Ref. No.Ex/PG/IEE/T/1210A/2024

M.Tech (I.E.E.) 2nd Semester Examination, 2024

SUBJECT: Dynamic System Control and Optimization

Time: Three hours Full Marks 100

Answer **all** questions. Q1, Q2 for CO1, Q3, Q4 for CO2 and Q5, Q6 for CO3.

Marks are allotted for detailed work out.

Q.No.		Marks
1.	For a system described by $x(k+1)=Fx(k)+Gu(k),\ x(k_0)=x^0$, find a) the eigenvalues, b) generalized eigenvectors c) Vandermonde matrix and d) the exponential F^k using Cayley Hamilton technique, modal matrix or Inverse z-transform for $F=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}$.	10
2.	a) Find state model for the following: i) $\dddot{y}+6\ddot{y}+11\dot{y}+6y=u$ OR ii) $y(k+2)+3y(k+1)+2y(k)=5u(k+1)+3u(k)$	10
	b) Obtain a controllable companion form representation and initial condition vector of the differential equation $\dddot{y}+6\ddot{y}+11\dot{y}+6y=\dot{u}+4u$.	5
	c) Find the Jordan canonical realization of the pulse transfer function $\dfrac{z+6}{z^3+5z^2+7z+3}$ and draw the state diagram.	5
3.	(i) a) For the system below, find 1) control sequence {u(0),u(1)} to drive the system from $x(0)^T=\begin{bmatrix}1 & 0\end{bmatrix}$ to $x(2)^T=\begin{bmatrix}-1 & 0\end{bmatrix}$, 2) the state x(0) when y(0)=1, y(1)=0, y(2)=-1, y(3)=2 for $u(k)=(-1)^k$, $k\geq 0$. $x(k+1)=\begin{bmatrix}-2 & -1\\ -1 & -2\end{bmatrix}x(k)+\begin{bmatrix}0\\ 1\end{bmatrix}u(k);\ y(k)=\begin{bmatrix}1 & -1\end{bmatrix}x(k)$ **OR** b) Comment on the controllability of the continuous time system with $A(t)=\begin{bmatrix}1 & e^{-t}\\ 0 & -1\end{bmatrix}, b=\begin{bmatrix}0\\ 1\end{bmatrix}$ $at\ t=0$. If the system is controllable, then find the minimum energy control to drive it from x(0)= $[0\ 0]^T$ to x(1)=$[1\ 1]^T$ at t=1.	10
	(ii) For the system with a) $A=\begin{bmatrix}0 & 1\\ 0 & -5\end{bmatrix}, B=\begin{bmatrix}0\\ 1\end{bmatrix}$ OR b) $A=\begin{bmatrix}-0.01 & 0\\ 0 & -0.02\end{bmatrix}, B=\begin{bmatrix}1 & 1\\ -0.004 & 0.002\end{bmatrix}$, determine the corresponding discrete- time matrices F and G. Using these matrices, determine the controllability matrix and the minimum time control sequence for the discrete-time system.	10

[Turn over

Q.No.		Marks
4.	(i) a) Find the Lyapunov function V(**x**) that ensures asymptotic stability of the system $A = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix}$. Determine the upper bound on the time it takes this system to go from the initial state $x(0)=[1 \quad 1]^T$ to within the area defined by $x_1^2+x_2^2= (0.25)^2$. **OR** b) For the unity feedback system with open loop TF 5/[s(s+1)(s+2)], show that the closed loop system is asymptotically stable using R-H criterion. Further, using bilinear transformation, show that the use of a sampler and ZOH in the forward path destabilizes the sampled-data system. (ii) a) Consider the continuous time state space system with $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $u = \pm 1$. Determine the phase plane trajectories for both inputs. **OR** b) Consider the continuous time state space system with $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Determine the equation of the isocline corresponding to trajectory slope m = 1. Also determine the equations of the asymptotes, if any.	10 10
5.	Consider the continuous time state space system with $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. In order to transfer the system with any initial state to the origin, determine the optimal control a) which minimizes $J = \int_0^\infty (x_1^2 + u^2)dt$. **OR** b) which minimizes $J = \int_0^\infty x_1^2 dt$ when $k_1 = 1$ in $u = -[k_1 \; k_2]x$. **OR** c) satisfies $\lvert u(t) \rvert \leq 1$.	10

Q.No.		Marks
6.	(i) a) Formulate the two point boundary value problem , which yields the optimal control u*(t) for the system $\dot{x}_1 = x_2;\ \dot{x}_2 = x_1 + (1 - x_1^2)x_2 + u$; $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and the performance index $J = \frac{1}{2}\int_0^2 (2x_1^2 + x_2^2 + u^2)\, dt$ when (i) u(t) is not bounded and (ii) $\lvert u(t) \rvert \le 1$.	10
	OR	
	b) Find the optimal control $u^*(t)$ for the system $\dot{x}(t) = 2x(t) + u(t)$; which minimizes the performance index $J = \frac{1}{2}\int_0^{t_1} (3x^2 + \frac{1}{4}u^2)\, dt$, where t_1 is specified.	
	(ii) a) Design a feedback controller with state feedback for a linear system described by the system matrices $A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}; B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ so that the eigenvalues of the closed loop system are at -1,-2, -3.	10
	OR	
	b) For a position servo system described by $F = \begin{bmatrix} 1 & 0.0787 \\ 0 & -.6065 \end{bmatrix}, g = \begin{bmatrix} 0.0043 \\ 0.0787 \end{bmatrix}$, determine a deadbeat control law. Further, assuming that only the angular position measurement is available, design a deadbeat observer for the system.	