MASTER OF MECHANICAL ENGINEERING EXAMINATION, 2024

(1st Year, 1st Semester)

STRESS AND DEFORMATION ANALYSIS

Time: **Three hours** Full Marks: **100**

Different parts of a question must be answered together.
Provide sketches wherever applicable. Missing data, if any, may be appropriately assumed.

Group – A (Answer any Two (2))

1(*a*). Derive Cauchy stress equation. Considering the stress components on a small rectangular element, prove that the cross-shears are equal and hence show that only 6 independent stress components are necessary to describe the state of stress at a point. [05+07]

1(*b*). Discuss what do you understand by principal stresses and principal planes. Write down the form of characteristic equation using the stress invariants and also write down the expressions for the stress invariants. Show that in general the characteristic equation has three real roots. [03+03+05]

1(*c*). Prove that the stress surface of Cauchy completely defines the state of stress at a point. [07]

2(*a*). The state of stress at a point within a stressed machine element is given by the following matrix: [10]

$$\begin{bmatrix} 60 & 40 & 30 \\ 40 & 40 & 20 \\ 30 & 20 & 20 \end{bmatrix} \text{ (in MPa)}$$

Transform the above set of Cartesian stress components into a new set of stress components using a new set of coordinates $Ox'y'z'$ where the new axes are defined by the following direction cosines:

	x	y	z
x'	2/3	2/3	-1/3
y'	-2/3	1/3	-2/3
z'	-1/3	2/3	2/3

2(*b*). The state of stress at a point in a stressed body is given by the following expressions: [08]
$\sigma_{xx} = 3x^2 - 3y^2 - z$, $\sigma_{yy} = 3y^2$, $\sigma_{zz} = 3x + y - z + 1.25$, $\tau_{xy} = z - 6xy - 0.75$, $\tau_{yz} = 0$ and $\tau_{zx} = x + y - 1.5$.
Determine whether equilibrium is satisfied, in the absence of body forces.

2(*c*). Explain what is meant by octahedral planes and octahedral stresses. Derive the expressions of octahedral stresses in terms of stress invariants and rectangular stress components. Explain what do you understand by state of pure shear. [04+05+03]

3(*a*). The displacement field for a body is given by the following relations: [08+04]

$$u = (x^2 + y^2 + 2) \times 10^{-4}$$
$$v = (3x + 4y^2) \times 10^{-4}$$
$$w = (2x^3 + 4z) \times 10^{-4}$$

(*i*). Determine the strain components at point (3,2,1) considering linear and nonlinear strain-displacement relationships.
(*ii*). Determine the strain at the above-mentioned point in a direction which is equally inclined to the three coordinate axes.

3(*b*). A strain field is given as follows: $\varepsilon_{xx} = 0, \varepsilon_{yy} = 0, \varepsilon_{zz} = 0, \gamma_{xy} = 0, \gamma_{yz} = ax, \gamma_{zx} = -ay$. Determine the displacement field from the above-mentioned strain components. [10]

3(*c*). Explain compatibility conditions. Derive the Saint Venant's equations of compatibility. [03+05]

[Turn over

Group – B (Answer any Two (2))

4(*a*). Assuming that the body force components are absent, prove that the following expressions are true. (The symbols have their usual meaning) [08]

$$(\lambda + \mu)\frac{\partial \Delta}{\partial x} + \mu \nabla^2 u = 0$$
$$(\lambda + \mu)\frac{\partial \Delta}{\partial y} + \mu \nabla^2 v = 0$$
$$(\lambda + \mu)\frac{\partial \Delta}{\partial z} + \mu \nabla^2 w = 0$$

4(*b*). Discuss the difference between isotropy and homogeneity. Prove that two independent constants are sufficient to describe the homogeneous linear isotropic material behaviour. Express the elastic modulus and Poisson ratio in terms of these two independent constants. [03+05+04]

5(*a*). Write down the statement and expression for the failure criterion corresponding to the following theories of failure. Also obtain the region of safety (for bi-axial state of stress) for these theories of failure: [02+03+04]

(*i*). Maximum Shear Stress Theory
(*ii*). Maximum Elastic Strain Theory

5(*b*). Prove that the criterion for failure according to Distortion Energy Theory is as follows: [11]

$$\frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \geq S_{yt}$$

6(*a*). Describe the basic characteristics of a strain gauge. Write down the names of some commonly used materials for strain gauges. [05+01]

6(*b*). Define sensitivity in the context of strain measurement and deduce an expression for circuit sensitivity. [02+04]

6(*c*). During pressure test of a thick-walled cylindrical pressure vessel at 100 MPa, the following axial and hoop components of strains were measured at the inside and outside surfaces.

At $r = r_i$: $\varepsilon_\theta = 800 \times 10^{-6}$, $\varepsilon_z = 550 \times 10^{-6}$

At $r = r_o$: $\varepsilon_\theta = 150 \times 10^{-6}$, $\varepsilon_z = 550 \times 10^{-6}$

The elastic modulus and Poisson's ratio of the pressure vessel material are 200 GPa and 0.30, respectively. Determine the stresses associated with these strains. [08]

7(*a*). Derive the general differential equation of a column and mention an assumed general solution form (considering arbitrary constants) of the derived differential equation. Mention the possible boundary conditions of the column along with the applicable end conditions. [06+02+01+02]

7(*b*). Obtain a solution and an expression for critical load for the general differential equation of a column with Clamped-Free boundary conditions. [09]

8. Write short notes on the following topics: [05 × 04 = 20]

(*i*). Generalized Hooke's Law
(*ii*). Principle of minimum total potential energy
(*iii*). Null and deflection method of strain measurement
(*iv*). Significance of failure theories in engineering design

--O--