Index No. **81/19/Maths./26**

Nonlinear wave structures in multispecies plasmas

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Abstract

In this thesis, we have considered the following problems on dust-ion acoustic (DIA) waves in a collisionless magnetized/ unmagnetized five components electron-positron-ion-dust (e-p-i-d) plasma consisting of warm adiabatic ions, static negatively charged dust particulates, nonthermal hot electrons, isothermal cold electrons and nonthermal positrons.

Problem-1: Here, we have derived a Korteweg-de Vries-Zakharov-Kuznetsov (KdV-ZK) equation to study the nonlinear behaviour of DIA waves in a collisionless magnetized five components plasma. It is found that the coefficient of the nonlinear term of the KdV-ZK equation vanishes along different family of curves in different compositional parameter planes. In this situation, to describe the nonlinear behaviour of DIA waves, we have derived a modified KdV-ZK (MKdV-ZK) equation. When the coefficients of the nonlinear terms of both KdV-ZK and MKdV-ZK equations are simultaneously equal to zero, then we have derived a further modified KdV-ZK (FMKdV-ZK) equation which effectively describes the nonlinear behaviour of DIA waves. Analytically and numerically, we have investigated the solitary wave solutions of different evolution equations propagating obliquely to the direction of the external static uniform magnetic field. We have discussed the effect of different parameters of the system on solitary waves obtained from the different evolution equations.

Problem-2: Here, we have discussed the stability of DIA solitary waves obtained from the KdV-ZK and different modified KdV-ZK equations which have been derived in **Problem-1**. We have used the small-k perturbation expansion method of Rowlands and Infeld [J. Plasma Phys. **3**, 567 (1969); **8**, 105 (1972); **10**, 293 (1973); **33**, 171 (1985)] to analyze the stability of the steady state solitary wave solutions of the KdV-ZK equation and different modified KdV-ZK equations. In this method, we want to find a nonlinear dispersion relation of the nonlinear evolution equation connecting the lowest order of the wave frequency and the wave number. This nonlinear dispersion relation helps to analyze the stability of solitary structures of the KdV-ZK equation and different modified KdV-ZK equations. We have found the instability condition and the growth rate of instability up to the lowest order of wave number. We have analyzed graphically the growth rate of instability of solitary waves of different evolution equations with respect to different parameters of the plasma system.

Problem-3: In this problem, we have considered the quasi-neutrality condition instead of the Poisson equation along with other hydrodynamic conservation equations as described in **Problem-1** to investigate the nonlinear behaviour of arbitrary amplitude DIA waves. We have applied the Sagdeev pseudo-potential approach and phase portrait analysis to confirm the existence of different DIA nonlinear wave structures. The plasma system contains eleven parameters and the nonlinear wave structures have been studied through the compositional parameter space consisting of these eleven parameters. The effects of these parameters on the amplitude of the nonlinear wave structures have also been investigated. We have extensively discussed the existence of negative potential solitary waves (NPSWs), positive potential solitary waves (PPSWs), positive potential supersolitons (PPSSs), negative potential double layers (NPDLs) and super-nonlinear periodic waves. We have also analyzed the coexistence of PPSWs and NPSWs, the coexistence of NPSWs and PPSSs, the coexistence of

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NPDL and PPSSs. For increasing values of some parameters, there exists a sequence of NPSWs converging to the double layer solution of the same polarity, whereas it is also observed that a sequence of NPSWs converging to the double layer solution for decreasing values of some other set of parameters involved in the system. Therefore, the amplitude of the NPDL solution can be regarded as an upper bound of the amplitude of the sequence of the NPSWs. The dependence of the amplitudes of the PPSWs and PPSSs on the different parameters of the system has also been critically investigated.

Problem-4: Here, we have derived a nonlinear Schrödinger equation to study the modulation instability of DIA waves propagating obliquely to the direction of the uniform static magnetic field in the plasma system as defined in **Problem-1**. The nonlinear dispersion relation of the modulated DIA waves has been analyzed to study the instability regions in the parameter planes. This nonlinear dispersion relation has also been used to investigate the maximum modulational growth rate of instability of the modulated DIA waves. We have discussed the effect of different parameters of the system on the existence of the instability regions and the maximum growth rate of instability.

Problem-5: In this problem, the energy integral derived by using Sagdeev pseudo-potential technique has been analyzed to investigate the existence of arbitrary amplitude DIA solitons including double layers and supersolitons in a collisionless unmagnetized plasma system whose constituents are the same as given in **Problem-1**. The graphical analysis of Sagdeev pseudo-potential shows the existence of PPSSs along with PPDLs and PPSWs whereas on the negative potential side, the system does not support negative potential supersolitons (NPSSs) but the existence of NPDLs, NPSWs, the coexistence of both PPSWs and NPSWs and super-nonlinear periodic waves have been established. To explain the existence of different DIA solitary structures, phase portraits of the dynamical system corresponding to the different DIA solitary structures have been drawn. With the help of phase portraits, the transition of PPSWs just before and just after the formation of PPDL has been discussed. We have discussed the effect of different parameters of the system on the amplitude of the different DIA solitary structures including supersolitons.

Problem-6: Here, we have investigated the arbitrary amplitude DIA nonlinear wave structures at the acoustic (sonic) speed $M = M_c$ in a collisionless unmagnetized plasma system as defined in **Problem-5**. The present plasma system confirms the existence of NPSWs, PPSWs and NPDLs at the acoustic speed $M = M_c$. We have studied the effect of different parameters of the system on the amplitude of PPSWs, NPSWs and NPDLs at $M = M_c$. We have also analyzed the difference between various DIA nonlinear wave structures at supersonic speed, subsonic speed and sonic speed through phase portraits of the dynamical system corresponding to nonlinear DIA wave structures.

The present thesis has been written by considering the following published articles: *Z Naturforsch. A* 77, Article No. 659 (15 pages) (2022), *Indian J Phys.* 98, 771-783(2024), *Plasma Phys. Rep.*, **49**, pp. 467 (17 pages) (2023), *Eur. Phys. J. Plus*, **138**, pp. 734 (20 pages) (2023)

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