

Abstract

Title of the thesis: Some Algebraic and Graph Theoretical Problems Related to Semigroup Theory

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In the thesis, some characterization problems related to ideals of a commutative semigroup have been studied. Also some new graphs over semigroups have been defined and various aspects of these graphs, especially the interdependence of the graph-theoretic properties and the algebraic properties have been studied.

Firstly, the notion of 2-absorbing ideals in a commutative semigroup has been considered. It is observed that every maximal ideal of a commutative semigroup is 2-absorbing but the converse is not true. Then a commutative semigroup in which 2-absorbing ideals are maximal has been characterized. The concept of 2-AB semigroup, in which 2-absorbing ideals are prime, has been introduced. Also a 2-AB semigroup has been characterized in terms of minimal prime ideal over a 2-absorbing ideal and some properties of these semigroups have been studied.

Secondly the notion of 2-absorbing primary ideals of a commutative semigroup has been introduced. The relation of 2-absorbing primary ideals with prime, maximal, semiprimary and 2-absorbing ideals has been established. Various characterization theorems on a commutative semigroup, in which 2-absorbing primary ideals are prime, maximal, semiprimary and 2-absorbing ideals have been obtained. Also some other important properties of 2-absorbing primary ideals of a commutative semigroup has been studied.

Then the concepts of 2-prime and weakly 2-prime ideals in a commutative semigroup have been introduced and studied. Then a semigroups where 2-prime ideals are prime has been characterized and also a semigroups where every ideal is weakly 2-prime has been characterized.

Next, the power graph $\mathcal{P}(S_M)$ over a monogenic semigroup S_M with zero element has been considered and studied. Various graph parameters of $\mathcal{P}(S_M)$ and topological indices based on distance of vertices have been determined. Finally, some graph parameters of the cartesian product $\mathcal{P}(S_M^1) \square \mathcal{P}(S_M^2)$ of graphs $\mathcal{P}(S_M^1)$ and $\mathcal{P}(S_M^2)$ has been computed.

Then the inclusion ideal graph $In(S)$ of nontrivial right ideals of a semigroup S with zero element has been considered. A semigroup S for which the graph $In(S)$ is complete, connected has been characterized. Also various graph parameters of $In(S)$ have been obtained. The values of n for which the graph $In(Z_n)$ is complete, triangulated, split, unicyclic, threshold have been determined and also minimal embedding of $In(Z_n)$ into compact orientable (resp. non-orientable) surface have been studied. Both upper and lower bounds for metric and partition dimension of inclusion ideal graph of a completely 0-simple semigroup has been provided. Finally, some graph parameters of the cartesian

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product of inclusion ideal graph of two monoids have been computed.

Also the inclusion ideal graph of multiplicative semigroup R_S of a commutative ring R with unity, denoted by $In(R_S)$, has been considered. A commutative ring R has been characterised for which the graph $In(R_S)$ is null, complete, connected, bipartiate, split, co-graph, unicyclic and outerplanar, planar, toroidal and bitoroidal. Also the diameter and girth of the graph $In(R_S)$ have been characterized.

Further, the notion of prime inclusion ideal graph $In_p(S)$ of nontrivial prime ideals of a commutative semigroup S has been introduced. Then a semigroup S has been characterized for which the graph $In_p(S)$ is null, complete or connected. Also various graph parameters, minimal embedding, metric and partition dimension of the prime inclusion ideal graph of the multiplicative semigroup Z_n of integers of modulo n have been studied.

Finally, three inclusion graphs related to ideals of a commutative semigroup have been considered and its relation with inclusion (resp. prime inclusion) ideal graph of a commutative semigroup have been observed. Also it is observed when the graphs related to ideals of a commutative semigroup are isomorphic.

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