Ref. No.: Ex/PG/ME/T/129A/2024

M.E. MECHANICAL ENGINEERING FIRST YEAR SECOND SEMESTER - 2024

FINITE ELEMENT ANALYSIS IN ENGINEERING

Time: 3 Hrs **Full Marks: 100**

Any missing data may be assumed with suitable justification
Symbols/notations carry its usual meanings
All Questions Carry Equal Marks

(ANSWER ANY FIVE QUESTIONS)

1. Consider a flexure element (**Fig. 1**) of length L as a single finite element with two nodes denoted by **1** and **2** and each of the two nodes having two degrees of freedom, one being rotation and other translation. Derive an expression for the stiffness matrix of the flexure element with usual notations. (20) [CO1]

2. Consider the triangular element with three nodes as shown in **Fig. 2** with the nodal displacements specified. Evaluate the strain-displacement matrix and highlight the significance of a constant strain triangle (CST) element. (20) [CO2]

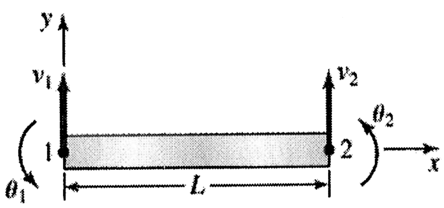

Fig. 1

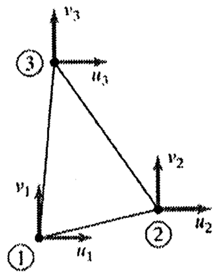

Fig. 2

3. Considering a four-node quadrilateral element, derive the strain-displacement matrix with usual notations, assuming isoparametric formulation. (20) [CO2]

4. (a) Explain the difference between consistent and lumped mass matrix.
(b) Derive an expression for the consistent mass matrix of a two-node bar element shown in **Fig. 3** with usual notations. Assume the displacements and applied forces to be time dependent. (5+15) [CO3]

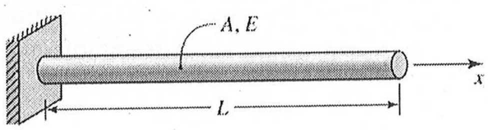

Fig. 3

[Turn over

5. What is meant by Rayleigh Damping? Using the normalization principle and assuming Rayleigh damping, evaluate an expression for the damping factor in case of structural damping. Explain the variations in the damping factor considering lower and higher modes. (5+15) [CO3]

6. Derive the mathematical statement of orthogonality of the principal modes of vibration using standard notations. (20) [CO3]

7. Using the orthogonality principle, explain the principle of normalization of the mass and stiffness matrices such that

$$[\emptyset]^T[M][\emptyset] = [I] \; \& \; [\emptyset]^T[K][\emptyset] = [\omega_n^2]$$

Where, $[\emptyset]$ is the normalized modal matrix, $[I]$ is the identity matrix and $[\omega_n^2]$ is a diagonal matrix having the diagonal terms equal to the squares of the natural circular frequencies of the principal modes of vibration. (20) [CO3]

8. Derive an expression for the strain displacement matrix in case of an axisymmetric element with usual notations. Assume triangular elements for the analysis. (20) [CO2]

9. For the axisymmetric element shown in **Fig 4,** determine the stiffness matrix assuming E=210GPa and ν=0.30. The nodal coordinates are specified. (20) [CO2]

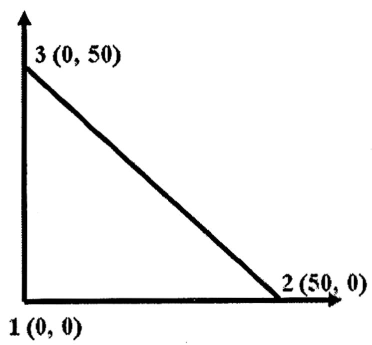

Fig. 4

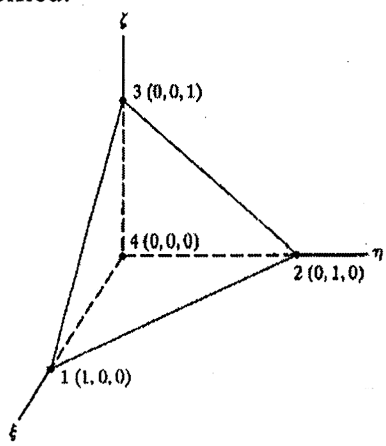

Fig. 5

10. (a) Draw a degenerated shell element defined in the (ξ, η, ζ) plane.
(b) Derive the Jacobian for a 3D element defined in the natural coordinates as shown in **Fig. 5** by applying isoparametric formulation. (5+15) [CO4]

11. Write Short notes on (**Any Four**) (5X4=20) [CO1]
- (a) Isoparametric formulation.
- (b) Numerical Integration.
- (c) Axisymmetric element.
- (d) h-refinement and p-refinement.
- (e) Steps in FEM analysis.