

Abstract

Title of the thesis: A Study of Some Aspects of Ideals in Semirings, Ternary Semirings and Ternary Hypersemirings

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In this thesis, we have enlightened some aspects of ideals in semirings, ternary semirings and ternary hypersemirings.

Firstly, the concepts of 2-prime and n -weakly 2-prime (resp. weakly 2-prime) ideals in a commutative semiring have been introduced and studied. Characterization of valuation semirings in terms of 2-prime ideals has been obtained. Semirings where 2-prime ideals are prime and semirings where every proper ideal is n -weakly 2-prime (resp. weakly 2-prime) ideals have also been characterized.

Secondly, the concepts of 1-absorbing prime ideals and weakly 1-absorbing prime ideals of commutative semirings have been introduced. Some important properties, results and various characterizations of 1-absorbing prime (resp. weakly 1-absorbing prime) ideals have been established. The relationships among 1-absorbing prime ideals, prime ideals, 2-prime ideals and 2-absorbing ideals have also been investigated.

After that, the notion of n -ideals has been introduced and studied. Semirings in which every proper ideal is n -ideal has been characterized. The characterization of entire semirings in terms of n -ideals has been established. Moreover, $\$n\$$ -ideals under various contexts of constructions such as homomorphic images, localizations, direct products, and expectation semirings have been studied.

Then the notions of left bi-quasi ideals and bi-quasi ideals of a ternary semiring have been introduced, which is an extension of the concept of bi-ideals in a ternary semiring. The concepts of minimal left bi-quasi ideals and left bi-quasi simple ternary semiring have also been studied. Characterization of regular ternary semirings in terms of left bi-quasi ideals have been obtained.

Next, the concept of 3-prime ideals in a ternary semiring has been introduced. Further, the concept of quasi 3-primary ideals as a generalization of 3-prime ideals and primary ideals in a commutative ternary semiring has been introduced. The relationships among prime ideals, 3-prime ideals, primary ideals, quasi primary ideals and quasi 3-primary ideals have been established. Various properties and examples concerning 3-prime ideals and quasi 3-primary ideals have been studied. Analogous theorems to the primary avoidance theorem for quasi 3-primary ideals have been established.

After that, the smallest strongly regular relation δ^* has been studied on a ternary hypersemiring S , so that the quotient structure is a ternary semiring. The notion of a

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fundamental ternary semiring with respect to the fundamental relation δ^* on a ternary hypersemiring S has been introduced. Then the connection between ternary hypersemirings and ternary semirings via the fundamental relation has been established.

Finally, the notions of prime hyperideals, primary hyperideals and maximal hyperideals in ternary hypersemirings have been introduced and some of their important properties analogous to the properties in hypersemirings have also been studied. A ternary semiring $P.(S)$ has been constructed from strongly distributive ternary hypersemiring S . Then an inclusion preserving bijection between the set of all prime hyperideals of S and the collections of all prime total subtractive ideals of $P.(S)$ has been established. The concepts of prime and primary avoidance theorems in ternary hypersemirings for C -ternary hyperideals have also been generalized.

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