M.E. (Water Resources & Hydraulic Engineering) Examination, 2024

(2nd Semester)

COMPUTATIONAL HYDRO DYNAMICS

(Paper-I)

Time : Three Hours Full Marks : 100

Answer any ***four*** questions.

1. a) Find a root of the equation $x^3 - x - 11 = 0$, using the bisection method in three stages.

   b) Find by Newton-Raphson's method, the real root of the equation $x + \log_{10} x = 3.375$ correct to two decimal places.

   c) Find root of the equation 2x – log x = 7, using method of false position. Correct to two decimal places.

   d) Apply Gauss elimination method to solve the equations $x + 4y - z = 5$; $x + y - 6z = -12$; $3x - y - z = 4$.

6+6+7+6 = 25

2. a) State different advantages and disadvantages of experimental, theoretical and computational approaches for solving non-linear problems.

   b) What do you mean by explicit and implicit approaches?

   c) Define Dirichlet and Neumann boundary conditions.

   d) Derive the following expression, which is a third order accurate one-sided difference

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{1}{6\Delta x}\left(-11u_{i,j} + 18u_{i+1,j} - 9u_{i+2,j} + 2u_{i+3,j}\right) + O(\Delta x)^3$$

9+4+3+9 = 25

3. a) Explain what you mean by "well posed" and "ill posed" problems?

   b) Describe FTBS, BTCS, and CTFS schemes with the help of Stencil Diagrams.

   c) Give examples of elliptic, hyperbolic and parabolic partial differential equations as found in fluid dynamics. Briefly explain the characteristics of such equations.

   d) Consider the function $f(x,y) = e^x + e^{2y}$. Consider the point (x,y) = 2,1. Assume $\Delta x = \Delta y = 0.1$.

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(i) Calculate the exact values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at this point.

(ii) Calculate the same using 1st order accurate forward and backward differences. Also, calculate the percentage difference compared with the exact values.

(iii) Calculate the same using 2nd order accurate central differences. Also, calculate the percentage difference compared with the exact values.

3+4+6+(3+6+3) = 25

4. a) By Taylor series expansion show how can you obtain 2nd order accurate finite difference expressions for the term $\frac{\partial^2 u}{\partial x \partial y}$ at the forward, backward and central boundaries of the flow domain.

b) What do you mean by reflection boundary condition? Explain with an example.

c) By Polynomial approach show how can you obtain 1st order, 2nd order and 3rd order accurate finite difference expressions for the terms $\frac{\partial u}{\partial x}$ at the forward boundaries of the flow domain.

11+2+12 = 25

5. a) By Taylor series expansion show how can you obtain 1st order and 2nd order accurate finite difference expressions for the terms $\frac{\partial u}{\partial x}$ at the forward, backward and central boundaries of the flow domain.

b) Find out the Tridiagonal Matrix and the solution of the system of equations using the Crank-Nicolson form. Choose to distribute seven grid points along the x-axis.

11+14 = 25

6. a) Distinguish between Discretisation Error, Round-off Error and Truncation Error.

b) Give the physical and mathematical classification of partial differential equations with examples.

c) What do you mean by the stability of a finite different expression?

d) A tank measures 5m × 6m in plan, and has a rectangular thin-plate weir, width b = 250 mm, If the initial head of water over the weir is 85 mm, how long will it take for the water to drain down to a head of 35 mm over the weir? Take C_d = 0.65. Find out the solution using analytical and CHD approaches and compare the results.

6+6+2+11 = 25