Ref No.Ex/PG/ME/T/116J/2023

MME 1st semester Examination 2024

SUBJECT: Advanced Dynamics

Time: Three hours | Full Marks 100

ANSWER ANY **FOUR** QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

1(a) Prove that the derivative of an unit vector is represented by –

$$\frac{d\hat{\varepsilon}}{du} = \hat{n}\frac{d\theta}{du}$$

Explain the meaning of the terms.
Hence obtain the expression for derivative of $\hat{\varepsilon}_t$ where $\hat{\varepsilon}_t$ is unit vector along the tangent.
Using the above, prove that

$$\bar{a} = \frac{v^2}{\rho}\hat{\varepsilon}_n + \frac{dv}{dt}\hat{\varepsilon}_t$$

Explain the physical meaning of the terms

(b) Mathematically state the relation between Conservative force $\bar{F}$ and potential function ϕ
Hence show that, if a force is conservative curl (F)=0
If a radial force is represented by - $\bar{F}_r = -\omega^2 r$,
Show that the force is conservative

(15+10=25)

2. Obtain the expression for Rotation Matrix [R] for finite rotation about x, y and z.
Hence prove that finite rotation is not a vector.
In $[S] = [R_\psi][R_\theta][R_\phi]$
Obtain the expressions for $[R_\psi], [R_\theta], [R_\phi]$
Discuss how [S] specifies the angular position of the rigid body.
Hence discuss how you can deduce $\frac{d\bar{r}}{dt} = \bar{\omega} \times \bar{r}$ ($\bar{r}$ – Vector fixed in Rigid body)
from the expression of [S]

3. Define constraint and state its types with equations.
Define generalized co-ordinate in Lagrangian Mechanics.
What is the condition for the differential constraint – $a_x dx + a_y dy + a_z dz = 0$ to be holonomic?
Define virtual displacement in terms of possible displacement. Hence show that it is a contemporaneous displacement.
Is a virtual displacement consistant with constraints?
Prove that –

$$\delta x_i = \sum_j \frac{\partial x_i}{\partial q_j} \delta q_j$$

$$v_i = \sum_j \frac{\partial x_i}{\partial q_j} \dot{q}_j + \frac{\partial x_i}{\partial t}$$

$$\frac{\partial x_i}{\partial q_j} = \frac{\partial v_i}{\partial \dot{q}_j} \quad , \quad \frac{d}{dt}\left(\frac{\partial x_i}{\partial q_j}\right) = \frac{\partial v_i}{\partial q_j}$$

Where x is Cartesian co-ordinate and q is generalised co-ordinate.
Prove that generalised force is given by -

$$Q_j = \sum_{i=1}^{3n} X_i \frac{\partial x_i}{\partial q_j} \quad , \quad X_i - \text{Cartesian comp. of force}$$

4. Explain the principles of Kane's dynamics with reference to the problem shown in Figure Q4

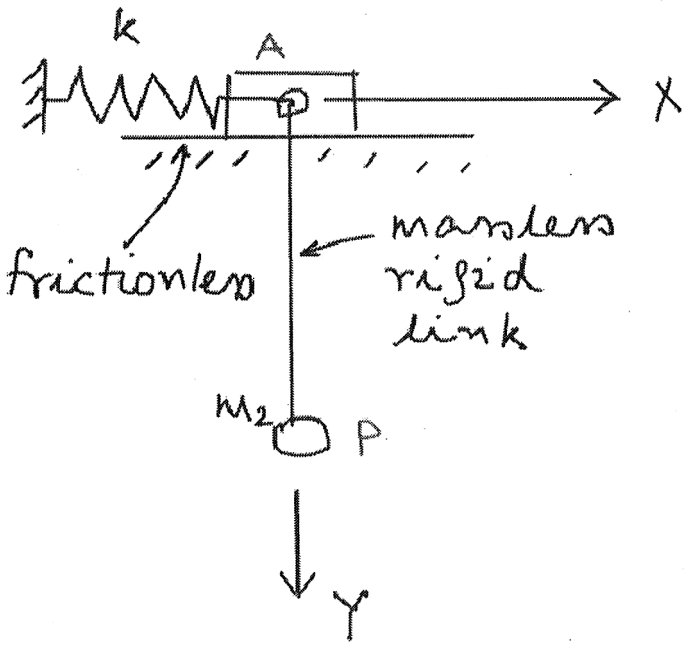

Figure Q4

5. Figure Q5 shows rigid bodies executing plane motion and connected by revolute joints. Describe the step by step method for automated generation of equations of motion

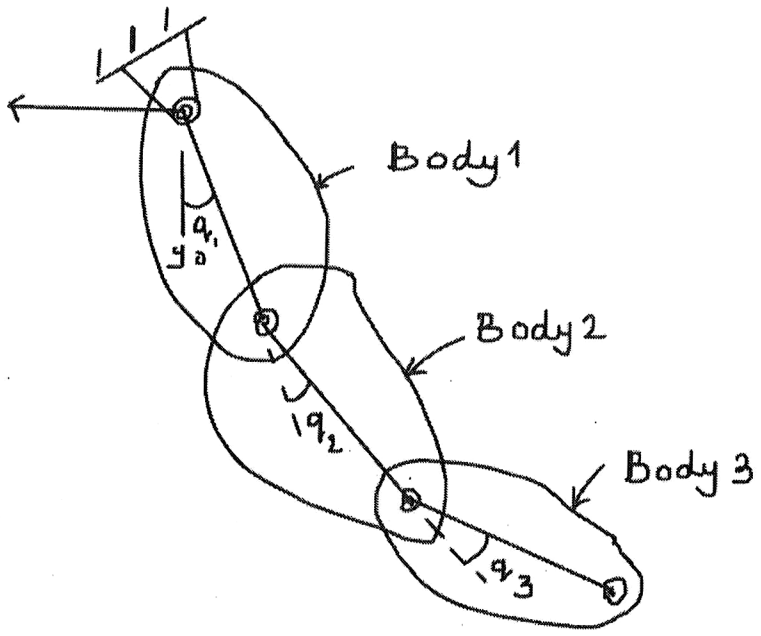

Figure Q5