Mathematical Modeling of Electro-osmotic Flow and Heat Transfer in the Rough Micro-channels

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Abstract

This thesis investigates the mathematical modeling of electroosmotic flow and heat transfer in microchannels and microtubes, especially focusing on how surface roughness affects these processes. It also examines the impact of magnetic fields, system rotation, and porous media on electroosmotic flow behavior, considering their interaction with surface roughness. This research study is motivated by the increasing importance of microfluidic systems, especially in applications like micro-heat exchangers, microreactors, micropumps, and lab-on-a-chip devices.

The thesis comprises an introductory chapter (**Chapter 1**) that explains microfluidics and electrokinetic phenomena. Subsequently, six main chapters explore various aspects of the new research findings, giving a detailed analysis of the problem.

In **Chapter 2**, the combined effects of electroosmosis and surface roughness on rotating electrothermal flow in a microchannel under a magnetic field are studied. The governing equations, including the Navier-Stokes equations, energy equation, and the Poisson-Boltzmann equation for EDL potential, are formulated. The analytical solutions are obtained using the separation of variables method combined with cosine Fourier series expansion. The analysis shows that surface roughness significantly affects flow patterns by increasing wall shear stress and friction, leading to a higher Poiseuille number. The Coriolis force from rotation increases entropy generation, causing more internal irreversibilities.

Chapter 3 investigates the electromagnetohydrodynamic flow of two immiscible fluids in a hydrophobic microchannel with a topographically charged rough surface. The mathematical model includes the Navier-slip and the changing zeta potential at the fluid-solid interface. It also considers the balance of viscous stress and zeta potential jump at the liquid-liquid interface. Analytical solutions are obtained using the perturbation technique. The investigations show that surface roughness intensifies flow disturbances, leading to the formation of microfluidic droplets. The zeta potential difference at the fluid-fluid interface can cause one layer to move faster than the other. Increased surface roughness intensifies flow disturbances, leading to microfluidic droplet formation.

In **Chapter 4**, the heat transfer characteristics associated with combined electromagnetohydrodynamic flow in a microchannel with regular wavy rough walls through a porous medium are examined. The solutions for the velocity and potential distribution equations are obtained using

the perturbation technique. The analytical solution for temperature distribution, considering Joule heating, is derived to study the thermal characteristics. The study reveals that the mean velocity decreases with increasing Darcy number but has an enhancing effect on the applied magnetic field, while the rate of heat transfer increases with an increase in Joule heating effects and Hartmann number.

Chapter 5 examines how complex wavy rough walls affect thermo-fluidic transport with combined electromagnetohydrodynamic flow in a porous microchannel. The governing equations for thermo-fluidic transport are solved analytically using the perturbation technique, with the volumetric flow rate derived analytically and the Nusselt number computed numerically. The study shows that the velocity profiles become fully developed in a transverse electric field, and the rate of heat transfer enhances with an increase in the roughness of the microchannel surface.

In **Chapter 6**, the thermal transport characteristics of nanofluid through a wavy microchannel with anisotropic porous medium under electromagnetohydrodynamic effects are analyzed. The governing equations for the electric double layer (EDL) potential, velocity, and temperature distributions are solved using numerical methods. The results are validated against analytical asymptotic solutions. The study reveals that anisotropy and permeability ratio of the porous medium significantly affect flow and temperature profiles, with Forchheimer inertial effect generating frictional heating and slug flow behavior.

Chapter 7 studies the electromagnetohydrodynamic pumping in a fluid-saturated anisotropic porous microtube with a rough surface. The analytical solutions are found to solve the governing equations, including the linearized Poisson-Boltzmann and Navier-Stokes equations, for this problem at low Reynolds numbers. The analysis reveals that the EMHD pumping rate is affected by a second-order term $(-\varepsilon^2\eta)$, where $\eta = \eta_1 + \eta_2$, indicating the effect of roughness on the mean pumping rate. The study shows that surface roughness alters the boundary layer near the microtube wall, leading to a thinner boundary layer and increased pumping rate, and higher anisotropic permeability introduces uneven resistance forces, leading to the formation of smaller eddies and boluses.

In the concluding **Chapter 8**, the thesis provides a concise summary of the important findings, emphasizing the influence of surface roughness on electroosmotic flow and heat transfer. The practical implications for lab-on-a-CD applications are highlighted, and avenues for future research, including further exploration of rough microchannels and microtubes, are outlined.

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