Ex/PG/PROD E/T/111B/2024

M. E. PRODUCTION ENGINEERING 1st YEAR 1st SEMESTER EXAMINATION,

THEORY OF OPTIMIZATION

Time: Three hours **Full marks:** 100

Answer any FIVE questions

1.(a) Derive the corresponding LP model for a two-person zero-sum game. (8)

(b) State the basic assumptions while solving a two-person zero-sum game problem. (4)

(c) Solve the following game problem: (8)

Player B / Player A	I	II	III
I	1	-1	-1
II	-1	-1	3
III	-1	2	-1

2. Minimize $Y = 4x_1^2 + 5x_2^2$ subject to $x_1 + x_2 = 9$ using the following optimization methods: (5x4)
   (i) Direct substitution method,
   (ii) Lagrangian multiplier method,
   (iii) Penalty function method, and
   (iv) Constrained variation method.

3.(a) State the limitations of an LP problem. (5)

(b) A manufacturer makes two animal feeds M and N, and employs three workers W_1, W_2 and W_3. The time taken by the workers (in hours) to produce each kg of each feed and the times for which each worker is available are given below: (15)

Worker	Item M	Item N	Time available
W_1	4	3	24
W_2	-	1	4
W_3	5	3	30

The contributions of M and N (per kg) to profit are Rs. 120 and Rs. 180 respectively. Determine the idle times for all the workers. Suddenly worker W_1 leaves and the operations are run by the remaining two workers. Determine the change in profit margin.

4.(a) Solve the following LP problem: (14)

Minimize $Z = x_1 - 3x_2 + 2x_3$

Subject to $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

(b) With numerical examples, highlight the occurrence of (i) multiple optimal solution and (ii) infeasible solution in a given LP problem. (3x2)

5.(a) Determine the dimensions of an open box of maximum volume that can be constructed from an A4 sheet 210mm X 297mm by cutting four squares of side x from the corners and folding and gluing the edges. (10)

(b) A market analysis group studying car purchasing trends in a certain region has concluded that on average, a new car is purchased once every 3 years. The buying patterns are described by the following matrix: (10)

Car	Small	Large
Small	80%	20%
Large	40%	60%

The elements of the above matrix are to be interpreted as follows: The first row indicates that of the current small cars, 80% will be replaced with a small car, and 20% with a large

[Turn Over

car. The second row implies that 40% of the current large cars will be replaced with small cars, while 60% will be replaced by large cars. Construct a stochastic matrix that will define a Markov chain model of these buying trends. If there are currently 40000 small cars and 50000 large cars in the region, what will be the contribution in 12 years time?

6.(a) A radio manufacturer finds that he can sell x radios per week at Rs. P each, where $P = 2(100 - (x/4))$. His cost of production of x radios per week is Rs. $(120x + (x^2/2))$. Show that his profit is maximum when the production is 40 radios per week. Also find his maximum profit per week. (10)

(b) Solve the following problems by geometric programming: (10)
Minimize $Z = 2x_1^2x_2^{-3} + 8x_1^{-3}x_2 + 3x_1x_2$, $x_1, x_2 \geq 0$

7.(a) Minimize $y = 4x_1^2 - 2x_1 + 3x_1x_2 + 5x_2^2 - 4x_2$ subject to $x_1 + x_2 \geq 1$ (10)

(b) Find the maximum of the function $f(x_1, x_2) = 2x_1 + x_2 + 12$ subject to $g(x_1, x_2) = x_1 + 2x_2^2 - 5 = 0$ using Lagrangian Multiplier method. (10)