M. TECH. COMPUTER TECHNOLOGY FIRST YEAR SECOND SEMESTER EXAMINATION, 2024
THEORY OF COMPUTING

Time: Three hours **Full Marks: 100**

Answer any five questions

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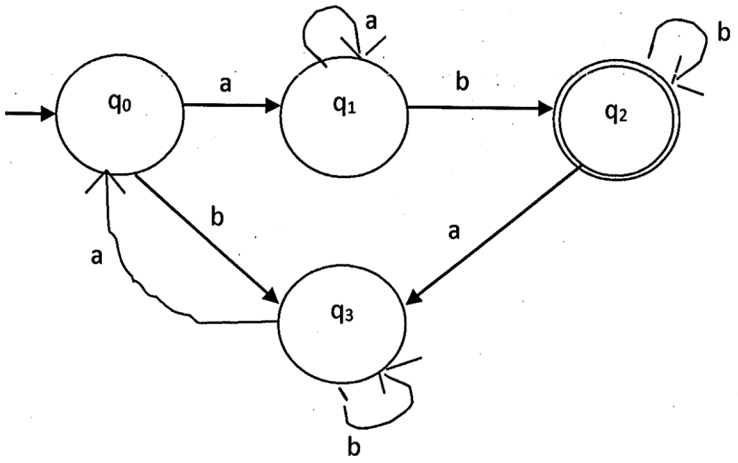
1. Design Turing Machines for the following languages
   a) L= $\{0^n1^n2^n \mid n \geq 1\}$
   b) Palindrome over $\{a, b\}^*$

10 + 10

2. Construct PDA accepting by a final state for the following languages
   a) (i) L= $\{a^nb^ma^n \mid m, n \geq 1\}$
   (ii) L= $\{a^n\, b^m : m < n\}$
   b) Construct a Context Free Grammar for each of the following with proper justification
   (i) L= $\{a^mb^{m+n}c^n \mid m, n \geq 1\}$
   (ii) L= $\{w \mid n_a(w) = 2n_b(w)\}$
   (where $n_a(w)$ means number of occurrences of symbol 'a' in string w)

(4 + 4) + (6 + 6)

3. a) Show that
   (i) any regular set is closed under transpose.
   (ii) any regular set is closed under complementation.
   b) For the following DFA, find an equivalent regular expression



   c) Find a regular expression for
   (i) $L = \{ab^nw \mid n \geq 3, w \in \{a, b\}^+\}$
   (ii) $L = \{a^nb^m \mid n > 4, m \leq 3\}$

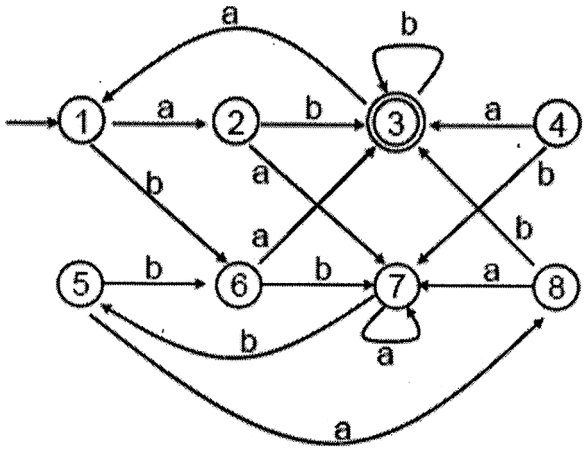
(3 + 3) + 6 + (4 + 4)

4. a) Show that "There exists a language over $\sum$ that is not recursively Enumerable".
   b) Let A_{TM} = {(M, w) | The TM accepts w}. Show that A_{TM} is undecidable.
   c) State Cook's theorem with all terms explained.

8 + 8 + 4

5. a) Find the union of two langauges over {a,b}
   i. all strings with at least one a
   ii. all strings with exactly two b's

   b) Design a grammar for L= $\{0^n1^n2^n \mid n \geq 1\}$ confirming that it does work.
   c) Minimize the following DFA



(7 + 5 + 8)

6. a) Taking language L = $\{a^kb^k \mid k \geq 0\}$ as an example and three machines, M_1, M_2, and M_3, it can be shown that the runtime depends on a computational model. Machines, M_1, M_2, and M_3 are described as

- 1-tape Turing Machine M1: $O(n^2)$
- 1-tape Turing Machine M2: O(n log n)
- 2-tape Turing Machine M3: O(n)

Design, explain the acceptability of the language L, and derive time complexity for machines M_1, M_2, and M_3.

b) Show that a k-tape Turing machine can be simulated in a 1-tape Turing machine with polynomial overhead.

12 + 8

7. Construct a Turing machine for the function f(m,n) = m × n
(Validate the construction taking at least one valid and another invalid string)

20

8. Prove that
   a) If Language L is recursive then its complement $\bar{L}$ is also recursive.
   b) If Languages L and $\bar{L}$ are Recursively Enumerable, then L is recursive.
   c) The intersection of Recursive and Recursively Enumerable Languages are Recursively Enumerable.
   d) The union of two Recursive Languages are recursive.

(4 × 5)

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