

Study of Some Aspects of Clean Semirings

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Abstract

Rings in which every element is the sum of an idempotent and a unit are said to be clean rings and this notion was introduced by W.K. Nicholson in the study of exchange rings. Since then various generalizations of clean rings have been obtained by many authors. The algebraic theory of semirings has experienced remarkable growth in recent years. A semiring, which extends the concepts of a ring and a distributive lattice, has seen significant development. In this thesis we have introduced the concept of clean semiring and exchange semiring as a generalization of clean ring and exchange ring. We also aim to shed light on different generalizations of clean semiring. Here some characterizations of certain classes of clean semirings are studied.

First, the concept of clean semiring and exchange semiring is discussed. The concept of clean ring was introduced by W.K. Nicholson in his study of exchange rings. In this thesis we have introduced the concept of clean semiring as a generalization of clean ring. A semiring is said to be clean if its every nonzero element can be written as the sum of an idempotent and a unit. We have studied the notion of clean semiring and obtained some important characterizations of clean semiring. We have also studied the notion of exchange semiring and found out the connection between clean semiring and exchange semiring.

In this thesis, we have introduced and studied the concept of strongly clean semiring. Let S be a semiring. An element $a \in S$ is called strongly clean if $a = e + u$ with e an idempotent in S and u a unit in S such that $eu = ue$. A semiring S is said to be strongly clean if every nonzero element of S is strongly clean. We have mainly studied the notion of strongly clean semiring and obtained some important characterizations of strongly clean semiring in connection with exchange semiring, antisimple semiring and additively regular semiring.

We have also introduced the concept of k -unit clean semiring which generalizes the notion of clean ring as well as clean semiring. Let S be a semiring with identity 1. An element $a \in S \setminus \{0\}$ is said to be a k -unit if there exist $r_1, r_2 \in S$ such that $1 + r_1a = r_2a$ and $1 + ar_1 = ar_2$. Basically, k -units are the generalization of units in a semiring with zero element. An element $a \in S$ is called k -unit clean if $a = e + u$, where e is an idempotent and u is a k -unit of S . A semiring S is said to be k -unit clean if every nonzero element can be written as the sum of an idempotent and a

k -unit. We have obtained some important characterizations of k -unit clean semiring in connection with exchange semiring, antisimple semiring and inverse semiring.

Let S be a semiring. An element $r \in S$ is said to be k -regular element if there exist $r_1, r_2 \in S$ such that $r + rr_1r = rr_2r$. An element $a \in S$ is called k -regular clean if $a = e + r$, where e is an idempotent and r is a k -regular element of S . A semiring S is said to be a k -regular clean semiring if every element can be written as the sum of an idempotent and a k -regular element of S . In this thesis, we have introduced the concept of k -regular clean semiring which generalizes the notion of clean ring, clean semiring and k -unit clean semiring. We have obtained some important characterizations of k -regular clean semiring in connection with antisimple semiring, additively inverse semiring and zeroic semiring.

Motivated by the work of k -unit clean semiring and clean index of a ring, in this thesis, we have introduced the concept of k -unit clean index of a semiring S . If $S = \begin{bmatrix} A & M \\ O & B \end{bmatrix}$ be the formal triangular matrix semiring and k -unit clean index of S is finite then, in this thesis, we have determined k -unit clean index of A and B . Finally, we have characterized the semirings of k -unit clean indices 1 and 2, with the help of some other class of semirings.

We have introduced the concept of nil clean semiring. A semiring is said to be nil clean semiring if every element can be written as the sum of an idempotent and a nilpotent element. We have obtained some important results of nil clean semiring in connection with duo semiring, k -duo semiring, exchange semiring, k -semipotent semiring.

We have introduced the concept of nil clean index of a semiring S . If $S = \begin{bmatrix} A & M \\ O & B \end{bmatrix}$ be the formal triangular matrix semiring and nil clean index of S is finite then we have determined nil clean index of semirings A and B . We have also determined the relation between nil clean index and k -unit clean index of a semiring S . Finally, we have characterized the semirings of nil clean indices 1 and 2, with the help of some other class of semirings.


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