

Generalized Skew Derivations and Related Additive Maps in Prime and Semiprime Rings

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Abstract

For as long as rings and algebras have been studied for their own sakes, it has been a problem of interest to determine the consequences of various special identities and conversely, to find sufficient conditions on a given ring which ensure that a specified identity holds. Ring derivation is a branch of algebra in which we study about the structure of additive maps as well as structure of rings by analyzing some functional identities involving additive maps. These additive maps are derivation, skew derivation, generalized derivation, generalized skew derivation, b -generalized derivation, multiplicative generalized derivation, multiplicative (generalized) derivation, etc. It is well known that there is a strong relationship among the functional identities involving derivations and generalized derivations and the structure of the rings. This thesis is mainly intended to find out the structure of above mentioned additive maps satisfying some functional identities on different subsets of prime and semiprime rings. A simple and well known functional identity $[d(x), x] = 0$ for all $x \in R$, where R is a prime ring and d is a nonzero derivation on R , was studied by Posner [6]. On that article, he got a wonderful structure of R . Henceforth, several researchers studied several functional identities and got marvelous results. We have also studied some problems in this line of investigation. This thesis contains seven chapters. Chapter-wise brief information is given below:

Chapter 1 is basically devoted for introductory purpose. Some basic definitions, preliminaries and prerequisites have been collected from other references which are needed for the development of the subsequent chapters in this thesis.

Dhara and Ali [3] gave a precise definition of multiplicative (generalized) derivation and studied some standard situations. In Chapter 2, we have studied some identities of multiplicative (generalized) derivation of a semiprime ring. Some examples have been given at the end of this chapter concluding that semiprimeness hypothesis in the theorems are not superfluous.

The study of commuting and centralizing maps was initiated by Posner [6]. There, he proved that if a prime ring has a nonzero centralizing derivation, then the prime ring is commutative. Brešar [1] generalized Posner's result by considering co-centralizing derivations and also proved that, if two nonzero derivations of a prime ring R are co-centralizing on R , then R must be commutative. De Filippis et al.[5] and Carini et al.[2] generalized result of Brešar [1] acting on a noncentral Lee ideal. In Chapter 3, we have inspected an identity involving three generalized derivations on a noncentral Lee ideal of R , which extends the above results.

A number of authors have studied some functional identities involving noncentral valued multilinear polynomial. In chapter 4, we have studied the identity $d(F^2(x)x) = xG^2(x)$ for all $x \in f(I)$; where f is a noncentral valued multilinear polynomial, I is a two sided ideal of a prime ring R , d is a derivation and F, G are generalized derivations of R .

Chapter 5 has dealt with another identity involving derivations and generalized derivations acting on multilinear polynomials in prime ring.

Eroğlu and Argaç [4] studied the identity $F^2(x)x \in C$ for all $x \in f(R)$, where C is the extended centroid of R , F is a generalized derivation of a prime ring R and f is a multilinear polynomial. Recently, Yadav [7] has described all possible forms of the maps, when $F^2(x)d(x) = 0$ for all $x \in f(R)$, where d is a derivation and F is a generalized derivation of R . Chapter 6 has been dedicated to study the above identity with central value.

Lastly, in Chapter 7, we have studied an identity involving X -generalized skew derivation in prime rings.

References

- [1] M. Brešar : Centralizing mappings and derivations in prime rings, *J. Algebra*, vol. 156 (1993), 385-394.
- [2] L. Carini, V. De Filippis and G. Scudo : Product of generalized derivations with commuting values on a Lie ideal, *Differential Geometry, Algebra, and Analysis; Springer Proceedings in Mathematics & Statistics*, 2020.
- [3] B. Dhara and S. Ali : On multiplicative (generalized)-derivations in prime and semiprime rings, *Aequ. Math.*, vol. 86, No. 1-2 (2014), 65-79.
- [4] M. P. Eroğlu and N. Argaç : On identities with composition of generalized derivations, *Canadian Math. Bull.*, vol. 60, No. 4 (2017), 721-735.
- [5] V. De Filippis and F. Rania : Commutating and centralizing generalized derivations on Lie ideals in prime rings, *Math. Notes*, vol. 88, No. 5 (2010), 748-758.
- [6] E. C. Posner : Derivation in prime rings, *Proc. Amer. Math. Soc.*, vol. 8 (1957), 1093-1100.
- [7] V. K. Yadav : Identities involving generalized derivations in prime rings, *Rend. Circ. Mat. Palermo, II. Ser.*, vol. 71 (2022), 259-270.

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