Ref. No.: Ex/PG/CTLE/T/112A/2024

MASTER IN CONTROL SYSTEM ENGINEERING

1ST YEAR 1ST SEMESTER EXAMINATION, 2024

Subject: CONTROL SYSTEM ENGINEERING **Time: Three Hours** **Full Marks: 100**

Part I (50 marks)

Use a separate answer-script for each part

Question No.

Question 1 is compulsory

Answer ***Any Two*** questions from the rest (2×20)

Marks

Q1 Answer ***Any One*** : (a) ***or*** (b)

(a) A mechanical system, as shown in Figure Q1-(a), is subjected to 2 lb of force (step-input) at time t=0. The resulting response (displacement x in ft) exhibits a peak overshoot of 9.5% at time t =2 sec and finally settles down to a value of 0.1 ft as shown in Figure Q1-(b). 10

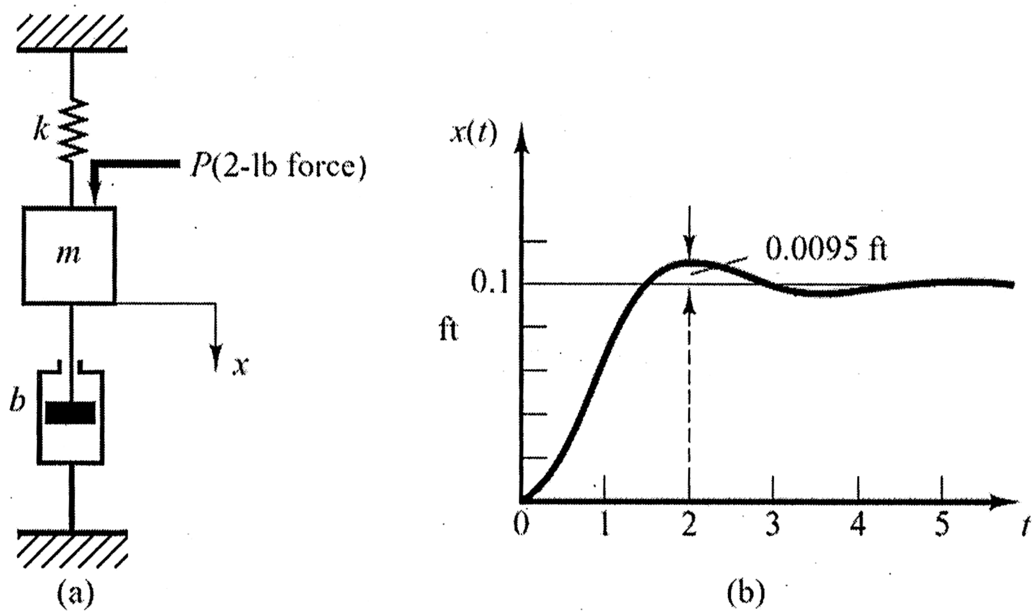


Figure Q1

Determine m, b, and k of the system.

The displacement x is measured from the point where spring and mass are at static equilibrium.

OR

[Turn over

Ref. No.: Ex/PG/CTLE/T/112A/2024

Q1 (b) The block diagram of a control system is shown in Figure Q1(b) with

$$G(s) = \frac{100}{(1+0.1s)(1+0.5s)}$$

(i) Find the step-, ramp-, and parabolic-error constants. Assume the system to be stable. 6+4

(ii) Determine the minimum steady-state error that can be achieved with a unit-ramp input by varying the values of K and K_t.

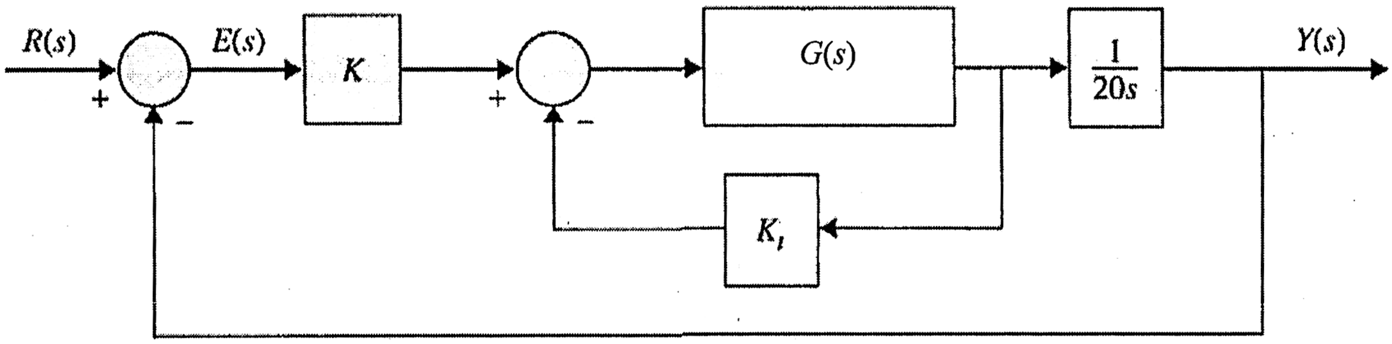


Figure Q1(b)

Q2 (a) Consider the system shown in Figure Q2(a), which involves velocity feedback. Determine the values of the amplifier gain K and the velocity feedback gain K_h so that the following specifications are satisfied: 14

(i) Damping ratio of the closed-loop poles is 0.5.

(ii) Settling time ≤ 2 sec.

(iii) Static velocity error constant $K_v \geq 50$ sec $^{-1}$.

(iv) $0 < K_h < 1$.

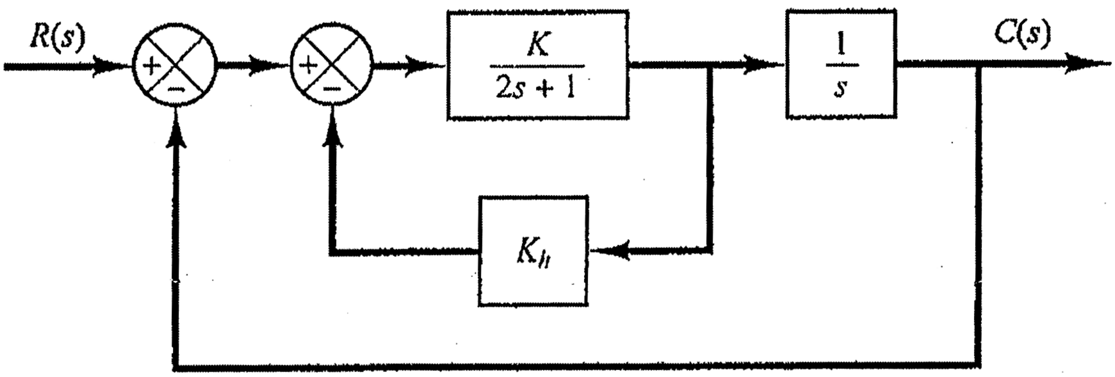


Figure Q2(a)

(b) Show that the root loci for a unity negative feedback system with

$$G(s) = \frac{K(s^2+6s+10)}{s^2+2s+10}$$ 6

will be the arcs of a circle centered at the origin with radius $\sqrt{10}$.

Ref. No.: Ex/PG/CTLE/T/112A/2024

Q3 (a) Consider the system shown in Figure Q3(a). Plot the root loci as the value of k varies from 0 to ∞. 8

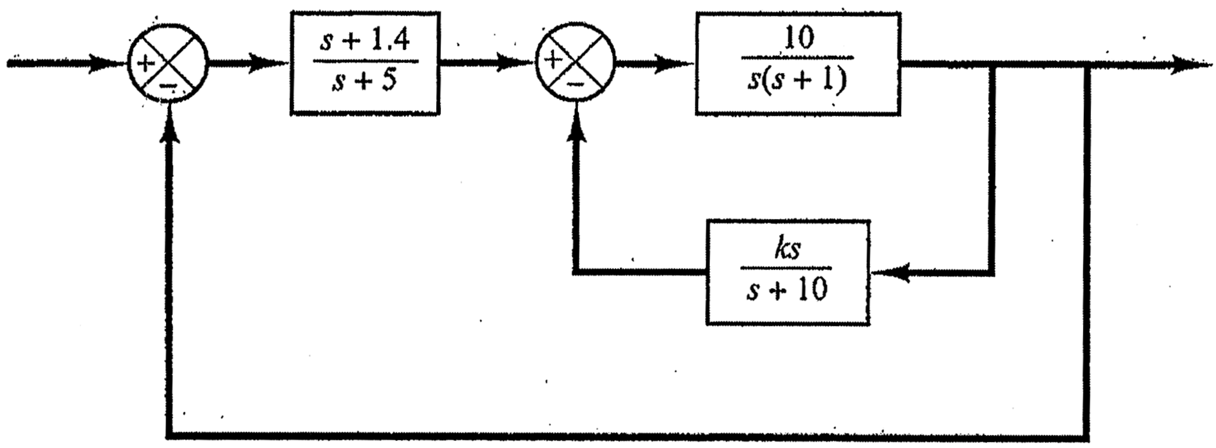


Figure Q3(a)

(b) Consider the servo system with tachometer feedback shown in Figure Q3(b). Determine the ranges of stability for K and K_h. (K_h must be positive.) 6

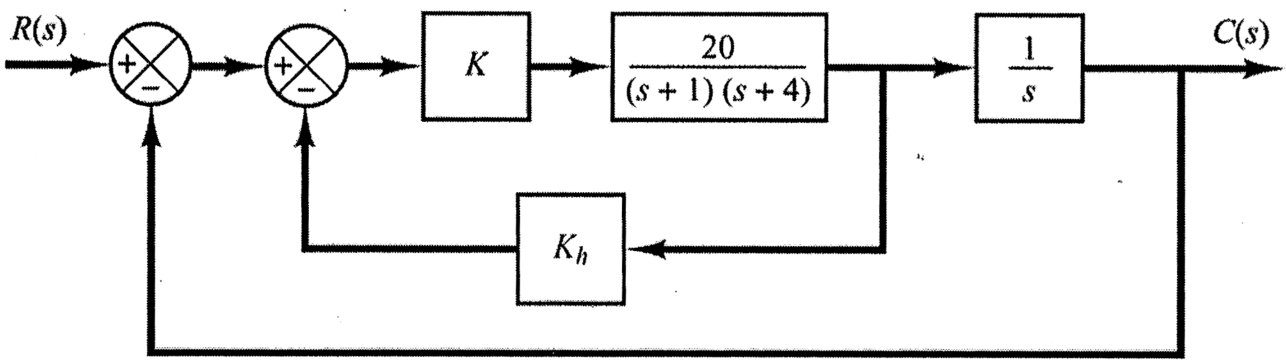


Figure Q3(b)

(c) Show how the following aspects of a control system can be improved by feedback: 3+3

(i) speed of response, (ii) sensitivity to parameter variations.

Q4 (a) Define "Type" of a system? For a unity feedback system define the following terms and find the expressions for steady-state error in response to Step, Ramp and Parabolic inputs in terms of these constants: 2+6

(i) Static position error constant

(ii) Static velocity error constant

(iii) Static acceleration error constant

Ref. No.: Ex/PG/CTLE/T/112A/2024

Q4 (b) A control system with a PD controller is shown in Figure Q4(b). Find (analytically) the values of K_P and K_D so that the ramp-error constant K_v is 1000 and the damping ratio is 0.5. 6

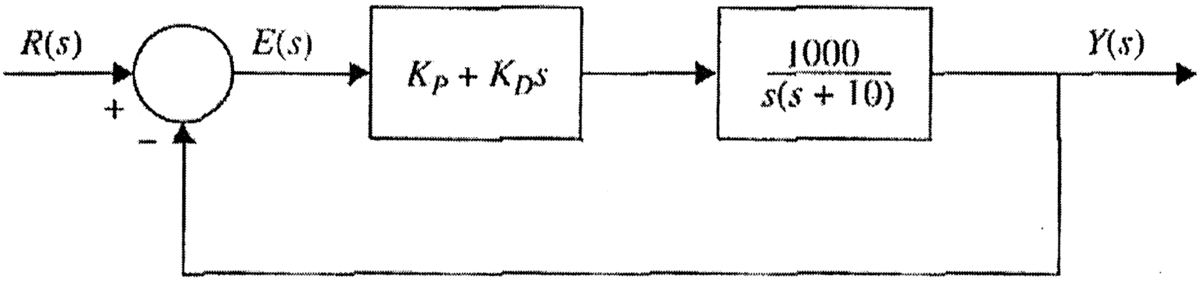


Figure Q4(b)

(c) Obtain the transfer function of the electrical network shown in Figure Q4(c).
Determine whether this is an electrical *lead network* or *lag network*? 4+2

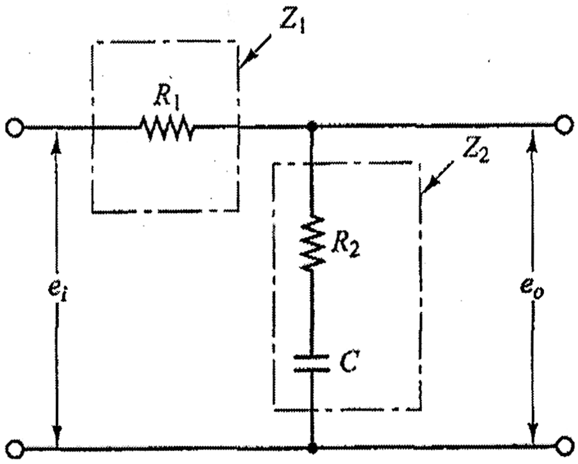


Figure Q4(c)

Q5 (a) What is *Set Point Kick* phenomenon associated with PID Controllers? What are the components responsible for such behavior? 2+2

Show, with the help of block diagrams, how the PID control configuration needs to be modified to eliminate such phenomena. 6

(b) Describe, stating the assumption, the Ziegler Nichols method of PID controller tuning based on unit step test. 4

(c) Consider a control system in which a PID controller is used to control the plant 6

$$G(s) = \frac{1}{s(s+2)(s+4)}$$

Determine the parameters of PID controller by Ziegler-Nichols tuning rule.

Ref. No.: Ex/PG/CTLE/T/112A/2024

MASTER IN CONTROL SYSTEM ENGINEERING

1ST YEAR 1ST SEMESTER EXAMINATION, 2024

Subject: CONTROL SYSTEM ENGINEERING **Time: Three Hours** **Full Marks: 100**

Part II (50 marks)

Question No.	Answer *Any FIVE* questions (5×10)	Marks

Q1 (a) Comment, by inspection, on linearity and time-invariance property of the following system: 2

$$\ddot{y} + 5\cos y + 2\dot{y} + y^2 = -u$$

(b) For the following Matrix **A**: 4+4

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(i) Find the eigenvalues λ_1, λ_2, λ_3 and λ_4 of the matrix **A**.

(ii) Obtain a transformation matrix ***P***, such that

$$\boldsymbol{P^{-1}AP} = diag\ (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

Q2 (a) Consider the system defined by 5+1

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}, \quad y = \boldsymbol{Cx}$$

where,

$$\boldsymbol{A} = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 0 \\ -1 & 0 & -4 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{C} = [1\ 1\ 0]$$

Derive the transfer function $Y(s)/U(s)$. Comment on the stability of the system.

(b) Find $x_1(t)$ and $x_1(t)$ of the system described by 4

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where the initial conditions are

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Q3 (a) What is detectability? 1

(b) Discuss Popov-Belevich-Hautus (PBH) test for observability. 2

(c) Given a system in state equation form as 7

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu},$$

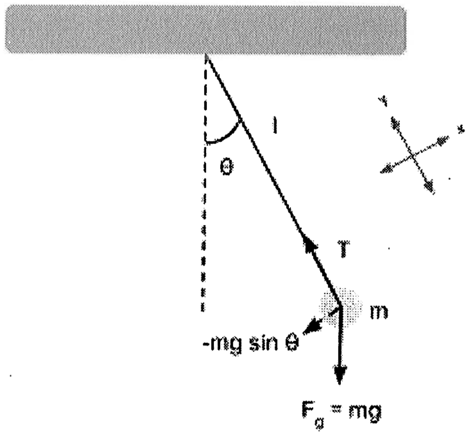
with, $\boldsymbol{A} = \begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Determine, with the help of Popov-Belevich-Hautus test the controllability of the system.

Ref. No.: Ex/PG/CTLE/T/112A/2024

Q4 (a) State the difference between a Forced Equilibrium and a Natural Equilibrium. 2

(b) Consider a simple pendulum system, as shown in the figure, with
bob of mass $\boldsymbol{m}$,
string of length $\boldsymbol{l}$.
Let the tension in the string be $\boldsymbol{T}$ at an angle $\boldsymbol{\theta}$.
(i) Derive the dynamical equation of the system.
(ii) Find out the equilibrium points and discuss their characteristics in terms of stability. 4+4



Q5 (a) A system is described by the following equation

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$$

Obtain a state space representation of this system. Comment on controllability of the system. 3+2

(b) Consider a system defined by state equation

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}, \quad \text{with } \boldsymbol{A} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \text{ and } \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Design, using Ackermann's formula, a state feedback controller to place the closed-loop poles at $s = -2 \pm j0$. 5

Q6 (a) Draw the block diagram for full-order observer-based state feedback control scheme. 5

(b) Derive the equation to describe the error dynamics of a full-order observer. 5

Q7 A system is given by the following state and output equations

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}, \quad y = \boldsymbol{Cx},$$

$$\text{with } \boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \text{ and } \boldsymbol{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The system is compensated using a full-order observer based state-feedback control with the following desired criterion: 10

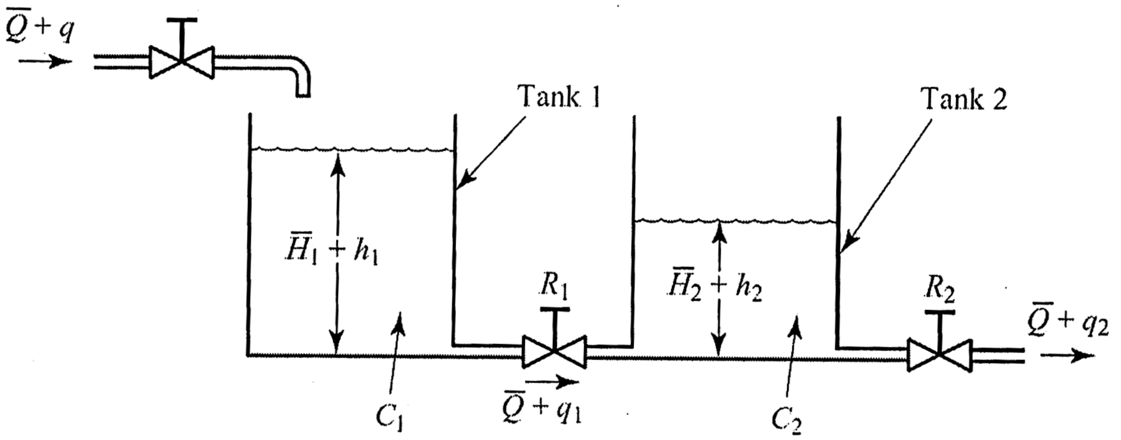
i. Location of closed-loop poles at $s = -3, -5$.
ii. Location of observer poles at $s = -10 \pm j2$,

Design the observer. Determine the necessary state-feedback gain matrix $\boldsymbol{K}$.

With necessary block diagram represent the compensated system.

Ref. No.: Ex/PG/CTLE/T/112A/2024

Q8 (a) Consider the system as shown in Figure, where the two tanks interact with each other.



6+2

$\overline{Q}$: Steady-state flow rate
$\overline{H}_1$: Steady-state liquid level of tank 1
$\overline{H}_2$: Steady-state liquid level of tank 2

(i) Derive the nonlinear state-space equations for the system.

(ii) Briefly discuss the control problem involved herein.

(b) What is *Controllability Gramian*? Explain its significance over Kalman controllability matrix. 2