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Study of Functional Identities Involving Generalized Derivations and Related Additive Maps in Prime Rings

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Abstract

Motivated by ordinary derivative in several branches of mathematics the notion of derivation of ring was introduced long back. But the study of derivation got its momentum after the work of Posner [4]. For as long as rings and algebras have been studied for their own sakes, it has been a problem of interest to determine the consequences of various special identities and conversely, to find sufficient conditions on a given ring which ensure that a specified identity holds. It is well known that there is a strong relationship between the functional identities (FI) involving derivations, generalized derivations and the structure of the rings. Ring derivation is a branch of algebra in which we study about the structure of additive maps as well as structure of rings by analyzing some functional identities involving additive maps. These additive maps are derivation, skew derivation, generalized derivation, generalized skew derivation, X-generalized derivation, X-generalized skew derivation, X- generalized (α, β)-derivation, etc. The main objective of the thesis is to study some functional identities and generalized functional identities in prime and semiprime rings. Let R be an associative ring. Then a simple example of functional identity is the identity $[f(x), x] = 0$ for all $x \in R$, where $[x, y] = xy - yx$ and $f : R \to R$ is a mapping. In 1957, Posner [4] studied a special kind of functional identities in rings by taking the above function as a derivation. Much more recently, in [1], Brešar proved that if f is any additive mapping satisfying the identity studied by Posner [4] in prime ring R, then f must be in the form $f(x) = \lambda x + \xi(x)$, where $\lambda \in C$ and $\xi : R \to C$ is an additive mapping, C is the extended centroid of R.

This thesis contains eight chapters and in each chapter we study different type of identities under certain conditions. Chapter-wise brief information is given bellow:

Chapter 1 is basically an introduction to some basic definitions, preliminaries and prerequisites which are collected from other references and those are needed for the development of the subsequent chapters of this thesis.

Two additive maps $F, G : R \to R$ are said to be co-commuting (co-centralizing) on R if $F(x)x - xG(x) = 0$ for all $x \in R$ (resp. $F(x)x - xG(x) \in Z(R)$ for all $x \in R$).

In Chapter 2, we study an identity involving three generalized derivations in prime rings for the generalization of the concept of co-commuting maps.

There are many papers in literature which have studied mapping behave like a derivation as well as a homomorphism. El Sofy [3] introduced the concept of homoderivation maps on a ring R. An additive mapping H from R into itself is called homoderivation if $H(xy) = H(x)H(y) + H(x)y + xH(y)$ for all $x, y \in R$. Now, we can define Jordan homoderivation maps in a ring R. An additive mapping H

from R into itself is called Jordan homoderivation if $H(x^2) = H(x)H(x) + H(x)x + xH(x)$ for all $x \in R$.

In **Chapter 3**, we study the Jordan homoderivation behavior of three generalized derivations in prime rings. More precisely, we study the identity $F(x^2) = G(x)^2 + H(x)x + xH(x)$ for all $x \in f(R)$, where R is a prime ring and F, G, H are three generalized derivations.

Chapter 4 is devoted to the study of generalized derivations with annihilator conditions in prime rings.

In **Chapter 5**, we study an identity involving two generalized skew derivations in prime rings for generalization of commuting maps on prime rings.

In this line of investigation, De Filippis [2] introduced the new map *b-generalized skew derivation*. We note that b-generalized skew derivation generalizes the concept of generalized skew derivation as well as b-generalized derivation.

In **Chapter 6**, we study an identity involving two b-generalized skew derivations in prime rings.

Over the last few decades, several authors have investigated the relationship between the commutativity of the ring R and some specific types of maps of R. The first result in this context is due to Posner [4], who proved that if a prime ring R admits a nonzero centralizing derivation d, then R must be commutative.

In **Chapter 7**, we study some commutativity theorems involving multiplicative (generalized)-derivations in prime and semiprime rings. Some examples are given at the end of this chapter concluding that semiprimeness hypothesis in the theorems are not superfluous.

Lastly, in **Chapter 8**, we study some functional identities involving b-generalized (α, β)-derivations in prime rings to extend many known results in literature.

References

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