BACHELOR OF ARTS EXAMINATION, 2024

(2nd Year, 1st Semester)

ECONOMICS

(Advanced Statistics)

Time : *Two Hours* *Full Marks* : 30

Answer *any* three from the following four questions.

1. *(a)* Let X be a random variable with moment generating function

$M(t) = [(2 + e^t) / 3]^9, -\infty < t < +\infty$.

Calculate the expectation of X and variance of X. 4

(b) A box contains 10 marbles out of which θ are white and the rest are red. We want to test the hypothesis $H_0 : \theta = 5$ against $H_1 : \theta = 4$. Suppose that H_0 is rejected if two marbles taken at random with replacement, are both red. Calculate probability of type I error. 4

(c) Arnab is shooting at a target. The probability of a hit is 0.4. What is the probability that his 10th trial results in the second hit. 2

2. *(a)* If the joint density function of X and Y is given as

$f(x, y) = 2, 0 < x < 1, 0 < x < y < 1,$

0, otherwise

What is the expectation of X? 5

(b) Describe the steps to use random sampling numbers for drawing a sample of size 7 from a population of size 30. Consider SRSWOR. 5

3. *(a)* Suppose a pair of numbers (x, y) is chosen randomly from the interval [0, 1], what is the probability that $y \leq x^2$? 4

(b) Test whether the following data is compatible with the assumption that there is no association between the weight and colour of the eyes : 6

Weight (in kg)	Colour of the eyes		
	Black	Brown	Blue
Less than 50	10	20	20
50–65	12	22	30
65–80	15	30	40
More than 80	8	10	8

4. *(a)* The random variable y has a probability density function

$$f(y) = (1 - \theta) + 2\theta y \text{ for } 0 < y < 1$$

$$= 0, \text{ otherwise}$$

Suppose y_1 and y_2 are two sample observations drawn independently from this distribution. *(i)* Suggest a method of moment estimator of θ based on sample observations. *(ii)* Find maximum likelihood estimator of θ based on these sample observations. 3+3

(b) Suppose X is a random variable following any distribution with mean μ and standard deviation σ.

(i) What is the maximum value of $P\{|x - \mu| \geq 4\sigma\}$?

(ii) If X follows normal distribution what is the value of $P\{|x - \mu| \geq 4\sigma\}$? 1+3

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