Master of Arts Examination, 2024

(1st Year, 1st Semester)

ECONOMICS

[Econometrics I]

Time : Two Hours | Full Marks : 30

Answer ***any five*** of the following questions : 6×5=30

1. Consider the model $y = \alpha + \beta x + u$, $E(u) = 0$, where x is endogenous. Let z be an instrument for x, with $E(zu) = 0$. Let w be another random variable uncorrelated with z but correlated with $u : u = \gamma w + v$, with $E(v) = 0$, $E(w\,v) = 0$. Let $\tilde{\beta}$ and $\hat{\beta}$ be two estimators of β using instruments z and (z, w), respectively. Demonstrate which of $\tilde{\beta}$ and $\hat{\beta}$ is (asymptotically) more efficient.

2. Suppose family i chooses annual consumption c_i (in dollars) and annual contribution to a charitable fund q_i (in dollars) to solve the problem

$$\max_{c,q}\; c + a_i \log(1+q)$$

subject to the constraint $c + p_i q \le m_i$; $c, q \ge 0$, where m_i is the annual income of family i, p_i is the price of one dollar of charitable fund (where $p_i < 1$ because of tax-deductibility of charitable contributions) and this price differs across families because of different marginal tax rates and different state tax codes, $a_i \geq 0$ determines marginal utility of charitable contributions. Consider m_i and p_i to the family in this problem.

a) What is the optimal solution for q_i? 3

b) Define $y_i = 0$ if $q_i = 0$ and $y_i = 1$ if $q_i > 0$. Suppose $a_i = \exp(\mathbf{z}_i\gamma + v_i)$, where $\mathbf{z}_i$ is a $J \times 1$ vector of observable family traits and v_i is unobservable. Assume that v_i is independent of $(\mathbf{z}_i, m_i, p_i)$ and v_i / σ has symmetric distribution function $G(.)$, where $\text{var}(v_i) = \sigma^2$. Show that,

$P(y_i = 1 \mid \mathbf{z}_i, m_i, p_i) = G\left[(\mathbf{z}_i\gamma - \log p_i)/\sigma\right]$. 3

3. Consider the following regression

$$E(y \mid x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_2^2.$$

Let the population expectations of x_1 and x_2 be $E(x_1) = \mu_1$ and $E(x_2) = \mu_2$. Let α_1 denote the average partial effect (across the distribution of explanatory variables) of x_1 on $E(y \mid x_1, x_2)$ and let α_2 denote the same for x_2.

Prove that $\text{Avar}(\hat{\alpha}_1) = \text{Avar}(\tilde{\alpha}_1) + \beta_3^2(\sigma^2 / N)$, where $\hat{\alpha}_1$ is the sample estimator of α_1, $\hat{\alpha}_1$ is the estimator of α_1 if

we knew the value of μ_1 and μ_2, $\sigma^2 = \text{var}(x_2)$ and N is the sample size.

4. Suppose in a sample of n observations drawn from a binomial population with parameter p, the number of successes turn out to be x. Find the maximum likelihood estimate of p.

5. Prove that for the logit model the expression $\dfrac{f^2(\mathbf{x}_i\beta)}{F(\mathbf{x}_i\beta)(1 - F(\mathbf{x}_i\beta))}$ achieves its maximum value when $\mathbf{x}_i\beta = 0$ and decreases monotonically as $|\mathbf{x}_i\beta|$ increases.

6. Prove that convergence in probability implies convergence in distribution.

7. Consider the three equation model: $y = \beta x + u$, $x = \lambda u + \varepsilon$ and $z = \gamma\varepsilon + v$, where the mutually independent errors are normally distributed with zero mean and variances σ_u^2, σ_ε^2 and σ_v^2, respectively.

a) Is $\hat{\beta}_{OLS}$ consistent? What is the asymptotic bias in $\hat{\beta}_{OLS}$ if any?

b) Is $\hat{\beta}_{IV}$ consistent if z is used as an instrument for x?

3+3=6