Abstract

"Classical and quantum analysis of gravitational singularity: A study of Raychaudhuri Equation"

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The thesis consists of seven chapters and is entirely devoted to the classical and quantum aspects of the Raychaudhuri equation in different background geometries. The work presented in the thesis not only shows the application of the Raychaudhuri equation in identifying the gravitational singularity but also suggests some classical and quantum mechanical tools to mitigate the singularity problem. The novelty of the work lies in the fact that it considers a plethora of background geometries and modified gravity theories namely, $f(R)$ gravity in inhomogeneous and isotropic background; $f(T)$ gravity in homogeneous and isotropic background; anisotropic universe characterized by Kantowski-Sachs model; Bouncing model of Universe etc. where the modified Raychaudhuri equation and Focusing Theorem (that follows as a consequence of the Raychaudhuri equation) has been investigated. The work also points out the quantum aspects of Raychaudhuri equation via its Lagrangian and Hamiltonian formulation and gives an account of Wheeler-DeWitt quantization in quantum cosmology alongwith the formulation of quantum Bohmian trajectories as two important quantum mechanical tools to find the possible escape routes from these problemmatic singularities at the classical level. Chapter 1 deals with the overview of relativistic cosmology while the subsequent chapters (2-8) comprise of the research work done. Finally, Chapter 9 summarizes the whole content and discusses some of its future prospects. Raychaudhuri equation hints that singularity is inevitable in Einstein gravity via the Focusing theorem which requires positiveness of the Raychaudhuri scalar for possible convergence (also known as Convergence Condition CC) of a bundle of geodesic. If there is a singularity in a space-time manifold, then a congruence of geodesic tend to focus at the singularity. Thus a singularity always implies focusing. So if we can avoid this focusing by making the Raychaudhuri scalar negative we can avoid singularity also. Motivated by this, Raychaudhuri equation and corresponding CC have been formulated in modified gravity theories where the Raychaudhuri scalar may be made negative under certain physical assumptions. This is because the field equations for modified gravity differ from those of Einstein gravity. Hence there may arise certain conditions under which focusing and hence singularity might be avoided. The role of anisotropy in Focusing theorem in case of anisotropic model described by Kantowski-Sachs metric has been explored. The thesis further brings out some inherent mathematical and cosmological properties of bouncing model using the Raychaudhuri equation and discusses its consequences in emergent scenario and Wormholes. Moreover it shows the entire cosmic evolution from the point of view of Raychaudhuri equation. Quantum formulation of RE has also got ample motivation. This is because, it is generally speculated that quantum effects which become prominent in strong gravity regime may alleviate the singularity problem at the classical level. To be more precise, a quantum replica of Raychaudhuri equation and classical geodesic might be helpful in identifying the existence of singularity and also to resolve them. The thesis gives emphasis on Wheeler DeWitt quantization and Bohmian trajectory formulation corresponding to the quantum Raychaudhuri equation of which the former quantize the geodesic flow and the later replaces the classical geodesics by quantum Bohmian trajectories.

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