion: an empirical analysis. *Review of economics and statistics*, *84*(3), 433-444.

Lancaster, K. (1979). *Variety, Equity, and Efficiency: Product Variety in an Industrial Society* (Vol. 10). New York: Columbia University Press.

Li, S., Liu, Y., Su, J., Luo, X., & Yang, X. (2023). Can e-commerce platforms build the resilience of brick-and-mortar businesses to the COVID-19 shock? An empirical analysis in the Chinese retail industry. *Electronic Commerce Research*, *23*(4), 2827-2857.

Lucas, R. E., & Rossi–Hansberg, E. (2002). On the internal structure of cities. *Econometrica*, *70*(4), 1445-1476.

Madden, P., and Pezzino, M. (2011). Oligopoly on a Salop Circle with Centre. *The BE Journal of Economic Analysis and Policy*, *11*(1), 1-30.

Malykhin, N., & Ushchev, P. (2018). How market interactions shape the city structure. *Regional Science and Urban Economics*, *71*, 122-136.

Mantena, R., & Saha, R. L. (2012). Co-opetition between differentiated platforms in two-sided markets. *Journal of Management Information Systems*, *29*(2), 109-140.

Martimort, D., and Stole, L. (2009). Market participation in delegated and intrinsic common-agency games. *The RAND Journal of Economics*, *40*(1), 78-102.

Martin, P., and Ottaviano, G. I. (2001). Growth and agglomeration. *International Economic Review*, *42*(4), 947-968.

Mathews, N. M. (2018). Impact of Shopping Malls on Small Retail Outlets-A Study in Kollam City, Kerala. *International Journal of Research and Analytical Reviews*, *5*(3), 1408x – 1417x.

Matsumura, T., & Shimizu, D. (2005). Spatial Cournot competition and economic welfare: a note. *Regional Science and Urban Economics*, *35*(6), 658-670.

Matsushima, N. (2001). Cournot competition and spatial agglomeration revisited. *Economics Letters*, *73*(2), 175-177.

Mishra, S. K., & Sarkar, S. (2023). Can social media platforms charge their users?. *Economics Letters*, *233*, 111415.

Montagna, C. (2001). Efficiency gaps, love of variety and international trade. *Economica*, *68*(269), 27-44.

Mukherjee, S., & Mukherjee, V. (2023). "Love for variety," outside option and extensive margin of demand. *International Journal of Economic Theory*, *19*(3), 425-449.

Mukherjee, S., & Mukherjee, V. (2020). Tax Incidence of Two-Sided Monopoly Platform. In M. S. de Vries, J. Nemec and V. Junjan (eds.), *From Policy Design to Policy Practice* (1st ed., pp. 161 – 177). Bratislava, Slovak Republic: NISPAcee Press.

Mussa, M., & Rosen, S. (1978). Monopoly and product quality. *Journal of Economic Theory*, *18*(2), 301-317.

Naidu, K., & Naidu, C. K. (2016). Shopping malls: a growing phenomenon in Indian retail industry. An empirical study of cities of Chennai and Coimbatore. *International Journal of Recent Research Review*, *1*(3), 15-19.

Neven, D., & Thisse, J. F. (1990). On quality and variety competition. JJ Gabszewicz, J.-F. Richard, LA Wolsey, eds. Economic Decision Making : Games, Econometrics and Optimization. Amsterdam : North-Holland.

Nyarko, Y., & Pellegrina, H. (2022). From bilateral trade to centralized markets: A search model for commodity exchanges in Africa. *Journal of Development Economics*, *157*, 102867.

Ottaviano, G., Tabuchi, T., and Thisse, J. F. (2002). Agglomeration and trade revisited. *International Economic Review*, *43*(2), 409-435.

Ottaviano, G. I., and Thisse, J. F. (2002). Integration, agglomeration and the political economics of factor mobility. *Journal of Public Economics*, *83*(3), 429-456.

Ottaviano, G., and Thisse, J. F. (2004). Agglomeration and economic geography. In *Handbook of regional and urban economics* (Vol. 4, pp. 2563-2608). Elsevier.

Oyamada, K. (2020). Love of variety in trade models with product differentiation. *Journal of Global Economic Analysis*, *5*(2), 1-62.

Perloff, J. M., & Salop, S. C. (1985). Equilibrium with product differentiation. *The Review of Economic Studies*, *52*(1), 107-120.

Rajagopalan, S., & Xia, N. (2012). Product variety, pricing and differentiation in a supply chain. *European Journal of Operational Research*, *217*(1), 84-93.

Rajagopal. (2009). Growing shopping malls and behaviour of urban shoppers. *Journal of Retail & Leisure Property*, *8*, 99-118.

Rochet, J. C., & Tirole, J. (2006). Two-sided markets: a progress report. *The RAND Journal of Economics*, *37*(3), 645-667.

Rochet, J. C., & Tirole, J. (2004). *Two*-Sided Markets: An Overview. Mimeo, IDEI, Toulouse, France, March. Retrieved from https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=1181ee3b92b2d6c1107a5c899bd94575b0099c32

Rochet, J. C., & Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association*, *1*(4), 990-1029.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, *98*(5, Part 2), S71-S102.

Roson, R. (2005). *Platform Competition with Endogenous Multihoming*. Retrieved from https://core.ac.uk/download/pdf/6264532.pdf

Roy, S. (2014). Emergence of shopping malls and its impact on the hawkers' market economy: A case study of Kolkata city. *Indian Journal of Applied Research*, *4*(8), 295-298.

Salem, M. A., & Nor, K. M. (2020). The effect of COVID-19 on consumer behaviour in Saudi Arabia: Switching from brick and mortar stores to E-Commerce. *International Journal of Scientific & Technology Research*, *9*(07), 15-28.

Salop, S. C. (1979). Monopolistic competition with outside goods. *The Bell Journal of Economics*, *10*(1), 141-156.

Schiff, N. (2015). Cities and product variety : evidence from restaurants. *Journal of Economic Geography*, *15*(6), 1085-1123.

Schivardi, F., & Viviano, E. (2011). Entry barriers in retail trade. *The Economic Journal*, *121*(551), 145-170.

Sekhani, R., Mohan, D., & Roy, S. (2019). Inclusive urban eco-systems: A case study on street vending businesses in markets of Kolkata, India. *Asian Journal of Social Science*, *47*(4-5), 581-606.

Shaked, A., & Sutton, J. (1982). Relaxing price competition through product differentiation. *The Review of Economic Studies*, *49*(1), 3-13.

Shimomura, K. I., & Thisse, J. F. (2012). Competition among the big and the small. *The RAND Journal of Economics*, *43*(2), 329-347.

Singh, N., & Vives, X. (1984). Price and Quantity Competition in a Differentiated Duopoly. The *RAND Journal of Economics*, *15*(4), 546-554.

Smith, H., & Hay, D. (2005). Streets, malls, and supermarkets. *Journal of Economics & Management Strategy*, *14*(1), 29-59.

Smulders, S., and van de Klundert, T. (2003). Monopolistic competition and economic growth. In S. Brakman and B. Heijdra (Eds.), *The Monopolistic Competition Revolution in Retrospect*. Cambridge : Cambridge University Press.

Spanke, M. (2020). *Retail Isn't Dead: Innovative Strategies for Brick and Mortar Retail Success*. Springer International Publishing: Berlin/Heidelberg, Germany.

Spence, M. (1976). Product selection, fixed costs, and monopolistic competition. *The Review of Economic Studies*, *43*(2), 217-235.

Sweeney, K., Riley, J., & Duan, Y. (2022). Product variety in retail: the moderating influence of demand variability. *International Journal of Physical Distribution & Logistics Management*, *52*(4), 351-369.

Tan, G., & Zhou, J. (2021). The effects of competition and entry in multi-sided markets. *The Review of Economic Studies*, *88*(2), 1002-1030.

Teh, T. H., Liu, C., Wright, J., & Zhou, J. (2023). Multihoming and oligopolistic platform competition. *American Economic Journal: Microeconomics*, *15*(4), 68-113.

Tirole, J. (1988): *Theory of Industrial Organization*, MIT Press: Cambridge, Massachusetts.

Ushchev, P., Sloev, I., & Thisse, J. F. (2015). Do we go shopping downtown or in the 'burbs?. *Journal of Urban Economics*, *85*, 1-15.

Vialard, A., Ingleby, T., & Ring, P. (2017). *Fight for Frontage: A Shopping Mall On A High Street*. Retrieved from https://nrl.northumbria.ac.uk/id/eprint/32275/1/VialardIngleby Ring_2017_FightForFrontage.pdf. Accessed on October 10, 2024.

Vives, X. (1990). Nash equilibrium with strategic complementarities. *Journal of Mathematical Economics*, *19*(3), 305-321.

Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review*, *100*(4), 1642-1672.

Yang, X., & Heijdra, B. J. (1993). Monopolistic competition and optimum product diversity: Comment. *The American Economic Review*, *83*(1), 295-301.

Zhu, F., & Liu, Q. (2018). Competing with complementors: An empirical look at Amazon. com. *Strategic management journal*, *39*(10), 2618-2642.