Ref. No. : Ex/PG/ME/T/127A/2024

M.E. MECHANICAL ENGINEERING FIRST YEAR SECOND SEMESTER EXAM – 2024

Subject: ROTOR DYNAMICS **Time: 3 Hours** **Full Marks: 100**

Answer all questions. Carefully note the options for the alternative questions.

Any missing information can be assumed suitably with appropriate justification.

Q1. (a) Derive the equations of motion (in Cartesian coordinate system) for lateral vibrations of a Jeffcott rotor with mass-unbalance, mounted on two identical undamped flexible bearings (Fig. Q1a). Consider that m is the mass of the rotor, e is the unbalance eccentricity, k_S is the shaft stiffness, k_Y and k_Z are the stiffness coefficients in each bearing in XY and XZ-planes respectively, c is the constant damping coefficient against the vibration in both Y and Z directions and Ω is the constant spin speed of the rotor about the X axis. The force due to weight can be assumed as negligible compared to the force due to unbalance. **Clearly state other assumptions relevant to this derivation.** In the process show the equivalent stiffness of the system along Y- and Z-directions. **[8]**

(b) Find out the expressions of the non-dimensional magnification factors (ratio of steady-state synchronous response amplitude to unbalance eccentricity) and the relative phase angles between the excitation and response along each of the Y and Z directions in terms of the frequency ratio (ratio of the spin frequency to the natural frequency of vibration along the corresponding direction) and the damping ratio. **Show the variation of the unbalance response in terms of the plot of the magnification factors versus frequency ratio and the phase angle versus frequency ratio for different possible values of damping ratio.** **[10]**

(c) Find out the expressions of maximum possible magnification factor along any of these directions and the corresponding frequency ratio as a function of effective damping ratio in that direction. **[7]**

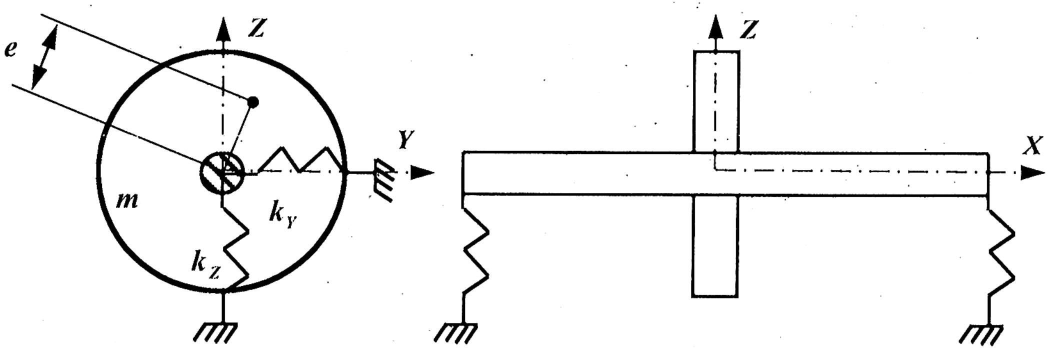

Fig. Q1a

Alternative to Question Q1

Q1. (a) A rigid symmetric cylindrical rotor is mounted on two identical isotropic undamped flexible bearings at its ends as shown in **Fig. Q1b**. m, J_P, J_T, L and Ω are the mass, principal polar mass moment of inertia, principal transverse mass moment of inertia, length of the rotor between bearings and spin speed of the rotor respectively. k is the stiffness of each bearing along each direction. Starting from the expressions of angular velocity vector of the system (the derivation of angular velocity vector is not required) in the rotor-fixed reference frame, derive the

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rotational kinetic energy of system. Consider gyroscopic effect in your derivation. Clearly show with neat sketches different coordinate systems used in the derivation. **[8]**

(b) Derive the equations of motion for free transverse vibration (for both the cylindrical and the conical mode) for such a system using Lagrange's principle. **[8]**

(c) How many natural frequencies are possible for this rotor at a given spin speed? Find out their expression and plot them versus spin speed in a Campbell diagram. Present your answer in terms of non-dimensional variables. Hint: you may use complex variables for finding the expressions of natural frequencies. **[9]**

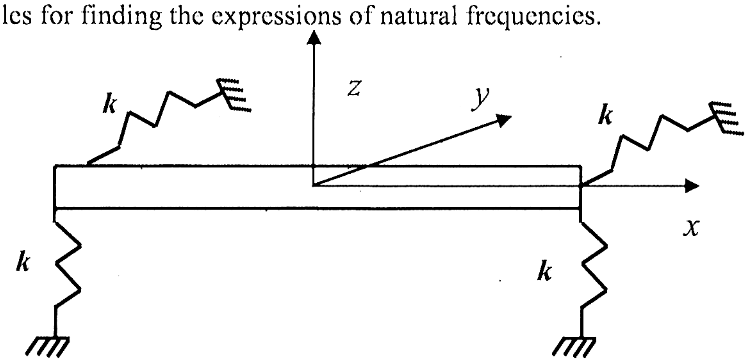

Fig. Q1b

Q2. (a) Consider a Jeffcott rotor mounted on a pair of identical journal bearings at its ends. Write down the relevant equations of motion for such a system under unbalance excitation. Assume standard symbols for mass, shaft stiffness, bearing stiffness and damping coefficients, unbalance eccentricity, spin speed etc. What is the order of the characteristic polynomial of the corresponding system of homogeneous equation? How many eigenvalues are possible for this system? **[6]**

(b) Derive the expression of **the dynamic stiffness matrix** for the steady state synchronous vibration of the above-mentioned rotor-bearing system due to unbalance excitation. Given are the mass of the rotor, spin speed, stiffness of the shaft, linearized stiffness and damping coefficients of the bearings. **[9]**

(c) Simplifying the Reynold's equation for hydrodynamic bearings, find out an expression for pressure distribution around the journal for the case of a plane cylindrical short journal bearing at steady state. **[10]**

<u>Alternative to Question Q2. (c)</u>

(c) Consider a rolling element bearing with zero radial clearance and rigid inner race. The bearing is supporting a journal with horizontal axis of rotation, which is acted on by a vertically downward force F and whose centre is displaced along the direction of the load only. From the relation $F_i = \left[\dfrac{\delta_i}{C}\right]^n$ between the contact force F_i on the i[th] rolling element and the radial displacement δ_i towards the direction of that rolling element, **show that the stiffness of the bearing under this condition is given by** $k = \dfrac{nF}{x}$, where x net displacement of the journal centre in the direction of F. The symbols n and C stand or their usual meaning in context of the contact force between rolling elements and the bearing races. **[10]**

Q3. (a) Consider a uniform prismatic rotor having a non-circular cross-section area with two non-identical principal area moments of inertia along two principal orthogonal directions of the cross-section. Derive relevant equations of motion for free vibration of such a system rotating at a constant spin speed. Discuss the possibility of instability for such a rotor at a speed in between two natural frequencies of transverse vibration. **[15]**

(b) Write short notes on oil-whirl and oil-whip phenomena in case of a rotor supported on oil-film bearings. **[10]**

Q4. (a) In the context of a rigid rotor write short notes, with relevant sketches, on (i) static-unbalance, (ii) quasi-static unbalance, (iii) couple unbalance, (iv) dynamic unbalance. **[10]**

(b) With relevant sketches explain the **single-plane balancing method without phase measurement** in case of a short rigid rotor. **[15]**