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On Some Wave Structure Interaction Problems In The Linearised Theory Of Water Waves

ABSTRACT

The present thesis focuses on investigating water wave propagation problems in water under the assumption of linearised theory and associated mathematical methods. Among the imporatant classes of water wave propagation problems, one of the most significant is the scattering of waves by obstacles with diverse geometric shapes. When a series of surface waves, originating from a considerable distance, encounters an obstacle within a fluid, it undergoes partial reflection off the obstacle and partial transmission either beneath or over it. In the context of the linearized theory of water waves, the determination of reflection and transmission coefficients constitutes a wave scattering problem. Here in the present thesis we have worked on the following scattering problems.

1. Propagation of surface gravity waves by a submerged thin elastic plate beneath an ice cover.
2. Effect of porosity on wave scattering by a vertical porous barrier over a rectangular trench.
3. Scattering of water waves by rectangular thick barriers in presence of surface tension.
4. Water wave scattering by thick rectangular slotted barriers in presence of ice cover.
5. Wave propagation in presence of a curved barrier with variable permeability.
6. Water wave interaction with a vertical wall with a gap submerged in deep water.

In the problem 1, a theoretical analysis involving surface gravity waves propagation by oblique incidence wave due to a thin elastic plate submerged in finite depth water with ice cover is studied. Applying Greens function technique, the boundary value problem is reduced to integral equations whose solutions are then utilised to find the reflection and transmission coefficients.

The interaction of surface wave with a vertical porous barrier over a rectangular trench is studied in problem 2 under the assumption of linearized theory of water waves. For the solution, the fluid region is divided into four subregions depending on the position of the barrier and the trench. Using the eigen function expansion of water wave potential in different regions along with suitable matching conditions at the interface of different regions, the problem is formulated in terms of three integral equations. Considering the edge conditions at the submerged end of the barrier and at the edges of the trench, these integral equations are solved using multi-term Galerkin approx- imation technique taking orthogonal Chebyshevs polynomials and ultra-spherical Gegenbauer polynomial as its basis function. Using the solutions of the integral equations, the reflection coefficient, transmission coefficient, energy dissipation coefficient and horizontal wave force are determined.

The influence of surface tension on an obliquely incident waves in presence of thick rectangular barriers present in water of uniform finite depth is discussed in problem 3. The

three different configurations of the thick barrier, viz., bottom-standing submerged barrier, fully submerged rectangular block and fully submerged block extending down to the bottom with a finite gap are considered. An appropriate multi-term Galerkin approximation technique involving ultraspherical Gegenbauer polynomial is employed for solving the integral equations arising in the mathematical analysis. The reflection and transmission coefficients are evaluated by utilizing the solution of the integral equations.
The problem 4 is concerned with scattering of surface water waves by a thick vertical slotted barrier of rectangular cross section with an arbitrary number of slots of unequal lengths present in water of finite depth with ice cover surface. The problem is formulated in terms of an integral equation in disjoint intervals, by suitable matching of eigen function expansion of water wave potential. Galerkin approximations involving ultraspherical Gegenbauer polynomials are utilised in the mathematical analysis for solving the integral equations to obtain a very accurate numerical estimates for the reflection coefficient.
The phenomena of scattering of waves by a circular arc shaped barrier with nonuniform porosity is studied in problem 5. The water region is considered to be infinitely deep or of finite depth. Based on judicious application of Greens integral theorem, the corresponding boundary value problem is reduced to a hypersingular integral equation of second kind. The boundary element method and the collocation method are adopted to solve the hypersingular integral equation and a good matching of the solutions obtained by two methods are ensured. The reflection coefficient and energy dissipation are evaluated by using the solution of the integral equation which are then studied graph- ically.
The problem of water wave scattering by a thin vertical wall with a gap submerged in deep water is studied using singular integral equation formulation. The corresponding boundary value problem is reduced to a Cauchy type singular integral equation of first kind in two disjoint intervals where the unknown function satisfying the integral equation has square root zero at the end points of the two intervals. In this case the solution exists if the forcing function satisfies two solvability conditions. The reflection coefficient is determined here using the solvability conditions without solving the integral equation and also the boundary value problem.

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