M.E. (Water Resources & Hydraulic Engineering) (6 Semester), 2024

(3rd Semester)

COMPUTATIONAL HYDRO DYNAMICS

(Paper- VII)

Full Marks : 100

Answer any ***four*** questions.

1. a) With suitable examples briefly explain the basic characteristics of an equilibrium problem and a marching problem.
   b) What do you mean by the stability of a finite different expression?
   c) Describe FTBS, FTCS, and FTFS schemes with the help of Stencil Diagrams.
   d) Briefly explain with examples Dirichlet and Neumann boundary conditions.
   e) Define consistency with a suitable example.
   f) What do you mean by Round-off Error?
   g) Consider Laplace's equation, given by $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$. Show that this is an elliptic equation.

5+3+6+3+3+2+3 = 25

2. a) Apply Gauss elimination method to solve the equations: $x + 4y - z = 5$; $x + y - 6z = -12$; $3x - y - z = 4$.
   b) Find root of the equation $2x^3 + x^2 - 20x + 12 = 0$, using bisection method in three stages.
   c) Find root of the equation $2x - \log x = 7$, using method of false position. Correct to two decimal places.
   d) Find by Newton-Raphson's method, the real root of the equation $x + \log10\ x = 3.375$ correct to two decimal places.

6+7+6+6 = 25

3. a) By von Neumann method find the stability condition for the first order wave equation as follows:- $\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}=0$
   b) Consider the function $f(x,y)=3e^{x/3}+e^{y/2}$. Consider the point $(x,y)=(0.1,0.08)$.
   Assume, $\Delta x=0.005,\ \Delta y=0.004$.
   (i) Calculate the exact values of $\partial f/\partial x$ and $\partial f/\partial y$ at this point.

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(ii) Calculate the same using first-order accurate forward and backward differences. Also, calculate the percentage difference compared with the exact values.

(iii) Calculate the same using second-order accurate central differences. Also, calculate the percentage difference compared with the exact values.

13+12 = 25

4. a) A tank measures 6m × 5m in plan, and has a rectangular thin-plate weir, width b = 225 mm, If the initial head of water over the weir is 90 mm, how long will it take for the water to drain down to a head of 40 mm over the weir? Take C_d = 0.685. Find out the solution using analytical and CHD approaches and compare the results.

   b) By Taylor series expansion show how can you obtain first order and second order accurate finite difference expressions for the terms $\frac{\partial u}{\partial y}$, and $\frac{\partial^2 p}{\partial x^2}$ at the forward and backward boundaries of the flow domain.

10+15 = 25

5. a) By Taylor series expansion, show how can you obtain first-order and second-order accurate finite difference expressions for the term $\frac{\partial^2 w}{\partial x \partial y}$ at the backward boundary.

   b) By the Polynomial approach determine the second-order accurate forward difference expressions for the term $\partial v/\partial x$.

   c) Consider the viscous flow of water over a river bed. At a given station in the flow direction, the variation of the flow velocity (u) in the direction perpendicular to the bed (the z-direction) is given by the expression $u = 100p(2p^2 - p + 0.5)$ where $p = z/h$ and h = total depth of water = 90 cm. The dynamic coefficient of viscosity of water at 20°C (μ) = 0.001002 kg/(m·s). Values of u in m/s were measured at discrete grid points with $\Delta z = 5$ cm. Calculate the shear stress (τ_o) at the wall using (a) first order, (b) second order and (c) third order accurate forward difference schemes of the Polynomial approach. Finally, compare these calculated finite-difference results with the exact value of bed shear stress.

7+5+13 = 25

6. a) Draw the stencil diagrams for the second order central difference scheme of $(\partial v/\partial z)_{i,j}$ with respect to *z*.

   b) Find out the Tri-diagonal Matrix and the solution of the system of equations using the Crank-Nicolson form of the 1-D linear heat conduction equation. Assume T is known at all grid points at the time level *n*. Choose to distribute seven grid points along *x* axis.

3+22=25