Ref. No. Ex/PG/IEE/T/116A/2024

M.Tech (I.E.E.) 1st Year 1st Semester Examination, 2024

SUBJECT: Mathematical Methods in Instrumentation

Time: Three hours | Full Marks 100

Answer all questions. Q1a,4,6:CO1, Q1b,2,5:CO2, Q1c,3,7:CO3

Q.No.		Marks
1.	Write short notes with illustrations on: a) Lagrange equations of motion OR State space model of a continuous stirred tank reactor (CSTR) OR an inverted pendulum b) Properties of fields and vector spaces OR Inner product, vector norm and matrix norm c) Solution of LTI discrete -time state equations OR linear time-varying continuous-time state equations	5x3=15
2.	a) Determine rank, nullity and the bases for range space and null space of the matrices for i) $A = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 6 & 8 & 3 & 4 \\ 9 & 12 & 4 & 6 \end{bmatrix}$, ii) $B = \begin{bmatrix} 2 & 1 & 9 \\ 4 & 1 & 18 \\ -6 & -3 & -27 \end{bmatrix}$	10+10=20
	OR	
	b) Find the eigenvalues, eigenvectors and Jordan form for the matrices $A = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.	10+10=20
3.	a) (i)Linearize the Watt's governor equations given as $\dot{x}_1 = x_2\,, \dot{x}_2 = \frac{1}{2}N^2x_3^2 sin2x_1 - g\ sin\ x_1\ - \frac{b}{m}x_2, \dot{x}_3 = \frac{k}{l}cos\ x_1 - \frac{\tau}{l}.$	5
	ii) For the dynamical system modeled by $\dot{x}_1 = x_2\,, \dot{x}_2 = -x_1\,,$ find the time T it takes the system to move from the state $x(0)=[0\ 1]^T$ to the state $x(T) = [1\ 0]^T$.	5
	OR	
	b) In terms of $N_S(x) = (2/\pi)\ [\sin^{-1}(1/x) + (1/x)\cos(\sin^{-1}(1/x))]$. Show that the describing function for the limiter is 0 for $M \leq A$ and $K\ N_s(M/A)$ for $M > A$. Derive the describing function for a dead zone. Show diagram and state assumptions.	5+5
4.	a) For the system of differential equations $\dot{x}_1 = x_1 + x_2 + 1, \dot{x}_2 = -x_2 + 2,$ i) find the equation of the isoclines corresponding to the trajectory slope m=1 ii) find the equations of the asymptotes.	10
	b) (i) Draw the phase portrait of the spring mass system $m\ d^2x/dt^2 + kx = 0$ and state the interpretations.	10
	OR	
	ii) State the van der Pol equation and use Bendixson's theorem to interpret the presence or absence of limit cycle.	10

[Turn over

Q.No.		Marks
5.	a) Compute f(A)=exp(At) for $A = \begin{bmatrix} 0 & 2 \\ -2 & -4 \end{bmatrix}$ using Cayley-Hamilton technique or Inverse Laplace technique.	5
	b) Determine F^k for a discrete-time system with $F = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ using Cayley-Hamilton technique or Inverse Laplace technique.	6
6.	a) (i) Draw and discuss properties of time-invariant systems. What are zero-state and zero-input linearity? ii) Prove that a system described by the state model of the form $\dot{x}(t) = Ax(t) + Bu(t);\ y(t) = Cx(t) + Du(t)$ where A, B, C and D are constant matrices, is linear and time-invariant. Use the transition function. OR	6+6
	b) i) Derive the state space model of a mixing tank with supporting diagram. ii) Derive the relations for the system matrices of a sampled data system with supporting diagram.	6+6
7.	a) For the system $\dot{x}(t) = \begin{bmatrix} 1 & e^{-t} \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t);\ x(0) = 0$, i) find the homogeneous system solution and the corresponding state transition matrix; ii) find the output *x(t)* for a unit step input applied at t=0. OR	6+6
	b) Consider an unity feedback position servo system with a symmetric saturation nonlinearity controller with u=4 for e=4. The plant dynamics is an integrator cascaded with 1/(s+1). Find the system response to a step input r(t)=10. Assume zero initial conditions for output position and velocity.	12